Ptolemaeus
Arabus et Latinus

THE FIRST LATIN TREATISE OF PTOLEMY'S ASTRONOMY: THE *ALMAGESTI MINOR* (c. 1200)

Henry Zepeda

BREPOLS

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Ptolemaeus Arabus et Latinus

Texts

Volume I

Ptolemaeus Arabus et Latinus (PAL) is a project of the Bavarian Academy of Sciences and Humanities and the University of Würzburg. As part of the Academies' Programme, PAL is jointly funded by the Federal Republic of Germany and the Free State of Bavaria.

PAL is sponsored by the Union Académique Internationale (UAI).

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Cover design by Hilde Verhelst, T'Hi, Lier, Belgium.

Cover illustrations:
Woodcut (detail) from *La geografia di Ptolemeo Alessandrino*, Venice, 1548. Copy from Munich, Bayerische Staatsbibliothek. Aries (detail) from al-Ṣūfī's *Book of Constellations* (Latin), MS Paris, Bibliothèque de l'Arsenal, 1036, fol. 20r.

D/2018/0095/178

ISBN 978-2-503-58137-8

Printed on acid-free paper.

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Acknowledgments

I owe a deep debt of gratitude to many people who have helped me in the long process of writing this book. First and foremost, Steve Livesey has been an excellent advisor, teacher, mentor, and friend, and my wife, Elizabeth, answered innumerable questions about the finer points of Latin grammar to me and checked the entirety of my translation with great care. The other members of my doctoral committee, Rienk Vermij, Kerry Magruder, and Roberta Magnusson, gave me feedback on the research that led to this book. Another special thank you must go to Gerry and Linda Etzkorn, who taught me the fundamentals of reading and editing manuscripts while they hosted me at their home for two weeks. David Juste has provided guidance during the whole process of writing this book, and the use of his catalogue of Ptolemaic manuscripts (to be published soon) has been invaluable. Our work is so intermingled that we often do not know which of us made which discoveries. Additionally, I owe a debt of gratitude to Benno van Dalen, especially for his computer programs. Menso Folkerts and Richard Lorch, whose work on medieval mathematics has been the foundation for so much of my own work, have dispensed much good advice and made valuable corrections and suggestions, as have Charles Burnett and Philipp Nothaft. The other members of the Ptolemaeus Arabus et Latinus research team, Dag Nikolaus Hasse, María José Parra Pérez, Bojidar Dimitrov, Stefan Georges, José Bellver, and Stefan Müller have offered a great deal of help. Daniel Nodes has helped decipher some of the most obscure Latin passages. Many of my thoughts on the *Almagesti minor* were developed during my summer working with Vincenzo De Risi and the other members of his 'Modern Geometry and the Concept of Space' research group. I must thank the librarians of countless libraries, especially at the Institut de recherche et d'histoire des textes, the Huntington Library, the University of St. Louis' Vatican Film Library, the Monumenta Germaniae Historica, and the Universitätsbibliothek Erfurt. Of course, the errors in this book are my own.

For Elizabeth and Julia

Sigla

B	Berlin, Staatsbibliothek Preussischer Kulturbesitz, Ms. lat. qu. 510
Ba	Basel, Universitätsbibliothek, F.II.33
D	Dresden, Sächsische Landesbibliothek – Staats- und Universitätsbibliothek, Db. 87
Da	Darmstadt, Hessische Landes- und Hochschulbibliothek, 1987
E	Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 4° 356
E_1	Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 2° 383
F	Florence, Biblioteca Medicea Laurenziana, Conv. Soppr. 414
K	Cracow, Biblioteka Jagiellońska, 1924
L	London, British Library, Harley 625
L_1	Leipzig, Universitätsbibliothek, 1475
M	Munich, Bayerische Staatsbibliothek, Clm 56
Me	Memmingen, Stadtbibliothek, 2° 2,33
N	Nuremberg, Stadtbibliothek, Cent. VI.12
P	Paris, Bibliothèque nationale de France, lat. 16657
P_7	Paris, Bibliothèque nationale de France, lat. 7399
P_{16}	Paris, Bibliothèque nationale de France, lat. 16200
Pr	Prague, Národní Knihovna České Republiky, V.A.11 (802)
R	Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1012
R_1	Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1261
T	Toledo, Archivo y Biblioteca Capitulares, 98–22
W	Vienna, Österreichische Nationalbibliothek, 5266
W_1	Vienna, Österreichische Nationalbibliothek, 5273
W_2	Vienna, Österreichische Nationalbibliothek, 5292

Part I

Introduction

Overview

The *Almagesti minor* is a Latin summary of part of Ptolemy's *Almagest* that was written during the first two decades of the thirteenth century or possibly in the late twelfth century. There are short descriptions of it in two thirteenth-century texts. The *Speculum astronomiae* states, 'Also from these two books [i.e. the *Almagest* and Albategni's *De scientia astrorum*], a certain man assembled a book in the style of Euclid, the commentary of which [book] contains the opinions of both Ptolemy and Albategni, which thus begins: *Omnium recte pholosophantium* [*sic*] *etc*.'[1] In his *Biblionomia*, Richard of Fournival describes the work as follows: 'The book of the extraction of the elements of the science of the stars from Ptolemy's book the *Almagest* made by Walter of Lille up to the end of the sixth book.'[2] These descriptions emphasize three features of this astronomical book. Firstly, the *Almagesti minor* strips the *Almagest* down to its 'elements', the core of Ptolemy's argumentation, and reorganizes this material after the model of Euclid's *Elements* into lists of principles followed by proofs of general propositions. Secondly, it covers only the first six books of the *Almagest*, which are on the preliminaries to astronomy, spherical astronomy, the sun, the moon, and eclipses. The *Almagesti minor* does not treat the fixed stars or the planets. Thirdly, it also supplements Ptolemy's astronomy with theories and proofs from Arabic scholars, in particular Albategni (i.e. al-Battānī). Chiefly because of its organization into propositions and its emphasis on geometrical proofs, the *Almagesti minor* had a substantial impact upon astronomical works through the thirteenth, fourteenth, and fifteenth centuries.

[1] 'Ex hiis quoque duobus libris collegit quidam vir librum secundum stilum Euclidis, cuius commentarium continet sententiam utriusque, Ptolemaei scilicet atque Albategni, qui sic incipit: *Omnium recte pholosophantium* [*sic*] *etc*.' Zambelli, *The Speculum astronomiae and Its Enigma*, (Latin text from edition by Stefano Caroti, Michela Pereira, and Paola Zambelli), pp. 212–14.

[2] 'Liber extractionis elementorum astrologie ex libro Almagesti Ptolomei per Galterum de Insulla usque ad finem sexti libri ex eo.' A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 169.

Chapter 1

Title, Date, Origin, and Author

Title

This work has been referred to under a variety of titles by medieval scholars and modern researchers. Although scholars of the twentieth and twenty-first centuries have favored the title 'Almagestum parvum', neither of the two parts of this title can be justified.[1] First, the typical Latin name for Ptolemy's work was not 'Almagestum', but rather 'Almagesti', which was almost always treated as an indeclinable name by medieval scholars. Secondly, there are only eight manuscripts that use 'Almagesti' in conjunction with a form of the adjective 'parvus' to refer to this work in medieval sources, and among these there is no consensus that it should be the neuter 'parvum.' One (P_{16}) uses the masculine 'parvus'; one (Oxford, Bodleian Library, Ashmole 424) uses 'parvum'[2] in a context calling for an accusative, so it could be either masculine or neuter; three (Erfurt, UFB, Dep. Erf. CA 2° 375; Cambridge, University Library, Ee 3.61; and Oxford, New College, 281) write the title in contexts calling for the genitive or the ablative, and they correspondingly use 'parvi' or 'parvo', which also could be either masculine or neuter; and only three (*F*, *D*, and *Ba*) use 'parvum' for the nominative. Thus, there is little historical evidence for either part of the title commonly used by scholars today. On the other hand, the title 'Almagesti minor' or 'Minor Almagesti' is found in nine of the manuscripts bearing the work or excerpts from it (*P*, R_1, *Pr*, *Me*, L_1, P_{16}, *M*, *W*, and Vienna, ÖNB, 5258), and there are references using this title in at least seven other medieval sources – a note on the *Almagest* in Paris, BnF, lat. 7257, f. 10r,

[1] Among the many works of scholarship that use 'Almagestum parvum' exclusively or frequently are the following: A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', pp. 142–47; Haskins, *Studies in the History of Mediaeval Science*, p. 104; Lorch, 'The Astronomy of Jābir ibn Aflaḥ', p. 92; Lorch, 'Some Remarks on the *Almagestum parvum*'; Pereira, 'Campano da Novara autore dell'*Almagestum parvum*'; Zambelli, *The Speculum astronomiae and Its Enigma*, pp. 50, 107, 187 n. 15, and 214 n.; Byrne, *The Stars, the Moon, and the Shadowed Earth*, pp. 2, 118–19, 126, 158–59, 171, 197–98, 254; and Byrne, 'The Mean Distances of the Sun.' I referred consistently to the *Almagesti minor* as the 'Almagestum parvum' both in Zepeda, *The Medieval Latin Transmission of the Menelaus Theorem*, and in 'Euclidization in the *Almagestum parvum*.' North, *Richard of Wallingford*, generally uses 'Almagesti abbreviatum' but also refers to it as the 'Almagestum parvum' (e.g. vol. I, p. 49).

[2] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 145.

another note on the *Almagest* in Vatican, BAV, Pal. lat. 1365, f. 13v, John of Sicily's *Scriptum super canones Azarchelis*,[3] John of Genoa's *Canones eclipsium*,[4] the 1338 catalogue of the Sorbonne,[5] Bernard Chorner's *Almagesti Ptolomei abbreviatum*,[6] and John of Gmunden's *De sinibus, chordis et arcubus*.[7] There are a number of other titles given to this work in the manuscripts with the *Almagesti minor* and works that cite it. Among these are the following: *Liber Almagesti* (*B*, *K*, Johannes Andree Schindel's *Almagest* notes,[8] and Florence, Biblioteca Riccardiana, 885), *Liber Almagesti demonstratus* (R_1, *D*, the 1338 Sorbonne catalogue[9]), *Almagesti abbreviatum* (*L*, *M*, Vienna, ÖNB, 5258, gloss on canons for Toledan tables,[10] Bernard Chorner's commentary,[11] Richard of Wallingford's *Albion*,[12] Schindel's *Canones pro eclipsibus*,[13] and Albert of Brudzewo's *Commentariolum*[14]), *Commentarius Alberti Magni* (Johannes Schindel's *Canones pro eclipsibus Solis et Lune*,[15] Schindel's notes on the *Almagest*,[16] and Albert of Brudzewo's *Commentariolum*[17]) and a number of other titles and descriptions found in only single sources.[18]

Dating

The *Almagesti minor* depends upon Gerard of Cremona's translation of the *Almagest* (as I will show below), but Gerard likely made his translation over a lengthy period of time, perhaps beginning in the mid twelfth century and still working on it until his death in 1187. Because it is likely that the *Almagesti minor*'s author used the earlier version of Gerard's translation, this dependence can only provide an imprecise *terminus post quem* for the *Almagesti minor* of

3 Pedersen, 'Scriptum Johannis de Sicilia.'
4 Paris, BnF, lat. 7322, f. 41v.
5 Delisle, *Le Cabinet des manuscrits,* tome III, p. 75.
6 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 66v.
7 Busard, 'Der Traktat De sinibus, chordis et arcubus', p. 95.
8 Cracow, BJ, 619, f. 93v.
9 Delisle, *Le Cabinet des manuscrits*, tome III, p. 88.
10 Oxford, Bodleian Library, Auct. F.3.13, f. 217r.
11 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 66v.
12 North, *Richard of Wallingford*, vol. I, p. 248.
13 Vienna, ÖNB, 5415, f. 143v. This actually occurs in an addition to the work.
14 L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 23.
15 Vienna, ÖNB, 5415, f. 141v.
16 Cracow, BJ, 619, f. 69v.
17 L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 44.
18 'In speram' in P_7, 'Epitome Alberti in Almagesti Ptolomei' in W_2, 'Epythomatis super Astronomia Albategni' in Utrecht, Universiteitsbibliotheek, 6.A.3 (725), 'Parvum Almagesti Ptolomei demonstratum per Campanum' in *D*, and 'Liber Ieber' in *Ba*.

c. 1150.[19] The other sources of the *Almagesti minor* either are no later or cannot be dated, thus providing no further evidence for our dating of the work. The earliest manuscripts containing the *Almagesti minor* are from the thirteenth century and suggest that it was written at the latest in the 1240s. P_7 is from the first half of the thirteenth century; *K* is also likely from the first half of the thirteenth century; *P* was written between *c.* 1225 and 1260; P_{16} was written *c.* 1246–47; *B* is a manuscript of the mid thirteenth century that was probably written before 1249; and *F* may have been written before 1263. A similar endpoint for the range of time in which the *Almagesti minor* was written is given by Richard of Fournival's *Biblionomia*, which was most likely written around 1250, definitely between the time Richard became chancellor in 1240 and his death in 1260.[20]

Evidence that the *Almagesti minor* was written earlier is provided by the *Astrologia* of Guillelmus Anglicus, best known for his *De urina non visa*. The *Astrologia*, which begins, 'Quoniam astrologie speculatio …' and ends, '… de motibus que docentur in ipso auctore', is found in six copies: Erfurt, UFB, Dep. Erf. CA 2° 394, ff. 136r–140v; Erfurt, UFB, Dep. Erf. CA 4° 357, ff. 1r–21r; Paris, BnF, lat. 7298, ff. 111v–124v; Seville, Biblioteca Capitular y Colombina, 5–1-25, once on ff. 1–33 and a second time incompletely on ff. 110v–128v; and Vienna, ÖNB, 5311, ff. 42r–52v. In the *Astrologia*, there is a passage bearing a close resemblance to a passage in the *Almagesti minor* that is derived from Albategni's *De scientia astrorum*. The three corresponding passages all discuss the length of the year as determined by the Egyptians and Babylonians, Hipparchus, Ptolemy, and Albategni. The following table gives the relevant readings from these three works. Unique similarities between *De scientia astrorum* and the *Almagesti minor* are italicized, similarities between the *Almagesti minor* and the *Astrologia* are underlined, and any similarities between *De scientia astrorum* and the *Astrologia*, as well as any words in the *Astrologia* that could not have been derived from the *Almagesti minor*, are emboldened.

[19] Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog*, vol. II, pp. 2–3.

[20] For information on the *Biblionomia* and its date, see A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 119; Roy, 'Richard de Fournival, Auteur du *Speculum Astronomie*', pp. 165 and 167; Lucken, 'La *Biblionomia* de Richard de Fournival', pp. 90–94; and Lucken, 'La *Biblionomia* et la bibliothèque de Richard de Fournival.' Zambelli, *The Speculum astronomiae and Its Enigma*, p. 107, states, 'The *Biblionomia* was certainly written before 1243'; and Pereira, 'Campano da Novara autore dell'*Almagestum parvum*', p. 769, claims that the date of composition is 1243. Unfortunately, neither of these two provide evidence for their claims. For a discussion of the date of Richard's death, see Lepage, *L'Oeuvre lyrique de Richard de Fournival*, p. 11.

De scientia astrorum, Ch. 27[21]	*Almagesti minor* III.1	*Astrologia*[22]
Aegyptiorum etenim et[23] *ex* Babylonia *vetustissimi* quidam eam ex 365 diebus et quarta ultraque[24] parte *ex* 130 *diei partibus* constare dicebant.	Cum Egyptiorum *antiquissimi ex* Babylonia sicut per suas considerationes deprehenderunt ipsum ex ccclxv diebus et quarta diei et una parte *ex* cxxx *diei partibus* constare dixerunt,	Babilonici et Egptii perceperunt annum ex 365 diebus et quarta diei et una 130 parte diei.
A passage of 98 words: Ptolemaeus autem illos haec … in signorum circulo.	n/a	n/a
Abrachis autem longitudinem temporis anni 365 diebus et quarta diei parte solummodo constare confirmavit, licet hoc minus esse probasset sed[25] quod Ptolemeus eum dixisse recitavit cum eius omnia dicta collegit. Dixit etenim tempus anni fore 365 diebus minus quam quarta veraciter,	Abrachaz vero super cuius considerationem operatus est Ptolomeus ex ccclxv diebus et quarta diei tantum.	Abrachis autem quem imitatur Ptolomeus in considerationibus tantum ex 365 diebus et quarta diei inter quos fluxerant 285 anni Egiptiorum.
A passage of 166 words: eo quod aestivale solstitium … cuius crastinum fuit dies quarta in Alexandria,	n/a	n/a
post hoc[26] Ptolemaeus 285 annis Aegyptiacis transactis observavit. … A passage of 128 words … Tempus ergo anni quod his duabus observationibus depraehensum est fuit 365 dierum et quarte unius diei minus una parte *ex* 300 unius diei part*ibus*,	*Post hec* Ptolomeus ab hac quantitate anni in ccc annis unum diem excepit, et annum Solis esse ex ccclxv diebus et minus quam quarta quantum est una pars *ex* ccc diei part*ibus* per suam considerationem et considerationem Abrachaz, inter quas fuerunt cclxxxv anni Egyptiaci deprehendit.	Sed Ptolomeus **qui scripsit anno Nabuchodonosor 880**[27] in 300 annis unum diem post excepit[28] inveniens annum per suas considerationes ex 365 diebus et quarta diei minus 300a diei parte.

[21] Albategni, *De scientia astrorum*, 1537 ed., ff. 26v–27v, with selected variants from *P*, ff. 25v–26r.

[22] Vienna, ÖNB, 5311, 43rb, with selected variants from Erfurt, UFB, Dep. Erf. CA 2° 394, f. 136v

[23] *P*: *om*.

[24] *P*: unaque.

[25] *P*: secundum.

[26] *P*: hec.

[27] Erfurt, UFB, Dep. Erf. CA 2° 394: 886.

[28] Erfurt, UFB, Dep. Erf. CA 2° 394: attenditur.

A passage of 221 words: quod est una pars … quod est 186 annorum.	n/a	n/a
Post hoc etiam in Aracta observavimus invenimusque per unam nostrarum observationum autumnalium in qua confisi fuimus secundum quod per instrumentum apparuit quod fuit post Ptolomaei praedictam observationem autumnalem 743 annorum Solem per *aequidiei punctum autumnalem* transisse anno 1194 ex annis Adilcanari **qui sunt post mortem Alexandri 1206 annorum**	Deinde vero a Ptolomeo post dccxliii annos observavit *Albategni punctum equinoctii* et per intervallum duarum considerationum, sue scilicet et Ptolomei, *tempus* ann*i* ccclxv die*rum* et *xiiii* minutorum et xxiiii secundorum fore deprehendit.	Albategni post Ptolomeum 743 anno invenit annum ex 365 diebus **et quarta diei parte et 34**[29] minutis et 24 secundis hore. **Et fuit Albategni anno Alexandri 1191**.
… A passage of 161 words …		
Erit ergo *tempus* ann*i* verissimum 365 die*rum* et *14* minutorum et 26 secundarum fere.		

In the compared passages, the *Almagesti minor* and the *Astrologia* have many similarities that are not shared with *De scientia astrorum*. They address the same parts of Albategni's work and leave out identical passages. When Hipparchus is first mentioned, they both have relative clauses expressing Ptolemy's use or imitation of his predecessor. They have the identical phrase 'in ccc annis unum diem excepit', and they share many smaller linguistic similarities. Additionally, both the *Almagesti minor* and the *Astrologia* follow this passage with short discussions of the theory of trepidation while *De scientia astrorum* does not. While the passages in the two Latin works are derived from the passage in *De scientia astrorum*, they clearly have a closer relationship to each other. Next, it should be noted that *De scientia astrorum* and the *Almagesti minor* have several common features that are not found in the *Astrologia*: they have the synonyms 'vetustissimi' and 'antiquissimi'; they both report Albategni's observation of an equinox, while Guillelmus does not say whether Albategni made an equinoctial or solsticial observation; they both give the correct length for Albategni's year, while the two manuscripts of the *Astrologia* that I have seen contain incorrect values; and they have several similarities of wording such as 'unaque parte ex 130 diei partibus' and 'et una parte ex cxxx diei partibus' as opposed to the *Astrologia*'s 'et una 130 parte diei.' On the other hand, the

[29] Erfurt, UFB, Dep. Erf. CA 2° 394: 24.

Astrologia shares almost nothing with *De scientia astrorum* that is not common to all three passages. Furthermore, almost all of the content of the *Astrologia*'s passage is also in the *Almagesti minor*. There are small exceptions where Guillelmus claims that Ptolemy wrote 880 (or 886) years after Nabonassar and that Albategni made his observations in 'anno Alexandri 1191'; however, both of these dates are incorrect and do not match the dates that Albategni gives for the observations of Ptolemy and himself in this passage of *De scientia astrorum*. Guillelmus' sources for his statements that Ptolemy wrote in the 880th or 886th year of Nabonassar and that Albategni lived in the 1191th year of the Seleucid Era are perhaps *De scientia stellarum* Ch. 51 and *Almagest* III.8, where other (non relevant) observations of these two men made in those years are reported.[30] The inclusion of these dates suggests that Guillelmus may have had knowledge of *De scientia stellarum*, but the irrelevance of these dates to the calculations for the length of the year indicates that he was not using the relevant passage here and was instead using the *Almagesti minor* as his main source. It is possible that both the passages in the *Almagesti minor* and the *Astrologia* depend upon an unknown summary of *De scientia astrorum*, but in the absence of such a work, it is most reasonable to conclude that Guillelmus used the *Almagesti minor*.

There are other similarities between the *Astrologia* and the *Almagesti minor*, but they are not close enough to determine dependency. That Guillelmus relies heavily upon the *Almagesti minor* for only this one passage is not inexplicable. The *Astrologia* is more in the genre of canons than theoretical astronomy, although it does contain some geometrical representations and proofs. Guillelmus also did not need to rely on the *Almagesti minor* for most of the *Astrologia* since he had several other sources, including the *Almagest* and the canons to the Toledan Tables. In the *Astrologia* and *De urina non visa*, Guillelmus shows a penchant for referring to authorities, so the passage of *Almagesti minor* III.11 in which the Egyptians, Babylonians, Hipparchus, Ptolemy, and Albategni are all discussed would have been particularly appealing to him.[31]

It is known that Guillelmus wrote the *Astrologia* in 1220, as can be seen from the colophon in Seville, Biblioteca Capitular y Colombina, 5–1-25, f. 31r: 'Explicit astrologia magistri Verberillini ciuis Massiliensis qui Anglicus est natione professione medicus ex scientie merito astronomus appellatus compilata

[30] Albategni, *De scientia astrorum*, 1537 ed., f. 79r-v; and Ptolemy, *Almagest*, 1515 ed., f. 34r.

[31] In *De urina non visa*, Guillelmus refers to Ptolemy's *Quadripartitum* and *Centiloquium*, Alcabitius, Albumasar, Messahala, and to 'Egiptorum antiqui', and in the *Astrologia*, he mentions Ptolemy's *Quadripartitum*, the *Almagest*, the *Centiloquium*, Geber, Albumasar, Euclid's *De visu* and the *Elements*, Arzachel's canons and the Toledan Tables, Thebit, Alpetragius, Theon, and a *Liber de triangulis,* as well as the Egyptians, Babylonians, Hipparchus, and Albategni in the passage from the *Almagesti minor*.

per ipsum anno domini 1220 et scripta per me Ioannem Mariam de Albinis de Argarta anno domini 1472 die mensis februarii.' This date for the work accords well with the dating for Guillelmus' other works, which range from around 1220 for *De urina non visa*, which contains calculations for a date in December 1219, to 1231 for his translation of Arzachel's *Saphea*.[32] The dependence of the *Astrologia* upon the *Almagesti minor* thus establishes that the latter was written before 1220, and the *Almagesti minor*'s use of Gerard of Cremona's translation of the *Almagest* show that it was composed after the mid twelfth century.

Authorship

In addressing the question of the author's identity, I must first treat an interesting hypothesis about the *Almagesti minor*'s origin that is suggested by Richard Lorch on the basis of three observations of his.[33] First, the *Almagesti minor* has only a loose connection to Gerard of Cremona's translation of the *Almagest*, so it is possible that parts of the work were written using another translation, perhaps the Sicilian translation of the *Almagest* from the Greek, which was thought by some historians to have been written by Hermann of Carinthia in the mid twelfth century. Secondly, the *Almagesti minor* employs some words that are derived from Greek. Thirdly, the *Almagesti minor* sometimes only has outlines of proofs, and the enunciations are sometimes found alone or with different proofs. This is reminiscent of the 'Adelard II' version of Euclid's *Elements*, which is thought to be the work of Hermann's colleague, Robert of Ketton.[34] From these Lorch sets out his theory:

> In conclusion, it is tentatively suggested here that the preface, most of the earlier part of the book (where the proofs are short), the enunciations and perhaps the introductions to books II–VI were the work of a scholar in the Hermann-Robert circle in the mid-twelfth century and that the treatise was filled out later on the basis of a form of Gerard's translation of the *Almagest*. The whole was finished by about 1200.'[35]

Lorch's hypothesis, which he only offers 'tentatively', proves to be unlikely. To the first of Lorch's supporting observations, there are closer similarities in wording to Gerard's translation than Lorch realized, and there are a few traces of Gerard's translation even in Book I of the *Almagesti minor*. Furthermore, it is doubtful that Hermann of Carinthia was the author of the Sicilian translation of the *Almagest*.[36] In regard to Lorch's second point, there are some words

[32] Moulinier-Brogi, *Guillaume l'Anglais, le frondeur de l'uroscopie médiévale*, p. 24.

[33] Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 431–33.

[34] Busard and Folkerts, *Robert of Chester's (?) Redaction*. For the argument that the author is Robert of Ketton, not Robert of Chester, see Burnett, 'Ketton, Robert of, (fl. 1141–1157).'

[35] Lorch, 'Some Remarks on the *Almagestum parvum*', p. 434.

[36] Haskins, *Studies in the History of Mediaeval Science*, pp. 53–54.

derived from Greek; however, these words could very well have been known from other sources, and there is at least one 'Grecised' version of an Arabic-to-Latin translation.[37] The third point about the comparison to the Adelard II version of the *Elements* has truth to it; however, much more would be needed to show that the two texts have a similar origin. Furthermore, while the preface is of a very different style from the remainder of the work and perhaps was composed by a different scholar, the slight changes in vocabulary and style found throughout the bulk of the *Almagesti minor* are consistent with the supposition of a single author. There is no more difference in vocabulary and the rates of usage of each word than is to be expected from the range of subject matter and the difference in roles between the enunciations/corollaries and the proofs.[38] In the absence of any strong evidence for the theory of dual authors, it is more reasonable to only assume the existence of one author.

The manuscripts containing the *Almagesti minor* bear attributions to five men, all of which prove to be incorrect. It is said to be the work of Albategni in three of the manuscripts with the work or excerpts of it: *F*, Vienna, ÖNB, 5258, and Utrecht, Universiteitsbibliotheek, 6.A.3 (725). These last two manuscripts are very late ones – Vienna, ÖNB, 5258 is from the second half of the fifteenth century and the manuscript from Utrecht dates from *c.* 1500. Although *F* was probably written before 1263, the attribution is at the start of the work in a later hand that appears to have been written in 1304. Another attribution to Albategni is offered by John of Sicily in his *Scriptum super canones Azarchelis*: '… in quarto libro Minoris Almagesti, quem abbreviavit Albategni.'[39] This work is from around 1290–95, so it is the earliest evidence for Albategni's authorship.[40] Yet another attribution to Albategni is found in one manuscript of the fifteenth-century work *Compositio duorum instrumentorum*.[41] It is certain that these attributions to Albategni are incorrect and that

[37] Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 33–34 and 124–41.

[38] To aid in the comparison of vocabulary and usage, I separated the text of the *Almagesti minor* into the parts that Lorch suggests are the works of different authors, i.e. one file containing Book I and the enunciations and corollaries of Books II–VI, and another file containing the proofs of Books II–VI. I then generated lists of keyword density for the enunciations and for the proofs. After excluding words that are only used infrequently, I used the word-frequency calculator again to find the words that appear frequently in one set of text but not in the other. While there were many such words, their appearance in only one set was consistent with the theory of a single author. No technical words were used frequently in one and not at all in the other. I came to similar results when I compared the enunciations of *Almagesti minor* VI to their proofs. As a comparison, I then performed the same process of comparing word choice and frequency in Book VI of Gerard's translation of the *Almagest* and *Almagesti minor* VI. The differences there were quite obvious.

[39] Pedersen, 'Scriptum Johannis de Sicilia', 52, p. 135 (J287c).

[40] Pedersen, 'Scriptum Johannis de Sicilia', 51, p. 10.

[41] Munich, BSB, Clm 367, f. 42r.

the work was originally written in Latin. It uses Gerard of Cremona's translation of the *Almagest*, and it includes a reference to Jesus 'qui est rex regum et dominus dominantium.'[42] Secondly, the *Almagesti minor* contains many references to Albategni, which obviously would not occur if he were the author.[43] The cause of this misattribution could perhaps be Hermann of Carinthia's preface to his translation of Ptolemy's *Planisphere*, in which it is stated that Albategni summarized the *Almagest*.[44]

The Latin origin of the work also immediately disqualifies the attributions to Geber (i.e. Jābir ibn Aflaḥ) found in *Ba*, which dates from the mid fourteenth century, and in a table of contents added to E_1. Although Aleksander Birkenmajer, Carlo Nallino, and Richard Lorch made it abundantly clear that work was not composed by Jābir, several modern scholars have confused the *Almagesti minor* with the *Liber super Almagesti*, the Latin translation of Jābir's *Iṣlāḥ al-Majisṭi* (the *Correction of the Almagest*), beginning 'Scientia species habet …', and have misattributed the *Almagesti minor* to him.[45] Finding Nallino's concise arguments that the *Almagesti minor* was an original Latin work unconvincing, Francis Carmody posited that it could indeed be a translation of a work by Jābir.[46] He later included it among Jābir's works, noting that it is 'considered spurious but for no valid reason.'[47] Lynn Thorndike and Pearl Kibre also attributed the work to him in their influential *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, which is perhaps the chief reason for the persistence of this error to the present.[48]

An attribution to Thomas Aquinas is found only in a single manuscript of the fifteenth century, *M*. Thomas was born *c.* 1225 and is thus much too late to have been the author. Even if the composition of the *Almagesti minor* were

[42] *Almagesti minor* III.17. The use of Gerard's translation was known to Nallino, *al-Battānī*, vol. I, p. xxvii, and A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 144; however, Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 423–30, claimed that it is not clear which translation of the *Almagest* the author used. The passage referring to Jesus was reported by A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 144.

[43] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', pp. 142–43.

[44] '… Almagesti quidem Albateni commodissime restringit.' Burnett, 'Arabic into Latin in Twelfth-Century Spain', p. 110. The verb 'stringo' can mean 'to summarize', and it seems to me that 'restringo' must mean the same here. Burnett, p. 111, understands it in the same way: 'Al-Battānī has appropriately made the *Almagest* more concise (?).'

[45] On this issue, see Nallino, *al-Battānī*, vol. I, p. xxvii; and Lorch, 'The Astronomy of Jābir ibn Aflaḥ', p. 92.

[46] Carmody, *Al-Biṭrūjī: De motibus celorum*, pp. 29–30.

[47] Carmody, *Arabic Astronomical and Astrological Sciences*, p. 164. Carmody also lists Gerard of Cremona as the translator of the *Almagesti minor*.

[48] Thorndike and Kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, column 1006. Recent instances of this misattribution are found in Byrne, *The Stars, the Moon, and the Shadowed Earth*, pp. 2, 118, 157–59, and 254; and Byrne, 'The Mean Distances of the Sun', pp. 206 and 211.

to harmonize with his biography, it is very clear from the work's content and style that it was not written by Thomas.

An attribution to another Dominican, Albertus Magnus, is more credible but is still doubtlessly an error. Albertus had an interest in astronomy and mathematics, and he was also born early enough (before 1200) that he could have conceivably been the author of the *Almagesti minor* sometime before 1220. However, this is highly unlikely. First, he would have had to have written it quite early in his career, and the organization and style of the *Almagesti minor* does not match that of his other works. Secondly, Albertus' authorship is attested to only in W_2, which is a very late manuscript from the sixteenth century, and in the writings of Johannes Schindel in the early fifteenth century, as well as in later texts based upon Schindel's. In the margins of Cracow, BJ, 619, a manuscript of the *Almagest* from which he lectured from 1412 to 1418, Schindel copied almost the entirety of the *Almagesti minor*, and he gives Albertus Magnus as the author several times.[49] In his *Canones pro eclipsibus solis et lune* written in 1433, Schindel continues to attribute the work to Albertus.[50] Because there are only these late attestations to his authorship, they almost surely stem from attempts to attach the name of a prestigious authority to the anonymous work. A further indication that Albertus was not the author is the lack of any mention of his name in *D*, a manuscript that was owned by a Dominican in Cologne in the 1330s and perhaps several decades earlier. If Albertus were in fact the author of the *Almagesti minor*, one would suspect that a fellow member of his order living a short time later in a place where he spent so much of his life would most likely have attributed it correctly to him.

The attributions to Campanus of Novara are more promising because he clearly had the interest and skill in mathematics and astronomy to compose the work. There are two pieces of evidence for his authorship. The first is in *D*. This manuscript could have been written in the late thirteenth century, but the attribution is not in the scribe's hand. The second testimony is in a note about William of Moerbeke from the late fourteenth or fifteenth century found in Oxford, Bodleian Library, Ashmole 424: '… ipse autem socius fuit magistri Campani qui fecit parvum almagesti et commentavit geometriam Euclidis.'[51] Aleksander Birkenmajer laid out the case against Campanus' authorship, arguing under the belief that the beginning of Campanus' known career in the 1260s clashed with the *Almagesti minor*'s inclusion in the *Biblionomia*, which he supposed to have been composed around 1250.[52] However, Campanus is known to have been active in the 1250s and there is much uncertainty about

[49] E.g. Cracow, BJ, 619, ff. 69v, 93v, 117r, and 126v.

[50] E.g. Vienna, ÖNB, 5415, ff. 137v and 141r-v.

[51] Black, *A Descriptive, Analytical, and Critical Catalogue of the Manuscripts Bequeathed unto the University of Oxford by Elias Ashmole*, column 340.

[52] A. Birkenmajer, 'Bibliothèque de Richard de Fournival', pp. 145–46.

Campanus' early life, as well as some slight evidence that he could have started his astronomical work as early as 1232.[53] Accordingly, Michela Pereira was able to argue that Campanus was the author of the *Almagesti minor*.[54] Pereira's confidence in Campanus' authorship was bolstered by her mistaken belief that two other manuscripts, *Pr* and *Me*, bore attributions to Campanus.[55] Agostino Paravicini Bagliani and Paola Zambelli found Pereira's argument convincing.[56] However, now that the *Almagesti minor*'s new *terminus ante quem* of 1220 has been established, this claim for Campanus' authorship proves to be unfounded. If Campanus were the *Almagesti minor*'s author, he would have had to be born before 1200, which would make him nearly 100 years old when he died in 1296.[57] Furthermore, Campanus wrote a set of glosses on the *Almagest*, and it seems unlikely that he would have written two works on the same subject that do not exhibit any close similarities.[58]

There remains one further man to whom the *Almagesti minor* has been ascribed, Walter of Lille. None of the manuscripts containing the *Almagesti minor* say that it is by him; however, the first attribution, which is perhaps several decades earlier than any of the others, is to him. As stated above, Richard of Fournival's entry in the *Biblionomia*, which may have been written as early as 1240, reads, 'Liber extractionis elementorum astrologie ex libro Almagesti Ptolomei per Galterum de Insulla usque ad finem sexti libri ex eo.'[59] That Richard is generally accurate in his descriptions of works demands that we take this claim seriously. In an alternate proof of *Almagesti minor* I.7 in *T*, a thirteenth-century manuscript likely from northern France, the reviser cites a book on ratios by a Walterus Flandrensis.[60] This very likely refers to a work on compound ratios and their 'modes' with the incipit 'Proportio est rei …' that immediately precedes the *Almagesti minor* in *T*.[61] This *De proportionibus* has been

[53] Benjamin and Toomer, *Campanus of Novara*, pp. 3–5. The date 1232 comes from some tables attributed to him, but it fails to harmonize well with the remainder of the evidence for his biography, which suggests that he started to flourish in the 1250s.

[54] Pereira, 'Campano da Novara autore dell'*Almagestum parvum*', pp. 769–76.

[55] Pereira, 'Campano da Novara autore dell'*Almagestum parvum*', p. 772. Pereira's mistake is perhaps due to reliance upon Björnbo and Vögl, *Alkindi, Tideus und Pseudo-Euklid. Drei optische Werke*, p. 129 n. 3, which attributes the *Almagesti minor* in *Pr* to Campanus.

[56] Paravicini Bagliani, 'La scienza araba nella Roma del Duecento: Prospettive di ricerca', p. 153; Paravicini Bagliani, *Le Speculum astronomiae, une énigme?*, pp. 139–42; and Zambelli, *The Speculum astronomiae and Its Enigma*, pp. 48, 50, and 214.

[57] Benjamin and Toomer, *Campanus of Novara*, p. 9.

[58] Zepeda, 'Glosses on the *Almagest*.'

[59] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 169; and Roy, 'Richard de Fournival', pp. 165 and 167.

[60] *T*, f. 68ra.

[61] Edited in Busard, 'Die Traktate De Proportionibus von Jordanus Nemorarius und Campanus.' The modes of compound ratio are the valid rearrangements of the six terms in a statement that one ratio is composed of two other ratios.

attributed to Jordanus, but it has been established that this is an error.[62] The *Almagesti minor* and *De proportionibus* were often transmitted together. Of the 16 manuscripts containing *De proportionibus*,[63] five have the *Almagesti minor*: *B*, P_7, *T*, *K*, and W_2. For the sake of comparison, the *Almagesti minor* is not found in any of the 11 manuscripts containing Campanus' very similar treatise on compound ratio and the modes, and of the 13 manuscripts that contain one of the versions of the *De figure sectore* of Thebit (i.e. Thābit ibn Qurra), *R* is the only one that also has the *Almagesti minor* (and *R* only contains a short excerpt of Thebit's work). Furthermore, three of the five manuscripts containing the *Almagesti minor* and *De proportionibus* have them in succession. Additionally, in both Florence, Biblioteca Riccardiana, 885 and Peter of Limoges' gloss to the *Almagest* in P_{16}, there are references to a *De proportionibus*, which could very likely be this treatise that I believe is by Walter.[64]

Even after establishing that the text is by a 'Walter of Lille', we would have to identify which of the multiple Walters of Lille is the author. Searching through the possibilities, Aleksander Birkenmajer considered a Walter of Lille who was a chancellor of England in 1166–70, but he discarded him as a potential author of the *Almagesti minor* because he believed that the *Almagesti minor* must have been written after 1175.[65] Our revised *terminus post quem* of *c.* 1150 means that this Walter could possibly have been the author; however, there is nothing to indicate that this Walter had any interest in astronomy. Also, as will be discussed below, the evidence suggests that the work has a French, not English origin. The name 'Walter of Lille' is found another time in the *Biblionomia*: 'Galteri de Insula, dicti de Castelione, liber Alexandreidos.'[66] Although this Walter of Châtillon, as he was more commonly known, was well known for his poetry, especially his epic poem on Alexander the Great, the *Alexandreis*, much of his biography remains shrouded in mystery; however, enough is known to determine that he fits the most basic criteria to be the author of the *Almagesti minor*. He was born in the neighborhood of Lille, and was a student of Stephen of Beauvais at Reims and Paris. After running schools at Laon and Châtillon, he studied law at Bologna. He then worked for William, archbishop of Reims, to whom he dedicated the *Alexandreis*, started in the 1170s and probably finished in the 1180s. With the help of his patron, he became a canon at either Amiens, Beauvais, or Orléans (depending upon which source one trusts). In

[62] R. Thomson, 'Jordanus de Nemore: Opera', pp. 124–25, includes the work among 'Dubious Ascriptions.' Zepeda, 'Jordanus de Nemore and His Conception of Compound Ratios', consists of an extended argument against Jordanus' authorship.

[63] I thank Menso Folkerts for his personal list of medieval mathematical authors, works, and manuscripts. To his list, I add W_2, ff. 274r–275v.

[64] E.g. Florence, Biblioteca Riccardiana, 885, f. 115v; and P_{16}, f. 12r.

[65] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 146.

[66] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', p. 193.

whichever of these cities he was appointed a canon, he died of leprosy. The year of his death is unknown, but given his biography, it seems to be *c.* 1200.[67] If he was canon of the cathedral of Amiens, it would be more likely that Richard of Fournival did not make a mistake in attributing the *Almagesti minor* to him, because from 1240 or even earlier, Richard held a number of positions, including chancellor, at the same cathedral, where his half-brother was bishop.[68] Although Richard may have become a canon at Amiens decades after Walter's death, his relatively close institutional connection to Walter lends credence to his attribution. The fact that Walter of Châtillon was a poet should not be regarded as evidence against his authorship, especially since it was not unusual at this time for learned scholars to write poetry – e.g. Alain of Lille or Richard of Fournival. Walter of Châtillon's *Alexandreis* mentions some astronomical phenomena, e.g. an eclipse, and although the astronomical passages of Walter's poetry show no close linguistic similarities to the *Almagesti minor*, the difference in genres may explain this divergence.[69]

Evidence suggesting that the *Almagesti minor* was composed in northern France or that the region played an important role in its transmission strengthens Walter's claim of authorship. Of the eight manuscripts known to have been written in the thirteenth century, P, R_1, P_{16}, T, K, F, B, and P_7, the first five are known to have originated in northern France. Furthermore, because F appears to have been copied from P, which seems to have been present only in Amiens and Paris, it also seems to stem from one of those places. B and P_7 are thus the sole early manuscripts of the *Almagesti minor* that have no known probable connection to the area.

A further connection between the *Almagesti minor* and northern France is seen by comparing the *Almagesti minor* to some manuscripts of Gerard's translation of the *Almagest*. First, there is a similarity in the way of representing numbers. The author of the *Almagesti minor* appears to have usually used either words or Roman numerals to represent numbers. Seven of the manuscripts written before 1400 generally have Roman numerals, four generally have Arabic numerals, and two use the two forms of numerals in roughly equal portions. Additionally, mistakes show that manuscripts were copied from exemplars with Roman numerals; for example, in V.9, a scribe early in the transmission of the text must have read 'de lx' (with the preposition abbreviated) as 'dlx.' This mistake appears in K and E_1 in this manner, and it is found as

[67] The best discussion of his life and the text of the important medieval sources are found in Colker, *Galteri de Catellione Alexandreis*, pp. xi–xviii and 493–94. Also, see Meter, *Walter of Châtillon's Alexandreis Book 10 – A Commentary*, pp. 28–30.

[68] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', pp. 123 and 146; and Lucken, 'La *Biblionomia* et *la* bibliothèque', pp. 68–69.

[69] E.g. see Colker, *Gualteri de Castellione Alexandreis*, Liber III, ll. 463–521, pp. 86–88; Liber IV, ll. 376–77, p. 137; and Liber X ll. 329–39.

'560' in *B* and P_7. If *B* and P_7 did not ultimately depend upon an exemplar that used Roman numerals at this location in the text, their readings would not be explainable. Additionally, in V.19 the text has 'lx idest', and some manuscripts misread the letter 'i' abbreviated for 'idest' as part of the number. Thus, *B* has 'lxi', and this error is reflected in the reading '61', which is found in *M*, *N*, and *Ba*. Therefore, Roman numerals are original in most of the work. However, even in the manuscripts that predominately use Roman numerals (*P*, *T*, *K*, *D*, *R*, *L*, and W_2), which come from all three main families of the text's tradition, we find Arabic numerals in I.6, often very poorly and unsurely written. Three of the early manuscripts of Gerard of Cremona's translation of the *Almagest* (i.e. those that could date from the mid thirteenth century or earlier) similarly have Roman numerals throughout almost the entirety of the work but have many Arabic numerals in the exact passages of *Almagest* I.9 that correspond to the one in *Almagesti minor* I.6 that has Arabic numerals.[70] These three *Almagest* manuscripts are closely related members of the earlier version of Gerard's translation, Paul Kunitzsch's A-Klasse.[71] The first, Paris, BnF, lat. 14738, was composed in the late twelfth century in northern France, likely Paris. The second is P_{16}, which was copied in Paris in 1213, likely from BnF, lat. 14738. The third, Paris, BnF, lat. 7255, was composed in the first half of the thirteenth century, and although it was perhaps written in England, it appears to be very closely related to the other two and possibly was copied from one of them.[72] There are two other manuscripts, also from the A-Klasse, that have the same pattern of numerals – Parma, Biblioteca Palatina, 719 and Florence, BNC, Conv. Soppr. J.III.24 (San Marco 177); however, both of these date from the late thirteenth century or later, long after the *Almagesti minor*'s composition. The change of numbering styles at corresponding places in the *Almagesti minor* and this group of *Almagest* manuscripts with a connection to northern France is surely no coincidence.

Moreover, these three *Almagest* manuscripts held in Paris share another feature with the *Almagesti minor*. In *Almagest* III.3–5, Ptolemy explains much of the solar theory in terms of both the eccentric and the epicyclic model, and the author of the *Almagesti minor* follows this in III.5–6, 9–10, 13–14, and 15–16. Each pair of propositions consists of a proof concerning the eccentric model and a corresponding proof for the epicyclic model. However, *Almagesti minor* III.12, which is a proof in terms of the eccentric model, is not followed

[70] I was able to consult 20 manuscripts of Gerard's translation of the *Almagest* from the twelfth and thirteenth centuries. There are three that I have not seen: Private Owner, *olim* Robert B. Honeyman Jr., California, no. 14; Vatican, BAV, Vat. lat. 6788; and Wolfenbüttel, Herzog August Bibliothek, 147 Gud. lat. 4° (4451).

[71] Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog*, vol. II, pp. 5–6 and 12–16.

[72] Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog*, vol. II, p. 13. A closer examination of these manuscripts is needed to confirm the exact relationship of this manuscript to the others.

by the epicyclic proof that one would expect, although Ptolemy provided such an epicyclic proof at the end of *Almagest* III.4. The reason for the lack of an epicyclic proof in the *Almagesti minor* becomes evident when we turn again to the early *Almagest* manuscripts. Some *Almagest* manuscripts, including the same Parisian manuscripts discussed above, omit the last paragraph of III.4, which contains the very proof that is missing in the *Almagesti minor*.[73] The beginning of the *Almagest*, including I.9, is missing in another early Parisian manuscript of Gerard's translation, Paris, BnF, lat. 7268; it has Roman numerals throughout the surviving books of the *Almagest*, and it lacks the same paragraph at the end of III.4. It is very possible that it also shared the same pattern of numeral changes in I.9.

It thus appears that there was a group of manuscripts of Gerard's translation of the *Almagest* that had unique characteristics and many of the members of this group were in northern France around the time the *Almagesti minor* was written. The *Almagesti minor* shares the two characteristics of this group, so it appears that the author used one of the members of this group of *Almagest* manuscripts, perhaps in or near Paris. That the author was a man from Lille who lived in Amiens fits this situation well. Although the attribution to Walter of Lille is the only credible one and is, in fact, quite possible, it is unclear whether this is Walter of Châtillon. It is best to avoid the temptation to attach his name or even the more generic 'Walter of Lille' to the work prematurely. Until more evidence emerges, the matter of the *Almagesti minor*'s authorship remains unsettled.

[73] The omitted passage is written in the margin by Peter of Limoges in P_{16} and it is placed at the end of the *Almagest* in Paris, BnF, lat. 14738.

Chapter 2

Euclidean Style

The most conspicuous characteristic of the *Almagesti minor* is that it fits content from the *Almagest* into a new Euclidean framework.[1] As we have seen above, the author of the *Speculum astronomiae* pointed out that it was written 'secundum stilum Euclidis' and Richard of Fournival described it as 'Liber extractionis elementorum astrologie.'

Although there was a great variety in formats of the *Elements* in the Middle Ages, the axiomatic, deductive, and universal nature of Euclid's work was apparent in all versions. Euclid's style can be explained relatively easily because he has a very formal and bare format with a limited number of types of writing and he has no or very little informal discourse. Euclid's *Elements* begins most of its 13 books with a list of principles. These can be definitions, postulates, or common notions. The truth of the principles is not argued, nor are they explained in many of the medieval versions. The bulk of each book is made up of propositions and their proofs. The propositions or enunciations are stated in general terms. (Because 'proposition' is sometimes used to refer to the enunciation and the accompanying proof as a unit, I will use 'enunciation' to refer to the statement that is proved.) The proofs argue for the truth or validity of the enunciations using the principles and prior propositions. In the *Elements* there are only two types of these propositions, theorems and problems. The former concern facts about mathematical objects, and the latter are about the construction or finding of mathematicals that meet certain criteria.

There are many parts of a formal Euclidean proof. First, the enunciation states what is to be proved. In the medieval Latin versions of the *Elements*, it is usual for a theorem's enunciation to be stated as a sentence using an indicative main verb (e.g. 'Omnium duarum linearum inter se secancium omnes anguli contra se positi sunt equales.'[2]), and a problem's enunciation is expressed with an accusative plus infinitive construction (e.g. 'Triangulum equilaterum super datam lineam rectam collocare.'[3]). Enunciations are always expressed in general terms. Secondly, the exemplification sets out a particular example of the mathematical objects given universally in the enunciation. It is often a description of a geometrical figure. Note that there is still a sense of universality at play here;

[1] I explored this topic in more detail in Zepeda, 'Euclidization in the *Almagestum parvum*.'
[2] Busard and Folkerts, *Robert of Chester's (?) Redaction*, vol. I, p. 120.
[3] Busard and Folkerts, *Robert of Chester's (?) Redaction*, vol. I, p. 115.

if there is a line AB, it is not stated how long line AB is nor where line AB is, except in relation to other parts of the figure. In some versions of the *Elements*, this is generally introduced by a phrase such as 'exempli gratia' or 'verbi gratia.' Thirdly, the specification states what is to be proved or done in terms of this particular example. In medieval Latin versions of the *Elements*, it is often introduced by 'Dico quia' or 'Dico quod.' Sometimes the parts after the specification are introduced by the phrase 'rationis causa.' Fourthly, the construction lays out additional mathematical objects that are needed in the argument. In a problem, the construction of the sought quantity is included in this part of the proof. Fifthly, the argument uses the principles and prior propositions to lay out the logical steps between what is known and the conclusion. Sixthly, the conclusion is the endpoint of the argument. In the theorem, it is a restatement in particular terms of the enunciation, followed sometimes by a universal restatement of the enunciation. Seventhly, a corollary is a part of a proposition that is occasionally found in the *Elements*. It is another general expression that is proved true or valid from the proof although it is not the proof's main objective. An example is found in *Elements* III.15 (in the numbering of medieval versions; III.16 in Heiberg's edition of the Greek); the corollary to this proposition reads (in the 'Adelard II' version), 'Corollarium. Unde eciam manifestum est omnem lineam rectam a termino diametri cuiuslibet circuli ortogonaliter ductam circulum ipsum contingere.'[4] While corollaries are placed at the ends of the proofs in some versions of the *Elements*, in others they are placed after the enunciations. Corollaries are often introduced by 'Unde manifestum quod …' or similar wording. Eighthly, the figure is an important part of most proofs, especially in the geometrical books of the *Elements*. It often conveys information that is not expressed in the text. In the more concise medieval versions of the *Elements*, its importance is even higher because it stands in place of many textual parts of the proof that are left unstated. Not all parts of a proof are found for each proposition. In some of the Latin versions of the *Elements*, especially the 'Adelard II' version, the proofs are very concise. For example, after the enunciation of *Elements* I.15, there is only the figure and the brief proof, 'Per XIII^{am}.'[5] Thus this proposition only has the enunciation, the figure, and an outline of the argument. An enunciation and at least some of the remaining six parts are always found.

In the late twelfth century and the first half of the thirteenth century, there was interest in using Euclid's works as a model for both mathematical and non-mathematical works, e.g. Jordanus de Nemore's works on a multitude of mathematical topics and Nicholas of Amiens' *De arte catholicae fidei*.[6] To

[4] Busard and Folkerts, *Robert of Chester's (?) Redaction*, vol. I, p. 143.

[5] Busard and Folkerts, *Robert of Chester's (?) Redaction*, vol. I, p. 120.

[6] Høyrup, 'Jordanus de Nemore: A Case Study'; and Evans, 'Boethian and Euclidean Axiomatic Method.'

be clear, there were other mathematical works translated into Latin that could have served as examples of this type of theoretical mathematics, e.g. Menelaus' *Sphaerica* or Theodosius' *Sphaerica*, but the *Elements* was more popular and prestigious.

The style of Ptolemy's *Almagest* is different in some important aspects. Like the *Elements*, the *Almagest* is divided into 13 books, but each of these books is divided not into lists of principles and propositions, but into chapters. The text in each chapter flows and is only broken by tables. Like the *Elements*, the *Almagest* contains formal mathematical writing, but it also includes much informal discourse on observations, natural philosophy, and metaphysics, as well as informal discourse on the relationships of the various parts of the *Almagest*. Ptolemy does not begin his work with a list of principles; instead, he argues for the importance of astronomy in the first chapter, outlines his whole book in the next, and devotes the next six chapters to arguments for cosmological principles on mathematical, observational, and metaphysical grounds. Formal mathematical writing does not appear until the ninth chapter.

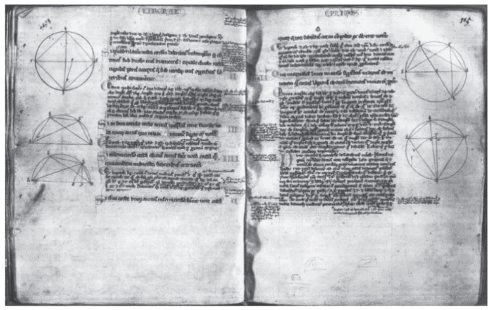
Even the formal mathematical writing in the *Almagest* is quite different from that of the *Elements*. While the *Almagest* does have theorems and proofs, which are sometimes set out quite formally, it has four other types of mathematical writing, as Nathan Sidoli has shown.[7] A 'metrical analysis' is a proof that 'provides the theoretical justification for the derivation of a numerical value given through computation as opposed to a geometric object given through construction.'[8] This type of proof is found frequently in the *Almagest*, as are computations and tables, two other types of formal mathematical writing. Ptolemy's computations are usually not just a set of arithmetical operations. Instead, they are usually closely related to metrical analyses; they often utilize a figure and give geometrical reasons for performing operations. Furthermore, the classical parts of propositions can be distinguished in them, and it is evident that Ptolemy intended his readers to generalize ways of calculating from his computations. For example, in *Almagest* I.13, Ptolemy gives computations to find the declinations of arcs of 30° and 60° of the ecliptic, and then he says that we will calculate similarly for the other arcs from 1° to 90°.[9] A last type, which Sidoli refers to as a 'description', is found less often in the *Almagest*. A description provides a mathematical model for some physical phenomena. While there are some chapters in the *Almagest* that consist wholly of non-mathematical writing or of a single type of mathematical writing, a typical chapter of the *Almagest* contains non-mathematical discourse surrounding several instances of some of these six types of mathematical writing.

[7] Sidoli, *Ptolemy's Mathematical Approach*.

[8] Sidoli, *Ptolemy's Mathematical Approach*, p. 17.

[9] *Almagest*, 1515 ed., ff. 10r-v.

Although the astronomical content is not wholly axiomatic, deductive, or general and the material is not completely amenable to a Euclidean style, the author of the *Almagesti minor* makes many kinds of changes to emphasize these characteristics. His attempt to Euclidize the *Almagest* would have been immediately apparent to medieval readers from the lists of principles at the start of each book and from the arrangement of the rest of the material into propositions with proofs. In many manuscripts this would have been apparent from the layout on the folios almost without reading a word.



Folios of Euclid's *Elements* and the *Almagesti minor* in *B* (Berlin, Staatsbibliothek Preussischer Kulturbesitz, Ms. lat. qu. 510, ff. 9v–10r and 114v–115r, with permission)

In comparison with the *Almagest*, the axiomatic nature of the *Almagesti minor* would have been very obvious to medieval readers. Ptolemy begins the *Almagest* with 8 non-mathematical chapters, but the *Almagesti minor* has only a short preface providing in a few sentences the same cosmographical principles to which Ptolemy devotes 6 entire chapters. Because the principles are given so succinctly and without argumentation, they appear more as axioms or principles of astronomy whose truth does not need to be argued (at least not in the science of astronomy). The author also writes, 'Confidence in these things is brought about so securely that if anyone unjustly finding fault should deny them, he would not unworthily be judged to be either a quibbler consciously denying the truth or a madman.'[10] In the other books, the appearance of an axiomatic science is heightened by the presence of true lists of principles, unlike the preface's flowing text. In the definitions, there are many uses of 'est', 'dicitur', and 'vocatur', which were common in Euclidean definitions. Also, the words used to refer to postulates are reminiscent of the *Elements*. The *Almagesti minor*'s author follows Adelard of Bath and 'Adelard II' in using 'petitiones' for postulates, and the *Almagesti minor*'s 'communia' is similar to the phrase

[10] Book I, preface: 'Hiis firme adeo fides conciliata est ut si quis iniuste calumpnians obviet, aut cavillator verum scienter inficians aut mente captus non indigne estimetur.'

'communes animi concepciones' that Robert of Ketton uses to refer to common notions.[11] Of course, the majority of the starting points of the arguments in the *Almagesti minor* are not among the listed principles, but the appearance of an axiomatic science is enhanced by such lists.

The deductive aspects of the astronomical content are also emphasized by the addition of more internal references to the 'mathematical toolbox', which include propositions from more elementary works such as the *Elements* and the works on spherics by Theodosius' and Menelaus, as well as the *Almagesti minor*'s principles and prior propositions. Many of the internal references are to numbered propositions (e.g. see I.6 and I.13), so it is clear that the author numbered the propositions of each book himself. The emphasis upon the parts of proofs also draws attention to deduction. Despite the Euclidean format, some inductive or observational content of the *Almagest* is retained. There are propositions devoted to instruments and their use and on the finding of astronomical parameters, e.g. I.15, III.1, IV.1–4, IV.6, V.1–2, V.4, V.11, and V.15. Other propositions such as III.3 and V.3 concern modeling phenomena with geometrical figures. The *Almagesti minor* also has passages that describe how tables are laid out and used, e.g. in V.9, V.21, VI.1, and VI.24. The reasoning is often approximative, not rigorously exact. For example, the chord of 1° is not found exactly in I.6, and V.19 involves a near proportionality, not an exact one ('… as the difference of the other distances of the epicycle from the earth's center is to the greatest difference, thus approximately is the excess of the parallax occurring because of that distance to the excess resulting from the greatest difference'). There are even enunciations that justify simplifications or that state that the objective is an approximation, e.g. V.10, V.26, VI.3, and VI.7.

The enunciations play a large role in making the content from the *Almagest* more general. The commitment of the *Almagesti minor*'s author to expressing things in a universal manner is especially apparent in lengthy and complicated enunciations such as that of I.13:

> With two arcs of great circles each less than a semicircle descending from one common point on the surface of a sphere, and with two other ⟨arcs⟩ of not smaller circles reflected from the remaining endpoints of these ⟨descending arcs⟩ into the same ⟨descending arcs⟩ by intersecting each other, each of the reflected arcs will pierce the ⟨descending⟩ arc conterminous with the other in such a way that the ratio of the chord of the arc doubling the lower part of the pierced arc to the chord of the arc doubling the upper part of the same pierced arc is produced from a twofold ratio, i.e. from that which the chord of the arc doubling the lower part of the reflected arc that is conterminous with that pierced arc has to the chord of the arc doubling the remaining part of that same reflected arc, and the ratio which the chord of the

[11] Busard and Folkerts, *Robert of Chester's (?) Redaction*, vol. I, p. 115; Busard, *The First Latin Translation of Euclid's Elements Commonly Ascribed to Adelard of Bath*, p. 32.

> arc doubling the lower part of the other descending arc has to the chord of the arc doubling that whole arc of which it is a part.[12]

Expressing this in terms of a figure, as Ptolemy does, is not only shorter, but much clearer; however, the author decided that universality was of more importance than conciseness or clarity. Generality is also emphasized in the *Almagesti minor* by the small amount of actual numbers in the text. The most evident example of this is that the *Almagest*'s many tables are not given. While the first six books of the *Almagest* have 23 tables containing approximately 10,000 values, the *Almagesti minor* has a single table with 8 values. While there are particular values in the text of the *Almagesti minor*, the number of these is almost insignificant compared to the number of values in the *Almagest*. Even a proposition such as II.25 that is about a particular angle remains on the general level. Another of the strategies that the *Almagesti minor*'s author uses to emphasize universality is the conversion of computations into metrical analyses. As was stated above, Ptolemy's computations in the *Almagest* are often very similar to full proofs and were intended to be generalized by the reader; therefore, the conversion of one of them to a metrical analysis requires very little change. However, the simple transformation raises the argument to a higher level of epistemological certainty. Another place in which the *Almagesti minor*'s author's shift from particulars to universality is especially clear is the treatment of the properties of different latitudes. In *Almagest* II.6 Ptolemy discusses 39 different latitudes, giving the degrees of latitude and the number of hours of the longest day for each; however, in *Almagesti minor* II.7–13, the author gives non-numerical properties of only the four most significant latitudes (i.e. the equator, the Tropic of Cancer, the Arctic Circle, and the pole) and the classes of latitudes between these.

There are some negative consequences of the Euclidization of the *Almagest*. Ptolemy's exposition of the high status of astronomy among the sciences and its relation to ethics is omitted, as are his arguments for the cosmological principles. The arrangement into propositions makes it more difficult to see how units of mathematical writing fit together into larger arguments. For example, *Almagesti minor* I.7–9 and 11 are lemmata for I.13–14, but the hierarchy between them is no longer apparent. Similarly, while Ptolemy has separate chapters (*Almagest* II.11–13) for the angles between the ecliptic and the meridian,

[12] I.13: 'In superficie sphere duobus arcubus magnorum orbium semicirculo divisim minoribus ab uno communi termino descendentibus aliisque duobus non minorum orbium ab illorum reliquis terminis in eosdem sese secando reflexis, utervis reflexorum alterius conterminalem arcum sic figet ut proportio corde arcus duplicantis inferiorem portionem arcus fixi ad cordam arcus duplicantis superiorem eiusdem fixi portionem producatur ex gemina proportione, ex ea videlicet quam habet corda arcus duplicantis inferiorem arcus reflexi portionem qui ipsi fixo conterminalis est ad cordam arcus duplicantis reliquam eiusdem reflexi portionem, et ea proportione quam habet corda arcus duplicantis inferiorem alterius descendentis arcus partem ad cordam duplicantis arcum ipsum cuius pars est totalem.'

angles contained between the ecliptic and the horizon, and angles contained between the ecliptic and circles of altitude, the 15 propositions of the *Almagesti minor* that correspond to these chapters (II.22-36) are numbered sequentially and are not grouped together. Another downside of Euclidization is the lack of practicality. While the *Almagesti minor* includes rules that theoretically instruct one how to perform many calculations, tables are needed for the real-world practice of astronomy. For example, without a table of chords or of sines, one would have to perform an immense amount of calculation to complete even the most basic task of determining right ascensions.

The attempt of the *Almagesti minor*'s author to strike some balance between practical and theoretical aspects of astronomy can be seen in his attitude towards calculation. While many of the *Almagest*'s computations are turned into metrical analyses and there is much less actual calculation with numbers reported in the *Almagesti minor*, there is still much discussion of calculation on a general level. The metrical analyses are proofs of the validity of certain arithmetical processes for calculating values of arcs and times. This is sometimes very clear, e.g. in I.16 and I.17. In many of the metrical analyses, e.g. II.1, the final logical steps of the argument, e.g. from a proportion to an algorithm such as the 'rule of three', are left implicit, but it is still clear that the proofs are about calculation. Thus, while a proposition's enunciation often only expresses that a quantity can be found or is known, the author follows the enunciation with a corollary expressing a rule of calculation set forth in general terms. Therefore, it is abundantly clear that the arithmetical rules for finding the values of certain astronomical distances or times are a result (even if expressed as a secondary goal) of the proposition. The *Almagesti minor*'s author did not create this type of rule – such rules are common in astronomical canons, but the inclusion of them in a work of theoretical astronomy does appear to be a true innovation.

There were earlier medieval astronomical works that show some Euclidean features. In the *Liber super Almagesti*, Geber criticizes Ptolemy for mixing practical and theoretical matters. Geber has some lists of definitions and some formal propositions, and he also stays on the general level for almost his entire work, only rarely mentioning any specific values.[13] The Dresden *Almagest*, which only survives in one incomplete copy, was a twelfth-century Latin translation of the Arabic *Almagest* that incorporates some added enunciations and references to the mathematical toolbox.[14] There are no indications that the *Almagesti minor*'s author knew either of these works. Furthermore, neither the Dresden *Almagest* nor the *Liber super Almagesti* are nearly as Euclidized as the *Almagesti minor*. In the centuries following its composition, several astronomers followed the lead of the *Almagesti minor*'s author, as will be seen below in the chapter on the *Almagesti minor*'s influence.

[13] Geber, *Liber super Almagesti*, Nuremberg: Johannes Petreius, 1534.

[14] *D*, ff. 1r–71r. Grupe, *The Latin Reception of Arabic Astronomy*.

Chapter 3

Sources

As the title suggests, the *Almagesti minor*'s main source is the *Almagest*. This is readily apparent both from the fact that each book follows the content of the *Almagest* with only minor deviations from Ptolemy's order of presentation and from the more than 100 references to Ptolemy by name. Lorch attempted to find the version of the *Almagest* that was used by comparing a handful of passages of the *Almagesti minor* with the Sicilian translation of the *Almagest*, the Dresden *Almagest*, and both Gerard of Cremona's first and revised translations.[1] He did not find a clear connection between the *Almagesti minor* and any of the known Latin translations. Of the passages of the *Almagesti minor* that he selected for his comparisons (from the preface I.6, I.9, I.15 II.3, and V.1), he was only able to find a couple of instances of 'striking words common to Gerard and the *Almagestum parvum*' in a single passage, a result that he considered 'a poor harvest from such a long passage.' On the other hand, he found that the manner in which the figures are labeled matches that in Gerard's translation.[2] He concluded, 'In general, if one of the Gerard texts is the basis of the *Almagestum parvum*, the compiler must have been at some pains to change the terminology as much as possible. The alternative is another source.'[3] He also writes, 'We are left with the conclusion that the compiler either had some other access to the *Almagest* in addition to Gerard or deliberately and radically altered the wording, perhaps with the intention of simplifying or modernizing it.'[4]

After comparing each proposition to Gerard's translation of the *Almagest*, I have been able to establish that the *Almagesti minor*'s author did indeed use this version of the *Almagest*. Lorch appears to have been correct when he suggested that the author may have obscured his use of Gerard's translation. In most of the work, the debt to Gerard is not obvious; however, there are some passages in which the dependency is undeniable. For example, compare the proof of *Almagesti minor* II.24 with the corresponding passages, *Almagest* II.10, in the Latin translations of Ptolemy's work (I have italicized the most conspicuous parallels):

[1] Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 423–30.
[2] Lorch, 'Some Remarks on the *Almagestum parvum*', p. 408.
[3] Lorch, 'Some Remarks on the *Almagestum parvum*', p. 430.
[4] Lorch, 'Some Remarks on the *Almagestum parvum*', p. 431.

Dresden Almagest[5]	Translation from Greek[6]	Gerard's Translation of the *Almagest A-Klasse*[7]	*B-Klasse Variants*[8]	*Almagesti minor*
Angulus autem qui fit in duobus punctis duarum conversionum sectione circuli signorum et circuli meridiei est rectus.	His preconsideratis	Et post scientiam eorum que premisimus,		
Sit namque circulus meridiei qui transit super quatuor polos ABCD et medietas circuli signorum AEC,	esto meridianus quidem circulus ABGD. Eius autem qui per media animalia semicirculus AEG,	describam circulum orbis meridiei, supra quem sint A, B, G, D, et medietatem circuli orbis signorum, supra quam sint A, E, G.		Sit denuo circulus meridianus ABGD et medietas circuli signorum AEG.
sitque punctus A punctus conversionis yemis.	puncto A subiacente hiberno tropico.	*Et sit punctum* ipsum *A tropicum hiemale*,	ipsum A] A ipsum	*Et sit punctum A tropicum hiemale*
Dico quia angulus DAE est rectus.				
Racio: faciemus enim punctum A polum et circulabimus longinquitate lateris quadranguli medietatem circuli DEB,	Atque polo A spacio vero tetragoni latere scribatur ABD semicirculus.	*et describam supra polum A secundum spacium lateris quadrati medietatem circuli*, supra quam sint B, E, D.		*et describam super polum A secundum spatium lateris quadrati medietatem circuli* BED.
tunc circulus ABCD transiet super polum circuli DEB et super polum	Quoniam ergo ABGD meridianus et per eius qui est ABG polos et	Et quia orbis meridiei, qui est *ABGD*, *est descriptus supra* duos *polos AEG*	supra] *super*	Quia ergo circulus meridianus *ABGD est descriptus super* utriusque circuli

[5] Grupe, *The Latin Reception of Arabic Astronomy*, pp. 320–21. I have capitalized diagram letters.

[6] Florence, BNC, Conv. Soppr. A.V.2654, f. 11v.

[7] Paris, BnF, lat. 14738, f. 29v.

[8] Vatican, BAV, Vat. lat. 2057, f. 27v–28r. Because Classes A and B are so close, I only note non-orthographical variants.

⟨circuli⟩ AEC. Igitur arcus DE est quadrans circuli et cordat angulum DAC, igitur angulus DAC est rectus	per eius qui est BED scriptus est, tetartimorii ED perifieria. Rectus est ergo DAE angulus.	*BED*, *erit arcus ED quarta circuli. Angulus ergo DAE erit rectus.*		*AEG BED polos*, *erit arcus ED quarta circuli. Angulus ergo DAE erit rectus.*
et est qui est in puncto conversionis estatis demonstracione none figure huius sermonis. Igitur angulus qui est in puncto conversionis est rectus	Rectus autem per premonstrata et sub estivo tropico puncto factus,	*Et propter* hoc cuius iam precessit declaratio, erit etiam *angulus qui est apud tropicum estivum rectus*,		*Et propter* idem est *angulus qui aput tropicum estivum rectus*,
et hoc est quod demonstrare voluimus.	quod oportebat ostendere.	et illud est quod oportuit nos declarare.		et hoc est quod oportuit demonstrari.

Another example is the second part of *Almagesti minor* II.33 and its source in *Almagest* II.12:

Dresden Almagest[9]	Translation from Greek[10]	Gerard's Translation of the *Almagest*[11]	B-Klasse[12] (variants only)	*Almagesti minor*
Rursus figurabimus quasi illam figuram precedentem	Adiaceat rursum descripcio similis	Describam quoque similem huius forme.		
facientem dumtaxat punctum A in medio celi in illorum duorum temporum uno – dico – cum fuerit prescitus punctus circuli signorum orien-	ita tamen ut orientalis quidem porcionis medium celi tenens punctus, hoc est A, australior sit G puncto qui ad verticem.	Et *sit* punctum *A portionis orientalis in medio celi in parte meridiana a puncto G* supra summitatem capitum,		*Sit* rursum *A portionis orientalis in medio celi in parte meridiana a puncto G*,

9 Grupe, *The Latin Reception of Arabic Astronomy*, p. 329.

10 Florence, BNC, Conv. Soppr. A.V.2654, ff. 13r-v.

11 Paris, BnF, lat. 14738, f. 32r.

12 Vatican, BAV, Vat. lat. 2057, f. 30r. Again, because Classes A and B are so close, I only note variants, and orthographical variants are ignored.

talis australem puncto C super capita,				
et punctum B in medio celi in alio tempore, quando fuerit prescitus punctus circuli signorum occidentalis, septentrionalem puncto C.	Eius vero que ad occidentem porcionis qui celi medium tenet, hoc est B, borealior sit eodem.	*et* sit *punctum B portionis occidentalis* que est in medio celi a *parte septentrionali* puncti G.		*et punctum B portionis occidentalis* in *parte septentrionali*.
Dico quia duo anguli CEF LGB [in]sim[ul] sunt equales duobus rectos quasi duplo anguli DEF.	Dico quoniam ambo simul anguli GEZ et LIB duobus DEZ maiores sunt duobus rectis.	Dico ergo quod ambo anguli qui sunt ex GEZ et LHB sunt maiores duplo anguli DEZ secundum duos angulos rectos.	secundum-rectos] per quantitatem duorum angulorum rectorum	Dico quod similiter accidit.
Racio: monstrabimus enim ut monstravimus in tribus figuris huic prepositis				
quia duo anguli DGC DEC sunt equales, et duo anguli DGC DGL insimul sunt equales duobus rectis.	Quoniam enim angulus quidem DIG equalis est angulo DEG, ambo autem simul DIG et DIL duobus rectis equales sunt,	*Angulus namque DHG equatur angulo DEG. Duo vero anguli DHG et DHL equantur duobus angulis rectis*.		*Angulus namque DHG equatur angulo DEG. Duo vero anguli DHG et DHL equantur duobus angulis rectis*;
Igitur duo anguli DEC GDL sunt equales duobus rectis angulis,	et ambo igitur simul DEG et DIL anguli duobus rectis sunt equales.	Ergo duo anguli DEG et DHL simul equantur duobus rectis.	duobus rectis] rectis duobus	
DEF est angulus DGB,	Sit autem et DEZ angulus idem angulo DIB.	*Angulus autem DEZ est equalis angulo DHB*.		*angulus autem DEZ est equalis angulo DHB*.
ergo duo anguli CEF LGB	Quare et ambo simul GEZ et	*Quapropter erunt duo*	DEG] *DEZ*	*Quapropter erunt duo*

insimul sunt plus quam duo anguli DEC DGL	LIB anguli duobus simul DEZ et DIB angulis	*anguli GEZ et LHB* maiores duobus angulis DEG *et DHB*,		*anguli GEZ et LHB* superantes duos angulos *DEZ et DHB*
	hoc est bis eo qui est DEZ maiores sunt ambobus simul DEG et DIL angulis	scilicet maiores duplo anguli DEZ secundum duos angulos *DEG et DHL*,	secundum-angulos] per *quantitat*em *duorum angulorum*	aut duplum unius eorum *quantitat*e *duorum angulorum DEG et DHL*,
quasi duo anguli DEF.	qui sunt duobus rectis equales.	*qui sunt equales duobus angulis rectis*,		*qui sunt equales duobus rectis*,
Monstravimus vero quia duo anguli DEC DGL sunt equales duobus rectis angulis, igitur duo anguli CEF LGB sunt maius quam duo recti anguli quasi duplo anguli DEF				
et hoc est quod demonstrare voluimus.	Quod oportet ostendere.	et illud est *quod oportuit demonstrar*e.		*quod oportuit demonstrar*i.

The dependency upon Gerard's translation is apparent and does not require a phrase-by-phrase explanation. While in these passages, the *Almagesti minor* is slightly closer to the B-Klasse of Gerard's translation, the differences between Gerard's classes are not significant enough to determine which was used by the author of the *Almagesti minor*. I argued earlier that the author used one of the members of a group of *Almagest* manuscripts that have the same numeral pattern as the *Almagesti minor* and omit or misplace the last paragraph of III.4, but these are in the A-Klasse. Also, *Almagesti minor* I.15 has the two words 'tornatiles piramidales' where Gerard's A-Klasse has only 'piramidales' and his B-Klasse has only 'tornatiles.' The issue of which version of Gerard's translation was used is thus not a simple matter. The evidence suggests that the *Almagesti minor* could possibly depend upon both the A-Klasse and the B-Klasse. This could be explained if a lost member of the group of *Almagest* manuscripts that represent numbers in the same manner as the *Almagesti minor* also bore readings from the B-Klasse or if the author of the *Almagesti minor* used a manuscript from each class. Complicating matters, some manuscripts of Gerard's translation, e.g. Venice, Biblioteca Nazionale Marciana, lat. VIII.10 (3266) and *Me*, contain a mixture of the two versions, but these are not obviously closer to the *Almagesti minor* than either class. The issue should become clearer when an edition of Gerard's translation has been completed.

Although there are only a small number of passages that show as close of a connection as the examples above, the author's use of Gerard's translation is seen in propositions of all six books. Some propositions have only a few words that show that the author was consulting the *Almagest* as he wrote. For example, *Almagesti minor* I.4 has the phrase 'AD facta communi' which is very similar to 'facta AD communi' from Gerard's translation, and *Almagesti minor* I.5's way of referring to an arc's supplement, 'residui arcus de semicirculo', is very close to the *Almagest*'s 'arcus residui semicirculi.'[13] In isolation, such slight commonalities could be attributed to coincidence, but the proven connection of *Almagesti minor* II.24 and II.33 with Gerard's translation makes it much more certain that the author of the *Almagesti minor* was consulting Gerard's translation also for these early propositions.

Returning to the comparison of the passages of *Almagesti minor* II.24 and II.33 to Gerard's translation, we see that while the author of the *Almagesti minor* copies some passages of Gerard's translation verbatim, he still deviates frequently from his source. Some of the changes were probably made by mistake. For example, there is no passage in *Almagesti minor* II.33 paralleling the *Almagest*'s 'Ergo duo anguli DEG et DHL simul equantur duobus rectis.' Other changes appear to have been made for the sake of brevity or clarity; however, some changes do not have perceivable reasons. *Almagesti minor* II.24's proof starts with 'Sit denuo circulus meridianus' instead of Gerard's 'describam circulum orbis meridiei.' There is no obvious reason for the change from 'describam' to 'sit', especially considering the fact that the *Almagesti minor*'s author starts proofs several times with 'describam' (i.e. in II.31, II.33, III.5, III.7, IV.8, IV.9, V.9, and V.22). Similarly, further in *Almagesti minor* II.24, the author writes 'circulus meridianus' in place of 'orbis meridiei' in Gerard's translation, but he shows no reluctance to use 'orbis meridiei' elsewhere (i.e. in *Almagesti minor* II.31–32 and II.35). In the *Almagesti minor*, it is clear that many propositions relied upon Gerard's translation, but there are only a relatively small number of propositions in which much wording is retained from Gerard's translation. This suggests that the author intentionally reworded the material from his source even when there was no need to do so for the sake of simplicity, conciseness, or clarity.

[13] *Almagest* I.9 (1515 ed., f. 6r).

The author does much the same thing with his second most used source, *De scientia astrorum*, which is Plato of Tivoli's translation of al-Battānī's *Zij*, written in Syria *c.* 900.[14] This translation exists in at least nineteen manuscripts and was printed in 1537 and 1645.[15] In the section on the *Almagesti minor*'s date of composition above, we have seen how one passage depends upon Plato's translation. In that same proposition, III.1, there are other passages that are very similar to ones in the source. For example, the *Almagesti minor*'s 'quia tunc aer purior est' is very close to *De scientia astrorum* Ch. 27's 'eo quod tunc aer est clarior et purior.'[16] For a longer comparison of parallel passages that clearly show the *Almagesti minor*'s reliance upon Plato's translation, compare the following passages:

De scientia astrorum Ch. 39	*Almagesti minor* V.28
51r: In diversitate autem *aspectus Lunae in latitudine*,	Diversitatem *aspectus Lune in latitudine* predicto modo colligere.
si Luna in meridionali parte *a* puncto *zenith capitum* fuit, *cum Lunae* pars *in coeli medio* fuerit, *diversitas aspectus Lunae erit* in parte *meridiana*.	Et *si cum Lune* gradus *in medio celi* erit, Luna *a cenit capitum* meridiana fuerit, *diversitas aspectus Lune* – in latitudine dicetur – et *erit meridiana*.
Si autem Lunae locus in circulo medii coeli *versus* septentrionalem a puncto zenith capitis fuerit, *diversitas aspectus* Lunae *in latitudine erit* in parte septentrionali,	Et *si versus* septemptrionem, *diversitas aspectus – in latitudine* dicetur – et *erit* septemtrionalis.
et semper fere erit meridiana in regione eius[17] *latitudo maior* fuerit *declinatione Solis et latitudine Lunae* septentrionali.	*Et fere semper erit meridiana in* hiis climatibus quorum *latitudo maior* est maxima *declinatione Solis et Lune latitudine*.
Cumque vera Lunae latitudo, *et diversitas aspectus* Lunae *in eadem parte* fuerint, *eas in unum collige*. *Si vero diversae fuerint*, *minorem de maiori deme*, residuique partem addisce, *et quod post augmentum vel diminutionem fuerit erit Lunae latitudo* per instrumentum visa.	*Cumque vera* visi loci *Lune* in longitudine *latitudo et* hec *diversitas aspectus in ea*ndem *parte*m fuerit, *eas in unum collige*. *Si vero diverse fuerint*, *minorem de maiori deme*. *Et quod post augmentum vel diminutionem fuerit erit latitudo Lune* visa, quam propter solares eclipses querimus.

[14] An edition of the Arabic and a Latin translation are found in Nallino, *al-Battānī*. Plato's translation of the *Zij*, which has the incipit 'Inter universa liberalium artium studia…', is known by a number of titles, but following other modern scholars, I refer to it as '*De scientia astrorum*.'

[15] Carmody, *Arabic Astronomical and Astrological Sciences*, pp. 129–30, lists only eight manuscripts, but David Juste informs me that he knows nineteen.

[16] Albategni, *De scientia astrorum*, 1537 ed., f. 26v.

[17] Surely a scribal error for 'cuius.'

De scientia astrorum Ch. 44	*Almagesti minor* VI.21
68v: … *si* minuta *quae* sunt *inter Solem* et Lunam minus [5] *minutis casus* initii indefiniti fuerint, *a Luna Solem ante* tempus initii *indefinit*i *occultari non* dubites.	2nd paragraph: Quod *si* quantitas *que* tunc erit inter Solem et visum locum Lune minor fuerit ipsis *minutis casus*, *a Luna Solem ante* principium *indefinit*um *occultari non* est dubitatio.
… et *si minuta quae inter Solem et Lunam fuerint plura minutis casus* extiterint *ad locum*, in quod *aliquid Solis occultari possit*, *Lunam nondum pervenisse* cognoscas.	… Quod *si minuta que* sunt *inter Solem et* visum locum Lune *fuerint plura* definitis *minutis casus*, *ad locum in* quo *aliquid Solis occultari possit nondum Lunam pervenisse* certum est.
69r: … *Lunam praeteriisse locum in quo* Solem *occultare debuit* non ignores.	…constat *Lunam preteriisse locum in quo* primo nichil de Sole *occultare debuit*.
… et *si* minuta *quae tunc inter Solem et* Lunam fuerint minus *minutis casus* extiterint, *Lunam nondum pervenisse ad locum in quo sic a Sole separatur*, *quod* eum *occultare* non *possit*, nullatenus ambigas.	… Quod *si* quantitas *que tunc* est *inter Solem et* visum locum Lune minor est definitis *minutis casus*, *Lunam nondum pervenisse ad locum in quo sic a Sole separatur quod* nichil eius *occultare possit* manifestum est.
71v: *Post hoc superflu*um quod *inter aspectus* temporis *medii diversitatem et diversitatem* uniuscuiusque *duorum temporum fuerit addiscens*, *eorum unumquodque per Lunae* superationem *partire*. *Et quod exierit erunt partes horae*.	4th paragraph: *Post hec superflu*a que *inter diversitatem aspectus medii* eclipsis *et diversitatem* utriusque *duorum temporum fuerint addiscens*, *eorum unumquodque per Lune* veram superlationem ad horam *partire*. *Et quod* utrinque *exierit erunt partes hore*.
Horas ergo casus superius *inventas in duobus locis* scribe, et *alteri locorum alteram partem divisionum ex superfluo diversitat*is *inventam superadde*, *alteri* vero *locorum alteram* divisioni partem superadiunge.	*Horas igitur casus* indefiniti absolute *inventas in duobus locis* servans, *alteri locorum alteram partem divisionum ex superfluo diversitat*um *inventam superadde*, et *alteri locorum alteram*.
De hinc istarum horarum casus post augmentum	Cum ergo horas casus sic equatas in duobus locis habueris,
maiorem partem accipiens, eam ex horis mediae eclipsis minue, si medietas eclipsis versus occidentem fuerit, quod esse non dubites, cum *longitudo mediae eclipisis ab ascendente plus 90 fuerit*, minorem vero partem horarum casus post augmentum horis mediae eclipsis superadde,	[*This section is located after the sign *** below*] Quod *si longitudo medie eclipsis ab ascendente plus xc* gradibus *fuerit*, conversam facies, scilicet quod maius est a tempore medie eclipsis demes et quod minus est addes
acsi versus orientalem partem eclipsis fuerit, quod cum *longitudo mediae eclipsis ab ascendente minus 90 fuerit*, evenire manifestum est, minorem illarum duarum partium ex horis *mediae eclipsis deme*, maiorem vero partem horis mediae eclipsis superadde.	eas que minus sunt tempori *medie eclipsis deme* et eas que plus temporis sunt super medium eclipsis adde. Ita dico si *longitudo medie eclipsis ab ascendente minus xc* gradibus *fuerit*. ***

Hoc autem ideo quia *duorum terminorum longior semper iuxta medium coeli debet esse.*	propter hoc scilicet quod *duorum terminorum longior iuxta medium celi semper esse debet*.

Identical wording has been italicized. The likenesses extend much further in these passages, and the dependence cannot be denied. Again, as with passages derived from Gerard's translation of the *Almagest*, some of the changes can be explained by a desire for simplicity or clarity. For example, in excerpts from *Almagesti minor* VI.21, we see that many words and phrases are taken directly from the corresponding sentences of *De scientia astrorum* Ch. 44, and that a similar sentence structure is used. But, it appears that our author has purposely changed some of the wording, e.g. 'quantitas' for 'minuta', 'visum locum Lune' for 'Lunam', 'principium indefinitum' for 'tempus initii indefiniti', and 'est dubitatio' rather than 'dubites.' Some of these changes may have been done to make subtle changes to the meaning. For example, our author may have preferred 'quantitas' over 'minuta' because he was more concerned with the actual arc and not the measurement of that arc. Other changes, however, appear to have been made for stylistic reasons, or perhaps the author simply wanted to produce his own text largely in his own words even if he was relying closely upon a work open before him. For example, there is not much of a difference in the meaning of 'est dubitatio' and 'dubites', but our author chose to make a change in wording. This sort of alteration of a text for no apparent reason other than producing one own's text is seen elsewhere in medieval astronomy. For example, a large percentage of Richard of Wallingford's *Quadripartitum* paraphrases his sources, but he changes almost all of the wording from his sources without changing the meaning.[18]

The *Almagesti minor* refers to some of al-Battānī's tables; however, it appears that the author did not know these tables as part of *De scientia astrorum*. The surviving manuscripts of *De scientia astrorum* do not include tables.[19] The most convincing evidence that the *Almagesti minor*'s author did not have a copy of *De scientia astrorum* that included the tables is that in VI.3 he talks about one of al-Battānī's tables, but instead of attributing it to its maker, he writes that it is among the Toledan Tables. If, as appears most likely, the author of the *Almagesti minor* did not have al-Battānī's tables collected together, the instances in which al-Battānī's tables are described or mentioned must be explained. Some of these instances could be due simply to the *Almagesti minor*'s author's use of the text of *De scientia astrorum*. Thus, when he writes in IV.16 that Albategni '… ita in

[18] Zepeda, *The Medieval Latin Transmission*, pp. 260–67.

[19] Nallino, *al-Battānī*, vol. I, p. lv, states that Plato omitted the tables in his translation, but Nallino also concludes here from Plato's inclusion of solar, lunar, and planetary positions at the end of his translation of Savasorda's *Liber embadorum*, that Plato did in fact know and use Albategni's tables.

tabulis scripsit', the author may merely be paraphrasing *De scientia astrorum* Ch. 30's '... quodque remansit in tabulis scripsimus.'[20] In other instances, the author could have found tables of al-Battānī included among the Toledan Tables that matched descriptions in *De scientia astrorum*. For example, in *Almagesti minor* V.21, the author's description of parallax tables goes beyond what one could learn from reading Albategni's own description of them; however, from Albategni's text, the *Almagesti minor*'s author may have recognized the relevant tables in the Toledan Tables, and then based his own description of the tables upon both Albategni's text and his own first hand experience with the tables.

As stated above, the *Almagesti minor*'s author seems to have known tables of al-Battānī from the Toledan Tables, and his knowledge of the Toledan Tables is confirmed by explicit citations in *Almagesti minor* III.1, IV.14, and VI.3. The first of these ('... et super hoc Arzacel tabulas motuum Toleti novissime composuit') makes it clear that our author considered Arzachel (i.e. al-Zarqālī) to be the author of the tables, which was a common supposition at the time, and thus further references to Arzachel in III.15 and III.11 can be understood to refer to the Toledan Tables. Euclid's *Elements* is another source of the *Almagesti minor*. Many references explicitly mention the name Euclid, e.g. in *Almagesti minor* I.1, I.2, I.6, I.12, II.21, III.8. Because the *Elements* were so well known, a few references do not even include the name Euclid or the title of his most famous work. For example, in I.6 there are references merely to 'per terciam sexti et ultimam eiusdem' and in I.4 we find the justification, 'per heleufugam', which is a name for *Elements* I.5. The author also refers to another work of pure mathematics, Theodosius' *Sphaerica*, which was translated into Latin by Gerard of Cremona.[21] There are references to Thebit in *Almagesti minor* I.15 and III.1. In the first of these, the author probably uses *De motu octave spere*, and in the latter, the references appear to be to *De anno solis* and again to the *De motu octave spere*; however, there are serious doubts about whether these two works attributed to Thebit in the Middle Ages were indeed composed by him.[22]

Other astronomers who are mentioned in the work are known second-hand. For example, Hipparchus (called 'Abrachis', as was common in medieval texts) is mentioned in III.1 and other places, but the information about him comes from Ptolemy and Albategni. Similarly, Theon of Alexandria is mentioned in V.21, but the *Almagesti minor*'s author's source is *De scientia astrorum*. There

[20] Albategni, *De scientia stellarum*, 1537 ed., f. 35r.

[21] Kunitzsch and Lorch, *Theodosius*, *Sphaerica*.

[22] An edition of *De anno solis* and editions of two versions of *De motu octave spere* are found in Carmody, *The Astronomical Works of Thabit b. Qurra*. This book has errors and is arranged very confusingly, so I use the edition of *De motu octave spere* found in Millás Vallicrosa, *Estudios sobre Azarquiel*, pp. 496–509. Concerning the doubts that either of these two works are by Thābit, see Morelon, *Thābit ibn Qurra: Œuvres d'astronomie*, pp. xix and lii–liii.

are other sources not explicitly cited that appear to have been used, because of the similarity of content or of wording. These include Martianus Capella's *De nuptiis Philologiae et Mercurii* (see commentary on the Preface below), Raymond of Marseilles' *Liber cursuum planetarum* (see commentary on III.1 and III.11), and the canons to the Toledan Tables (see commentary on V.18).

Chapter 4

Major Changes in Content from the *Almagest*

As stated earlier, the Euclidean style of the *Almagesti minor* is a major change from the *Almagest*. There are additionally a number of changes in content, some related to the style change, some unrelated. For the ease of finding the innovations, mistakes, and deviations from the *Almagest*, an overview of the more significant changes in content, as well as major rearrangements and omissions, is provided here.

The following parts of *Almagest* I–VI are omitted or modified to such an extent that the correspondence is faint (most small omitted passages are not noted):

Book and Chapter of the *Almagest*	Folios in 1515 ed.	Content
I.1–8	1r–5r	preface, outline of the book, and arguments for the cosmological principles
I.9's 1st section	5r-v	transition and outline
I.10–11	6v–8v	discussion of table of chords, and the table itself
I.12's last part	10v	table of declinations
II, 1st part and Ch. 1	11v–12r	list of chapters, a transition between books, and a general discussion of longitude and latitude
II.4	12v–13r	at which latitudes the sun can be directly overhead and how often and when this will occur
II.8	17v–18v	tables of oblique ascensions
II.13	22r–26r	tables of arcs of altitude and angles contained by circle of altitude and ecliptic, and discussion concerning the tables and latitude and longitude of places on earth
III's 1st part	26r	list of chapters
III.2	28v–29r	tables of the sun's mean motion
III.4's last paragraph	32r	size of the greatest solar anomaly according to epicyclic model
III.7	33v	table of sun's anomaly
Addition after III and IV's 1st part	35r-v	tables concerning eras and list of chapters
IV.4	37v–39v	tables of moon's mean motions
IV.5's 1st section	40r	discussion of 1st and 2nd lunar anomalies and outline of Ptolemy's choice to first ignore the 2nd

IV.9's 1st paragraph	44r	introduction to the correction of moon's mean motion in latitude, discussion of problems with Hipparchus' methods, and comment on methodology
Most of IV.9's last paragraph and IV.10	45r	table of the moon's first anomaly and discussion of this table
IV.11	45r–46v	problems with Hipparchus' calculations of lunar anomaly
V's 1st part	46v	list of chapters
V.8's table	51v	table of moon's complete anomaly
End of V.10 and V.11	53r	transition and introduction to section on parallax
V.18	58r	parallax table
Section of V.19	59r	discussion of mistakes made by Hipparchus
VI's 1st section and VI.1	60v	list of chapters, introduction to topic of mean conjunctions and oppositions
VI.3's table	61v–62v	tables of mean conjunctions and oppositions
VI.7's 1st paragraph	66r-v	description of eclipse tables
VI.8	68v–69v	eclipse tables
VI.9, 2nd half	69v–70r	discussion of Hipparchus' mistakes in determining the moon's mean motion of latitude
VI.12	72r	table of inclinations of eclipses
VI.13	72v	table of parts of horizon to which eclipses are inclined

The following is a list of propositions of the *Almagesti minor* that have significant deviations from Gerard's translation of the *Almagest*.

I.9: In this proposition, the author first uses sines while Ptolemy never uses them. Both sines and chords of double arcs are used in later propositions, but after I.16 the *Almagesti minor* generally uses sines.

I.14: The author provides a proof of the conjunct Menelaus Theorem while Ptolemy only states the conclusion without offering a proof.

I.15: The content of the proposition, about the use of instruments to determine the ecliptic's maximum declination, is placed before the Menelaus Theorem and its lemmata in the *Almagest*. Also, the author includes parameters from other astronomers.

I.16: In this proposition, as well as in I.17, II.18, and II.30, the author deals with compound ratios differently than Ptolemy does in the corresponding passages of the *Almagest*. This proposition is also the first of many to have a corollary offering a rule of calculation.

II.1–3: The order of these first proofs of Book II does not match that of the corresponding proofs in the *Almagest*.

II.6: The corollary relies on rules of Albategni regarding gnomon shadows, the proof aims at demonstrating the validity of these rules, and parts of the proof are given in much more detail than in the *Almagest*.

II.9–12: The manner of grouping climes into classes is similar to Albategni's and differs from Ptolemy's treatment of the climes.

III.1: This proposition leaves out much of Ptolemy's discussion of the investigation of the length of the year. It also reports values of Albategni, Thebit, and the Toledan Tables, and it presents the theory of trepidation and the hypothesis that the year is not a constant amount of time.

III.11: The author adds parameters of Albategni and Arzachel for the sun's eccentricity and apogee. In this and many other propositions, the author solves right triangles by making a circle whose radius is the triangle's hypotenuse, while Ptolemy solves them by making the hypotenuse a diameter. The *Almagesti minor*'s method is better suited for sine tables, while the *Almagest*'s method is more amenable to chord tables.

III.13: This proposition in the *Almagesti minor* has some errors.

III.17: This is the first proposition that is primarily derived from Albategni's *De scientia astrorum* although there are some differences from Albategni's method.

III.19–25: This group of propositions on the equation of time is much more detailed than the *Almagest*'s corresponding section, and the author contradicts the *Almagest* at points.

IV.1: The author provides a figure and uses it to explain some of Ptolemy's statements regarding the problems caused by the moon's proximity to earth.

IV.3: This includes an extra proof concerning the return of the moon's irregularity.

IV.5–6: Regarding the choice of eclipses that will lead to good values for the moon's diversity, the author separates what is intermingled in the *Almagest*, and he adds more detailed explanation, using an additional geometrical figure in the first of these two propositions.

IV.13: The author reports Albategni's value for the size of the moon's epicycle.

IV.14: The last paragraph is on Albategni and the Toledan Tables' values for the moon's mean motion of diversity and mean motion of longitude.

IV.16: This proposition includes a paragraph on Albategni's values for the moon's mean motion of latitude.

IV.19: Unlike Ptolemy, the author provides a separate discussion of the motion of the nodes.

V.6: This proposition, which is about the greatest apparent quantities of the moon's second irregularity for any location on the eccentric, gives material that Ptolemy provides much later. This would be V.10 if the author followed Ptolemy's order. The proposition's proof also utilizes a different case than Ptolemy does.

V.8: Unlike Ptolemy but like Albategni, the author separates the argument for finding the equation of portion from that of finding the moon's true position.

V.9: This proposition is largely composed of rules for calculating the moon's true place that are not in the *Almagest*. These rules are at least partially based on ones of Albategni. It also describes a table for finding the moon's true position that is not among the *Almagest*'s tables.

V.10: The author follows Albategni in arguing that the equation of portion during conjunctions and oppositions cannot be ignored, as Ptolemy claims.

V.11: The description of the parallactic instrument or triquetrum is closer to Albategni's than to Ptolemy's.

V.12: Perhaps using Albategni's *De scientia astrorum*, the author is clearer about requirements for a certain observation for finding the moon's parallax than Ptolemy is.

V.14: The author gives an outline of what appears to be an original proof for finding the distance to the moon wherever it is on the epicycle and eccentric. Also, he deviates from Ptolemy's order of presentation by giving here the distance of the moon at the four 'termini.'

V.18: This proposition has the author's own short paragraph on the volume of spheres. He also uses Albategni's recalculation of the relative sizes of the earth, moon, and sun and their distances using Albategni's values for the apparent sizes of the moon and sun. He also provides rules for finding the sun and moon's apparent diameters from their hourly motions that perhaps come from the canons to the Toledan Tables.

V.19: This proposition has many rules for the calculation of parallax that are not from the *Almagest*. Some of these are taken from Albategni, but some are the author's original work, as is part of the explanation of the table of parallax.

V.20: The author begins with his own paragraph on the difference of the moon and sun's parallax. He also provides Albategni's method of calculating the solar parallax from Ptolemy's tables but in accordance with Albategni's parameters.

V.21: The author appears to have created his own geometrical proofs for finding the latitude in longitude and in latitude. He has a paragraph on Theon's parallax tables, probably taken from Albategni and the Toledan Tables.

V.22: In this proposition on the moon's parallax when it is not on the ecliptic, the author provides a proof for a second case that Ptolemy does not address.

V.25: The author makes several mistakes in presenting a rule of Ptolemy's regarding the moon's parallax when it has latitude.

V.26: The author takes this content about the negligible difference between the moon's motion on the declined circle and the ecliptic from *Almagest* VI.7 and places it much earlier.

V.27–28: These propositions about the moon's apparent longitude and latitude rely upon Albategni and only loosely correspond to passages in the *Almagest*. This content is given earlier in the *Almagest*, not at the end of Book V.

VI.1: The author offers his own additional instructions regarding tables of mean syzygies, and he also describes tables from the Toledan Tables and provides his own instructions for these.

VI.2: The author provides an additional way of finding the sun or moon's hourly motion. He also misinterprets one of Ptolemy's rules and gives instructions for an Albategnian table.

VI.3: The author gives a method for finding the time and place of a true conjunction that he wrongly attributes to Ptolemy. He also devotes multiple paragraphs to Albategni's methods for finding the same with some added explanation of his own, and he discusses a table from the Toledan tables.

VI.4–5: Unlike in the *Almagest*, lunar eclipse limits are treated before solar ones and geometrical explanations are added. It appears that the author also calculated eclipse limits using Albategnian parameters. The author also improves Ptolemy's method for determining solar eclipse limits.

VI.6: This proposition adds a geometrical figure that it uses to show that eclipses can be repeated in the sixth month.

VI.7: This proposition on the apparent size of the moon and the earth's shadow only corresponds loosely to a passage in the *Almagest* that is placed much later. The author here remains closer to Albategni, but he explains in more detail than this source.

VI.8–11: Unlike Ptolemy, the author puts this content in terms of a geometrical figure, and he also performs many calculations using Albategni's parameters. In VI.10–11, the author uses a non-Ptolemaic value for the apogee (probably taken from the Toledan tables).

VI.13: This proposition on the digits of a lunar eclipse has no corresponding passage in the *Almagest* and only a very loose parallel in *De scientia astrorum*.

VI.14–23: The order in which the author discusses matters does not follow that of the *Almagest*.

VI.14: Much of this proposition regarding the minutes of immersion and delay in a lunar eclipse corresponds to rules given by Albategni. As in *De scientia astrorum*, the slant of the moon's transit during an eclipse is factored in when calculating minutes of immersion and delay, but unlike this source, this proposition includes geometric proofs. It also separates the investigations of distances and times, which are intertwined in *De scientia astrorum*.

VI.15: This proposition on the significant times of lunar eclipses corresponds only loosely to a passage in the *Almagest*, and it is more closely dependent upon *De scientia astrorum*.

VI.16: The author follows Albategni's method of finding the moon's hourly apparent motion (with some differences) while Ptolemy has a rather different procedure.

VI.17: This proposition on apparent conjunctions corresponds only loosely to the *Almagest*, and more closely to *De scientia astrorum*. The author adds a figure representing space and time that he uses to explain matters.

VI.18: This proposition on the digits of a solar eclipse has no closely corresponding passage in the *Almagest*. The author takes rules of Albategni and adds his own geometrical representation.

VI.19: In this proposition on the minutes of immersion in solar eclipses, the author uses a different figure than Ptolemy's, and some of it is derived from a rule and table of Albategni.

VI.20: This proposition on a solar eclipse's three times and minutes of immersion corresponds to no passage in the *Almagest*. It is taken from *De scientia astrorum*, but unlike his source, the author gives a geometrical representation of his source's rules.

VI.21: This proposition on the times of a solar eclipse first provides a method of Albategni's. It includes a procedure of Ptolemy's, but even this is presented in Albategni's wording.

VI.22: In the course of paraphrasing Ptolemy's manner of finding the area of the moon obscured in an eclipse, the author includes an alternative way of finding a quantity taken from Albategni.

VI.25: In this proposition on the direction of the darkness in an eclipse, the author adds geometrical figures and uses them to paraphrase Ptolemy.

Chapter 5

The Manuscripts

Sigla

Group 1.A

P Paris, Bibliothèque nationale de France, lat. 16657
R_1 Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1261
F Florence, Biblioteca Medicea Laurenziana, Conv. Soppr. 414

Group 1.B

Pr Prague, Národní Knihovna České Republiky, V.A.11 (802)
Me Memmingen, Stadtbibliothek, 2° 2,33
L_1 Leipzig, Universitätsbibliothek, 1475
N Nuremberg, Stadtbibliothek, Cent. VI.12

Group 2

P_7 Paris, Bibliothèque nationale de France, lat. 7399
B Berlin, Staatsbibliothek Preussischer Kulturbesitz, Ms. lat. qu. 510
Da Darmstadt, Hessische Landes- und Hochschulbibliothek, 1987
E Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 4° 356
T Toledo, Archivo y Biblioteca Capitulares, 98–22
E_1 Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 2° 383
W_1 Vienna, Österreichische Nationalbibliothek, 5273

Group 3.A

K Cracow, Biblioteka Jagiellońska, 1924
P_{16} Paris, Bibliothèque nationale de France, lat. 16200
D Dresden, Sächsische Landesbibliothek – Staats- und Universitätsbibliothek, Db. 87
R Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1012
L London, British Library, Harley 625
W_2 Vienna, Österreichische Nationalbibliothek, 5292

Group 3.B

M Munich, Bayerische Staatsbibliothek, Clm 56
W Vienna, Österreichische Nationalbibliothek, 5266

Group 4

Ba Basel, Universitätsbibliothek, F.II.33

Grouping, Contamination, and Stemma

There are 23 existing manuscripts that contain the text in its entirety or close to its entirety. Lorch was able to distinguish three main groups by collating the preface and by seeing which manuscripts contain an addition and an alternate passage in I.6.[1] In order to test this tripartite division, I checked each manuscript for completeness and noted the appearance of significant and conspicuous variants in all manuscripts. After collating passages where the differences in families seemed most apparent, i.e. the preface, I.4, I.6, and V.26, as well as passages of III.4 and VI.1, I was able to group the manuscripts and reveal some of their relationships. Unfortunately, a large amount of contamination hinders the construction of a complete stemma.

Group 1

The members of Group 1, which Lorch identified by similar readings in the preface, also share several unique omissions that show their common ancestry. These include omissions in Book III's list of principles ('Celestia corpora … mobilia'), III.1 ('verum tempus solstitii vel equinoctii'), IV.12 ('Libra xxv gradus … Sol in'), IV.16 ('corrigantur'), and VI.3 ('epicicli etiam … reflexione diametri'). A multitude of unique variant readings also confirm the existence of this group (e.g. 'iustior' in V.26 and 'deinde mutata' in VI.1). While this group is tightly knit and very few significant differences appear, it can nevertheless be broken down into two subgroups.

Manuscripts *P*, *F*, and R_1, which make up Group 1.A, are extremely close to each other. While their proximity is clear at a glance, evidence includes unique omissions in I.13 ('eiusdem reflexi … quam') and II.4 ('dividas'), the corruption of the same passage in III.4 (where the text should read 'a Z in H'), the same mistaken reading in IV.10 ('eadem' for 'cadit'), and an addition in Book IV's list of principles ('qualiter moveri'). Many of the figures are almost identical and contain the same errors (e.g. the figures of I.6, I.13, I.14, II.6). Even a couple of identical unlabeled, incomplete figures are found in the three manuscripts (VI.14 and VI.25). These three manuscripts also contain the same note and two figures concerning planetary models at the conclusion of the work. *F* and R_1 probably descend from *P*. The *Almagesti minor* in *P* is followed by a star catalogue in another hand, and the *Almagesti minor* and this catalogue do not appear to have originally been part of one manuscript; however, *F* contains this same star catalogue after the *Almagesti minor*, both in the same hand. *P* has some omissions in II.30 and VI.25 that are not in *F* or R_1, but the text is supplied in the margins of *P*. The other manuscripts could have been copied

[1] Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 416–19. Lorch describes these as being 'just before I 9' and 'a little before this' (p. 418), but these two sections are indeed in I.6.

from *P* after these corrections were made. Because R_1 has many of the omitted passages supplied in the margins and the addition in Book IV's list of principles is deleted, it appears that it was corrected against a manuscript from another group. This makes it difficult to determine its relationship within the group. It is possible that it was copied from *P* and then corrected. R_1 and *F* share several common features that are not found in *P* (e.g. both have the same superfluous lines in the figures of I.7–8), but it is clear that one does not descend from the other because *F* has omissions that are not found in R_1 in I.8 (e.g. 'Unde habemus propositum') and because *F* has figures that show very close similarities to *P* that are not in R_1 (e.g. I.13 and III.6). All three manuscripts may have originated in northern France around the middle of the thirteenth century.

Pr, *Me*, L_1, and *N* form another close, clearly defined subgroup, Group 1.B. While the existence of this group is made clear by numerous shared variants, especially telling are a unique addition in I.9 ('Et ex hoc habebis propositum cum adiutorio 15^e^ prime partis, 29^e^ primi, et quarte sexti') and unique omissions in II.33 ('Sed hii duo … ex DHG') and VI.1's enunciation ('Solis et Lune'). There are also additions in I.16 ('poteris invenire') and III.4 ('sed angulus AEB semper minor est … est angulo DZG', in the text of *Me* and L_1 and the margins of *Pr* and *N*) that are found only in this group and in *M* and *W*. The inclusion of these variant readings in the latter two manuscripts is probably due to contamination, as I will discuss later. *Me* and L_1 are extremely close to each other, and while the omission of a large passage in II.33 and of the figure for V.25 in L_1 show that it is not the exemplar of *Me*, *Me* is very likely the source of L_1. It is clear that *Pr* was checked against a manuscript from another group because it contains text in its margins in IV.10 that is omitted in all other members of Group 1 except *N* ('et locus Lune in medio eclipsis secunde punctum B'). *N* contains the text as it is found in *Pr* for this passage (most witnesses have '… secunde *tempore* punctum B'). Further evidence that suggests that *N* descends from *Pr* is found in *N*'s incorporation into the text of an explanatory gloss on II.34 ('Quia declinatio puncti … ad gradum medii celi') in the margin of *Pr*. The connection between these two, however, is difficult to determine because *N* appears to have been corrected against a manuscript from another group; for example, all the other members of Group 1 have omissions in III's list of principles ('Celestia corpora … esse mobilia' – *P* does have this first passage added in the margin but in a later hand) and in IV.12 ('in Libra xxv gradus … esset Sol'), but these texts are written in *N*'s margins in the scribe's own hand.

In addition to the unique omissions of Group 1.A listed above, there is further evidence that Group 1.B does not come from Group 1.A. For example, the addition of 'corollarium' in I.1 in *P* and *F*, their omission of a definition in Book V ('Diversitas aspectus Lune in longitudine … in celo'), and *F*'s omission in I.8 ('Unde habemus propositum') establish that Group 1.B does not descend from them. The inclusion of notes in the margins of some of the manuscripts in Group 1.B that are not in R_1 but are in *P*, *B*, and other manuscripts suggests that, barring contamination, Group 1.B does not come from R_1. Stronger evidence is that in I.6 R_1 has 'eorumdem notam' corrected into 'chordam notam' in the scribe's hand, while the manuscripts in Group 1.B have the same corrupt text 'eorumdem notam chordam' that is in *P*. Also, *F* and R_1 have an omission of text in I.6 ('et minor quam pars una puncta 2 secunda 50') that is in Group 1.B.

Group 2

This group, consisting of P_7, *B*, *Da*,[2] *E*, *T*, E_1, and W_1, is Lorch's second group, which he established on their similarity of text in the preface. Its members show a much greater diversity than the manuscripts of the first group. Uniting characteristics include an added phrase and a short alternate phrase in the preface ('aut potius deviet', which is also found in *M*, *W*, and *Ba*; and 'in huiusmodi disciplina parum exercitatus', also in *M* and *W*), a unique mathematical correction in II.18 ('elevationum sumpte … totius quarte'), and a unique reversal of clauses in II.7 ('in superiori est dies … est nox'). *T* has several of its own proofs in Book I, but the remainder of the text shows that it is a member of this group. Within this more amorphous Group 2, a large number of shared variants show that P_7 *B*, and *Da* are closely related, e.g. these are the only three manuscripts to add 'alios' before 'quinque erraticos' in the preface, and they all omit Book V's definition 'Media oppositio … cursum medium.' Also, *B* and P_7 have a unique omission in V.25 ('remanebit arcus LZ … super BZ et'). It is clear that P_7 is not copied from *B* because *B* has an omission in II.7 ('quandoque ad meridiem') while P_7 has the text, and also because there is an omission in V.18 in *B* ('in epicyclo pene … et v minuta') while the text is supplied in the margin of P_7 in what appears to be the scribe's hand, which could not have occurred if *B* were the sole exemplar. In fact, *B* appears to have been copied from P_7. I collated the entire text from these two manuscripts, and there was not a single instance that showed that *B* could not have been copied from P_7. While this fact alone makes it relatively certain that *B* descends from P_7, more positive evidence is that P_7 makes multiple corrections of the text that are then found in *B* (for example, in II.6 'cordam' is corrected in P_7 above the line into 'sinum', which is the reading found in *B*; in II.26 'antepremissam' is corrected in P_7 into '23', which is the reading in *B*; and in III.17 P_7's scribe corrects 'undecima' into '13' and '15ª' into '13', and these latter readings are found in *B*). An omission in *B* and P_7 of text in IV.17 that is found in *Da* ('Capitis in prima eclipsi … a nodo') is an example of the evidence that *Da* was not copied from either of these. *B*, P_7, *Da*, *E*, and *T* share a number of characteristics: an

[2] David Juste brought this manuscript, which Lorch did not know, to my attention.

added reference in II.24 ('secundum Teodosium de speris', slightly different in *T*) and omissions in II.14 ('Sit ergo … signum Arietis'), II.30 ('cum G sit … equinoctiali nota et', also in *P*), IV.10 ('ad ED … est nota ergo', *T* has the text supplied in the margin), V.7 ('equalem que est … A longitudine longiore', also in W_2), and V.9 ('equalis epicicli est longitudo longior'). There are a number of variants found in other pairs or subgroups of manuscripts of Group 2, but not in all. *Da*, *E*, and *T* all omit a definition of Book IV ('Circuitiones Lune in longum tempore diversas esse', *T* has it supplied in the margin, and it is also omitted in *R* and *Ba*). *B*, P_7, *Da*, and *E* have a unique addition in II.17 ('super L polum et super T') and an alternate text in II.12 ('e contrario in arcu opposito', *T* has a large omission here and thus has neither reading). *B*, P_7, and *E* have extra figures for V.9. P_7, *E*, and *T* share an addition in V.10 ('quantitas esse rerum', deleted in P_7), and these three manuscripts also skip III.6 and then place it later in the text.

E_1 and W_1 are very close to each other, as is clear from numerous variants, including an omission at the beginning of Book III ('Communia quedam … est aptior', supplied by the scribe in the margin in W_1) and the unique arrangement of principles at the start of Book V (the third and fourth definitions are placed after the other definitions). That W_1's scribe supplied the text omitted at the start of III in the margin and that he included passages that are omitted in E_1 in I.6 ('EG' in the sentence starting 'Linea etiam GE …') and V.9 (10^{th}-11^{th} paragraphs: 'servatam radicem divide. Quod si arcus … per v partes et xv minuta multiplica et per', in W_1's margin) shows that E_1 cannot be W_1's sole exemplar. These two manuscripts share few variants with every other member of Group 2, but they have a few similar variants that are also found in *T*. Among these are an addition in V.19 ('reliqua fac sicut in Luna', also found in the text of *M* and *W* and added later in margin of W_2) and a misplacement of a passage of V.20 ('Diversitatem vero aspectus Solis … et hoc quidem prope verum.'). In E_1 and *T*, a section of V.21 ('Et dico quod arcus KN … sive angulo KHN') is found in V.20 in the place of the missing passage, which in turn is placed in V.21 (after the text of V.21 in *T* and after 'quare MT est quarta circuli' in E_1). In W_1, none of V.21 is put into the text of V.20, and the omitted passage of V.20 is placed in the middle of V.21 (after 'sive angulo KHN') and is also supplied in the scribe's hand on a small added leaf.

Group 3

Lorch's Group C is established primarily on the inclusion of an alternate passage ('Unde corda AG … merito reputari' in place of 'Sed ad hunc numerum … fuerit postponitur') and a large addition ('Quia tamen earum numerus … tabule ordinentur') in I.6. All the manuscripts of this group share these significant variants and many others, including additions in I.4 ('quia anguli DAB … in equali circuli portione', 'quia AE nota … cum diametrus sit nota', and 'per sextum Euclidis') and the same misreading in II.20 ('angulis' for 'ianuis'); however, an examination of the preface and the sounding texts from Books V and VI indicates that there are two subgroups, Group 3.A consisting of *K*, *D*, P_{16}, *L*, *R*, and W_2,[3] and Group 3.B made up of *M* and *W*.

Among Group 3.A, W_2 is definitely copied from *K*. Besides almost always agreeing with *K*, W_2 has a large jump in the text from IV.12 ('considerationem Ptolomei de-') to V.13 ('superficie circuli altitudinis'), which agrees perfectly with page breaks in *K*. It is clear that the scribe of W_2 accidentally turned from page 72 to page 105 in his exemplar, *K*, and then supplied the text from *K*'s pages 73–104 after VI.25. *D* is also close to *K*, as is shown by examples such as the same mistaken 'stabulis' for 'tabulis' in IV.14, the misspellings 'epiclo' and 'eplclici' in the same locations in the text of III.4, and an omission in IV.3 ('et medietas unius gradus', which was later supplied above the line in *K*). P_{16} is also very similar to *K* and *D*; for example, they all have the same addition in I.10 ('eorum ad cordam dupli arcus alterius'). *L* and *R* are not as close to the other members of the group. For example, *R* does not have two short additions that are in the rest of Group 3.A in III.16 ('et propter hoc HA ad AL nota') and VI.5 (the table of eclipse limits), and *L* has many small unique variants. *D*, *R*, and *L* share some common unique readings in I.6: they all have 'eadem proportione prima' for 'eodem teorumate'; they all share some of the same garbled Arabic numerals in the sentences following 'Unde corda AG …'; *D* and *R* have 'quis' for the first 'quia'; and *L* and *D* both have '106' instead of '120.' Unique readings in *R* (e.g. the omissions of 'duple scilicet GA proportio ad eandem EA minor quam' in I.6, 'prope ortum Solis … prior coniunctio fuerit' in VI.11, and 'ad notitiam loci … ipsum tempus inventum' in VI.1) and *L* (e.g. the omission of 'arcus EG … in secunda eclipsi' in IV.17) make it impossible that they are the exemplars for any of the other surviving manuscripts. That figures for I.15 are in *K* but not *D*, *L*, or *R* suggests, but does not necessitate, that *K* is not the source of any of these.

The members of Group 3.B, *M* and *W*, are very similar to each other and rarely differ. *W* has a few omissions that are not in *M* (e.g. '55 cordam que residuo … partibus 124 punctis 7 secundis' in I.6 and 'qui ipsi fixo … eiusdem reflexi portionem' in I.13), so *W* cannot be *M*'s exemplar; however, *W* is likely copied from *M*.

Group 4

Ba is a rather difficult manuscript to place. It was copied very badly by a scribe who clearly did not understand the meaning of the text. It contains alternate proofs for much of Book I, and the order of the text is in disarray. It does

[3] Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 416, mistakenly put *K* in his first group, and he did not know about the existence of P_{16}.

not fit the distinguishing criteria for any of the families described above, so it seems to be the sole member of a fourth group. *Ba* does, however, share some variants with other families. For example, in I.6 it contains one sentence of Group 3's alternate text ('Unde corda … et secundas 3 ['8' in Group 3] fere'); however, it also shares many variants with members of Group 2, such as readings for the concluding formulae of II.33–34 ('et hoc est quod intendimus' and 'et hoc est propositum') that only occur in it and in *E*.

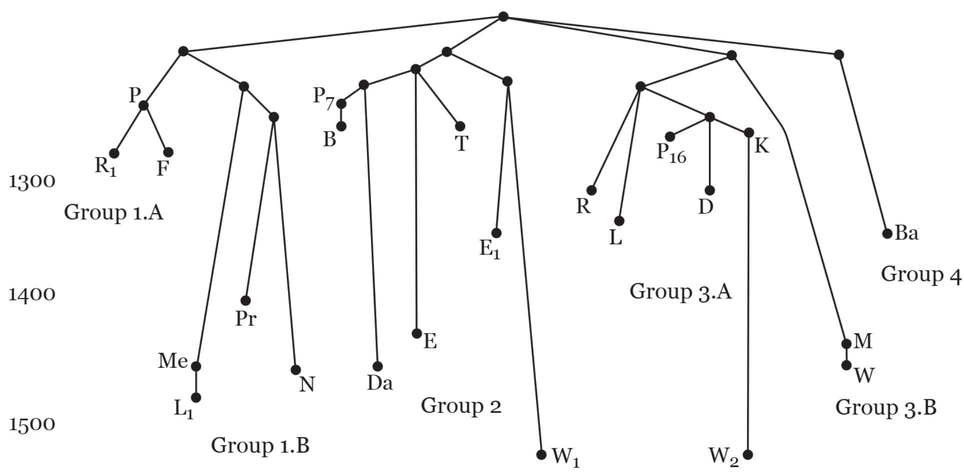
With a work such as the *Almagesti minor*, a stemma cannot be created with certainty through the standard practices of critical edition.[4] In the case of a mathematical commentary, scribes could often deduce the correct reading of the text even when their exemplars contained errors. When a mathematical or linguistic problem was noticed, some scribes seem to have consulted another witness. It also appears that some scribes copied the *Almagesti minor* while reading Gerard's translation of the *Almagest* and Plato of Tivoli's translation of al-Battānī's *Zij*, and thus errors would be caught much of the time. For example, all the witnesses have 'extreme' in VI.11, while the corresponding passage of the Gerard's translation of the *Almagest* uses the word 'postrema'; however, *M*'s scribe corrected 'extreme' to 'postreme', which is close to the *Almagest*'s reading. The use of the *Almagest* by scribes of the *Almagesti minor* is also clear from the figures. In the drawing for I.1, *K*, *D*, *R*, and W_2 have an added point E near point Z that should not be there, but there is another point labeled E in Gerard's translation of the *Almagest*. Other examples of reliance upon Gerard's translation of the *Almagest* are the relabeling of I.14's figure in *M*, *N*, and *Pr* to match that of the *Almagest*, and the inclusion of a figure taken from the *Almagest* for II.16 in *Me* and L_1. In III.11, it appears that the *Almagesti minor* misreported Albategni's value for the sun's apogee, but *N* gives the correct value.

An even larger problem is the contamination from the use of multiple exemplars. Definite proof of such contamination is found at the end of I.7, where an addition that is found in Group 3.B ('per similitudinem triangulorum … ut prius') is also found in W_2, which doubtlessly was copied from *K*, a manuscript that lacks the addition. This same addition is also found in the margins of W_1, but not in E_1 or any of the other members of Group 2. Additionally, while *K* has a variant reading in III.7 ('a longitudine longiori'), W_2's scribe corrects it back to the standard reading ('et longitudinem longiorem'). W_2 also includes in its margin some text of V.7 that is omitted in its exemplar ('vera epicicli … a longitudine longiore'), which shows that its scribe also consulted a manuscript from Group 2, 3.B, or 4. There are other similarities between manuscripts far

[4] Similarly, Benjamin and Toomer, *Campanus of Novara*, pp. xiv–xv, found that high levels of contamination in Campanus' *Theorica planetarum* hindered them from constructing a stemma and that they were only able to sort the manuscripts into groups.

apart in the stemma that lead me to suspect that contamination was common. For example, *P*, P_7, and *B* share the same set of notes. Group 3.B has several similarities with other groups that suggest contamination: an explanatory addition in III.4 that is in Group 1.B ('angulus AEB semper … angulo DZG semper'); an alternate passage of a few sentences in VI.1 that is also found in E_1 and W_1 (where the standard text has 'Quotiens ergo … cum hoc numero');[5] an addition in V.5 that is also found in Group 2 ('super centrum D cuius diameter ADG'); an added sentence in V.19 that is also in E_1, W_1, and *T* ('Reliqua fac sicut in Luna'); and a mistaken reference to Thebit instead of Theon in V.21 that is also in *T*. Additionally, in the margins of *Pr*, its scribe supplies some text that is missing in IV.10 in the rest of Group 1 except *N* ('et locus Lune in medio eclipsis secunde tempore punctum B'). Similarly, R_1 has in the margin a principle of Book V ('Diversitas aspectus Lune in longitudine est … in celo') that is missing in the text in all the members of Group 1.A. Moreover, *M* and *W* are in Group 3, but *M* has an addition to II.34 found only in it, *Pr* and *N* ('Quia declinatio puncti … ad gradum medii celi'), and *M* and *W* have an alternative text in VI.1 ('Quotiens ergo mediam coniunctionem … Si vero') that is similar to one in *T* and E_1.

Without the assurance that there is no contamination, many different possible stemmata can be derived from the same evidence. The stemma presented here is the most likely, but it is not the only interpretation that could be derived from the textual and codicological evidence.



Manuscript Descriptions

More complete codicological information for the *Almagesti minor* manuscripts will soon be available in David Juste's catalogue of the Latin Ptolemaeus

[5] *T* and W_2 have some of the alternate readings here.

astronomical and astrological corpus.[6] For each manuscript, I first give the shelfmark, the date, the folios upon which the work appears, followed by any title that the work is given and any incipit or explicit that differs from the standard 'Omnium recte philosophantium ...' and '... tenebrarum sic se habent.' I then provide additional information on the origin and provenance of the manuscript and on the state of the *Almagesti minor* in this work, e.g. blank or misplaced folios, any large omissions or alternate texts, whether the text is accompanied by glosses, whether diagrams are generally lacking, and any other relevant characteristics of the text in the manuscript. I also report the inclusion of other works in the manuscript if they may illuminate the relationships of manuscripts to each other or how the *Almagesti minor* was employed.

Group 1.A

P **Paris, Bibliothèque nationale de France, lat. 16657**

Between *c.* 1225 to 1260. 82v–132r. '... tenebrarum sic se habent. Explicit hic sextus liber et sexti glosa textus.'

The title 'Minor Almagesti' is given in another hand (82v).

This is the specimen of the *Almagesti minor* that is included in the *Biblionomia* of Richard de Fournival.[7] This manuscript is in three parts that were once separate and that were all commissioned by Richard for his own library. Since Richard was born in 1201, this manuscript is very unlikely to have been written before the mid 1220s, but it must have been written before Richard wrote the *Biblionomia* probably *c.* 1250, and definitely before his death in 1260.[8] When the *Biblionomia* was written, the manuscript's first part, which contains Albategni's *De scientia astrorum*, was still a separate manuscript, but the two other parts, containing the *Almagesti minor* and a star table that uses Gerard's translation of the *Almagest* but with modified values, were already bound together.[9] The grouping of folios shows that folio 82 containing the preface was not original, and a close examination of the hands shows that the preface was not written by the scribe who wrote the rest of the text. Proposition numbers are written in the margins, but it appears that many of them were lost when the folios were trimmed. After Richard's death, Gerard of Abbeville owned this manuscript, and he donated it to the Sorbonne.[10] Book I has marginal glosses, some perhaps in the scribe's hand and others in a later hand. This manuscript primarily has Roman numerals (Arabic numerals in I.6).

[6] I have relied heavily upon the drafts of his book, which he has generously shared with me. David Juste's manuscript descriptions can also be seen on the Ptolemaeus Arabus et Latinus website (www.ptolemaeus.badw.de).

[7] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', pp. 169–70.

[8] For more on Richard and his *Biblionomia*, see Section II above on the dating of the *Almagesti minor*.

[9] A. Birkenmajer, 'La Bibliothèque de Richard de Fournival', pp. 169–70.

[10] *P*, f. 82r.

The text is followed by a note and two diagrams concerning planetary models (132r-v): 'Not{a} P est centrum terre, O centrum excentri … a primo argumento ad hoc quod diximus.' This same note is found in *F* and R_1. The text following this note is a star catalogue (133r–146v) that was not originally part of the same manuscript.

R_1 Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1261

2nd half of thirteenth century. 1r–49r. 'Incipit liber primus Almagesti minoris. Omnium recte philosophantium … tenebrarum sic se habent. Explicit liber sextus Almagesti minoris.'

While the scribe uses the title 'Almagesti minor', Peter de Limoges lists the work on a fly leaf as 'Liber Almagesti demonstratus libri 6.'

Besides its relationship to *P*, there is much evidence that this manuscript was written in northern France. It was written in one hand by the same scribe who wrote part of Paris, BnF, lat. 7434, which was given by Peter of Limoges to the Sorbonne, and Aleksander Birkenmajer has argued that R_1 originates from France and perhaps has its own connection to the Sorbonne.[11] It includes occasional glosses by Peter of Limoges, including one that refers to Geber.[12] In two notes, Peter refers to specific folia and columns of an *Almagest* manuscript; these references match the foliation of P_{16} exactly.[13] Because the first of these notes also refers to Campanus' *Theorica planetarum*, which has an accompanying letter of dedication to Urban IV, who was pope from 1261–64, Peter wrote his glosses in the early 1260s or later. There is at least one note in another hand. The same note and diagrams that immediately follow the *Almagesti minor* in *P*, which was owned by Richard of Fournival, are also found in this manuscript. It has further connections to Richard. It contains his *Nativitas* (59r–60v), and it has similarities to Edinburgh, Royal Observatory, Cr. 1.27, which is known to have been owned by him and that was also later owned by the Sorbonne.[14] This manuscript generally uses Arabic numerals.

F Florence, Biblioteca Medicea Laurenziana, Conv. Soppr. 414

2nd half of thirteenth century or 1st half of fourteenth century (but likely before 1263). 1r–45r. '… tenebrarum sic se habent. Explicit hic sextus liber et sexti glosa textus.'

[11] Alexandre Birkenmajer, 'Pierre de Limoges, commentateur de Richard de Fournival', pp. 22–23.

[12] R_1, f. 19r.

[13] R_1, ff. 31v and 68v. The first states, 'Hec figure melius facta est in Almagesti libri 5 capitulo 13, hoc est folio 73 columna 3, et declaratur per figuram Lune positam in fine tractatus instrumenti Campani.' The second is in the margins of Euclid's *Elements* I.42, and the reference to the *Almagest* reads, 'Et nota quod hoc correlarium supponit Ptolomeus in Almagesti 6 libro, folio 90 columna prima, ad inveniendum aream trianguli.'

[14] Rouse, 'Manuscripts Belonging to Richard de Fournival', p. 255.

Before the text begins there is added in a later hand 'Incipit liber Albategni qui dicitur Almagesti parvum' (1r).

Although cataloguers provide a wide range of possible dates for this manuscript, it can perhaps be dated from a marginal note in perhaps another hand that gives the current year as 1263.[15] Another note added to the manuscript (in what appears to be the same hand as the added incipit) gives the year 1304.[16] There are some short marginal notes accompanying the *Almagesti minor*. The text is followed by the same note and figures on planetary models that are in the above two manuscripts (45v). Following this note is the same star table that is found in *P*. That the star table and the *Almagesti minor* were not originally together in *P* but are in the same hand in *F* suggests that *F* was copied from *P*. While this manuscript has many Arabic numerals in Books I–II, it usually uses Roman numerals.

Group 1.B

Pr **Prague, Národní Knihovna České Republiky, V.A.11 (802)**

Fourteenth or early fifteenth century (before 1432). 1r–59v. '… tenebrarum sic habent. Ave gratia plena, Dei genitrix, virgo, ex te enim ortus est. Scriptoris votum, Virgo, tu respice totum. Explicit liber Almagesti minoris et Deo gratias.'

This has many marginal notes, but many are rather faded and very difficult to decipher. Many appear to be in the scribe's hand and partially match those found in *P*. A set of notes was written in the margin by Johannes Andree Schindel, whose manuscript of the *Almagest* (Cracow, BJ, 619) contains excerpts from the *Almagesti minor* that he used in his lectures given in 1412–18. Among the notes in *Pr* in Schindel's hand is the report of a series of observations performed on 11–12 March 1431 and 10–11 March 1432 in order to demonstrate the procedure for finding the year's length.[17] He remarks that the resulting value for the year is off by a significant amount but that he does not care, since he merely wants to show the method. Many of the diagrams are also very faded and difficult to see. Space is left for the initial letter of each paragraph, but these letters were never added. The text is followed by several folios of notes on astronomy and perspectiva, among which are four reworkings of proofs from the *Almagest* I.9 and I.12 or the *Almagesti minor* I.4–7 (62v). These are written for relettered diagrams, but because these diagrams are not given and the script is rather faded, these are very difficult to make out. This manuscript employs Arabic numerals.

15 *F*, f. 63v.
16 *F*, f. 60r.
17 *Pr*, ff. 14v–15r.

Me Memmingen, Stadtbibliothek, 2° 2,33

Fifteenth century. 152r–198v. '… tenebrarum sic se habent. Explicit liber sextus Minoris Almagesti.'

The text stops after the enunciation of I.6 (152v) and restarts at the beginning of the work on the following folio (153r). Perhaps the reason is that the scribe realized that he put a 'Q' instead of a 'D' in the rubrication for I.6. Another work in the manuscript gives the date 1466, but this section of the manuscript is in another hand and thus cannot be used to accurately date the copying of the *Almagesti minor*.[18] There are some marginal notes, mostly short ones. At least one or two notes match those in *P*, and some that are not in *P* match ones in L_1. There is an added diagram at the beginning of the work that is also found in *Pr* and L_1. This manuscript uses Arabic numerals.

L_1 Leipzig, Universitätsbibliothek, 1475

2nd half of fifteenth century. 2r–51v. '… tenebrarum sic se habent. Explicit liber sextus Minoris Almagesti.'

Although several dates are given in this manuscript, there are several hands and thus these dates cannot be used to precisely date the portion with *Almagesti minor* in it. This text is accompanied by many marginal notes in what appears to be the same hand as the scribe. Some of these notes match those in *P*. Two small leaves containing only figures were added later: f. 5 has a figure for I.14 that is poorly drawn earlier, and f. 14 has the figures for V.3 and V.5, which should have been on ff. 28r-v. A note and a diagram for I.1 are given on f. 1v, and there is a small blank leaf (11bis). Many initial letters in the text are omitted although space was left for them. This manuscript uses Arabic numerals.

N Nuremberg, Stadtbibliothek, Cent. VI.12

c. 1459. 1r–66v. '… tenebrarum sic se habent et cetera. Laus Deo, qui mihi favisti ceptis imponere finem. Laus et honor tibi sint astrorum aeterne volutor.'

Regiomontanus wrote this manuscript in Vienna. He wrote many marginal notes. This manuscript is the only one with a unique addition in II.34 in the text. *M* also has this addition, but on an extra small piece of parchment bound into it. This manuscript uses Arabic numerals.

Group 2

P_7 Paris, Bibliothèque nationale de France, lat. 7399

1st half of thirteenth century. 15v–93v. '… tenebrarum sic se habent. Explicit liber sextus.'

[18] *Me*, f. 230r.

This is titled 'In Speram' in a later hand (15v).

This manuscript, which originated in England, contains notes perhaps in the hand of the scribe and at least one note, which refers to Regiomontanus, in a later hand.[19] Some of the notes match those in *P*, *B*, and others. Pseudo-Jordanus' *De proportionibus* is immediately before the *Almagesti minor*. The manuscript includes Campanus' *De figura sectore* (94r-v), but the folio on which it was written is much larger and was clearly written separately before being bound in this manuscript. III.6 was skipped but the original scribe noticed his mistake and wrote it on a separate folio (34v). This mistake, however, led to a misnumbering of the remaining propositions in III. This manuscript usually uses Arabic numerals but also has many Roman numerals.

B Berlin, Staatsbibliothek Preussischer Kulturbesitz, Ms. lat. qu. 510

Mid thirteenth century, probably before 1249. 114r–175v. '... tenebrarum sic se habent.'

On a flyleaf this is listed as 'Almagesti libri 6.' 'Liber Almagesti primus' is given in rubricated text (114r).

The *Almagesti minor* was copied from P_7. Many notes and calculations are written in both the scribe's hand and another hand on f. 113v immediately preceding the text and in the margins throughout the work. Many of these notes are not legible (at least not in my reproductions). Some of the marginalia and interlinear notes are the same as those found in *P*, P_7, and other manuscripts. A note discusses the conversion of years from Christian to Arabic eras and gives a value for the year AD 1249.[20] This manuscript was owned by an English family, the Langfords, and the names of Richard, Edward, and George Langford appear in marginalia along with the years 1611 and 1613 with English writing and mention of the Langford's church at Gresford.[21] This manuscript has Pseudo-Jordanus' treatise *De proportionibus* immediately following the *Almagesti minor*. The rubrication and initials stop in Book V although space was left for the initials throughout the work. The scribe was perhaps from Spain or Portugal since he often doubles consonants, which is a common characteristic of Iberian orthography, and he also generally follows the southern custom of omitting the letter 'h' at the start of syllables (e.g. 'protrao' for 'protraho'). Some of the notes for II.15–16 referring to the diagrams make it clear that the notes were not written for *B*, but were copied into it from another manuscript. Most of the propositions in Books V–VI are not numbered or are misnumbered. The text usually has Arabic numerals, but Roman numerals occur frequently.

19 P_7, f. 5r.

20 *B*, f. 141r.

21 *B*, ff. 25r, 163v, and 167v.

Da Darmstadt, Hessische Landes- und Hochschulbibliothek, 1987

Fifteenth century. 2r–38v. The text ends in the middle of V.9: '… et quo provenerit numeracio ibi erit verus locus Lune.'

The scribe, a Master Anthonius, added some marginal glosses and three additions within the text, most of which concern tables. The first addition follows Book III and gives the name of the scribe: 'Explicit liber tercius. Sequuntur quedam addiciones quas ego Magister Anthonius hic inseravi [*sic!*]. Aditio. Tabula equacionis dierum cum noctibus suis sic componitur. Quare arcum … si fuerit posita e converso fiat e converso etc.' (23v–24r). The next is after IV.3: 'Addicio. Ad inveniendum medium motum Lune in una die, numerum dierum equalium lunacionis … in quibus coniunctiones ad statum similem reducuntur' (25v). The last addition is after the V.9: 'Addicio. Ad componendum tabulam equationis centri Lune, sic fac. Primo … et cum instrumentis materialibus invenitur. Expliciunt addiciones mee' (37v–38v). It has five parts that explain how to find the values for five of the columns of the table of lunar anomalies described in the paragraph of the *Almagesti minor* that starts 'Artificium vero tabularum …' A sixth section of this addition addresses the likeness of this table to the tables of the planets' anomalies and their stations, topics which the *Almagesti minor* does not cover. Anthonius' marginal note at the end of the last addition giving the next words in the *Almagesti minor* V.9 shows that Anthonius' exemplar did not end where he did.[22] Many figures are omitted in this manuscript. Arabic numerals are used.

E Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 4º 356

Early fifteenth century. 1r–101v. The text ends abruptly mid-sentence in VI.8: '…notus est arcus a nodo usque ad terminos eclipticos.'

This contains a star table verified for 1400, so perhaps this was written then or soon after. This manuscript divides I.6 and I.15 each into two propositions, so the remaining proposition numbers in Book I are off. There are only occasional, short marginal notes by the scribe. Some additional diagrams are added in the margins, sometimes with notes stating that they are not original. This manuscript uses Arabic numerals.

T Toledo, Archivo y Biblioteca Capitulares, 98–22

Thirteenth century. 67ra-80vb. '… tenebrarum sic se habent. Explicit liber sextus.'

Although this may be an early manuscript, it shows more differences from the standard text than almost any other manuscript. The text has alternate or additional proofs for each proposition from I.1–14. The alternate text of

[22] *Da*, f. 38v: 'Sequitur "artificium vero etc." et est de textu.'

I.7 includes a reference to Ametus' *Epistola de proportionibus*, which follows the *Almagesti minor* in *T* and to the 'librum Walterum Flandrensem (*corr. in* Walteri Flandrense†ri†) de proportionibus.'[23] This latter reference may refer to the Pseudo-Jordanus *De proportionibus* that immediately precedes the *Almagesti minor*. It appears that the scribe was a skilled mathematician who intended to write the proofs in his own words, but he quickly drops this project and then only adds steps or, in the case of I.13–14, extra related proofs, which are derived from proofs from Thebit's work on the sector figure.[24] After the first fourteen propositions, the scribe follows the standard text. Blank spaces were left after I.6 and I.16 (ff. 67v–68r and 69r). Perhaps the scribe intended to add the *Almagest*'s tables of chords and of declinations. III.6 is skipped and mistakenly placed in the middle of III.19. *E* and P_7 also misplace this proposition. Passages from V.20 and V.21 are switched. There are many glosses. The text is followed by a note in another hand on Ptolemy's preface to the *Almagest*. This manuscript usually uses Roman numerals, but it uses Arabic numerals in the alternate proofs in Book I.

The scribe seems to have reworked at least one other work. This manuscript includes a version of Euclid's *De speculis* that had been revised in order to make the original text more 'Euclidean' (*De speculis* must have appeared to lack rigor in comparison to the *Elements*), and it includes a French phrase that suggests that the reviser was northern French.[25] A note providing the value of the manuscript in Parisian solidi is another piece of evidence that the manuscript is French.[26] This manuscript was owned by the cathedral of Toledo.

E_1 Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 2° 383

Fourteenth century. 1r–51v.

This is perhaps an English manuscript, and it was given by Master Henricus Runen to the college Porta Celi in Erfurt.[27] Lorch reports that the title 'Liber magni astrologi in astronomicis' is given on f. 1r and that this work is also described in a table of contents added by a later hand as 'liber quidam astronomicus ... cuius autor est Geber magnus astrologus.'[28] In this manuscript, the proofs of III.13 and III.14 are reversed, but there are notes making readers

23 *T*, f. 68ra.

24 Richard Lorch, *Thābit ibn Qurra. On the Sector-Figure*.

25 *T*, ff. 87v–97v. Among the changes are reworkings of proofs, the '[f]ormalization of proof steps', references to other theorems and principles, and examination of more cases, according to Takahashi, 'A Manuscript of Euclid's *De Speculis*', pp. 76 and 80.

26 *T*, f. 65v.

27 Schum, *Beschreibendes Verzeichnis der Amplonianischen Handschriften-Sammlung zu Erfurt*, p. 269.

28 Lorch, 'Some Remarks on the *Almagestum parvum*', p. 417; and 'The Astronomy of Jābir ibn Aflaḥ', p. 92. Unfortunately, neither of these are visible in my reproductions.

aware of the mistake. Passages from V.20 and V.21 are also switched. The text is only accompanied by a few short notes. The scribe used a mixture of Arabic and Roman numerals.

W_1 Vienna, Österreichische Nationalbibliothek, 5273

1527. 35v–90v. '... tenebrarum sic se habent. Explicit.'

The part of this manuscript containing the *Almagesti minor* was written by Johannes Vögelin in 1527 at the University of Vienna.[29] There are some short notes in Vögelin's hand. He misplaced a section of V.20 and inserted it in the middle of V.21. He noted this mistake in the margin and then rewrote this omitted section on a separate leaf (between ff. 73 and 74). This manuscript is very similar to E_1. Both have the same unusual ordering of definitions at the start of Book V and the same unique omissions in VI.1. Vögelin, however, consulted another exemplar in addition to his main one from Group 2.B. The text omitted in E_1 in VI.1 was also left out in W_1, but then it was supplied in the margins. Also, an addition to I.7 that only *M*, *W*, and W_2 have in the text is written in the margin of W_1. This suggests that Vögelin used W_2 as his second exemplar, which is not surprising since it was his main exemplar for several texts on calendar reform in W_1.[30] Vögelin uses Arabic numerals.

Group 3.A

K Cracow, Biblioteka Jagiellońska, 1924

Thirteenth century, perhaps before 1250. Pp. 9–163. '... tenebrarum sic se habent. Explicit liber sextus.'

Another hand has titled the work 'Almagesti Ptholomei' (inside front cover and p. 9).

This manuscript was written in France, probably Paris. It has only short marginal and interlinear notes. It contains the Pseudo-Jordanus *De proportionibus* that was likely written by Walter of Lille. This manuscript normally uses Roman numerals, but it has Arabic numerals in I.6.

P_{16} Paris, Bibliothèque nationale de France, lat. 16200

c. 1246–47. 5r–96r. 'Formam celi spericam esse...'

The glossator who copied out the *Almagesti minor* refers to it as 'parvus Almagesti' (5v, 42v, and 89v). Peter of Limoges calls it the 'parvus Almagesti' (7r) and the 'Almagesti minor' (20v and 47r).

This is a manuscript of Gerard of Cremona's translation of the *Almagest*, copied in December 1213 probably from Paris, BnF, lat. 14738, which was then

[29] W_1, ff. 138v and 257r.

[30] Nothaft, 'The Chronological Treatise *Autores Kalendarii*', pp. 3 and 30.

at St. Victor's in Paris.[31] P_{16} was in the Sorbonne by 1338. This manuscript contains many marginal notes in at least two hands, which are often hard to distinguish. Among these notes is a set that contains almost the entirety of the *Almagesti minor*. These notes appear to be in the same hand that wrote three notes that can be dated to 1246–47. The first of these notes states that this book began to be read in 1246, and the others perform calculations for the year 1247.[32] This glossator describes the source of the excerpts from the *Almagesti minor*: 'He sunt propositiones extracte (*corr. in* protracte) per 6 libros huius libri sumpte ex libro qui dicitur parvus Almagesti cum commentis scilicet hec {est} prima.'[33]

The glossator copied the text of the *Almagesti minor* fairly loosely and made many small changes and rewordings of the text, especially at the beginnings and ends of the proofs. He often gives a short comment on proofs, instructing the reader which diagrams to use or whether proofs are only approximations. He also consistently replaces 'circulus signorum' with 'zodiacus.' One of the few major differences in the text is that the preface is not given in its normal form, but is converted into a list of principles instead.[34] Also, much of I.6 is not included, I.14 has an outline of the proof, I.15 has the enunciation followed by an alternate text, and I.17's proof has only excerpts from the standard text accompanied by some new commentary. This *Almagest* commentator also started to write extra definitions for the longitude of the moon and the diversity of the moon among the definitions of Book IV (46v). The order of some propositions is changed. II.2 is given after II.4. III.14 and 15 are given in reverse order and are given each other's numbers. III.19 and III.20 are reversed, but retain their standard numbering. At the beginning of Book IV, the order of the definitions of 'equalis lunatio' and 'mensis' is reversed. V.13 is placed after V.15. The propositions at the end of Book VI appear in this order: VI.17, 23, 22, 24, 18, 19, 20, 21, 25. The numbering of propositions also sometimes differs from the standard count. Many of the propositions in Book II continue the numbering from Book I, e.g. II.1 is numbered as the 18th proposition and as the first of the second book, and II.10–13 are numbered 11–14. The glossator points out that the *Almagesti minor* is missing a proposition concerning the

31 A note on a flyleaf (f. IIv) of P_{16} states, 'Liber iste fuit scriptus et perfectus ad exemplar beati Victoris Parisiensis anno domini M°cc°xiii mense decembri.' Although the year in this note appears now as 'mcclxiii', the 'l' is a medieval addition, and the original date accords with the appearance of the manuscript's writing and decoration; see Samaran, Marichal, *et al.*, *Catalogue des manuscrits en écriture latine portant des indications de date, de lieu ou de copiste*, t. III, p. 513.

32 P_{16}, ff. 1r, 45v, and 46v.

33 P_{16}, f. 5v.

34 P_{16}, f. 5r.

epicyclic model after III.12, but notes that adding a proposition would change the numbering of the propositions.[35] The glossator used Arabic numerals.

This manuscript also contains glosses and foliation that are seen to be the work of Peter of Limoges from a comparison to his notes in Paris, BnF, lat. 7320 and R_1. In these notes in P_{16}, Peter cites Book III of the *Summa de temporibus* of Giles of Lessines (often attributed falsely to Roger Bacon), which was finished in 1264 or 1265, so he must have written his notes between then and his death in 1306.[36] Interestingly, in R_1 Peter provides references to specific passages of the *Almagest* by column and folio that show that he was using P_{16}, and in P_{16} he cites *Almagesti minor* II.18 and 'that which he noted there.'[37] Indeed, this proposition is not glossed by Peter in P_{16}, but it is thoroughly glossed by him in R_1. It is clear then that Peter went back and forth between these two manuscripts, consulting the *Almagest* in P_{16} as he read the *Almagesti minor* in R_1, and consulting the *Almagesti minor* in R_1 as he reread the *Almagest* and wrote notes on it in P_{16}. That Peter had access to copies of the *Almagesti minor* from at least two of the groups of witnesses (R_1 from Group 1.A and P_{16} from Group 3) harmonizes well with the theory that the *Almagesti minor*'s early history was centered in Paris or northern France. Peter warns the reader about the scholar who added the *Almagesti minor* to P_{16}, calling him a 'blind' and 'deceptive glossator.'[38] Because Peter refers to the *Almagesti minor* as an authority multiple times, the exhortations to avoid whatever this commentator had written must be taken to apply only to this man's own interpretations, not to the entire *Almagesti minor*.[39] In fact, Peter states that certain of the notes of the earlier glossator are trustworthy.[40]

35 P_{16}, f. 42v.

36 P_{16}, f. 45v; Steele, *Opera hactenus inedita Rogeris Baconis, Fasc. VI*, p. xxvi; and Nothaft, 'Origen, Climate Change, and the Erosion of Mountains', p. 54. In his notes, Peter cites a number of other works, including Campanus' *Theorica planetarum* (P_{16}, ff. 56v, 67r, and 145v), Geber (ff. 48r, 56v, and 71r), and a *De proportionibus* (f. 12v). The note containing the last of the references explains a feature of compound ratios by refering to the second proposition of this 'Liber de proportionibus', and both Campanus' treatise on ratios and the one I believe was written by Walter each have relevant content in their second propositions.

37 'Nota quod figure hic posite bene facte sunt, et tota littera plane patet per commentum 18 propositionis secundi Almagesti minoris et per id quod ibi notavi' (f. 20v).

38 'Quicquid dicat iste cecus glosator qui in hoc libro nihil intellexit sed margines huius libri falsitatibus denigrando fedavit hic et ubique fere per totum noli verbis seu glosis eius attendere si non vis errare' (f. 20v). 'Nota quod quicquid dicat iste trufatorius glosator, actor in toto hoc quarto libro non ponit…' (f. 56v).

39 P_{16}, ff. 7r, 20v, and 47r.

40 E.g. he adds 'Hec notula vere est' near one of the notes of the earlier commentator (f. 26v).

D Dresden, Sächsische Landesbibliothek – Staats- und Universitätsbibliothek, Db. 87

Late thirteenth or early fourteenth century. 104r–161v. 'Incipit Almagesti demonstratum de sex primis libris Ptolomei. Omnium recte...' The text breaks off in VI.25, ending, '... arcus autem orizontis inter gradum orientem vel gradum occidentem et circulum equinoctialem.'

The work is described in another hand as 'Parvum Almagesti Pt{olomei} demonstratum per Campanum' (268v), and the top of each folio bears the book number with 'Almagesti demonstrati.'

As the text ends abruptly in the middle of a sentence in the last proposition, it is clear that only one folio with the remainder of the text has been lost. This manuscript was owned by the Dominican Berthold of Moosburg while he was teaching at his order's school in Cologne before he moved to Nuremberg in 1346.[41] There is only a single marginal note.[42] This manuscript includes the sole surviving witness of a translation of the *Almagest* made by Abd al-Masīḥ of Winchester.[43] This manuscript's scribe generally uses Roman numerals, but it has Arabic numerals in I.6.

R Vatican, Biblioteca Apostolica Vaticana, Reg. lat. 1012

Thirteenth or fourteenth century. 1r–73r. '... tenebrarum sic se habent. Explicit liber sextus. Amen dico amen. Totus liber continet 156 conclusiones.'

There are proposition numbers in the margin, but they do not match the standard numbering in much of Books I and II because of misnumbering problems and then in Books V and VI because of misplaced folios. A number of folios are misplaced, and to read the text in the correct order, one should read:

1r–47v	Preface to mid V.9
60r–71v	mid V.9 to mid VI.1
48r–59v	mid VI.1 to mid VI.23
72r–73r	mid VI.23 to VI.25

Although the scribe states that there are 156 'conclusiones', there are only 150 propositions in the *Almagesti minor*. There are a few notes in the margins in a later hand. The work is followed by an excerpt from Gerard of Cremona's translation of Thebit's *On the Sector Figure* (73r-v).[44] *R* generally uses

[41] *D*, f. 268v; and Ruh, Keil, *et al.*, *Die Deutsche Literatur des Mittelalters: Verfasserlexikon, Band 1*, p. 816.

[42] *D*, f. 104v.

[43] Burnett, 'Abd al-Masīḥ of Winchester'; Grupe, 'The "Thābit-Version" of Ptolemy's *Almagest*'; and Grupe, *The Latin Reception of Arabic Astronomy*.

[44] Knobloch, 'La Traduction Latine du Livre de Thābit ibn Qurra.'

Roman numerals, but it has some scattered Arabic numerals. Arabic numerals are found in I.6, with many misreadings.

L London, British Library, Harley 625

c. 1341. 85r–123r, 132r–136v. '… tenebrarum sic se habent. Explicit. Explicit' (123r).

The title 'Almagesti abbreviatum' is given by Simon Bredon (1*v), and it is also listed in a table of contents as 'Libri Almagesti 6 abbreviati' (1*r).

This manuscript, which originated in England, probably Oxford, was part of a manuscript that was bequeathed by Simon Bredon to Merton College, and was later owned by John Dee, and then divided in the seventeenth century.[45] The original manuscript contained tables written for Oxford for 1341–44, which suggests that it was written in 1341 or slightly earlier.[46] If one disregards this evidence, it is manifest that *L* was made between 1326 and 1347 because it includes Richard of Wallingford's *Albion*, which was composed in 1326, and because it has a marginal note relating observations made in 1347.[47] The scribe omitted V.7–19 but then the same scribe supplied this large passage later in the manuscript on 132r–136v. At the start of the misplaced section, we find the title 'Hec conclusiones sunt de libro quinto Almagesti abbreviati.' Much of the manuscript is known to have been written by Simon Bredon, who wrote a commentary on the *Almagest c.* 1340 that uses the *Almagesti minor*.[48] The hand in which the *Almagesti minor* is written appears very similar to Simon's known hand. It would make perfect sense that Simon copied out the *Almagesti minor* around the time he wrote his own commentary. It is at least known that Simon wrote the many corrections and notes in the *Almagesti minor*'s margins.[49] This manuscript primarily has Roman numerals, but it has a small number of Arabic numerals throughout the work and many in I.6 (although with many mistakes).

W_2 Vienna, Österreichische Nationalbibliothek, 5292

Early sixteenth century. 1r–65v. '… tenebrarum sic se habent. Explicit liber sextus' (53v).

On the covers is found 'Epitome Alberti in Almagesti Ptolomei' and 'Albertus Magnus in Almagesti Ptolemei.'[50]

45 Watson, 'A Merton College Manuscript Reconstructed.'

46 Watson, 'A Merton College Manuscript Reconstructed', p. 217.

47 *L*, f. 3v.

48 Watson, 'A Merton College Manuscript Reconstructed', pp. 216–17.

49 Snedegar, 'The Works and Days of Simon Bredon', p. 296 n. 34. Only one of these notes (f. 102v) is of a substantial size.

50 Roland, *Die Handschriften der alten Wiener Stadtbibliothek,* p. 117.

The scribe omitted a large section of text from mid IV.12 to mid V.13 on f. 29r. The missing section is placed after the end of the work (53v–65r). This misplaced section matches with folio changes in *K*, so it is clear that this manuscript was copied from *K*. This manuscript contains many marginal corrections, including ones by the scribe and by Johannes Vögelin (as can be seen by a comparison of hands with W_1), but it has only very few notes on the text. Vögelin corrected the text against a second exemplar, probably W_1. He had used W_2 as his exemplar for three other works in W_1.[51] In this manuscript there are additions in V.5 and VI.1 that are not in *K*, and text that is omitted in *K* in V.7 is added in the margins of W_2.[52] There are a few corrections and notes in a later hand, including ones pointing out the correspondence of I.3, IV.16, and V.11 respectively to *Epitome Almagesti* I.4, IV.15 and V.14. Between folios 56 and 57, there is pasted a small leaf with a table of shadow lengths for a gnomon of 12 units, which has no relation to the *Almagesti minor*. The manuscript includes the *De proportionibus* that is perhaps by Walter of Lille.[53] Despite the manuscript's late date, W_2's scribe follows *K* in using Roman numerals normally, but Arabic numerals in I.6.

Group 3.B

M Munich, Bayerische Staatsbibliothek, Clm 56

1434–36. 3r–120r. '... tenebrarum sic se habent. Explicit Almagesti minor finitus anno christi 1434°.'

The scribe calls the work 'Almagesti minor', but it is listed as the 'Almagesti abreviatum per magistrum Thomam de Aquino et continet libros sex' in a table of contents written in perhaps a later hand (1v).

The *Almagesti minor* was finished in 1434 by the scribe, Reinhardus Gensfelder of Nuremberg, who wrote other parts of the manuscript in Salzburg in 1436.[54] In the late 1300s, Reinhardus began to study at the University of Prague, where he became a Master of Arts in 1408. He is known to have been in Salzburg in 1434–36, but he was in Vienna in 1433.[55] So, he was most likely in one of these two cities when he copied the *Almagesti minor*. The manuscript was given by Johannes Fleckel in 1457 to the Dominican convent in Vienna before he made his profession.[56] Fleckel (or 'Flekel') was from Kitzbühel in Tyrol, and he made his determination at University of Vienna on 1 January,

[51] Nothaft, 'The Chronological Treatise *Autores Kalendarii*', pp. 3 and 30.

[52] W_2, ff. 38v (VI.1), 60r (V.5), and 61v (V.7).

[53] W_2, ff. 274r–275v.

[54] *M*, ff. 3r and 153v.

[55] Durand, *The Vienna-Klosterneuburg Map Corpus*, pp. 44–48; and Pilz, *600 Jahre Astronomie in Nürnberg*, p. 50.

[56] *M*, f. 3r.

1438.[57] Although this manuscript contains the alternate and added sections in I.6 that characterize Group 3, a note is added on a small leaf between f. 98 and f. 99 that gives the standard text for the passage that is altered in the main text. This standard text is similar, but not identical, to that of *Pr*. This added leaf says that the corrected text comes from the 'exemplari Magistri Iohannis.' While John is too popular of a name for us to determine to whom this refers, it could be John of Gmunden or one of the several other Master Johns who taught astronomy or mathematics in Vienna, or perhaps it could mean Johannes Andree Schindel, whom Reinhardus probably knew from Prague, and who wrote notes in *Pr*. An addition to II.34 that is on a small leaf added after folio 28 is only found here and in *Pr* and *N*. In addition to these similarities with Schindel's copy of the *Almagesti minor*, the inclusion in this manuscript of the relatively rare *Tractatus de quantitate trium solidorum* of 'Magistri Iohannis Schindl', (as he is named in the table of contents on 1v) gives us reason to entertain the possibility that Schindel let Reinhardus correct his text against one that he possessed. Three other small leaves are inserted among the folios of the *Almagesti minor*: the first after f. 6 contains a note, the second is a blank leaf after f. 65, and the third follows f. 69 and has figures replacing those of V.7, which Reinhardus found unsatisfactory. Besides the aforementioned notes, Reinhardus only wrote a few very short corrections and notes in the margins. This manuscript and *W* contain an alternate text for a passage in VI.1 that suggests a connection with *T* or E_1. The regular text, however, is also given in the margin in Reinhardus' hand. This manuscript uses Arabic numerals.

W Vienna, Österreichische Nationalbibliothek, 5266

1434. 176ra-228va. '... tenebrarum sic se habent et cetera. Explicit Almagesti minor finitus in vigilia conceptionis gloriosissime dei genitricis virginis matris Marie per me Martinum Mospekch artium baccalarium in alma universitate studii Wyenniensi anno domini m° quadringentesimo tricesimoquarto.'

The work is listed in the table of contents as 'Almagesti minor Ptolomei' (1v).

This manuscript was written by several scribes in the first half of the fifteenth century and was owned by Klosterneuburg.[58] The scribe of the portion containing the *Almagesti minor* was Martinus Mospekch (or Mospeck), who also copied the *Almagest* (Klosterneuburg, Stiftsbibliothek, 682) in 1434. Martinus matriculated as a 'pauper' to the University of Vienna in October, 1428, and he had his determination at the University of Vienna on 13 Octo-

[57] *„Wiener Artistenregister" 1416 bis 1447*, p. 110. Fleckel gave another astronomical manuscript written by Reinhardus, Munich, BSB, Clm 10662, to the Viennese Dominicans (see Durand, *The Vienna-Klosterneuburg Map Corpus*, p. 46).

[58] *W*, f. 134r.

ber, 1433.[59] He was later a notary of Friedrich III and a pastor in Perndorf (i.e. Berndorf bei Salzburg), and several documents that he witnessed from 1438 to 1481 exist.[60] Martinus omitted the proof of I.16 and the enunciation of I.17, but he inserted them on a small leaf. There are no marginal notes in *W*. The scribe used Arabic numerals throughout the *Almagesti minor*. As noted above, *M* has an addition unique to it and some members of Group 1, and both *M* and *W* contain an alternate text for a passage in VI.1 similar to that in some members of Group 2. A possible answer for this confused situation is that Reinhardus and Martinus Mospekch were part of a group, perhaps including Schindel, who were studying the *Almagesti minor* in 1434, and among themselves they had manuscripts from Groups 1, 2, and 3 that they were able to consult.

Group 4

Ba **Basel, Universitätsbibliothek, F.II.33**

Mid fourteenth century. 221r–244r. '... tenebrarum sic se habent et cetera. Explicit liber Ieber (erased but still legible) per manus Engelberti. Deo gracias.'

The scribe refers to the work as 'liber Ieber' (232v and 244r), and it is described in the medieval table of contents as 'Parvum Almagesti' on a flyleaf (Iv).

The table of contents shows that the *Almagesti minor* was originally bound at the beginning of the manuscript.[61] Although written in a neat hand, the text in this manuscript would have been very difficult for a reader to understand. The text contains such a great number of nonsensical readings (e.g. 'in 3' for the verb 'intres' in VI.1) that some passages would have made little sense, and further confusion would have been caused by the several mid-sentence jumps to new sections. A frustrated reader wrote at the end, 'Falsissimi scriptoris quia non est verbum correctum nisi fuerit malum exemplar.'[62] Folios were also bound in the wrong order, although the misarrangement likely occurred when the work was moved to the end of the manuscript. The following table shows the order of the text in this manuscript.

Current Foliation	*Almagesti minor*	Correct Sequence
221r to 224v	Preface to II.19	I
225r to 228v	V.19 to VI.10	VII
229r to 231r (line 4)	III.21 to IV.10	III

[59] Gall, *Die Matrikel der Universität Wien*, p. 162; and *„Wiener Aristenregister" 1416 bis 1447*, p. 94.

[60] For example, [1438] Munich, Bayerisches Hauptstaatsarchiv, KU Raitenhaslach, Nr. 667; and [1481] Salzburg, Landesarchive-Urkunden Salzburg, Erzstift (1124–1805), OU 1481 XII 17.

[61] *Ba*, f. Iv.

[62] *Ba*, f. 244r.

231r (line 4) to 231r (line 27)	IV.14 to IV.15	V
231r (line 27) to 232r (line 10)	IV.10 to IV.14	IV
232r (line 10) to 236v	IV.15 to V.19	VI
237r to 240v	II.20 to III.21	II
241r to 244r	VI.10 to VI.25	VIII

The misplacement of sections IV and V was perhaps the result of carelessly copying another manuscript with misplaced folios. The resulting chaos would have made it especially difficult, if not impossible, for a reader to understand IV.10–15, especially IV.10, 14, and 15. This manuscript has its own proofs for *Almagesti minor* I.1–9. It lacks most of I.13. This manuscript has a few short notes and corrections in the margins in more than one hand. The scribe normally uses Arabic numerals, but occasionally he uses Roman numerals.

Chapter 6

Manuscripts Containing Excerpts of the *Almagesti minor*

Eleven manuscripts contain passages of *Almagesti minor* I.15, V.1, and V.11, discussing the construction and use of instruments. Several of these can be grouped. The first six manuscripts are related, as can be seen by the short addition at the end of the second excerpt. Four of these manuscripts form a subgroup, the members of which contain three excerpts. The first excerpt is *Almagesti minor* V.1 without the enunciation. The second is the passage on the second instrument of *Almagesti minor* I.15 with a small addition to the last sentence. The third is taken from *Almagesti minor* V.11. It follows the standard text of the first two paragraphs with only small changes, but it then paraphrases the third paragraph, using little of the wording of the *Almagesti minor*. The variants in these excerpts are most similar to those of Group 2.

Milan, Biblioteca Ambrosiana, H.75 sup.

Between 1284 and the early fourteenth century. 67ra-68rb. 'Incipit tractatus de compositione armillarum. Queruntur primum due armille convenientis mensure …' '… vicinior vero est consideratio' (67ra-va); 'Incipit compositio instrumenti per quod habetur tropicorum et remotio sumitatis capitum ab equinoctiali. Sume laterem ligneum vel lapide⟨u⟩m …' '… diligenter attende et per hoc distantiam tropicorum et remotionem summitatis capitum ab equinoctiali contemplaberis si Solis umbram in omni meridie circa maxime solsticium observaveris' (67va-vb); and 'Compositio instrumenti per quod reperitur diversitas aspectus Lune in latitudine. Sumantur tres regule recte et planissime …' '… arcus inquam deprehensus inter locum latitudinis Lune et visum locum Lune' (67vb-68rb).

The manuscript, which includes a work authored in 1284, was owned in the sixteenth century by Gian Vincenzo Pinelli.[1]

Paris, Bibliothèque de la Sorbonne, 595

Fourteenth century. 62ra-63vb. 'Incipit opus armillarum Ptholomei. Regula. Queruntur primo due armille convenientis mensure …' '… vicinior est consideracio' (62ra-63ra); 'Opus instrumenti declinationis Solis. Sume laterum ligneum vel lapideum …' '… et remocionem summitatis capitum equinoctiali

[1] Milan, Biblioteca Ambrosiana, H.75 sup., f. Ir.

contemplaberis si Solis umbram in omni meridie circa utrumque maxime solsticium observaberis vel observaveris' (63ra-rb); and 'Instrumentum diversitatis aspectus Lune. Sumantur tres regule recte et planissime quadrilatarum superficierum …' '… in ipsa hora elevetur linea HM et revolvatur linea FL tamdiu donec per utrumque foramen Luna compa-' (63rb-vb).

The text is cut off midword in the paraphrase of the third paragraph of *Almagesti minor* V.11. There are figures of the second instrument of *Almagesti minor* I.15 and V.11.

Munich, Bayerische Staatsbibliothek, Clm 10661

Fifteenth or sixteenth century. 171r–172r. 'Incipit opus armillarum. Queruntur primum due armille mensure orbiculares …' '… vicinior vero est consideratio' (171r); 'Opus instrumenti declinationis Solis. Summe laterem ligneum vel lapideum …' '…et remotionem summitatis capitum ab equinoctiali contemplaberis et si Solis umbram in omni meridie circa utrumque maxime solstitium observaveris' (171v); and 'Opus instrumenti quo latitudo Lune et distantia centri Lune a terra deprehenduntur. Summantur tres regule recte et planissime …' '… arcus inquam deprehensus inter locum latitudinis Lune et visum locum Lune. Finis. Amen' (171v–172r).

These passages are accompanied by marginal notes, and there are figures accompanying the two last excerpts. Where there is a reference in the text to 'in libro primo', the scribe added 'scilicet Almagesti' above the line, which is an indication that the scribe was not copying these excerpts from a manuscript with the whole *Almagesti minor*.

Oxford, Bodleian Library, Canon. misc. 61

Fifteenth century. 9r–10v? 'Opus armillarum. Queruntur primo 2 armille convenientis mensure …' (9r); 'Opus instrumenti declinationis Solis cum figura. Sume laterem ligneum vel lapideum …' (9v); and 'Opus instrumenti quo latitudo Lune et distancia centri Lune a terra deprehenditur. Sumantur 3 regule recte et planissime …' (10r).

Having not been able to see this manuscript, I here rely upon a short catalogue description.[2] It is clear, however, that these texts are *Almagesti minor* V.1, the passage on the second instrument in I.15, and V.11, and from a comparison of the headings and incipits with those in Munich, BSB, Clm 10661, it is almost certain that the excerpts match the others in this group.

The following two manuscripts have excerpts of *Almagesti minor* V.1 without its enunciation and the description of the second instrument of I.15 with the same small addition to the last sentence that is found in Milan, BA, H.75 sup., Paris, BS, 595, and Munich, BSB, Clm 10661.

[2] Coxe, *Catalogi Codicum Manuscriptorum Bibliothecae Bodleianae.* Part 3, col. 473.

Paris, Bibliothèque national de France, lat. 7295

1430–50. 77r-v. 'Opus armillarum Ptolomei capitulo primo dictionis quinte Almagesti. Querantur primo due armille convenientis mensure …' '… vicinior vero est consideracio' (77r); and 'Composicio instrumenti declinacionis Solis. Opus instrumenti declinationis Solis. Sume laterem ligneum vel lapideum …' '… et remocionem summitatis capitum equinoxialis comtemplabis si Sol umbram in omni meridie circa utrumque maxime solsticium observaveris' (77r-v).

Much of this manuscript, including the folia with these excerpts, was written by Henricus Arnault (or Henri Arnaut) de Zwolle, who was a student of Jean Fusoris and physician to the Duke of Burgundy. Henricus is known to music historians for his notes on and drawings of musical instruments in this manuscript, but he clearly shared his teacher's interest in astronomical instruments. In fact, Jean Le Fèvre, who owned the manuscript in the sixteenth century, describes Henricus as physician of the dukes of Burgundy and an 'astrologus profundissimus.'[3]

Salamanca, Biblioteca Universitaria, 2662

Late fourteenth century. 49va-50ra. 'Incipit opus armillarum. Queruntur primo due armille convenientis mensure …' '… vicinior est consideratio' (49va-50rb); and 'Opus instrumenti declinationis Solis. Sume laterem ligneum vel lapideum …' '… et remotionem summitatis capitum equinociali contemplaberis si Solis umbram in omni meridie circa utrumque maxime solstitium observaberis vel observaberi' (50ra).

The following pair of manuscripts form another group. They both have *Almagesti minor* I.15 and V.1 in their entirety. Given that the excerpts in the first of these manuscripts were copied by the scribe of *M*, it is not surprising that they also belong to Group 3.B.

Vienna, Österreichische Nationalbibliothek, 5418

1433–34. 184r–189v. 'Maximam declinacionem per instrumenti artificium et consideracionem reperire. Paratur itaque lamina quadrate forme …' '… et magis visui quam auditui credendum' (184r–185v); and 'Locum stelle secundum longitudinem et latitudinem artificio instrumenti deprehendere. Queruntur primum due armille …' '… vicinior vero est consideracio. Explicit compositio et utilitates armillarum' (187r–189v).

This manuscript was written in 1433–34 by Reinhardus Gensfelder, who also was the scribe of *M*.[4] These excerpts are listed in a table of contents on f. Iv as 'tractatus de composicione instrumentorum inventionis maxime declina-

[3] Paris, BnF, lat. 7295, lat. 7295, f. 1r.

[4] Vienna, ÖNB, 5418, ff. 24v, 110r, 124r, and 204v.

cionis' and 'tractatus de composicione armillarum cum suis utilitatibus.' The instruments for finding the ecliptic's declination are depicted on ff. 184v, 185r, and 186r. Ff. 186v and 188r-v are blank. The section of *Almagesti minor* V.1 on the use of the armillary sphere, which begins, 'Constructo tandem et secundum hunc modum …', is introduced with the title 'Sequitur utilitates prefati instrumenti.'[5]

Vienna, Österreichische Nationalbibliothek, 5303

c. 1519–20. 256r–259r. 'Maxima declinacionem per instrumenti artificium et constructionem reperire. Paratur itaque lamina …' '… et magis visui quam auditui credendum' (256r–257v); and 'Locum stelle secundum longitudinem et latitudinem artificio instrumenti deprehendere. Queruntur primum due armille …' '… vicinior vero est consideracio. Explicit compositio et utilitates armillarum.' (258r–259r).

The excerpts in this manuscript are almost definitely copied from Vienna, ÖNB, 5418. Not only are the excerpts the same, but they have identical explicits and the same figures depicting the instruments for observing the ecliptic's declination. From f. 130r to 279r, this manuscript includes works contained in Vienna, ÖNB, 5418 in the same order and even with some of the same marginal notes and colophons referring to the years 1433–34.[6] This manuscript is written in several hands, but these excerpts appear to be in the same hand as the *Albion*, which was written in 1519–20 (see ff. 351v and 359v).

The following three manuscripts have excerpts relating to instruments, but show no manifest connection to each other or to the manuscripts listed above.

Oxford, Bodleian Library, Ashmole 345

Fourteenth century. 21r–22r. 'Queritur primum due armille convenientis mensure …' '… ubi diversitas aspectus non impedit vicinior est consideracio.'

This excerpt consists of *Almagesti minor* V.1 without its enunciation. The text is closest to Group 2.

Vienna, Österreichische Nationalbibliothek, 5258

2nd half of the fifteenth century. 75r–77r. 'Instrumenta observatoria que in Almagesto ponuntur. Almagesti abbreviato libro primo capitulo 15mo docetur de instrumento per quod maxima declinatio Solis reperitur. Et est in forma ista propositio. Maximam declinationem per instrumentum artificium et considerationem reperire…' '… et magis visui quam auditui credendum' (75r-v); 'Alma-

5 Vienna, ÖNB, 5418, f. 189r.
6 Vienna, ÖNB, 5418, ff. 143r, 146r, 147r, 223, and 271v.

gesti minori libro quinto capitulo primo docet instrumentum armillarum fieri. Et est propositio. Locum stelle secundum longitudinem et latitudinem …' '… vicinior vero est consideratio' (75v–76v); and 'In eodem, capitulo undecimo libri quinti. Latitudo Lune maxima qualiter per instrumentum deprehendi potuit patefacere…' '… et similiter ex altera parte orbis signorum cognita est' (76v–77r).

The excerpts are from the *Almagesti minor* I.15, V.1, and V.11, and they are described in a table of contents as 'De instrumentis observatoriis que in Almagesto ponuntur.'[7] A later hand added the name 'Albategni' at the top of 75r. While the second passage follows the standard text closely, the first and third passages have many omissions and paraphrases. The excerpts are closest to Group 1.B. The manuscript was partially written by Regiomontanus, but he did not write the excerpts from the *Almagesti minor*. The manuscript was owned by Willibald Pirckheimer, who sold it to Johannes Schöner in 1522.[8]

Jena, Thüringer Universitäts- und Landesbibliothek, El. f. 73

Early sixteenth century, before 1536. 182ra-vb. 'Locum stelle secundum longitudinem et latitudinem artificio instrumenti deprehendere. Queruntur primum due armille …' '… et ita locum longitudinis et latitudinis ut prius cognosces.'

This excerpt consists of *Almagesti minor* V.1 except the last paragraph and with a small addition at the end of the second paragraph. The division of the text into columns at the bottom of f. 182v makes it difficult to realize in what order the text is to be read. The text is closest to Group 3.A although there are not enough variants in this short passage to be certain about this. This manuscript was written by Johann Volmar, who studied at the University of Cracow and taught at the University of Wittenberg from 1519 until his death in 1536.

The following pair of manuscripts with Gerard of Cremona's translation of the *Almagest* have excerpts of the *Almagesti minor*.

Florence, Biblioteca Medicea Laurenziana, Plut. 89 sup. 57

2nd half of thirteenth century, before 1295. 8v, 9v–10v, 29r–30r, 34r–35r, 36r–38r, 49v–53r, 54r, 55r, 56r–58v, 59v, 65r-v, 67v, 69r–71v, 74r, and 88v. The excerpts begin with I.1's enunciation: 'Data circuli diametro …'; and they end with V.19's enunciation, '… a cenith capitum elongationem certam demonstrare.'

This manuscript of Gerard's translation of the *Almagest* breaks off mid-sentence in *Almagest* VI.5. The manuscript's margins contain many notes, including excerpts from the *Almagesti minor*. One note provides the sun's place according to its mean motion for the middle of the year 1295 at the Porta

[7] Vienna, ÖNB, 5258, f. 1r.
[8] Vienna, ÖNB, 5258, notes on inner front and back covers.

Latina, presumably the gate in Rome, and a colophon added in another hand states that this was the book of a Magister Thadeus Arduvinis de Florentia.[9] This perhaps refers to Taddeo Alderotti, who was a well-known professor of medicine at the University of Bologna, who was born between 1206 and 1215 and who died at the beginning of June, 1295.[10] However, if he were the owner, two things appear odd: first, the inclusion of 'Arduvinis', which is not a normal name for Alderotti, and second, the marginal calculation performed for a location in Rome regarding the sun only one month after Taddeo Alderotti's death. A more likely scenario is that the Thaddeus mentioned is the Thaddeus of Florence who wrote a letter probably after 1320 and before 1348, complaining that he damaged his eyes by looking at an eclipse.[11] The excerpts from the *Almagesti minor* belong to Group 3.A and consist almost entirely of the enunciations of I.1–6, II.16-V.1 (except II.28, III.1–2, IV.4, and IV.6), and V.19. When applicable, the corollaries are included, except that II.29 only has two words of the corollary.[12] There are occasionally excerpts of more than the enunciations. II.27 contains the first word of the proof, III.23 has the first sentence of the proof, and III.24 offers the complete proposition.[13] IV.19 is preceded by a statement that Ptolemy did not address the topic at hand, and it is followed by a summary of the proof.[14] The enunciations are sometimes numbered, and while these often correspond to the standard numbering, there are frequent discrepancies. There are other enunciations among the marginalia that do not come from the *Almagesti minor*. For example, in addition to the enunciation of *Almagesti minor* I.4, there is another enunciation for the same proof in different words, and, while the enunciations of *Almagesti minor* I.7–8 are not included, there are other enunciations for these proofs that share little similarity in wording to those of the *Almagesti minor*.[15]

Oxford, New College, 281

Fourteenth century. 28r–30r, 32v, 48r–48v, 49v–50v, 51v–54r, 55v, 56v–58r, and 76v–77r. The excerpts begin with II.16's enunciation: 'Propositio 16[a]. Cuiuslibet porcionis circuli declivis ascensionem in spera declivi invenire. Ecce ratio operationis …'; and they end with IV.19's enunciation: 'Propositio 19. Non

[9] Florence, BML, Plut. 89 sup. 57, ff. 57r and 100v.

[10] Siraisi, *Taddeo Alderotti and His Pupils*.

[11] Siraisi, *Taddeo Alderotti and His Pupils*, p. 42 n. 81. This letter is found in Mattioli, *Il Beato Simone Fidati*, pp. 436–38. Mattiola suspected that the Thaddeus who wrote the letter is the artist Taddeo Gaddi, but this seems unlikely since Gaddi continued to paint after Simone Fadati's death, apparently with full use of his sight.

[12] Florence, BML, Plut. 89 sup. 57, f. 36r.

[13] Florence, BML, Plut. 89 sup. 57, ff. 36r and 56v.

[14] Florence, BML, Plut. 89 sup. 57, f. 71v.

[15] Florence, BML, Plut. 89 sup. 57, ff. 10r and 17v.

facit Ptholomeus intentionem huius propositionis scilicet ex premissis sequitur. Medium motum capitis draconis elicere.'

These excerpts are found among the marginalia to Gerard of Cremona's translation of the *Almagest* in Oxford, New College, 281. The commentator refers to the source of the excerpts: 'in parvo Almagesti.'[16] These excerpts consist of enunciations, with their corollaries when applicable. These are not all numbered, but those with numbers match those of the *Almagest*. The enunciations included are II.16–18, II.21–26, II.33, III.3-IV.1, IV.17–19.[17] The corollary of II.26 is incomplete.[18] Only a few enunciations are accompanied by further commentary. For example, after the enunciation of IV.19, the glossator notes that this proposition has no corresponding passage in the *Almagest* and provides a paraphrase of *Almagesti minor* IV.19's proof.[19] A number of variants show a close connection between these excerpts and those in Florence, BML, Plut. 89 sup. 57, and some of the excerpts are accompanied by the same short commentary in both manuscripts. For example, before the enunciation of *Almagesti minor* II.20, both add, 'Non ponitur manifeste in littera, sed ex prehabitis potest haberi.'[20] Both manuscripts also share some of the same marginal and interlinear notes that are not related to the *Almagesti minor*.[21] As all of the excerpts in this manuscript are also found in the Florence, BML, Plut. 89 sup. 57, this manuscript is able to directly descend from that one.

Lastly, the following two manuscripts each have different excerpts from the *Almagesti minor* that are not related to instruments.

Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 2° 375

Mid to late fourteenth century. 85r–86v, 93v–94v, 97r-v, and 103r. The excerpts begin with I.1's enunciation: 'Data circuli dyametro ...'; and they end with II.15's enunciation: '... note erunt ascensiones omnium.'

The excerpts of the *Almagesti minor* are in the margins of a section of Gerard's translation of the *Almagest* (I.9 to mid II.12) that are found on ff. 85r–88r, 93r–100v, 102r–112v (f. 101 is an inserted folio with the scribe's notes on the *Almagest*). The marginalia is in the scribe's own hand, and he acknowledges their source: 'Parvi Almagesti breviato cum commento.'[22] The excerpts consist of the enunciations and proofs of I.1–7 and the enunciations

[16] Oxford, New College, 281, f. 52r.

[17] The first is found on Oxford, New College, 281, f. 28r and the last is on f. 77r.

[18] Oxford, New College, 281, f. 30r.

[19] Oxford, New College, 281, f. 77r.

[20] Oxford, NC, 281, f. 57r and Florence, BML, Plut. 89 sup. 57, f. 56r.

[21] For example, the same note starting 'Diversitas secundum elevationes ...' is found on Oxford, NC, 281, f. 57r and Florence, BML, Plut. 89 sup. 57, f. 56v.

[22] Erfurt, UFB, Dep. Erf. CA 2° 375, f. 85r.

of I.8–14, I.17, II.2–5, and II.14–15. The proofs often stray from the standard version of the text, following instead the rewording of Peter of Limoges in P_{16}, and as in P_{16} only the first paragraph of I.6's proof is given. Moreover, much of the marginalia that is not taken from the *Almagesti minor* and many interlinear notes match ones that Peter wrote in P_{16}; therefore, it is probable that the *Almagest* and the excerpts from the *Almagesti minor* were copied directly from that manuscript. There are a few other generalized enunciations in the marginalia, but they do not come directly from the *Almagesti minor* or are so altered that they can no longer be seen as excerpts; for example, for I.16 we find, 'Cuiuslibet puncti ecliptice declinationem per catham coniunctam invenire', which is similar to the standard enunciation, but only shares four words with it and lacks the corollary.[23] This section of the *Almagest* in this manuscript is followed by the 'Erfurt Commentary' (113r–126v), which is described in the following chapter, and a short super-commentary upon that (127r–129v). Of the *Almagest*'s tables, only two incomplete columns of the table of arcs and chords were inserted by the scribe. He left space for the remainder of this table and for the other tables. Also, few diagrams after *Almagest* II.5 are provided.

Cambridge, University Library, EE 3.61

Sixteenth century. 55r-v. 'Libro 5 parvi Almagesti propositione 18. Elongationem Lune a centro terre cognosces iuxta terminos prius positos, et quum eam habueris, a quolibet gradu epicycli unum minutum …' '… erit diversitas aspectus in circulo altitudinis' (55r); 'Item propositione 20 eiusdem. Scias angulum ex circulo altitudinis et orbe signorum causatum …' '… et habes diversitatem aspectus in longitudine' (55r); and 'Item propositione 24. Queres primo arcum distantie gradus ecliptice in quo est Luna …' '… et illud quoque rarissime eveniet' (55r-v).

Although most of this manuscript was written in the fifteenth century, notes related to the *Almagest*, beginning with 'Kata coniuncta potest haberi per numerum ut patet per triangulum …' (54v), were added to the manuscript in the sixteenth century.[24] After a note on the Menelaus Theorem and a paraphrase of a passage in *Almagest* V.17, there are the three passages taken from the *Almagesti minor*. The first, which is said to be from V.18, paraphrases and takes excerpts from the paragraph of V.19 that begins 'Cum autem per operationis methodum…' It tells how to calculate the moon's parallax on the circle of altitude. The second excerpt, which is said to be from V.20, is an excerpt

[23] Erfurt, UFB, Dep. Erf. CA 2° 375, f. 95r.

[24] Hardwick, Babington, *et al.*, *A Catalogue of the Manuscripts Preserved in the Library of the University of Cambridge*, vol. II, p. 114, dates both the manuscript and another note (f. 7v) that is in the same hand as the excerpts from the *Almagesti minor*.

from V.21's paragraph starting 'Operationis modus est …' It concerns the calculation of the moon's parallax in longitude when it is on the ecliptic. The third excerpt, which is said to be from V.24, consists of excerpts and paraphrases of the last two paragraphs of V.25, which concern the angles needed to find the parallax in longitude and latitude when the moon is not on the ecliptic. These excerpts are copied loosely with short omissions and many rewordings, so it is not clear from which group of witnesses they were derived.

Some other manuscripts contain excerpts of the *Almagesti minor*, but because these are minor or are incorporated into larger works, they will be treated in the following chapter.

Chapter 7

Influence of the *Almagesti minor*

The *Almagesti minor* had an impact upon a large number of astronomical works of the thirteenth-fifteenth centuries.[1] Its use in Guillelmus Anglicus' *Astrologia* has been discussed above. Some findings and descriptions of these works are provided here, but most require much further study, and many more examples of the *Almagesti minor*'s influence will surely come to light in the future. Already, the impact of the *Almagesti minor* upon the astronomy of the centuries following its writing is manifest. There are approximately 20 works, surviving in more than 140 manuscripts, that include excerpts from the *Almagesti minor*, summarize parts of it, or reference it.

Almagest Manuscripts

The *Almagesti minor* had an influence upon *Almagest* manuscripts. Manuscripts of Gerard's translation of the *Almagest* contain references to the *Almagesti minor*, excerpts from it, and passages that are very similar to those in the *Almagesti minor*. Four such manuscripts, Paris, BnF, lat. 16200, Florence, BML, Plut. 89 sup. 57, Oxford, NC, 281, and Erfurt, UFB, Dep. Erf. CA 2° 375, have such extensive excerpts of the *Almagesti minor* in their margins that they were described above in the sections on manuscripts containing the *Almagesti minor* or excerpts of it. Additionally, Cracow, BJ, 619, which contains notes of Johannes Andree Schindel is described later in this chapter.

There is also a set of notes based upon the *Almagesti minor* included among the marginalia found in Paris, BnF, lat. 7256 and almost identically in Vatican, BAV, Barb. lat. 336. Incidentally, these manuscripts' marginalia are especially noteworthy because they contain Campanus of Novara's gloss upon the *Almagest*.[2] The enunciations in these manuscripts are only found for Books I–II. While about 10 of the enunciations are taken from the *Almagesti minor* with no (or only trivial) modifications and several others show a dependency upon the wording of the *Almagesti minor*, some are worded very differently. The order in which the enunciations are given is also different, as the glossator follows the order in which topics are treated in the *Almagest*, even when the *Almagesti minor* presents them in another order. The glossator also occasionally joins two of the *Almagesti minor*'s enunciations into one or separates one

[1] Lorch, 'Some Remarks on the *Almagestum parvum*', pp. 421–23, briefly discussed several of the following works.

[2] Zepeda, 'Glosses on the *Almagest*.'

into two. He adds one enunciation with a lengthy corollary, II.5, that has no corresponding enunciation in the *Almagesti minor*.[3] A few enunciations (I.10, II.20, II.25, and II.34) are part of notes that include proofs or added commentary, but these are not taken from the *Almagesti minor* and at least two of the proofs were composed by Campanus.

Correspondence of Enunciations

Paris, BnF, lat. 7256 & Vatican, BAV, Barb. lat. 336	*Almagestum parvum*
Book I	Book I
1–2	1
3–7	2–6
8	15
9–10	7–8
10 (bis)	9
11–15	10–14
16–17	16–17
Book II	Book II
1–2	2–3
3	1
4	4
5	--
6	5
7–8	6
9–20	7–18
21–22	19
23	20
24	19
25–35	21–31
36	32–33
37–39	34–36

[3] Paris, BnF, lat. 7256, f. 13r: 'Dato puncto orbis signorum arcum orizontis interceptum inter ortum eius et ortum equatoris in regione cuius latitudo sit data investigare. Unde manifestum est quod cognito loco Solis scietur differentia diei illius et diei equalis. Patet iterum quod si sinum latitudinis regionis ducatur in sinum declinationis puncti orbis signorum dati et productum dividatur per sinum complementi declinationis eiusdem; itemque quod exierit ducatur in sinum quarte et productum dividatur per sinum complementi latitudinis regionis, exibit sinus medietatis excessus dierum equalis et illius. Adhuc quoque manifestum est quod si sinum declinationis puncti eiusdem ducas in sinum quarte et productum dividas per sinum complementi latitudinis regionis, exibit sinus arcus orizontis intercepti inter ortum puncti illius et equatoris.'

References to the *Almagesti minor* are also found in other *Almagest* manuscripts. First, a reference to the *Almagesti minor* is found in Paris, BnF, lat. 7257, a manuscript of the thirteenth century. The glossator wrote a long note on how one finds one of the six quantities in a statement of composition (i.e. when it is known that a ratio is composed of two other ratios) when the other five are known. After this complex discussion, he writes, 'Facilius tamen fient omnes hee operationes per regulas Minoris Almagesti.'[4] The commentator also adds a note to *Almagest* VI.6 concerning parallax: '… patet in parvo Almagesti in commento illius † rationis† Solis eclipsim iterari.'[5] There is at least one other note that is likely based upon the *Almagesti minor* – in what appears to be the scribe's hand, there is a note reporting Albategni's values for the sun and moon's diameter at their respective apogees and perigees.[6] Secondly, a reference to the *Almagesti minor* is found among the notes in another *Almagest* manuscript, Vatican, BAV, Pal. lat. 1365, which was written by Mengotus Itelbrot in France in 1385. In a note on finding declinations of arcs of the ecliptic, Mengotus writes, 'Sed auctor Minoris Almagesti faciliorem ponit operationem et sunt hec verba eius: "Si sinus inchoate portionis ab equinoctiali cuius finalis puncti declinatio queritur, ducatur in sinum maxime declinationis, productum dividatur per sinum quarte, exibit sinus quesite declinationis."'[7] Thirdly, in Cracow, BJ, 589, finished in 1495, the scribe, Henricus Griffinus Ragnetensis, provides the enunciation of *Almagesti minor* I.1, the last definition of *Almagesti minor* II, and the enunciation of II.21.[8] Perhaps drawing upon *Almagesti minor* III.1, Henricus also reports a value for Albategni's length of the year.[9] Lastly, another possible use of the *Almagesti minor* is found in Melbourne, State Library of Victoria, RARES 091 P95A, f. IIv. In the fly leaves of this thirteenth-century manuscript containing the *Almagest*, later scribes added Campanus' *De figura sectore* and what are called 'conclusiones Almagesti.' These are enunciations and a few outlines of proofs, most of which correspond to ones in *Almagest* I.9. The first enunciation is similar to that of the *Almagesti minor*, but the others show less similarity.[10] There are also more enunciations than in the correspond-

[4] Paris, BnF, lat. 7257, f. 10r.

[5] Paris, BnF, lat. 7257, f. 49v.

[6] Paris, BnF, lat. 7257, f. 54v.

[7] Vatican, BAV, Pal. lat. 1365, f. 14r.

[8] Cracow, BJ, 589, ff. 6r and 23v. The date and the scribe's name are found on f. 206v.

[9] Cracow, BJ, 589, f. 39v.

[10] I give the text of the first two for the sake of comparison: 'Data circuli dyametro latus decagoni, exagoni, pentagoni, tetragoni, triangulique reperire. Hec probatur per sexta secundi, per unam sexti, per penultimam primi, per 8, 9, 10 tercii decimi, et alia media. Si quadrangulo circulus inscribatur, quod fit ex ductu dyametrorum inse equum est ei quod fit ex ductu oppositorum laterum inse. Hec probatur per 4 et 15 sexti.'

ing sections of the *Almagesti minor*, and its use of versed sines is also quite a difference from the usual trigonometry of the *Almagesti minor*.

Robert Grosseteste's *Compotus*

Robert Grosseteste, the renowned English bishop, theologian, and philosopher, is thought to have written his *Compotus* in 1220–25, perhaps in Paris, and his work shows some of the earliest use of the *Almagesti minor*.[11] The *Compotus* exists in at least 29 manuscripts and was printed in Venice in 1518.[12] In the first chapter, Grosseteste includes a definition of the year: 'Annus est reditio solis ab aliquo puncto in zodiaco fixo ad idem punctum, ut ab eodem solsticio ad idem solsticium, vel ab eodem equinoctio ad idem equinoctium', which is very similar to that found in *Almagesti minor* III.1.[13] Grosseteste goes on to discuss varying lengths of the year given by Hipparchus, Ptolemy, Albategni, and Thebit, reflecting the similar discussion in *Almagesti minor* III.1.[14] *Compotus* Ch. 4 contains two definitions taken from the *Almagesti minor* IV's definitions of the lunar month and of a mean lunation; Grosseteste writes, '… et est mensis lunaris tempus equalis lunationis. Equalis autem lunatio est reditus lune ad solem secundum utriusque cursum medium.'[15]

Bishop Guillelmus' *Tractatus super armillas*

Fermo, Biblioteca communale, 85, ff. 110r–113v, contains a treatise on the armillary sphere written by Guillelmus, the bishop of Laon, in 1264. It begins 'Incipit tractatus in compositione et opere armillarum ad inveniendum loca planetarum et aliarum stellarum. Querantur due armille orbiculares convenientis mensure …' In this witness it ends:

> … sicut superius fuerit predeterminatum. Explicit tractatus Guillelmi episcopi Laudunensis super armillas scriptus anno domini 1263 perfecto et de inperfecto menses 10 dies 10, cuius finis fuit vigilia beati Martini episcopi, quod est 4 idus novembris. Et in illo anno imperfecto fuerunt multe coniunctiones planetarum et multe impressiones, et apparuit una de cometis in Cancro que Dominus Ascone appellatur, a qua exibat radius in longitudine 90 graduum, que exivit a zodiaco gradiens contra stellas

[11] These findings about Grosseteste's use of the *Almagesti minor* have been confirmed by Philipp Nothaft, who has been working with Alfred Lohr on a new edition and translation of the *Compotus*: Alfred Lohr and C. Philipp E. Nothaft, *Robert Grosseteste: Compotus; Edition, Translation, Commentary*, expected 2019. An older edition is found in Steele, *Opera hactenus inedita Rogeris Baconis*, pp. 212–67.

[12] S. Thomson, *The Writings of Robert Grosseteste, Bishop of Lincoln 1235–1253*, pp. 95–96.

[13] Steele, *Opera hactenus inedita Rogeris Baconis*, pp. 213.

[14] Steele, *Opera hactenus inedita Rogeris Baconis*, pp. 214–16.

[15] Steele, *Opera hactenus inedita Rogeris Baconis*, p. 232.

> et signorum successionem ultra Arietis regionem. In anno sequenti fuit coniunctio Saturni et Iovis in signo humano in regione videlicet Grecorum.[16]

Guillelmus, who had been made bishop in 1261 and who died *c.* 1270, perhaps became interested in making an armillary sphere because of the Great Comet of 1264. His text relies heavily upon *Almagesti minor* V.1, incorporating the bulk of this proposition, but Guillelmus adds a much greater level of detail about the instrument's construction and use. Because *Almagesti minor* V.1 often circulated by itself or with other excerpts concerning instruments, it is possible that Guillelmus had access only to such an excerpt from the *Almagesti minor*.

Glosses to Canons for the Toledan Tables

Oxford, Bodleian Library, Auct. F.3.13, ff. 201r–219v, contains the canons on the Toledan tables that begin, 'In nomine domini scito quod annus lunaris …', and the text is accompanied by many marginal notes.[17] These notes were probably written in Oxford in 1271, as there are notes giving the distance between Oxford and Toledo and conversions for 25 March, 1271.[18] In the margin by a canon on the sun's apparent diameter, the glossator refers the reader to a proposition of the 'libri quinti Abreviati Almagesti.'[19] Additionally, in a note which is also found accompanying the same canons in Paris, BnF, lat. 7281, f. 24r, the glossator gives a rule for finding the place of the sun and moon at a true conjunction more accurately. He attributes this rule to Albategni, but his source is the fourth paragraph of *Almagesti minor* VI.3, not *De scientia astrorum*, as is clear from a comparison of the corresponding passages.

De scientia astrorum Ch. 42[20]	*Almagesti minor* VI.3	Gloss[21]
Quod si locus Solis a Lunae loco differt, superfluum quod inter eos ex gradibus minutis accipe,	Opus vero *Albategni* est ut si non convenerint Sol et Luna in eodem minuto post equationes premisso modo factas, *distantia* que inter eos *reperta* fuerit *sumatur*.	Secundum *Albategni* verius fit equatio veri loci Lune equando prius portionem Lune sic. *Sumatur distantia* inter vera loca *reperta* hic per opus canonis,

[16] Fermo, Biblioteca comunale, 85, f. 213v.

[17] Pedersen, *The Toledan Tables*, Canons Ca, pp. 189–323.

[18] Oxford, Bodleian Library, Auct. F.3.13, ff. 201r and 215r.

[19] Oxford, Bodleian Library, Auct. F.3.13, f. 217r. The proposition number in the reference is not visible in the reproductions to which I have access, but the context (Pedersen, *The Toledan Tables*, Ca185, pp. 304–05) suggests that it is *Almagesti minor* V.18.

[20] Albategni, *De scientia stellarum*, 1537 ed., ff. 59r-v.

[21] Oxford, Bodleian Library, Auct. F.3.13, f. 212r.

	Et per eam portio equetur videlicet *duplica*ndo distantiam et per eam *accipi*endo equationem portionis que et *puncti equatio* dicitur, et *addend*o eam *super portionem si coniunctio vera futura est post media*m vel subtrahendo si post.	*et per eam duplica*tam *accipi*atur *equat*[io][22] *puncti* in tabula equationis Lune, que quidem equatio est distantia in epiciclo inter augem mediam et veram. Et hec equatio *addend*a est *super portionem si* distantia fuerit Solis, tunc enim *vera coniunctio* vel preventio *futura est post medi*u*m*. Et eadem equatio est minuenda a portione si distantia fuerit Lune. Et sic habetur portio equata.
et eorum sextam octavamque partem addisce. Quod si superfluum ex Sole fuerit, illius sextam et octavam portionem Lunae superadde. Quod si Lunae fuerit, ex ea deme. Et quod post augmentum vel diminutionem Lunae portio fuerit, erit portio aequata.	Quod si velis, *distantie reperte* sextam et octavam *par*tem *accip*e. Nam *hec est fere equatio addenda vel* subtrahenda *portioni* sicut experientia temptatum est.	Vel sic potest equari portio, ut *distantie reperte* sexta et octava *par*s *accipi*atur quia *hec est fere equatio addenda portioni vel* minuenda ab ea.
		Sexta vero pars et octava accipi poterit multiplicando distantia ipsam per 14 et productum dividendo per 48, cuius ratio patet per 19 propositionem septimi Euclidis et per hoc quod 14 sunt sexta et octava pars 48.
Intra ergo cum ea in tabulam aequationis Lunae in duas numeri lineas, et quod in eius directo fuerit ex aequatione simplici in secunda tabularum descripta sume. Et [si] haec portio minus 180 fuerit, hanc aequationem ex aequali motu Lunae et ex motu latitudinis minue; si vero plus 180 portio fuerit, eis superadde. Et quod aequalis Lunae motus post	Per hanc ergo *equata*m *portione*m simplicem equationem Lune sumens, locum Lune ut prius verifices *addendo* scilicet *vel* subtrahendo simplicem equationem *medio* cursui Lune. Et loco Lune sic verificato uteris vice prioris verificationis, verificationem vero Solis non mutabis.	Cum *portione* igitur sic *equata* intrandum est in tabulam equationis Lune. Et eandem *addendo vel* minuendo de *medio* loco, ut docetur hic in canone, habebitur verus locus Lune verius secundum Albategni quam secundum doctrinam canonis hic, que quidem doctrina consona est doctrine Ptholomei.

[22] The note has 'equata.'

augmentum vel diminutionem fuerit, erit locus Lunae verus.		
… Post hoc superfluum quod inter Solem et Lunam fuerit per Lunae superfluum partire. Et quod ex horis vel ex unius horae parte fuerit, erunt horae superflui, serva eas.	*Distantiam itaque Solis et Lune hoc modo repertam divides per veram super*l*ationem Lune,* et operaberis per cetera ut prius.	*Distantia*[m] *itaque* inter verum locum *Solis* et verum locum *Lune hoc modo repertam divides per veram superationem Lune* ut docetur in canone.

Other notes outlining methods for finding the location of the sun and moon at true conjunctions are similar to other ones in *Almagesti minor* VI.3, and are likely derived from the *Almagesti minor*.[23]

John of Sicily's *Scriptum super canones Azarchelis*

John of Sicily used the *Almagesti minor* often in his *Scriptum super canones Azarchelis de tabulis Toletanis*, which he wrote in Paris between 1290–95.[24] The work, which exists in 12 manuscripts, was a commentary upon the canons to the Toledan Tables that begin, 'Quoniam cuiusque actionis …'[25] Almost nothing is known about John of Sicily; however, from his sole work, it is inferred that he was 'a conventional schoolman, widely read for his purpose, though not particulary gifted mathematically', but that his work was nonetheless 'an important digest of contemporary astronomical reading.'[26] John refers explicitly to the *Almagesti minor* three times. The first of these references is to a definition at the start of *Almagesti minor* III, although John gives the wrong book number: 'Unde et in quarto libro Minoris Almagesti, quem abbreviavit Albategni, definitur medius motus hoc modo: Motus stellae medius est cum tota et integra eius revolutio secundum aequalia tempora per aequales motus fuerit distributa.'[27] John's only change is that he converts the definition into a complete sentence. The other explicit references are only to books of the *Almagesti minor*, not to specific proposition numbers; however, it is clear that the first of these, which occurs in a discussion of the length of the year, refers to *Almagesti minor* III.1, and the second, which is in John's treatment of the equation of time, refers to *Almagesti minor* III.25.[28] Despite the small number of men-

[23] Oxford, Bodleian Library, Auct. F.3.13, f. 212v.

[24] Pedersen, 'Scriptum Johannis de Sicilia', 51, p. 10.

[25] These canons are edited as the 'Canons Cb' in Pedersen, *The Toledan Tables*, pp. 331–568.

[26] Pedersen, 'Scriptum Johannis de Sicilia', 51, pp. 14–15.

[27] Pedersen, 'Scriptum Johannis de Sicilia', 52, p. 135, section J287c.

[28] Pedersen, 'Scriptum Johannis de Sicilia', 52, pp. 138 and 199, sections J292 and J411. In the first of these John again provides the wrong book number.

tions of the *Almagesti minor*, John used it as a source frequently throughout the astronomical section of his text. Fritz Pedersen writes, '… explicit references to Alm. Min. are rarer. Conversely, Alm. Min. seems to be the source most consistently used for wording, either by itself or adduced as an auxiliary where excerpts from the others turn out to present difficulties in exposition or terminology.'[29] Because Pedersen carefully notes dozens of parallel passages of the *Scriptum super canones* and the *Almagesti minor*, these will not be covered here in great detail.[30] John's use of *Almagesti minor* III.1 to learn of Thebit's length of the year and his use of *Almagesti minor* III.11 to report various astronomers' values for the sun's eccentricity and apogee are atypical;[31] John normally uses the *Almagesti minor* for definitions and for rules of calculating various values. Besides the passage with the definition from *Almagesti minor* III mentioned above, there are also passages that rely upon the lists of principles of *Almagesti minor* II and *Almagesti minor* V.[32] The majority of excerpts dependent upon the *Almagesti minor* are on rules of calculation. Among these are rules from *Almagesti minor* II.36, III.17, IV.7, V.9, and VI.14.[33] There are no excerpts of geometrical arguments, although passages taken from V.9 retain the mention of line EB.[34]

Commentary in Florence, Biblioteca Riccardiana, 885

A unique *Almagest* commentary that is based upon the *Almagesti minor* is found in Florence, Biblioteca, Riccardiana, 885, ff. 109r–123v. The portion of this manuscript with the *Almagesti minor* dates from the late thirteenth or the early fourteenth century. The text begins, 'Omnium recte phantium [*sic!*] verisimilibus coniecturis etc.' It is unclear how far the commentary continued since in this manuscript it ends midsentence at the end of a folio in the proof corresponding to *Almagesti minor* II.35 with '… quod erit notum per 18^{am} huius secundi A sit.' On the first folio of the work, another hand has written the title 'Almagesti.' The work includes references to Euclid's *Elements*, Theodosius' *Sphaerica*, and a *De proportionibus* (it is unclear whether Campanus' treatise or the one by Pseudo-Jordanus, which I have argued is the work of Walter of Lille). When discussing the approximation of the chord of 1°, the author

[29] Pedersen, 'Scriptum Johannis de Sicilia', 51, p. 54.

[30] See Pedersen, 'Scriptum Johannis de Sicilia', 51, pp. 55–56 and 100–16.

[31] Pedersen, 'Scriptum Johannis de Sicilia', 52, p. 151, section J311.

[32] Pedersen, 'Scriptum Johannis de Sicilia', 52, pp. 74, 131–32, and 210, sections J146, J280, and J433.

[33] Pedersen, 'Scriptum Johannis de Sicilia', 52, pp. 151–53, 161–62, 162–64, 210, 220, and 237–38, sections J312, J328, J330, J332, J433, J451, and J493.

[34] Pedersen, 'Scriptum Johannis de Sicilia', 52, pp. 163–64, section J332.

writes, '… sicut patet in tractatu nostro de modo operandi.'[35] Identifying this treatise on calculation may help in finding the author of this commentary.
This commentary borrows its structural elements from the *Almagesti minor*. The work's first folio contains the first few words of the *Almagesti minor*'s preface and the remainder of it is left blank. Presumably, the commentator intended to write the remainder of the preface or his own version of it in this space. The commentator arranges the bulk of the work around the enunciations of *Almagesti minor* I.1-II.35, which are copied without substantial modifications, and he also includes the list of principles for Book II. Most of the proofs, however, are given in different wording than that of the *Almagesti minor*. The text of I.15 is an exception, as it consists of the entirety of *Almagesti minor* I.15 with an additional paragraph, and some of the other proofs contain sentences or phrases from the *Almagesti minor*. The excerpts have many of the variant readings of Group 3.A (e.g. an omission in the enunciation of I.5 that is characteristic of this group). In the proofs there are internal references to propositions, but they are frequently inconsistent with the marginal numbering of the propositions, which agrees with the *Almagesti minor*. No figures are included.

In a few passages, the commentator proceeds differently than does the author of the *Almagesti minor*. The discussion of calculating the values of chords of various arcs in I.6 is much briefer. The reason for the brevity appears to be that the commentator discusses these calculations in more depth in his treatise 'De modo operandi.'[36] In I.15 the commentator adds instructions of how to determine a meridianal line in order to set up one of the two instruments used to find the ecliptic's maximum declination.[37] II.5 contains a brief proof using several of Theodosius' propositions, but then, 'quia modus iste procedendi non est modus Tholomei', he also provides a second proof that is closer to the corresponding passage in the *Almagest*.[38] The commentator treats II.33's two cases in the *Almagest*'s order, not the *Almagesti minor*'s, and he does not have an error made in this proof by the *Almagesti minor*'s author.[39]

The Erfurt Commentary

An anonymous *Almagest* commentary that relies heavily upon the enunciations of the *Almagesti minor* is the 'Erfurt Commentary.'[40] It is found in four manuscripts: Dijon, Bibliothèque municipale, 441, ff. 212r–232v; Erfurt,

[35] Florence, Biblioteca Riccardiana, 885, f. 111r.
[36] Florence, Biblioteca Riccardiana, 885, ff. 110v–111r.
[37] Florence, Biblioteca Riccardiana, 885, f. 114r.
[38] Florence, Biblioteca Riccardiana, 885, ff. 116r-v.
[39] Florence, Biblioteca Riccardiana, 885, ff. 123r-v.
[40] Zepeda, *The Medieval Latin Transmission*, pp. 184–221 and 493–572.

UFB, Dep. Erf. CA 2° 375, ff. 113r–126v; Erfurt, UFB, Dep. Erf. CA 2° 393, ff. 63r–80v; and Vatican, BAV, Pal. lat. 1380, ff. 116r–138v (incomplete). The work begins, 'Data circuli dyametro latera decagoni, pentagoni, hexagoni, tetragoni, et trianguli omni ab eodem circulo circumscriptorum reperire. Pro probatione …', and the explicit appears to be '… erit angulus DEA notus orientalis super orizontem, quod est propositum.' Dijon, BM, 441 has a preface of about 175 words that is not found in the other manuscripts. It begins 'Quelibet circumferentia circuli secundum astrologos …'[41] Its last four proofs are not found in the other manuscripts and appear to have been added by another scholar. This addition begins 'Cum fuerint duo puncta orbis signorum equalis elongationis ab uno et eodem tropico …' and ends '… proportionaliter intelligendum est de aliis signis in quolibet climate etc. Et sic est expleta dictio secunda Almagesti.'[42] The manuscripts all date from the mid or late fourteenth century, and although it could possibly be earlier, it was probably composed in the early or mid fourteenth century, definitely before 1366 when Vatican, BAV, Pal. lat. 1380 was written.[43] Although Erfurt, UFB, Dep. Erf. CA 2° 375 contains parts of *Almagest* I–II with excerpts from the *Almagesti minor* written in the margins, these excerpts cannot be the source of the similarity between the Erfurt Commentary and the *Almagesti minor*. A short super-commentary on the first book of the Erfurt Commentary is found in Erfurt, UFB, Dep. Erf. CA 2° 375, ff. 127r–129v.

The Erfurt Commentary only treats *Almagest* I.9 to II.11, although Dijon, BM, 441's addition consists of commentary on *Almagest* II.12–13. The author of this commentary provides many additional lemmata and related proofs. He includes several extra proofs related to compound ratio in the commentary on the plane Menelaus Theorem in *Almagest* I.12, and at the beginning of Book II, he adds a section consisting of many proofs related to geographical questions, such as proofs for calculating the longitudinal width of the earth's dry portion or the distance between two points on the earth. Most of the Erfurt Commentary is arranged in the enunciation and proof format that is found in the *Almagesti minor*, but there are sections that are not formal mathematics. The enunciations of the Erfurt Commentary include ones that are very similar to those of *Almagesti minor* I.1, 7, 9–13 and II.1, 3–17, 19, and 21–29. The corollaries are generally not given; exception are the inclusion of corollaries taken from *Almagesti minor* II.15, II.28, and II.29. Other enunciations are much closer to those found among the gloss in Paris, BnF, lat. 7256 and Vatican, BAV, Barb. lat. 336. For example, both this gloss and the Erfurt Commentary have 'Omnis quadrilateri circulo inscripti quod sub duabus eius

[41] Dijon, Bibliothèque municipale, 441, f. 213r.
[42] Dijon, Bibliothèque municipale, 441, ff. 232v and 233v.
[43] Schuba, *Die Quadriviums-Handschriften der Codices Palatini Latini*, p. 111.

dyametris continetur equum aggregato duarum superficierum a duobus lateribus contentarum', which is not very similar to the wording of the corresponding *Almagesti minor* I.2.[44] Besides the enunciations and the three corollaries mentioned above, there are no close similarities to the *Almagesti minor*, so it is probable that the author of the Erfurt Commentary did not use the *Almagesti minor* itself, but a manuscript of the *Almagest* with some excerpts of it in the margins.

Richard of Wallingford's *Quadripartitum*, *De Sectore*, and *Albion*

The *Almagesti minor*'s influence appears often in the works of Richard of Wallingford, one of the most well-known astronomers of medieval England. Richard, who was born in 1291 or 1292 and died in 1336, studied at Oxford as a youth, and became a monk at St. Albans. After being ordained to the priesthood, he returned to Oxford *c.* 1317, and on his return to St. Albans in 1327, he was elected abbot. Although he had struggles both with members of his community and with the laity, and despite having caught 'leprosy', Richard was able to accomplish much during his abbacy, including writing more astronomical works and overseeing the building of a clock.[45] Richard used the *Almagesti minor* in three of his works, the *Quadripartitum*, *De sectore*, and the *Albion*.[46] The first of these works, as its name suggests, consists of four parts: the first on trigonometry, the second and third on compound ratio and the modes, and the fourth on the Menelaus Theorem and spherical astronomy, i.e. three-dimensional problems. The bulk of the work consists of excerpts or paraphrases of other texts.[47] Richard wrote the *Quadripartitum* before 1326, and it survives in 9 manuscripts.[48]

In *Quadripartitum* I.11, Richard provides three methods of finding the chord of 1°. The first of these is that found in *Almagesti minor* I.6, which Richard cites: 'Quod ostendam tibi, ut promisi, primo per modum quo ostendit commentator hoc super ultimam proposicionem primi *Almagesti*, capitulo 6° …'[49] Although most of this is in other words and Richard arrives at a slightly different value for the chord of 1° ($1^p\ 2'\ 50''\ 20'''$ instead of $1^p\ 2'\ 50''$), the method aligns with the *Almagesti minor*'s. Most tellingly, Richard writes,

44 Paris, BnF, lat. 7256, f. 5r; and Erfurt, UFB, Dep. Erf. CA 2° 375, f. 113v. The latter omits the 'est.'

45 North, *Richard of Wallingford*, vol. II, pp. 1–16.

46 Editions, translations, and analysis of these works are found in North, *Richard of Wallingford*.

47 Zepeda, *The Medieval Latin Transmission*, pp. 261–67.

48 North, *Richard of Wallingford*, vol. II, pp. 23 and 32–39. A reworking of Part I with the incipit 'Cognito sinu recto …' is found in Vienna, ÖNB, 5303, ff. 27r–31r, but this does not contain the part on the method from the *Almagesti minor*.

49 North, *Richard of Wallingford*, vol. I, pp. 48–50.

'… quod minus est 2 terciis unius tercie in errore, quare multo minus quam in uno secundo? Sed in inquisicione cordarum quod minus est quam unum secundum abicitur …', which is clearly taken from the *Almagesti minor*'s 'quia minus quam in duabus terciis unius tercii error erit, quare multo minus quam in uno secundo, sed in inquisitione cordarum quod minus quam secundum fuerit postponitur.'[50] Richard here repeats a mistake of the *Almagesti minor* in saying that the error must be less than 2/3 of a 1‴ when the proper value should be 2/3 of 1″. Although the other two methods for finding the chord of 1° are more precise, Richard seems to have found them unnecessarily complex and states that the method from the *Almagesti minor* is more commendable.[51]

Richard returns to the *Almagesti minor* in Part IV. *Quadripartitum* IV.16 is an excerpt, albeit with some short additions and slight changes in wording, of the whole of *Almagesti minor* II.35, i.e. the enunciation, proof, and rule. Richard even includes the *Almagesti minor*'s internal reference to the '18am proposicionem', which would have perplexed readers since without mentioning the title of the *Almagesti minor*, it would have appeared that Richard was citing the 18th proposition of his own work.[52] *Quadripartitum* IV.18–20 are the rules of *Almagesti minor* I.16–17 respectively.[53] *Quadripartitum* IV.21 is the enunciation and corollary of *Almagesti minor* II.1, and *Quadripartitum* IV.21 is the enunciation, corollary, and last sentence of *Almagesti minor* II.2.[54] *Quadripartitum* IV.22–23 have the enunciations and corollaries of *Almagesti minor* II.3–4, and Richard provides proofs that are partly paraphrases and partly taken directly from the *Almagesti minor*.[55] *Quadripartitum* IV.24 consists of *Almagesti minor* II.16's enunciation and corollary.[56]

In 1335 Richard made a revision of the *Quadripartitum*, which is entitled *Tractatus de sectore* in the sole, difficult to read manuscript containing it.[57] This work retains Richard's use of *Almagesti minor* I.6 without major changes.[58] Richard rewrote Part IV very differently from the *Quadripartitum*, but he still has rules for calculating declinations and right ascensions with some of the

[50] North, *Richard of Wallingford*, vol. I, pp. 50. I have reinserted the mathematically mistaken 'tercie' from the critical apparatus.

[51] North, *Richard of Wallingford*, vol. I, p. 52.

[52] North was among those confused by this reference, and he misunderstood a number of other references to the *Almagesti minor*. See North, *Richard of Wallingford*, vol. I, pp. 151 n. 1, 153 n. 5, and 155 n. 1, and vol. II, p. 78.

[53] North, *Richard of Wallingford*, vol. I, p. 152.

[54] North, *Richard of Wallingford*, vol. I, p. 154.

[55] North, *Richard of Wallingford*, vol. I, pp. 154–56.

[56] North, *Richard of Wallingford*, vol. I, p. 158.

[57] North, *Richard of Wallingford*, vol. II, p. 123.

[58] North, *Richard of Wallingford*, vol. I, p. 173; Cambridge, University Library, Gg 6.3, f. 59v.

wording of *Almagesti minor* I.16–17.[59] Richard also includes a paraphrase of *Almagesti minor* II.16's corollary and *Almagesti minor* II.4's enunciation without its corollary.[60] There do not appear to be further uses of the *Almagesti minor*, but much of the remainder of the work is illegible.

Richard cites the *Almagesti minor* more frequently in Part I of his *Albion*, which he composed in 1326.[61] Well over 30 manuscripts survive with Richard's own version or one of several revisions made by other astronomers including John of Gmunden. The *Albion* treats an instrument of the same name that was developed by Richard, but it also includes theory and tables.[62] Richard provides several references to corresponding passages in the *Almagesti minor* even when his passages do not appear to be derived from those of the earlier work. Some of the numbering of these is odd. For example, in *Albion* I.1, he writes, 'Hec conclusio prima equivalenti ponitur in *Almagesti abbreviato* libro 3º capitulo 7º' although the context fits *Almagesti minor* III.11.[63] In the following conclusion, he gives the equivalent proposition as '*Almagesti abbreviato* libro 3º commento 5º', but it should be *Almagesti minor* III.13.[64] The reason for the errors in numbering is unclear. At other times, his references match the standard numbering of the *Almagesti minor*. In *Albion* I.8, he references *Almagesti minor* III.8 and IV.9.[65] In the preamble before his section on eclipses, Richard directly quotes from the beginning of *Almagesti minor* III: 'Dicit commentator *Almagesti* libro 3º: In principio communia quedam premittenda sunt, quia hic modus demonstracioni est aptior.'[66] *Albion* I.12 provides a paraphrase of the rule in *Almagesti minor* II.30, and Richard refers to that proposition, albeit confusedly.[67] *Albion* I.13 has a non-problematic reference to *Almagesti minor* II.36, but in it Richard also writes, '… ut patet 5º *Almagesti* capitulo 19º, et commento

[59] Cambridge, University Library, Gg 6.3, ff. 77r-v.

[60] Cambridge, University Library, Gg 6.3, f. 78r.

[61] North, *Richard of Wallingford*, vol. II, p. 23.

[62] The most popular version of this work was the reworking made by John of Gmunden, which survives in over 20 witnesses. An edition is being made by Alena Hadravová and Petr Hadrava. I consulted one witness, Vatican, BAV, Pal. lat. 1369, ff. 1r–53v, and it includes all the references to the *Almagesti minor*, and John of Gmunden even added another short excerpt from the *Almagesti minor*, i.e. he placed the enunciation of *Almagesti minor* II.30 at the beginning of *Albion* I.12.

[63] North, *Richard of Wallingford*, vol. I, p. 248.

[64] North, *Richard of Wallingford*, vol. I, p. 250.

[65] North, *Richard of Wallingford*, vol. I, p. 272.

[66] North, *Richard of Wallingford*, vol. I, p. 282.

[67] North, *Richard of Wallingford*, vol. I, p. 284, has '… et primo *Almagesti* de figura sectore, commento 30º.' North, *Richard of Wallingford*, vol. II, p. 151, reports, 'John of Gmunden mistakes the reference to Ptolemy, giving *Almagest* II.30, "de figura sectore".' John of Gmunden was correct, and very likely Richard originally referred to the proper proposition of the *Almagesti minor*.

11° …'[68] The latter part of this would seem to refer to *Almagesti minor* V.11, but the context calls for a reference to *Almagesti minor* V.26. Even more perplexing is a reference during *Albion* I.15, a proposition regarding how the parallax on the circle of altitude increases. Richard refers to 'quod ponit commentator in *Almagesti* libro 5°, capitulo 5°.'[69] Neither *Almagesti minor* V.5 nor its propositions corresponding to *Almagest* V.5 would make sense in this context. In *Albion* I.16, I.18, I.20, and I.21, there are other apparent references to the *Almagesti minor* that do not fit the standard numbering;[70] while it is unclear to which passage of the *Almagesti minor* Richard refers in I.16 (if indeed he is referring to this work), the contexts of the references in I.18, I.20, and I.21 suggest that Richard may have intended to refer to *Almagesti minor* V.19, V.26, and V.18 respectively. In *Albion* I.17, Richard refers to *Almagest* V.19 and to the 'commentatorem ibidem', which may refer to *Almagesti minor* V.22, which corresponds to a passage of *Almagest* V.19.[71] *Albion* I.19 refers twice to *Almagesti minor* VI.4, and Richard writes, 'Et nota quid dicit hic commentator, quod omnes indifferenter utuntur hic lineis rectis pro arcubus, propter hoc quod insensibilis est eorum differencia in tam modica quantitate', which is almost a quotation of *Almagesti minor* VI.4's 'Nam indifferenter arcus ut rectas hic ponimus eo quod non sit sensibilis differentia eorum in tam modica quantitate.'[72] In *Albion* I.22, Richard refers to *Almagesti minor* VI.14.[73] Again, the reason that Richard sometimes refers to the *Almagesti minor* in accordance with the standard numbering and sometimes does not is obscure.

John of Genoa's *Canones eclipsium*

Another work that relies upon the *Almagesti minor* is John of Genoa's *Canones eclipsium*.[74] This work is found in the following manuscripts: Douai, Bibliothèque municipale, 715, ff. 32r–35r (or 36r); Florence, BML, Ashburnham 132, ff. 73r–76r; London, British Library, Royal 12.C.XVII, ff. 214r–216v; Melk, Stiftsbibliothek, 601, ff. 196–97; Oxford, Bodleian Library, Digby 97, ff. 125r–128v; Paris, BnF, lat. 7281, ff. 206r–208r (or 208v); and Paris, BnF, lat. 7322, ff. 39v–41v. Its incipit is 'Ad sciendum eclipsim Solis primo quere…' Its sixth chapter ends '… quia magis prolixum est quam difficile et ideo de hoc ad presens supersedeo', followed by, 'Expliciunt canones eclipsium quas Magister Iohannes de Ianua conpilavit extrahendo eos partim a canonibus commu-

68 North, *Richard of Wallingford*, vol. I, p. 284.
69 North, *Richard of Wallingford*, vol. I, p. 286.
70 North, *Richard of Wallingford*, vol. I, pp. 288–90.
71 North, *Richard of Wallingford*, vol. I, p. 288.
72 North, *Richard of Wallingford*, vol. I, pp. 288–90.
73 North, *Richard of Wallingford*, vol. I, p. 292.
74 Nallino, *al-Battānī*, vol. I, p. xxvii attributes this work to Iohannes de Capua, but in a cross-referenced passage (p. xxxvi) he correctly attributes it to 'Iohannes de Ianua.'

nibus, partim ab Albategni, partim a Minori Almagesti, partim a Magistro Iohanne de Scicilia inscripto suo super tabulas Toletanas et specialiter quantum ad puncta eclipsis, minuta casus ac etiam minuta more, 1332 incompleto 22[a] die Ianuarii. Laus Deo etc.'[75] In the manuscripts from Douai and Paris, and perhaps others, but not in the Melk manuscript, there are added chapters after the colophon. Whether these are by John is unclear, but perhaps the work should be considered to include the nine chapters and that the explicit should be '… et ideo non video necessitatem repetendi. Explicit de eclipsibus.'[76] Besides the colophon, there is one passage with explicit references to the *Almagesti minor*. The sixth chapter begins:

> Circa predicta sciendum quod Albategni capitulo 42° ponit alium modum equandi veram coniunctionem subtrahendo a longitudine sextam et octavam et idem modus repetitur in Minori Almagesti in libro 5 capitulo de equatione vere coniunctionis; tamen modus hic positus est precisior et ideo non repetivi modum Albategni. Secundo sciendum circa diversitatem aspectus in longitudine quod istum modum extraxi ex quibusdam diffinitionibus positis in principio libri 5 Minoris Almagesti licet inquerendo eandem diversitatem aspectus non viderim, nec ibi nec in Albategni nec alibi.[77]

In what way John's manner of finding the parallax of latitude is reliant upon the definitions at the start of *Almagesti minor* V is not clear.

Simon Bredon's Commentary on the *Almagest*

Simon Bredon, who was born *c.* 1300 and died in 1372, was a fellow of Merton College between 1330 and 1341, after which he studied medicine. He later received the patronage of the Earl of Arundel and the archbishop of Canterbury, presumably for his abilities as a doctor, and he was granted a number of positions in the Church. During his time at Merton College, he appears to have focused on mathematical sciences, especially astronomy and astrology, and *c.* 1340 Simon wrote a commentary on the *Almagest* that uses enunciations similar to those of the *Almagesti minor*, but with his own proofs and commentary.[78] The entirety of the work is not extant, and only two manuscripts have large portions of it: Oxford, Bodleian Library, Digby 168, ff. 21r–39r; and

75 Paris, BnF, lat. 7322, f. 41v.

76 Paris, BnF, lat. 7281, f. 208v.

77 Paris, BnF, lat. 7322, f. 41r.

78 For an overview of Simon's life, see Snedegar, 'The Works and Days of Simon Bredon', pp. 285–309. Note that although a recension of Ptolemy's *Quadripartitum* is attributed to him, it has recently been shown that he merely copied one translation into the margins of a manuscript containing another translation; see Vuillemin-Diem and Steele, *Ptolemy's Tetrabiblos in the Translation of William of Moerbeke*, pp. 3–4. I have discussed Simon's commentary and partially edited it in Zepeda, *The Medieval Latin Transmission*, pp. 282–301 and 637–86.

Oxford, Bodleian Library, Digby 178, ff. 39r–86v.[79] A small section is found in in a fifteenth-century manuscript, Cambridge, University Library, Ee 3.61, ff. 43r–45r.[80] Although Digby 168 appears to be in Simon's own hand, it does not include the entire work, and it is difficult to ascertain what is part of the commentary and what is not. Digby 168, ff. 21r-v has the text of Gerard of Cremona's translation of the *Almagest* from the preface, beginning 'Quoniam princeps nomine Albuguafe in libro suo …', until early in the third chapter. In the margin, there is written in what appears to be Simon's hand, 'Editio Bredonis de Almagesti.'[81] From the old foliation, it is clear that three folia are missing after this. The next folio begins mid-sentence in the commentary on *Almagest* I.12. In Digby 178, the scribe added the title 'Commentum Magistri Symonis Bredon super aliquas demonstrationes Almagesti' at the top of f. 42r, where the commentary on *Almagest* I.12 begins, 'Nunc superest ostendere quanta sit maxima declinatio ecliptice ab equinocciali.' However, there is commentary on *Almagest* I.9–11 that immediately precedes this in this manuscript that I believe is part of Simon's text. This section on trigonometry, which begins, 'Arcus dicitur pars circumferencie circuli …', is misattributed to Richard of Wallingford in a table of contents on a flyleaf;[82] however, this fits together relatively harmoniously with what follows, and it is also very similar to a note on the *Almagest* written by John Farley that refers to Bredon's way of finding the chords of various arcs. In both this note and the trigonometrical section that I believe is Simon's work, the chords are found in both a geometrical and arithmetical manner and the numbers are expressed as very large numbers (e.g. instead of rounding things off to minutes or seconds, there are numbers expressed in fractions as small as 60^6, which requires using hundreds of trillions).[83] Thus it appears that Simon started his work by taking the pref-

[79] Snedegar, 'The Works and Days of Simon Bredon', p. 296 states that Paris, BnF, lat. 7292, ff. 334r–345v also contains a portion of this commentary, but these folia contain excerpts from Gerard's translation of the *Almagest*.

[80] In Zepeda, *The Medieval Latin Transmission*, p. 285, I mistakenly wrote that this manuscript contains early sections that are not found in the other two manuscripts.

[81] I have compared the hand to that of the glosses in Oxford, Bodleian Library, Digby 179, which are said to be in Simon's hand. See Watson, 'A Merton College Manuscript Reconstructed', p. 216 n. 2; and Vuillemin-Diem and Steele, *Ptolemy's Tetrabiblos in the Translation of William of Moerbeke*, pp. 3 and 5.

[82] North, *Richard of Wallingford*, vol. II, p. 37, states, 'The ascription is doubtless mistaken.' From the author's interest in calculation of extremely large values, North, *Richard of Wallingford*, vol. II, pp. 37 and 387, argues that this is a work by Lewis of Caerleon.

[83] Oxford, New College, 281, f. 8v. It might be objected that the trigonometry does not match well with Simon's practice in the astronomical part of the commentary, e.g. it discusses versed sines which he never or rarely uses in the part of the commentary commonly attributed to him. A similar phenomenon is found in Richard of Wallingford's *Quadripartitum*. In Part I of that work, Richard treats versed sines at length; however, he rarely uses them in the

ace and cosmological chapters straight from Gerard's translation of the *Almagest* and only really commenced his own commentary with *Almagest* I.9. Simon's commentary continues through *Almagest* III, and it concludes with an excerpt from the very end of *Almagest* III in Gerard's translation: '… 5. Quod est inter annos Christi et annos Arabum, 621 [anni] 6 [menses] 15 [dies].'

Like the *Almagesti minor*, Simon's commentary is arranged into propositions. Simon appears to have attempted to put most of his sources into his own words, and he did not refer to the *Almagesti minor*; however, his debt to the earlier work is undeniable. His use of it is clearest in the enunciations. For example, compare the following:

Simon Bredon II.10: 'Cuiuslibet anguli speralis supra polum alicuius circuli consistentis ad quatuor rectos proporcio est sicut arcus eiusdem circuli qui angulo predicto subtenditur ad circumferenciam eius totam.'[84]	*Almagesti minor* II.21: 'Proportio speralis anguli supra polum alicuius circuli consistentis ad iiii rectos est sicut arcus eiusdem circuli qui ei subtenditur ad totam circumferentiam.'
Simon Bredon III.12: 'Maximam differentiam veri motus Solis ad medium et in quanta elongatione a longitudine longiori in ecentrico fuerit indagari.'[85]	*Almagesti minor* III.12: 'Maximam differentiam diversi motus Solis ad motum medium et in quanta elongatione a longitudine longiore in ecentrico ceciderit notificare.'
Simon Bredon III.19: 'Dies naturales anni inter se invicem duabus de causis inequales esse convincere. Unde quidam dies differentes et quidam mediocres nominantur.'[86]	*Almagesti minor* III.18: 'Dies anni duabus de causis inequales esse invicem necessario comprobatur. Unde patet quosdam dies differentes dici, quosdam mediocres.'

The dependence and mindful modification are simultaneously clear in these examples. In the last example of corresponding enunciations, Simon purposely clarifies that the days concerned are natural days, not mean ones, and he also simplifies the grammar of the corollary. While Simon's use of the *Almagesti minor* is often obscured by his rewording, more than a third of the approximately 80 enunciations share similar wording with those of the *Almagesti minor*.

Besides the enunciations, there are some other clear instances of Simon's reliance upon the *Almagesti minor*. Simon includes Ptolemy's definition of a

astronomical Part IV, and indeed he frequently uses the less sophisticated chords of double arcs instead of sines (see North, *Richard of Wallingford*, vol. I). Another potential objection is that the numbering of propositions does not quite match. There are 11 propositions in the trigonometric section, but the first of the propositions expressly attributed to Simon is numbered 13. Such a slight misnumbering could be a simple mistake or perhaps due to an omitted proposition.

[84] Oxford, Bodleian Library, Digby 168, f. 27r.

[85] Oxford, Bodleian Library, Digby 168, f. 34v.

[86] Oxford, Bodleian Library, Digby 168, f. 36r.

year, but his wording is much closer to the *Almagesti minor*'s restatement than to Ptolemy's definition:

Simon Bredon III.1: 'Secundum Tholomeum annus est reditus Solis ab aliquo puncto zodiaci ad eundem ut a solsticio ad idem solsticium vel ab equinoctio ad idem equinoctium. Illa enim puncta secundum eum digniora sunt aliis …'[87]	*Almagesti minor* III.1: 'Tempus vel quantitas anni est reditus Solis ab aliquo puncto circuli signorum ad idem ut a puncto solstitiali ad idem aut a puncto equinoctiali ad idem. Hec enim notabiliora et digniora sunt in circulo.'	*Almagest* III.1: 'Diffiniam autem dies anni, quod est tempus motus Solis ab aliquo punctorum fixo immobili huius orbis secundum continuitatem signorum donec redeat ad idem punctum.'[88]

In the same proposition, Simon mentions Thebit's trepidation model in wording similar to that of *Almagesti minor* III.1; he writes, 'Unde propter huius inequalitatem annorum et propter diversitatem {etiam} que in maximis Solis declinationibus reperitur, posuit Thebit Benthoraz motum octave spere super duos parvos circulos super capita Arietis et Libre quorum diameter est 8 gradus 37 minuta 26 secunda.'[89] This is very similar to the *Almagesti minor*'s:

> Huius ergo diversitatis causam Tebit Benchoraz coniectans necnon et illius diversitatis que in declinationibus reperitur, motum octave spere ante et retro supra duos circulos parvos supra caput Arietis fixum et caput Libre fixum descriptos quorum diameter est viii gradus et xxxi minuta et xxvi secunda deprehendit. Et hunc motum qui inferioribus quoque speris communis est diversitatem annorum efficere necnon et diversitatem declinationum maximarum que reperitur indicavit.

In this section of his commentary, Simon also reports the lengths of the year found by the 'the oldest of the Egyptians from Babylon', a mistake clearly derived from *Almagesti minor* III.1, and as in the *Almagesti minor*, he also mentions in this context that Arzachel made his tables for the meridian of Toledo.[90] Another clear example of the use of the *Almagesti minor* is in III.18 of Simon's commentary, where Simon writes, 'Eligas ergo pro radice tua annos alicuius secte vel rei note ut puta annos diluvii vel pocius annos Christi …' which is clearly derived from *Almagesti minor* III.17's 'Elige ergo annos alicuius viri noti vel rei note quos radicem velis constituere, ut Augusti vel Alexandri aut potissimum annos Christi qui est rex regum et dominus dominantium.'[91] Additionally, although Simon uses different language and has more sophisticated proofs, he follows the *Almagesti minor* in treating the equation of time in much more detail than Ptolemy does.[92]

87 Oxford, Bodleian Library, Digby 168, f. 31r.
88 *Almagest*, 1515 ed., f. 26v.
89 Oxford, Bodleian Library, Digby 168, f. 32r.
90 Oxford, Bodleian Library, Digby 168, f. 32r.
91 Oxford, Bodleian Library, Digby 168, f. 36r.
92 Simon's commentary III.19–24; see Oxford, Bodleian Library, Digby 168, ff. 36r–39r.

Commentary on Geber's *Liber super Almagesti*

An anonymous commentary on Geber's *Liber super Almagesti* is found in a single manuscript that appears to have originated in a university setting in northern France or England in the second half of the thirteenth century, Paris, BnF, lat. 7406, ff. 114ra-137vb.[93] This work begins, 'Geber in libro 30 figurarum ad probationem …', and it ends '… et fecit currere illud secundum semitam indagationis subtilis.' Most of the commentary is arranged into propositions and proofs. It begins at the start of the *Liber super Almagesti* and ends in the middle of the first chapter of Book IV. The commentator's own voice is heard less and less, and from f. 136ra to the end of the work, the text is copied verbatim from the *Liber super Almagesti*.

The commentator utilized the *Almagesti minor* for many of the enunciations. This use of the *Almagesti minor* begins with an enunciation derived from *Almagesti minor* I.1: 'Dato circulo latera decagoni, exagoni, pentagoni, tetragoni, trianguli omnium equilaterorum et equiangulorum ab eodem circulo circumscriptorum reperire.'[94] Among the enunciations are ones clearly derived from *Almagesti minor* I.16, its corollary, I.17, II.6, II.7–8, II.10–14, II.16, II.21, and II.33–34. That the enunciations of *Almagesti minor* II.7–8, 10–13, and 21 are included is especially revealing because there are no corresponding passages in Geber's *Liber super Almagesti*. Other enunciations have wording that is similar to those of the *Almagesti minor*, but because the author generally uses his own wording, determining all cases of dependency is difficult.

The proofs are generally Geber's, Ptolemy's, or occasionally the commentator's own creation, and among them are few clear instances of dependency upon the *Almagesti minor*. In introducing the second instrument for finding the ecliptic's declination, the commentator writes, 'Paratur etiam aliud instrumentum commodius sic.'[95] This is very similar to the *Almagesti minor* I.15's 'Paratur et aliud commodius et facilius instrumentum', and no similar wording is found in the *Almagest* or the *Liber super Almagesti*'s passage on this instrument. Thus, at the end of this discussion of the ecliptic's declination, the commentator reports values from Albategni and Arzachel as does *Almagesti minor* I.15, and he writes, 'Quapropter diligenter est ad hec inspiciendum et magis usui quam auditui est credendum …', which is almost straight from the final sentence of *Almagesti minor* I.15.[96] Similarly, at the end of his treatment of the length of the year, the commentator has a section on trepidation and various

[93] Lorch, 'The Astronomy of Jābir ibn Aflaḥ', pp. 102–03, briefly describes this commentary. The dating of the manuscript comes from tables for the years 1273–1320 found on Paris, BnF, lat. 7406, ff. 30v–32r, and the location comes from the calendar on ff. 83r–85v.

[94] Paris, BnF, lat. 7406, f. 118vb.

[95] Paris, BnF, lat. 7406, f. 121vb.

[96] Paris, BnF, lat. 7406, f. 122ra. The 'usui' is almost surely a misreading of the *Almagesti minor*'s 'visui.'

astronomers' values for the length of the year that is very close contentwise to the last two paragraphs of *Almagesti minor* III.1, and which concludes, '… et credat magis visui circa hoc quam auditui', which is reminiscent of the sentence of *Almagesti minor* I.15 mentioned above.[97]

Bernard Chorner's *Almagesti Ptolomei abbreviatum*

A commentary on the first two books of the *Almagest* based upon the *Almagesti minor* is attributed to a Bernard Chorner. The work is imperfect in the sole manuscript containing it, Prague, Archiv Pražského Hradu, L.XLVIII (1292), ff. 60r–69v. The first folio of the work (including perhaps the preface and presumably I.1–2 and I.3's enunciation) is lost, so it begins with the proof of the third proposition: 'Sint AB et AG nota. Erit quotque per secundum corollarium prime huius corde BG …' The work ends with proposition II.34, corresponding with the similarly numbered proposition of the *Almagesti minor*. The last words of the text and the explicit read, '… et cum HED sic notus per 30 huius, erit HEA notus. Sicque patet correlarium. Explicit Almagesti Ptolomey abreviatum Bernhardi Chorner quondam Iacobi de Tyrnavia.' A 'Jacobus cappellanus de Tyrna', probably the author of this work, matriculated on 14 April, 1385 at the University of Vienna, which is relatively close to Tyrnavia (i.e. the Slovakian city of Trnava).[98] 'Bernhardus Chorner' presumably is a name that Jacob of Tyrnavia took on. Jacob, aka Bernard, probably wrote this at least a few years after starting at the University of Vienna, but definitely before 1410, which date is found in another colophon in the manuscript.[99] The hand is perhaps that of Johannes Andree Schindel, who lectured upon the *Almagest* at the University of Prague from 1412 to 1418 and who was in Vienna in the first decade of the fifteenth century, as is discussed below.

Bernard's commentary has the enunciations of the *Almagesti minor* and occasionally excerpts from the proofs. The enunciations are given in the same order as the *Almagesti minor* with the exception of I.15. The numbering of the propositions, however, is not identical and is inconsistent. The numbering is carried on continuously for the proofs corresponding to *Almagest* I–II.5. The numbering then breaks off, but some figures for propositions corresponding to *Almagesti minor* II.22–34 are numbered as in the source. The proofs of Bernard's commentary are generally more detailed than those of the *Almagesti minor* (I.11 is an exception), and they include many more internal references

97 Paris, BnF, lat. 7406, f. 129ra.

98 Gall, *Die Matrikel der Universität Wien*, p. 17. Other men with the same name Jacobus de Tyrna matriculated in 1413 and 1418, as did a Bernardus Tirnaw de Syrndorf in 1448. Perhaps a Petrus Cherner de Tirnavia who matriculated in 1415 was a relative (Gall, *Die Matrikel der Universität Wien*, pp. 100, 108, 121, and 262).

99 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 58ra.

and references to other texts, including Euclid's *Elements*, Campanus' *De figura sectore*, and Theodosius' *Sphaerica*. Some of the proofs use entirely different wording than the *Almagesti minor*; however, others have almost the exact wording of the source, but with additional references and explanations. This is especially clear in Propositions 7–8 corresponding to *Almagesti minor* I.7–8.[100] Unlike the *Almagesti minor*, there are frequent short passages of non-mathematical text between the propositions. In these, Bernard either discusses parts of the *Almagest* that are not mathematical or he provides transitions between chapters or books. For example, after describing how the table of chords is made, Bernard writes descriptions of the chapters of the *Almagest*, 'Capitulum 11, de positione arcuum et cordarum est in tabulis. 12^{m} capitulum ostendit …'[101] Such chapter explanations remedy one of the deficiencies of the *Almagesti minor* – the loss of a clear understanding of the relationship between proofs.

The following are some other differences from the *Almagesti minor* that stand out:

- There are two additional lemmata to I.6: 'Si fuerit proportio primi ad secundum sicut proportio tercii ad quartum et proportio secundi ad quintum maior quam proportio quarti ad sextum, erit primi ad quintum maior quam tercii ad sextum' and 'Si proportio primi ad secundum maior fuerit quam tercii ad quartum et tercii ad quartum maior quam quinti ad sextum, erit proportio primi ad secundum maior quam quinti ad sextum.'[102]
- Bernard gives the enunciation of *Almagesti minor* I.15 before the Menelaus Theorem and its lemmata, and does not number it as a proposition. His description of the instruments used to find the maximum declination is very abbreviated, but it concludes with a passage taken almost directly from the end of *Almagesti minor* I.15: 'Nam Yndi invenerunt eam 24 graduum, Ptolomeus 23 graduum 51 minutorum et 20 secundorum, et Arzahel 23 graduum 33 minutorum et 30 secundorum. Ideo sollerter adhuc est inspiciendum et magis visui quam auditui credendum.'[103]
- In I.13, which is numbered 13 and 14, Bernhard points out that Ptolemy does not prove the Menelaus Theorem universally, and he refers the reader to Campanus' *De figura sectore*.[104] Bernard adds a lemma for the Menelaus Theorem that is not found in the *Almagest* or the *Almagesti minor*: '15. Si linea in semicirculo nulla parte aput dyametrum terminata arcum resecaret, si arcus inter ipsam et dyametrum fuerit equalis, ipsam dyametro equidistare necesse est; si autem inequales, ex qua parte fue-

[100] Prague, Archiv Pražského Hradu, L.XLVIII (1292), ff. 61r-v.
[101] Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 61r.
[102] Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 60r.
[103] Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 61r.
[104] Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 62r.

rit arcus minor eas concurrere necesse est.'[105] Bernard provides *Almagesti minor* I.14's enunciation, numbered here as 16, but he provides no proof, only the comment: 'Istam conclusionem non credo in Almagesti minori vel maiori demonstratam esse, quare eam hic non demonstrare [*sic*]. Sed [qui] velit eam demonstrare, recurrat ad figuram sectionis Campani.'[106]

– Proposition 24, corresponding to *Almagesti minor* II.6, has a different corollary, more rules for calculation, and has a longer proof with different figures.[107] The enunciation of the proposition corresponding to *Almagesti minor* II.12 is worded rather differently, but is still clearly taken from the previous work.[108]
– Bernard points out that the proofs of *Almagesti minor* II.15–16 are different than those of the 'Maior Almagesti', and while he outlines the proofs of the *Almagesti minor*, he gives Ptolemy's versions of the proofs in more detail.[109]
– There is an added corollary to *Almagesti minor* II.17: 'Unde manifestum quod arcus circuli magni a polo venientis per punctum communem orizontis et paralelli transeuntis per finem portionis ab equinoctiali incepte terminat differentiam ascensionum eiusdem portionis in spera recta et declivi incepta a communi puncto orizontis et equinoctialis.'[110]
– There is no proof for the proposition corresponding to *Almagesti minor* II.20; Bernard merely states that it is clear enough.[111]
– After the proof corresponding to *Almagesti minor* II.28, Bernard adds a very vivid explanation that involves imagining a large man with his head at the north pole and his feet on the south pole who uses his arms to turn the universe. He then contrasts this with how we see the motions of a 'sphera materialis.'[112]
– Bernard noticed that *Almagesti minor* II.33 had errors, and he adds what the enunciation should say:

  Si punctum medians celum orientalis portionis meridionale fuerit septentrionaleque alterum, anguli qui proveniunt ad punctum dictum superant duplum anguli

105 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 62r.

106 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 62v.

107 Prague, Archiv Pražského Hradu, L.XLVIII (1292), ff. 64r–65v.

108 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 66r: 'Sub linea circuli artici vel antartici umbra in aliquo die ad omnem partem fecltitur et fit dies 24 horarum et dies sine nocte, et ex opposito nox sine die, et quanto distantia cenit ab hac linea maior versus polum tanto maius tempus abiit sine nocte et ex opposito sine die.'

109 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 66v.

110 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 67r.

111 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 67v.

112 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 68r-v.

> ex meridiano arcu ad idem punctum facti quantitate duorum rectorum. Si vero punctum celum medians orientalis portionis septentrionale fuerit meridionaleque alterum, anguli qui proveniunt ad punctum dictum superantur a duplo anguli ex meridiano ad idem punctum facti quantitate duorum rectorum.[113]

He fixed the proof as well.

Schindel's Lectures on the *Almagest* and his *Canones pro eclipsibus*

Johannes Andree Schindel, born *c.* 1370, matriculated at the University of Prague in 1395 and became a master in 1399. He taught mathematics and studied medicine in Vienna in 1407–09, but by 1410 he had returned to Prague, where he served as rector of the university and where he was involved in the making of the famous astronomical clock. During the Hussite Wars, he left Prague, staying for a time in Olomouc in Moravia. He was in Nuremberg from 1423 to perhaps 1436, and he also served as a physician for the Emperor Sigismund. After peace was reached in 1436, he returned to Prague, and he died between 1455–58. During his lifetime, Schindel wrote several theological, astronomical, and medical texts, none of which appears to have had a wide circulation.[114] He lectured on the *Almagest* at the University of Prague from 1412–18, and his manuscript of the *Almagest* containing his marginal notes, Cracow, BJ, 619, survives.[115] Among Schindel's notes on the *Almagest* are many excerpts from the *Almagesti minor*. He notes, 'Hec sunt sunt [*sic!*] suppositiones commentarii quod incipit "Omnium recte philosophantium", quod credo esse Alberti Magni.'[116] Schindel attributes the work several other times to Albertus.[117]

The preface is not given, but Schindel's use of the phrase 'machine mundi' suggests that he consulted it.[118] Schindel includes the lists of principles from the beginnings of Books III–VI and all of the *Almagesti minor*'s enunciations except I.15, V.5–6, V.27–28, VI.13, VI.15–18, VI.20–21, and VI.24–25. One definition from *Almagesti minor* II is included (the definition of spherical right angle), but it is placed after II.20. IV.19 and V.15 are unique in that some of

113 Prague, Archiv Pražského Hradu, L.XLVIII (1292), f. 69r.

114 For information on Schindel's life, see Spunar, *Repertorium Auctorum Bohemorum, Tomus I*, pp. 133–40; and Durand, *The Vienna-Klosterneuburg Map Corpus*, pp. 41–44.

115 This is clear from a horoscope for the beginning of his course and a note at the end of the *Almagest* (Cracow, BJ, 619, ff. Iv and 272r).

116 Cracow, BJ, 619, f. 69v.

117 For example, Cracow, BJ, 619, f. 93v: 'Sequuntur diffinitiones Alberti precedentes quintum librum Almagesti.'; f. 117r: '20[a]. Diversitatem aspectus Lune ad Solem in circulo altitudinis presto summere. Hoc addit Albertus.'; and f. 126v: 'Albertus dicit in suo commentario quod sicut se habet sinus maxime latitudinis …'

118 Cracow, BJ, 619, f. 13v.

the text of the proof is also given.[119] The numbering of propositions is off for I.7–14. Some enunciations are not given in the order in which they are found in the *Almagesti minor*: II.1 is after II.3; V.26 is given after V.21; VI.2 and VI.3 are reversed; VI.4 and VI.5 are reversed; VI.19 precedes VI.14; and VI.22 and VI.23 are reversed. A few enunciations, V.17 and VI.1, are given twice. Occasionally Schindel summarizes parts of the *Almagesti minor* (e.g. he discusses a method of Albategni that is described in *Almagesti minor* VI.5), and he notes that V.20–21 are added to the *Almagest*, i.e. they do not have corresponding passages.[120] The text in this manuscript has some variants that are unique to Group 1, such as the omission of the second supposition of Book III, and more specifically readings unique to Group 1.B, such as an omission in one of the definitions of Book IV. Since Schindel also wrote notes in *Pr*, these excerpts are perhaps copied from that manuscript or they may have been copied from the same exemplar.

More of how Schindel taught the *Almagest* can be gained from Prague, Archiv Pražského Hradu, O. I (1585), which contains notes of Johannes Borotin both as a student and teacher.[121] Borotin's notes that he took while he was attending Schindel's lectures on the *Almagest* are included among these (ff. 138r–161v). These cover only parts of *Almagest* I–II, and they are not in order and include many blank leaves – Borotin was not included in Schindel's list of his most zealous students;[122] however, there is still a passage that shows one way in which Schindel used the *Almagesti minor*. In his treatment of oblique ascensions, Schindel referenced propositions or demonstrations of the *Almagest* using the numbering of his excerpts from the *Almagesti minor* in Cracow, BJ, 619.[123]

Johannes Andree Schindel also used the *Almagesti minor* in his *Canones pro eclipsibus solis et lune per instrumentum ad hoc factum inveniendis*.[124] This work exists in 3 manuscripts: Nuremberg, Stadtbibliothek, Cent. V.58, ff. 116v–121v; Vienna, ÖNB, 5412, ff. 161r–169r; and Vienna, ÖNB, 5415, ff. 133r–141r. The work's incipit is 'Partes instrumenti circulosque et lineas pro sequentibus facilius intelligendis …' Schindel's work ends, 'Et illud quod est inter primum almuri et secundum est semidyameter Lune etc.'[125] In the two

119 Cracow, BJ, 619, ff. 90v and 111r.

120 Cracow, BJ, 619, ff. 126v and 117r.

121 For more on this manuscript and Borotin's life, see Burnett, 'Teaching the Science of the Stars in Prague University.'

122 Cracow, BJ, 619, f. 272r.

123 E.g. Prague, Archiv Pražského Hradu, O. I (1585), ff. 150r, 151r, 153v–154r, 155r, and 161r-v.

124 An edition of Schindel's text and a similar reworking of Richard of Wallingford's *Albion* made by John of Gmunden is being made by Alena Hadravová and Petr Hadrava.

125 Vienna, ÖNB, 5415, f. 141r.

Vienna manuscripts, the text is continued, but it is a later addition to the text, as a reader of Vienna, ÖNB, 5415 noted in the margin.[126] This added part begins, 'Pro diversitate aspectus per arcus et angulos invenienda …', and it ends with a table in which the last numbers are 49, 9, and 60, and the last sentence of its text before the table is 'Verbi gratia anno domini 1433º 17 die Iunii erit eclipsis Solis hora post meridiem quarta et aliquot minuta, cuius ascendens est ut sequitur.' Because the two instances of dates, both to 1433, are in the fourth part, they can only provide a *terminus ante quem* for Schindel's composition of the first three parts. The author of the addition is not known. It could be Schindel's work, or perhaps it was composed by Reinhardus Gensfelder from Nuremberg, who is the scribe of Vienna, ÖNB, 5415.

As its name suggests, this work consists primarily of rules for determining when eclipses will occur and how they will appear. Most of the rules involve the use of an instrument and are not arithmetical rules of calculation. Much of the work is dependent upon Richard of Wallingford's *Albion*. Although most of the work is devoted to the use of an instrument, it is divided into propositions with enunciations, and some of the propositions do not involve the instrument. The work is divided into parts. In the Vienna manuscripts, whose division and numbering I follow, the first part has 6 propositions, the second has 6, the third has 16 with an added, unnumbered proposition, and the added fourth part has 9 propositions followed by a discussion concerning compound ratios and a reworking of the third proposition of Part III. The Nuremberg manuscript divides the work into only two parts, as it unites what are the first two parts in the other manuscripts, and does not have the additions.

In both the original text and the addition, most of the uses of the *Almagesti minor* are acknowledged. In these references, Schindel and the author of the addition (if another person) refer to the *Almagesti minor* as a work of Albertus Magnus. In Schindel's original part of the work, there are only a few uses of the *Almagesti minor*. After describing how to find the size of a specific arc with the instrument in III.3, Schindel gives another way of finding it in III.4. This proposition consists of the *Almagesti minor* II.35's enunciation and a paraphrase of its 'operatio arismetica.'[127] The rule in III.10 is very close in wording to *Almagesti minor* V.28; however, both are very similar to *De scientia astrorum* Ch. 39, so it is unclear which of the earlier works was Schindel's source.[128]

The added section shows a closer dependence upon the *Almagesti minor*. The first added proposition, which is perhaps to be considered the 17th proposition of Part III, has the enunciation of *Almagesti minor* II.34. It also refers to *Almagesti minor* II.36 and has an arithmetical rule that is is perhaps derived

[126] Vienna, ÖNB, 5412, ff. 169r–174r; and Vienna, ÖNB, 5415, ff. 141r–146r.

[127] Vienna, ÖNB, 5415, f. 137v.

[128] Vienna, ÖNB, 5415, f. 139r; and Albategni, *De scientia stellarum*, 1537 ed., f. 51r.

from that proposition's rule.[129] IV.2 has once again the enunciation of *Almagesti minor* II.35, which was also given in III.4. It also provides *Almagesti minor* II.35's rule and outlines its geometrical proof.[130] IV.3 provides *Almagesti minor* II.36's enunciation and rule.[131] IV.4's enunciation is a rephrasing of that of *Almagesti minor* V.17 with clear similarities in wording, and IV.4's arithmetical rule combines a paraphrase of *Almagesti minor* III.17 with an excerpt from *Almagesti minor* V.19.[132] Similarly, IV.5 has an enunciation based upon that of *Almagesti minor* V.13, and it provides an arithmetical rule derived from *Almagesti minor* V.9 and V.19.[133] IV.6 takes its enunciation and directions from *Almagesti minor* V.19.[134] IV.7's enunciation and rule are derived from *Almagesti minor* V.20.[135] IV.8 is taken from *Almagesti minor* V.21 with some changes.[136] IV.9's enunciation corresponds to *Almagesti minor* V.22, but it then gives the geometric proof of *Almagesti minor* V.25, much of it word for word.[137] Following IV.9, there is a discussion of finding unknown quantities when a ratio is known to be composed of two others. The example that the author uses makes it clear that this passage is a commentary on *Almagesti minor* II.35.[138]

Many excerpts from Schindel's *Canones pro eclipsibus* are also found in an anonymous work titled *Compositio duorum instrumentorum*. Like Schindel's *Canones*, this work relies heavily upon Richard of Wallingford's *Albion*. It consists of sections on the construction of two instruments followed by 31 chapters on the use of the instruments. This work is found in at least seven manuscripts: L_1, ff. 226ra-230r; Melk, Stiftsbibliothek, 601, ff. 162ra-174va; Munich, BSB, Clm 221, ff. 246v–249r; Munich, BSB, Clm 367, ff. 32r–47r; Vienna, ÖNB, 5228, ff. 53v–57r; Vatican, BAV, Pal. lat. 1340, ff. 60va-73va; and Vatican, BAV, Pal. lat. 1381, ff. 198r–203r. The work begins 'Pro faciliori modo habendo et multiplici labore …', but it is not completely clear where the work ends. Munich, BSB, Clm 221 and Pal. lat. 1381 have the same condensed version of the text, ending '… habebis duracionem tocius eclipsis.' This version, which has only about half of the chapters on the use of instrument, lacks many passages, and paraphrases or adds to others. L_1 has only the parts of the text related to the construction of the first instrument and the first four numbered chapters. Munich, BSB, Clm 367 has 31 chapters and ends with '… contin-

129 Vienna, ÖNB, 5415, f. 141r.
130 Vienna, ÖNB, 5415, f. 141v.
131 Vienna, ÖNB, 5415, ff. 142r-v.
132 Vienna, ÖNB, 5415, f. 142v.
133 Vienna, ÖNB, 5415, ff. 142v–143v.
134 Vienna, ÖNB, 5415, f. 143v.
135 Vienna, ÖNB, 5415, ff. 143v–144r.
136 Vienna, ÖNB, 5415, f. 144r.
137 Vienna, ÖNB, 5415, ff. 144r–145r.
138 Vienna, ÖNB, 5415, ff. 145r-v.

gunt circulum umbre etc.' The Melk manuscript continues past this, but it is unclear whether the last parts are part of the original text or are additions. Either way, the text or the additions in this manuscript conclude, '… similiter etiam de aliis debet procedere. Sequitur figura.'[139] The work appears to have been written in the mid fifteenth century, as the manuscripts containing the work all date from the fifteenth century or the early 16th, and Munich, BSB, Clm 367 has the colophon 'anno domini nostri mº ccccº lxxiiii in Perusia.'[140] Chapter 8 contains the text of *Canones pro eclipsibus* III.4, which is based upon and refers to *Almagesti minor* II.35. The reference to Albertus' 35th comment of the second book is included, but in Munich, BSB, Clm 367, 'Albategni' is found instead of Albertus' name.[141] L_1, Munich, BSB, Clm 221, and Pal. lat. 1381 lack this chapter.[142]

John of Gmunden's *Tractatus de sinibus, chordis et arcubus*

John of Gmunden, born between 1380 and 1385, was extremely important to the history of astronomy in the early fifteenth century. He receive his B.A. and M.A. from the University of Vienna in 1402 and 1406 respectively. He was ordained to the priesthood, became a canon of Stephansdom in Vienna, and was later appointed pastor at Laa an der Thaya. John lectured on mathematics and astronomy many times in 1406–25 and again in 1431 and 1434. He also held a number of positions at the University of Vienna, and he was an important member of a circle of scholars interested in mathematics, astronomy, and cartography until his death in 1442.[143] As noted above, he reworked Richard of Wallingford's *Albion*. He also wrote a trigonometrical treatise, the *Tractatus de sinibus, chordis et arcubus*, in 1437. The work, which survives in its entirety in four manuscripts, is written in two parts, the first of which provides a way of making trigonometrical tables based upon Arzachel.[144] The second part is devoted to Ptolemaic trigonometry, and shows the influence of the *Almagesti minor*. At the beginning of the second part, John acknowledges his source for the geometrical declarations: '… praemittam 6 propositiones quae

139 I have not been able to see Vienna, ÖNB, 5228 or Vatican, BAV, Pal. Lat. 1340.

140 Munich, BSB, Clm 367, f. 47r.

141 Melk, Stiftsbibliothek, 601, ff. 167ra-b; and Munich, BSB, Clm 367, f. 42r.

142 It remains to be seen whether this chapter is found in the other two manuscripts.

143 For the details of John of Gmunden's life, see Grössing, 'Zur Biographie des Johannes von Gmunden.'

144 An edition of the work can be found in Busard, 'Der Traktat De sinibus, chordis et arcubus.' Also, see Folkerts, 'Die Beiträge von Johannes von Gmunden zur Trigonometrie.' The full work is found in Innsbruck, Servitenkloster, I.b.62, ff. 87r–100v; London, British Library, Addit. 24071, ff. 51r–70v (or perhaps 71v); and Vienna, ÖNB, 5268, ff. 84r–97v. The first half of the treatise, which does not use the *Almagesti minor*, is found also in Vienna, ÖNB, 5277, ff. 69r–90v.

etiam praemittuntur in principio primi libri Almagesti minoris.'[145] The enunciations of *Almagesti minor* I.1–6 including the corollary of I.1 are given with very few changes in wording; however, the proofs and calculations are much more detailed than those of the *Almagesti minor*. Most of the proofs use no language taken directly from the *Almagesti minor*, but the sixth proposition has several sentences or phrases from *Almagesti minor* I.6. For example, compare the following sentences:

De sinibus, chordibus et arcubus: Sic enim minus quam in duabus tertiis unius tertii error erit quare multo minus quam in uno secundo, sed in inquisitione cordarum quod minus quam secundum fuerit postponitur.[146]	*Almagesti minor* I.6: … quia minus quam in duabus terciis unius tercii error erit, quare multo minus quam in uno secundo, sed in inquisitione cordarum quod minus quam secundum fuerit postponitur.
De sinibus, chordibus et arcubus: … facilis ergo est secundum praemissorum tenorem cordarum ad suos arcus agnitio.[147]	*Almagesti minor* I.6: Facilis est ergo secundum premissorum tenorem cordarum ad arcus suos agnitio.

The *Tractatus super propositiones Ptolemaei de sinubus et chordis* attributed to Georg Peurbach, which is found in Vienna, ÖNB, 5203, ff. 124r–128r, and which was printed in 1541 (Nuremberg, Johannes Petreius) and 1561 (Basel, Henricus Petrus and Petrus Perna), consists of excerpts from this work, including the enunciations and most of the portions taken from the *Almagesti minor*.[148]

Paul of Gerresheim's *Expositio*

Paul of Gerresheim's *Expositio practice tabule tabularum et propositionum Ptolomei pro compositione tabule sinuum et cordarum necessariarum* is yet another treatise on trigonometry that utilizes excerpts from the *Almagesti minor*. This work, the entirety of which only survives in the author's own hand in Brussels, Bibliothèque Royale, 1022–47, ff. 184v–197v, begins, 'Necessitatem et utilitatem tabule sinuum et cordarum astronomorum signifer Ptolomeus ostendit …', and the text, which is followed by a table, ends, '… et in hoc terminatur consideraciones compositionis tabule sinuum et cordarum. Sequitur nunc ipsa tabula rectificata anno domini 1443'.[149] According to a biographical note found

[145] Busard, 'Der Traktat De sinibus, chordis et arcubus', p. 95.

[146] Busard, 'Der Traktat De sinibus, chordis et arcubus', p. 109.

[147] Busard, 'Der Traktat De sinibus, chordis et arcubus', p. 109.

[148] This work's incipit is 'Sinuum, chordarum et arcuum noticia ad coelestium motuum cognitionem …', and the explicit is '… secundum praemissorum tenorem chordarum ad suos arcus cognitio.'

[149] Brussels, Bibliothèque Royale, 1022–47, ff. 184v and 196r. A small fragment including little more than the enunciation of the first proposition is found in Brussels, Bibliothèque Royale, 2962–78, ff. 211v–212r.

in Brussels, Bibliothèque Royale, 2962–78, f. 160r, Paul was a doctor of theology, a canon of St. Gereon, and the pastor of St. Laurence in Cologne.[150] He is described in this note as a 'mathematicus et astronomicus maximus' and is said to have made his own tables of mean motions for Cologne. Paul, who entered the University of Cologne in 1422, was chosen to be rector twice, and he died in 1470.[151] Paul's *Expositio* has three sections: on arithmetical operations with sexagesimal numbers, on finding chords of arcs, and a table of sines calculated to the fourth sexagesimal place. The second of these parts is a summary of *Almagest* I.9, and Paul uses the enunciations (and corollary) of *Almagesti minor* I.1–6. There is no other trace of the *Almagesti minor*, so it is likely that Paul used an *Almagest* manuscript that had these enunciations in the margins.

Peurbach and Regiomontanus' *Epitome Almagesti*

Georg Peurbach, who studied at the University of Vienna and who taught at the *Bürgerschule* of Vienna, was one of the most renowned astronomers of the fifteenth century; however, he was eclipsed by his pupil, Johannes Regiomontanus. In 1460 at the bidding of Cardinal Bessarion, Peurbach began to write the *Epitome Almagesti*, but after Peurbach's death the following year, it was completed by Regiomontanus.[152] In his dedicatory letter to Cardinal Bessarion, Regiomontanus writes that Peurbach was scarcely able to finish six books before he died.[153] Thus the portions of the work that correspond to the *Almagesti minor* were composed by Peurbach; however, Regiomontanus added a preface and six cosmological chapters to the beginning of the first book, and it is unknown whether he added other passages or to what degree he revised Peurbach's work in the first six books. The *Epitome Almagesti* became very popular. Not only does it survive in 11 manuscripts, but it was printed three times, in 1496 (Venice, Johannes Hamman), 1543 (Basel, Henricus Petrus), and 1550 (Nuremberg, Johannes Montanus and Ulricus Neuber).

The book has deep ties to the *Almagesti minor*. The work is arranged in propositions instead of chapters and includes no tables. Some definitions are also included, e.g. near the beginning of Book II. Textual dependence on the *Almagesti minor* is apparent in many places in the work. Because the first six books (omitting the early chapters known to be added by Regiomontanus) are

[150] Durand, *The Vienna-Klosterneuburg Map Corpus*, pp. 61 and 129, claims that Paul studied in Vienna *c.* 1440 and is credited with bringing material relating to maps from Vienna to Cologne; however, this claim seems to have little evidence to support it.

[151] Masai and Wittek, *Manuscrits datés conservés en Belgique, Tome III: 1441–1460,* p. 20, no. 237.

[152] My transcription of Venice, BNM, Fondo antico lat. Z. 328 can be found at www.ptolemaeus.badw, and my critical edition of the *Epitome Almagesti* will be finished in the near future.

[153] Venice, BNM, Fondo antico lat. Z. 328, f. 2r.

so close in style and content to the *Almagesti minor*, it appears that Peurbach intended the first six books of the *Epitome Almagesti* to be little more than a paraphrase of the *Almagesti minor* and that Regiomontanus is the source of most of the differences.[154] In fact, it may have been the case that it is precisely because the *Epitome Almagesti* is so similar to the *Almagesti minor* while it is more complete and comprehensive, that it replaced the earlier work and was primarily to blame for the decline of interest in the *Almagesti minor* by the end of the fifteenth century. In the following century, astronomers such as Copernicus and Erasmus Reinhold, often used and referred to the *Epitome Almagesti*, but not the *Almagesti minor*.[155]

While few enunciations match those of the *Almagesti minor* verbatim and some are worded very differently, over 25 enunciations in the first six books of the *Epitome Almagesti* show a dependency upon the earlier work. The following are a small selection of examples of the enunciations that show Peurbach's use of the *Almagesti minor*:

Epitome Almagesti	*Almagesti minor*
I.7: Data circuli diametro latera decagoni, exagoni, pentagoni, tetragoni atque trianguli isopleurorum eidem circulo inscriptorum reperire.[156]	I.1: Data circuli diametro latera decagoni, pentagoni, exagoni, tetragoni, atque trianguli omnium ab eodem circulo circumscriptorum reperire.
II.12: Sub omni paralello versus septentrionem ab equatore, bis tantum fit dies equalis nocti in anno et dies estivi hibernis longiores, noctes breviores; et quanto ab equinoctiis distantiores tanto estivo productiores, hiberni correptiores; et quedam stelle apparentes semper, quedam occulte semper, et distantia cenith ab equinoctiali equalis altitudini poli.[157]	II.8: Sub omni alia linea equidistante linee equinoctiali bis tantum dies fit equalis nocti in anno; et dies estivi hibernis prolixiores, noctes vero breviores; et quanto ab equinoctio distantiores dies estivi productiores, hiberni vero correptiores; et quedam stelle apparentes semper, quedam occulte semper; et distantia cenit ab equinoctiali equalis altitudini poli.
V.21: Proportiones trium corporum Solis, terre, et Lune ad invicem assignare.[158]	V.18: Magnitudinem Solis et magnitudinem Lune metiri, et trium corporum Solis, Lune, et terre proportiones adinvicem assignare.

[154] Swerdlow and Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus* pp. 51–52, come to similar conclusions about the relationship between the two works.

[155] The extant to which Copernicus relied upon the *Epitome Almagesti* is made clear in Swerdlow and Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus*. Reinhold's *Commentary on Peurbach's Theoricae novae planetarum* (printed in Wittenberg in 1542, 1553, 1580, and 1601) refers to the *Epitome Almagesti* several times.

[156] Venice, BNM, Fondo antico lat. Z. 328, f. 6r.

[157] Venice, BNM, Fondo antico lat. Z. 328, f. 14r.

[158] Venice, BNM, Fondo antico lat. Z. 328, f. 45v.

VI.15: Transitum Lune in circulo declivi inequales arcus in ecliptica secare, verum differentiam longitudinum in ambobus circulis admodum parvam esse.[159]	V.26: Motum Lune in circulo declinante et in circulo signorum arcus differentis longitudinis efficere necesse est, sed differentia admodum parve quantitatis esse convincitur.

While not as clear as the first three examples, the fourth shows traces of the influence of the *Almagesti minor*. The use of 'admodum parvam esse' and 'admodum parve quantitatis esse' in the same context, while there is no similar phrase in the corresponding passage of the *Almagest*, establishes that there is a connection here between the enunciations of the *Epitome Almagesti* and the *Almagesti minor*.

There are a large number of other features of the *Epitome Almagesti* that show dependence upon the *Almagesti minor*. The similarities include the following.

- In the standard version of the *Epitome Almagesti*, there are cosmological propositions corresponding to the first chapters of the *Almagest*; however, these are not found in one of the earliest manuscripts, Venice, BNM, Fondo antico lat. Z. 329, and they appear to have only been added by Regiomontanus at a late stage in the text's composition. Thus in Peurbach's first version, it appears that the work began with a proposition corresponding to *Almagesti minor* I.1. That Peurbach only completed six books, matching the *Almagesti minor* closely, suggests the possibility that he merely summarized the *Almagesti minor* and that most of the differences in content in the first six books are due to Regiomontanus' revision.
- Peurbach treats the ecliptic's maximum declination after the Menelaus Theorem, as in the *Almagesti minor*, not before it as in the *Almagest*.
- Peurbach makes a switch from chords of double arcs to sines in I.23, his proposition on finding declinations, which mirrors the change in trigonometric styles of *Almagesti minor* (although sines are occasionally mentioned before this in the *Almagesti minor*).
- In *Epitome Almagesti* II.11–18, climes are treated more as they are in the *Almagesti minor* than in the *Almagest*.
- In *Epitome Almagesti* III.3, Peurbach reports varying opinions on the length of the year, echoing *Almagesti minor* III.1. Much of this passage is closer to the common source of these passages, Albategni's *De scientia astrorum* Ch. 27, but that both commentaries leave Ptolemy to discuss the same passage in Albategni does not appear to be a coincidence.[160] Furthermore, both commentaries immediately follow this with discus-

[159] Venice, BNM, Fondo antico lat. Z. 328, f. 56r.

[160] Albategni, *De scientia stellarum*, 1537 ed., ff. 26v–27v.

sions of Thebit's theory of trepidation and his value for the length of the year.

– As in the *Almagesti minor*, there is no proof corresponding to Ptolemy's last proof in *Almagest* III.4.

– In *Epitome Almagesti* III.13, Peurbach follows *Almagesti minor* III.11 in reporting parameters of Albategni and Arzachel. Although the *Almagesti minor* cannot be Peurbach's sole source for this passage, some of the wording matches that of the earlier work:

Epitome Almagesti: 'Arzachel autem licet motum medium variaverit tamen eandem quam Albategni invenit ecentricitatem.'[161]	*Almagesti minor*: 'Arzachel vero licet variaverit motum medium, eandem tamen quam Albategni invenit centrorum differentiam.'

– Peurbach also follows the *Almagesti minor* in devoting several propositions (III.22–30) to the equation of time in much greater detail than Ptolemy does. In these propositions, Peurbach takes some wording directly from the *Almagesti minor*. For example, compare the following corresponding proofs:

Epitome Almagesti III.24[162]	*Almagesti minor III.21*
Locus ille secundum varietatem orizontium varius est; in omni tamen regione ante tropicum estivalem et post tropicum hiemalem deprehenditur. ... Vide itaque quanta sit portio ecliptice inter hec duo loca et quanta sit huius portionis obliqua ascensio. ...	Locus qui queritur secundum climata variatur; in omni tamen climate ante punctum tropicum estivum et post tropicum punctum hiemale deprehenditur. ... Vide ergo portio circuli signorum inter hec duo loca quanta sit aut ex parte Libre aut ex parte Arietis, et cum quanta portione equinoctialis elevetur. ...
Quantum autem ex hac causa sola dies mediocres addunt super differentes per portionem ecliptice in qua est Aries tantum differentes addunt super mediocres per reliquam portionem ecliptice. Ex hoc constat quod dies differentes maiores addunt super dies differentes minores duplum collecte differentie. ... Palam etiam quod differentia sic inventa augmentum diei solsticialis super diem equinoctialis excedit.	Et quia quantum dies mediocris addit super dies differentes ex parte Arietis tantum dies differentes addunt super diem mediocrem ex parte Libre, palam quod dies differentes maiores addunt super dies differentes minores duplum collecte differentie. Palam etiam quod differentia sic inventa augmentum maxime diei regionis super diem equinoctialem excedit ...

– In *Epitome Almagesti* IV.12, Peurbach includes findings of Albategni concerning the moon's mean motion of anomaly, as *Almagesti minor* IV.14 does, and he uses some language from this source.

161 Venice, BNM, Fondo antico lat. Z. 328, f. 25v.
162 Venice, BNM, Fondo antico lat. Z. 328, f. 28v.

- Like *Almagesti minor* IV.19, *Epitome Almagesti* IV.17 is on the motion of the moon's nodes, a topic that does not receive its own discussion in the *Almagest*.
- As in *Almagesti minor* V.10, *Epitome Almagesti* V.12 includes an argument, derived from Albategni, that ignoring the equation of portion at true syzygies can lead to perceptible errors.
- Like *Almagesti minor* V.11, *Epitome Almagesti* V.13–14 describe Ptolemy's triquetrum in terms of a geometric figure, using some of the *Almagesti minor*'s language.
- The proof of *Epitome Almagesti* V.20 begins, 'Compertum dixit Ptolemeus quod Luna ...' The 'compertum' is most likely in the text because Peurbach started to copy the 'compertum est' of the *Almagesti minor* V.17 before deciding to rephrase the sentence.
- Like *Almagesti minor* V.18, *Epitome Almagesti* V.21 has a section on Albategni, and while Peurbach must have consulted *De scientia astrorum* for this passage, some sections of it are taken directly from the *Almagesti minor*.
- *Epitome Almagesti* V.25 not only has an enunciation very similar to that of *Almagesti minor* V.20, but its explanation of how one subtracts the sun's parallax from the moon's, which is not in the *Almagest* and which is explained differently in *De scientia astrorum*, is from *Almagesti minor* V.20. It is also similar to this proposition of the *Almagesti minor* in that it brings up a way to rectify the sun's parallax from Ptolemy's tables according to Albategni at the same spot.
- *Epitome Almagesti* V.26 has a geometrical proof similar to that of *Almagesti minor* V.21, while there is not a corresponding one in the *Almagest*.
- *Epitome Almagesti* VI.4 begins with a brief, 'more certain way' of finding the moon's true carrying beyond for a certain hour that comes from *Almagesti minor* VI.2 and that is not in the *Almagest*.
- In *Epitome Almagesti* VI.7–8, Peurbach follows *Almagesti minor* VI.4–5 in providing eclipse limits attributed to Albategni although no such limits are reported in the text of *De scientia astrorum*. The value of the solar eclipse limits match those calculated by the author of the *Almagesti minor*, but the lunar eclipse limits are slightly different.
- *Epitome Almagesti* VI.16, which is on the digits of a lunar eclipse, has no clearly corresponding passage in the *Almagest*, but it does correspond to *Almagesti minor* VI.13.
- In *Epitome Almagesti* VI.17, Peurbach includes a way of finding the minutes of a lunar eclipse more accurately by taking the slant of the moon's path into account, as does *Almagesti minor* VI.14.

- *Epitome Almagesti* VI.18, which is on the times of a lunar eclipse, only has a loosely corresponding passage in the *Almagest*, but it does correspond to *Almagesti minor* VI.15.
- In *Epitome Almagesti* VI.22, Peurbach uses a geometric figure in his directions for finding visible conjunctions, as does *Almagesti minor* VI.17.
- *Epitome Almagesti* VI.23, on the digits of a solar eclipse, corresponds to *Almagesti minor* VI.18, but there is no parallel passage in the *Almagest*.
- *Epitome Almagesti* VI.29 uses a geometric figure that is closer to that of *Almagesti minor* VI.25 than to the figure of the corresponding passage in the *Almagest*, and Peurbach also uses some of the wording of the *Almagesti minor*, such as 'flexus tenebrarum.'

Albert of Brudzewo's *Commentariolum super theoricas novas planetarum Georgii Purbachii*

Albert (or Wojciech) of Brudzewo was an important figure in the history of the University of Cracow in the late fifteenth century.[163] Born in 1445 or 1446, he entered the university in 1468 and he studied and taught there for most of his life, which ended in 1495. He became a bachelor in 1470, a master in 1474, and a bachelor of theology in 1490. He also held positions in the university, including dean of the Arts Faculty. Albert lectured upon many subjects including arithmetic, perspectiva, logic, and natural philosophy, but he is most well-known for his work in astronomy and astrology. Of the most interest for our study are his lectures upon Peurbach's *Theorica nova planetarum* in 1483, repeated in 1488, which make up his *Commentariolum super theoricas novas planetarum Georgii Purbachii*.[164] The *Commentariolum* is noteworthy both because it shows the adoption of Georg Peurbach's *Theoricae novae planetarum* with its three-dimensional models and also because in it Albert emphasized the problem of the lack of physical models using the regular motion of physical spheres to explain some of the mathematical models used in Ptolemaic astronomy. Although he was by no means the first to discuss this problem, because of his highlighting of this issue in the years immediately preceding Copernicus' time at the University of Cracow, much scholarly attention has been paid to Albert's *Commentariolum*.[165] It is very likely that Copernicus knew Albert

[163] For an overview of his life in English, see Pawlikowska Brożek, 'Wojciech of Brudzewo.'

[164] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, followed by others including Pawlikowska Brożek, 'Wojciech of Brudzewo', p. 69, gives the date of composition of the work as 1482, but no reason is given for this.

[165] In just the last few years, the following articles have been published: Barker, 'Albert of Brudzewo', pp. 125–48; Malpangotto, 'La critique de l'univers de Peurbach développée par Albert de Brudzewo'; Malpangotto, 'The Original Motivation for Copernicus's Research'; Sylla, 'The Status of Astronomy as a Science in Fifteenth-Century Cracow', esp. pp. 70–76.

and that he read the *Commentariolum*. It has been stated that in his 1493 lectures on the *Theorica planetarum*, Simon Sierpic read the *Commentariolum*.[166] While it is possible that Copernicus attended these lectures or that he was taught by Albert privately, this cannot be confirmed. While the importance of the *Commentariolum* in the formation of Copernicus' thought is perhaps not as strong as has been argued, the work clearly reflects at least part of the astronomical environment that Copernicus encountered during his time at Cracow.[167] The entire *Commentariolum* exists in five manuscripts (an excerpt is found in a sixth) and it was printed twice in Milan in 1494 and 1495 by Uldericus Scinzenzeler, one of Albert's students.[168]

In the *Commentariolum* Albert uses the *Almagesti minor* quite often although these two works are in different genres of astronomical writing. Both deal with theoretical astronomy, but the *Commentariolum*, in line with the *theorica* tradition, generally treats matters on the qualitative level. It includes only a few proofs or rules for calculation, and figures usually serve as models or examples, not as components of proofs. Albert refers to the *Almagesti minor* as the 'Abbreviatum Almagesti' or more commonly only as the 'Abbreviatum', and he attributes it to Albertus Magnus. This title probably comes from Richard of Wallingford's *Albion*, which Albert cites in the *Commentariolum*, and the attribution comes from Johannes Andree Schindel's notes in the margin of Cracow, BJ, 619. This manuscript was brought to Cracow by Alexius

[166] Knoll, *'A Pearl of Powerful Learning': The University of Cracow in the Fifteenth Century*, p. 397; and Malpangotto, 'The Original Motivation', pp. 393, 404. The evidence for this claim seems incomplete.

[167] Malpangotto, 'The Original Motivation', p. 403, also reaches the conclusion: 'For Copernicus, the perfect regularity and circularity of motions upon which Brudzewo had insisted as a necessity became the basis upon which he founded his search for an alternative solution…'; however, while the issue of irregular motions in Ptolemaic astronomy may partially explain Copernicus' motives in seeking out a new model of the universe, the motivation of Copernicus remains a controversial issue. Malpangotto also sees Albert as a critic of Peurbach, and this interpretation leads her to translate and interpret some passages in ways that allow her to see critiques and even 'personal disappointment' (e.g. 'The Original Motivation', pp. 372 and 387) where I see only Albert's agreement with Peurbach. Barker, 'Albert of Brudzewo', p. 137 and Sylla, 'The Status of Astronomy as a Science in Fifteenth-Century Cracow', p. 78, also see Albert as fundamentally agreeing with Peurbach.

[168] An edition of the *Commentariolum* is found in L. Birkenmajer, *Commentariolum super theoricas novas planetarum*. Because Birkenmajer did not know all the surviving exemplars, also see Markowski, *Astronomica et Astrologica Cracoviensia ante Annum 1550*, pp. 11–13; and Malpangotto, 'The Original Motivation', pp. 403–09. Additionally, Jacob of Würzburg's copy of Peurbach's *Theoricae novae planetarum*, Munich, BSB, Clm 51, ff. 72r–88, contains numbers and letters in alphabetical order marking the lemmata upon which Albert commented in the *Commentariolum*. Jacob at least had access to a manuscript of the *Commentariolum*, but because he was at the university of Cracow in the 1480s, he could have attended Albert's lectures.

de Polonia, one of Schindel's students named by Schindel as one of his most engaged students.[169] That Albert depended upon this particular manuscript of the *Almagest* is clear because at the beginning of his treatment of the lunar orbs, he gives principles from the beginning of *Almagesti minor* IV, prefacing them, 'Antequam autem accedetur littera, quasdam suppositiones praemittere videtur esse non inutile, ex quibus Luna argui et concludi potest, plures habes orbes. Et hae suppositiones sunt de Commentario, seu Abbreviato Ptolemaei, quod creditur esse Magni Alberti (quod incipit: "Omnium recte philosophantium").'[170] This second sentence is obviously copied from one that Schindel has in his note at the beginning of *Almagest* IV that contains the principles of *Almagesti minor* IV: 'Hec sunt sunt [*sic!*] suppositiones commentarii quod incipit "Omnium recte philosophantium", quod credo esse Alberti Magni.'[171] Albert's use of this manuscript is also shown by his inclusion of a figure depicting the phases of the moon that is copied from one in Schindel's marginal notes.[172] While many of Albert's uses of the *Almagesti minor* could come from the excerpts found in Schindel's manuscript, others could not; consequently, he must have had access to another witness of the *Almagesti minor*.

Albert quotes many of the principles from the beginning of the books of the *Almagesti minor*. He makes a great deal of the 'maxim' in *Almagesti minor* III that explains that the motions of celestial bodies are simple and uniform, quoting it three times, once in the section on the sun and surprisingly twice in the section on the planets, which are not treated in the *Almagesti minor*.[173] He also quotes four of the postulates of *Almagesti minor* IV concerning the apparent irregularities in the moon's motion through the zodiac.[174] He quotes the definitions of the moon's true place in the heavens and in the ecliptic from *Almagesti minor* IV, and he gives similar definitions for the planets.[175]

Albert quotes almost the entirety of the text of *Almagesti minor* V.2 to show why Ptolemy gave an eccentric to the moon.[176] This passage is more detailed than the corresponding section of the *Almagest*, which is perhaps why Albert uses it. In other places he only quotes the enunciation and corollaries, not the proofs. Albert quotes the enunciation of *Almagesti minor* III.4, and he follows

[169] Cracow, BJ, 619, f. 272r. I must thank David Juste for this information on Alexius.

[170] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 44–45.

[171] Cracow, BJ, 619, f. 69v.

[172] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 68; and Cracow, BJ, 619, f. 98r.

[173] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 23, 85, and 89. Note that Birkenmajer was only able to determine a few quotations from the *Almagesti minor* and often did not know where in the text they ended.

[174] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 45. These are the second, third, fourth, and fifth principles of *Almagesti minor* IV.

[175] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 73 and 102.

[176] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 47–48.

this with a proof, but it is taken from Gerard of Cremona's translation of the *Almagest*, not the *Almagesti minor*.[177] Similarly, Albert quotes *Almagesti minor* III.5's enunciation and corollary, but follows them with the corresponding proof using the *Almagest* itself as his source.[178] The inclusion of any demonstration at all is seen to be a deviation from Albert's *modus operandi*; he gives Ptolemy's position for the solar apogee and gives references to the relevant chapter of the *Almagest* and to *Almagesti minor* III.11. He then writes, 'Go there for a mathematical proof of this, or to the first part of the *Albion*, for it is not our present intention because of the expenditure of effort to treat each thing demonstratively, but in some things it will be enough to show the place to which you may withdraw it.'[179] Accordingly, Albert refers his students to *Almagesti minor* III.12, 13, and 15 for the proofs concerning the solar equation.[180] He quotes the enunciation of *Almagesti minor* IV.1.[181] He quotes the enunciation of *Almagesti minor* III.4 a second time, but in a discussion of why the moon has an epicycle.[182] He also quotes the quite lengthy enunciation and corollary of *Almagesti minor* V.7, which he follows with a reference to its proof.[183]

Two of Albert's usages of another source, Richard of Wallingford's *Albion*, include references to the *Almagesti minor*. He quotes from *Albion* I.18, which states that the *Almagesti minor* corrects Ptolemy's values for the moon's apparent diameter, and he includes Richard's incorrect reference to the fourth comment of *Almagesti minor* V.[184] He then quotes from *Albion* I.19 concerning the ratio of the moon's radius to the radius of the earth's shadow and paraphrases Richard's reference to the 'Commentator of the *Almagest*.'[185]

Epitome of the *Almagesti minor*

Utrecht, Universiteitsbibliotheek, 6.A.3 (725), ff. 4r–10r, contains a commentary on the *Almagest* that consists largely of excerpts from the *Almagesti minor*.

[177] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 33.

[178] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 42–43.

[179] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 38: '... ut patet per eumdem dictione tertia capitulo quarto et in Abbreviato [Almagesti] per undecimam propositionem. Ibi ergo recurre pro demonstratione huius mathematica, aut ad primam partem Albeonis, non est enim praesentis intentionis propter dispendium singula demonstrative tractare, sed in quibusdam satis erit, locum, ad quem te referas, ostendere.'

[180] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 40.

[181] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 46.

[182] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 50.

[183] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, pp. 64–65.

[184] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 134; and North, *Richard of Wallingford*, vol. I, p. 288. The correct passage of the *Almagesti minor* is probably V.19.

[185] L. Birkenmajer, *Commentariolum super theoricas novas planetarum*, p. 135. North, *Richard of Wallingford*, vol. I, p. 288.

The work begins 'Incipit Liber Almagesti Ptholomei abbreviatus. Prefatio sex continens conclusiones. Omnium recte philozophantium ...' The last folio of the text has been trimmed and some of the text is lost. The last remaining legible words are '... pariformiter de duobus reliquis triangulis orthogoniis duarum.' In addition to the title given in the incipit, another is given later: 'Incipit liber tertius Epythomatis super Astronomia Albategni.'[186] It is clear that the work was composed in the second half of the fifteenth century or the early sixteenth century because it cites the *Epitome Almagesti*, which was finished in 1462, and the manuscript can be dated to the late fifteenth century or the early sixteenth century. The author is unknown. Most of the propositions are lacking their figures.

This text is a mixture of excerpts, summaries, and commentary. The text follows Group 1. It contains the preface with the principles numbered. It then contains enunciations of *Almagesti minor* I. Of the first 13 propositions, only I.1 and I.6 provide more than the enunciation, and these only have very short comments and excerpts from the *Almagesti minor*. I.6 is divided into two propositions – the second is 'Propositio septima. Non est ergo inconveniens chordam unius arcus ponere partem 1 puncta 2 secunda 50. Unde manifestum est quod arcus dimidii chorda gradus punctis concluditur fere 31 et secundis 25.'[187] Therefore, the following propositions of Book I are not numbered in accordance with the *Almagesti minor*. Propositions I.14–18, corresponding to *Almagesti minor* I.13–17, include the text of the proofs taken from the *Almagesti minor*, with some changes such as the inclusion of a more recent value for the maximum declination of the ecliptic, which is probably taken from Peurbach and Regiomontanus' *Epitome Almagesti* I.16, and the omission of particular values in I.17–18, corresponding to *Almagesti minor* I.16–17.[188] The writer usually notes the correspondences between each proposition and the *Epitome Almagesti*.[189] Book II's definitions are summarized and the enunciations up to II.6 are given. Only the proof of II.3 is included, and even for this the writer mentions that the corresponding proof in the *Epitome Almagesti* is more universal.[190] The writer gives no excerpts from II.7–36, explaining that these are explained well in the *Almagest*, the *Epitome Almagesti*, and Regiomontanus'

[186] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 7v.

[187] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 4v.

[188] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), ff. 6r-v.

[189] This commentary's references to the propositions from *Epitome Almagesti* I show that the compiler used a version of Peurbach and Regiomontanus' work with numbering as found in Cracow, BJ, 595 and perhaps other manuscripts, not that found in Venice, BNM, Fondo Antico lat. Z.328 and the 1496 printed edition.

[190] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 7r.

'Problems of the general table of the first mobile or *directorii*.'[191] Book III only has the principles (minus the second) and the enunciations of the first two propositions directly taken from the *Almagesti minor*. A summary concerning some of the values for the length of the year is given for III.1 and a note explains that the *Epitome Almagesti*'s propositions match clearly up to the ones of III and that the *Epitome Almagesti* is sufficient.[192] Only the first seven postulates of Book IV are given, followed by the enunciation and first two sentences of the proof of *Almagesti minor* V.15. After blank ff. 8v–9v, there follows what is titled 'Propositiones ad planetarum motuum equationes facilime fabricandas.' This primarily consists of the enunciation and excerpts from *Almagesti minor* III.17, but there are also many of the commentator's own additions.[193]

[191] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 7r. This refers to Regiomontanus' canons for his tables of the first mobile and his tables of directions, which were printed in 1490 (Augsburg, Erhard Ratdolt) and in 1514 (Vienna, Iohannes Winterburger for Leonardus and Lucas Alantse).

[192] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 7v.

[193] Utrecht, Universiteitsbibliotheek, 6.A.3 (725), f. 10r.

Chapter 8

Editing Methods

In my attempt to produce the text of the *Almagesti minor* in a form close to the original and to portray the development of the different manifestations of the text, I report the readings from a representative witness from each of Groups 1.A, 1.B, 2, 3.A, and 3.B. These are *P*, *N*, P_7, *K*, and *M*. I generally selected a manuscript in each group that appeared to be close to the original text and that does not contain an excessive amount of careless errors. I made an exception and chose a late manuscript from Group 1.B because it was written by Regiomontanus and it may be of use to some scholars to have the text as he wrote it. Although *Ba* is the sole witness of Group 4, I do not count it among my principle witnesses because reporting all of its numerous variants, which are often due to extreme carelessness, would be of little use and would obscure more important variants; however, when there is uncertainty over which reading from my witnesses is the best, I also turn to *Ba* and to E_1, which could be considered a member of a subgroup of Group 2 consisting of it and W_1. In orthographical matters, I generally follow my principle witness, *P*, which was chosen because it is an early manuscript from northern France, where the work was likely composed, and it is one of the *Almagesti minor* manuscripts that shares characteristics of some of the early northern French manuscripts of Gerard of Cremona's translation of the *Almagest*. Despite the probability that *P* is close to the original text, it contains a number of careless errors, which is not surprising given that its scribe probably produced this manuscript because he was hired by Richard de Fournival, not because he was genuinely interested in the content. Alternate proofs and unique passages from manuscripts are included in the Appendix.

I use the stemma that I have proposed when weighing variants. I do not treat the representatives of the groups equally. Because *N* is a member of a late group and was written by a scribe who had the knowledge to correct the text, it is not as reliable of a witness for Group 1 as *P* is. I also generally take *K* as a more reliable witness of Group 3 than *M* because of the likelihood of contamination in *M*. Thus, in the first sentence of the preface, for example, I suspect that 'non solum' is an addition although it is in members of two of the three groups that I consider (*P*, *N*, and *M*). Its absence in P_7 and *K* suggests that it should not belong in Groups 2 and 3. In such cases I turn also to *Ba* and E_1 as additional arbiters, and in this case they confirm my suspicion that 'non solum' is an addition. Whenever I consult these sixth and seventh witnesses, I report their variants in parentheses to make it clear that I do not report all significant variants as I do for my five principle witnesses.

I attempt to be generous in my inclusion of variants so that readers who disapprove of my editorial choices can easily recreate the text according to their own criteria, but I exclude some types of variants that tell us little about the history of the text and that clutter the apparatus. I disregard most orthographical variants. I typically follow the spelling of *P*, and only note orthographical variants for names and a few rare words. As is typical of medieval writing, there are many variations in spelling, and even in the same manuscript, several spellings of the same word can be found. I report marginalia or interlinear writing only when it supplies part of the text of the *Almagesti minor* or if it is inserted in the text in another witness.

Readers should be aware that my editorial choices include the following standardizations and deliberate omissions of minor variants:

- The punctuation is my own. My guiding principle has been to punctuate according to the expectations of an English reader.
- *P* and other manuscripts sometimes have '-ci-' before vowels in words that would have '-ti-' in Classical Latin. Although this is normal in medieval Latin, I standardize to '-ti-' both in the text and the apparatus for the sake of uniformity and because the two letters are often indistinguishable in the manuscripts. However, in instances where *P* has '-ti' where standard orthography and the other manuscripts have '-ci' (e.g. 'superfities' instead of the more common 'superficies'), I put the more common '-ci-.'
- In some words, the letters 'm' and 'n' are both acceptable spellings (e.g. the prefixes 'com-'/'con-' and 'in-'/'im-' in some words and 'quamdiu'/'quandiu'). Although almost always abbreviated in ways that could be either, both spellings of the prefix 'con-'/'com-' are found. I standardize to 'com-' in all instances in the text and apparatus. Similarly, the prefix 'in-'/'im-' has been standardized to 'im-' before letters 'm' and 'p.' In other words that could have either 'm' or 'n', I follow *P*, and if it is ambiguous, I choose what seems the more usual spelling of the word. I do not report substitutions of these two letters in the apparatus.
- The letters 'y' are 'i' are often used interchangeably. In *P*, the letter 'y' is frequently written where other scribes and even *P*'s scribe normally write 'i.' In the text, I standardize the following spellings to ones with 'i': 'semydyameter', 'tropycus', 'pyramydales', 'clyma', 'ymago', 'epicyclus', 'hyemalis', and 'Euclydis.' I retain 'y' in words usually spelled with it (e.g. 'physica', 'ypothesis', 'Egypti-', and 'Amphytritis').
- *P* often has a single consonant when there is clearly accepted spelling with a doubled consonant found in standard dictionaries that is also found in the other witnesses (e.g. 'agregatur' instead of 'aggregatur' or 'Sagitarii'

instead of 'Sagittarii'). In such cases, I put the more common spellings in the text and ignore the variant spellings.

- For third declension adjective ablative endings, one finds the endings '-e' or '-i' (e.g. *P* has both 'longiore' and 'longiori'). I follow what is in the main witness if that is clear. If that is not clear, I add endings according to the general practice of the scribe in the surrounding text or according to the other witnesses. I then ignore all other variants that differ only in having the other '-e' or '-i' ending.
- As I wrote above, I do not record most orthographical variants in the apparatus. To be more precise, I do the following:
  - While witnesses, especially *N*, sometimes have 'eque-', I always standardize to 'equi-.'
  - I do not note variants that are obvious misspellings of non-technical words (e.g. 'lenea' for 'linea' or 'costituta' for 'constituta').
  - Except in names, I have not noted variants that add or omit 'h' at the beginning of a syllable or after the letters 't' or 'c.'
  - I ignore the following variant readings: 'apud'/'aput', 'sed'/'set', 'caput'/'capud', 'velut'/'velud', 'nichil'/'nihil', 'auctor'/'autor', 'hee'/'he', 'hii'/'hi', 'hiis'/'his', and 'sexqui-'/'sesqui-'/'sexqu-'/'sesqu-.'
  - I ignore variants with doubled consonants or single consonants where two are given in the principle witness.
  - I ignore variants that add or omit a 'p' to an 'm' or an 'n' (e.g. 'calumpnians', 'septemptrionalis').
  - I ignore variants that interchange 's' and 'z' (e.g. 'orizon'/'orison'). 'Orison' is the spelling used in *M* and *N*, but others spell it with a 'z.'
  - I ignore variants that have 'i' for 'y' and vice versa.
  - I ignore variants that have '-ae' or 'ę' (both occur rarely in *N*) for '-e.'
  - P_7 sometimes has the endings '-qum' and '-qus' (e.g. 'reliqum' and 'equs'). I do not report variants that differ only in this regard, and I standardize other variants to '-quum' and '-quus.'
- I ignore variants that consist of mere reorderings of letters referring to lines, arcs, etc. of the figures that make no mathematical difference (e.g. 'GD'/'DG').
- I ignore variants for some words that are commonly used interchangeably and that do not affect the meaning, including 'et'/'etiam', 'igitur'/'ergo', 'super'/'supra' (also when used as prefixes).

- I do not report variants that are merely different manners of reporting the same number (see section on numbers below for more details).
- I also ignore variants that reflect situations in which scribes immediately realized and corrected their mistakes; e.g. when the normal text has 'Word A' followed by 'Word B' and a manuscript has '~~Word B~~ Word A Word B', it appears that the scribe skipped 'Word A', noticed his mistake before he wrote more than 'Word B', and then deleted 'Word B' and wrote the two words in the order that he saw in his exemplar.

These are the types of standardizations that I make and the insignificant variants that I leave out of the apparatus. When two variant readings differ only according to such differences, I do not give separate entries in the apparatus. I spell the variant as it is found in the first manuscript in this order: *P*, P_7, *K*, *M*, *N*. For example, instead of writing in the apparatus 'semicirculo] semycirculi *P* semicirculi *N*', I would only write 'semicirculo] semycirculi *PN*'; or instead of writing 'ccli] 269 P_7 cclxix *K*', I would write 'ccli] 269 P_7K.' When there is a correction in a witness that involves only the types of variants listed above, I do not present it in the apparatus.

There are a few specific words that are particularly problematic because of either ambiguous abbreviations or the inconsistent use of similar words. I follow witnesses when possible and attempt to follow what seems to be the general practice. However, the reader should be aware that there is often a degree of ambiguity in such cases.

Among the troublesome words, there are a few words that are often written in an abbreviated form but without any of the usual signs of abbreviation. Thus 'equinoc' and 'lon lon' (for 'equinoctium'/'equinoctialis' and 'longitudo longior') are found sometimes with a raised dot following the words to indicate abbreviation and are sometimes found with no such dot or mark (this occurs frequently in *K*). Given the great number of times these words are used in the *Almagesti minor*, it seems reasonable to assume that the scribes intended the readers to expand this word (even if they did not always write any sign that the word needed to be expanded) and felt that the intended word would be obvious. Unfortunately, 'equinoc' seems to be used by the scribes as the abbreviation for 'equinoctium' and 'equinoctialis', and both expansions make sense; e.g. 'punctum equinoc' could be 'punctum equinoctii' or 'punctum equinoctialis.'[1] Likewise, the same abbreviation is used both for 'longitudo', 'longior', and 'longum.' To compound the difficulty, different endings are often possible (e.g. 'ab arcu' could be followed by either 'equinoctiali' or 'equinoctialis'). For

[1] A similar use of several different similar formulations is found in Plato of Tivoli's translation of Albategni. For example, a short passage in Ch. 28 (*De scientia stellarum*, 1537 ed., f. 28r) uses forms of 'aequinoctium', 'punctum aequinoctii', and 'punctum aequinoctiale.'

'equinoc-', the relatively few times that the word is expanded shows no discernible system, but when the witnesses point to a particular reading that makes sense, I select it and only note variants that cannot be expanded in the same way. I generally expand various abbreviations of 'equinoc-' as forms of 'equinoctialis', and I only choose the word 'longum' when the word is spelled out in a witness or is abbreviated unambiguously.

The endings of forms of 'gradus' and 'minuta' are often not provided although more than one case makes grammatical and mathematical sense (e.g. 'arcus est 10 gradus et 30 minuta' or 'arcus est 10 graduum et 30 minutorum'). Whenever the ending is clear, I report the reading either in the main text or the apparatus. Otherwise, I supply the ending that seems to make the most sense. When these words occur in sets, e.g. '50 grad- et 10 minut-' and one of them has a clear ending, I have supplied the other one with an ending in the same case. Occasionally, mismatched sets are found (e.g. '50 gradibus et 10 minuta').

Another troublesome set of words is 'septentrio' and 'septentrionalis', which are often used interchangeably although one is a noun and the other is adjective. The endings are often left for the reader to supply, and it can be impossible to determine with certainty which word is intended. I opt for the noun when there is no other noun and the adjective when there is a noun.

Another messy situation involves the words for diameter. None of the main witnesses are consistent. For the nominative singular, we find not only 'diameter' but also 'diametros' in all of the five main witnesses except P_7, as well as 'diametrus' in *P*, *K*, and P_7. We also find 'diametrum' in these three, but perhaps these are mistakes. The gender is only consistent in *N*, which always treats it as feminine. The others also treat 'semidiameter' as feminine, e.g. in V.7, but more often treat it as masculine. While feminine adjectives and 'diametros'/'diametrus' are common near the beginning of the work, by Book VI most witnesses use the masculine 'diameter' exclusively. The '-os' ending is used as genitive in I.6, but 'diametri' is found for genitives everywhere else in the book. There is also a lack of clarity because abbreviations are used that could be expanded in more than one way. For example, *K* has the ending '-us' clearly written only once, has the ending '-os' twice, and could be expanded either way six times. Given this confusion, even the scribe very possibly did not have certainty over which ending was intended. *P* shows a slight preference for '-os.' In unclear situations, I either follow a witness that is clear, or I hesitantly expand the ambiguous cases with the more common '-os.' It is very unclear whether the chaotic state of this word is reflective of the original text or whether it is the result of scribes changing the text to their preferred form of the word(s).

'Eclipsis' is another word that could be interpreted in different ways. When it is spelled completely, the accusative singular is normally 'eclipsim' but sometimes 'eclipsem' is employed, especially in *P*. Also, either 'eclipsem', 'eclipsim', and 'eclipsis' all make sense following 'medium.' In ambiguous cases, I follow the more normal practices, i.e. when the word is clearly accusative, I expand as 'eclipsim', and I use the expression 'medium eclipsis.'

Several scribes had difficulty with the word 'epiciclus' when they first encountered it in *Almagesti minor* III. It is found in the following incorrect forms, which I do not report in the apparatus:

P: epiticlum, epicliclus, epiclicli, epicliclo, piciclum

P_7: epicipli

K: epicirculi, episciclo, epiclici, epiclo, eplclici, epiclichi, episcicli, episcicli, episciclum

There are similarly a great variety in the way names are spelled. I only report the variant spellings of the 5 main witnesses in the critical apparatus.

Numbers

There is a great variety in the manner in which numbers are written. All witnesses have a mixture of spelled numbers and numbers given in numerals. *P* and *K* generally use Roman numerals, but they also contain some Arabic numerals (see I.6 and also once in VI.9). P_7, *M*, and *N* generally have Arabic numerals, but P_7 also uses Roman numerals often. *M* and *N* often write fractions in the form '2/3' while the other manuscripts normally use numerals with endings or spell them out in words. In addition to the spelled, Roman, or Arabic variations of a number, there can be different ways of specifying an ending. For example, the witnesses could have 'ii', 'ii^{a}', '2', '2^{a}', or 'secunda.' Noting four variants for almost every number in the text would be burdensome, so I generally follow my primary witness *P* and give no variants that refer to the same number. I ignore the endings added to cardinal numbers (e.g. 'xxiiii^{or}' is written as 'xxiiii'), and I allow simple numerals to be understood as fractions or ordinals depending on the context (e.g. 'xii' can be understood to mean 'duodecima' or 'duodecim'). I have expanded ordinal numbers up to 'duodecima' as words in both the text and apparatus. When two manuscripts have variant readings that differ only in way of referring to the same number, I combine them into one entry in the apparatus. When the incorrect ending is on a fraction or ordinal, I report it in the apparatus. When *P* has an omission that includes numbers, the text is often found in the margins in Arabic numerals. In such instances and when *P*'s number is contradicted by the other witnesses, I give the Roman numerals from *K* and do not note the use of Arabic numerals, or when the reading from *K* is also unable to be used, I put the numbers in Roman numerals because that is surely what was in the original.

Most manuscripts number the propositions in each book in some manner, and although some manuscripts have some deviations, the numbering is very consistent among the manuscripts. There are also internal references, many of which confirm the numbering of propositions. There are some references that are inconsistent with the numbering of propositions found in the manuscripts, but these seem most likely to be simple mistakes on the part of the author or a scribe early in the text's transmission, especially since some of the propositions referred to by another number are also referred to by the standard number elsewhere. My numbers agree with those given in most manuscripts, but in the apparatus I do not report the variant ways in which these numbers are expressed, errors in numbering, or the absence of numbering.

Abbreviations in the Apparatus

⟨…⟩		mark additions by the editor
† …†		mark uncertain words or letters
add.	*additur*	word(s) are added
add. et del.	*additur et deletur*	word(s) are added but then deleted
adnot.	*adnotatur*	the text is given above line or in margin but appears to have been intended as a note, not as a part of the text
corr. in	*corrigitur in*	the text has been corrected into the text that follows
corr. ex	*corrigitur ex*	the text has been corrected from the text that follows
del.	*deletur*	the text is deleted, erased, or expunged
iter.	*iteratur*	word(s) are given twice
iter. et del.	*iteratur et deletur*	word(s) are given twice but then one is deleted
marg.	*margine*	text is given in the margin
s.l.	*supra/sub lineam*	word(s) are written above or below the line

Figures and Labels

In the figures and the text, I have put all letters labeling or referring to points in the figures in capital letters to clearly distinguish them although the witnesses have lowercase letters. In the manuscripts diagram letters are not always differentiated, but they are sometimes indicated by points before and after or by lines over them. I do not attempt to replicate or note the presence of such markers.

The importance of figures has been pointed out strongly in recent years, but many issues remain about how to approach them.[2] While I have found some figures to be useful in illuminating the transmission of the text (see section on the relationships of the manuscripts above), it quickly became apparent that the figures did not always reflect the manuscripts' grouping or dependence

[2] See Saito and Sidoli, 'Diagrams and Arguments in Ancient Greek Mathematics'; and De Young, 'Editing a Collection of Diagrams Ascribed to Al-Ḥajjāj.'

upon each other. A complicating factor is that when they were drawing figures, scribes turned not only to their exemplars of the *Almagesti minor*, but also to those from the *Almagest*, as was discussed above in Ch. 5. I have chosen a descriptive methodology that does not attempt to recreate archetypal figures.[3] I present figures taken from one of my witnesses that are clear and that harmonize with the text. I turned to *P* first, but when there were major problems, I selected figures from *K*, *B*, *M* or *N*. I recreated the figures by drawing over images taken from the manuscripts using the program DRaFT.[4] There are some obvious changes between my recreations and the original figures (e.g. I draw on a computer with thin, straight lines and circles instead of thick, sometimes crooked, hand-drawn lines and curves, I use capital letters, and I sometimes move the locations of the labels for clarity); however, I recreate many of the mistakes and imperfections of the originals that do not make the proofs obscure (e.g. perpendicular lines that are clearly drawn obliquely). I note the changes in the figures of main witnesses, and I report the more significant changes in the remaining manuscripts – I ignore some small differences such as one or two labels that are different or lines that do not meet quite as they should. I also ignore many differences in appearance that have no mathematical significance. For example, I do not report that some manuscripts have I.2's figure rotated 180° and that some do not draw angles ABE and DBG of equal size.

The Translation

The translation provides a more accessible version of the content of the *Almagesti minor* and also presents my interpretation of unclear passages in the Latin. I have attempted to remain as literal as possible with some concessions for the sake of clarity and conciseness. For example, I often take the freedom of adding or ignoring 'and' and forms of 'to be' to make the meaning clearer. Any other words that I insert in the translation for the sake of clarity in the English are marked by pointed brackets. Short explanatory comments are marked by square brackets. Also, for the sake of fluidity in English, I frequently translate the tense and mood of verbs non-literally. I also occasionally translate prepositional phrases as adverbs, translate adjectives as relative clauses, and separate relative clauses into their own sentences. Additionally, while I think it is often best to avoid modern symbolism when expressing medieval mathematics, I make some small concessions for the sake of conciseness. I express numbers of measurement in Arabic numerals and simplify fractions, which are often expressed in ways that sound clumsy to our ears. For example, instead of the literal 'that which will consist of a half and a quarter [degrees]' for 'qui ex media et quarta

[3] My approach is similar to that found in Kunitzsch and Lorch, *Theodosius*, *Sphaerica*.
[4] Developed by Ken Saito and available at http://greekmath.org/draft/draft_index.html.

constabit', I have simply '45′.' I use the symbols in the format $1^{\circ}\ 1'\ 1''\ 1'''\ 1^{iv}$ etc. for the degrees and subsequent sexagesimal divisions of arcs, and $1^{p}\ 1'\ 1''$ etc. for the parts that are 1/120 of the diameter and the subsequent sexagesimal divisions of straight lines. I also use more common astronomical terms instead of replicating the longer and sometimes clumsy Latin phrasing for these concepts.

Part II

Critical Edition and Translation

Sigla of MSS Used in Edition

P	Paris, Bibliothèque nationale de France, lat. 16657
P_7	Paris, Bibliothèque nationale de France, lat. 7399
K	Cracow, Biblioteka Jagiellońska, 1924
M	Munich, Bayerische Staatsbibliothek, Clm 56
N	Nuremberg, Stadtbibliothek, Cent. VI.12
Ba	Basel, Universitätsbibliothek, F.II.33
E_1	Erfurt, Universitäts- und Forschungsbibliothek, Dep. Erf. CA 2° 383

⟨Liber I⟩

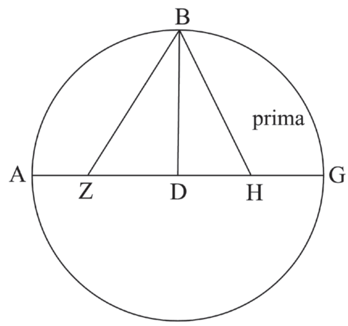
Omnium recte philosophantium verisimilibus coniecturis et credibilibus argumentis sed et firmissimis rationibus deprehensum est formam celi spericam esse motumque ipsius orbicularem circa terram undique secus globosam in medio imoque defixam. Que quidem etsi omnium cadentium tam gravitate corporis quam quantitate ponderis sit maxima ideoque immobilis, ipsius tamen crassitudo comparatione infinitatis applani respectuque distantie fixorum luminum insensibilis, et vicem centri obtinere physica indagatione comperta est. Ad hec duos principales et sibimet invicem contrarios motus superiorum sane animadverti, etiam fides oculata comprobavit, quorum alter semper ab oriente in occidentem pari et eadem concitatione per circulos et inter se et ad eum qui omnium spatiosissimus equinoctialem paralellos totum mundane machine corpus movet et agitat, cuius circumvolutio circa celestis spere polos indefesse consistit. Alter e contrario Solem et Lunam et quinque erraticas circa alios diversosque polos circumducit et torquet. Hiis firme adeo fides conciliata est ut si quis iniuste calumpnians obviet, aut cavillator verum scienter inficians aut mente captus non indigne estimetur. Que cum ita sint superest ut propositum aggrediamur.

1 Liber I] Minor Almagesti *marg. (probably other hand) P* Almagesti Ptholomei *marg. (other hand) K* primus *marg. N* **2/18** Omnium – aggrediamur] *other hand, folio likely added later P* **2** philosophantium] phylosophantium non solum *PMN* (phylosophantium BaE_1) coniecturis] *om. PKN* (coniecturis BaE_1) et credibilibus] credibilibus P_7 credibilibusque *KM* **3** formam celi] *del. N* spericam esse] esse spericam P_7 **4** ipsius] eius *N* secus] sicut *M om. N* **5** etsi] non *add. et del. K* gravitate] quantitate P_7N **6** quantitate – maxima] gravitate ponderis maximaque sit P_7 **7** applani] adplani *M* ad plani *N* **8** obtinere] optinere P_7K physica] *corr. in* phylosofica (*other hand*) *P* indagatione] ratione *N* comperta] compertum *M* **9** sibimet] sibi *N* contrarios – superiorum] diversos superiorum motus P_7 **10** fides oculata] occulta fides P_7 fides occulta *N* comprobavit] *corr. in* approbavit *N* **11** et^1] atque P_7 concitatione] contentione *K* eum] illum P_7 **12** spatiosissimus] spatiosissimus est P_7 est spatiosissimus *M* mundane] meridiane P_7 **12/13** machine – circumvolutio] corpus machine movet atque exagitat cuius revolutio P_7 **13** et] *corr. in* atque (*other hand*) *M* **14** e contrario] vero P_7 vero e contrario *M* quinque erraticas] alios quinque erraticos P_7 **15** firme adeo] adeo firme *M* firme] *corr. ex* ferme *P* ferme P_7 adeo fides] fides adeo P_7 conciliata] *corr. in* consiliata *M* consiliata *N* **16** iniuste] etiam iuste *P* etiam iniuste *N* obviet] aut potius deviet *add.* P_7 obviet vel potius deviet *M* inficians] inficiens *K* **16/17** aut^2 – captus] aut in huiusmodi disciplina parum excercitatus P_7 in huiusmodi disciplina parum exercitatus aut mente captus *M* **17** sint] constent P_7

Book I

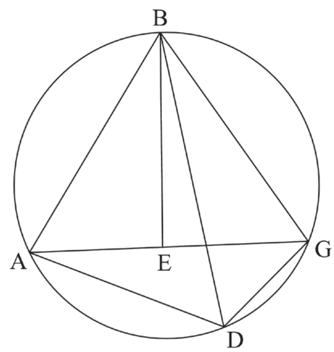
It has been discovered by probable inferences and credible arguments but also by the most firm proofs of all those rightly philosophizing that the form of the heavens is spherical and that its motion is circular around the earth, ⟨which is⟩ from all sides a sphere fixed at the middle and lowest ⟨point⟩. Indeed, although it [i.e. the earth] is the greatest of all falling things both by the heaviness of body and by the quantity of weight and for that reason is immobile, its size in comparison to the limitlessness of the outermost sphere[1] and with respect to the distance of the fixed lights is imperceptible, and it is found by physical investigation to stand in the place of a center. In addition to these things, I observed carefully – and evident confidence confirmed – these two principal and contrary motions of the higher ⟨bodies⟩. One of these moves and revolves the whole body of the universal machine always from east to west by an even and constant motion through circles parallel both to each other and to the equator, which is the largest of all, the revolution of which [i.e. the universal machine] remains unwearyingly upon the poles of the celestial sphere. The other ⟨motion⟩ leads around and turns the sun, moon, and the five planets in the opposite direction about other, different poles. Confidence in these things is brought about so securely that if anyone unjustly finding fault should deny them, he would not unworthily be judged to be either a quibbler consciously denying the truth or a madman. Because these things are such, it remains for us to advance to the objective.

[1] This should not be taken to mean that the author believes that the heavens are literally infinite.



1. Data circuli diametro latera decagoni, pentagoni, exagoni, tetragoni, atque trianguli omnium ab eodem circulo circumscriptorum reperire. Unde manifestum est quod si nota fuerit circuli diameter, et prenominata latera erunt nota, corde quoque que residuis semicirculi arcubus subtenduntur note erunt.

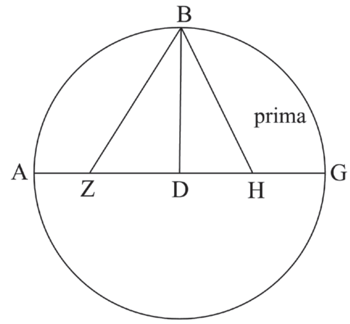
Lineetur enim super AG diametrum semicirculus ABG, sitque DB a centro perpendiculariter erecta, H medius punctus DG, ZH equalis BH subtense angulo recto. Dico quia ZD est latus decagoni et ZB latus pentagoni. Ratio per sextam secundi Euclidis et penultimam primi et nonam tertii decimi. Patent cetera per tricesimam tertii et penultimam primi.



2. Si quadrilaterum infra circulum describatur, rectangulum quod continetur sub duabus eius diametris est equale duobus rectangulis pariter acceptis que sub utrisque eius oppositis lateribus continentur.

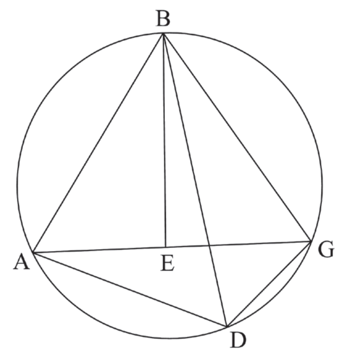
Esto enim quadrilaterum cuius duo diametri AG et BD infra circulum descriptum, fiatque angulus ABE equalis angulo DBG. Erit igitur ABD angulus equalis EBG angulo communiter adiecto EBD, sed etiam ADB et EGB anguli sunt equales quia super eundem arcum consistunt. Ergo propter similitudinem triangulorum unde accidit proportionalitas laterum, quod fit ex AD in BG equum est ei quod

20 tetragoni] *corr. ex* tragoni *K* **21** circumscriptorum] circumscriptibilium P_7 *corr. ex* descriptorum *M* **22** reperire] corollarium *add.* PP_7 **23** diameter] diametrus (*corr. ex* diametris P_7) P_7K **24** erunt] *corr. ex* erint *K* **25** subtenduntur] intenduntur *P* note erunt] erunt note P_7N *corr. in* erunt note *M* erunt] sunt *K* **26** diametrum] diameter *M* dyametro *N* semicirculus] semycirculis *P* semicirculus et sit *K* **28** medius punctus] punctus medius *MN* DG] DG et *M* **29** quia] quod *KMN* **30** sextam] sexta *P* secundi] secundi libri P_7 **31** et[1]] et per *M* decimi] decimi Euclidis *M* Patent cetera] cetera patent P_7MN **32** primi] primi Euclidis *M* **33** infra] intra *KM* **35** duabus] duobus PP_7 (duabus BaE_1) **36** utrisque] utriusque *P* **37** oppositis lateribus] lateribus oppositis *PN* **38** enim] *om. N* duo] due *N* **39** descriptum] descripti *M* **40** angulus ABE] ABE angulus P_7 **40/41** angulo DBG] DBG angulo P_7 **41/42** EBG angulo] angulo EBG P_7 **43** ADB] *corr. in* ABD *M* EGB] *corr. ex* AGB *K* anguli] *om. N* **44** eundem arcum] arcum eundem *PN* (eundem arcum E_1) consistunt] consistunt (*corr. in* consistent) per 20^{am} tertii *M* Ergo – similitudinem] propter similitudinem ergo *PN* unde] inde *M om. N* **45** quod[1]] quare quod *N* ex] ex ductu *PMN* (ex E_1) est] *om. M*



1. With the diameter[2] of a circle given, to find the sides of a decagon, pentagon, hexagon, quadrilateral, and triangle all circumscribed by the same circle. Whence it is manifest that if the diameter of a circle is known, the said sides will be known and also the chords that subtend the remaining arcs of a semicircle [i.e. the supplements] will be known.

For let semicircle ABG be drawn upon diameter AG, and let there be DB erected perpendicularly from the center, H the middle point of DG, and ZH equal to BH, which subtends a right angle. I say that ZD is the side of a decagon and ZB is the side of a pentagon. The proof is through the sixth of the second of Euclid, the penultimate of the first [i.e. *Elements* I.47], and the ninth of the 13th. The rest[3] are clear through the thirtieth of the third[4] and the penultimate of the first.



2. If a quadrilateral is described under a circle, the rectangle that is contained under its two diameters is equal to the two rectangles taken together that are contained under each of its ⟨pairs of⟩ opposite sides.

Indeed, let there be a quadrilateral, whose two diameters are AG and BD, drawn under a circle, and let there be made angle ABE equal to angle DBG. Therefore, angle ABD will be equal to angle EBG with EBD added to both, but angles ADB and EGB are also equal because they stand under the same arc. Therefore, because of the similitude of the triangles from which proportionality of the sides occurs, what comes from AD in BG is equal to that which is contained under BD and GE.

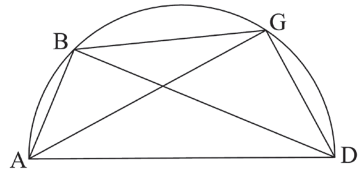
[2] The participle shows that 'diametro' is feminine. *N* is the only of our main witnesses that consistently uses a feminine noun for the line through the center of a circle. Others use masculine and feminine forms inconsistently.

[3] This refers to the sides of the square and the triangle, as well as chords of the supplements.

[4] This is *Elements* III.31 in modern editions.

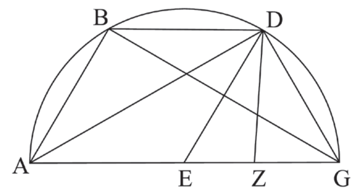
continetur sub BD et GE. Consimili ratione quod continetur sub BD et AE equatur ei quod fit ex AB in GD. Restat ergo per primam secundi Euclidis argumentari.

3. Si in semicirculo corde arcuum inequalium certe fuerint, corda quoque arcus quo maior minorem superat erit nota.



Sint enim AB et AG nota; ergo et DB et GD quia subtenduntur residuis arcubus in semicirculo note sunt. Et quia diameter semicirculi nota, per proximam argue.

4. Si in semicirculo corda alicuius arcus fuerit nota, corda quoque que eius medietati subtenditur erit nota.

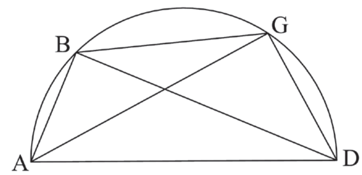


Ex ypothesi BG nota cuius arcus medius punctus D. Ergo AB nota cui sit equalis AE. Ergo AD facta communi erit ED equalis tam BD quam DG. Unde anguli super E et G equales per heleufugam. Quare demissa perpendiculari DZ, erit GZ equalis EZ, et GZ nota. Diametros quoque nota, inter quas DG proportionalis, quare et ipsa nota.

46 continetur[1] sub] fit ex *K* BD[1]] *corr. ex* BGD P_7 Consimili ratione] consimili de causa P_7*M* pari causa *K* **47** equatur] equum est *K* Restat – Euclidis] per primam ergo secundi Euclydis restat *PN* secundi] secundi libri P_7 **49** arcuum] *corr. ex* arcuum P_7 **50** certe] note *KM* fuerint] fuerunt P_7 **51** superat] suparat *N* **52** Sint] sit *K* nota – et[2]] note ergo *PN* **52/53** DB – quia] BD nota et que *KM* GD et BD erunt note quia *N* **54** note sunt] nota est P_7 quia quadratum DA valet duo quadrata reliquorum laterum propter angulum rectum ad circumferentiam *KM om. N* quia] *om. N* **54/55** diameter – nota] semycirculi dyameter notus *P* **54** diameter] diametrus P_7*K* **55** semicirculi] circuli *N* proximam] *corr. ex* primam *K* argue] quod BG est nota *adnot. K* quod BG est nota *add. M* **56** corda[1] – nota] alicuius arcus corda nota fuerit *PN* arcus] *s.l. K* eius] eiusdem P_7*M* **58** ypothesi] ypostesi *M* BG] est *add.* (*s.l. K*) *KM* medius] medinus *P* **59** D] scilicet D *M* **60** erit] *s.l. P* **61** DG] per iiii primi quia anguli A sunt equales cum sint in portionibus equalibus *add. s.l.* (*other hand P*) PP_7; quia (*corr. ex* unde *M*) anguli DAB (et *add. M*) DAG sunt equales quia sunt (super *M*) in (*om. M*) equali circuli portione *add. KM*; similiter DAB DGB similiter DBG DEB ergo DAB DAE *add. et del. K*; et latera AB (et *add. M*) AD sunt equalia lateribus AD AE *add.* (*s.l. K*) *KM* **61/62** Unde – heleufugam] *del. K om. M* E – G] *corr. ex* EZ (*perhaps other hand*) *P* **62** heleufugam] elnef†...† P_7 ellefugam *K* helefugam *N* demissa] *corr. ex* dimissa P_7 **63** perpendiculari] *corr. ex* pendiculari *M* DZ] DE *M* **64** GZ] est *add.* (*s.l. K*) *KM* nota[1]] quia AE nota que equalis est AB, et ita EG nota cum diametrus (dyameter *M*) sit nota *add. KM* Diametros] diametrus P_7 quoque] que *N* DG] *corr. ex* BG *P* DG est *K* DG (*corr. in* BG) est *M* BG est *N* proportionalis] proportionalis per sextum (sextam *M*) Euclidis *KM* quare] ergo P_7*K corr. ex* ergo *M* **65** nota] nota est *M*

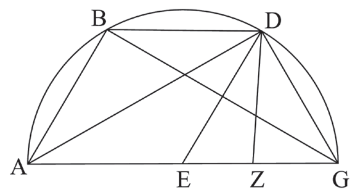
By a very similar proof, what is contained under BD and AE is equal to that which comes from AB in GD. Then it remains to argue through the first of the second of Euclid.

3. If chords of unequal arcs in a semicircle are known, the chord of the arc by which the greater exceeds the smaller will also be known.



For let AB and AG be known; therefore, both DB and GD are known because they subtend the arcs remaining in a semicircle. And because the diameter of the semicircle is known, argue through the last proposition.

4. If the chord of any arc in a semicircle is known, the chord that subtends its half will also be known.

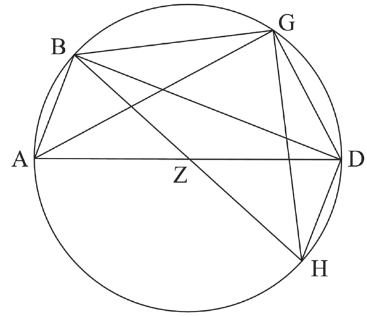


From hypothesis BG is known, the middle point of which arc is D. Therefore, AB will be known, to which let AE be equal. Then, with AD made common, ED will be equal as much to BD as to DG.[5] Whence the angles upon E and G are equal through the *heleufugam* [i.e. *Elements* I.5].[6] Therefore, with perpendicular DZ dropped, GZ will be equal to EZ, and GZ will be known. The diameter is also known, between which [i.e. GZ and AG] DG is the proportional, therefore it is also known.

[5] The critical apparatus here may be hard to comprehend. An explanatory note ('quia ... equalibus'), which is also found above the line in *P*, made its way into the text in *KM*. *K* then includes in the text what is supposed to be a further explanation ('similiter ... DAE'). This explanation duplicates what is already in the text, so it was deleted, but whoever deleted it also deleted the next sentence of the authentic text ('Unde ... heleufugam'). A further explanatory note ('et latera ... AE') was written above the line in *K* and this was copied into the text in *M*.

[6] See Paul Kunitzsch, '"The Peacock's Tail"' pp. 206–8.

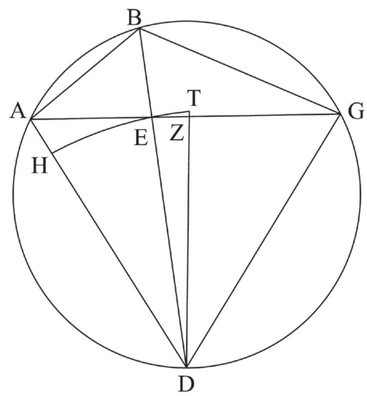
5. Si due corde duorum arcuum in semicirculo fuerint note, corda quoque que toti subtenditur arcui composito ex illis duobus arcubus erit nota.



Ex ypothesi et AB et BG nota. Facta ergo tam AZD quam BZH circuli diametro, erit tam BD quam GH nota. Et quia AB nota, nota est et DH. Ergo cum sit BGDH quadrilaterum circulo inscriptum cuius duo diametri noti et tria latera nota, per secundam erit et quartum notum scilicet DG. Ergo et corda residui arcus de semicirculo AG videlicet nota est, quod erat propositum.

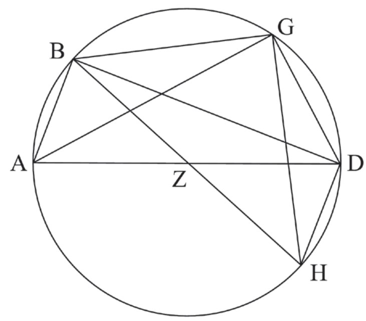
6. Due linee inequales in circulo si protrahantur, maioris ad minorem quam arcus longioris ad arcum brevioris minor est proportio.

Primo angulum ABG linea BD per medium partiatur. Lineis deinceps AG et AD et DG protractis, quia ergo angulus ABG per medium divisus est, lineas AD et GD constat fieri equales. Linea etiam GE longiore existente quam EA, in lineam EG perpendicularem DZ protrahimus. Quia ergo AD quam DE et DE quam DZ longiores sunt, circulus ad centrum D et ad distantiam DE circumductus lineam AD procul dubio secabit. Linea etiam DZ ulterius protracta, ipsum circulum HET signabunt. Quia ergo sector DET triangulo DEZ maior est, sed etiam triangulum DEA eo sectore qui est DEH constat fieri maiorem, erit per octavam quinti Euclidis trianguli DEZ ad triangulum DEA proportio minor ea que est sectoris DET ad sectorem DEH. Sed sectoris



68/69 composito – arcubus] *om. K* composito ex illis P_7 **69** erit nota] nota erit *P* **70** et[1]] *om. N* nota] note sunt *N* **72** AB] HB *KM* nota[2]] *om. N* **73** est] etiam *KM* BGDH] BDGH P_7 quadrilaterum] *corr. ex* quadratum (*perhaps other hand*) *P* **74/75** duo – noti] due dyametri note *N* **75** secundam] *perhaps corr. ex* tria *P corr. in* tertiam *M* **75/76** erit et] erit *P* igitur erit *N* **76** Ergo] *s.l. P* **76/77** AG videlicet] scilicet AG *N* **77** nota est] est nota P_7 nota *KM* erit nota *N* quod – propositum] quod proposuimus P_7 que proponebatur *K* quod proponebatur *M* quod est propositum *N* **78** linee inequales] inequales linee P_7K **79** arcus] archus *K* brevioris] breviorem *M* est] erit P_7K **81** AD] AB *M* **82** fieri] esse *KM* quam] linea *add.* (*s.l. K*) *KM* **83** protrahimus] protrahimus DE *M* **84** ergo] tam *add.* (*s.l. other hand P*) PP_7N **87** secabit] *corr. ex* stabit *M* **88** ulterius] altius PP_7 **89** HET] HEZ *PMN* signabunt] significabunt P_7 signabit *corr. in* secabit *N* sector] sector portio *PM* portio *add. et del. N* **90** maior est] est maior P_7 **92** erit] ea *del. M* erit igitur *N* **94** ea] proportione *K corr. ex* EA *M*

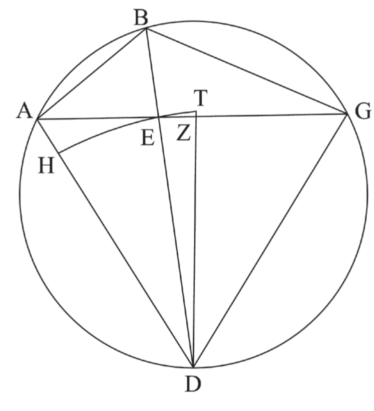
5. If two chords of two arcs in a semicircle are known, the chord that subtends the whole arc composed of those two arcs will also be known.



From hypothesis both AB and BG are known. Therefore, with both AZD and BZH made diameters of the circle, both BD and GH will be known. And because AB is known, DH is also known.[7] Therefore, because quadrilateral BGDH is inscribed in a circle, whose two diameters are known and three sides are known, through the second ⟨proposition⟩ the fourth ⟨side⟩, i.e. DG, will also be known. Therefore, also the chord of the remaining arc of a semicircle, i.e. AG, is known, which had been proposed.

6. If two unequal lines are drawn in a circle, the ratio of the larger to the smaller is less than that of the longer arc to the shorter arc.

First, let line BD divide angle ABG in half. Then, with lines AG, AD, and DG drawn in turn, because angle ABG was divided in half, it is evident that lines AD and GD are made equal. Also, with line GE being longer than EA, we draw perpendicular DZ onto line EG.



Then, because AD is longer than DE and DE is longer than DZ, the circle drawn around with center D and distance DE will doubtlessly cut line AD. Also, with line DZ extended further, ⟨letters⟩ HET will designate that circle. Then, because sector DET is greater than triangle DEZ but also it is evident that triangle DEA is made greater than that sector that is DEH, through the eighth of the fifth of Euclid,[8] the ratio of triangle DEZ to triangle DEA is less than that which is of sector DET to sector DEH. But ⟨the ratio⟩ of sector to sector is that

[7] In the argument of the *Almagest*, DH is known because BD is known. *KM* follow this argument, but it appears that our author reasoned to this in a different manner – through the equality of vertical angles at the center of the circle.

[8] This proposition in the *Elements* is not directly applicable since it only concerns three quantities, but one could use it as the basis for an argument *a fortiori*.

ad sectorem que sui anguli ad suum angulum. Ergo per primam sexti minor est proportio EZ linee ad EA quam anguli ZDE ad EDH. Ergo coniunctim, ergo duple scilicet GA proportio ad eandem EA minor quam dupli anguli scilicet GDA ad eundem EDA angulum. Proportio ergo disiunctim. Restat ergo per tertiam sexti et ultimam eiusdem argumentari.

Nunc quorsum hec tendant declarabimus. Interest presentis negotiationis cuiuslibet arcus noti respectu 360 graduum, que est universalis omnium circulorum partitio, invenire cordam notam respectu 120 partium diametri, ad quem numerum omnis diametros secta intelligitur. Cuius rei cognitio non minus utilis quam difficilis.

Igitur ex prime speculationis ratione arcum 36 graduum habere cordam partium 37 punctorum sive minutorum 4 secundorum 55 sollers practicus inveniet, est enim ea corda latus decagoni; cordam vero pentagonicam que arcui 72 graduum subtenditur componi ex partibus 70 punctis 32 et secundis fere tribus; sed et latus exagoni supra quod arcus 60 graduum curvatur 60 itidem partibus terminari. Ad eundem quoque modum quia latus tetragoni existens 90 partium corda quadratum medie diametros potentialiter duplat, latus item trigonale existens 120 graduum corda medie diametros quadratum potentialiter triplat, illud quidem partibus 84 punctis 51 secundis 10 fere concludi, istud autem partibus 100 et tribus punctis 55 secundis 23 equari, diligens examinator compariet manente dico predicta diametri in 120 equas portiones sectione. Ad hec ex eodem teorumate cum sit corda arcui 36 graduum subtensa ex parti-

96 suum] *om. N* primam sexti] primam scilicet octavam quinti *M* **97** EA] lineam EA *MN* EDH] EDH angulum *M* **98** duple] dupli *KM* **99** EDA] GDA *M* GDA *corr. in* ADE *N* Proportio] Proportio est *PN* disiunctim] disiuncti *P* **101** tendant] *corr. ex* intendant P_7 negotiationis] negotii *MN* **102** universalis] communis *corr. ex* †...† *P* communis *N* **103** cordam notam] eorumdem notam cordam (*the last word in marg. P*) *PN* eorumdem (*marg.*) cordam notam *M* (cordam notam BaE_1) **104** omnis] universaliter omnis refertur *KM* diametros] diametrus generaliter P_7 diametri *M* secta] *corr. in* sectio *M* intelligitur] *om. KM* cognitio] agnitio *PN* **106** speculationis] idest propositionis *adnot.* (*marg. perhaps other hand P, s.l.* P_7) PP_7 propositionis *N* arcum] arcum qui est *MN* graduum] gradus *N* **107** sive] *del. M* 4] 9 *P* 55] *corr. ex* †...† *P* **108** enim] autem *N* pentagonicam] pentaconicam *K* **109** 72] *corr. ex* †...† *P* componi] componitur *N* 32] *corr. ex* †...† *P* et] *om. KM* **110** sed] *om. KM* **111** quoque] ergo P_7 quia] *om. N* tetragoni] *om.* P_7 **112** partium] *corr. in* graduum *M* quadratum – diametros] medie diametros quadratum *KM* (*text confirmed by* BaE_1) diametros] dyametri *N* potentialiter duplat] duplat potentialiter *PN* **113** existens] ens PP_7 *iter. et del. M* quod est *N* 120] *corr. ex* †...† *P* graduum] partium P_7 *om. K* diametros] dyametri *N* **114** illud – concludi] *marg.* (*perhaps other hand*) *M* 51] 15 *P corr. in* 53 *M corr. ex* †...† *N* istud] illud *PN* **115** partibus] *om. N* 55] *corr. ex* †...† *P* 23] *corr. in* 33 (*other hand*) *P* 33 P_7 34 *corr. in* 33 *M* equari] *s.l. K* **116** dico] dice *P* 120] *corr. ex* †...† *P* equas] *om. KM* portiones] *corr. ex* ditones (*other hand*) *P* divisiones *N* **117** Ad hec] ad hoc *M* adhuc *N* eodem teorumate] eadem proportione prima *KM* teorumate] teoh†emat†e P_7 graduum] *om. P* which is of one ⟨sector's⟩ angle to the other's angle. Therefore, though the first of the sixth ⟨of Euclid⟩, the ratio of line EZ to EA is less than that of angle ZDE to EDH. Then *coniunctim*; therefore, the ratio of the double, i.e. GA, to the same EA is less than that of the double angle, i.e. GDA, to the same angle EDA. Then the ratio ⟨is taken⟩ *disiunctim*. Then it remains to argue through the third of the sixth and the last of the same.

Now we will declare in what direction these things proceed. It is the concern of the current business to find the chord, known in respect of the 120 parts of the diameter (by which number every diameter is understood to be divided), of any arc known in respect of 360 degrees, which is the universal division of all circles. The knowledge of which matter is not less useful than it is difficult.

Therefore, from the proof of the first proposition, a clever and practical man will find that an arc of 36° has a chord of 37^p 4′ 55″,[9] for that chord is the side of a decagon; indeed, ⟨he will find⟩ that the pentagonal chord, which subtends an arc of 72°, is composed of approximately 70^p 32′ 3″; but also that the side of a hexagon, upon which an arc of 60° is curved, is likewise bounded by 60^p. In the same way also, because the side of a square, being the chord of 90°, potentially [i.e. its square] doubles the square of half of the diameter,[10] and likewise, ⟨because⟩ the triangular side, being the chord of 120°, potentially triples the square of half of the diameter, the diligent examiner will establish[11] that that [i.e. the chord of 90°] is indeed bounded by approximately 84^p 51′ 10″ and also that that [i.e. the chord of 120°] is equaled by 103^p 55′ 23″ – I mean with the said division of the diameter in 120 equal parts remaining. In addition to these things, from the same theorem, because the chord subtending the arc of 36° is

9 This and many of the following Arabic numerals in this passage appear to be written over the original numerals which appear to have been written in a strange form in *P*.

10 Here and in the next clause, 'diametros' must be genitive. While these two instances could be mistakes, there are other words of Greek origin that have '-os' for the singular genitive.

11 'Compariet' appears to be an unorthodox form of the verb 'comparo.'

bus 37 punctis 4 secundis 55, cordam que residuo arcui de semicirculo scilicet arcui 144 graduum partibus 124 punctis 7 secundis 37 fere terminandam esse sobrius vestigator agnoscet.

Amplius ex sequentium demonstratione constat ad certorum arcuum differentias cordas multas posse inveniri. Qua quidem ratione corda arcus 12 graduum reperienda est, hiis inquam que sunt arcuum 60 atque 72 cordis precognitis. Deinceps plurimas diversorum arcuum cordas invenire inventas secundum arcum mediare sciemus, utpote arcus 12 partium cordam, et deinde arcus 6 partium, nec minus quoque trium, eius tunc qui habet partem et dimidiam, et deinde qui ex media et quarta constabit. Docet autem hec observatio unius partis et medie cordam ex parte una punctis 34 secundis 15 constare, retenta dico dicta diametri divisione; ad eundem denique modum arcus medie partis et quarte cordam puncta 47 habere secunda 8. Amplius ex sequenti apodixi ratum est secundum arcum unius partis et medie et eius cordam quamlibet cordam multiplicis arcus posse inveniri. Nam eo arcu duplicato vel triplicato et deinceps omnes corde note occurrent.

Verum cordam unius gradus sub certa veritate nulla deprehendit ratio. Quamvis enim ad arcum unius gradus et medii corda constiterit, eius tertie partis corda sub numeri compoto nullatenus scibilis est. Eius tamen rei notitia presenti intentioni necessaria est. Summo igitur studio et industria, quamvis non verissime tamen omnis sensibilis erroris periculo depulso, unius gradus corda per cordam unius gradus et medii sed etiam per medii et quarte in hunc modum reperta est.

118 55] *corr. ex* †...† *P* **119** graduum – 124] *marg.* (*other hand*) *P* graduum] graduum subtenditur *KM* (graduum *Ba* graduum subtenduntur E_1) 124] 114 *N* 37 fere] fere 37 P_7 **120** sobrius] subtilis *K corr. ex* subtilis *M* vestigator] investigator P_7N agnoscet] cognoscet *corr. ex* cognoscit P_7 noscet *K* **121** ex sequentium] exequentium *P* ad] *corr. in* ex *P* ex P_7MN (ad BaE_1) certorum] ceterorum *M* differentias] *corr. in* differentiis (*s.l. P*) PP_7 differentiis *MN* (differentias BaE_1) **122** inveniri] invenire *M* corda arcus] *corr. ex* arcus corda *K* graduum] *om. K* **123** est] *om.* P_7 inquam] inquam cordis *N* que sunt] sunt *corr. in* que sunt corda *M* que] qui *P* 72 cordis] *s.l. K* 72 partium cordis *M* **125** utpote] ut *K* utpute *M* partium] graduum *K corr. ex* graduum (*other hand*) *M* deinde] deinceps *KM* **126** eius tunc] item eius *KM* **127** media] dimidia *KMN* (media BaE_1) **128** cordam] cordam invenire *KM* parte] *corr. ex* partes *M* **129** dico dicta] inquam predicta P_7 itaque dicta *KM* ad] et ad P_7K denique] *om. KM* **130** cordam] corda *P* secunda] secundas P_7M **131** ratum] *iter. et del.* P_7 secundum] scilicet *M* eius cordam] cordam eius P_7 **132** cordam] cordam certam *K* multiplicis] *corr. ex* maioris *K* inveniri] invenire *MN* et] et sic *N* **133** occurrent] occurrunt PP_7M (occurrent BaE_1) **135** arcum] cordam P_7 **136** compoto] *corr. ex* concepoto P_7 *corr. ex* composito *M* computo *N* **138** non] nisi P_7 **139** gradus] *om. K* medii[1]] dimidii *KM*

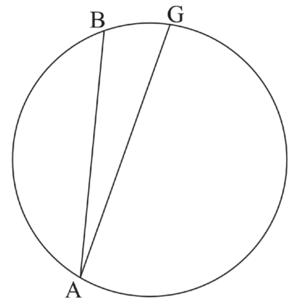
37^p 4′ 55″, the sober investigator will realize that the chord that ⟨subtends⟩ the remaining arc of a semicircle, i.e. the arc of 144°, should be bounded by approximately 124^p 7′ 37″.[12]

Further, by the proof of the following things [i.e. I.3], it is evident that many chords can be found for the differences of the known arcs. By which proof, indeed, the chord of the arc of 12° should be found – I say with these chords known that are of the arcs of 60° and 72°. Then we will know how to find several chords of different arcs by halving an arc, as the chord of the arc of 12°,[13] and then of the arc of 6°, and no less also of 3°, then of 1° 30′, and then of 45′. Moreover, this observation teaches that the chord of 1° 30′ consists of 1^p 34′ 15″ – I mean with the said division of the diameter retained – and finally in the same way that the chord of 45′ is 47′ 8″. Further, from the proof of the following, it is judged that by the arc of 1° 30′ and its chord, any chord of a multiple arc [i.e. of arcs that are multiples of 1° 30′] can be found. For by that arc doubled or tripled and so on, all the chords will appear known.

However, no proof grasps the chord of 1° with exact truth. For although the chord for the arc of 1° 30′ has been made evident, the chord of its third part is by no means knowable under the reckoning of number. Nevertheless, knowledge of this matter is necessary for the current purpose. Therefore, by the highest study and diligence – although not most exactly, yet with the danger of any perceptible error expelled – the chord of 1° is found through the chord of 1° 30′ and of 45′ in this way.

[12] This should be 114^p 7′ 37″ to match the *Almagest*.

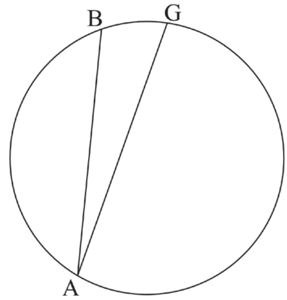
[13] The author generally uses ‘pars’ to denote the parts of the diameter and ‘gradus’ to denote the parts of the circumference, but he often uses ‘pars’ for a chord’s measurement, and more than once, he uses ‘gradus’ with reference to the measurement of a straight line.



Protrahimus in circulo cordam AB unius partis, AG vero unius gradus et medii. Quemadmodum ergo supradictum est quia AG ad AB quam arcus maioris ad arcum minoris minor est proportio. AG autem arcus ad AB arcum sexquialter est, linea ergo AG ad AB necessario quam sexquialtera minor erit. Constat autem cordam AG gradum unum puncta 34 secunda 15 habere; unde corda AB maior quam gradus et puncta 2 secunda 50 profecto constabit. Unus namque gradus cum 34 punctis et secundis 15 gradum unum puncta 2 secunda 50 integraliter sexquialterat. Rursus AB lineam arcus medii gradus et quarte, ipsam vero AG ad unum gradum cordas statuimus. Igitur arcus AG ad AB sesquitertius est. Sed palam ex supradictis cordam AB punctis 47 secundis 8 concludi. Sed ad hunc numerum scilicet puncta 47 secunda 8 sexquitertius numerus est hic, pars una puncta 2 secunda 50 tertia 40. Ergo corda unius gradus maior est quam pars una puncta 2 secunda 50 et minor quam pars una puncta 2 secunda 50 tertia 40. Non est ergo incongruum cordam unius gradus ponere partem unam puncta 2 secunda 50, quia minus quam in duabus tertiis unius tertii error erit, quare multo minus quam in uno secundo, sed in inquisitione cordarum quod minus quam secundum fuerit postponitur.

Unde manifestum quoniam arcus dimidii gradus corda punctis 31 secundis 15 fere concluditur. Ad cuius quantitatis exemplar reliquas que inter duas certas cordas binatim cadunt possumus sine sensibili errore deprehendere. Namque duorum graduum cordam eius que est dimidii ad unius et dimidii facit cognosci adiectio. Duorum item graduum atque dimidii corda poterit

141/142 AB – AG] unius partis scilicet AB aliam *KM* **142** medii] dimidii scilicet AG *KM* **144** est] erit P_7 **145** sexquialter] sesquialtera *MN* **146** ergo] *om.* P_7K AG – AB] *corr. ex* AB ad AG P_7 **146/147** necessario – minor] necessario minor quam sexquialtera P_7 quam sesquialtera minor necessario *KM* **148** secunda] secundas P_7K unde] unde cum *M* **149** AB] sit *add.* (*s.l. K*) *KM* et] *om.* P_7 puncta 2] *iter. et del.* P_7 secunda] secunde *K* **151** secunda] secundas *K* **152** et quarte] *om. N* vero] *s.l.* (*perhaps other hand*) *P* **153** cordas] cordam *PMN* (cordas BaE_1) statuimus] constituimus *KM* AB] BG *MN* sesquitertius] sequitertius *PM* **154** palam] patet *K* **154/161** Sed – postponitur] *alternate text (provided in the appendix) KM* **155** secunda] secundas P_7 numerus est] est numerus P_7 est *N* **156** gradus] *corr. ex* arcus P_7 **157** una[1] – pars] *marg.* (*other hand*) *P* **158** ergo] autem *s.l.* P_7 **159** secunda 50] 50 secunda *N* quia minus] minus †autem† (*s.l.*) P_7 unius tertii] secundi unius *N* **160** in[2]] *om. P* **162** manifestum] manifestum est *KMN* arcus – corda] corda arcus dimidii gradus *KM* **163** concluditur] terminatur *KM* reliquas] *corr. ex* relinquas *M* **164** certas] *s.l. K* sine – errore] *om. KM* **165** cordam] *om.* P_7 **166** cognosci] internosci *K* item] *corr. ex* tunc *P* atque] et *MN* corda] *om. K*



We draw in a circle the chord AB of 1° and indeed AG of 1° 30′. Therefore, in the way described above, the ratio of AG to AB is less than the ratio of the greater arc to the smaller arc. But arc AG to arc AB is sesquialter, so line AG to AB will necessarily be less than sesquialter. And also it is evident that chord AG has 1^p 34′ 15″;[14] whence chord AB will certainly be established to be more than 1^p 2′ 50″. For 1^p 34′ 15″ is completely sesquialter to 1^p 2′ 50″. In turn, we set up that the chords are line AB of an arc of 45′ and indeed AG for 1°. Therefore, arc AG is sesquitertiate [i.e. 4/3] to AB. But it is clear from what has been said above, that the chord AB is bounded by 47′ 8″, but the sesquitertiate number to this number, i.e. 47′ 8″, is this: 1^p 2′ 50″ 40‴. Therefore, the chord of 1° is greater than 1^p 2′ 50″ and less than 1^p 2′ 50″ 40‴. It is not unfitting, therefore, to posit that the chord of 1° is 1^p 2′ 50″ because the error will be less than 40^{iv}[15] and therefore much less than 1″, but anything that is less than 1″ is disregarded in the finding of chords.

Whence it is manifest that the chord of the arc of 30′ is defined by approximately 31′ 15″.[16] By the model of which quantity, we are able to discover the remaining chords that fall two by two between two known ones without perceptible error. For the addition of 30′ to 1° 30′ makes known the chord of 2°. Also, the chord of 2° 30′ will be able to be found if we remove the difference

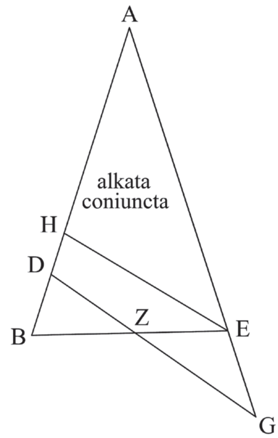
[14] The author here uses the noun ‘gradus’ to refer to 1/120 of the diameter.

[15] The value here should be 40‴, which is the difference between the upper and lower limits for the chord of 1°. If the lower limit, 1^p 2′ 50″, is taken as the size of the chord of 1°, we know that the deviation from the true value can at the most can be 40‴, which is 2/3 of 1″, not of 1‴ as the author mistakenly writes.

[16] To follow Ptolemy’s table of chords, this number should be 31′ 25″.

deprehendi, si ab arcu trium partium ad medie partis differentiam sequestremus. Et ad hunc modum de ceteris. Facilis est ergo secundum premissorum tenorem cordarum ad arcus suos agnitio.

7. Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus ab earum descendentium reliquis terminis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proportio ipsius fixe ad eam sui partem que supra fixionem est producatur ex duabus proportionibus, ex una dico proportione quam habet sibi conterminalis reflexa ad eam sui partem que sectioni interiacet et fixioni, et alia proportione quam habet alterius reflexe inferioris sub sectione portio ad eam totam cuius pars est lineam.

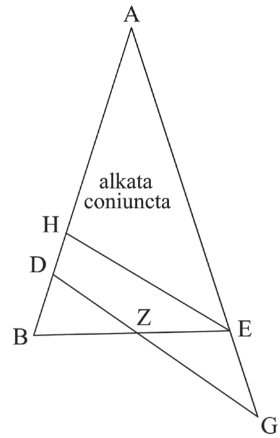


Exempli gratia proportio linee GA ad EA producitur ex proportione linee GD ad lineam ZD et proportione linee BZ ad lineam BE. Sit enim EH equidistans GD; quare proportio GA ad EA tanquam proportio GD ad EH, inter quas ZD linea statuatur media, cuius proportio est ad HE tanquam BZ ad BE.

8. Duabus rectis lineis ab angulo uno descendentibus aliisque duabus sese secantibus ab earum descendentium reliquis terminis in easdem reflexis, utralibet reflexarum alterius conterminalem sic figet ut proportio portionum fixe, inferioris dico partis ad superiorem, producatur ex duabus proportionibus, ex una inquam proportione quam habet sibi conterminalis reflexe inferior sub sectione portio ad reliquam partem que sectioni interiacet et fixioni, et alia proportione quam habet relique descendentis inferior sub fixione portio ad eam totam cuius pars est lineam.

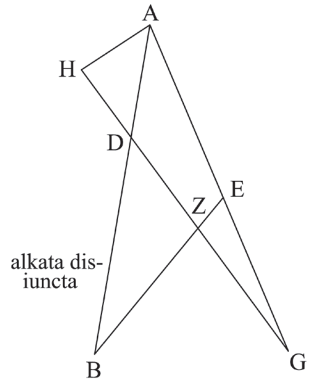
167 partium] arcum *add. et del. P* ad] *del. K* ad medie] arcum dimidie *M* arcum ad dimidie *N* differentiam] diversitatem *K* sequestremus] sequestramus *N* **168** Facilis] nunc facilis *M* est ergo] ergo est P_7 est *N* **169** arcus suos] suos arcus *N* agnitio] cognitio P_7*K large addition here* (*provided in the appendix) KM* **170** rectis lineis] lineis rectis P_7 angulo uno] uno angulo *KM* **177** eam] illam P_7 **178** alia] ea *KM* **179** inferioris] inferior P_7 **181** lineam] linea *M* **185** ad^2] *om.* P_7 **186** statuatur media] media statuatur *KM* cuius] eius *P* **187** BE] per similitudinem triangulorum HEB et DZB. Probatur per quartam sexti Euclidis ut prius *add. M* **188** rectis lineis] lineis rectis P_7 **190** portionum] proportionum P_7 portionis *M* **191** producatur] producatutur P_7 ex duabus] *marg. M* **193** et alia] aliaque *N* **194** habet] *om. N* descendentis] *corr. ex* descendentes *K* descendentes *M* fixione] sectione *KM* **195** lineam] linea *M* between the arc of 3° and the arc of 30′. And in this way concerning the rest. Therefore, knowledge of chords for their arcs is easy according to the way of proceeding of what has been put forth.

7. With two straight lines descending from one angle and with two other lines cutting each other reflected from the remaining endpoints of those descending lines into the same ⟨descending lines⟩, each of the reflected lines will pierce the line conterminous with the other in such a way that the ratio of that pierced line to that part of it that is above the piercing point is produced from two ratios – I mean from one ratio that the reflected line conterminous with it has to that part of it that lies between the intersection and the piercing point, and from another ratio that the other reflected line's lower part under the intersection has to that whole line of which it is a part.



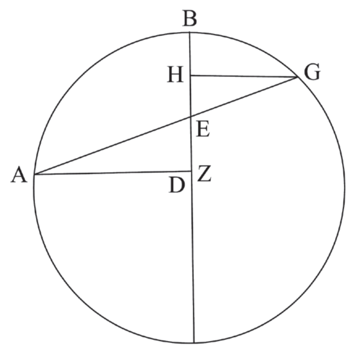
For example, the ratio of line GA to EA is produced from the ratio of line GD to line ZD and the ratio of line BZ to line BE. For let there be EH parallel to GD; therefore the ratio of GA to EA is as the ratio of GD to EH, between which let line ZD be set up as a middle, the ratio of which is to HE as BZ is to BE.

8. With two straight lines descending from one angle and with two other lines cutting each other reflected from the remaining endpoints of those descending lines into the same ⟨descending lines⟩, each of the reflected lines will pierce the line conterminous with the other in such a way that the ratio of the parts of the pierced line – I mean of the lower part to the upper – is produced from two ratios – I mean from one ratio that the conterminous reflected line's lower part under the intersection has to the remaining part that lies between the intersection and the piercing point, and another ratio that the remaining descending line's lower part under the piercing point has to that whole line of which it is a part.



Exempli gratia proportio GE ad EA producitur ex proportione GZ ad ZD et proportione BD ad BA lineam. Protrahatur enim a puncto A linea equidistans BE donec concurrat cum linea GDH. Quare proportio GE ad EA tanquam proportio GZ ad ZH, inter que statuatur medium ZD, cuius proportio est ad DH tanquam BD ad DA. Quare coniunctim ZD ad ZH sicut BD ad BA. Unde habemus propositum.

9. Si in circulo continui arcus sumantur et uterque minor semicirculo, diametrus producta a communi eorum termino lineam rectam reliquos eorumdem terminos continuantem secabit secundum proportionem corde dupli arcus unius ad cordam dupli arcus alterius.



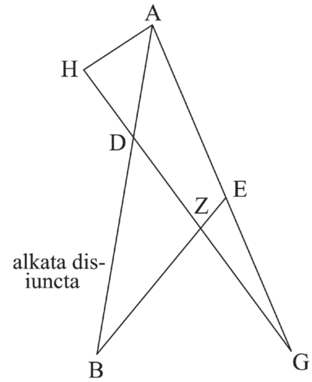
Fiat enim GH linea perpendicularis super semidiametrum BD et sit medietas corde arcus duplicantis arcum GB. Item sit AZ perpendicularis super eandem diametrum et sit sinus arcus AB. Quare fient trianguli GEH et AEZ similes.

10. Si unus arcus notus in duos dividatur fueritque nota proportio corde dupli arcus unius ad cordam dupli arcus alterius, ambo ipsi erunt noti.

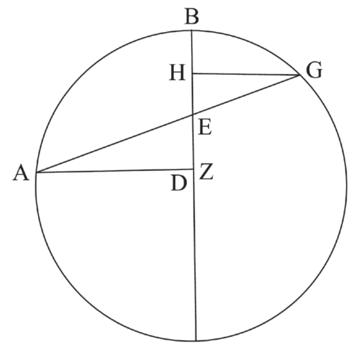
Sit DZ perpendicularis ad cordam arcus AG noti. Quare totus triangulus ZDA lineis et angulis notus. Item proportio GE ad EA per premissam et ypo-

196 gratia] vel causa *adnot. s.l. P* causa P_7*K* proportio] proportio que *K* EA] *corr. ex* EHA P_7 **197** et] et ex *MN* **200** GDH] GD *K corr. ex* GH *M* **201** proportio] *s.l.* (*different hand*) *P* ad] d *P iter. M* **203** tanquam] tanquam proportio *N* DA] DH *P corr. ex* DH P_7*K* **204** BD ad] *marg. M* BA] *corr. ex* BH P_7 **204/205** Unde – propositum] unde habes propositum *K* unde habemus propositum et cetera *M* et cetera *N* **206/207** arcus – semicirculo] sumatur arcus et uterque semicirculo minor *N* **207** diametrus] dyametros *MN* **209** reliquos] *om. N* **212** enim] ergo *PN* GH linea] GH P_7 linea GH *N* **213** semidiametrum] diametrum P_7*K* **214** arcus duplicantis] duplicantis arcus P_7 arcus duplantis *K* AZ] AD P_7*KM* **215** super] ad *K corr. ex* ad *M* diametrum] *corr. ex* semidiametrum *K* semidyametrum *N* **216** sinus arcus] medietas corde (*om. M*) arcus duplantis arcum *KM* **217** AEZ] AED *KM* similes] Et ex hoc habebis propositum cum adiutorio xv sexti. *adnot.* (*other hand P*) PP_7 Et ex hoc habebis propositum cum adiutorio 15^{e} prime partis, 29^{e} primi, et quarte sexti *add. N* **220** unius] *s.l. P* alterius] eorum ad cordam dupli arcus alterius *add. et del. K* **221** ipsi] illi *KM* **222** Sit] exempli gratia sit P_7 Quare] qualiter *P* **223** ZDA] ZDA et *K* notus] notis *M* proportio] *om.* P_7

For example, the ratio of GE to EA is produced from the ratio of GZ to ZD and the ratio of BD to line BA. For let a line parallel to BE be drawn from point A until it meets line GDH. Therefore, the ratio of GE to EA will be as the ratio of GZ to ZH, between which let there be set up middle ZD, the ratio of which is to DH as BD to DA. Therefore, *coniunctim* ZD to ZH as BD to BA. Whence we have what was proposed.



9. If in a circle contiguous arcs are taken and each is less than a semicircle, the diameter produced from their common point will cut the straight line joining their remaining endpoints according to the ratio of the chord of double one arc to the chord of double the other arc.

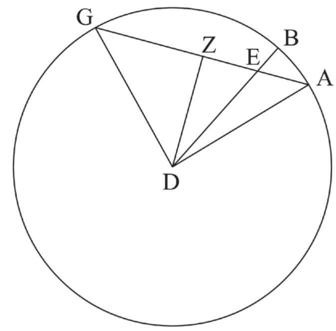


For let line GH be perpendicular to semidiameter BD and let it be half of the chord of the arc doubling arc GB. Likewise, let AZ be perpendicular to that same diameter and let it be the sine of arc AB. Therefore, triangles GEH and AEZ will be similar.

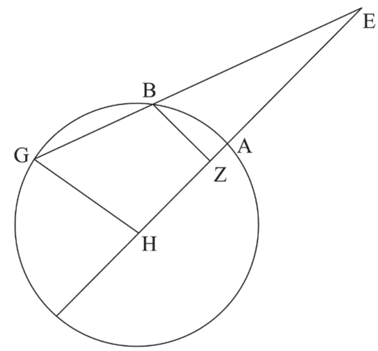
10. If one known arc is divided in two and the ratio of the chord of double one arc to the chord of double the other arc is known, both of these ⟨chords⟩ will be known.

Let there be DZ perpendicular to the chord of known arc AG. Therefore, the whole triangle ZDA is known in terms of lines and angles. Also, the ratio of GE to EA is known through the preceding ⟨proof⟩ and the hypothesis.

thesim est nota. Ergo proportio coniuncta GA ad EA addita unitate denominationi proportionis disiuncte fiet nota. Ergo AE nota, ergo EZ et DZ et ED linee note respectu diametri circuli magni. Ergo omnes anguli trianguli ortogonii EZD noti sunt per circulum ei circumscriptum respectu duorum rectorum, ergo respectu iiii. Dempto ergo angulo ZDE nunc noto ab angulo ZDA prius noto, relinquitur angulus EDA notus. Quare arcus AB notus, ergo et reliquus GB notus.

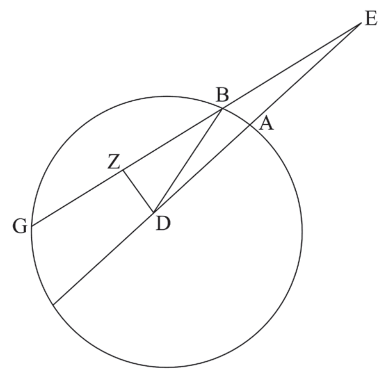


11. Si ab uno termino arcus semicirculo minoris linea ipsum arcum secans educatur donec cum diametro per reliquum eiusdem arcus terminum extracta concurrat, fiet proportio linee preter centrum transeuntis ad partem sui extrinsecam sicut proportio corde dupli arcus de quo sermo est ad cordam dupli arcus illius quem educte linee includunt.



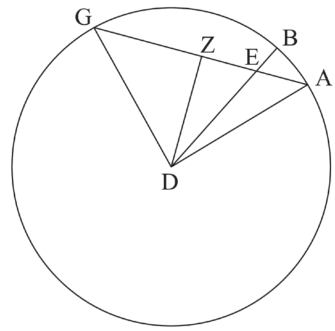
Esto igitur GH sinus arcus GA cui equidistat BZ sinus arcus BA interclusi lineis concurrentibus, quarum altera GBE preter centrum transiens arcum GA secat, altera HAE secundum diametrum extracta. Fiet ergo triangulus GEH totalis similis triangulo BEZ partiali.

12. Si arcus dicto modo divisi lineis ut prescriptum est donec concurrant eductis maior portio nota fuerit et proportio corde dupli arcus ipsius divisi ad cordam dupli arcus lineis eductis inclusi constiterit, ipse arcus inclusus notus erit.

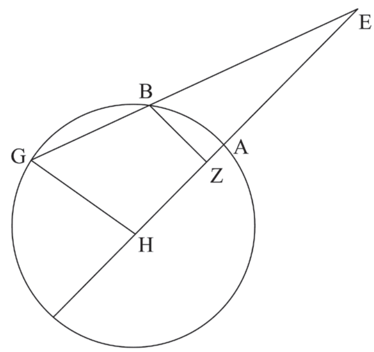


224 est nota] nota est *PN* **225** denominationi] denominatori *P* denominatore *N* **226** proportionis disiuncte] disiuncte proportionis *N* **227** et ED] *s.l.* P_7 **228** diametri] *om. K* **229** ortogonii] ortogoni *KM* **230** noti sunt] sunt noti P_7 noti] *corr. ex* nati *P* **232** noto] *corr. ex* nota *K* **235** et] *om.* P_7K GB] GD *KM* **237** arcum secans] secans arcum *N* **238** cum] *om. N* **239** eiusdem] eundem *P* arcus – extracta] terminum extracto P_7 concurrat] *corr. ex* concur†...† *K* **240** fiet] *corr. ex* fiat *K* **243** sermo est] est sermo *MN* **244** quem] que *P* **245** igitur] *om.* P_7K **246** equidistat] equidistet *N* interclusi] inclusi P_7 **248** preter] *corr. ex* comperter *M* **250** GEH] GEB *P* **251** BEZ] HEZ *P* BEZ partiali] partiali BEZ *KM* **252** arcus] alicuius scilicet *add. s.l. K* dicto] predicto P_7 **253** eductis] eductus *P* **254** portio] *s.l. P* **256** constiterit] constituerit *P om. N* **257** notus erit] erit notus *N*

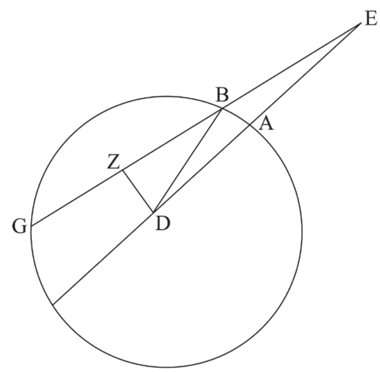
Therefore, *coniunctim* the ratio of GA to EA will be known with unity having been added to the denomination of the disjunct ratio. Therefore, AE will be known, so lines EZ, DZ, and ED will be known with respect to the diameter of the large circle. Therefore, all the angles of right triangle EZD are known through the circle circumscribing it with respect to two right angles, and therefore with respect to four. Therefore, with known angle ZDE subtracted from angle ZDA known earlier, angle EDA remains known. Therefore, arc AB is known, so also the remainder GB is known.



11. If from one endpoint of an arc less than a semicircle, a line cutting that arc is extended until it meets the extended diameter ⟨that passes⟩ through the remaining endpoint of that same arc, the ratio of the line crossing away from the center to its extrinsic part will be as the ratio of the chord of double the arc about which the discussion is to the chord of double that arc that the extended lines enclose.



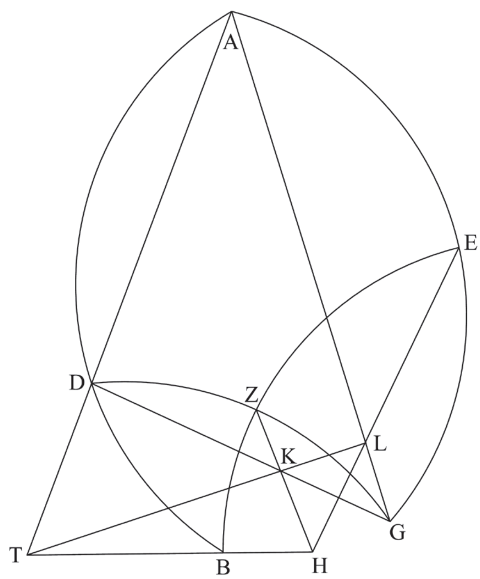
Accordingly, let GH be the sine of arc GA, parallel to which let there be BZ, the sine of arc BA enclosed between the meeting lines, of which ⟨lines⟩ one GBE crossing away from the center cuts arc GA, and the other HAE extended in line with the diameter. Therefore, the whole triangle GEH will be similar to partial triangle BEZ.



12. If the greater part of an arc divided in the said manner by lines extended until they meet, as was already drawn, is known and if the ratio of the chord of double that divided arc to the chord of double the arc enclosed by the extended lines is known, the enclosed arc will be known.

Esto ZB medietas corde arcus GB noti nota. Item DB nota, quare totus triangulus DZB ortogonius notus est lineis et angulis. Item proportio GE ad BE nota per proximam et ypothesim, quare per penultimam tertii Euclidis EA nota. Ergo angulus trianguli ortogonii qui angulus est EDZ notus. A quo dempto angulo BDZ noto, relinquitur angulus ADB notus; ergo et arcus AB notus.

13. In superficie sphere duobus arcubus magnorum orbium semicirculo divisim minoribus ab uno communi termino descendentibus aliisque duobus non minorum orbium ab illorum reliquis terminis in eosdem sese secando reflexis, utervis reflexorum alterius conterminalem arcum sic figet ut proportio corde arcus duplicantis inferiorem portionem arcus fixi ad cordam arcus duplicantis superiorem eiusdem fixi portionem producatur ex gemina proportione, ex ea videlicet quam habet corda arcus duplicantis inferiorem arcus reflexi portionem qui ipsi fixo conterminalis est ad cordam arcus duplicantis reliquam eiusdem reflexi portionem, et ea proportione quam habet corda arcus duplicantis inferiorem alterius descendentis arcus partem ad cordam duplicantis arcum ipsum cuius pars est totalem.

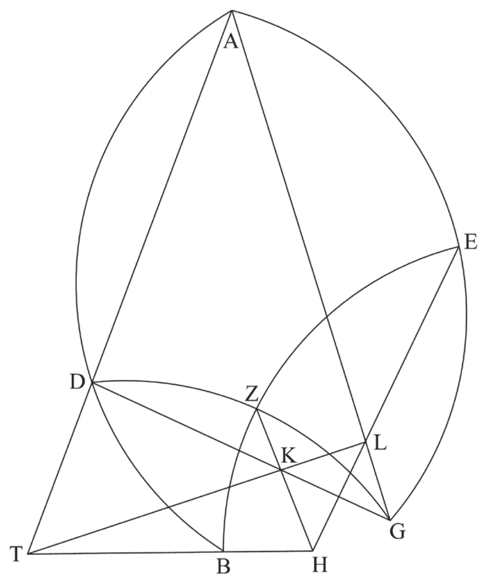


Evidentie gratia, arcus magnorum orbium AB et AG in superficie spere describimus, inter quos alii duo BE et GD sese intersecent aput Z. Dico ergo quod proportio corde duplicantis GE ad cordam arcus ipsius EA dupli ex gemina proportione componitur sicut in kata disiuncta, ex ea videlicet quam habet corda arcus ad GZ

259 ortogonius] portione *P* orthogonus *M om. N* est] et *K* est et *M* **260** nota] est nota *s.l.* P_7 proximam – ypothesim] ypothesim et proximam *N* tertii] *corr. in* primi *N* EA] *corr. in* ED P_7 ED *N* est *add.* (*s.l. K*) *KM* **261** ortogonii] orthogoni *M* EDZ] EZD *P corr. ex* EZD P_7*K* EZD est *M* **262** relinquitur angulus] angulus relinquitur P_7 arcus] *corr. ex* angulus *K* **263** AB] AB est P_7 **265** aliisque] illiisque P_7 **266** orbium] *s.l.* P_7 **267** utervis] utrius *P* ulterius *M* utriusque *N* **269** ea] eadem P_7 **271** qui] que *MN* **271/273** eiusdem – quam] *om. P* **272** et] et ex *MN* **273/274** arcus duplicantis] duplicantis arcus *PN* **277** pars est] est pars P_7 **281** duo] *om. N* **282** intersecent] intersecant *PN* Z] punctum Z P_7 **283** quod] quia *K* **283/284** duplicantis] duplantis *K* **284/285** arcus ipsius] ipsius arcus *PN* **285/286** proportione] propositione *M* **287** disiuncta] diiuncta *P* **288** ad] *om.* P_7

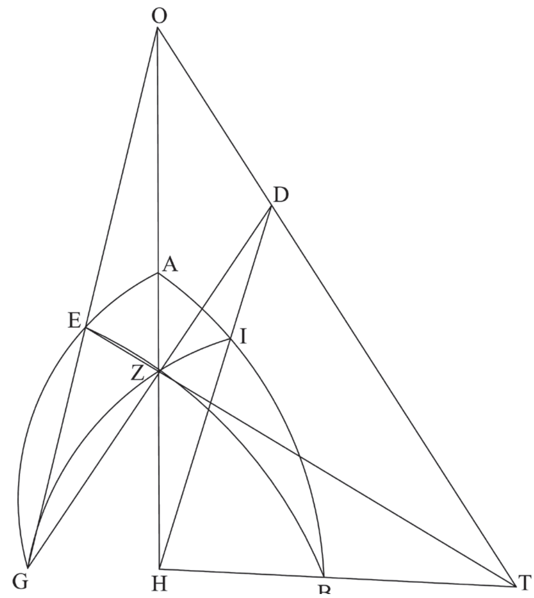
Let ZB be the known half of the chord of known arc GB. Likewise, DB is known; therefore, the whole right triangle DZB is known both in lines and angles. Also, the ratio of GE to BE is known through the last proposition and the hypothesis; therefore, EA will be known through the penultimate proposition of the third of Euclid. Therefore, the right triangle's angle, which is angle EDZ, is known. With known angle BDZ subtracted from that, angle ADB remains known; therefore, arc AB is also known.

13. With two arcs of great circles each less than a semicircle descending from one common point on the surface of a sphere, and with two other ⟨arcs⟩ of not smaller circles reflected from the remaining endpoints of these ⟨descending arcs⟩ into the same ⟨descending arcs⟩ by intersecting each other, each of the reflected arcs will pierce the ⟨descending⟩ arc conterminous with the other in such a way that the ratio of the chord of the arc doubling the lower part of the pierced arc to the chord of the arc doubling the upper part of the same pierced arc is produced from a twofold ratio, i.e. from that which the chord of the arc doubling the lower part of the reflected arc that is conterminous with that pierced arc has to the chord of the arc doubling the remaining part of that same reflected arc, and the ratio which the chord of the arc doubling the lower part of the other descending arc has to the chord of the arc doubling that whole arc of which it is a part.



For the sake of clarity, we will describe arcs of great circles AB and AG on the surface of a sphere, between which let two others BE and GD intersect at Z. Then I say that the ratio of the chord of double GE to the chord of double arc EA is composed of a twofold ratio as in dupli ad cordam arcus ipsum ZD duplantis et ex ea que est corde arcus qui est duplus ad DB ad cordam arcus ad ipsum BA duplicis. Ratio. Centro spere H posito ab ipso ad notas B Z E — circulorum dico sectiones — linee ducantur. Linee rursum AD et HB descendentes ad notam T conveniant. Sed etiam due GA et GD linee eas que sunt HZ et HE ad puncta K et L secantes protrahantur. Sic ergo in una recta linea sunt note tres scilicet T K L. Nam sunt et in superficie trianguli AGD quantumlibet extensa et in superficie circuli relicti BZE, quarum superficierum communis sectio linea. Hac igitur linea protracta restat ex kata disiuncta et nona bis et undecima semel assumpta propositum colligere.

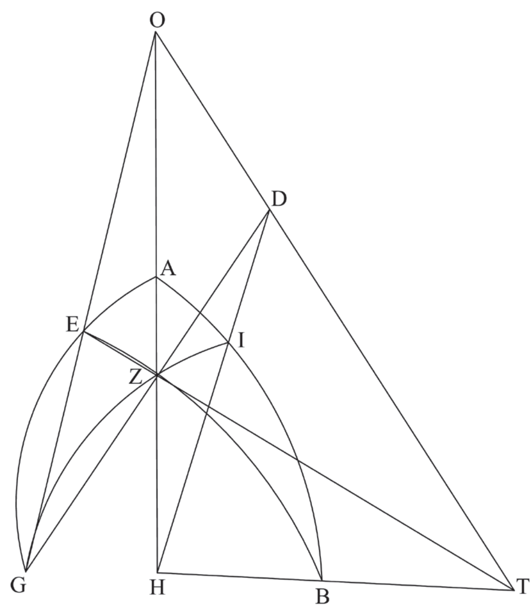
14. In superficie spere iiii arcubus supradicto modo depictis, fiet ut proportio corde arcus duplicantis unum descendentium totalem ad cordam arcus duplicantis superiorem ipsius descendentis portionem componatur ex gemina proportione, ex ea videlicet quam habet corda duplantis arcum totum ab eiusdem descendentis termino reflexum ad cordam duplam illam ipsius reflexi portionem que sectioni interiacet et fixioni, et alia proportione quam habet corda arcus duplantis inferiorem sub sectione alterius reflexi portionem ad cordam arcus duplicis ad eundem reflexum cuius pars est totum.



Evidentie gratia, proportio corde arcus duplicantis arcum GA ad cordam arcus duplicantis arcum EA componitur

289 duplantis] duplicantis *MN* (duplicantis E_1) corde] *corr. ex* corda *K* qui] que *M* **290** ad ipsum] ae†rum† *P* ad arcum *N* (ad ipsum E_1) Ratio] *corr. ex* nam *M* H] *s.l. K* **291** notas] notos *P* (notas E_1) Z E] E Z P_7 **292** due] *s.l.* P_7 **293** puncta – L] K et L puncta *PN* **295** quantumlibet] quantum L et *P* extensa] protensa *PN corr. ex* protensa *M* (extensa E_1) **296** BZE] GZE *P* communis sectio] sectio communis *N* Hac] hac hac *corr. in* secta hac *N* **297** et[1]] *s.l. M* undecima – assumpta] quinta semel assumptam *N* **299** iiii] *om. N* supradicto] predicto *PN* **305** reflexum] reflexu *P* **306** duplam – ipsius] illam duplantem (*corr. ex* duplam) ipsius P_7 duplantem illius *KM* duplantem ipsius *N* (duplam illam ipsius *Ba*E_1) **307** que] que inter *M* **310** duplantis] duplicantis P_7*MN* (duplantis *Ba* duplicantis E_1) **313** duplicis] *corr. in* dupl†icatis† *M* **316** proportio] *s.l.* P_7 **318** arcus] *om. K* the disjunct kata, i.e. from that ratio which the chord of the arc double GZ has to the chord of the arc double ZD and from that ratio which is of the chord of the arc that is double DB to the chord of the arc double BA. Proof. With H supposed as the center of the sphere, let lines be drawn from it to points[17] B, Z, and E – I mean the intersections of the circles. Again, let descending lines AD and HB meet at point T. But also let there be drawn two lines GA and GD cutting HZ and HE at points K and L. Thus therefore, there are three points, i.e. T, K, and L, in one straight line. For they are both in the plane of triangle AGD extended however far and in the plane of the remaining circle BZE, the intersection of which planes is a line. Accordingly, with this line drawn, it remains to deduce what was proposed from the disjunct kata, the ninth ⟨proposition⟩ taken twice, and the 11th once.

14. With four arcs depicted in the abovesaid way on the surface of a sphere, it will happen that the ratio of the chord of the arc doubling one of the whole descending ⟨arcs⟩ to the chord of the arc doubling that descending arc's upper part is composed of a twofold ratio, i.e. out of that which the chord of double the whole arc reflected from the endpoint of the same descending arc has to the chord double[18] that reflected arc's part that lies between the intersection and the piercing point, and from another ratio that the chord of the arc doubling the other reflected arc's lower part below the intersection to the chord of the arc double the whole reflected arc of which it is a part.



For the sake of clarity, the ratio of the chord of the arc doubling arc GA to the chord of the arc doubling arc EA is composed of a

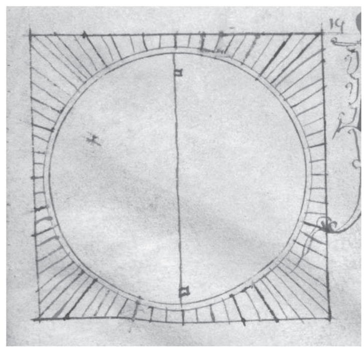
[17] The noun *nota, -ae, f.* means a sign, letter, or marker, and thus seems to refer more to the letter marking the point than to the point itself, but the context suggests that it was meant as a synonym of *punctus* or *punctum*.

[18] To be mathematically correct, this should say 'the chord of the doubling [arc]', but it appears that the author mistakenly had either 'duplam' or 'duplantem' where he should have had 'duplantis.'

ex gemina proportione, scilicet ex proportione corde arcus duplicantis arcum GI ad cordam arcus duplicantis arcum ZI et ex proportione corde dupli arcus BZ ad cordam dupli arcus BE. Ratio. A spere centro H linee per sectiones circulorum A B I educantur donec singule cum singulis preter centrum transeuntibus ad notas O D T conveniant. Quas tres notas in eadem esse linea conveniet. Nam sunt et in superficie trianguli GZE indefinita et in superficie circuli relicti BA — superficie dico quantumlibet extensa. Hac igitur linea protracta ODT, per kata coniunctam et undecimam argue quod proponitur.

15. Maximam declinationem per instrumenti artificium et considerationem reperire.

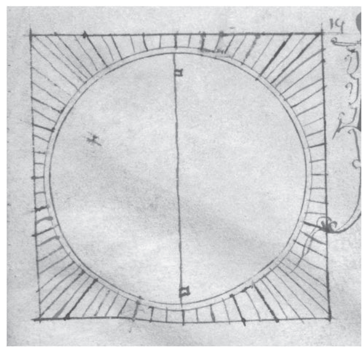
Paratur itaque lamina quadrate forme cubitalis vel eo amplius mensure ad unguem polita et planissima, in cuius una superficie circulus ut modicum extra labrum relinquatur describitur. Ipsumque labrum in circuitu in ccclx partes equissime linea in centro semper posita dividitur, et queque pars in minuta quot capere poterit subdistinguitur. Deinde ad circuli descriptionem cavatur et cavata aptissime planatur. Post hec minoris quantitatis et forme orbicularis nec minus plana queritur lamina ad spissitudinem labri in alia relicti spissa ut cum ei super centrum inserta fuerit, in una cum labro fiat superficie. Et in huius minoris duobus punctis per diametrum oppositis due erigantur equales et per omnia sibi similes pinne sic ut linea secans utramque per medium pinnulam erecta sit super diametrum. Et a duobus terminis diametri due in directum promineant lingule in extremitate sua gracillime, quarum erit officium ut cum minor lamina infra maiorem super centrum rotata fuerit, lingule sectiones partium in labro diametraliter oppositas numerent et indicent.



320 corde] *s.l. P* duplicantis] duplantis P_7K GI] *corr. ex* GD *M* GD *N* **321** duplicantis] duplantis P_7 arcum] *om. M* ZI] *corr. ex* ZD *M* ZD *N* BZ] BE *M* **322** BE] BZ *PM* spere centro] centro spere *N* per sectiones] *corr. ex* sectionis *M* **323** A – I] ab L *M* et ab L *del. N* educantur] educantur et a G et E P_7 **324** O] E *M* eadem] *iter. et del. K* esse linea] linea esse *N* conveniet] conveniat P_7 convenient *M* **325** indefinita] infinita P_7 **326** ODT] *corr. in* EDT *M* ad T *N* **327** coniunctam] iunctam *K* et] et per *M* undecimam] quintam *N* **328** Maximam] maximam Solis *N* **330** forme] figure *N* **331** unguem] uguem *M* **332** in[1] – partes] in 360 partes in circuitu *N* **334** subdistinguitur] subdistinguatur P_7 subdistinguntur *M* **336** hec] hoc non (*second word s.l.*) *M* hoc *N* **337** nec] non P_7 **338** queritur] quare *P* quam *MN* **340** in] illud P_7 fiat] in *add. et del. P* **342** oppositis] oppositus P_7 due] *om. N* erigantur] eriguntur *PN* (erigantur BaE_1) **343** ut] *s.l. K* **345** duobus] duabus *P* directum] directam *P* **346** in] *om. M* sua] *s.l. P* **347** lingule] linguule *N* **348** labro] *corr. ex* libro *K* oppositas] oppresso *K* twofold ratio, i.e. of the ratio of the chord of the arc doubling arc GI to the chord of the arc doubling arc ZI and of the ratio of the chord of double arc BZ to the chord of double arc BE. Proof. Let lines be extended from the sphere's center H through the circles' intersections A, B, and I until each meets each ⟨line⟩ crossing away from the center [i.e. GE, GZ, and EZ] at points O, D, and T. It will be agreed upon that these three points are on the same line. For they are both in the unlimited plane of triangle GZE and in the plane of the remaining circle BA – I mean the plane extended however far. Then, with this line ODT drawn, prove what is proposed through the conjunct kata and the 11th ⟨proposition⟩.

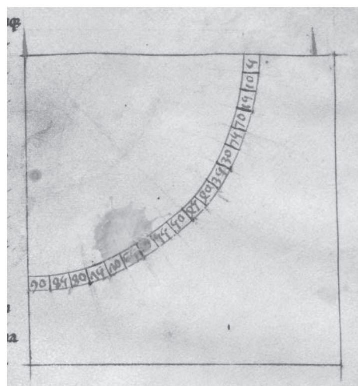
15. To find the maximum declination through the crafting and observation of an instrument.

Accordingly, a plate is prepared in the shape of a square, a cubit or greater in size, polished exactly and very even, in one surface of which a circle is described in such a way that a little remains beyond the rim [i.e. the circle does not extend to the perimeter of the plate]. And that rim is divided into 360 degrees on the circumference very equally by a line always placed on the center, and each degree is subdivided into as many minutes as is possible to take. Then it is hollowed out to the circumference of the circle, and what is hollowed out is made even very exactly. Afterwards, a sheet of smaller size is sought, of a circular shape, not less even, and thick as the thickness of the rim remaining in the other ⟨sheet⟩, so that when it is inserted in this ⟨other⟩ one upon the center, it is in one plane with the rim. And in two diametrically opposite points of this smaller one, let there be set up two fins equal and similar in all ways to each other thus that the line cutting each little fin through the middle is set up upon the diameter. And from the two endpoints of the diameter, let two projections ⟨that are⟩ very slender at their ends jut out in line, the task of which ⟨two projections⟩ will be that when the smaller plate is rotated within the greater upon the center, the projections number and indicate the divisions of degrees diametrically opposite on the rim.



Eis ergo ita paratis et minore maiori ut in ea volvi possit centraliter inserta, quotiens opus erit per eas operari, latus lamine quadrate super lineam meridianam in plano protractam erectum constituemus superficie minoris incluse ad meridiem obversa. Sicque aptabimus et firmabimus ut latus suppositum orizonti equidistet et superficies erecta a meridiano non declinet, quorum primum arte livelli efficies, secundum experientia perpendiculi. Solis ergo umbram circa utrumque solstitium in omni meridie observans, tamdiu volves interiorem rotulam donec superior pinna totam inferiorem obumbret. Et per hoc duorum tropicorum distantiam cuius medietas est maxima declinatio, necnon et distantiam puncti in summitate capitum ab equinoctiali deprehendes.

Paratur et aliud commodius et facilius instrumentum. Laterem scilicet ligneum vel lapideum vel eneum quadratum quere cubitalis latitudinis et apte altitudinis ut super latus sine tortuositate et inclinatione erigi possit, sitque una superficierum levissima et equalis. Positoque centro in uno angulorum super ipsum quartam circuli describe. Et ab eo centro duas lineas rectas angulum rectum continentes et quartam circuli includentes protrahe, et quartam circuli in xc partes et unamquamque partium in minuta quot poteris partire. Deinde duas pinnulas tornatiles piramidales equales longitudine et grossitie quere. Et unam centro ortogonaliter infige et alteram extremitati linee a centro descendentis. Quo expleto erige instrumentum super latus suum duabus pinnis ad orientem conversis et ea que in centro est superiori et alia deorsum inferiori. Sitque superficies in qua fixe sunt obversa orienti.

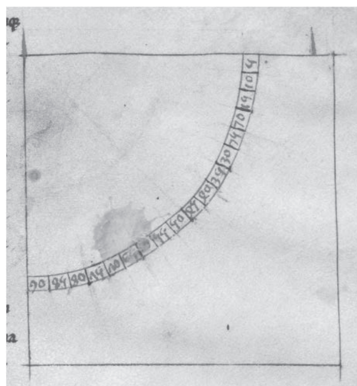


Tunc perpendiculo a superiori pinna in inferiorem demisso ad meridiani superficiem et orizontis equidistantiam adapta, umbramque pinnule in centro exis-

349 ut] ita ut *N* **350** operari] operaris *N* **352** ad meridiem] *om. N* meridiem obversa] *corr. in* orientem versa *K* orientem versa *M* ut] *corr. ex* et *M* suppositum] suppremum P_7N superpositum *K* supinum *M* (suppositum BaE_1) **354** livelli] libelli *M* secundum] *corr. ex* scilicet *K* experientia] experientiam *PMN* **355** utrumque] utramque *PM* volves] *corr. ex* volvens *K* **358** capitum] capitis *M* **359** Laterem scilicet] laterem vel *M* latereum *N* **360** vel[1]] *om. N* quere] quarte P_7 figura *N* **363** describe] describere *P* **364** et] *om.* P_7 **365** xc] *corr. ex* xi *K* **367** pinnulas] pinnas P_7 **368** piramidales] *corr. ex* pyramydes *s.l. P* equales] equales in *M* **369** et[1]] *om. PN s.l. K* grossitie] grossiore *P* unam] unam in *PN* **371** expleto] completo *PN* **372** pinnis] pinnis eius *N* **373** conversis] *om. N* et] et sit *KM* **374** superiori] *corr. in* superior *K* superior *M* alia] alta P_7 inferiori] *corr. in* inferior *K* inferior *M* **376** Tunc] cum P_7 in inferiorem] *om. N* in] *s.l. M* **377** equidistantiam] equidistantem P_7 adapta] instrumentum *add. s.l. K* adaptata instrumentum *M*

Then, with these things prepared in this way and with the smaller ⟨plate⟩ inserted in the greater centrally such that it is able to be turned in it, whenever there will be a need to work with them, we will set up the side of the square plate upright on the meridian line drawn on level ground, with the surface of the enclosed smaller ⟨plate⟩ turned towards the meridian.[19] And thus we will adjust and support it so that the side placed under is parallel to the horizon and the set up surface does not tilt away from the meridian; the first of which you will bring about by the art of the level, and the second by a test of the plumb line. Then, observing the sun's shadow at every noon around each solstice, you will turn the little inner disk for so long a time until the upper fin casts a shadow upon the whole lower one. And through this you will discover the distance between the two tropics, the half of which is the maximum declination, as well as the distance of the point at the zenith [*lit*., the highest point of the heads] from the equator.

Also, another instrument is prepared more conveniently and easily. Seek a square block of wood, stone, or bronze, a cubit wide and of a suitable height so that it may be set up on its side without twisting and tilting, and let one of the surfaces be very smooth and even. And with a center supposed on one of the corners, describe a quarter circle upon it. And from that center draw two straight lines containing a right angle and enclosing a quarter circle, and divide the quarter circle into 90 degrees and each of the degrees into as many minutes as you are able. Then seek two little fins turned as pyramids equal in length and thickness. And fasten one in the center perpendicularly and the other at the extremity of the line descending from the center. With that completed, set up the instrument upon its side, with its two fins facing the east, and with the one at the center higher and the other lower down. And let the surface to which they are fastened face the east. Then, with a plumb line sent down from the upper fin to the lower one, align ⟨the instrument⟩ with the plane of the meridian and parallel to the horizon, and



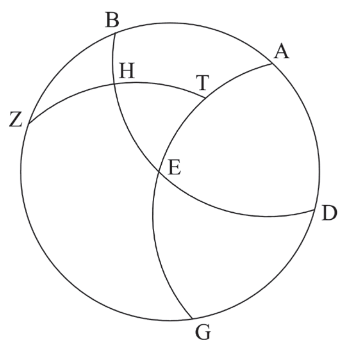
[19] This is an ambiguous phrase in the text because 'ad meridiem' most obviously means 'towards the south', but the meaning must be to have this surface in the plane of the meridian. To make this meaning more clear, *K* and *M* have the revised reading, 'towards the east.'

tentis quorsum in meridie cadat diligenter attende. Et per hoc sicut superius distantiam tropicorum et remotionem summitatis capitum ab equinoctiali contemplare.

Notandum autem quod diversitas aliqua in maxima declinatione reperta est a diversis consideratoribus in suis temporibus. Nam Indi invenerunt eam esse xxiiii graduum, Ptolomeus xxiii graduum et li minutorum et xx secundorum, Albategni vero xxiii graduum et xxxv minutorum, Arzazel quoque xxiii graduum et xxxiii minutorum et xxx secundorum. Ideo sollerter adhuc est inspiciendum et magis visui quam auditui credendum.

16. Cuiuslibet puncti in circulo declivi cuius discessus ab equinoctiali est notus declinationem invenire. Unde manifesta est hec regula: si sinus portionis ab equinoctiali inchoate cuius finalis puncti declinatio queritur ducatur in sinum maxime declinationis productumque dividatur per sinum quadrantis, exibit sinus quesite declinationis.

Describo circulum per polos circuli equinoctialis et etiam declivis transeuntem ABG, infra quem equinoctialis medietas AEG et medietas circuli declivis BED ad notam E se intersecantes locentur. Et nota E vernale designet equinoctium, punctus vero D hiemale solstitium, et nota B estivale. Polus equinoctialis circuli nota Z. Arcus EH a declivi abscisus xx partes contineat. Deinde arcum ZHT magni circuli circumduco. Est ergo propositum arcus HT quantus sit agnoscam. Cum ergo in huiusmodi figura duo arcus AZ et AE a communi termino descendant inter quos duo alii ZT et EB ad notam H intersecantur, et ZT quadrans sit equalis EB quadranti, per kata coniunctam facto ergo sinu

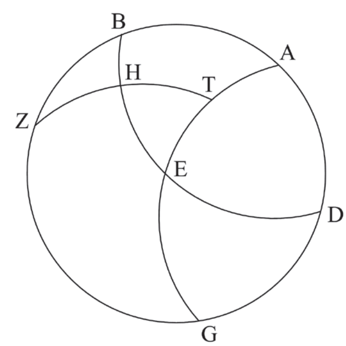


378 in – cadat] cadat in meridie *N* **379** summitatis] summitatem *M* capitum] capitis *M* **381** reperta] comperta *MN* **382** a] *om. N* consideratoribus] considerantibus P_7N Indi] Indei *P* eam] *om. K* **383** Ptolomeus] Ptholomeus P_7N et[1]] *s.l. K* secundorum] secundarum *PK* **384** xxiii[1]] *corr. ex* xxxiii *P* et] *om. M* xxxv] *corr. ex* xxv *K* Arzazel] Arzachel P_7N Arzacel *corr. in* Arzachel *K* Arzahel *M* quoque] vero P_7 autem *N* xxiii[2]] *corr. ex* xxxiii *P* 33 P_7 **385** et[1]] *om.* P_7 secundorum] secundarum *PK* adhuc est] adhuc P_7 ad hoc est *M* est ad hoc *N* est] *om.* P_7 **387** in] *om. P* discessus] dicessus P_7 **388** manifesta – hec] et manifestum est hac P_7 portionis] alicuius portionis *MN* **390** productumque] *corr. ex* punctum *P* **392** circuli equinoctialis] equinoctialis circuli P_7 equinoctialis *N* declivis] circuli declivis *N* **393** AEG] ABG *P* **394** BED] BDE *KM* Et] que P_7K designet] *corr. ex* designat *K* **397** circuli – Z] punctus Z circuli *N* a] *om. K* abscisus] abcisus *N* **398** xx] 30 P_7 **399** magni] maximi P_7 circumduco] circumducto *M* **400** quantus – agnoscam] quantitatem cognoscere *N* agnoscam] agnoscere P_7 **401** huiusmodi] huius *M* duo] et *N* **403** intersecantur] se intersecant P_7K **405** facto ergo] factoque P_7 facto *K* carefully watch where the shadow of the little fin at the center falls at noon. And through this as above, consider the distance of the tropics and the distance of the zenith from the equator.

Moreover, it is to be noted that some difference in the maximum declination was found by different observers in their own times. For the Indians found it to be 24°, Ptolemy 23° 51′ 20″, indeed Albategni 23° 35′, and Arzachel 23° 33′ 30″. For that reason, it still should be be observed cleverly, and sight is to be trusted more than hearing.

16. To find the declination of any point on the ecliptic [*lit.*, declined circle] whose distance from the equator is known. Whence this rule is manifest: if the sine of the part beginning from the equator of which the declination of the final point is sought be led into the sine of the maximum declination, and the product be divided by the sine of a quadrant, the sine of the sought declination will result.

I describe the circle ABG passing through the poles of the equator and the ecliptic also, below which let there be placed half the equator AEG and half the ecliptic BED intersecting at point E. And let point E mark the vernal equinox, and indeed point D the winter solstice, and point B the summer ⟨solstice⟩. The pole of the equator is point Z. Let arc EH which is cut off from the ecliptic contain 20°.[20] Then I describe arc ZHT of a great circle. Then what is proposed is that I discern how great arc HT is. Then, because in a figure of this kind two arcs AZ and AE descend from a common point between which two others ZT and EB intersect at point H, and quadrant ZT is equal to quadrant EB, therefore, through the conjunct kata, with the sine of arc BE made a middle between the



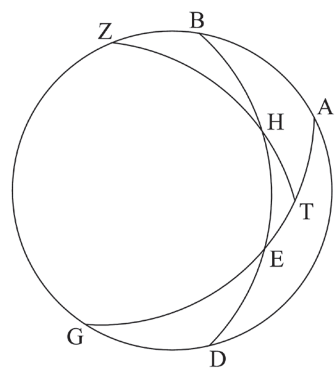
[20] This value should be 30° to accord with Ptolemy and the values given later in this proposition.

arcus BE medio inter sinum HE et sinum HT arcus, erit proportio corde dupli arcus HE ad cordam dupli arcus HT que est corde dupli arcus AZ ad cordam dupli arcus AB. Unde manifestum si sinus HE ducatur in sinum AB productumque dividatur per sinum arcus AZ, exibit sinus arcus HT. Sinum voco medietatem corde dupli arcus.

Posito igitur arcu AB duplicante ex partibus xlvii punctis xlii secundis xl, secundum quod Ptolomeus distantiam inter duos tropicos invenit, invenies ipsum TH arcum ex partibus xi punctis xl fere componi. Ad hunc modum cuiuslibet gradus finalis puncti declinationem in circulo declivi.

17. Cuiuslibet portionis circuli declivis elevationem in spera recta invenire. Unde patet regula: si sinus perfectionis maxime declinationis ducatur in sinum declinationis portionis inchoate ab equinoctiali linea cuius portionis queritur elevatio, productumque dividatur per sinum perfectionis declinationis illius portionis, et quod exierit ducatur itidem in sinum elevationis unius quadrantis, productumque dividatur per sinum maxime declinationis, exibit sinus quesite elevationis.

Elevatio portionis circuli declivis est arcus equinoctialis qui cum ipsa portione incipit et desinit oriri. Ad huius rei expositionem supradicta figura in exemplum denuo assumatur. Est enim propositum quantus sit arcus ET agnoscere qui est elevatio arcus EH. Cum ergo in huiusmodi figura AZ et AE arcus duo a communi termino descendant inter quos ZT et EB alii duo se intersecant ad punctum H, quare per kata disiunctam proportio sinus ZB ad BA constat ex proportionibus ZH ad HT et ET ad EA. De sinibus eorum arcuum loquor. Quare sinus ZB si ducatur in sinum HT, primum scilicet in quartum, et productum dividatur per sinum ZH tertium, exibit linea cuius proportio ad sinum arcus BA secundi sicut sinus ET ad sinum EA, quinti scilicet ad sextum. Ergo si linea illa ducatur in sinum EA qui est elevatio unius quadrantis et dividatur per sinum AB qui est maxima declinatio, exibit sinus ET quesite elevationis.

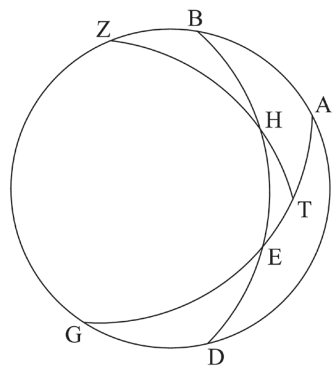


407 que] qui *P* **412** Ptolomeus] Tholomeus P_7 **413** arcum] *om. K* modum] invenies *add. s.l.* (*other hand*) *K* **414** declivi] declivi poteris invenire *MN* **415** in – recta] *om. K* **419** exierit] exit *corr. ex* exiit *K* **420/421** quesite elevationis] *corr. in* elevationis quesite *M* **423** huius] cuius *KM* supradicta] supraposita *KM* **423/424** in exemplum] *om. N* **424** agnoscere] cognoscere P_7 **425** qui] que *N* **426** huiusmodi] huius *M* duo] *om. N* **428** intersecant] *corr. ex* intersecent P_7 **428/429** punctum H] H punctum *N* **429** kata] cata *K* **430** sinus] *om. N* **431** ad^2] *corr. ex* et *K* EA] AE *KM* **432** sinus] *s.l. K* **433** HT] ZT HT *P* **436** arcus] arcum *P* **438** sinum] *s.l. K* sine of HE and the sine of arc HT, the ratio of the chord of double arc HE to the chord of double arc HT will be that ⟨ratio⟩ that is of the chord of double arc AZ to the chord of double arc AB. Whence it is manifest that if the sine of HE is led into the sine of AB and the product is divided by the sine of arc AZ, the sine of arc HT will result. I call the half of a chord of double an arc a 'sine.'

Therefore, with the arc doubling AB supposed to be 47° 42′ 40″ because of the fact that Ptolemy discovered the distance between the two tropics, you will find that that arc TH is composed of approximately 11° 40′. In this way ⟨you will find⟩ the declination of the final point of any degree on the ecliptic.

17. To find the elevation in the right sphere of any part of the ecliptic. Whence this rule is clear: if the sine of the complement of the maximum declination be led into the sine of the declination of the part beginning from the equator of which part the elevation is sought, and the product be divided by the sine of the complement of the declination of that part, and what results be led likewise into the sine of the elevation of one quadrant, and the product be divided by the sine of the maximum declination, the sine of the sought elevation will result.

The elevation of a part of the ecliptic is the arc of the equator that begins and finishes rising with that part. For the exposition of this matter, let the aforesaid figure be taken up again in an example. For what is proposed is to discern how great is arc ET, which is the elevation of arc EH. Then, because in a figure of this kind, two arcs AZ and AE descend from a common endpoint between which two others ZT and EB intersect at point H, therefore, through the disjunct kata, the ratio of the sine of ZB to BA consists of the ratios of ZH to HT and of ET to EA. I speak about the sines of these arcs. Therefore, if the sine of ZB is led into the sine of HT, i.e. the first into the fourth, and the product is divided by the sine of ZH, the third, there will result a line whose ratio to the sine of arc BA, the second, is as the sine of ET to the sine of EA, i.e. of the fifth to the sixth. Therefore, if that line be led into the sine of EA, which is the elevation of one quadrant, and be divided by the sine of AB, which is the maximum declination, there will result the sine of ET, the sought elevation.



Posito ergo arcu EH xxx graduum, invenies arcum TE partibus xxvii punctis l terminari. Quod si arcum EH ponas esse partium lx, reperies arcum TE ex partibus lvii punctis xliiii. Ex hiis ergo constans est quod prima zodiaci pars duodecima ortus sui sive ascensionis tempus partibus xxxvii punctis l — linee dico equinoctialis — terminat; secunda xxix partibus punctis liiii. Unde palam quod tertie ipsius duodecime elevationi relinquuntur de equinoctiali linea partes xxxii puncta xvi. Nam ascensus cuiuslibet quarte zodiaci quarte cuilibet de recto circulo adequatur quod ex circulo per polos equinoctialis transeunte poterit deprehendi. Et vide quod uni quarte accidit alteri accidere necesse est, dum circulus equinoctialis orizonti recte spere ortogonaliter insistat. Sufficit ergo inquisitio elevationum unius quarte ad habendum omnes. Evidenter igitur ex hiis deprehenditur quot horis rectis pars zodiaci circa meridianum circulum ubique locorum et ab orizonte recte spere transierit.

439 arcum] *s.l. P* **440** esse] *om. N* TE] *corr. ex* DE P_7 **441** constans est] constat *M* zodiaci pars] pars zodiaci P_7 **442** xxxvii] *corr. in* xxvii *K* 27 *MN* (36 *Ba* 37 E_1) **443** xxix] *corr. ex* xxx *P* **444** tertie] tertius P_7 ipsius duodecime] duodecime ipsius *N* **445** puncta] puncti *K* quarte[1] – quart[2]e] zodiaci quarte PP_7K quarte zodiaci *N* (quarte zodiaci *Ba* quarte zodiaci quarte E_1) **449** elevationum] elevationis *MN* **450** circa] certa *P corr. ex* certa P_7 **451** ubique] ubicumque *N* transierit] transeat *N*; explicit primus liber *add.* P_7 explicit liber primus *add. M* primus liber finit *add. N* (explicit primus liber *add. Ba*)

Therefore, with arc EH supposed to be 30°, you will find that arc TE is bounded by 27° 50′. But if you suppose that arc EH is 60^p, you will find that arc TE is 57° 44′. From these, therefore, it is evident that the first twelfth of the zodiac bounds its time of rising or ascension by 37° 50′[21] – I mean of the equator; the second by 29° 54′. Whence it is clear that 32° 16′ of the equator remain for the elevation of its third twelfth. For the ascension of any quarter of the zodiac is made equal to any quarter of the right circle, which will be able to be discovered by the circle passing through the poles of the equator. And see that it is necessary that what occurs for one quarter occurs for another, as long as the equator stands perpendicularly to the right sphere's horizon. Therefore, an investigation of the elevations of one quarter is sufficient for having all. Therefore, from these things it is grasped clearly by how many right hours a part of the zodiac passed the meridian – at whatever different points – and the right sphere's horizon.

[21] To match the *Almagest* and to make mathematical sense, the value should be 27° 50′.

⟨Liber II⟩

Orizon declivis est cui polus elevatur.

Spera declivis est vel obliqua hiis qui orizonte declivi utuntur.

Cenit capitum est punctum summitatis capitum et est polus orizontis.

Latitudo regionis est distantia cenit capitum ab equinoctiali, et est arcus meridiani inter cenit capitum et circulum equinoctialem interceptus.

Longitudo regionis est distantia eius ab orientis vel occidentis principio, et est arcus paralelli ad equinoctialem inter cenit capitum et eum circulum qui super Amphytritis circuitum in celo est dispositus.

Locus notus dicitur cuius longitudo et latitudo nota.

Speralis angulus dicitur angulus ex duobus arcubus in superficie spere proveniens.

Speralis angulus rectus dicitur cui sub duobus arcubus maiorum orbium contento quarta circuli supra cuius polum ipse angulus consistit subtenditur.

1. Arcum diei minimi vel maximi in quovis climate per notam poli altitudinem cognoscere. Unde manifestum quod si sinus altitudinis poli ducatur in sinum maxime declinationis, et productum dividatur per sinum perfectionis maxime declinationis, et quod provenerit ducatur in semidiametrum, productum dividatur per sinum perfectionis altitudinis poli, exibit differentia mediata minime diei ad equinoctialem diem.

Sit ergo meridiei circulus ABGD infra quem orientalis medietas orizontis BED sed etiam equinoctialis AEG designentur. Australem polum nota Z, hiemale solstitium ascendens in orizonte nota H notet. Deinde circuli per utrumque polum transeuntis quarta ZHT deducatur. Quia ergo H et T note motu suo paralellos in spera describunt circulos, et spere revolutio super polos

1 Liber II] liber secundus *marg. (other hand) P* secundus *marg. K* incipit secundus P_7MN **3** est – obliqua] est P_7 vel obliqua est *KM* declivi] obliquo *N* **4** capitum[1]] capitis P_7N punctum] punctus *MN* capitum[2]] capitis P_7M **5/6** Latitudo – interceptus] *This definition is placed after the following one.* P_7 **5** capitum] capitis P_7N **6** capitum] capitis P_7 et] *corr. ex* ab *K* circulum equinoctialem] equinoctialem circulum *KM* equinoctialem *N* **7** eius] *om. N* **8** capitum] capitis P_7 qui] qui est P_7 **9** Amphytritis] Amphitrias P_7 Amphitritis *KM* Amphitrios *N* (Amphitritis *Ba* Amphtritis E_1) circuitum – celo] in celo circuitum *N* circuitum] *corr. ex* circulum *M* dispositus] *corr. ex* depositus *K* **11** angulus[2]] *iter. K* superficie spere] spera *N* **15** diei] *corr. in* circuli *P* circuli diei *N* vel] et *M* **18** provenerit] proveniat P_7 semidiametrum] et *add.* (*marg. K*) *KMN* **19** mediata] *marg. P* **20** minime] minimi P_7 **22** BED] *corr. ex* BDE *K* **23** solstitium] nota *add. et del. P* notet] notat *P* nota *N* **24** utrumque] eundem *MN* quarta] *corr. ex* quarata P_7 **25** suo] *iter. et del. M* paralellos – circulos] in spera paralellos describunt *N*

Book II

A declined horizon is one to which the pole is raised.

The sphere is declined or oblique to those who live with a declined horizon.

The zenith is the highest point above ⟨their⟩ heads [*lit*., the zenith of heads is the point of the highest part of the heads], and it is the pole of the horizon.

The latitude of a region is the distance of the zenith from the equator, and it is the arc of the meridian cut off between the zenith and the equator.

The longitude of a region is its distance from the most eastward or westward point [*lit*., from the beginning of the east or west], and it is an arc of the ⟨circle⟩ parallel to the equator between the zenith and that circle that is laid out in the heavens above the circumference of Amphytritis.[1]

A known place means one whose longitude and latitude are known.

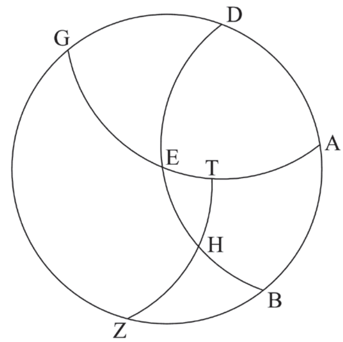
A spherical angle means an angle resulting from two arcs on the surface of a sphere.

A right spherical angle means one that, contained by two arcs of great circles, is subtended by a quarter of the circle upon the pole of which that angle stands.

1. To know the arc of the shortest or longest day in any climate through the pole's known altitude. Whence it is manifest that if the sine of the pole's altitude be led into the sine of the maximum declination, and the product be divided by the sine of the complement of the maximum declination, and ⟨if⟩ what results be led into the radius, and the product be divided by the sine of the complement of the height of the pole, the half of the difference between the shortest day and the equinoctial day will result.

Then let there be the meridian ABGD below which let the eastern half of the horizon be designated BED and also the equator AEG. Let point Z mark the south pole and point H mark the winter solstice ascending on the horizon. Then let ZHT, a quarter of the circle passing through each pole, be led down. Then, because points H and T describe parallel circles on the sphere

[1] Amphitrite was the mythical wife of Poseidon. The circuit of Amphitritis is the ocean surrounding the entire earth and passing through the four cardinal points. Robert Grosseteste uses the term in his *De sphaera*, 'Intelligatur circulus magnus cingens corpus terrae sub utroque polo, et alius circulus magnus cingens corpus terrae sub aequinoctiali circulo, secundum situm horum duorum circulorum cignunt duo maria totam terram; et illud, quod cingit terram sub polis amphitrites vocatur, reliquum vero vocatur oceanus. Haec duo maria dividunt terram in quattuor partes quarum una sola inhabitatur'; see Baur, *Die philosophischen Werke des Robert Grosseteste*, p. 24. This term was later used by John of Sicily; see Pedersen, 'Scriptum Johannis de Sicilia', 52, p. 132, section J280.

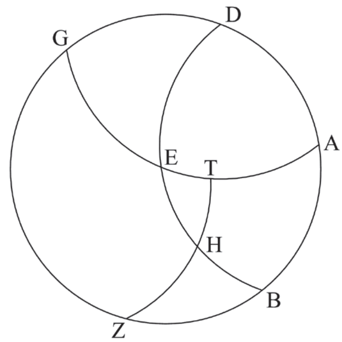


utriusque circumducitur, constat notas H et T ad arcum AB meridiani circuli uno et eodem tempore pariter devenire propter similes paralellorum circulorum portiones. Tempus autem quo nota H ad medium celum ab ortu suo conscendit est quantitas arcus TA de linea equinoctiali. Tempus autem a medio sub terra celo ad oriens est quantitas arcus GT. Quod inde apparet quia ipsius diei tempus est quantitas arcus ad TA duplicis, noctis vero tempus est quantitas arcus qui ad GT duplus est. Est ergo arcus TE differentia equinoctialis et minimi diei, cum E sit medius punctus arcus AG ad quem punctum oritur Aries vel Libra. Hiis ita se habentibus vide quod inter duos arcus AZ et AE due quarte circulorum se intersecant scilicet EB et TZ. Quare per kata disiunctam proportio sinus ZB ad BA producitur ex proportione sinus ZH ad HT et sinus ET ad EA. Sed primum est notum et secundum propter altitudinem poli notam, et tertium propter maximam declinationem notam esse, et quartum similiter, sextum vero quia est quarta circuli. Quapropter et quintum notum erit.

2. Arcum orizontis in quovis climate qui est inter ortum tropici et equinoctialem per assignatum minimi diei arcum investigare. Unde patebit quod si ducatur sinus dimidii arcus diei minime in sinum perfectionis maxime declinationis, productumque dividatur per sinum quadrantis, exibit sinus perfectionis arcus orizontis qui est inter ortum utriusque tropicorum et circulum equinoctialem. Similique ratione inveniri potest distantia ortus cuiuslibet signi vel gradus ab equinoctiali.

Premissa dispositione sicuti est manente arcum HE querimus. Quare per kata coniunctam conversis proportionibus proportio AT ad AE, de sinibus loquor, producitur ex proportione sinus BH ad sinum BE et eiusdem BE sinus proportione ad sinum HZ. Sed ex eisdem proportionibus constat proportio

26 utriusque] *om.* *N* **28** devenire] pervenire *MN* **29** similes] similes scilicet P_7 **30** celum] celi *M* **31** quantitas] *om.* *N* **33** sub – celo] celo sub terra *N* oriens] *corr. ex* orientis *K* **33/35** est – tempus] *marg.* P_7 **34** Quod] et *K* inde] non *N* apparet] *corr. ex* appareat *K* **36** noctis vero] vero noctis *P* *corr. ex* vero noctis *K* **37** duplus est] est duplus P_7 minimi] minime *PN* **38** medius punctus] punctus medius *N* **41** EA] eam *P* **43** maximam declinationem] magnam declinatione *K* esse] *del.* *K* *om.* *MN* (*om.* BaE_1) **44** erit] erit et cetera *M* erit] erit et cetera *M* **46** minimi] minime *N* **47** minime] *om.* *N* **49** utriusque] utriuslibet P_7K (utriusque *Ba* utriuslibet E_1) tropicorum – circulum] et *N* **52** dispositione] dispositio *P* *corr. ex* dispositio *K* sicuti] sicut *N* arcum HE] arcu HT *P* arcuum HE P_7 **53** coniunctam] coniunctam et P_7 sinibus] similibus *P* **54** et] ad *add. et del.* *K* eiusdem] eundem *P* BE sinus] sinus BE *N*

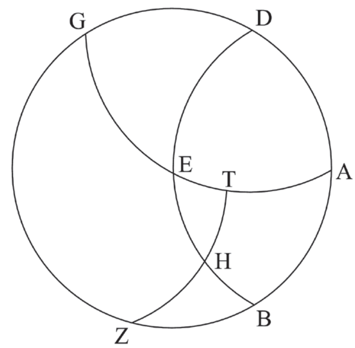


by its own motion, and a sphere's revolution is turned upon the poles of each, it is evident that points H and T arrive together at arc AB of the meridian at one and the same time because of similar parts of parallel circles. Moreover, the time in which point H ascends to the middle heaven from its rising is the quantity of arc TA of the equator. Moreover, the time from the middle heaven under the earth to the rising is the quantity of arc GT. Which thence is apparent because the time of that day is the quantity of the arc double TA, and indeed the time of the night is the quantity of the arc that is double GT. Therefore, arc TE is the difference[2] between the equinoctial and shortest day because E is the middle point of arc AG, to which point Aries or Libra rises. With these things disposed in this way, see that between the two arcs AZ and AE two quarters of circles intersect, namely EB and TZ. Through the disjunct kata, therefore, the ratio of the sine of ZB to BA is produced from the ratio of sine ZH to HT and of the sine of ET to EA. But the first is known and the second because of the pole's known altitude, and the third because of the known maximum declination, and the fourth similarly, and indeed the sixth because it is a quarter circle. For this reason, also the fifth will be known.

2. To find the arc of the horizon in any climate that is between the tropic's rising and the equator through an allotted arc of the shortest day. Whence it will be clear that if the sine of half the arc of the shortest day be led into the sine of the complement of the maximum declination, and the product be divided by the sine of a quadrant, there will result the sine of the complement of the arc of the horizon that is between the rising of each of the tropics and the equator. And by a similar proof, the distance of the rising of any sign or degree from the equator can be found.

Keeping the previous arrangement as it is, we seek arc HE. Therefore, through the conjunct kata with the ratios reversed, the ratio of AT to AE, I speak of the sines, is produced from the ratio of the sine of BH to the sine of BE and the ratio of the same sine of BE to the sine of HZ. But the ratio of the

[2] This should be 'half the difference.' *K*'s scribe added above the line 'mediata supple', but the mistake seems to have been original.



sinus HB ad HZ; ergo proportio sinus AT ad sinum AE est sicut proportio sinus HB ad sinum HZ. Ergo si primum ducas in quartum et cetera. Sed primum ex ypothesi quia arcus TA medietatis diei minime est tempus, et quartum notum quia maxima declinatio nota, et secundum notum quia est quarta circuli. Ergo tertium notum, ergo eius arcus scilicet HB notus. Ergo reliquus de quarta scilicet HE arcus notus est, quod proponebatur.

Posito ergo quod dies longissima xiiii horis rectis et media terminetur ut est in Rodos insula, invenies arcum EH partes xxx de ccclx continere.

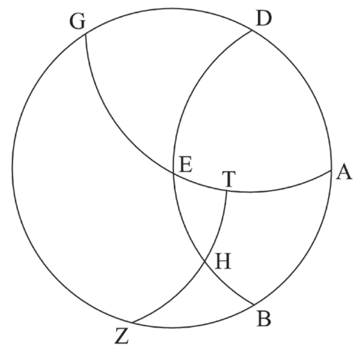
3. Altitudinem poli per arcum diei minimi notum presto indagare. Regula. Si sinum differentie medie diei minimi ad equinoctialem diem ducas in sinum perfectionis quarte orizontis, productumque dividatur per sinum arcus orizontis qui est inter ortum tropici et equinoctialem, atque quod exierit ducatur in sinum quadrantis, productumque dividatur per sinum arcus medii minimi diei, exibit sinus altitudinis poli.

Supraposita figura denuo assumpta quantitatem arcus ZB que est altitudo poli querimus. Igitur per kata disiunctam proportio sinus ET arcus ad sinum arcus AT componitur ex proportione sinus EH ad HB sinum et proportione sinus ZB ad sinum ZA. Quare si ducas primum in quartum et productum dividas per tertium, exibit quiddam quod sic se habebit ad secundum sicut quintum ad sextum. Sed tria nota, duo enim per ypothesim, tertium quia est quarta circuli; ergo quartum notum est, quod intendebamus.

Posito ergo arcum diei minimi habere horas rectas ix et dimidiam, invenies altitudinem poli esse fere xxxvi graduum.

4. Arcum orizontis qui est inter ortum tropici et equinoctialem per altitudinem poli notam reperire. Unde patet regula: si sinum maxime declinationis

57 sicut] sinus *del.* P_7 **59** primum] primum notum P_7 ypothesi] notum est eo *add. marg.* *N* (est notum *add. Ba text confirmed by* E_1) **60** quia] quod *PN* qui *KM* (quia *Ba*E_1) TA] tam *corr. in* est *K* est *M* medietatis] *corr. ex* medietas P_7 **61** tempus] *corr. in* notum *K* notum *M* **64** reliquus] reliquum P_7 **65** scilicet – arcus] circuli scilicet arcus HE *N* **68** partes] partium *N* **69** minimi] minime *PN* **71** quarte orizontis] arcus orisontis qui est inter ortum tropici et equinoctialis orisontis (*last word del.*) *N* productumque] *corr. ex* productum (*same hand*) *P* orizontis^2] medii orizontis *P* **72** exierit] exiet *N* **73** minimi] *s.l.* P_7 **75** Supraposita] supposita P_7 **76** proportio] proportio l *P* l *add. et del.* *K* sinum] sinus P_7 **77** AT] *corr. ex* ET *K* **78** sinum] proportionem P_7 ducas] dividas *P* **79** quiddam] quidam *M* quoddam *N* habebit] habet *N* **80** quintum] quintus *M* nota – enim] sunt etenim *N* quia] *corr. ex* qui *K*



sine of HB to HZ consists of the same ratios; therefore, the ratio of the sine of AT to the sine of AE is as the ratio of the sine of HB to the sine of HZ. Therefore, if you lead the first into the fourth, etc. But the first ⟨is known⟩[3] from hypothesis because arc TA is the time of half of the shortest day, and the fourth is known because the maximum declination is known, and the second is known because it is a quarter of a circle. Therefore, the third is known, so its arc, i.e. HB, is known. Therefore, its complement, i.e. arc HE, is known, which was proposed.

Therefore, with it supposed that the longest day is bounded by 14 1/2 right hours, as is on the island of Rhodes, you will find that arc EH contains 30° of 360.

3. To track down the pole's altitude through the known arc of the shortest day at hand. Rule. If you lead the sine of half the difference between the shortest day and the equinoctial day into the sine of the complement of the ⟨arc on the⟩ horizon ⟨between the risings of the equator and tropic⟩, and ⟨if⟩ the product be divided by the sine of the arc of the horizon that is between the tropic's rising and the equator, and ⟨if⟩ what results be led into the sine of a quadrant, and the product be divided by the sine of the arc of half the shortest day, the sine of the pole's altitude will result.

With the figure given above taken up again, we seek the quantity of arc ZB, which is the pole's altitude. Accordingly, through the disjunct kata the ratio of the sine of arc ET to the sine of arc AT is composed of the ratio of the sine of EH to the sine of HB and the ratio of the sine of ZB to the sine of ZA. Therefore, if you lead the first into the fourth and divide the product by the third, something will result that will be disposed thus to the second as the fifth to the sixth. But three are known, for two through hypothesis and the third because it is a quarter circle; therefore, the fourth is known, which we intended.

Therefore, with it supposed that the arc of the shortest day has 9 1/2 right hours, you will find that the pole's altitude is about 36°.

4. To find the arc of the horizon that is between the tropic's rising and the equator through the pole's known altitude. Whence this rule is clear: if

[3] This is needed for the meaning, but the manuscripts point to it being omitted by the author.

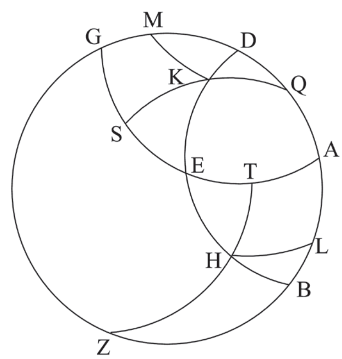
ducas in semidiametrum, et productum dividas per sinum perfectionis altitudinis, exibit sinus arcus orizontis qui inter tropicum et equinoctialem deprehenditur.

Resumatur eadem figura. Nota quantitate arcus ZB querimus arcum orizontis EH. Igitur per kata coniunctam conversis proportionibus propter arcus EB et ZT equales esse, constat sinum AB ad sinum AZ eandem proportionem habere quam sinus TH ad sinum EH. Sed primum notum est quia est sinus perfectionis altitudinis poli note, et secundum quod est semydiameter circuli, sed etiam tertium quia est sinus arcus maxime declinationis. Quare quartum notum.

Simili modo est cognoscere quemlibet arcum orizontis inter quemcumque gradum circuli declivis et equinoctialem deprehensum eo quod cuiuslibet gradus declinatio ex premissis est nota.

5. Quilibet duo circuli paralelli circulo equinoctiali eiusdem longitudinis a duobus tropicis sive ab ipso equinoctiali equales arcus orizontis resecant ex utraque parte equinoctialis, et fit alternatim nox unius diei alterius equalis.

Repetita itidem eadem figura, in ipsa duos circulos HL et KM paralellos equinoctiali describimus, et notam Q polum septentrionalem, et ab eo per notam K quartam circuli magni QKS. Quia ergo circuli KM et HL eiusdem longitudinis sunt ab equinoctiali, eos equales esse constat et orizontem quia circulus magnus est equales arcus ab eis abscindere. Item SG equalis est arcui TA quia similes eorum equales sunt; relinquitur ergo arcus SE equalis arcui ET. Sed et arcus HT arcui KS propter declinationes equas esse. Sed et angulus KSE angulo HTE eo quod uterque circulus erectus est super equinoctialem. Quare basis basi equalis, scilicet arcus EK arcui EH, quod proposuimus.

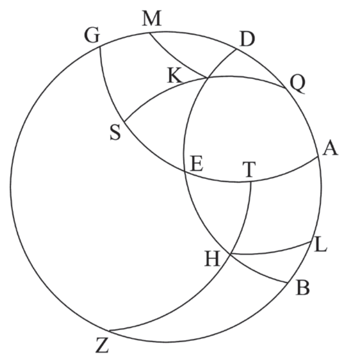


86 semidiametrum] *corr. ex* diametrum *K* dividas] *om. P* altitudinis] altitudinis poli *N* **87** qui] est *add. et del. P* **90** propter] et propter (portio *add. et del.*) P_7 **91** EB] *corr. in* EH *M* ZT] ET *PM* esse] *om.* P_7KM (esse BaE_1) ad] *iter. P* **92** sinus[1]] sinum P_7K est[1]] *om.* P_7 **93** quod] qui *P* quia P_7N semydiameter] semidyametrum PP_7 **94** quia] qui *P om. N* **96** Simili] *corr. ex* sit *K* est] *iter.* P_7 **99** Quilibet] cuilibet *P* paralelli] parabelli *K* **100** orizontis] de orisonte obliquo *marg. N* **100/101** resecant – equinoctialis] ex utraque parte equinoctialis resecant P_7 **102** Repetita] recepta P_7 **104** Q] *om. P* Q quasi P_7 *s.l. K* (Q E_1) septentrionalem] septentrionis *N* **105** eo] *corr. in* ea *M* **106** QKS] QKL *N* **107** eiusdem] *corr. ex* eius *P* ab equinoctiali] ad equinoctialem *PMN* (ab equinoctiali BaE_1) **108** eos] ipsos *MN* **110** Item] unde *KM* TA] *corr. ex* TH *N* **112** HT] AT *P* **113** equas esse] equales esse P_7 equas equalis (*marg.*) est *K* equales TE *M* **114** circulus] *corr. ex* angulus *M* **115** quod] *s.l. K* you lead the sine of the maximum declination into the radius and divide the product by the sine of the complement of the altitude, there will result the sine of the arc of the horizon that is caught between the tropic and the equator.

Let the same figure be taken again. With the quantity of arc ZB known, we seek the arc EH of the horizon. Accordingly, through the conjunct kata with the ratios reversed, because arcs EB and ZT are equal, it is evident that the sine of AB has the same ratio to the sine of AZ that the sine of TH has to the sine of EH. But the first is known because it is the sine of the complement of the pole's known altitude, and the second because it is the radius of a circle, and also the third because it is the sine of the arc of the maximum declination. Therefore, the fourth will be known.

In a similar way, it is ⟨possible⟩ to know any arc of the horizon caught between any degree of the ecliptic and the equator because the declination of any degree is known from what has been set forth [i.e. I.16].

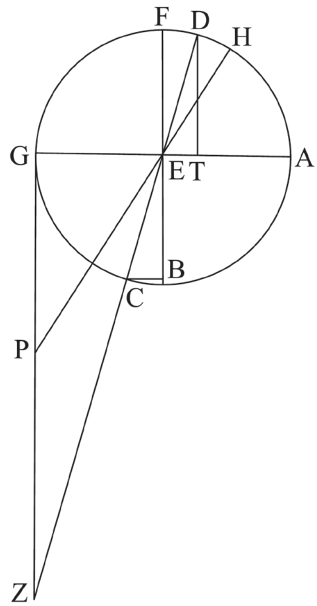
5. Any two circles parallel to the equator at the same distance from the two tropics or from the equator cut off equal arcs of the horizon on both sides of the equator, and the night of one alternately is made equal to the day of the other.



With the same figure repeated in the same way, we describe in it the two circles HL and KM parallel to the equator, and ⟨we draw⟩ point Q the north pole, and from it through point K ⟨we draw⟩ the quarter of a great circle QKS. Then, because the circles KM and HL are of the same distance from the equator, it is evident that they are equal and that the horizon cuts off equal arcs from them because it is a great circle. Likewise, SG is equal to arc TA because ⟨arcs⟩ similar to them are equal; therefore, arc SE remains equal to arc ET. But also arc HT ⟨is equal⟩ to arc KS because the declinations are equal. But also angle KSE ⟨equals⟩ angle HTE because each circle is set up ⟨perpendicularly⟩ upon the equator. Therefore, base is equal to base, i.e. arc EK to arc EH, which we proposed.

6. Nota Solis altitudine proportionem umbre iacentis ad gnomonem erectum vel umbre verse ad gnomonem iacentem invenire; et conversim nota proportione umbre ad gnomonem altitudinem Solis indagare. Regula: si sinum perfectionis altitudinis ducas in partes gnomonis quantaslibet, et productum dividas per cordam altitudinis, exibunt partes quantitatis umbre similes partium gnomonis; et e converso, si radicem duorum quadratorum gnomonis et umbre cum nota sint extrahas, et per eam id quod ex ductu gnomonis in semidiametrum provenit dividas, exibit sinus quesite altitudinis.

Sit ergo circulus altitudinis ADG supra centrum E, et AEG linea a summitate capitis perpendiculariter demissa supra lineam GZ, que linea orizontis intelligitur. Et est quidem super terram locata; propter insensibilem tamen terre quantitatem ad celum, centrum constituitur. Et sit EG gnomo erectus et D altitudo Solis ab F quasi orizonte. Erit ergo radius Solis per summitatem gnomonis DEZ et longitudo umbre GZ. Propter similitudinem ergo triangulorum ET ad TD eadem que EG ad GZ. Cum ergo ET sinus altitudinis notus, et DT sinus perfectionis altitudinis notus, et quantitas gnomonis nota, erit quartum scilicet umbra nota. Pari ratione si EB sit gnomo iacens et BC umbra versa ponatur.

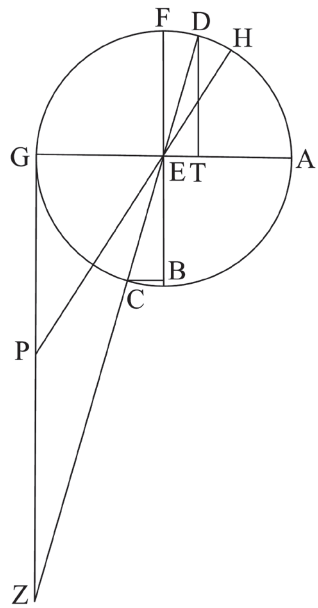


Rursum si GE et GZ sint nota, ergo EZ basis que subtenditur angulo recto nota, cuius ad ED semidiametrum est proportio ut GE ad ET. Simili modo HF arcus potest innotescere

116 Nota] data *N* gnomonem] *corr. ex* gomonem *s.l. P* erectum] erectam P_7 **117** gnomonem] *corr. ex* gomonem *s.l. P* conversim] conversum *P corr. ex* conversio *K* **118** Regula] ratio P_7 si sinum] sinum *corr. ex* si non *P corr. ex* sinum *K* **120** dividas] dividis P_7 cordam] *corr. in* sinum P_7N (cordam BaE_1) **121/122** et umbre] vel umbre *M om. N* **122** et – eam] per eamque (*corr. ex* eam) P_7 **124** Sit] si *P* **126** demissa] dimissa P_7 **127/128** est – locata] E quidem super terram locatur P_7 (*text confirmed by Ba* est quidem super terram locatur E_1) **127** quidem] quasi *M* **129** terre quantitatem] quantitatem terre *N* celum] *corr. in* celum subtus *K* celum subtus *corr. ex* circulum subtus *M* **130** D] *om. N* **131** F] *corr. ex* †B† *M* Erit ergo] ergo erit *M* **134** triangulorum] triangulorum proportio P_7 TD] DTB *PMN* proportio *add. s.l. K* **134/135** eadem que] sicut *MN* **136** notus] sit *add.* (*marg. K*) *KM* notus est *N* **140** EZ] *corr. ex* EB *M* **141** subtenditur] subtendit *PK* (subtenditur BaE_1) **143** innotescere] ignotescere *P*

6. With the sun's altitude known, to find the ratio of the horizontal shadow to the upright gnomon or of the turned shadow to the horizontal gnomon; and conversely, with the ratio of the shadow to gnomon known, to track down the altitude of the sun. Rule: if you lead the sine of the complement of the altitude into however many parts of the gnomon [i.e. into whatever number of parts the gnomon is divided into], and you divide the product by the chord[4] of the altitude, there result the parts of the quantity of the shadow similar to the parts of the gnomon; and conversely, if you extract the root ⟨of the sum⟩ of the two squares of the gnomon and the shadow, because they are known, and ⟨if⟩ you divide by that what results from leading the gnomon into the radius, the sine of the sought altitude will result.

Then let there be the circle of altitude [i.e. the meridian] ADG upon center E, and line AEG sent down from the zenith perpendicularly upon line GZ, which is understood to be the line of the horizon. And it is, in fact located above the earth; nevertheless, because of the imperceptible quantity of the earth to the heavens, it is set up as the center.[5] And let EG be the upright gnomon[6] and D the altitude of the sun from F as the horizon. Therefore, the sun's ray through the top of the gnomon will be DEZ and the shadow's length will be GZ. Therefore, because of the similarity of triangles, ET to TD is the same ⟨ratio⟩ as EG to GZ. Then, because ET, the sine of the altitude is known, and DT, the sine of the complement of the altitude, is known, and the gnomon's quantity is known, the fourth, i.e. the shadow, will be known. ⟨Proceed⟩ by a like proof if EB is the horizontal gnomon and BC is placed as the turned shadow.



If in turn GE and GZ are known, then base EZ, which subtends a right angle, will be known. The ratio of this [i.e.

[4] This should read 'sine' to make mathematical sense, but the error seems to have been in the original text. It probably derives from the confusing terminology in Plato's translation of al-Battānī, which uses 'chorda' to mean 'sine' in his rule (Albategni, *De scientia astrorum* Ch. 10, 1537 ed., f. 14r). This is not unusual for Albategni; he wrote earlier, '… et ne in sequentibus haec nobis iterare necesse sit, edicimus omnem tractatum nostrum sive mentionem cordarum de medietatis cordis oportere intelligi, nisi aliquo proprio nomine signaverimus, quod et cordam integram appellabimus, unde frequentius non multum indigemus' (Albategni, *De scientia astrorum* Ch. 3, 1537 ed., f. 7r).

[5] The reading here is found in almost all the witnesses, but P_7's reading 'And E indeed is placed upon the earth…' also makes sense astronomically.

[6] The noun here is 'gnomo, gnonomis', not 'gnomon, gnomonis.'

per umbram GP. Si ergo H sit maxima Solis in meridie altitudo et D minima, erit DH distantia duorum tropicorum et eius medietas maxima declinatio circuli declivis.

7. Sub linea equinoctiali omnes dies sunt equales noctibus et sibi invicem, et omnes stelle ortum habent et occasum, et umbre meridiane quandoque ad meridiem quandoque ad septentrionem quandoque nusquam declinant.

Ibi enim orizon et ipsum equinoctialem et omnes ei paralellos super quos fiunt revolutiones Solis in omni die et nocte semel dividit equaliter. Et quia orizon dividit superius emisperium ab inferiori, et latio Solis in inferiori emisperio est nox, in superiori emisperio est dies, erunt arcus diurni equales arcubus nocturnis. Et quia Solis revolutio ex motu spere equalis est in illis, erunt dies noctibus equales. Et quia similes sunt omnes arcus diurni sibi invicem et in similibus equales transitus, erunt omnes dies sibi invicem equales et noctes similiter. Et quia orizon iste super polos primi motus transit super quos fit revolutio stellarum omnium, omnes sursum emergunt et omnes occidunt. Et quia umbra semper cedit in oppositum luminis, cum Sol est ab equinoctiali in parte meridiana, fit umbra septentrionalis et e converso. Et cum est in ipso equinoctiali quod bis contingit in anno, quia tunc super capita fertur, umbra nusquam declinat.

8. Sub omni alia linea equidistante linee equinoctiali bis tantum dies fit equalis nocti in anno; et dies estivi hibernis prolixiores, noctes vero breviores; et quanto ab equinoctio distantiores dies estivi productiores, hiberni vero correptiores; et quedam stelle apparentes semper, quedam occulte semper; et distantia cenit ab equinoctiali equalis altitudini poli.

Ponamus ad hoc circulum meridianum ABCD, et duos polos primi motus A D, et lineam AD loco orizontis in spera recta, et CG loco equinoctialis, HI et KL et MN loco equidistantium ei. Quia vero sub omni alia linea, hoc est in spera declivi, polus unus elevatur super orizonta et alius deprimitur, sit QP loco orizontis declivis. Palam ergo quod quia magni circuli spere sunt ori-

144 umbram] umbras P_7 **145** distantia] distantia a *P* **146** declivis] declivis et cetera *M* **147** equales] sunt *add. et del. K* **148** et[3]] *om. M* umbre] umbre quandoque *N* meridiane] *corr. ex* meridie *P* **148/149** quandoque – meridiem] *om. PN* (*om. Ba* quandoque ad meridiem E_1) **149** nusquam] numquam *K* **150** Ibi] ubi *P* **151** in] *om. N* **152** emisperium] *corr. ex* empireum *K* **152/153** in – dies] in superiori est dies, latio Solis in inferiorum hemisperio est nox P_7 **152** emisperio] *marg. P* **153** in] et in *N* **156** equales transitus] equalis transitus P_7 transitus *M* transitus equales *N* **156/157** noctes similiter] similiter noctes *N* **157/158** fit revolutio] revolutio fit *PN* **159** cedit] cadit *MN* Sol] *om. N* **160** cum] *om.* P_7 **161** anno] et *add. s.l.* P_7 **161/162** umbra – declinat] nusquam declinat umbra *N* **164** prolixiores] longiores *MN* **166** semper[1]] *om. N* quedam[2]] quedam vero *N* **167** cenit] zenith *M* **168** ABCD] *corr. ex* ABCG *M* **169** A] A et P_7 loco[1]] circulum *N* CG] *corr. in* TG *N* **170** HI] *om. N* Quia] qui *PN* **171** et] *om. N* **172** magni circuli] circuli magni *N*

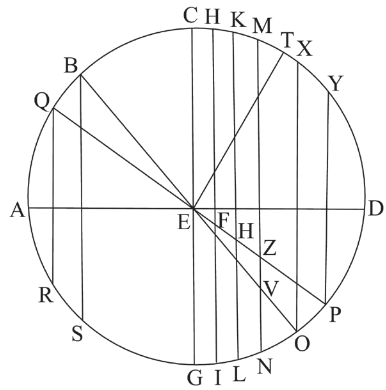
EZ] to radius ED is as GE to ET. In a similar way arc HF is able to become known through shadow GP. Therefore, if H is the greatest altitude of the sun on the meridian and D the smallest, DH will be the distance between the two tropics, and its half will be the maximum declination of the ecliptic.

7. Under the equator all days are equal to nights and to each other, all stars have a rising and setting, and the noon shadows decline sometimes towards the south, sometimes towards the north, and sometimes nowhere.

For there the horizon divides equally the equator itself and all its parallels, upon which the revolutions of the sun are made one time in each day and night. And because the horizon divides the hemisphere above from the lower, and the carrying of the sun in the lower hemisphere is night and in the upper hemisphere is day, the diurnal arcs will be equal to the nocturnal arcs. And because the sun's revolution caused by the sphere's motion is uniform in them, the days will be equal to nights. And because all the diurnal arcs are similar to each other and equal passages are in similar ⟨arcs⟩, all the days will be equal to each other and similarly nights ⟨will be equal to each other⟩. And because that horizon crosses upon the poles of the first motion, upon which the revolution of all the stars is made, all rise upwards and all set. And because the shadow also falls back opposite the light, when the sun is on the south side of the equator, the shadow is made north, and conversely. And when it is at the equator itself, which happens twice in a year, because it is then carried overhead, the shadow declines nowhere.

8. Under any other line parallel to the equator, the day is equal to the night only twice a year; the summer days are longer than those of winter, but the nights are shorter; as they are more distant from the equinox, the summer days are longer, but the winter ones shorter; certain stars are visible always, certain are hidden always; and the distance of the zenith from the equator is equal to the pole's altitude.

Let us place for this the meridian ABCD and the two poles A and D of the first motion, and line AD for the horizon in the right sphere, and CG for the equator, and HI, KL, and MN for the parallels to it. But because under any other line, i.e. in the declined sphere, one pole is raised above the horizon and the other is depressed, let QP be for the declined horizon. Therefore, it is clear that because the horizon and equator are great circles of the sphere, they

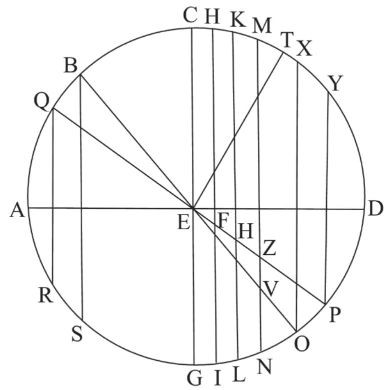


zon et equinoctialis per equalia se secant ut QP CG, reliquos vero omnes quia per polos A D non transit orizon inequaliter secat ad puncta F H Z. Fiunt ergo arcus diurni nocturnis maiores versus polum septentrionalem D, et noctes e converso. Et cum Sol transit per equinoctialem, fit arcus diurnus EC equalis nocturno EG, ideoque dies equales noctibus tantum. Et quia ZM arcus maior est quam qui ex eo sumitur similis arcui HK ut ex Theodosii De speris, maior est revolutio super hunc quam super illum. Ideoque dies maior et sic deinceps, tempus HK maius quam tempus FH, et hoc quam tempus EC; e contrario in diebus hibernis. Et quia quicquid est a PY versus polum D est super orizontem semper, erunt stelle in hac parte celi apparentes semper, et quia quicquid est a QR versus polum A sub orizonte, semper erunt stelle in hac parte celi occulte semper. Sit autem ET perpendicularis super QP; erit ergo T cenit capitum, et est TP quarta circuli, et similiter CD quarta circuli. Subtracto communi DP poli altitudo equalis est CT distantie cenit ab equinoctiali.

9. Sub remotiori linea ab equinoctiali maior est inequalitas dierum et noctium, et maior pars celi apparens semper et maior pars celi occulta semper. Quippe quia maior est remotio, maior est poli elevatio ut si sit BO orizon. Quare arcus VM maior arcu ZM, et ideo dies die maior. Atque arcus ODX apparens semper qui utique maior est arcu PDY.

10. Sub omni linea cuius distantia minor ab equinoctiali maxima declinatione, umbre meridiei ad utramque partem alternatim declinant et bis in anno declinatione carent.

173 secant] secare *N* **174** QP] et *add.* (*s.l. K*) *KMN* (QP *Ba*E_1) CG reliquos] CD reliqui *P corr. in* TG reliquos *N* **178** converso] converso versus polum meridionalem *N* **179** Sol transit] transit Sol *N* **180** EC] *om. P s.l. K* TE *s.l. N* (*om. Ba* E E_1) **181** equales] equalis P_7*M* tantum] tunc scilicet *add. s.l. K* **183** ut] ut patet *MN* Theodosii] Theodosio *N* Theodi *M* **184** speris] habetur *add. marg. K* **186** HK] *corr. ex* KHK *P* maius] *marg. M* **187** EC] ET *N* a] intra *s.l. K* PY] PV *P* PX *corr. in* QR *M* PX *N* **188** est – orizontem] super orizontem est *M* erunt] erunt ergo *M* **188/189** apparentes – quia] semper apparentes et *M* **189** QR] *corr. ex* GR *K* Q†R† *corr. in* QP *M* polum] polum est super orizontem *P* A[2]] A est *MN* orizonte] est *add. s.l. K* **191** capitum] capitis P_7*M* CD] TD *corr. in* TQ *M* Subtracto] subtracto ergo P_7 **192** DP] DT P_7 DT remanet DP *M* DP remanet *N* est CT] CT *M* CT et *corr. ex* Z et *N* **193** et] *om. P* **194** semper[1]] in parte universali in qua polus elevatur *add. s.l. K* **195** Quippe] su†s†pple *P* supple *N* **196** Quare] quia P_7 VM] NM *M* arcu] AM *corr. in* a†nni† *P* die] diei *M* **197** arcu] *om.* P_7 PDY] PDY et cetera *M* PDX *N* **198** minor – equinoctiali] *corr. in* ab equinoctiali minor *N* declinatione] declinatione Solis *MN*



cut each other in half as QP and CG, but because it does not pass through poles A and D, the horizon cuts all the remaining ⟨circles⟩ unequally at points F, H, and Z. Therefore, the diurnal arcs are made greater than the nocturnal arcs towards the north pole D, and the nights conversely. And when the sun crosses the equator, the diurnal arc EC is made equal to nocturnal arc EG, and for that reason days are equal to nights only ⟨there⟩. And because arc ZM is greater than that ⟨arc⟩ which is taken from it similar to arc HK as from Theodosius' *De speris*,[7] the revolution upon this is greater than upon that. And for that reason the day is greater, and thus in succession, the time of HK is greater than the time of FH, and this than the time of EC; and conversely in winter days. And because whatever is from PY towards pole D is always over the horizon, the stars in this part of the heavens will be always visible, and because whatever is from QR towards pole A is below the horizon, the stars in this part of the heavens will always be hidden. Moreover, let ET be perpendicular upon QP; therefore, T will be the zenith, and TP is a quarter circle, and similarly CD is a quarter circle. With what is common [i.e. arc TD] subtracted, the pole's altitude DP is equal to CT, the distance of the zenith from the equator.

9. Under a line more distant from the equator, the inequality of days and nights is greater, and a greater part of the heavens is always visible and a greater part of the heavens is always hidden.

Surely because the distance is greater, greater is the elevation of the pole as if BO is the horizon. Therefore, arc VM is greater than ZM, and for that reason ⟨its⟩ day is greater than ⟨that one's⟩ day. And arc ODX is always apparent, which is certainly greater than arc PDY.

10. Under each line whose distance from the equator is less than the maximum declination, the noon shadows fall alternately on each side [i.e. both north and south] and lack declination twice in the year.

[7] Theodosius, *Sphaerica*, II.19 (see Kunitzsch and Lorch, *Theodosius*, *Sphaerica*, pp. 169–71).

Nimirum quia Sol quandoque est septentrionalis a capite eorum, quandoque australis. Et bis in anno scilicet quando est in gradu cuius declinatio est equalis distantie que est inter ipsam lineam et equinoctialem, declinatione caret.

11. Sub linea cuius discessus equalis est maxime declinationi, umbra semel in anno declinatione caret, et umbra meridiana numquam declinat ad meridiem.

Tunc scilicet cum Sol est in capite Cancri, umbra in meridie flexu caret. Et quia Sol ab hoc loco numquam fit septentrionalis, umbra numquam cedit in meridiem. Ex quo etiam palam est quod sub omni linea discedente ab hac numquam umbra declinatione caret quia Sol numquam usque ad cenit capitum accedit, neque umbra cadet in meridiem quia Sol numquam fit ab ea septentrionalis.

12. Sub linea cuius discessio est ut poli zodiaci ab equinoctiali, umbra in aliquo die ad omnem partem orizontis flectitur; et fit spatium xxiiii horarum dies sine nocte et ex opposito nox sine die; et quanto discessus ab hac linea maior maius tempus abit sine nocte et ex opposito maius tempus sine die.

Hic enim principium Cancri numquam occidit, sed fit in superficie orizontis zodiacus. Et ideo cum Sol est in principio Cancri, circumgiratur, et umbra semper ex opposito, et fit tempus unius revolutionis sine occasu Solis. In maiori vero discessu ab hoc magis deprimitur orizon et abscindit arcum zodiaci numquam occidentem in quo quandiu Sol moratur, est dies sine nocte, et ex opposito abscindit arcum numquam orientem in quo quamdiu Sol existit, est nox sine die.

13. Sub polo medietas celi est apparens semper et medietas occulta semper, et anni spatium dies una cum nocte sua.

Ibi enim equinoctialis semper vertitur in superficie orizontis, et pars zodiaci septentrionalis fit super orizontem. Ideoque quamdiu Sol moratur in hac medietate, est dies sine nocte. Et medietas zodiaci australis est sub orizonte semper, et fit nox sine die. Et ita anni spatium dies una cum nocte sua.

14. In spera declivi quilibet duo arcus equales circuli declivis et equaliter a puncto equinoctii distantes equales habent ascensiones.

201 septentrionalis] *corr. ex* atrionalis P_7 **202** scilicet quando] Solis quando *P* *corr. ex* scilicet quandoque *K* quando Sol *MN* (scilicet quandoque BaE_1) est[1]] *s.l. K* **205** numquam] *corr. ex* nusquam *P* nusquam *N* **206** Tunc] *corr. ex* nunc *K* nunc *M* capite Cancri] *corr. ex* Capricorno *N* **207** septentrionalis] *corr. ex* atrionalis P_7 cedit] cadit *MN* **208** discedente] descendente *M* (descendente BaE_1) **209** usque] *om.* P_7 capitum] capitis P_7 **210** neque] nec *K* cadet] cadit *N* fit – ea] ab ea fit P_7 **214** linea] *om. PN* **216** Hic] sic *K* occidit] accidit *P* **217** circumgiratur] circumgirat *K* **219** abscindit] *corr. ex* ascindit *K* **220** occidentem] occidententem P_7 moratur] existit P_7 **220/222** ex – die] e contrario in arcu opposito P_7 **221** existit] *corr. ex* moratur *M* **223** medietas[1] – est] est medietas celi *N* **225/226** zodiaci septentrionalis] septentrionalis zodiaci *N* **226** septentrionalis] *corr. ex* atrionalis P_7 orizontem] orizontem semper P_7M semper *add. et del. K* (orizontem BaE_1) **227** medietate] mediet†re† P_7 medietas] *corr. ex* mediaetas P_7 **228** spatium] est *add.* (*s.l. K*) *KM* **230** equinoctii] equinoctiali *M*

Doubtlessly because the sun is sometimes north of the zenith [*lit.*, their head] and sometimes south. And twice in the year, i.e. when it is in the degree whose declination is equal to the distance that is between that line and the equator, it lacks declination.

11. Under the line whose distance is equal to the maximum declination, the shadow once in a year lacks declination, and the noon shadow never declines to the south.

Then, i.e. when the sun is at the beginning of Cancer, the shadow lacks an angle. And because the sun never occurs north of this place, the shadow never falls away to the south. From which it is also clear that under any line departing from this, the shadow never lacks declination because the sun never reaches the zenith, nor will the shadow fall to the south because the sun is never north of it.

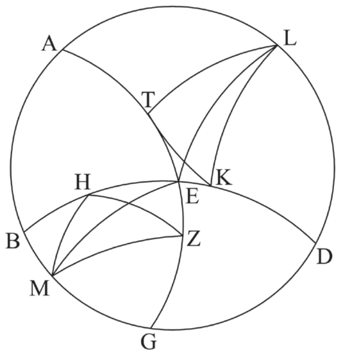
12. Under the line whose distance is as that of the pole of the zodiac from the equator, the shadow on any day is bent to every part of the horizon; there is an interval of 24 hours day without night and on the contrary, night without day; and as the distance from this line is greater, a greater time without night passes away, and on the contrary a greater time without day.

For here the beginning of Cancer never sets, but the zodiac is on the plane of the horizon. And for that reason, when the sun is at the beginning of Cancer, it is wheeled around, the shadow is always opposite, and the time of one revolution occurs without the setting of the sun. But in a greater distance from this ⟨latitude⟩, the horizon is depressed more and cuts off an arc of the zodiac never setting, in which as long as the sun remains, there is day without night, and on the contrary it cuts off an arc never rising, in which as long as the sun is, there is night without day.

13. Under the pole half of the heavens is always visible and half always hidden, and the length of a year is one day with its night.

For there the equator is always turned in the horizon's plane, and the northern part of the zodiac is above the horizon. And for that reason as long as the sun remains in this half, there is day without night. And the southern half of the zodiac is always below the horizon, and there is made night without day. And thus the space of a year is one day with its night.

14. In the declined sphere any two equal arcs of the ecliptic equally distant from the equinox point have equal ascensions.

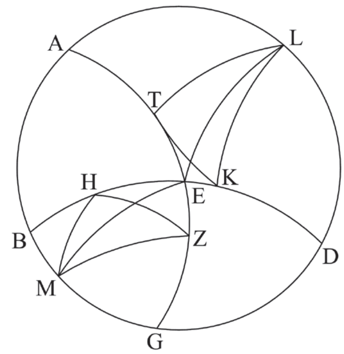


Sit ergo circulus meridianus ABGD, infra quem orizontis orientalis medietas BED sed equinoctialis AEG. Sitque HZ arcus circuli declivis inchoata a puncto equinoctiali et sit, si placet, signum Piscium. Et est punctum Z sectio communis equinoctialis et circuli declivis, finis Piscium et principium Arietis. Palam ergo quod arcus HZ oritur cum arcu EZ quia H et E puncta pariter veniunt ad orizonta. Dico quod cum arcu equinoctialis equali arcui EZ oritur signum Arietis. Sit ergo propter commoditatem figure arcus TK signum Arietis, et T idem punctum equinoctialis communis sectio. Palam ergo quod arcus TK oritur cum arcu equinoctialis ET. Dico ergo quod arcus EZ equalis est arcui ET. Sint itaque note M et L duo poli et ab eis arcus magnorum circulorum MH, ME, MZ, LT, LE, LK. Quia ergo triangulus MHZ equilaterus est triangulo LTK tum propter quartas magnorum circulorum, tum propter equales declinationes principii Piscium et finis Arietis, tum ex ypothesi. Sunt ergo anguli HMZ et TLK equales. Sed et arcus HE equatur arcui EK ex quinta huius libri; est ergo angulus HME angulo ELK equalis. Relinquitur ergo angulus EMZ equus angulo ELT. Et latera continentia hos angulos sunt equalia, ergo arcus EZ equus est arcui ET, quod intendebamus.

Pari modo quilibet duo arcus maiores vel minores propositis inchoati a puncto equinoctiali, si equales sunt, equos habent ortus. Et quia si ab equalibus equalia demantur et cetera, palam quod omnes equales et equaliter distantes a puncto equinoctiali equales habent ascensiones, quod proponitur.

15. Quilibet duo arcus circuli declivis equales et equaliter ab alterutro punctorum tropicorum distantes habent in spera obliqua ascensiones coniunc-

232 orizontis orientalis] orientalis orizontis P_7 sed] et *add.* (*s.l. K*) P_7KM (sed BaE_1) **233** Sitque] fitque PP_7 HZ] HAZ *corr. ex* AZ *P* **234** inchoata] inchoatus *KMN* (inchoata BaE_1) equinoctiali] equinoctii *N* **234/235** sit si] si sic *P* si *N* **235/236** est – communis] est Z punctum sectionis P_7 Z (*s.l.*) est punctum sectionis *N* **236** Z] *om. P* **237** finis] sinus *P corr. ex* fine *K* **239** quia – E] et quia H et K *N* **241** equali] equalis *P corr. ex* equalis *N* **241/242** Sit – Arietis] *om.* P_7 **241** Sit] fit *P* **242** commoditatem] comodi†e†tatem *N* idem] *del. K om. M* **243** equinoctialis] et declivis *add. s.l. K* TK] signum *add. et del. P* **244** arcui] *iter. et del. M* **245** duo poli] poli duo *N* MZ] MZ et *M* **246** LK] LR *K* Quia] qui *PN* equilaterus] equalis P_7 LTK] *corr. ex* LTR *K* **247** quartas] cartas *K* **248** et[2]] *iter. N* **249** EK] *corr. ex* ER *K* **250** HME] *corr. ex* HMT *K* **252** intendebamus] intendebatur *PN* (intendebamus BaE_1) **253** quilibet duo] et quilibet *N* **254** equinoctiali] equinoctii *N* ortus] *corr. ex* arcus *K* arcus *M* **255/256** a – equinoctiali] a puncto equinoctii *M* equinoctii punctis *N* (a puncto equinoctiali *Ba* a puncto equinoctii E_1) **256** equales] *marg. P* proponitur] fuit propositum *N* **257** Quilibet] cuilibet PP_7 (cuilibet *Ba* quilibet E_1)



Then let there be meridian circle ABGD, below which there is the eastern half of the horizon BED and the equator AEG. And let HZ be an arc of the ecliptic starting[8] from the equinox point, and if it pleases, let it be the sign of Pisces. And point Z is the intersection of the equator and the ecliptic, the end of Pisces and the beginning of Aries. Therefore, it is clear that arc HZ rises with arc EZ because points H and E come together to the horizon. I say that the sign of Aries rises with an arc of the equator equal to arc EZ. Then, because of the symmetry of the figure, let arc TK be the sign of Aries, and T is the same point, the intersection of the equator ⟨and the ecliptic⟩. It is clear, therefore, that arc TK rises with arc ET of the equator. I say then that arc EZ is equal to arc ET. Accordingly, let points M and L be the two poles and from them arcs of great circles MH, ME, MZ, LT, LE, and LK. Then, triangle MHZ is equilateral to triangle LTK because of the quarters of great circles, because of the equal declinations of the beginning of Pisces and the end of Aries, and from hypothesis. Therefore, angles HMZ and TLK are equal. But also arc HE is equal to arc EK from the fifth of this book; therefore, angle HME is equal to angle ELK. Therefore, angle EMZ remains equal to angle ELT. And the sides containing these angles are equal, so arc EZ is equal to arc ET, which we intended.

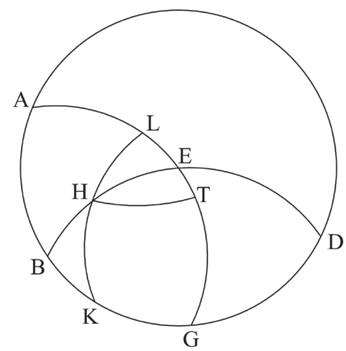
In a like way any two arcs greater or smaller than the proposed ones that begin from the equinox point, if they are equal, have equal risings. And because if equals are subtracted from equals etc., it is clear that all equal ⟨arcs⟩ equally distant from an equinox point have equal ascensions, which is proposed.

15. Any two equal arcs of the ecliptic equally distant from one or the other of the tropic points have ascensions in the oblique sphere together equal

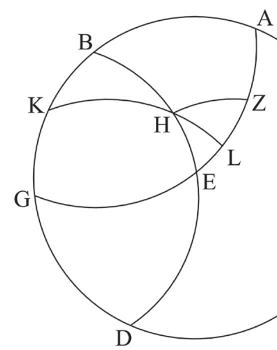
[8] The witnesses point towards a feminine ending, and indeed the noun ‘arcus, -us’ was feminine in some classical authors (see *Oxford Latin Dictionary*, p. 180); however, the author, as well as most mathematical writers, understood it to be masculine. There may have been confusion on the author’s part here or this is perhaps merely a scribal error that found its way into many of the surviving manuscripts.

tas equas eis ascensionibus quas idem arcus habent in spera recta coniunctis. Ex quo et premissa propositione manifestum quod si note fuerint ascensiones unius quarte in spera obliqua, note erunt ascensiones omnium.

Describemus ad hoc circulum meridiei in duobus locis ABGD, infra quem orizontis medietas BED et medietas circuli equinoctialis AEG. Et sit T punctum vernale, Z punctum autumpnale. Notandum autem quod cum orizon rectus per polos spere transeat et orizon declivis ipsum ad puncta equinoctialia secet, necessario cum polus septentrionalis elevetur super eum, inclinatur ab orizonte recto ad septentrionem et elevatur super eum ad austrum. Unde fit ut arcus zodiaci a vernali puncto inchoatus et citra initium Libre terminatus, quantuscumque sit, minorem moram faciat oriendo in orizonte declivi quam oriendo in orizonte recto. Simul enim hic et ibi incipit, sed hic tardius oriri desinit. E converso quilibet arcus ab auptumnali puncto inceptus et citra principium Arietis finitus maiorem moram facit ascendendo in spera declivi quam ascendendo in spera recta. Simul enim incipit hic et ibi, sed hic prius oriri desinit. Differentias ergo ascensionum equalium arcuum hinc inde sumptorum equales esse ostendemus.

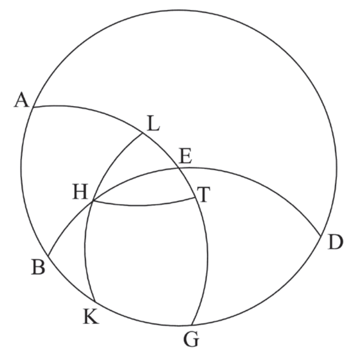


Et quia quilibet duo arcus equales ad punctum equinoctialem conterminales equales habent in quacumque spera eadem ascensiones, sit TH arcus quantuslibet circuli declivis ad vernale punctum T finitus, et sit si placet signum Piscium, et ZH equalis arcus signum Libre, et KHL quarta orizontis recti a polo K australi venientis. Oritur itaque arcus HT in spera declivi cum arcu

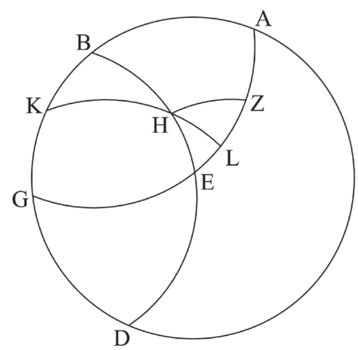


259 equas] *corr. ex* equales *M* idem – coniunctis] in spera recta habent coniunctas *N* idem] hiidem P_7 **260** propositione] proportione P_7 est *add.* (*s.l. K*) *KM* manifestum] manifestum est P_7 note fuerint] fuerint note *N* **260/261** ascensiones – quarte] unius quarte ascensiones *N* **261** note] *corr. ex* nocte *K* **262** Describemus] describamus *N* **263** medietas[1]] sit medietas *KM* T] *s.l. K* **264** Z – autumpnale] et Z punctum autumpnale *marg.* P_7 Notandum] nota P_7 **265** declivis] declivuus *P* declivum P_7 **267** elevetur] elevatur *MN* **270** fit] sit *N* **272** faciat] facit *K* **274** recto] rectam *N* Simul] similiter *K* **276** principium] initium P_7 **277** spera[1]] spera obliqua *N* **278** Simul] similiter *K* ibi] ibi oriri *N* sed] et *K* ergo] *corr. ex* enim *P* **279** hinc] hic P_7 **285** vernale – T] punctum T vernale *N* **286** Piscium] Piscis *K* et] *s.l. K om. M* **287** et] sit *M* to the conjoined ascensions that the same[9] arcs have in the right sphere. From which and the preceding proposition, it is evident that if the ascensions in the oblique sphere of one quarter are known, the ascensions of all will be known.

We will describe for this the meridian ABGD in two positions, below which ⟨we describe⟩ half of the horizon BED and half of the equator AEG. And let T be the vernal point, Z the autumnal point. Moreover, it must be noted that because the right horizon passes through the sphere's poles and the declined horizon cuts it at the equinoctial points, necessarily because the north pole rises above it, it inclines from the right horizon to the north and it is raised over it to the south. Whence it occurs that the arc of the zodiac beginning from the vernal point and bounded short of the beginning of Libra, however large it may be, takes less time for rising in the declined horizon than for rising in the right horizon. For it begins together here [i.e. in the right sphere] and there [i.e. in the oblique sphere], but here it finishes rising later. Conversely, any arc beginning from the autumnal point and ended short of the beginning of Aries takes more time for ascending in the declined sphere than for ascending in the right sphere. For it begins together here [i.e. in the right sphere] and there [i.e. in the oblique sphere], but here it finishes rising earlier. Therefore, we will show that the differences of the ascensions of equal arcs taken from one side and the other are equal.



And because any two equal arcs conterminous at the equinox point have equal ascensions in whichever same sphere, let there be arc TH however large of the ecliptic ending at the vernal point T, and let it be, if it pleases, the sign of Pisces, and ⟨let⟩ the equal arc ZH be the sign of Libra, and KHL a quarter of the right horizon coming from south pole K. Accordingly, arc HT rises in the declined sphere with arc ET and in the



[9] Note here that 'idem' is nominative plural, i.e. a spelling of 'iidem.'

ET et in spera recta cum arcu TL; est ergo differentia arcus EL. Rursum arcus ZH oritur in spera declivi cum arcu ZE et in spera recta cum arcu ZL; est ergo differentia arcus LE. Dico quod hee differentie sunt equales. Nam duo arcus HL et HL sunt equales propter eandem declinationem finis Libre et principii Piscium, et arcus ab orizonte recisi HE et HE cum sint idem equales, et angulus HLE utrobique rectus; ergo arcus EL arcui EL est equalis. Hoc enim similiter accidit in curvilineis maiorum orbium triangulis sicut in rectilineis cum angulus qui est ad H super polum equinoctialem non consistat et angulus qui est ad L sit rectus vel recto maior. Eodem modo constare potest de quibuslibet maioribus vel minoribus hiis arcubus sibi invicem equalibus.

Palam ergo quod si note fuerint ascensiones unius quarte, note erunt ascensiones omnium, quia ascensiones a principio Arietis usque ad initium Cancri, si note sunt, erunt note ascensiones ab initio Capricorni usque ad principium Arietis propter ascensiones equales esse; et erunt note ascensiones ab initio Cancri usque initium Libre sive ab initio Libre usque ad initium Capricorni quia cum has ascensiones notas in spera declivi quotlibet partium minimus ab ascensionibus earumdem partium in spera recta duplicatis prius notis, relinquuntur ascensiones quesite sumptarum partium. Et hoc est quod volebamus.

16. Cuiuslibet portionis circuli declivis ascensionem in spera declivi invenire. Regula operationis: si sinum altitudinis poli duxeris in sinum declinationis portionis inchoate ab equinoctiali puncto, et productum dividas per sinum perfectionis declinationis, et quod exierit itidem ducas in semidiametrum, productum dividas per sinum perfectionis altitudinis, exibit sinus differentie elevationum sumpte partis in spera recta et in spera declivi.

Resumpta superiori figura arcum EL querimus qui est differentia elevationum in spera recta et declivi attinens arcui zodiaci TH. Vides ergo quod in hac figura duo arcus AK et AE a communi termino A descendant inter quos duo

290/292 Rursum – LE] *marg.* P_7 **290** Rursum] rursus *N* **291** in[1] spera] infra *P* **292** sunt equales] equales sunt *P* **293** et HL] *marg.* (*other hand*) *P corr. in* et HT *M* **294** Piscium] Piscis *K* recisi] rescisi *K* sint] sit *PMN* (sint BaE_1) idem] hiidem P_7 equales] equalis *N* **296** curvilineis] curvis lineis *M* maiorum] maior *P* rectilineis] rectilineis est *N* **298** potest] *marg.* (*other hand*) *P* **299** hiis] *om.* *M* **300** fuerint] fiunt *P* **302** sunt] sint P_7 note ascensiones] ascensiones note *N* ad principium] ad initium (*s.l. K*) *KMN* ad] *om.* P_7 **303** esse] *del. K om. M* erunt note] note erunt *PN* **304** usque[1]] usque ad *M* sive – usque[2]] *s.l.* P_7 ab] hoc *add. et del. P* usque ad] ad *PN* usque *K* (usque ad *Ba* ad E_1) **305** has ascensiones] ascensiones has P_7 quotlibet] quodlibet *PM* partium] spatium *M* minimus] minueris P_7 *corr. in* minueris *K corr. in* minuimus *M* minuimus *N* **306** ascensionibus] ascensu *MN* earumdem] eorumdem P_7 duplicatis] *corr. ex* duplicantis *K* **309** Regula] ratio *P corr. ex* ratio *K* (regula *Ba* ratio E_1) **311** productum] et productum P_7 **312** altitudinis] poli *add. s.l. N* **313** in[2]] *om.* P_7K **314** arcum] arcuum *P* qui] que *PM* elevationum] *om. N* **315** et] et in spera *N* **316** duo[1]] *om.* P_7 a[1]] et *P* descendant] descendunt P_7N (descendat *Ba* descendant E_1)

right sphere with arc TL; therefore, the difference is arc EL. In turn, arc ZH rises in the declined sphere with arc ZE and in the right sphere with arc ZL; therefore, the difference is arc LE. I say that these differences are equal. For the two arcs HL and HL are equal because of the same declination of the end of Libra and the beginning of Pisces, the arcs HE and HE cut off from the horizon are equal because they are the same, and angle HLE is right in both instances; therefore arc EL is equal to arc EL. For this occurs similarly in curvilinear triangles of great circles as in rectilinear triangles because the angle that is at H does not stand upon the equator's pole and the angle that is at L is right or greater than right. In the same way it is able to be known concerning any arcs greater or smaller than these that are equal to each other.

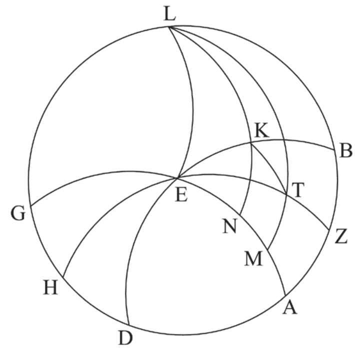
It is clear, therefore, that if the ascensions of one quarter are known, the ascensions of all will be known, because if the ascensions from the beginning of Aries to the beginning of Cancer are known, the ascensions from the beginning of Capricorn to the beginning of Aries will be known because the ascensions are equal; and the ascensions from the beginning of Cancer to the beginning of Libra or from the beginning of Libra to the beginning of Capricorn will be known because when we subtract these known ascensions of as many degrees as you please in the declined sphere from the doubled ascensions of these same parts in the right sphere known before, the sought ascensions of the taken parts remain. And this what we wanted.

16. To find the ascension of any part of the ecliptic in the declined sphere. Rule of operation: if you lead the sine of the pole's altitude into the sine of the declination of the part beginning from the point of the equinox, you divide the product by the sine of the complement of the declination, you lead what results into the radius in the same way, and you divide the product by the sine of the complement of the altitude, the sine of the difference between the taken part's elevations in the right sphere and in the declined sphere will result.

With the above figure [i.e. the first of the two diagrams of II.15] taken again, we seek arc EL, which is the difference between elevations in the right and the declined sphere pertaining to the arc of the zodiac TH. Then see that in this figure the two arcs AK and AE descend from a common endpoint A between alii KL et EB se invicem secant ad punctum H. Per kata ergo disiunctam cum hec quinque sint nota, KB altitudo poli primum, et BA secundum perfectio altitudinis, et KH tertium perfectio declinationis, et HL quartum declinatio sumpte partis, et EA sextum quarta equinoctialis, erit quintum EL notum. Quod si dempseris a TL noto quia est elevatio in spera recta, relinquitur ET notum, quod est elevatio quesita arcus HT in spera declivi.

Est alia via et faciliori idem deprehendere.

17. Differentiam ascensionum in spera recta et spera declivi eiusdem portionis per arcum circuli magni a polo venientis determinare.



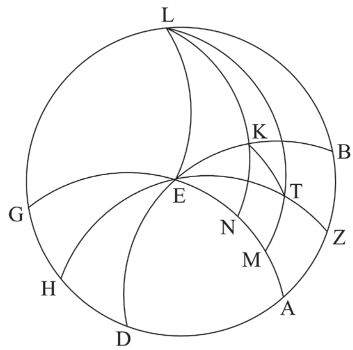
Ponam circulum meridianum ABGD et medietatem orizontis BED sed et equinoctialem AEG et medietatem circuli signorum HEZ. Et sit E punctum vernale communis sectio trium circulorum in situ, et nota L polus. Sumam ergo portionem a puncto vernali E iam exortam quantam voluero et sit ET, et describam quartam magni orbis LTM. Palam ergo quod portio ET oritur in spera recta cum arcu equinoctialis EM. Determinabo per quartam magni circuli cum quo arcu oritur in spera declivi. Describo ergo a puncto T arcum circuli equidistantis circulo equinoctiali donec secet arcum orizontis ad punctum K et sit TK, et super polum et punctum K quartam magni orbis LKN. Dico quod cum arcu MN oritur portio ET in spera declivi. Etenim oritur cum arcu equidistantis TK simili arcui MN, at cum eadem portione oriuntur similes equidistantium arcus in omni loco et omni tempore. Est ergo EN differentia ascensionum determinata per quartam magni circuli LKN transeuntem semper per commune punctum orizontis et equidistantis cuius distantia ab equinoctiali est ut declinatio portionis sumpte. Unde et arcus KN equalis est arcui TM.

18. Cuiuslibet portionis elevationem in spera obliqua alia via rationis invenire. Unde manifestum erit quod si sinus differentie equalis diei ad minimum ducatur in sinum elevationis sumpte portionis in spera recta et quod exierit

317 KL] *corr. ex* RL *K* **317/318** cum hec] hic *M* **318** sint nota] sint *P* sint nota scilicet *M* nota sint *N* KB] *corr. ex* RB *K* **319** declinationis] *corr. ex* altitudinis P_7 **320** erit] et *KM* EL] scilicet erit *adnot. s.l. K* erit *add. M* **321** a TL] ATL *PK* quia] quod *MN* **323** Est] est et *N* faciliori] facilior *KM* **325** declivi] obliqua *N* **332** in situ] insimul *KM* **335** ET[1]] *iter. et del. N* describam] describam super L polum et super T P_7 **338** arcum[1]] *marg. P* arcum[2]] circulum *N* **340** LKN] *corr. ex* LRN *K* Etenim] et EM *corr. ex* et E†N† *M* **341** equidistantis] equidistante *N* **342** et] et in *M* **343** ascensionum] ascensionis *M* **344** semper] *om. KM* et] *s.l. P* **345** KN] *corr. ex* RN *K* **346** TM] TM et cetera *M* which two others KL and EB intersect at point H. Through the disjunct kata, therefore, because these five are known: KB, the pole's altitude, the first; and BA, the second, the complement of the altitude; and KH, the third, the complement of the declination; and HL, the fourth, the declination of the taken part; and EA, the sixth, a quarter of the equator; ⟨then⟩ the fifth, EL, will be known. If we subtract this from TL known because it is the elevation in the right sphere, ET remains known, which is the sought elevation of arc HT in the declined sphere.

There is another, easier way to discover the same.

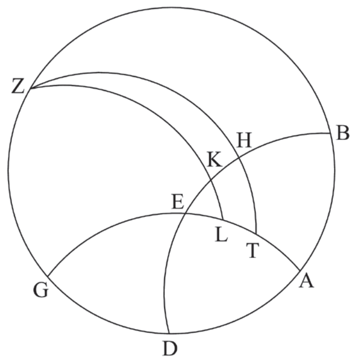
17. To determine the difference between the ascensions in the right sphere and the declined sphere of the same part through the arc of the great circle coming from the pole.



I will place meridian ABGD, half of the horizon BED, the equator AEG, and half of the ecliptic HEZ. And let E be the vernal point, the intersection of the three circles in position, and point L the pole. I will take, therefore, a part as large as I wish from the vernal point E already risen, and let it be ET, and I will describe the quarter of a great circle LTM. It is clear, therefore, that part ET rises in the right sphere with equinoctial arc EM. Through the quarter of a great circle, I will determine with what arc it [i.e. arc ET] rises in the declined sphere. Therefore, I describe from point T an arc of a circle parallel to the equator until it cuts the arc of the horizon at point K, and let it be TK, and upon the pole and point K, ⟨I describe⟩ a quarter of a great circle LKN. I say that part ET rises with arc MN in the declined sphere. For it rises with the parallel's arc TK similar to arc MN, but similar arcs of parallels rise with the same part in every place and every time. Therefore, EN, the difference between the ascensions, is determined by the quarter of a great circle LKN always passing through the intersection of the horizon and the parallel whose difference from the equator is as the declination of the taken part. Whence also arc KN is equal to arc TM.

18. To find any part's elevation in the oblique sphere by another way of reasoning. Whence it will be manifest that if the sine of the difference between the equal day and the shortest be led into the sine of the elevation of the taken

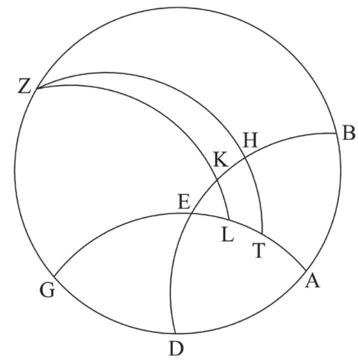
dividatur per sinum quadrantis, exibit sinus quesite differentie.



Reponam igitur scema circuli meridiani et dimidii orizontis et dimidii equinoctialis et poli meridiani qui sit Z. Et sit E punctum vernale, et sit ZHT determinans differentiam elevationum totius quarte ab initio Capricorni ad finem Piscium transiens per punctum commune orizontis et equidistantis tropici H. Est ergo ET tota differentia, et palam quod idem arcus ET est differentia dimidia diei equalis ad minimum. Sit iterum quarta magni circuli ZKL determinans differentiam elevationum portionis minoris quamcumque voluero, et sit Piscium, transiens per punctum K commune orizontis et illius equidistantis cuius distantia ab equinoctiali ut declinatio principii Piscium vel alterius portionis sumpte. Est ergo arcus EL differentia. Vides itaque arcus duorum magnorum orbium ET et TZ a communi puncto T venientium, inter quos alii duo EH et ZL se invicem secant super punctum K. Ergo per kata disiunctam proportionem ZH ad HT componunt proportio ZK ad KL et proportio EL ad ET – de sinibus loquor. Sed eandem proportionem componunt ut per ultimam prioris libri constat proportio ZK ad KL et proportio sinus totius quarte ad sinum elevationum sumpte portionis scilicet Piscis in spera recta. Ergo proportio sinus TE totalis differentie ad sinum differentie EL equalis est proportioni semidiametri ad sinum ascensionis Piscium in spera recta. Ex quatuor ergo proportionalibus tria sunt nota, primum propter arcum minimi diei notum esse, et tertium quia semidiameter est, et quartum propter ascensiones omnes in spera recta notas esse.

Collectis ergo de gradu in gradum huiusmodi differentiis usque ad completionem unius quarte, subtrahantur gradatim ab ascensionibus quarte in spera recta illius que est ab initio Arietis ad principium Cancri vel illius que est a capite Capricorni ad caput Arietis. Addantur vero ascensiones in spera recta

350 Reponam] deponam *K* **352** E] T *N* **353** determinans differentiam] *corr. in* differentiam determinans *P* differentiam determinans *N* **357** H] per punctum H *MN* **358** palam] palam est *MN* **359** diei equalis] equalis diei P_7 **360** differentiam] *corr. ex* differentias P_7 differentias *N* **362** distantia] declinatio *N* **363** duorum] duos P_7 **364** T] *s.l. P* **366** proportio[1]] *om.* P_7 **366/367** ad ET] *s.l.* P_7 **368/369** totius – recta] elevationum sumpte partis scilicet Piscis in spera recta ad sinum totius quarte P_7 (*the text is confirmed by Ba, but* E_1 *has text as in* P_7) **369** elevationum] elevationis *M* Piscis] Piscium *N* **371** Piscium] *om.* P_7 Piscis *K* proportionalibus] *corr. ex* proportionibus P_7 **372** sunt] *om.* P_7 **373** est] *om. MN* **375** ergo] vero *K* gradu] *corr. ex* gradus *P* gradum] gradus *P* huiusmodi] huius *M* **378** vero] ergo *M* ascensiones] ad ascensiones P_7



part in the right sphere and what results be divided by the sine of a quadrant, the sine of the sought difference will result.

Accordingly, I will place again the figure of the meridian, half the horizon, half the equator, and the south pole, which let be Z. And let E be the vernal point, and let there be ZHT determining the difference of the elevations of the whole quarter from the beginning of Capricorn to the end of Pisces, passing through H, the intersection of the horizon and the parallel of the tropic. ET, therefore, is the whole difference ⟨between right and oblique ascensions⟩, and it is clear that the same arc ET is half the difference between an equal day and the shortest. Again let there be the quarter of a great circle ZKL determining the difference between the elevations of whatever smaller part I will have wished, and let it be Pisces, passing through K, the intersection of the horizon and of that parallel whose distance from the equator is as the declination of the beginning of Pisces or of another taken part. And, therefore, arc EL is the difference. Accordingly, you see the arcs ET and TZ of two great circles coming from the intersection T, between which two others EH and ZL intersect at point K. Through the disjunct kata, therefore, the ratio of ZK to KL and the ratio of EL to ET compose the ratio of ZH to HT–I speak of the sines. But the ratio of ZK to KL and the ratio of the sine of the whole quarter to the sine of the elevation in the right sphere of the taken part, i.e. Pisces,[10] compose the same ratio [i.e. of the sine of ZH to the sine of HT], as is evident from the last ⟨proposition⟩ of the prior book. Therefore, the ratio of the sine of the whole difference TE to the sine of the difference EL is equal to the ratio of the radius to the sine of Pisces' ascension in the right sphere. Therefore, three of the four proportionals are known: the first because the arc of the shortest day is known, the third because it is the radius, and the fourth because all ascensions in the right sphere are known.

Then, with the differences of this kind obtained degree by degree to the completion of one quarter, let them be subtracted degree by degree from the ascensions in the right sphere of that quarter which is from the beginning of Aries to the beginning of Cancer or of that which is from the beginning of Capricorn to the beginning of Aries. And indeed let there be added the ascensions in the right sphere of that quarter which is from the beginning of Cancer

[10] This ratio should be inverted. The mistake occurs in all of the principle witnesses besides P_7 and E_1, so it seems to be the author's mistake.

illius quarte que est ab initio Cancri ad caput Libre vel illius que est a capite Libre ad principium Capricorni. Et sic invenientur omnes elevationes partium circuli declivis in spera obliqua, quod erat propositum.

19. Per notas ascensiones et locum Solis notum, quantitatem arcus diei et quantitatem arcus noctis et numerum equalium horarum diei vel noctis et tempora inequalium ascendensque et medium celi in omni hora reperire.

Quia enim magni circuli sunt circulus signorum et orizon, necessario semper per equalia se secant. Unde necessario ab ortu Solis ad occasum vi signa feruntur super terram, et ab occasu ad ortum vi signa sub terra. Quare in spera cuius diem querimus ascensiones medietatis zodiaci late super terram illa die sunt quantitas arcus diurni, quam cum minuimus a toto circulo, remanet quantitas arcus noctis eo quod in nocte et die completur una revolutio. Cum ergo acceperimus ascensiones a loco Solis in oppositum, fit quantitas diei; et cum acceperimus ab opposito Solis ad partem Solis, fit quantitas noctis.

Et quia equalis hora est ascensio xv graduum equalium idest equinoctialium, si quantitatem arcus diurni notam diviseris per xv vel nocturni similiter, exibit numerus equalium horarum diei vel noctis quam quesieris. Et si numerum equalium horarum diei dempseris de xxiiii, remanet numerus horarum noctis vel e converso quia dies cum nocte xxiiii horas equales continet propter revolutionem ccclx graduum.

Et quia inequalis hora duodecima pars diei dicitur quantacumque dies sit, tempus vero hore ascensio gradus equalis, palam quod si arcum diei in xii diviserimus, exibunt tempora que sunt quantitas hore inequalis diei, et de horis noctis similiter. Aut si volueris, considera secundum ascensiones quid intersit inter arcum diei in spera obliqua et arcum eiusdem diei in spera recta, et dimidie differentie sextam vel totius duodecimam accipe. Et si locus Solis septentrionalis fuerit, ad xv adde; et si meridionalis, de xv deme. Et fient tempora hore inequalis. Ratio ex premissis patens est. Et si quantitatem hore diurne de

381 quarte] *om. K* **383** propositum] propositum et cetera *M* **384** notas] ergo *add. et del. P* arcus] *corr. ex* ergo *P* **386** et] ad *M* **387/388** semper – se] se semper per equalia *N* **388** ad] usque ad *MN* **389** ad] usque ad *N* signa] *marg. P* **390** cuius] *corr. ex* circa P_7 late] latere *M* **391** sunt quantitas] quantitas sunt *N* quam cum] quem cum *M* quem si *N* minuimus] minuerimus P_7M *in other hand where original scribe left blank space K* (minuerimus *Ba* minuimus E_1) **392** ergo] *om. K* **394** opposito] appositio *P* ad – Solis[2]] *s.l. K* **395** hora est] est hora *P* equalium – equinoctialium] idest equinoctialium *P corr. ex* equalium idest equinoctium *K* equinoctialis *N* **396** diurni] diei P_7 nocturni] nocturnum *N* **397** equalium horarum] horarum equalium *N* **397/398** vel – diei] *s.l. K* **397** noctis] notis *K* quesieris] quesiveris *N* **398** diei] *marg. P* **399** continet] continent *N* **401** quia] *s.l. P* inequalis] equalis *K* **402** diviserimus] diviseris *N* **403** quantitas] tempora *N* **404** secundum ascensiones] ascensionum *M* **405** et[2]] *om. N* dimidie] *marg. P* **407** fient] fiunt *N* **408** Ratio] tunc *PN* ideo *M* (ratio BaE_1) **408/409** hore[2] – quantitas] *marg. P* to the beginning of Libra or of that which is from the beginning of Libra to the beginning of Capricorn. And in such a way there will be found all the elevations of the parts of the ecliptic in the oblique sphere, which had been proposed.

19. Through the known ascensions and the sun's known place, to find the quantity of the day's arc, the quantity of the night's arc, the number of equal hours of the day or night, the times of unequal ⟨hours⟩, and the ascendant and the middle heaven in every hour.

For, because the ecliptic and the horizon are great circles, they necessarily always cut each other in half. Whence from the sun's rising to its setting, necessarily six signs are being carried over the earth, and from setting to rising, six signs under the earth. Therefore, in the sphere whose day we seek, the ascensions of the half of the zodiac carried over the earth on that day are the quantity of the diurnal arc, which when we subtract from a whole circle, there remains the quantity of the arc of the night because one revolution is completed in a night and day. Therefore, when we take the ascensions from the sun's place to the opposite point, there is the quantity of the day; and when we take ⟨the ascensions⟩ from the point opposite the sun to the sun's degree, there is the night's quantity.

And because an equal hour is the ascension of 15°, i.e. of the equator, if you divide the known quantity of the diurnal arc by 15 or similarly of the night, the number of the equal hours of the day or night that you sought will result. And if you subtract the number of the day's equal hours from 24, the number of the night's hours remains or conversely because the day with the night contains 24 equal hours because of the revolution of 360°.

And because an unequal hour means the twelfth part of the day however long the day is, and indeed the time of an hour is an ascension of an equal degree,[11] it is clear that if we divide the day's arc into 12, there will result times that are the quantity of the day's unequal hour, and similarly about the night's hours. Or if you want, consider according to ascensions what lies between the day's arc in the oblique sphere and the arc of that same day in the right sphere, and take the sixth of half the difference or a twelfth of the whole ⟨difference⟩. And if the sun's place is north, add to 15; and if it is south, subtract from 15. And there will be the times of the unequal hour. The proof is clear from what has been set forth. And if you subtract the quantity of the diurnal hour from

[11] The meaning of this clause is obscure. Perhaps the best understanding of it is that the hours will be determined by equal arcs of the equator (the 'ascension' here).

xxx dempseris, remanebit quantitas hore nocturne. Hora enim diurna et hora nocturna semper complent xxx gradus propter revolutionem ccclx graduum in die et nocte.

Quod si volueris partem ascendentem in hora data, accipe horas ab ortu Solis in die vel ab occasu Solis in nocte et in suos gradus per multiplicationem redige, et exibit arcus equinoctialis circuli qui ab ortu vel occasu Solis sursum emersit. Vide ergo quanta portio zodiaci a loco Solis inchoata secundum successionem signorum cum hoc arcu exorta sit, et pars ad quam calculando perveneris ipsa est pars oriens. Et si volueris partem medii celi, sume horas a proximo meridie ad horam datam preteritas, et eas in suos gradus redige. Et fiet arcus equinoctialis qui a proximo meridie meridianum transiit. Quere ergo in spera recta cuius portionis a loco Solis sit illa elevatio, et pars ad quam numerando perveneris est pars medii celi. Pars vero opposita orienti est occidens, et que opponitur medio celi super terram est pars medii celi sub terra.

Aut si velis per partem ascendentem scire partem medii celi sub terra, quere ascensiones in spera declivi portionis ab initio Arietis usque ad partem orientem, et habebis gradum equinoctialis circuli qui cum parte ascendente venit ad ortum. Et quia semper ab orizonte ad medium celi est quarta equinoctialis circuli, deme ab illis ascensionibus lxxxx si fieri potest. Si minus, adde super id quod inveneris ccclx idest revolutionem unam, et ex toto subtrahe xc. Et relinquitur arcus equinoctialis qui ab initio Arietis meridianum sub terra transiit in ortu dato. Quere ergo in spera recta cuius portionis sit illa elevatio, et invenies partem mediantem celum sub terra. Et vice versa si per medium celi super terram cognitum scire velis partem orientem, ab elevationibus in spera recta aufer xc. Et quere in spera declivi cuius portionis residuum sit elevatio. Ecce ad quid utile est ascensiones circuli declivis noscere.

20. Datas horas temporales ad equales vertere et datas equales ad inequales reducere.

409/410 diurna – nocturna] nocturna et hora diurna *PN* **410** complent] continent *N* **413** in[1] – Solis[2]] *s.l. K* in[1]] si in *M* in[2]] si in *M* per] per suam *N* multiplicationem] multiplicem *P* **414** arcus – qui] portio equinoctialis circuli que *N* **414/415** sursum – Solis] *om. M* **414** sursum] *corr. ex* cursum *K* **415** emersit] emerserit P_7 Vide] unde *P* quanta] quota P_7 portio] proportio *P* secundum] vel *P* **416** arcu] arcu equinoctialis *N* et] *s.l. K* **417** proximo] *corr. ex* primo *K* **418** fiet] fiat *K* **419** ergo] *marg. P* **420/421** numerando perveneris] perveneris numerando *N* **423** partem medii] medium *N* medii] *iter. et del.* P_7 sub terra] *om.* P_7 *corr. in* supra terram *N* (sub terra BaE_1) **425** gradum] gradus *PN corr. ex* graduum *K* (gradum BaE_1) **426** venit] veniunt *N* orizonte] *corr. ex* oriente *M* oriente *N* (oriente *Ba* orizonte E_1) quarta] quarta pars *M* **426/427** equinoctialis circuli] circuli equinoctialis *N* **427** minus] *corr. in* non *N* **428** id] illud *M* **429** sub terra] *om.* P_7 supra terram *N* **430** cuius] *corr. ex* cuiusdam *N* **431** sub terra] *om.* P_7 *corr. ex* supra terram *N* **434** noscere] scire *N*

30, the quantity of the nocturnal hour will remain. For a diurnal hour and a nocturnal hour always complete 30° because of the revolution of 360° in a day and night.

But if you want the ascending degree in a given hour, take the hours from the sun's rising in the day or from the sun's setting in the night and convert them into their degrees through multiplication, and there will result the arc of the equator that has risen up from the sun's rising or setting. Therefore, see how great a part of the zodiac beginning from the sun's place according to the succession of signs has risen with this arc, and that degree which you reach by calculating is the rising degree. And if you want the degree of the middle heaven, take the hours gone by from the nearest noon to the given hour, and convert them into their degrees. And there will be made the arc of the equator that has crossed the meridian since the last noon. Then seek of what part from the sun's place that elevation may be in the right sphere, and the degree that you reach by computing is the part of the middle heaven. And indeed the degree opposite the rising is the setting, and what is opposite the middle heaven above the earth is the degree of the middle heaven under the earth.

Or if you want to know the degree of the middle heaven under the earth[12] through the ascending degree, seek the ascensions in the declined sphere of the part from the beginning of Aries to the rising degree, and you will have the degree of the equator that comes to the rising with the ascending part. And because from the horizon to the middle heaven is always a quarter of the equator, subtract 90 from these ascensions if it can be done. If ⟨the ascension is⟩ less ⟨than 90⟩, add 360, i.e. one revolution, upon that which you found, and subtract 90 from the whole. And there remains the arc of the equator from the beginning of Aries that has passed the meridian under the earth[13] in the given rising. Then see of what part this is the elevation in the right sphere, and you will find the degree halving the heavens under the earth.[14] And vice versa if you want to know the rising degree through the known middle heaven above the earth, subtract[15] 90 from the elevations in the right sphere. And seek of what part the remainder is the elevation in the declined sphere. See how useful it is to know the ascensions of the declined circle.

20. To turn given temporal hours into equal ones and to restore given equal ⟨hours⟩ to unequal ones.

[12] This should be 'above the earth', but the mistake appears to be original. The scribes of P_7 and *N* realized that there was a mistake.

[13] Again, this should be 'above the earth.'

[14] Again, this should be 'above the earth.'

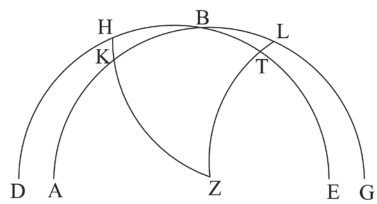
[15] This should say 'add.'

Datas nempe horas temporales multiplicando gradus effice, et ex gradibus dividendo in xv horas equales quotquot poteris restitue. Item datas equales in suos gradus ducito, et per tempora hore inequalis dividendo ad inequales reducito. Ratio in ianuis excubat.

21. Proportio speralis anguli supra polum alicuius circuli consistentis ad iiii rectos est sicut arcus eiusdem circuli qui ei subtenditur ad totam circumferentiam.

Hoc ex equisubmultiplicibus primi et tertii et item secundi et quarti sicut in sexto Euclidis de angulis planis facile comprobatur.

22. Omnes duo anguli ex duobus meridianis cum circulo signorum ad eandem distantiam a puncto equinoctiali provenientes quorum alter extrinsecus alter intrinsecus ex eadem parte sibi oppositus sunt equales.



Ponam ergo arcum equinoctialis circuli ABG et arcum circuli signorum DBE, et punctum B equinoctiale a quo duo arcus equales BH et BT. Et describam duos arcus meridianos super polum Z, qui sint ZKH et ZTL. Dico quod angulus ZHB equalis est angulo ZTE. Triangulus enim KHB equilaterus est triangulo TLB tum propter ypothesim, tum propter eandem declinationem, tum propter equales ascensiones. Ergo angulus KHB equalis est angulo LTB, qui equatur angulo ZTE quia sunt anguli contra se positi.

23. Omnes duo anguli ex duobus meridianis cum circulo signorum ad eandem distantiam a puncto tropico provenientes quorum alter extrinsecus alter vero intrinsecus ex eadem parte sibi oppositus equantur duobus rectis.

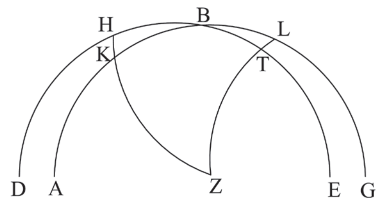
437 nempe] nampe P_7 **438** datas] datas horas *MN* **439** tempora] tempus *M* inequalis] inequaliter *K* **440** ianuis] angulis *KM* excubat] excubat et cetera *M* **441** 21] capitulum de scientia speralium angulorum *add.* P_7 speralis anguli] anguli speralis *corr. ex* talis anguli *K* anguli speralis *M* polum] polos *MN* consistentis] consistens P_7 **442** circuli] ad totam *add. et del. P* ei] *om.* P_7 ad totam] *s.l. P* **444** equisubmultiplicibus] *corr. in* equimultiplicibus *N* (equimultiplicibus *Ba* equisubmultiplicibus E_1) **445** sexto Euclidis] sexto Euclidis propositione ultima *M* sexti Euclidis ultima propositione *N* planis] planius *K* comprobatur] comprobatur et cetera *M* comprobabitur *N* **446** duo anguli] *corr. ex* anguli duo *P* **447** equinoctiali] equinoctialis *M* **448** alter intrinsecus] *marg.* P_7 oppositus] oppositi *MN* **451** equinoctiale] equinoctialem *M* **452** equales] *om.* P_7 BH – BT] *corr. in* BH et BL *but then corr. in* BK BT (*other hand*) *M* BT] HT PP_7N (HT *Ba* BT E_1) **453** sint] sunt *N* **454** ZKH] *corr. ex* et KH *K* et] *s.l. P* **455** ZTE] *corr. ex* ZHT *N* **456** equilaterus] equilaterum *K* **457** eandem declinationem] equales declinationes *N* **458** equalis est] est equalis *PN* **462** oppositus] oppositi *KM corr. in* oppositi *N* (oppositus BaE_1)

Truly, bring about degrees by multiplying the given temporal hours, and bring back however many equal hours you are able to from degrees by dividing by 15. Again, lead given equals into their degrees, and return them into unequals by dividing by the time of an unequal hour. The proof is evident [*lit.*, lies out in the doorway].

21. The ratio of a spherical angle standing upon the pole of any circle to four right angles is as the arc of the same circle that subtends it to the whole circumference.

This is proved easily from equisubmultiples[16] of the first and third and likewise of the second and fourth as in the sixth ⟨book⟩ of Euclid about plane angles.[17]

22. Any two angles resulting from two meridians with the ecliptic at the same distance from the equinoctial point, of which one is extrinsic, the other opposite to it intrinsic from the same side, are equal.



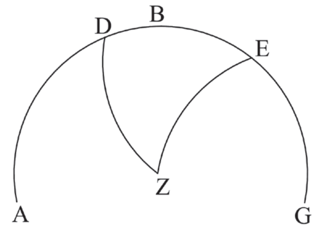
Then I will place an arc of the equator ABG, an arc of the ecliptic DBE, and an equinoctial point B, from which ⟨I place⟩ two equal arcs BH and BT.[18] And let me describe two arcs of the meridian upon pole Z, which let be ZKH and ZTL. I say that angle ZHB is equal to angle ZTE. For triangle KHB is of equal sides with triangle TLB because of hypothesis, because of the same declination, and because of equal ascensions. Therefore, angle KHB is equal to angle LTB, which is equal to angle ZTE because they are angles placed against each other [i.e. they are vertical angles].

23. Any two angles resulting from two meridians with the ecliptic at the same distance from a tropic point, of which one is extrinsic and indeed the other opposite it intrinsic from the same side, are equal to two rights.

[16] This should read 'equimultiples.'

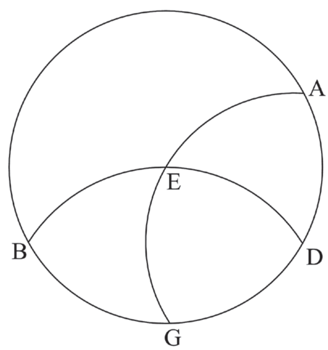
[17] *Elements* VI.33.

[18] Perhaps the author mistakenly had 'HT' here as many of the manuscripts do.



Sit iterum orbis signorum arcus supra quem ABG ex quo duo arcus equales a puncto tropico B DB et EB. Et sint duo arcus meridiani supra polum Z ZD et ZE. Dico quod angulus ZDB equus est angulo ZEG. Quoniam duo latera trianguli ZDE propter eandem declinationem sunt equalia, quare anguli ad basim DE sunt equales, quorum unus scilicet ZED cum angulo ZEG equatur duobus rectis.

24. Angulus ex circulo meridiano cum circulo signorum aput punctum tropicum proveniens rectus esse necessario comprobatur.



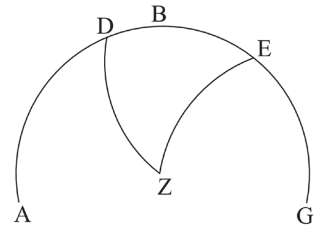
Sit denuo circulus meridianus ABGD et medietas circuli signorum AEG. Et sit punctum A tropicum hiemale et describam super polum A secundum spatium lateris quadrati medietatem circuli BED. Quia ergo circulus meridianus ABGD est descriptus super utriusque circuli AEG BED polos, erit arcus ED quarta circuli. Quare angulus DAE est rectus. Et propter idem est angulus qui aput tropicum estivum rectus, et hoc est quod oportuit demonstrari.

25. Maxima declinatione nota angulum ex meridiano et circulo signorum aput punctum equinoctii provenientem notum esse oportet. Unde patet quod si maximam declinationem addas super quartam vel ab ea subtrahas, exibit angulus quesitus.

Sit ergo ut solet circulus meridianus ABGD et infra eum medietas circuli equinoctialis AEG et medietas circuli signorum AZG. Et sit A punctum autumpnale, et describam supra polum A secundum spatium lateris quadrati semicirculum BZED. Propter hoc ergo quod circulus ABGD est descriptus

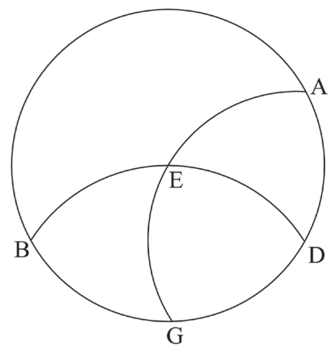
463 iterum] initium *M* arcus] *om.* P_7 **464** quem] commune *P* quo] sint *add.* (*s.l.* *K*) *KM* **465** B] B scilicet *M* **466** Z] *om.* *M* ZD] ZH *P* **467** ZDB] ZDE *N* **468** ZEG] ZEB P_7*N* *perhaps corr. in* ZED *K corr. in* ZED *M* (ZEG *Ba*E_1) duo] et *M* **469** equalia] *corr. ex* equalis *N* **470** basim] basam *P* basem P_7 **471** ZED] *corr. ex* ZDE P_7*N* **472** Angulus] angulus qui *M* circulo signorum] signorum circulo *KM* **475/476** punctum A] a P *P* A punctum *N* **476** describam] *corr. ex* describantur *K* describamus *M* **477** A] G *N* **478** Quia] quod *P* **480** AEG] AEG et *M* BED] BET *P* polos] secundum Theodosinum de speris *add.* P_7 pro Theodosius de speris *adnot. s.l.* *K* erit] *corr. ex* et *K* **481** ED] EB *N* DAE] EAB *corr. ex* DEA *N* **482** qui] qui est P_7*M* **484** oportuit demonstrari] oppositum est demonstrati *P* *corr. ex* opositum demonstrari *K* propositum est demonstratur *M* propositum est demonstrari *N* (oportet demonstrare *Ba* oportuit demonstrari E_1) **489** circuli] *om.* *N*

Again let there be an arc of the ecliptic upon which are ABG, from which there are two equal arcs DB and EB from the tropic point B. And let there be two arcs ZD and ZE of the meridian upon pole Z. I say that angle ZDB is equal to angle ZEG.[19] Because two sides of triangle ZDE are equals because of the same declination, therefore the angles at base DE are equal, of which one, i.e. ZED, with angle ZEG is equal to two rights.



24. The angle resulting from the meridian with the ecliptic at the tropic point is confirmed necessarily to be right.

Let there be again the meridian ABGD and half of the ecliptic AEG. And let point A be the winter tropic, and let me describe half circle BED upon pole A according to the distance of a square's side. Then, because meridian ABGD is described upon the poles of both circles AEG and BED, arc ED will be a quarter circle. Therefore, angle DAE is right. And because of the same, the angle that is at the summer tropic is right, and this is what was necessary to be demonstrated.

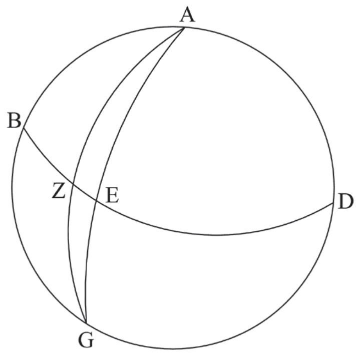


25. With the maximum declination known, it is necessary that the angle resulting from the meridian and the ecliptic at the equinox point is known. Whence it is clear that if you add the maximum declination to a quarter ⟨circle⟩ or subtract from it, the sought angle will result.

Then, as is the usual practice, let there be meridian ABGD and below it half of the equator AEG and half of the ecliptic AZG. And let A be the autumnal point, and let me describe semicircle BZED upon pole A according to the distance of a square's side. Then, because of this that circle ABGD is

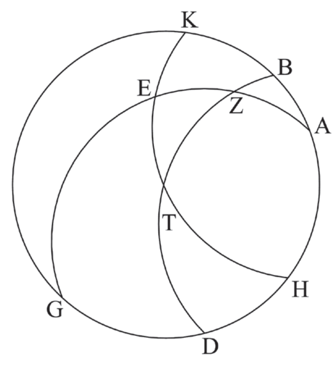
[19] This should read 'ZED' or 'ZEB', but the manuscript evidence points to the mistake being original.

super polos orbium AEG BED, erit uterque istorum arcuum AZ ED quarta circuli. Est ergo ZE maxima declinatio et est nota; ergo totus arcus ZD notus. Quare angulus DAZ notus respectu iiii rectorum. Reliquus ergo BAZ notus, quod oportuit demonstrari. Posito ergo quod maxima declinatio sit xxiii partes et li minuta, erit angulus BAZ lxvi partium et ix minutorum sicut in Almagesti constitutum est.

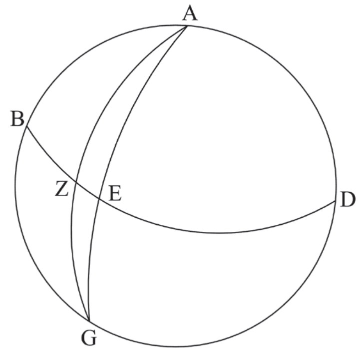


26. Quantitatem cuiuslibet anguli ex meridiano cum circulo signorum aput quodlibet punctum provenientis per notam puncti declinationem invenire. Unde liquet quod si declinationis puncti cuius angulus queritur sinum ducas in sinum perfectionis sumpte portionis a puncto equinoctiali, et productum dividas per sinum ipsius portionis, et productum iterum multiplices in semidiametrum, atque quod exierit dividas per sinum perfectionis declinationis, exibit sinus differentie duorum angulorum aput punctum propositum valentium duos rectos, quam si recto addideris vel subtraxeris, habebis utrumque.

Rationis causa, sit circulus meridianus ABGD et medietas equinoctialis AEG et medietas circuli signorum BZD. Et sit Z punctum autumpnale et arcus BZ pro libito sit signum Virginis. Et describam super polum secundum spatium lateris quadrati semicirculum HTEK. Quero ergo quantitatem KBT. Quoniam autem circulus ABGD est descriptus super polos AEG et super polos HEK, erit quilibet istorum arcuum BH BT EH quarta circuli. Et propter hanc formam proportio BA ad HA per kata disiunctam ex geminis ducitur proportionibus, una BZ ad ZT et alia TE ad EH – de sinibus intelligo. Sed quinque nota



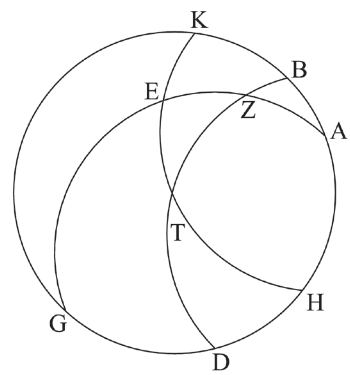
494 AZ] *corr. in* AE *K* AE *M* (AZ E_1) ED] propter hoc ergo quod circulus *add. et del.* P_7 AE *N* **495** et est] *om.* P_7 **497** rectorum] *corr. ex* angulorum *M* **499** xxiii] xxxiii *P* **500** minuta] minutum *M* **501** sicut] sic P_7 **502** constitutum] *corr. ex* constitum *N* est] est et cetera *M* **505/504** quodlibet] quemlibet *M* **510** propositum valentium] *corr. ex* valentium propositum *P* **511** quam] *corr. in* quem *M* **512** sit] *om. PN* fit *K* (sit BaE_1) medietas] *corr. ex* me†ridiei† *K* **514** libito] libita *P* libitu *N* **515** polum] B super *add. s.l.* P_7 B *add.* (*s.l. K*) *KN* A scilicet *add. M* (polum B *Ba* polum E_1) **516** HTEK] HETK *N* **517** KBT] KBT anguli *N* **521** ad] *om. P s.l. N* **522** ducitur] producitur *N* proportionibus] portionibus P_7 **524** sinibus] in *add. et del. K* intelligo] intego *corr. in* tego *N*



described upon the poles of circles AEG and BED, each of those arcs AZ and ED will be a quarter circle. Therefore, ZE is the maximum declination and it is known; therefore, whole arc ZD is known. Therefore, angle DAZ is known with respect to four right angles. Remainder BAZ, therefore, is known, which was necessary to be demonstrated. Therefore, given that the maximum declination is 23° 51′, angle BAZ will be 66° 9′, as was established in the *Almagest*.

26. To find the quantity of any angle resulting from the meridian with the ecliptic at any point through the known declination of the point. Whence it is certain that if you lead the sine of the declination of the point whose angle is sought into the sine of the complement of the part taken from the equinox point, you divide the product by that part's sine, again you multiply the product by the radius, and you divide what results by the sine of the complement of the declination, there will result the sine of the difference[20] between the two angles at the proposed point equaling two right angles. If you add to or subtract that ⟨difference⟩ from a right angle, you will have both ⟨of the angles at the point⟩.

For the sake of a proof, let there be meridian ABGD, half of the equator AEG, and half of the ecliptic BZD. And let Z be the autumnal point and let arc BZ be, as you wish, the sign of Virgo. And let me describe semicircle HTEK upon the pole ⟨B⟩ according to the distance of a square's side. I seek then the quantity of KBT. Because, moreover, circle ABGD is described upon the poles of AEG and upon the poles of HEK, each of those arcs BH, BT, and EH will be quarter circles. And because of this figure, through the disjunct kata, the ratio of BA to HA is led from twofold ratios, one of BZ to ZT and the other of TE to EH – I understand about sines. But five are known: BA because it is the declination of

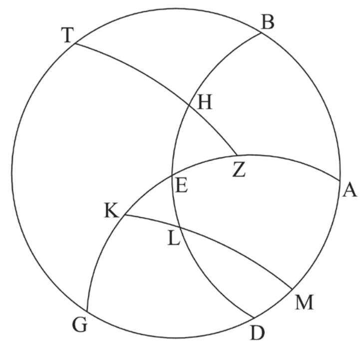


[20] This is not the difference between the two angles, but the difference between each of the angles and 90°.

sunt, BA propter declinationem principii Virginis, et AH propter perfectionem quarte, et BZ propter signum Virginis, et ZT quia est perfectio quarte, et EH quarta; relinquitur ergo ET notum. Quare et totus TK arcus et angulus cui subtenditur KBT notus. Igitur secundum Tholomei inventam declinationem erit angulus qui aput caput Virginis cxi partes, et qui aput caput Scorpii similiter propter equalem distantiam a puncto equinoctiali, et qui aput caput Tauri vel Piscium cum a duobus rectis illam quantitatem dempseris partes lxix ex antepremissa.

Pari modo si ponas punctum B principium Leonis lineis manentibus secundum suam habitudinem, invenies angulum in capite Leonis cii partium et xxx minutorum, et eum qui in capite Sagittarii similiter. Et cum a duobus rectis illum dempseris, occurret angulus qui in capite Geminorum vel in capite Aquarii partes lxxvii et partis medietas. Ad hunc modum in singulis sectionibus angulos unius quarte et per eos angulos aliarum trium poteris comprehendere. Atque hec est notitia angulorum omnium in orizonte recto et signorum circulo provenientium.

27. Omnes duo anguli ex uno orizonte declivi cum circulo signorum ad eandem distantiam a puncto equinoctiali provenientes quorum unus intrinsecus alter vero extrinsecus ex eadem parte sibi oppositus sunt equales.

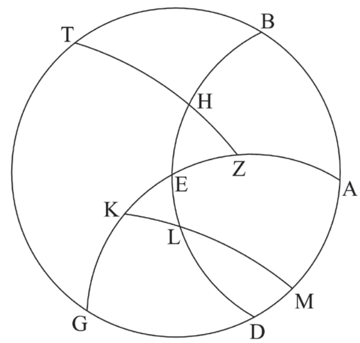


Propter hoc describo circulum meridianum ABGD et dimidium equatoris diei AEG et orizontis BED, et scribo duas portiones orbis signorum ZHT et KLM. Sitque utrumque Z K punctum autumpnale et arcus ZH equalis arcui KL. Dico quod angulus EHT equalis est angulo DLK. Latera namque trianguli EHZ sunt equalia lateribus trianguli EKL tum propter ypothesim, tum propter ascensiones equales, tum

525 Virginis] Virginum P_7 AH] *corr. ex* AB *P* **526** Virginis] et AH propter perfectionem quarte ABZ propter *add. et del. M* **527** angulus] *corr. ex* arcus P_7 **528** Tholomei] Ptolomei *K* Ptolomeum *corr. in* Ptolomei *M* Ptholomei *N* **529** Virginis] Virginis fit *N* Scorpii] Scorpionis *M* **530** equinoctiali] equinoctii P_7 **531** Piscium] Piscis et (*this last word s.l. K*) *KN* dempseris] depresseris P_7 remanebunt scilicet *adnot. s.l. K* remanebunt scilicet *add. M* partes] *om. N* **532** antepremissa] *corr. in* 23 P_7 **533** lineis manentibus] manentibus lineis P_7 **534** cii] 111 *MN* **536** occurret] occurrit *PMN* (occurret BaE_1) qui] est *add. s.l. K* **537** lxxvii] 70 *N* **539/540** signorum circulo] circulo signorum P_7 **541** uno] *om. P* **542** equinoctiali] equinoctii P_7 **543** alter vero] et alter *M* oppositus] oppositi *MN* **544** describo] scribo *K* meridianum] *om. N* **547** Sitque] sit *N* **548** Z] et *add. s.l. et del. K* Z et *M* **550** equalis est] est equalis P_7 DLK] ELK *K corr. ex* ELK *M* **551** trianguli] triangulorum *M* **552** trianguli] trianguli et *N* propter] *marg. P* the beginning of Virgo, AH because it is the complement, BZ because it is the sign of Virgo, ZT because it is the complement, and quarter circle EH; therefore, ET remains known. Therefore, also whole arc TK and the angle that subtends it, KBT, are known. Therefore, according to the found declination of Ptolemy, the angle that is at the beginning of Virgo will be 111°, and that which is at the beginning of Scorpio similarly ⟨is 111°⟩ because of the equal distance from the equinox point, and from what has been set forth before [i.e. II.23], when you subtract that quantity from two right angles, that ⟨angle⟩ which is at the beginning of Taurus or Pisces will be 69°.

In a like way, if you suppose point B to be the beginning of Leo with the lines remaining according to their disposition, you will find the angle at the beginning of Leo to be 102° 30′, and that which is at the beginning of Sagittarius similarly. And when you subtract that from two rights, the angle that is in the beginning of Gemini or in the beginning of Aquarius will present itself to be 77° 30′. In this way you will be able to grasp the angles of one quarter in the individual divisions, and through them the angles of the other three. And this is the knowledge of all the angles resulting from the right horizon and the ecliptic.

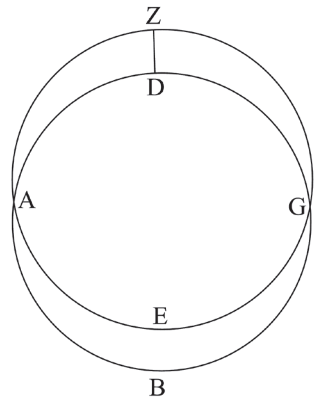
27. Any two angles resulting from one declined horizon with the ecliptic at the same distance from the equinox point, of which one is intrinsic, and indeed the other opposite it extrinsic from the same side, are equal.



For this I describe meridian ABGD, half of the equator AEG, and horizon BED, and I draw two parts of the ecliptic ZHT and KLM. And let both Z and K be the autumnal point, and let arc ZH be equal to arc KL. I say that angle EHT is equal to angle DLK. For the sides of triangle EHZ are equal to the sides of triangle EKL because of hypothesis, because of equal ascensions, and propter abscisiones orizontis equales. Ergo EHZ equalis est angulo ELK, quare angulus EHT residuus de duobus rectis equatur angulo DLK residuo.

28. Omnes duo anguli ex uno orizonte declivi cum circulo signorum aput puncta opposita orientis et occidentis extrinsecus cum intrinseco equantur duobus rectis. Unde colligitur quod duo quoque ad eandem distantiam a puncto tropico duobus rectis sunt equales. Quapropter notis angulis orientalibus unius medietatis ab Ariete in Libram, noti erunt anguli orientales alterius medietatis et una anguli occidentales in ambabus partibus.

Pono itaque circulum orizontis ABGD et circulum signorum AEGZ et puncta sectionum A G. Palam quod anguli ZAD et DAE equales sunt duobus rectis, angulus vero ZAD equatur angulo DGZ quia arcus maxime declinationis eorum circulorum DZ secat utriusque medietatem per equalia. Quapropter angulus DGZ et angulus DAE simul valent duos rectos. Et quia anguli ad eandem distantiam a puncto equinoctii sunt equales, accidit ut anguli quoque duo eiusdem a puncto tropico distantie – orientalis dico et occidentalis – duobus rectis sunt equales. Propter hoc ergo et premissam cognitis angulis orientalibus ab Ariete in Libram et orientales et occidentales in ambabus partibus erunt noti, et hoc est quod proponitur.

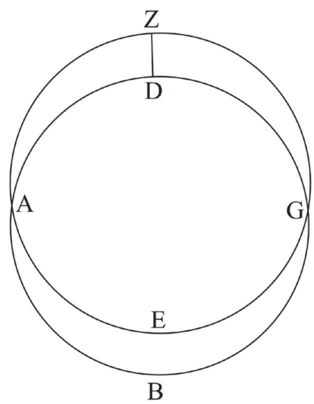


29. Nota poli altitudine et tropicorum distantia angulum ex concursu orizontis declivis et signorum circuli aput utrumque punctum equinoctii notum esse necesse est. Unde constat quod si differentiam que est inter regionis latitudinem et maximam declinationem cum latitudo maior fuerit a quarta circuli diminuas, vel cum minor fuerit adicias, relinquetur angulus sub capite Libre. A quo si quantitatem distantie inter duos tropicos abieceris, residuum erit angulus sub capite Arietis.

554 abscisiones] ascensiones *P* ascisiones *K* ascensiones *corr. in* portiones *M* portiones *N* (abscisiones *BaE_1*) orizontis] *corr. ex* orientis *P_7* EHZ] angulus EHZ *MN* ELK] *corr. ex* EKL *N* **555** residuo] residuo et cetera *M* **556** ex] in *PN* **557** occidentis] occidentis provenientes (*corr. ex* provenientis) *P_7* equantur] equatur *K* **561** una] pariter *MN* **563** signorum] *s.l.* *P_7* **565** ZAD] ZDA *P* **571** equinoctii] equinoctiali *N* **573** distantie] distante *P* dico] *om.* *MN* **578** orizontis] *corr. ex* orientis *P_7* **579** utrumque] *om.* *P_7* **580** esse] ei†us† *P* necesse est] oportet *N* **581** latitudo] altitudo *N* **582** adicias] additias *P* relinquetur] relinquitur *MN* because of equal parts cut off from the horizon. Therefore, EHZ is equal to angle ELK, therefore angle EHT, the remainder of two right angles, equals angle DLK, the remainder ⟨of two right angles⟩.

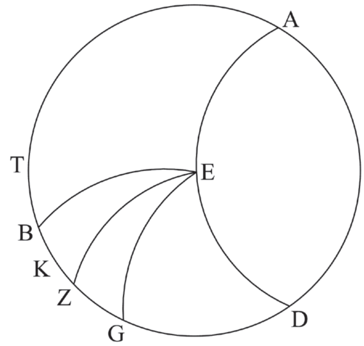
28. Any two angles from one declined horizon with the ecliptic at opposite points of the east and west, extrinsic with intrinsic, are equal to two right angles. Whence it is deduced that also the two at the same distance from a tropic point are equal to two rights. For this reason, with the eastern angles of one half from Aries to Libra known, the eastern angles of the other half and at the same time the western angles in both parts will be known.

Accordingly, I posit the horizon ABGD, the ecliptic AEGZ, and the intersections A and G. It is clear that angles ZAD and DAE are equal to two rights, and indeed angle ZAD is equal to angle DGZ because the arc of their circles' maximum declination DZ cuts the half of each in half. For this reason angle DGZ and angle DAE together equal two rights. And because angles at the same distance from an equinox point are equal, it occurs that also the two angles of the same distance from a tropic point – I mean the eastern and western – are equal to two rights.



Therefore, because of this and what has been set forth [i.e. II.27], with the eastern angles from Aries to Libra known, also the eastern and western in both parts will be known, and this is what is proposed.

29. With the pole's altitude and the distance of the tropics known, it is necessary that the angle from the meeting of the declined horizon and the ecliptic at each equinox point is known. Whence it is evident that if you subtract the difference that is between the region's latitude and the maximum declination from a quarter circle when the latitude is greater, or add when it is less, there will remain the angle at the beginning of Libra. If from this you subtract the quantity of the distance between the two tropics, the remainder will be the angle at the beginning of Aries.

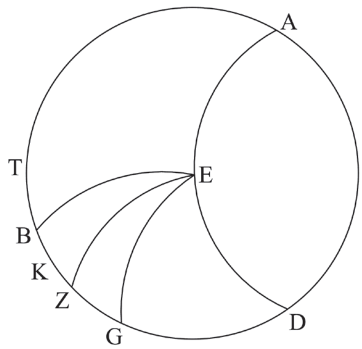


ABGD meridianus circulus infra quem orientalis medietas orizontis AED et quarta equatoris diei EZ et due quarte orbis signorum EB EG. Et sit punctum scilicet quod est quarte EB punctum autumnale, et quod est quarte EG punctum vernale, et punctum B tropicum hiemale sub terra, et punctum G tropicum estivum. Est ergo arcus GB tropicorum distantia notus, et eius medietas arcus BZ notus. Sitque latitudo regionis TZ maior sive KZ minor nota. Quare propter DT vel DK esse quartam circuli, erit uterque arcuum BD et GD notus. Et quia punctum E est polus meridiani, erit uterque angulus, scilicet BED qui est sub capite Libre et GED qui est sub capite Arietis, notus quia sunt cum dictis arcubus eiusdem quantitatis.

30. Quantitatem anguli ex concidentia orizontis et zodiaci aput quodlibet punctum per notum celi medium et eius declinationem notam investigare. Ratio. Si semidiametrum multiplices in sinum altitudinis gradus celi medii sub terra vel super terram, et productum dividas per sinum portionis que est inter orizontem et celi medium sub terra vel super terram prout contigerit eam portionem minorem esse quarta, exibit sinus et quesiti arcus et quesiti anguli.

Pingo circulum meridianum ABGD et infra eum medietatem orizontis orientalem BED et medietatem circuli signorum AEG. Et sit pro libito punctum E caput Tauri ad ortum venientis, et G celi medium sub terra, quod per ascensiones notas erit notum. Estque necessario secundum dictam positionem portio EG minor quarta. Describam autem super polum E secundum spatium lateris quadrati portionem orbis maioris ZHT. Et complebo duas quartas EGH EDT, et erit uterque duorum arcuum ZGD ZHT quarta circuli eo quod orizon BET est descriptus supra polum ZGD meridiani et supra polum ZHT orbis magni. Vides ergo a puncto T duos arcus TE et TZ magnorum orbium

585 ABGD] sit ABGD P_7*N* sit *add.* (*s.l. K*) *KM* (ABGD *Ba* sit ABG E_1) **587** diei] *om.* P_7 **588** punctum scilicet] punctum E *KN* E punctum *M* **591** B] *marg. P* **594** arcus BZ] BZ arcus P_7 **595/596** maior – minor] vel KZ (KT *M*) scilicet maior vel minor BZ (*in a later hand where originally a blank space was left K*) *KM* **596** nota] note *M* Quare propter] quare oportet *corr. ex* quare †DY† *K* quare oportet *M corr. ex* quapropter *N* erit] ergo *add.* (*s.l. K*) *KM* **597** E] *s.l. K* **598** capite Libre] Libre capite P_7*K* **599** notus] *om. PN s.l.* (*other hand*) *K* (*om. Ba* notus E_1) quantitatis] quantitatis et cetera *M* **601** investigare] vestigare *K* **602** Ratio] regula *MN* semidiametrum] diametrum *P* celi medii] medii celi *MN* **604** contigerit] contingit *MN* **605** quesiti^2] quesita P_7 **607** sit] sic *P* libito] libitu *N* punctum] punctus *KM* **608** et G] BG *M* per] propter *N* **609** erit notum] notum erit P_7 Estque] est quia *P* dictam] *corr. ex* differentiam *M* **610** portio] *om. N* autem] *om. N* **611** maioris] *corr. ex* maiori *K* **611/612** Et – ZHT] *marg.* P_7 **611** EGH] EGH et *M* **613** est] *om. M*

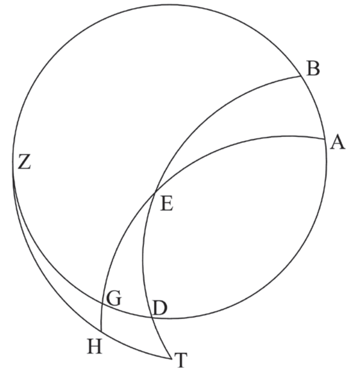


⟨Let there be⟩ meridian circle ABGD below which ⟨let there be⟩ the eastern half of the horizon AED, the quarter of the equator EZ, and two quarters of the ecliptic EB and EG. And let the point, i.e. that which is of quarter EB, be the autumnal point; that which is of quarter EG, the vernal point; point B, the winter tropic under the earth; and point G, the summer tropic. Therefore, arc GB, the distance between the tropics, is known, and its half arc BZ is known. And let the latitude of the region be known, TZ greater ⟨than the maximum declination⟩ or KZ smaller. Therefore, because DT or DK is a quarter circle, each of the arcs BD and GD will be known. And because point E is the pole of the meridian, each angle, i.e. BED, which is at the beginning of Libra, and GED, which is at the beginning of Aries, will be known[21] because they are of the same quantity with said arcs.

30. To find the quantity of the angle from the meeting of the horizon and the zodiac at any point through the known middle heaven and its known declination. The calculation. If you multiply the radius by the sine of the altitude of the degree of the middle heaven under the earth or over the earth and you divide the product by the sine of the part that is between the horizon and the middle heaven under the earth or above the earth according to whether it happens that that part is less than a quarter circle ⟨or not⟩, the sine both of the sought arc and of the sought angle will result.

I depict meridian ABGD and below it the eastern half of the horizon BED and half of the ecliptic AEG. And let point E be, as you wish, the beginning of Taurus coming to its rising, and G the middle heaven under the earth, which will be known through the known ascensions. And according to the said situation, part EG is necessarily less than a quarter circle. Moreover, I will describe part of a great circle ZHT upon pole E according to the distance of a square's side. And I will complete the two quarter circles EGH and EDT, and each of the two arcs ZGD and ZHT will be quarter circles because horizon BET is described upon the pole of meridian ZGD and upon the pole of great circle ZHT. You see, therefore, the two arcs TE and TZ of great circles descend-

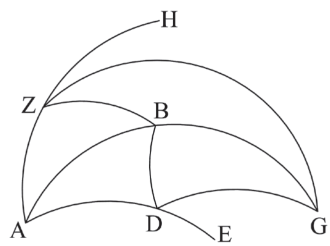
[21] The 'notus' perhaps was not in the original, but it is needed to make sense of this sentence.



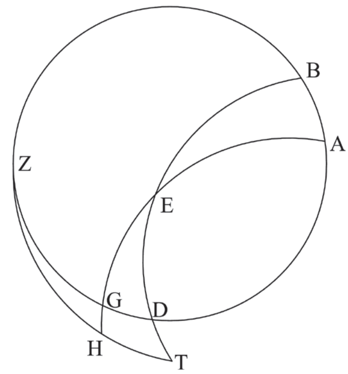
descendentes inter quos alii duo se secant super punctum G. Igitur per kata coniunctam conversis proportionibus, erit proportio sinus TH ad sinum TZ sicut sinus GD ad sinum GE. Sed tria nota sunt. TZ propter esse quartam circuli. GD propter declinationem gradus medii celi et latitudinem regionis esse notam. Nam cum Z sit polus orizontis, erit distantia in arcu meridiano ZGD ab equinoctiali nota, et cum G sit celi medium, erit eius quoque distantia in eodem arcu ab equinoctiali nota. Et propter hoc arcus GZ notus, quare perfectio quarte scilicet GD nota, et ipsa est altitudo partis celi medii ab orizonte. EG vero propter notam esse portionem inter orizontem et celi medium. Igitur primum notum HT cuius arcus quantitas est anguli quesiti quantitas. Eia, age ad hunc modum in ceteris sectionibus.

31. Omnes bini arcus binorum orbium altitudinis a polo orizontis egressi ad duo puncta circuli signorum eiusdem a puncto tropico distantie, cum ipsa etiam a circulo medii diei ante et post secundum equalia tempora destiterint, sunt equales et faciunt angulos cum circulo signorum extrinsecum et intrinsecum ex eadem parte sibi oppositum equales duobus rectis.

Describam itaque orbem meridiei supra quem sint ABG, et sit punctum B polus orizontis et G polus equinoctialis. Et ponam duas portiones orbis signorum ADE et AZH. Et sint puncta Z et D eiusdem longitudinis a puncto tropico et secundum equalia tempora distent a linea medii diei ABG ante et post, hoc est secundum equales arcus equidistantis equinoctiali. Post hec protraham duos arcus orbium altitudinis a puncto B BZ et BD. Et dico quod ipsi sunt equales et quod angulus BDE cum



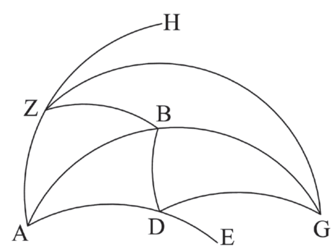
616 coniunctam] *corr. ex* disiunctam *K* **621** celi] circuli P_7 **623** distantia] eius distantia P_7M **624/626** cum – Et] *marg. P om.* P_7 **624** celi medium] medium celi *M* **625** eodem arcu] circulo meridiano *N* **626** equinoctiali] equinoctio *K* **627** est] *s.l. P* celi medii] medii celi *PN* **627/628** vero – esse] propter notam eius P_7 **629** anguli] *corr. ex* angulis *K* Eia age] scilicet AGE *corr. in* scilicet HET *M* HET age *corr. in* age *N* ad] *corr. ex* in P_7 **632** duo puncta] *corr. ex* puncta duo *P* distantie] *corr. ex* distante *P* **633** circulo – diei] medii diei circulo *KM* destiterint] distiterint P_7M (discuerit *Ba* distiterint E_1) **636** ABG] ABGD *N* punctum] punctus *N* **637** et[1]] *om.* P_7 **639** puncto tropico] tropico puncto *N* **640** distent – linea] a linea distent *P* distant a linea *M* **641** secundum] per *MN* **642** equidistantis] equidistantes *M* **643** Post hec] et post hec *P* post hoc *M* et post *N* **644** B] *om. PKM* (*om. Ba* B E_1)



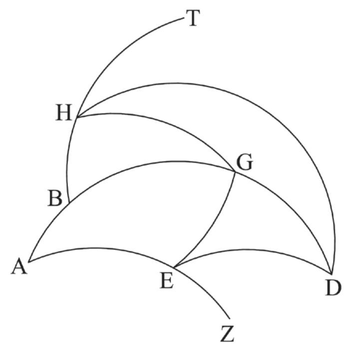
ing from point T, between which two others intersect at point G. Therefore, through the conjunct kata with the ratios reversed, the ratio of the sine of TH to the sine of TZ will be as the sine of GD to the sine of GE. But three are known. TZ [i.e. the first known quantity] because it is a quarter circle. GD [i.e. the second known quantity] because the declination of the middle heaven's degree and the region's latitude are known. For because Z is the horizon's pole, the distance on the meridian arc ZGD from the equator will be known, and because G is the middle heaven, also its distance on the same arc from the equator will be known. And because of this, arc GZ will be known, therefore the complement, i.e. GD, will be known, and that is the altitude of the degree of the middle heaven from the horizon. And indeed EG [i.e. the third known quantity] because the part between the horizon and the middle heaven is known. Therefore, the first ⟨quantity in the proportion⟩, HT, will be known, the quantity of which arc is the quantity of the sought angle. See! Work in this way in the other sections.

31. Any two arcs of two circles of altitude going from the horizon's pole to two points of the ecliptic of the same distance from a tropic point, when these ⟨points⟩ also stand away from the meridian according to equal times before and after, are equal and make angles with the ecliptic, an extrinsic and opposite it an intrinsic from the same part, equal to two rights.

Accordingly, I will describe the meridian upon which let there be ABG, and let point B be the pole of the horizon and G the pole of the equator. And I will posit two parts of the ecliptic ADE and AZH. And let points Z and D be of the same distance from the tropic point, and they stand away according to equal times from the meridian ABG before and after – i.e. according to equal arcs of a parallel to the equator. Afterwards I will draw two arcs of circles of altitude BZ and BD from point B. And I say that these are equal and that angle BDE with angle BZA

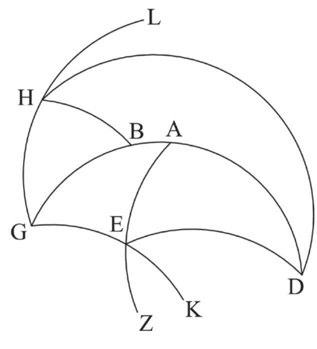


angulo BZA equantur duobus rectis. Propter hoc etiam describo duos arcus meridianorum GZ et GD. Quia ergo angulo ZGB et angulo BGD equales arcus pro paralello resecti subtenduntur, ipsi anguli quoque sunt equales. Quare BG linea facta communi duobus triangulis ZGB et GDB cum duo latera duobus sint equalia, erit basis BZ basi BD equalis, quod est unum ex propositis. Et angulus BZG equalis angulo BDG, sed ex xxii presentis libri angulus GZA et angulus GDE equantur duobus rectis. Ergo angulus BZA cum angulo BDE pariter equantur duobus rectis.



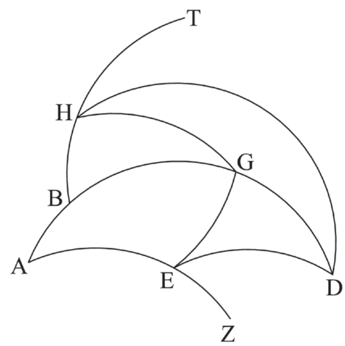
32. Omnes bini arcus binorum orbium altitudinis a cenit capitum egressi usque ad unum punctum circuli signorum cum ipsum a linea meridiei ante et post secundum equalia tempora destiterit, sive cenit capitum a punctis celum mediantibus septentrionale fuerit sive meridianum, sunt equales et faciunt angulos duos ad idem punctum duplo maiores pariter angulo ex concidentia meridiani et circuli signorum ad idem punctum proveniente.

Esto enim orbis meridiei ABGD et summitas capitum punctus G primo ex parte septentrionis et D polus equatoris diei. Et sint due portiones orbis signorum HB et AE, sitque H idem punctum quod E continuans duas portiones et secundum equalia tempora distans ante et post a linea meridiei. Et sint duo arcus orbium altitudinis GH et GE. Dico quod hii arcus sunt equales, et cum producti fuerint arcus meridianorum DH et DE, erunt anguli GHB et GEZ duplo maiores angulo DEZ sive angulo DHB. Quia ergo puncta H et E secundum equalia tempora distant a linea medii diei,

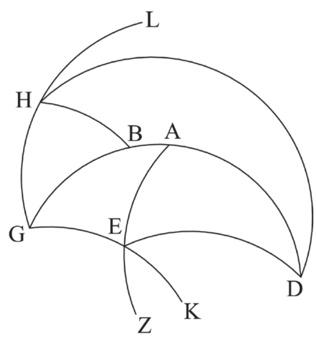


646 etiam describo] describo etiam *P_7K* **647** Quia ergo] ergo quia *P_7* angulo ZGB] angulo ZBG *P* angulus ZGB *M* BGD] *corr. ex* GBD *K* **648** pro] ex *P_7* anguli quoque] quoque anguli *P_7MN* **649** communi] linea communi *P* communis *M* ZGB] scilicet ZGB *KM* **650** BD] *corr. ex* AD *P* **651** xxii] 23ª *P_7MN* (xxiiª *BaE_1*) angulus[2]] *corr. ex* angulis *K* **657** meridiei] *corr. ex* meidiei *P_7* **658** destiterit] distiterit *P_7N* disteterint *M* (distent *Ba* disterint *E_1*) **660** fuerit] fuerit ab equinoctiali *N* **661** punctum] punctum zodiaci *M* zodiaci punctum *N* **663** circuli] *corr. ex* circulo *P_7* punctum] *om. N* **664** proveniente] *corr. ex* provenientem *P_7* provenientes *KM* **665** enim] *om. PN* capitum] capitis *P_7* **666** septentrionis] atrionis *P_7* portiones] proportiones *P* **667** AE] BE *PN* BE *corr. in* HE *M* (HE *Ba* AE *E_1*) **668** E] est *KM* **672** equales] *s.l. P* **675** DHB] DHE *N* **675/676** secundum – distant] distant equaliter *N* equals two rights. For this I also draw two arcs of meridians[22] GZ and GD. Therefore, because equal arcs cut off from a parallel ⟨to the equator⟩ subtend angle ZGB and angle BGD, these angles are also equal. Therefore, with line BG made common to the two triangles ZGB and GDB, because two sides are equal to two ⟨sides⟩, base BZ will be equal to base BD, which is one of the objectives. And angle BZG is equal to angle BDG, but from the 22nd[23] of the present book, angle GZA and angle GDE equal two rights. Therefore, angle BZA together with angle BDE equal two rights.

32. Any two arcs of two circles of altitude going from the zenith to one point of the ecliptic when it stands away from the meridian line before and after according to equal times are equal, whether the zenith is north or south from the points halving the heavens, and they make two angles at the same point, together greater by double than the angle resulting from the meeting of the meridian and the ecliptic at the same point.



For let there be meridian ABGD, point G the zenith first on the north side, and D the equator's pole. And let there be two parts of the ecliptic HB and AE, and let H be the same point as E, joining the two parts and distant from the meridian according to equal times before and after. And let there be two arcs of circles of altitude GH and GE. I say that these arcs are equal, and when arcs DH and DE of meridians are produced, angles GHB and GEZ will be greater by double than angle DEZ or angle DHB. Therefore, because points H and E stand away from the meridian according to equal times, angles GDH and GDE



[22] Here 'meridianus' is broadened to mean not only the great circle through the poles and the zenith, but to refer to other great circles passing through the poles.

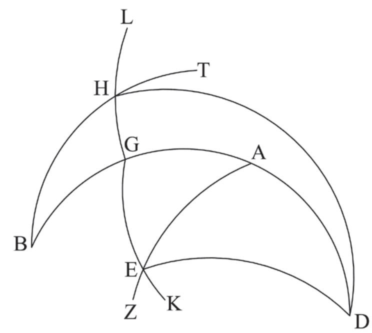
[23] This should refer to II.23, but the evidence points to the reading '22' here.

sunt anguli GDH et GDE equales. Facta ergo linea GD duobus triangulis communi erit linea GE equalis linee GH, et erit angulus GED equus angulo GHD. Sed et angulus DHB equalis est angulo DEZ; ergo ambo pariter GED et GHB sunt equales angulo DEZ. Quapropter ambo anguli GHB et GEZ totus equantur duplo anguli DEZ, quod intendimus.

Sit item cenit G meridianum a punctis celum mediantibus A et B. Dico ergo quod similiter accidit, scilicet quod duo anguli KEZ et LHB equantur duplo anguli DEZ. Angulus enim DEZ equalis est angulo DHB immo idem. Sed et angulus DEK equatur angulo DHL; ergo totus angulus LHB equatur duobus angulis simul DEZ et DEK. Quapropter duo anguli LHB et KEZ equales sunt duplo anguli DEZ.

33. Quod si unum punctorum celum mediantium sive orientalis portionis sive occidentalis meridianum fuerit a cenit capitum et alterum septentrionale, anguli qui proveniunt ad punctum dictum superant duplum anguli ex arcu meridiano ad idem punctum facti quantitate duorum rectorum. Ex quibus omnibus colligitur quod si noti fuerint anguli antemeridiani et arcus in omni declinatione a principio Cancri usque ad principium Capricorni, noti erunt et arcus et anguli eorumdem signorum postmeridiani et una anguli reliquorum signorum et arcus ante et post meridianam lineam.

Describam formam predicte similem, et sit punctum A portionis orientalis in parte septentrionali a puncto G in linea medii celi, et B punctum portionis occidentalis in parte meridiana. Dico ergo quod duo anguli KEZ et GHB simul superant duplum anguli DEZ quantitate duorum rectorum. Ideo siquidem quod duo anguli KEZ et GHB simul superantur a duobus angulis DEZ et DHB vel a duplo unius eorum quantitate duorum angulorum DEK et DHG, sed hii duo anguli

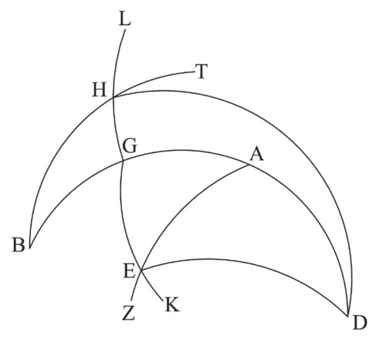


677/678 triangulis communi] angulis communis *M* **678** GH] GE *P* angulus] angulo *M* equus] equalis P_7 **679** DEZ] DEH *P* **680** angulo] *marg. M* **681** DEZ] *corr. ex* †...† *K* **682** item] igitur *M* meridianum] meridianus *PN* **683** KEZ] HEZ *P* **684** immo] quia *N* **685** DEK] DER *K* equatur[1]] *corr. ex* equantur *P* equantur *N* **686** simul] *perhaps added in a later hand K* **687** duplo anguli] anguli duplo *P* angulo duplo *N* DEZ] DEZ et cetera *M* **689** cenit capitum] cenith capitis P_7 czenith capitum *M* **690** anguli qui] qui anguli *M* superant] *corr. ex* separant *M* **691** facti] aut superantur ab eodem *add. marg. N* **692** omnibus] omnium *P* anguli] *s.l. P* antemeridiani] *corr. ex* ante meridianum *M* **694** postmeridiani] *corr. ex* post meridianum *M* **695** et[2]] *om. P* **697** parte] *corr. ex* partem P_7 **700** KEZ] *corr. ex* KEG *N* **701** simul] *corr. ex* similiter *K* superant duplum] superantur duplum (*corr. in* a duplo) P_7 superantur a duplo *N* (superant duplum BaE_1) DEZ] *corr. ex* DZ *N* **703** simul] *corr. ex* similiter *K* superantur] superant *P* **706** DHG] *corr. ex* DGH *N* **706/708** sed – DHG] *om. N* are equal. Therefore, with line GD made common to the two triangles, line GE will be equal to line GH, and angle GED will be equal to angle GHD. But also angle DHB is equal to angle DEZ; therefore, both GED and GHB together are equal to angle DEZ. For this reason, both angles GHB and the whole GEZ equal double angle DEZ, which we intended.

Likewise, let zenith G be south from the points A and B halving the heavens. I say, therefore, that it occurs similarly, i.e. that the two angles KEZ and LHB are equal to double angle DEZ. For angle DEZ is equal to, or more correctly, the same as, angle DHB. But also angle DEK is equal to angle DHL; therefore, whole angle LHB is equal to the two angles DEZ and DEK together. For this reason, the two angles LHB and KEZ are equal to double angle DEZ.

33. That if one of the points halving the heavens, whether of the eastern part or the western, will be south of the zenith and the other north, the angles that result at the said point exceed[24] double the angle made from an arc of the meridian at the same point by the quantity of two right angles. From all of which it is deduced that if the angles before the meridian and the arcs in each declination from the beginning of Cancer to the beginning of Capricorn are known, both the arcs and the angles of the same signs after the meridian and at the same time the angles of the remaining signs and the arcs before and after the meridian will be known.

I will describe a figure similar to the one spoken of before, and let point A be of the eastern part on the north side of point G on the meridian [*lit.*, line of the middle heaven], and B a point of the western part on the south side. Therefore, I say that the two angles KEZ and GHB together exceed double angle DEZ[25] by the quantity of two rights. Accordingly, ⟨it is so⟩ for that reason that the two angles KEZ and GHB together are exceeded by the two angles DEZ and DHB or by double one of them by the quantity of the two angles DEK and DHG. But these two

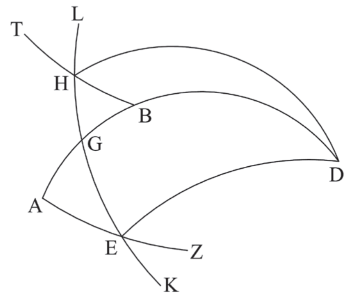


[24] To be universal, this enunciation would require an additional 'or are exceeded by' to be read here. Regiomontanus realized this and added a clause in *N* giving the alternative.

[25] This should read 'are exceeded by the double of angle DEZ', but the mistake appears to be original.

equantur duobus rectis eo quod duo anguli DEK et DEG equantur duobus rectis et ille qui est ex DEG equatur ei qui est ex DHG.

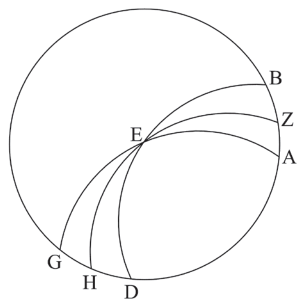
Sit rursum A portionis orientalis in medio celi in parte meridiana a puncto G, et punctum B portionis occidentalis in parte septentrionali. Dico quod similiter accidit. Angulus namque DHG equatur angulo DEG. Duo vero anguli DHG et DHL equantur duobus angulis rectis; angulus autem DEZ est equalis angulo DHB. Quapropter erunt duo anguli GEZ et LHB superantes duos angulos DEZ et DHB aut duplum unius eorum quantitate duorum angulorum DEG et DHL, qui sunt equales duobus rectis, quod oportuit demonstrari.



Palam ergo quod cum noti fuerint quilibet anguli antemeridiani ad quodlibet punctum, noti erunt postmeridiani ad idem. Et ex xxx cum noti fuerint secundum quamlibet longitudinem anguli a tropico ex quacumque parte meridiei, noti erunt anguli secundum eandem longitudinem ex parte altera. Et hoc est.

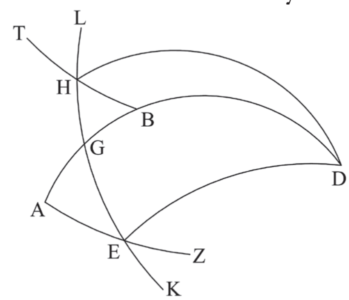
34. Quemlibet angulum ex concidentia circuli altitudinis cum circulo signorum aput punctum medians celum vel aput punctum orizontis et arcum quoque a summitate capitum ad utrumlibet notum esse oportet.

Pono circulum meridianum ABGD et infra eum medietatem orizontis BED et medietatem orbis signorum ZEH qualitercumque. Imaginemur itaque circulum altitudinis descriptum super A quod est summitas capitum et transeuntem per medium celi supra punctum Z. Dico quod arcus AZ est notus. Ideo scilicet quod arcus EZ notus est per xviiii huius, et declinatio puncti Z per



707/708 eo – rectis] *om. P* **709** A[1]] punctum A P_7 A punctum *N* **710** occidentalis] orientalis *PN* **713** angulis rectis] rectis angulis *PN* **716** DEZ] DEG *P corr. ex* DE†...† *K* **716/718** DEZ – angulorum] *marg. N* **717** quantitate] quantitatem *KM* (quantitate BaE_1) **718** DEG] *s.l.* (*other hand*) *K* **721** noti erunt] erunt noti P_7 *corr. ex* non erunt *K* **723** secundum] sed *K* parte altera] alia parte P_7 est] est propositum et cetera *M* est propositum *N* **727** Pono] ponam P_7 **728** BED] BDE *K* **729** ZEH] ZTH *P* ZHE *K* **733** punctum] *om.* P_7 **734** notus[1]] (Alii habent hic: Dico quod arcus AZ est notus *adnot. M*) Quia declinatio puncti (Z *add. M*) ab equinoctiali est nota (nota est *M*) et similiter latitudo regionis nota est. AZ ergo arcus est distantia (differentia *M*) cenith (zenit *M*) a gradu medii celi. Si ergo declinationem gradus (*om. M*) medii celi a latitudine regionis si gradus medii celi sit (signi *add. M*) septentrionalis, subtrahas, vel si sit gradus signi meridionalis, eidem superaddas, resultat quantitas AZ qui (que *M*) est arcus circuli altitudinis a cenith (zenit *M*) capitum usque ad gradum medii celi. *add.* (*on added leaf M*) *MN* Ideo] id *P* notus est] est notus *N* **735** xviiii] 18 P_7 *corr. ex* 14^{am} *M* angles equal two right angles because the two angles DEK and DEG equal two right angles and that which is from DEG is equal to it which is from DHG.

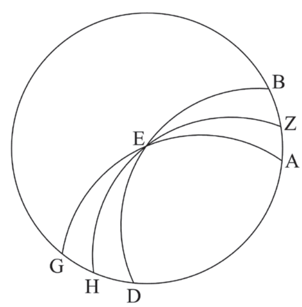
In turn, let A be of the eastern part in the middle heaven on the south side of point G, and point B of the western part on the north side. I say that it occurs similarly. For angle DHG is equal to angle DEG. And indeed the two angles DHG and DHL equal two right angles; moreover, angle DEZ is equal to angle DHB. For this reason, the two angles GEZ and LHB will exceed the two angles DEZ and DHB or double one of them by the quantity of the two angles DEG and DHL, which are equal to two rights, which was necessary to be demonstrated.



Therefore, it is clear that when any angles at any point before the meridian are known, the ones at the same ⟨point⟩ after the meridian will be known. And from the 30th ⟨proposition⟩[26] when they are known according to any distance of the angle from the tropic on whichever side of the meridian, the angles according to the same distance on the other side will be known. And this is ⟨what was proposed⟩.

34. It is necessary that any angle from the meeting of a circle of altitude with the ecliptic at the point halving the heavens or at a point on the horizon, and also the arc from the zenith to whichever point you please be known.

I place meridian ABGD and below it half of the horizon BED and half of the ecliptic ZEH in whatever way. Accordingly, let us imagine a circle of altitude described upon A, which is the zenith, passing through the middle heaven upon point Z. I say that arc AZ is known. For that reason that arc EZ is known through the 19th ⟨proposition⟩ of this, the declination of point Z ⟨is known⟩



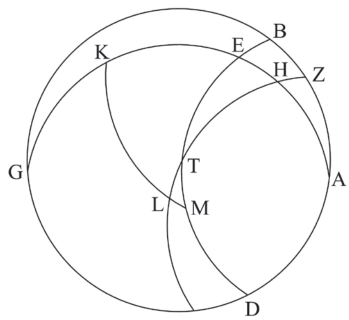
[26] This actually refers to II.31.

xv primi libri, et elongatio puncti A ab equatore diei quia est latitudo regionis. Et dico quod angulus AZE cum circulus altitudinis hic sit meridianus est etiam notus ex xxvi[a] presentis.

Rursus imaginemur circulum altitudinis descriptum supra punctum A et transeuntem per E quod est punctum orientis, scilicet AEG. Manifestum ergo quod arcus AE semper erit quarta circuli eo quod punctum A sit polus orizontis BED, et propter has causas erit angulus AED rectus semper. Sed et angulus DEH qui est ex orbe signorum et orbe orizontis semper notus ex xxx[a] presentis. Quare erit totus angulus AEH notus, et hoc est quod oportuit declarari.

35. Quantitatem arcus circuli altitudinis a summitate capitum ad quodlibet punctum circuli signorum invenire.

Conscribimus itaque orbem meridiei ABGD et infra eum medietatem orizontis BED et medietatem orbis signorum ZHT. Et sit punctum H caput Cancri secundum quodlibet tempus distans a linea meridiana et exempli causa sit distans secundum unam horam. Et punctum Z medians celum et punctum T orientis per xviii notum. Faciam ergo super summitatem capitis A et super caput Cancri H transire portionem circuli altitudinis AHEG. Scrutabor ergo quantitatem arcus AH. Est itaque sicut premisimus arcus ZT notus, et arcus HT notus cum H sit principium Cancri, et arcus AZ propter declinationem puncti Z et altitudinem poli notas notus, et arcus ZB quia est complementum quare notus. Hiis ergo cognitis vides quod proportio BZ ad BA aggregatur ex duabus, una scilicet que est EH ad EA quartam et alia que est TZ ad TH – de sinibus arcuum loquor. Cum ergo ceteri noti sunt, erit et arcus EH notus; ergo et reliquus AH notus.



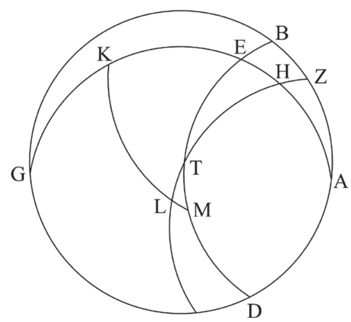
Regula operationis. Si sinum arcus meridiani deprehensi inter celum medium et orizontem multiplices in sinum arcus circuli signorum deprehensi inter orizontem et punctum circuli signorum ad quod circulus altitudinis deducitur, et

736 xv] xv[um] *P* equatore] equitore P_7 quia] *corr. ex* que *K* que *M* est] *om.* *N* **737** hic] HT *N* sit] sicut *M* **740** orientis] orisontis *N* **742** BED] BDE *K* **744** erit] *corr.* ex †ergo† *M* totus] *corr. ex* notus *P* AEH notus] AEB notis *P* **745** capitum] capitis *M* **748** ZHT] ZKT *KM* sit] si *M* H] B *P* **749** secundum] sed *PK corr. ex* sed *M* causa – distans[2]] tum sit differentia *K* **751** orientis] *corr. ex* orizontis *K* **753** capitis] capitum P_7*K* **754** AHEG] AHET *P corr. ex* AEHG P_7*N corr. ex* AHE *K* **755/756** Est – ZT] itaque sicut premisimus arcus ZT est *N* **759** notas] notam *N* **760** complementum] complentium *P* complementum quarte circuli *N* quare] quarte P_7*K corr. ex* quarte *M* **761** quartam] quartam circuli *MN* **763** AH] *corr. in* AB *M* **764** celum] celi P_7*MN* (celum *Ba*E_1) **766** et[1]] altitudinum *add. et del. N* through the 15th[27] of the first book, and the elongation of point A from the equator ⟨is known⟩ because it is the latitude of the region. And I say that angle AZE is also known from the 26th of the present because the circle of altitude here is the meridian.

In turn, let us imagine a circle of altitude described upon point A and passing through E, which is the point of rising, i.e. AEG. Then it is manifest that arc AE will always be a quarter circle because point A is the pole of horizon BED, and for these reasons angle AED will always be right. And also angle DEH, which is from the ecliptic and the horizon will always be known from the 30th of the present. Therefore, the whole angle AEH will be known, and this is what was necessary to be declared.

35. To find the quantity of the arc of a circle of altitude from the zenith to any point of the ecliptic.

Accordingly, we draw the meridian ABGD and below it half of the horizon BED and half of the ecliptic ZHT. And let point H be the beginning of Cancer distant according to whatever time from the meridian, and for example let it be distant according to one hour. And point Z halving the heavens and T, the point of rising, are known through the 18th.[28] Then I will make a part of a circle of altitude AHEG pass upon zenith A and upon the beginning of Cancer H. I will search, therefore, for the quantity of arc AH. Accordingly, as we set out, arc ZT is known, arc HT is known because H is the beginning of Cancer, arc AZ is known because of the known declination of point Z and the pole's known altitude, and arc ZB, because it is the complement, is therefore known. With these known, therefore you see that the ratio of BZ to BA is collected from two ⟨ratios⟩, i.e. one that is of EH to quarter circle EA, and another that is of TZ to TH – I speak about the sines of the arcs. Therefore, because the rest are known, arc EH will also be known; therefore, the complement AH will also be known.



Rule of operation. If you multiply the sine of the arc of the meridian caught between the middle heaven and the horizon by the sine of the arc of the ecliptic caught between the horizon and the point of the ecliptic to which the circle

[27] This should refer to I.16.

[28] This should refer to II.19 to match my counting.

productum dividas per sinum arcus circuli signorum intercepti inter orizontem et celi medium, exibit sinus perfectionis arcus quesiti, quam si a quarta dempseris, relinquitur arcus circuli altitudinis a summitate capitum ad punctum circuli signorum destinatum.

36. Quantitatem anguli ex concidentia circuli altitudinis cum circulo signorum ad quodlibet punctum a celi medio declinans perscrutari.

Resumamus positam figuram secundum habitudinem suam, et describamus super polum puncti H secundum spatium lateris quadrati portionem magni circuli KLM. Quia ergo orbis AHE est descriptus supra duos polos ETM et KLM, erit uterque duorum arcuum EM KM quarta circuli. Propter hanc ergo formam per kata disiunctam proportio sinus EH ad sinum EK componitur ex proportione sinus HT ad sinum LT et proportione sinus LM ad sinum MK. Sed quinque horum nota sunt. Relinquitur ergo LM notum; ergo et KL notum residuum quarte; ergo angulus LHK cui subtenditur notus. Quapropter et angulus AHT complementum duorum rectorum notus, quod volumus ostendere.

Opus. Longitudinem puncti destinati ab occidente de xc minue. Et sinum residui in sinum altitudinis puncti destinati ducito, quodque exierit per sinum longitudinis puncti destinati ab ascendente divide. Et quod fuerit in diametri dimidium multiplica, indeque collectum per sinum longitudinis puncti destinati a cenit capitum partire. Et quod exierit arcuabis, et arcum de xc minues, et residuum de clxxx. Et erit quantitas quesiti anguli. Ad hunc modum in ceteris punctis et arcus et angulos invenies. Atque hec est notitia omnium angulorum ex circulo altitudinis et orbe signorum quorum scientia necessaria est ad sciendum diversitatem aspectus Lune sine cuius notitia solares eclipses sciri est impossibile.

767 arcus] *corr. ex* altus P_7 intercepti] intercepta *PK corr. ex* intercepta P_7 **768** quam] quem *MN* **769** capitum] capitis *M* **770** destinatum] destinatus *N* **771** ex concidentia] *corr. ex* excidentia *P* **772** quodlibet] quemlibet *M* **774** H] *iter. et del. M* **775** orbis] AZB P_7 AHE] ABE *P corr. ex* †HB†E *N* ETM] *corr. ex* ATM *M* **776** erit] eritque *M* **778** et] et ex *MN* **780** quarte] *corr. ex* quare *P* **781** volumus] voluimus *N* **783** ab occidente] ab ascendente vel ab occidente P_7 (*om. Ba* ab ascendente E_1) **784** exierit] exibit *N* **785** ascendente] accidente *K corr. ex* occidente *M* divide] *s.l. P* fuerit] exierit *N* **787** cenit] czenit *M* capitum] *corr. ex* capite *K* exierit] exibit *N* **788** quesiti] quesititi *K* anguli] *corr. ex* circuli *P* **791** aspectus] *corr. ex* adspectus *K* **792** impossibile] impossibile et cetera *M*; explicit secundus liber *add.* P_7 finit secundus *add. N* of altitude is led down, and you divide the product by the sine of the arc of the ecliptic cut off[29] between the horizon and the middle heaven, the sine of the sought arc's complement will result. If you subtract that from a quarter circle, there remains the arc of the circle of altitude from the zenith to the appointed point of the ecliptic.

36. To search for the quantity of the angle from the meeting of the circle of altitude with the ecliptic at any point declining from the middle heaven.

Let us take the supposed figure again according to its disposition, and let us describe a part of great circle KLM upon the pole of point H according to the distance of a square's side. Then, because circle AHE is described upon the two poles of ETM and KLM, each of the two arcs EM and KM will be a quarter circle. Because of this figure, therefore, through the disjunct kata, the ratio of the sine of EH to the sine of EK is composed of the ratio of the sine of HT to the sine of LT and the ratio of the sine of LM to the sine of MK. But five of these are known. Therefore, LM remains known, so also KL, the complement, is known; therefore, angle LHK which it subtends is known. For this reason also angle AHT, the supplement, is known, which we wish to show.

The work. Subtract the distance of the appointed point from the setting[30] from 90. And lead the sine of the remainder into the sine of the altitude of the appointed point, and divide what results by the sine of the distance of the determined point from the ascendant. And multiply what that will be by the radius, divide what is obtained from this by the sine of the distance of the appointed point from the zenith. And you will arc what results, and subtract this arc from 90, and the remainder from 180. And there will be the quantity of the sought angle. In this way [i.e. the way here and the rule in II.35] you will find both the arcs and angles in the remaining points. And this is the knowledge of all the angles from the circle of altitude and the ecliptic, the knowledge of which is necessary for knowing the moon's parallax, without knowledge of which it is impossible that solar eclipses be known.

[29] The reading 'intercepta' is clearly the wrong form, but it is possibly the author's own mistake.

[30] This should be 'the rising point.'

⟨Liber III⟩

Communia quedam premittenda sunt quia hic modus demonstrationi est aptior.

Perpetuum motum orbicularem esse.

Celestia corpora perpetuo motu ideoque orbiculari esse mobilia.

Omnem motum celestis corporis simplicem et verum equabilem esse, hoc est super equos angulos in centro motus consistentes et in equales arcus cadentes equalibus fieri temporibus.

Motum Solis vel alterius planete in circulo signorum diversum apparere.

Motum stelle medium esse cum tota et integra eius revolutio secundum equalia tempora per equales motus fuerit distributa.

Hiis premissis quod proposuimus prosequamur.

1. Anni quantitatem per considerationes in instrumentis deprehendere.

Tempus vel quantitas anni est reditus Solis ab aliquo puncto circuli signorum ad idem ut a puncto solstitiali ad idem aut a puncto equinoctiali ad idem. Hec enim notabiliora et digniora sunt in circulo. Preparato itaque quadrante veridico sicut in primo libro diximus et per ipsum arcu qui est inter duos tropicos deprehenso, arcus ipse in duo equalia secetur, eritque punctus sectionis cum quadrans erectus fuerit super lineam medii diei in superficie equinoctialis circuli. Observandum itaque circa autumpnale equinoctium, quia tunc aer purior est, umbram in meridie cadentem donec puncto equinoctii quoad vicinius contingit apropinquet et hoc ante et post ipsum equinoctii punctum. Nota ergo erit utrimque per instrumenti bonitatem declinatio, et per declinationem fiet arcus circuli signorum utrimque notus. Cum ergo utrumque in unum collegeris, erit motus Solis diversus ad unam diem notus. Cum ergo

1 Liber III] liber tertius *marg.* (*other hand*) *P* incipit tertius P_7 tertius *K* incipit liber tertius *M* tertius incipit *marg. N* **2** quedam] quidem *K* demonstrationi] demonstrandi P_7 **2/3** demonstrationi – aptior] de materia communi aptior est demonstrationi *M* **5** Celestia – mobilia] *marg.* (*other hand P*) *PN* ideoque] et ideo *P* **6** equabilem] *corr. in* equalem *P* equalem *N* equabilem] *corr. in* equalem *P* equalem *N* **11** per – fuerit] et per equales motus fiunt *K* **12** prosequamur] prosequemur *K* **13** in] *om. P* deprehendere] comprehendere *N* **15** solstitiali] solstitii *M* aut] vel P_7 equinoctiali] equinoctii *M* **16** notabiliora] notabilia *K* quadrante] quadrato *N* **17** arcu] arcum *PKM* (arcum *Ba* arcus E_1) **18** secetur] secatur *K* punctus] punctum P_7K **19** lineam] linea P_7 in superficie] insuficie *K* **20** Observandum] conservandum *K* **20/21** quia – est] hoc *K* **21** est] *s.l. P* **21/22** quoad vicinius] *corr. ex* quod advicimus *K* **22** contingit] contingerit *M* apropinquet] apropinquat P_7K hoc] *om. PN* hoc et P_7 **24** fiet] fit *K om. N* utrimque] *corr. ex* uterque *M* **24/25** Cum – notus] *marg.* (*other hand*) *K om. N* **24** ergo] secundum proportionem *add. et del. M*

Book III

Certain common ⟨notions⟩ should be premised because this manner is more suitable for demonstration.

Perpetual motion is circular.

Celestial bodies are mobile by a perpetual, and for that reason circular, motion.

Every simple and true motion of a celestial body is uniform, i.e. it is made upon equal angles standing on the motion's center and falling on equal arcs in equal times.

The motion of the sun or another planet in the ecliptic appears irregular.

A star's motion is mean when its whole and complete revolution is distributed according to equal times through equal motion.

With these things having been set forth, let us describe in detail what we have proposed.

1. To discover the quantity of the year through observations with instruments.

The time or quantity of a year is the sun's return from some point of the ecliptic to the same, as from a solstice point to the same or from an equinox point to the same. For these are the more remarkable and worthy ⟨points⟩ in the circle. Accordingly, with a truthful quadrant having been prepared as we said in the first book and with the arc[1] that is between the two tropics having been discovered through it, let that arc be cut into two equals, and when the quadrant is erected upon the line of the middle day, the point of division will be in the equator's plane. Accordingly, the shadow falling at noon should be observed[2] around the autumnal equinox, because the air is purer then, until it approaches the equinox point as nearly as it occurs both before and after that equinox point. The declination on each side, therefore, will be known through the instrument's quality, and through the declination, the arc of the ecliptic on each side will be known. Therefore, when you combine both of these into one, the sun's irregular motion for the one day will be known. Therefore, when

[1] Most of the witnesses have this in the wrong case.

[2] The construction here of an impersonal gerundive with an accusative object is unusual, but the *Almagesti minor*'s author uses it here and in III.25. It was also found occasionally in Classical Latin (see *Gildersleeve's Latin Grammar*, § 427).

secundum proportionem totalis arcus circuli signorum ad utramlibet suarum partium tempus diei diviseris, erit punctum temporis quo Sol per equinoctiale punctum transierit notum. Eodem modo punctum temporis reversionis Solis ad idem punctum equinoctii innotescat. Quantitas ergo temporis inter utrumque deprehensa tempus anni esse perpenditur. Pari modo per solstitiale punctum et maximam declinationem perpendi potest quantitas anni, sed commodior et certior est equinoctialis observatio quia Sol circa equinoctium velocior est et ideo in brevi tempore maiorem habet in declinatione diversitatem, circa solstitium vero tarde et minime diversitatis est declinatio.

Attamen tempus anni ad verum deprehendi propter fallaciam que sensui per instrumentum accidit non contingit. Et cum per multos annos id in quo error est collectum fuerit, erit sensibilis differentia, et precedet vel subsequetur tempus solstitii aut equinoctii verum tempus solstitii vel equinoctii secundum computationem sensibiliter. Si ergo hoc tempus anni semper ut estimavit Ptolomeus idem est nec diversum, verius deprehendetur per duas magni intervalli considerationes et plurium reversionum quam per propinquas duas.

Porro definitum anni tempus diversum esse nec per omnia equale digne estimari potest. Cum Egyptiorum antiquissimi ex Babylonia sicut per suas considerationes deprehenderunt ipsum ex ccclxv diebus et quarta diei et una parte ex cxxx diei partibus constare dixerunt, Abrachaz vero super cuius considerationem operatus est Ptolomeus ex ccclxv diebus et quarta diei tantum. Post hec Ptolomeus ab hac quantitate anni in ccc annis unum diem excepit, et annum Solis esse ex ccclxv diebus et minus quam quarta quantum est una pars ex ccc diei partibus per suam considerationem et considerationem Abrachaz, inter

26 proportionem] signorum *add. et del. M* **27** diei] *om. N* equinoctiale] equinoctialem *M* **29** innotescat] innotescet P_7N innotescit *corr. in* innotescet *M* (innotescat BaE_1) temporis] *marg.* (*perhaps other hand*) *P* **30** tempus – perpenditur] tempus anni esse deprehenditur P_7 tempus esse anni perpenditur *K* tempus sive quantitas anni esse perpenditur *M* quantitas anni perpenditur esse *N* solstitiale] solsticialem *M* **31** perpendi potest] perpenditur *N* sed] itaque *K* **32** est] *s.l. K* **32/33** velocior est] velociorem PP_7K movetur velocius *MN* (velocior est *Ba* velociorem E_1) **33** declinatione] diversitate declinationis *N* **34** solstitium vero] vero solstitium movetur *M* vero] *om. N* diversitatis] diversitas *K* **35** tempus] opus *corr. ex* post *K* verum] unum *M* **37** error est] est error *K* precedet – subsequetur] precedat vel subsequatur *P* **38** aut] vel *PMN* verum – equinoctii[2]] *om. PN marg. M* (*om. Ba text confirmed by* E_1) **39** Ptolomeus] Tholomeus P_7 **40** est] esset *PMN* sit P_7 (est BaE_1) **41** per] *om. M* **42** definitum] diffinitum P_7KM anni tempus] tempus anni P_7N per omnia] omnino *N* **43** Egyptiorum antiquissimi] Egyptianorum antiquissimorum *K* suas] duas P_7 **44** deprehenderunt] deprehendunt *K* quarta diei] diei quarta *M* **45** diei partibus] partibus diei *M* partibus *N* constare] *marg.* (*perhaps other hand*) *P* Abrachaz] Abrachis *MN* **46** Ptolomeus] Tholomeus P_7 hec] hoc *MN* **47** Ptolomeus] Tholomeus P_7 quantitate] *corr. ex* consideratione *P* **48** diebus] *om. P* **49** diei partibus] diebus partibus *K* partibus diei *N* Abrachaz] Abrachis *MN* (Abrachis *Ba* Abrachaz E_1)

you divide the time of the day according to the ratio of the whole arc of the ecliptic to either of its parts, the point of time when the sun passed through the equinox point will be known. In the same way, let the point of time of the sun's return to the same equinox point be made known. Therefore, the quantity of time caught between both is assessed to be the time of a year. In a like way, the quantity of the year can be assessed through a solstice point and the maximum declination, but the equinoctial observation is more convenient and more certain because the sun is faster[3] around the equinox, and for that reason it has a greater difference in declination in a short time, but around the solstice ⟨it moves⟩ slowly and the declination is of least difference.

But yet the time of the year does not happen to be grasped truthfully because of the deception that befalls the senses through the instrument. And when that in which the error is is combined through many years, there will be a perceptible difference, and the true time of the solstice or equinox will visibly precede or follow the time of the solstice or equinox according to computation. Therefore, if this time of a year is, as Ptolemy judged, always the same, and not irregular, it will be grasped more truly through two observations of a great interval and of several returns than through two subsequent ones.

On the other hand, it can be judged worthily that the precise time of the year is irregular and not uniform through all things. When the oldest of the Egyptians from Babylon,[4] as they grasped from their observations, said that it consisted of 365 + 1/4 + 1/130 days[5], and indeed Abrachaz [i.e. Hipparchus], upon whose observation Ptolemy worked, ⟨said that it consisted⟩ of only 365 1/4 days. Afterwards Ptolemy removed from this quantity one day in 300 years, and he discovered through his observation and the observation of Abrachaz, between which were 285 Egyptian years, that the sun's year is of 365 + 1/4

[3] The reading 'velociorem' found in most of our witnesses appears to be a mistake arising from a confusion between an 'e' with a line over it signifying 'est' and the accusative ending to the adjective.

[4] This clearly wrong passage probably stems from the author's incorrect copying of Albategni's phrase 'Egiptiorum etenim ex Babilonia vetustissimi quidam' (as found in *P*, 25v, which differs slightly from that found in the 1537 ed., f. 26v).

[5] Albategni, *De scientia astrorum*, 1537 ed., f. 26v, mistakenly has 1/131 instead of 1/130 as is found in *P*, f. 25v.

quas fuerunt cclxxxv anni Egyptiaci deprehendit. Deinde vero a Ptolomeo post dccxliii annos observavit Albategni punctum equinoctii et per intervallum duarum considerationum, sue scilicet et Ptolomei, tempus anni ccclxv dierum et xiiii minutorum et xxiiii secundorum fore deprehendit. Quare cum eisdem usi sint instrumentis philosophi et exceptio secundum annorum spatia non equaliter procedat, hoc tempus anni diversum contingere non indigne putabitur.

Huius ergo diversitatis causam Tebit Benchoraz coniectans necnon et illius diversitatis que in declinationibus reperitur, motum octave spere ante et retro supra duos circulos parvos supra caput Arietis fixum et caput Libre fixum descriptos quorum diameter est viii gradus et xxxi minuta et xxvi secunda deprehendit. Et hunc motum qui inferioribus quoque speris communis est diversitatem annorum efficere necnon et diversitatem declinationum maximarum que reperitur indicavit. Docuitque secundum motum spere inequalitatem anni omnibus temporibus et maximam declinationem invenire. Tempus ergo anni equale non est a solstitio ad idem solstitium vel ab equinoctio ad idem equinoctium, sed a puncto zodiaci secundum motum octave spere mobili ad idem Solis regressio, quod est a stella fixa ad eandem Solis reversio. Et hoc quidem tempus anni equale est ex ccclxv diebus et xv minutis et xxiii secundis, et super hoc Arzacel tabulas motuum Toleti novissime composuit.

2. Medium motum Solis ad quaslibet divisiones temporis scilicet annos collectos, annos disgregatos, menses, dies, horas, puncta horarum collocare.

Datum enim annum Egyptiacum sive Romanum aut annum Arabum cum anno Solis equali confer, et secundum eorum proportionem de ccclx gradibus, idest de toto circulo, sume. Et productum, quod est motus medius ad unum annum datum, per numerum annorum collectorum quicumque sit multiplica.

50 fuerunt] fuerint *K* a] *s.l. P* Ptolomeo] Tholomeo P_7 **51** dccxliii annos] annos dccxliii *PN* Albategni] Abategni P_7 et] *om. M* **52** Ptolomei] Tholomei P_7 anni] agni *K* et[2]] *corr. ex* in *K* **53** xxiiii] *corr. ex* 23 *M corr. in* 26 *N* secundorum] secundarum *PK marg. M* Quare] *corr. ex* quale *K* **54** sint] sunt *N* **55** indigne] *corr. ex* digne *K* **56** Tebit Benchoraz] Tebith Bencoraz P_7 Thebit Beuchoraz *K* Thebith Benchorath *M* Tebith Benchorath *N* coniectans necnon] *corr. ex* concitans n†...† non *K* et] *om. KN* **57** reperitur] *corr. in* reperit *K* spere] *marg. N* **57/58** ante – parvos] super duos parvos circulos ante et retro *N* **57** ante et] *corr. ex* an†ni† *K* **58** parvos] *corr. ex* p†...† *K* **59** gradus] *corr. ex* grad†ibus† *K* et xxxi] et 30 *M* minuta] *corr. ex* minutorum P_7 secunda] *corr. ex* s†...† *K* **60** inferioribus] in inferioribus P_7 **61** efficere] proficere P_7 diversitatem declinationum] declinationum diversitatem *K* **62** Docuitque] locumque *K* spere] octave spere P_7 **63** temporibus] *s.l.* (*perhaps other hand*)*P* ergo] vero *PN* **63/64** anni equale] equale anni *MN* **64** equinoctium] equinoctium tantum *N* **66** regressio] reversio *N* **67** xv] xx *K* xxiii] *corr. ex* 22 *M* secundis] *corr. ex* secundi *K* **68** Arzacel] Arzachel P_7*N corr. in* Arzachel *K* Arzahel *M* (Arachel *Ba* Aracel E_1) tabulas] *corr. ex* tabula P_7 Toleti] *corr. ex* Topleti *K* Tolete *N* (Tholeti *Ba*E_1) **71** Datum] latum *P* aut] sive *PN* Arabum] *om. N* **72** equali] equari *P* **74** annorum] *corr. ex* annum *K*

– 1/300 days. Then indeed 743 years after Ptolemy, Albategni observed the equinox point and through the interval of two observations, i.e. his own and Ptolemy's, he discovered that the time of a year will be 365 14′ 24″ days.[6] Therefore, because the philosophers used the same instruments and what is removed[7] according to the space of years does not proceed equally, it will not be unworthily thought that this time of a year turns out to be irregular.

Therefore, Tebit Benchoraz [i.e. Thābit ibn Qurra], inferring the cause of this irregularity as well as of that irregularity that is found in the declinations, discovered the eighth sphere's motion forward and backward upon two little circles described on Aries' fixed beginning and Libra's fixed beginning, of which ⟨circles⟩ the diameter is 8° 31′ 26″.[8] And he showed that this motion, which is common also to the lower spheres, makes the irregularity of years, as well as the irregularity of the maximum declinations that is found. And he taught how to find the inequality of a year in all times and the maximum declination according to the sphere's motion. Therefore, the mean time of a year is not the sun's return from a solstice to the same solstice or from an equinox to the same equinox, but from a point of the zodiac, mobile according to the motion of the eighth sphere, to the same, because it is the sun's return from a fixed star to the same. And indeed this mean time is 365 15′ 23″ days, and upon this Arzacel very recently made the tables of motions of Toledo.

2. To set up the sun's mean motion for each division of time, i.e. collected years, separated years, months, days, hours, and fractions of hours.

Indeed, compare the given Egyptian, Roman, or Arabic year with the sun's mean year, and according to their ratio take ⟨an arc⟩ from 360°, i.e. from a whole circle. And multiply the result, which is the mean motion the one given year, by the number of collected years, whatever it may be. And always cast

[6] This should be 365 14′ 26″ days to match Albategni, *De scientia astrorum* (1537 ed., f. 27v).

[7] The normal definitions of 'exceptio' do not make sense here, but with the meaning in which the verb 'excipio' was used earlier in the passage in mind, the author seems to have meant 'what is removed.'

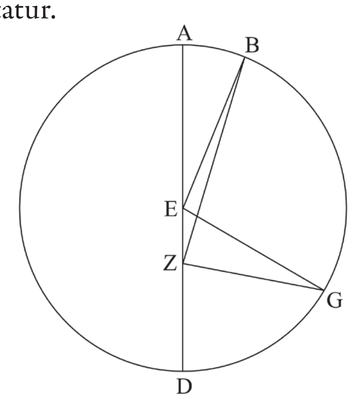
[8] This should be 8° 37′ 26″ to match *De motu octave sphere*; see Millás Vallicrosa, *Estudios sobre Azarquiel*, p. 498.

Et circulos integros semper proice, et erit medius motus Solis ad annos collectos. Ad annos vero expansos quoslibet motum unius anni preinventum duplicando, triplicando, et deinceps secundum consequentiam annorum motum constitues. Ad menses vero similiter scilicet secundum proportionem dati mensis ad annum Solis equalem de toto circulo accipies, et ipsum secundum processum mensium duplicabis, triplicabis, et sic deinceps usque ad expletionem mensium totius anni. Pari modo ad dies, horas, minuta horarum motum medium equaliter distributum adiunges.

3. Motum stelle diversum apparentem in signorum circulo propter duorum modorum utrumlibet posse contingere.

Unus duorum modorum est ut estimemus stellam ecentricum habere et in eius circumferentia corpus stelle equali motu circumferri. Alius modus est ut imaginemur stellam concentricum habere sed in eius circumferentia corpus stelle non permanere, verum etiam ipsam epiciclum habere, cuius centrum in circumferentia concentrici equaliter circumducitur, et corpus stelle nichilominus in circumferentia epicicli equaliter circumrotatur.

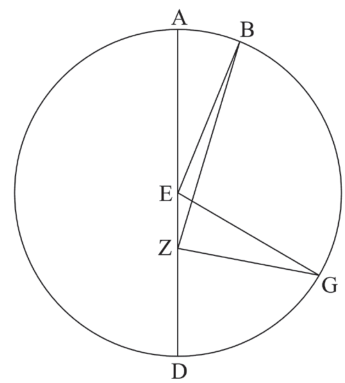
Sit ergo primum orbis ecentricus ABGD supra quem motus stelle equabilis cuius centrum sit E et eius diameter AED. Et sit punctum longitudinis longioris a terra punctum A, et D punctum longitudinis propioris terre. Sitque super diametrum AED nota Z a qua est aspectus oculorum quasi centrum mundi. Secabo ergo duos arcus equales AB et GD et protraham lineas BE BZ GE GZ. Dico ergo quod cum stella equabilem habeat motum in arcubus AB GD, diversum tamen apparenter habet in circulo signorum. Cum enim equales sint anguli AEB et



75 Et[1]] *corr. ex* per *K* semper] super *iter. et del. P* **76** quoslibet] quorumlibet *P* quotlibet *N* **77** et] et sic *N* **78** constitues] constitue *corr. ex* constituere *K* scilicet] *om. M* **79** Solis equalem] totum equalem Solis *N* **80** processum] precessum P_7 ad expletionem] impletionem P_7 ad completionem *KM* (ad completionem *Ba* ad expletionem E_1) **81** ad dies] addities *P* horas] *corr. ex* choras *K* horas et *MN* **82** medium] medium Solis *M* **83** diversum] *corr. ex* divisum *K* **86** stelle] telle *K* circumferri] conferri *P* **87** stellam] stellas *P* concentricum] concentrico *K* **88** etiam ipsam] in ipsum *K* habere] *om. K* **89** equaliter] equabiliter P_7K **90** equaliter circumrotatur] equabiliter circumferatur *K* **91** primum] primus *P* primo *N* ecentricus] centricus *P* **92** equabilis] equalis *MN* **97** aspectus] suspectus *P* oculorum] oculorum et *PN* **99** BZ GE] *s.l.* (*perhaps other hand*) *P* **100** equabilem] *corr. in* equalem *M* equalem *N* **101** AB GD] *corr. ex* AG BD *N* **101/102** tamen – habet] tam apparenter habet *P corr. ex* tantum apparentium *K* tamen apparentem habent *M* **102** sint] sunt *PM* out complete circles, and there will be the sun's mean motion for the collected years. And indeed, for whatever expanded years, you establish the motion by doubling, tripling, and so on according to the succession of years, the one year's motion already found. And similarly for months, i.e. you will take ⟨an arc⟩ from the whole circle according to the ratio of the given month to the sun's mean year, and according to the progression of months, you will double it, triple it, and so on until the completion of months of the whole year. In a like way, for days, hours, and minutes of hours, you will allot the mean motion distributed equally.

3. It happens that the motion of a star can appear irregular in the ecliptic because of either of two ways.

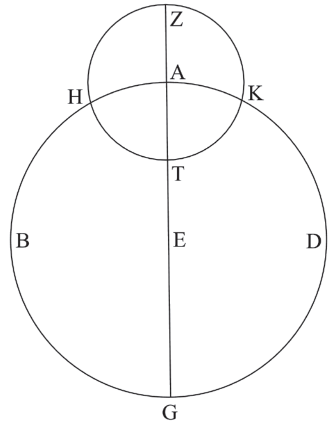
One of the two ways is that we judge that the star has an eccentric ⟨circle⟩ and that on its circumference the star's body is carried around with a uniform motion. The other way is that we imagine that the star has a concentric ⟨circle⟩, but the star's body does not remain on its circumference. Indeed, ⟨we imagine that⟩ it also has an epicycle, whose center is uniformly led around on the concentric's circumference, and likewise the star's body is spun around uniformly on the epicycle's circumference.



Then let there first be eccentric circle ABGD upon which the star's motion is uniform, the center of which let be E and its diameter AED. And let the apogee be point A, and D the perigee. And upon diameter AED let there be point Z from which is the eyes' sight, as the center of the world. I will, therefore, cut off two equal arcs AB and GD and draw lines BE, BZ, GE, and GZ. Then I say that because the star has uniform motion in arcs AB and GD, it nevertheless has visibly an irregular motion in the ecliptic. For, because angles AEB and

DEG, angulus AZB utroque minor est, sed et angulus DZG utroque maior. Ergo anguli ad Z inequales sunt, et cum Z sit centrum circuli signorum, cadunt in arcus circuli signorum inequales. Cum ergo super hos arcus vel angulos fiat stelle transitus in temporibus equalibus, accidit secundum aspectum in circulo signorum motus stelle diversus.

Similiter quoque accidit secundum alium modum. Sit enim circulus concentricus ABGD cuius diameter AEG, et super ipsum centrum A epiciclus ZHTK. Intelligamus ergo motum epicicli ab A in B, et interim sit motus corporis stelle in epicicli circumferentia. Cum ergo pervenerit ad utrumlibet punctorum Z T, nulla apparebit diversitas in circulo signorum quia super locum centri A conspicietur. Et cum alibi fuerit inter hec duo puncta, non erit ita. Sit enim quod pervenerit ad locum H in epiciclo. Cum ergo centrum A equali motu suo pervenerit ad punctum B, precedet corpus stelle locum illum secundum quantitatem arcus AH, et erit motus apparens maior medio. Et cum pervenerit ad punctum K in epiciclo, erit motus apparens minor secundum quantitatem arcus AK.

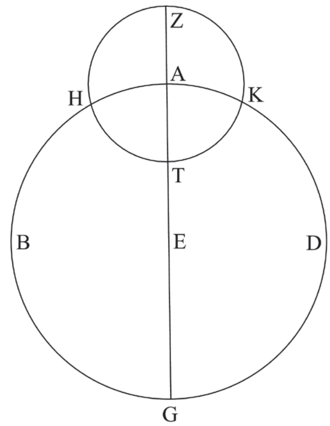


4. Secundum modum orbis ecentrici minor est motus apparens ad longitudinem longiorem et maior ad longitudinem propiorem, secundum vero epicicli modum ad utramque uterque potest accidere.

Resumamus ecentricum cum eadem dispositione. Angulus AZB semper minor est angulo GZD quia angulus AEB semper maior est angulo BZE, et idem, quia eius equalis, semper minor est angulo GZD. Sed super minorem angulum minor est motus et super maiorem maior in tempore equali.

103/104 sed – circuli] *om. P* Angulus vero DZG utroque maior est. Quare cum super eodem centro quiescant ipsi, super arcus inequales cadunt *marg. N* **103** maior] maior est P_7 **104** ad Z] ADZ *corr. in* AZB GZD *K* circuli] *om. M* **104/105** signorum – arcus[1]] *del. N* **105** inequales] equales *P om. N* arcus[2] – angulos] angulos et arcus *N* **109** modum] motum *K* Sit enim] sitque P_7 **112** B] G *K* interim] utrum *P* iterum *K corr. ex* iterum *M* **117** fuerit] fuerint *K* **118** pervenerit] pervenit *K* **120** A] ab *M* equali] equabili *P* motu suo] suo motu P_7K **122** medio] *s.l.* (*perhaps different hand*) *P* **122/123** Et[2] – secundum] *marg.* (*perhaps same hand*) *P* **122** pervenerit] pervenit PP_7 (pervenerit BaE_1) **122/123** in epiciclo] *om. N* **124** modum orbis] *corr. ex* orbis modum P_7 **125** propiorem] propinquiorem P_7 **126** utramque] utrumque *M* **127** Angulus] *om. N* **128** maior est] *corr. ex* minor est *K* est maior *M* BZE] sed angulus AEB semper minor est angulo DZG quia ei equalis DEG semper minor est angulo DZG *add. marg. N* **128/129** et idem] ideo *N* **129** idem] angulus AEB semper minor est angulo DZG quia equalis ei scilicet angulus DEG minor est angulo DZG semper *add. M* eius] ei *MN*

DEG are equal, angle AZB is less than each, but also angle DZG is greater than either. Therefore, the angles at Z are unequal, and because Z is the center of the ecliptic, they fall on unequal arcs of the ecliptic. Therefore, because the star's passage upon these arcs or angles occurs in equal times, an irregular motion of the star in the ecliptic occurs according to sight.



Also, it occurs similarly according to the other way. For let there be concentric circle ABGD, whose diameter is AEG, and epicycle ZHTK upon center A. Then let us understand that the epicycle's motion is from A to B, and that meanwhile the motion of the star's body is on the epicycle's circumference. Therefore, when it comes to either of the points Z or T, no irregularity will appear in the ecliptic because it will be seen upon the place of center A. And when it is in another place between these two points, it will not be thus. For let it have come to point H on the epicycle. Therefore, when center A has come to point B by its own uniform motion, the star's body precedes that place according to the quantity of arc A H and the apparent motion will be greater than the mean motion. And when it comes to point K on the epicycle, the apparent motion will be less according to the quantity of arc AK.

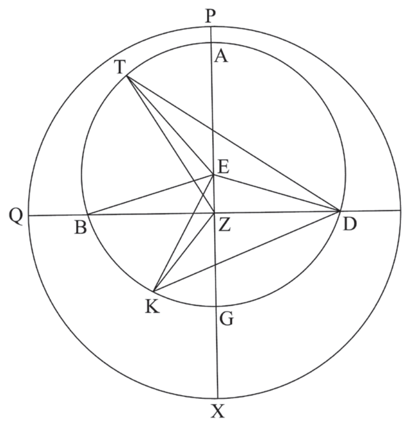
4. According to the eccentric's model [*lit*., way/manner], the apparent motion is less at the apogee and greater at the perigee, but according to the epicycle's model, either can happen at either.

Let us take the eccentric again with the same arrangement. Angle AZB is always less than angle GZD because angle AEB is always greater than angle BZE, and the same ⟨angle AEB⟩, because of its equal [i.e. angle DEG], is always less than angle GZD. But the motion is less upon a smaller angle and greater upon a greater in equal time.

Pono iterum concentricum cum epiciclo secundum priorem dispositionem. Dico quod ad eandem longitudinem et minor et maior motus apparens potest accidere. Ponamus enim centrum epicicli moveri ab occidente in orientem quod est ab A in B. Cum ergo motus corporis stelle fuerit similiter ab occidente in orientem a longitudine longiore quod est a Z in H, tunc motus stelle apparens erit maior ad longitudinem longiorem eo quod ambo motus sint versus eandem partem. Cum vero motus corporis stelle fuerit a longitudine longiore ab oriente in occidentem quod est a Z in K et e contrario motus epicicli, erit motus apparens minor ad longitudinem longiorem quia motus corporis stelle est contra motum epicicli deferentis stellam.

5. Maxima differentia apparentis motus in circulo signorum ad motum medium in ecentrico colligitur in directo puncti circuli signorum medii inter utramque longitudinem. Unde manifestum quod apparens permeatio unius quarte circuli signorum scilicet a longitudine longiore ad punctum medium maioris temporis est, et permeatio alterius quarte a puncto medio ad longitudinem propiorem minoris temporis est; et quod differentia huius temporis ad illud et illius ad hoc est sicut maior differentia collecta motus apparentis ad motum medium duplicata.

Describam itaque stelle orbem ecentricum ABGD supra centrum E, et diameter eius in longitudine longiore AEG. Et ponam in diametro centrum orbis signorum Z quo est aspectus oculorum et super ipsum perpendicularem BZD. Sitque stella super duas notas B D ut sit spatium motus apparentis in circulo signorum quarta a longitudine longiore. Dico quod ad has notas B D maxima differentia duorum motuum colligitur. Protraham enim duas lineas EB ED. Ex

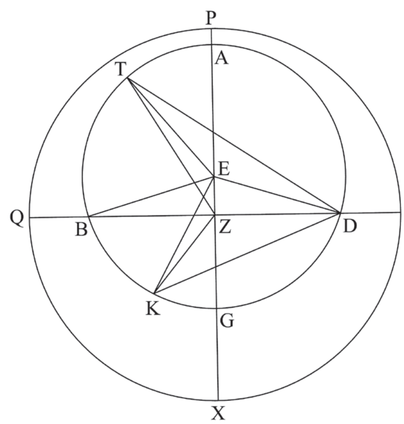


132/133 apparens – accidere] potest accidere apparens saltem *N* **133** occidente – orientem] oriente in occidentem *P corr. ex* oriente in occidentem *N* **134/135** occidente – orientem] *corr. ex* oriente in occidentem *N* **135** longiore] longiorem *K* est] est a X†I^{a†} †H† tunc stelle quod est *P* a Z] *corr. ex* †D†Z *K* in] *s.l. P* **136** sint] sunt P_7M **138** occidentem] occ†iden†dentem *K* a Z] a T *corr. ex* AG *K corr. in* a T *M* et] *om.* P_7 tunc *del. M* motus^2] *om. PN* **140** deferentis] *corr. ex* differentis *K* **141/142** motum medium] medium motum P_7 **142** in directo] *corr. ex* inducto *K* **146** quod] *om. N* **148** motum medium] medium motum *M* **149** Describam] describamus *N* itaque] quoque *M* ecentricum] *corr. ex* ad centricum *K* **150** diameter] dyametrum *M* **153** Z] *corr. ex* G *K* Z a *MN* **156** circulo] orbe *N* **158** ad] *om. P* apud *marg. N* notas] duas notas *PN* notas scilicet P_7 B] B et P_7 *corr. ex* G *K* **159** colligitur] concolligitur *K*

Again, I place a concentric with an epicycle according to the earlier arrangement. I say that at the same distance [i.e. at apogee or perigee] both a lesser and greater apparent motion can occur. For let us posit that the epicycle's center is moved from west to east, which is from A to B. Therefore, when the motion of the star's body is similarly from west to east from the apogee, which is from Z to H, then the star's apparent motion will be greater at apogee because both motions are in the same direction. However, when the motion of the star's body is from east to west from the apogee, which is from Z to K, and opposite the epicycle's motion, then the apparent motion will be less at the apogee because the motion of the star's body is against the motion of the epicycle carrying the star.

5. The maximum difference between the apparent motion on the ecliptic and the mean motion on the eccentric is obtained in the direction of the point of the ecliptic midway between the two apsides. Whence it is manifest that the apparent traverse of one quarter of the ecliptic, i.e. from the apogee to the mean point, is of a greater time; the traverse of the other quarter from the mean point to the perigee is of less time; and the difference of this time to that and of that to this is double the greatest difference obtained between the apparent motion and the mean motion.

Accordingly, I will describe the star's eccentric circle ABGD upon center E and its diameter AEG on the apogee. And I will place on the diameter the ecliptic's center Z, where the eyes' sight is, and upon that the perpendicular BZD. And let the star be upon the two points B and D so that the distance of the apparent motion in the ecliptic from the apogee is a quarter circle. I say that the two motions' maximum difference is obtained at these points B and D. For I will draw the two lines EB and ED. From this,



hoc ergo declaratur quod proportio EBZ anguli ad iiii rectos sicut arcus differentie ad totum circulum quoniam AEB est sub arcu motus medii AB et angulus AZB est sub arcu motus apparentis PQ et superfluum quod est inter eos est angulus propositus EBZ. Cum ergo ambo motus conveniant ad utramque longitudinem, dico quod non erigitur maior angulus in circulo ABGD utrolibet istorum supra lineam EZ. Erigantur enim duo anguli aput punctum T et aput punctum K, et protraham duas lineas TD KD. Vides ergo quod linea TZ maior est linea ZD; ergo maiori angulo subtenditur. Demptis ergo equalibus remanet angulus EDZ maior angulo ZTE. Item quia ZD linea maior est linea KZ, maiori angulo subtenditur. Demptis inequalibus ab equalibus, remanet angulus ZDE qui est equalis angulo ZBE maior angulo ZKE.

Cum hoc etiam declaratum est quod arcus AB qui est quantitas temporis apparentis permeationis a P in Q maior est arcu BG qui est quantitas temporis apparentis permeationis alterius quarte a Q in X; et quod differentia huius temporis ad illud et illius ad hoc est arcus differentie duorum motuum duplicatus quia angulus AEB qui est maioris temporis angulus superat angulum BEG qui est angulus minoris temporis duplo anguli EBZ qui est angulus differentie, et hoc est quod proponitur.

6. Iuxta modum epicicli cum equales in proportione semper fuerint motus epicicli in concentrico et stelle in epiciclo fueritque motus minor ad longitudinem longiorem, maxima differentia motus diversi ad motum medium colligitur cum apparuerit discessus stelle a longiori longitudine quarta circuli. Sequiturque quod apparens permeatio unius quarte ab auge maioris temporis sit, et permeatio alterius quarte ad augis oppositum est minoris temporis, fitque differentia duorum temporum sicut duplum differentie maxime diversi motus ad motum medium.

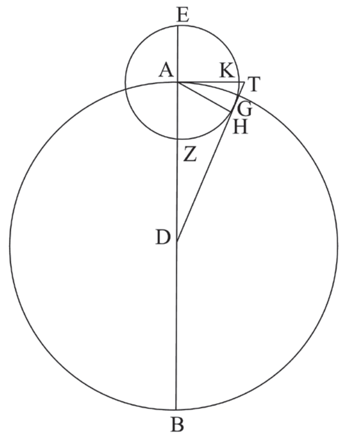
Esto circulus concentricus ABG supra centrum D et diameter ADB et epiciclus in superficie eadem EZH super centrum A. Ponamus itaque stellam super punctum H, cum discessus eius a longitudine longiore apparuerit quarta cir-

161 declaratur] patet *PN corr. in* patet *M* (declaratur *BaE*$_1$) anguli] qui est differentia duarum angulorum motus medii et motus apparentis *add. marg. N* rectos] est *add. s.l.* (*other hand*) *P* rectos sit *N* sicut] sicut proportio *N* **162** quoniam] quorum *N* **164** EBZ] *corr. ex* ABZ *K* ergo] vero *PN* **166** enim] *corr. ex* ergo *M* **168** ZD] *corr. ex* TD *K* **169** Item quia] itemque *K* linea^1] *om. PN* linea^2] *om. P*$_7$ **170** ab] sub *P* **171** ZBE – angulo^2] *om. N* **172** etiam] item *corr. ex* †in† *K* quantitas] quantitatis *N* **173** permeationis – Q] meationis AB inquam *corr. ex* quod meationis ap†ud† inquam *K* **174** a – X] *om. K* **176** temporis] partis *P*$_7$ **177** temporis] partis *K* EBZ] ABZ *K* **179** in – semper] semper in proportione *N* **180/181** minor – longiorem] minor ad longiorem longitudinem *P* ad longiorem longitudinem minor *N* longitudinem longiorem] longiorem longitudinem *P*$_7$ **182** discessus – longitudine] stelle discessus a longitudine longiore *M* **184** ad] ab *P* **187** diameter] dyametrum *N* ADB] ABD *K corr. ex* ABD *M* **188** EZH] *corr. ex* EHZ *M* A] *corr. ex* H *K* therefore, it is declared that the ratio of angle EBZ to 4 rights is as the arc of the difference to the whole circle because AEB is under the mean motion's arc AB and angle AZB is under the apparent motion's arc PQ and the difference that is between them is the proposed angle EBZ. Then, because both motions agree at each apsis, I say that a greater angle is not set up in circle ABGD upon line EZ on either side of them. For let two angles be set up at point T and at point K, and I will draw the two lines TD and KD. Therefore, you see that line TZ is greater than line ZD; therefore, it subtends a greater angle. Therefore, with equals subtracted, angle EDZ remains greater than angle ZTE. Likewise, because line ZD is greater than line KZ, it subtends a greater angle. With unequals subtracted from equals, there remains angle ZDE, which is equal to angle ZBE, greater than angle ZKE.

With this it has also been declared that arc AB, which is the quantity of the time of the apparent traverse from P to Q, is greater than arc BG, which is the quantity of time of the apparent traverse of the other quarter from Q to X; and that the difference of this time to that and of that to this is the doubled arc of the two motions' difference because angle AEB, which is the angle of greater time, exceeds angle BEG, which is the angle of less time, by double angle EBZ, which is the angle of the difference, and this is what is proposed.

6. According to the epicyclic model, when the motions of the epicycle on the concentric and of the star on the epicycle are always equal in ratio and the lesser motion is at the apogee, the greatest difference between the irregular motion and the mean motion is obtained when the star's distance from the apogee is seen to be a quarter circle. And it follows that the apparent traverse of the one quarter from apogee is of a greater time, and the traverse of the other quarter to perigee is of less time, and the difference of the two times is double the greatest difference between the irregular motion and the mean motion.

Let there be concentric circle ABG upon center D, diameter ADB, and epicycle EZH in the same plane upon center A. Accordingly, let us place the star upon point H, when its distance from the apogee is seen to be a quarter of

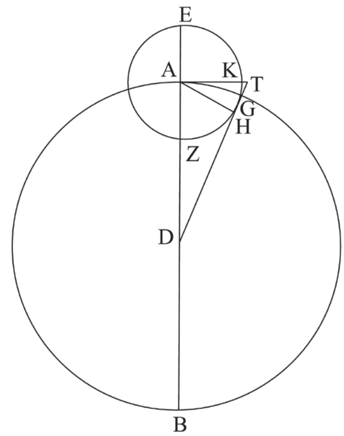


culi concentrici. Dico quod aput hunc punctum est maior differentia motus diversi ad motum medium. Protractis siquidem lineis AH et DHG erit linea DHG contingens circulum parvum. Motus enim medius a longitudine longiore continetur sub angulo EAH, propter hoc quod motus stelle in epiciclo et motus centri epicicli in concentrico sunt equalis velocitatis. Sed differentia que videtur inter motum medium et motum diversum continetur sub angulo ADH; remanet ergo motus diversus contentus sub DHA. Sed spatium huius motus erat quarta circuli, quare angulus DHA est rectus, ideoque linea DH contingens epiciclum. Quapropter arcus AG est maior differentia que contingit inter motum diversum apparentem et motum medium.

Palam ergo ex dictis quod arcus EH est tempus apparentis permeationis unius quarte que videtur ab auge, et arcus HZ est tempus apparentis permeationis alterius quarte usque ad oppositum augis. Est ergo illud tempus maius isto. Et dico quod differentia temporum est arcus AG duplicatus. Si enim eduxerimus lineam DHT et eiecerimus ortogonalem super lineam EZ, linea AKT, erit angulus KAH equalis angulo GDA; quare KH erit sicut arcus AG differentia duorum motuum. Sed arcus HE superat quartam hoc arcu KH, et quarta superat arcum HZ eodem. Palam ergo quod promisimus.

7. Cum equales in proportione fuerint tres motus semper, motus stelle in ecentrico, motus epicicli in concentrico, motus e contrario stelle in epiciclo, fuerintque equalis magnitudinis ecentricus et concentricus atque semidiameter epicicli equalis distantie centrorum illorum, quicquid accidit secundum unum modorum accidit secundum alterum. Quia par est motus medius, par est diversus, una est differentia motuum, unus et idem apparet stelle locus in circulo signorum.

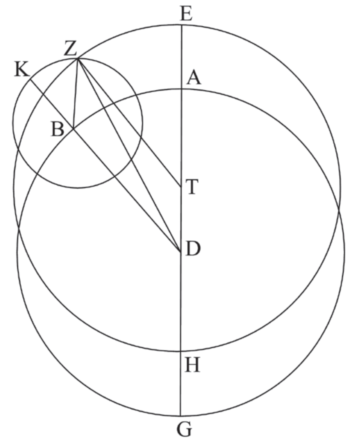
190 concentrici] ecentrici *P* centrici P_7 (concentrici *Ba* ecentrici E_1) hunc punctum] punctum hunc *K* **191** est maior] H (*corr. ex* Z) maior est *M* **193** erit – DHG[2]] *marg.* (*perhaps other hand*) *P* contingens] *corr. ex* continens P_7 **195** EAH] *corr. ex* EH P_7 **197** et] *om. P* **201** contentus] contemptus *K* concentricus *M corr. ex* contemptus *N* **202** Sed] *corr. ex* sub *P* **203** epiciclum] circulum epiciclum *N* **204** maior] *corr. ex* magior P_7 **207** unius – permeationis] *om.* P_7 videtur] producitur *K* HZ] *corr. ex* HA *N* **209** differentia] differentie *PN* **210** linea] lineam *M* **211** KAH] *corr. ex* KAGH P_7 sicut] *s.l. K* **212** Sed] cumque *KM* (cumque *Ba* sed E_1) **213** Palam ergo] ergo palam P_7 promisimus] proposuimus *K* premisimus *N* **214** in[1] – fuerint] fuerint in proportione *K* semper] scilicet *N* motus[2]] *s.l. P* **215** concentrico] *corr. ex* centrico *K* in[2]] *s.l. K* **218** par[1]] *corr. ex* parvus *P* par[2]] *corr. ex* parvus *P*



the concentric circle. I say that the greatest difference of the irregular motion from the mean motion is at this point. Accordingly, with lines AH and DHG drawn, line DHG will be tangent to the small circle. For the mean motion from the apogee is contained under angle EAH, because of this that the star's motion on the epicycle and the motion of the epicycle's center on the concentric are of equal speed. But the difference that is seen between the mean motion and the irregular motion is contained under angle ADH; therefore, the irregular motion remains contained under DHA. But the distance of this motion was a quarter circle, so angle DHA is right. And for that reason line DH is tangent to the epicycle. For this reason arc AG is the greatest distance that occurs between the irregular apparent motion and the mean motion.

Therefore, it is clear from what has been said that arc EH is the time of the apparent traverse of one quarter that is seen from apogee, and arc HZ is the time of the apparent traverse of the other quarter to perigee. Therefore, this time is greater than that. And I say that the difference of the times is double arc AG. For if we will draw line DHT and throw out a perpendicular upon line EZ, ⟨i.e.⟩ line AKT, angle KAH will be equal to angle GDA; therefore, KH, as also arc AG, will be the difference of the two motions. But arc HE exceeds a quarter circle by this arc KH, and a quarter circle exceeds arc HZ by the same. Therefore, what we promised is clear.

7. When the three motions, ⟨i.e.⟩ the star's motion on the eccentric, the epicycle's motion on the concentric, and the star's opposite motion on the epicycle, are always equal in ratio, and the eccentric and the concentric are of equal size and the epicycle's radius is equal to the eccentricity, whatever happens according to one of the models happens according to the other. Because the mean motion is equal, and the irregular motion is equal, and the difference of the motions is one [i.e. the same], one and the same place of the star is seen in the ecliptic.



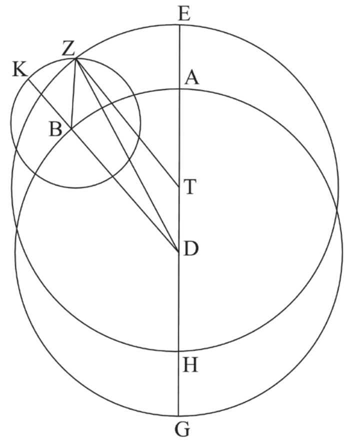
Describam ad hoc circulum concentricum ABG supra centrum D et circulum ecentricum ei equalem EZH supra centrum T et supra diametrum unam ambobus communem per duo centra et longitudinem longiorem quod est punctum E transeuntem. Et describam epiciclum KZ supra centrum B secundum spatium DT. Cum sumpsero arcum AB secundum libitum ex circulo ABG, dico ergo quod locus stelle est semper in sectione communi epicicli et ecentrici ut aput punctum Z, semper enim fiunt tres arcus similes KZ ZE AB epicicli ecentrici et concentrici. Protraham ergo ad hoc lineas ZT BZ DZ. Fiet igitur quadrilatera figura BZDT. Et quia opposita latera per ypothesim sunt equalia, erit ipsa equidistantium laterum. Erunt ergo tres anguli sibi invicem equales, duo scilicet BDA et ZTE, et propter hoc par est semper motus medius, et tertius angulus KBZ. Eritque apparens locus stelle super lineam DZ. Quare secundum utrumque modum est apparens motus stelle ad punctum Z et idem locus stelle in circulo signorum in directo huius puncti.

Dico etiam quod una est secundum utrumque modum motuum differentia. Quia secundum modum ecentrici angulus DZT est angulus differentie sicut preostendimus, et secundum modum epicicli est angulus differentie BDZ. At isti cum sint coalterni sunt equales. Palam ergo quod in omnibus elongationibus similiter erit quadrilaterum, namque semper fit equidistantium laterum, et quod motus stelle in epiciclo ecentricum signat nec ab eo discedit.

8. Et si inequalis magnitudinis fuerint ecentricus et concentricus dummodo proportionales fuerint eorum semidiametri ad distantiam centrorum ipsorum et semidiametrum epicicli, ceteris manentibus, omnia similiter accident.

Ad huius exemplum circumducam circulum concentricum ABG supra centrum D, sitque diameter eius super quem sit stella in utraque longitudine ADG, et epiciclus supra centrum B cuius puncti a longitudine longiori remo-

221 concentricum] *perhaps corr. ex* ecentricum P_7 **223** ecentricum – equalem] econcentricum ei equale *P* **225** et – longiorem] a longitudine longiori *K* **226** quod est] scilicet *N* **230** ergo] *om.* P_7 est semper] semper est *K* **232** – fiunt] semper fiunt *P* semper ettenim fiunt *M* et fiunt semper *N* (semper enim fiunt BaE_1) **233/234** epicicli – concentrici] ecentrici epicicli et concentrici P_7 epicicli concentrici et ecentrici *MN* **234** Fiet] fit *K* **236** Erunt ergo] ergo erunt *K* sibi] *om.* P_7 duo] DUC *P* **237** ZTE] TZE *M* par – semper] est semper par *M* **238** apparens locus] locus apparens P_7 lineam] linea P_7 **241** modum motuum] motum P_7 **242** modum] motum P_7 *om. N* ecentrici] econcentrici *P* **244** ergo] *om. M* **245** erit] *iter. et del. M* **249** similiter] simul *P* accident] accidunt *K* **251** quem] quam *N* **252** ADG] AGD *K*

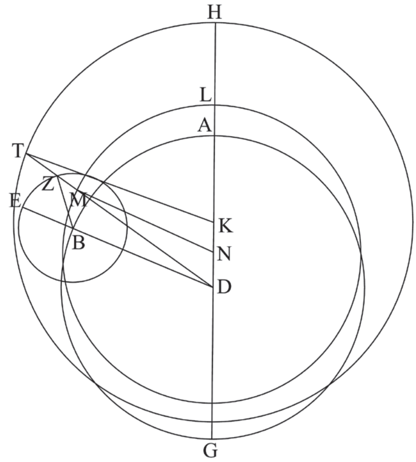


I will describe for this concentric circle ABG upon center D, and eccentric circle EZH equal to it upon center T and upon one diameter common to both passing through the two centers and the apogee, which is point E. And I will describe epicycle KZ upon center B according to distance DT. When I will have taken arc AB according to pleasure from circle ABG, I say then that the star's place is always at the intersection of the epicycle and eccentric as at point Z, for the three arcs KZ, ZE, and AB of the epicycle, eccentric, and concentric are always similar. Then, for this [i.e. to prove this] I will draw lines ZT, BZ, and DZ. And therefore quadrilateral figure BZDT will be made. And because the opposite sides are equal by hypothesis, it will be of parallel sides. Therefore, the three angles will be equal to each other, i.e. the two BDA and ZTE, and because of this the mean motion is always equal, and also the third angle KBZ. And the star's apparent place will be upon line DZ. Therefore, according to either model the star's apparent motion is at point Z and the same place of the star in the ecliptic is in the direction of this point.

I say also that the difference of the motions is one according to either model. Because according to the model of the eccentric, angle DZT is the angle of the difference as we showed before, and according to the model of the epicycle, the angle of the difference is BDZ. But these are equal because they are coalternate. It is clear, therefore, that in all elongations there will similarly be a quadrilateral, for it will always be made with parallel sides, and that the star's motion in the epicycle designates the eccentric and does not depart from it.

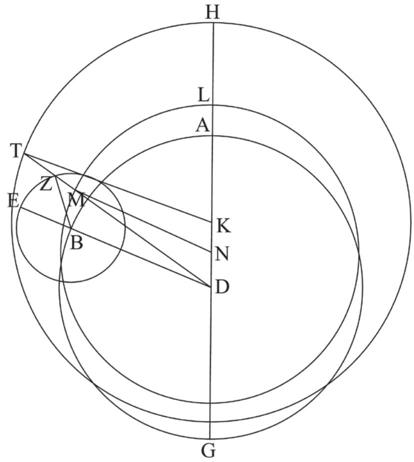
8. And if the eccentric and the concentric are of unequal size provided that their radii are proportional to the eccentricity and the epicycle's radius, with everything else remaining the same, all will occur similarly.

For an example of this, I will describe concentric circle ABG upon center D, and let ADG be its diameter upon which the star may be upon either apsis, and the epicycle upon center B, the distance of which point from the apogee tio est arcus AB secundum quantitatem quam voluerimus, et designatio epicicli EZ. Moveaturque stella in epiciclo secundum quantitatem EZ, qui arcus monstratur esse similis arcui AB eo quod motus ex ypothesi sunt equales. Protraham ergo lineas DBE et BZ et DZ. Quare duo anguli ADE et ZBE sunt equales semper, et stella secundum hunc modum videtur super lineam DZ.



Pono iterum circulos ecentricos, unum maiorem concentrico HT cuius centrum K super diametrum AG, et alium minorem LM supra centrum N. Et producam lineam directe DMZ ad punctum T et lineam DAL ad punctum H. Dehinc protraham duas lineas TK MN. Et sit proportio DB ad BZ lineam sicut linee KT ad KD et sicut proportio MN ad ND. Cum ergo angulus TDK sit equalis angulo BZD propter equidistantiam linearum BZ et DA, accidit ex septima sexti Euclidis quod anguli quibus latera proportionalia subtenduntur sunt equales, scilicet angulus BDT et angulus DTK et angulus DMN. Linee ergo BD et MN et TK sunt equidistantes. Quapropter erunt anguli ADB et ANM et AKT equales, et quia omnes aput centrum circulorum consistunt, erunt arcus qui eis subtenduntur similes, scilicet arcus AB et arcus HT et arcus LM. Eodem ergo tempore quo centrum epicicli secundum eius modum pertransit arcum AB et stella ex epiciclo arcum EZ, pertransit stella secundum alium modum de ecentrico maiore arcum HT et de ecentrico minore arcum LM. Quare videbitur secundum utrumque modum super lineam DMZT, sed in orbe epicicli cum fuerit super punctum Z, et in orbe ecentrici maioris cum fuerit supra punctum T, et in orbe ecentrici minoris cum fuerit supra punctum M, et hoc est quod volebamus.

253 AB] AD *P* **255** Moveaturque] videaturque *N* **257** monstratur] demonstratur *M* **258** motus] *iter. et del.* P_7 **259** DBE] DEB *K* **259/260** et[2] – duo] quare *corr. ex* quart- *K* **268** T] *corr. ex* K *N* DAL] *corr. ex* DHL *N* **269** sicut[1]] cuius P_7 linee] *om.* *N* KT] *corr. ex* KD P_7 proportio[2]] *om.* *N* **270** BZD] *corr. in* ZBD *N* **271** BZ] BM *PK* *corr. ex* BM P_7M (BZ *Ba* BM E_1) et DA] GA P_7 septima] *corr. in* sexta *N* sexti] *marg.* P_7 quod] vel *K* **273** MN] AM P_7 sunt] semper P_7 **274** ADB] ABD *K* ANM] AMN *M* equales] *om.* PP_7K (*om.* BaE_1) **274/275** omnes aput] *corr. ex* apud omnes *K* **275** centrum] centra P_7M **276** AB] *corr. ex* HB *M* **277** secundum – pertransit] pertransit secundum eius modum *N* modum] motum *MN* ex] in *PMN* (ex BaE_1) **278** secundum] et *P* modum] *corr. in* motum *M* **279/280** modum super] motum super *corr. ex* modum secundum *M* **279** modum] *corr. ex* lineam P_7 **281** T] *s.l.* *K*



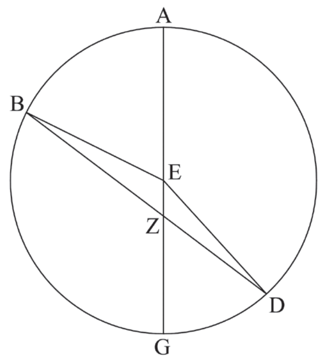
is arc AB according to a quantity that we wish, and the designation of the epicycle is EZ. And let the star be moved on the epicycle according to quantity EZ, which arc may be shown to be similar to arc AB because the motions are equal by hypothesis. Then I draw lines DBE, BZ, and DZ. Therefore, the two angles ADE and ZBE are always equal, and the star according to this model is seen upon line DZ.

Again, I posit eccentric circles, one HT greater than the concentric, whose center is K upon diameter AG, and another LM upon center N, smaller ⟨than the concentric⟩. And I will draw a line DMZ straight towards point T, and line DAL to point H. Then I will draw the two lines TK and MN. And let the ratio of DB to line BZ be as that of line KT to KD and as the ratio of MN to ND. Therefore, because angle TDK is equal to angle BZD because of the parallelness of lines BZ[9] and DA, it happens from the seventh of the sixth of Euclid that the angles that the proportional sides subtend are equal, i.e. angle BDT, angle DTK, and angle DMN. Therefore, lines BD, MN, and TK are parallel. For this reason, angles ADB, ANM, and AKT will be equal,[10] and because all stand at the center of circles, the arcs that subtend them will be similar, i.e. arc AB, arc HT, and arc LM. In the same time, therefore, in which the epicycle's center passes through arc AB according to its model and the star on the epicycle passes through arc EZ, according to the other model the star passes through arc HT of the greater eccentric and arc LM of the smaller eccentric. Therefore, it will be seen according to either model upon line DMZT, but on the circle of the epicycle when it is upon point Z, and on the circle of the greater eccentric when it is upon point T, and on the circle of the smaller eccentric when it is upon point M, and this is what we wanted.

[9] The 'BM' found in so many of the witnesses may have been the author's mistake.

[10] Because only *M* and *N* have 'equales', it was likely omitted by the author or early in the text's transmission although it is needed for the meaning here.

9. Stella in duobus punctis circuli signorum oppositis posita, iuxta modum ecentrici eadem vel nulla erit apparens a duabus longitudinibus in circulo signorum distantia, et eadem vel nulla erit duorum motuum differentia.



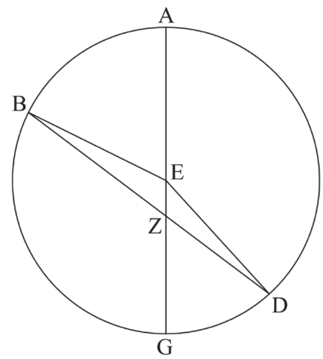
Si enim lineaverimus ecentricum ABG supra centrum E, cuius diameter super utramque longitudinem transiens AEG, et posuerimus aspectum oculorum super diametrum a puncto Z, stella que in terminis huius diametri circuli signorum posita nullam habebit a duabus longitudinibus distantiam, neque erit motuum differentia. Sed si alium quocumque modo duxerimus circuli signorum diametrum ut lineam BZD, tunc duo anguli aspectus AZB et DZG fiunt equales. Ideoque apparens distantia a duabus longitudinibus equalis, et est differentia motuum ad utrumque punctum una quia anguli differentie scilicet EBZ et EDZ sunt equales. Motus enim medius a longitudine longiore sub angulo AEB contentus maior est motu diverso sub angulo AZB contento quantitate anguli EBZ, et motus medius a longitudine propiore sub angulo GED contentus minor est motu diverso sub angulo DZG contento quantitate anguli EDZ. Et hoc est quod proponitur.

10. Iuxta modum epicicli stella supra rectam lineam a centro mundi epiciclum secando eductam in duobus locis sectionum posita, eadem vel nulla erit apparens a duabus longitudinibus in circulo signorum distantia, et eadem vel nulla erit motuum differentia.

Quia si lineaverimus concentricum ABG supra centrum D cuius diameter super utramque longitudinem ADG et epiciclum EZH supra centrum A et diametrum communem ET, stella quidem in terminis huius diametri posita nullam habebit a duabus longitudinibus distantiam nec motuum differentiam. Sed educta linea quocumque modo aliter a centro D per epiciclum ut est linea DHZ, stella in hiis duobus punctis H Z posita equales videbitur in circulo

283 duobus] et *add. et del. K* **286** ABG] ABDG *M* **288/287** utramque] *corr. ex* utrumque *K* **290** Z – que] Z (*corr. ex* A) stellaque *N* que] quod *P* quidem *KM* **292** neque] nec *K* **293** alium] aliam *MN* **295** BZD] BZD et duas lineas EB CD *M* **296** equales] equales per 15^{am} secundi *M* **297** distantia] *corr. ex* di†ffere†ntia *P* **299** et EDZ] *s.l. K* **300** AEB] DEB *P corr. ex* ZEB *M* contentus] contemptus *K* est] *om. PK s.l.* P_7*MN* (est *Ba om.* E_1) contento] contempta *K* **302** GED] GZD *P corr. in* GZD *M corr. ex* GZD *N* contentus] contemptus *K* contento] contempto *K* **305** secando] sequante *K* **308** Quia] quod *K* lineaverimus] *corr. ex* lineverimus *K* **309** ADG] AGD *K* EZH] CZH *P* **310** ET] EC *P* EG *corr. ex* E†C† *K* C *M* (et *Ba* ET E_1) quidem] que *M* **311** nec – differentiam] *iter. et del. P* **312** quocumque] quoque *K* **313** H] H et *M* posita] secundum *add. et del. N* videbitur] videbuntur *M*

9. With the star supposed at two opposite points of the ecliptic, according to the model of the eccentric, the apparent distance in the ecliptic from the two apsides will be the same or nothing, and the difference of the two motions will be the same or nothing.



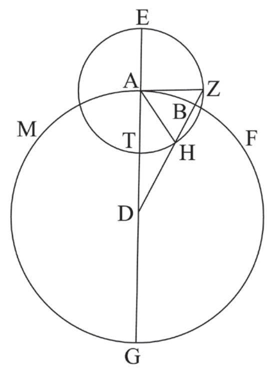
For if we draw upon center E eccentric ABG, whose diameter passing through each apsis is AEG, and we suppose the eyes' sight upon the diameter from point Z, the star that is placed on the endpoints of this diameter of the ecliptic will have no distance from the two apsides, nor will there be a difference of motions. But if we draw another diameter of the ecliptic in any way as line BZD, then the two angles of sight AZB and DZG are equal. And for that reason, the apparent distance from the two apsides is equal, and there is one difference of motions at each point because the angles of difference, i.e. EBZ and EDZ, are equal. For the mean motion from the apogee contained under angle AEB is[11] greater than the irregular motion contained under angle AZB by the quantity of angle EBZ, and the mean motion from perigee contained under angle GED is less than the irregular motion contained under angle DZG by the quantity of angle EDZ. And this is what is proposed.

10. According to the mode of the epicycle with the star supposed at the two places of division upon the straight line drawn by cutting the epicycle from the center of the world, there will be the same or no apparent distance from the two apsides in the ecliptic, and there will be the same or no difference of the motions.

Because if we draw upon center D concentric ABG, whose diameter upon each apsis is ADG, and the epicycle EZH upon center A and common diameter ET, indeed the star supposed at the endpoints of this diameter will have no distance from the two apsides nor a difference of motions. But with a line drawn in any other way from center D through the epicycle, as is line DHZ the star supposed at these two points H and Z will appear to have equal distances in

[11] From the manuscript witnesses, it appears that the needed 'est' was omitted by the author or was dropped early in the text's transmission.

signorum a longitudine longiore et a longitudine propiori habere distantias. Ea enim posita super punctum Z erit motus apparens a longitudine longiori contentus sub angulo AZD et arcus differentie AB. Et ea posita super punctum H erit motus apparens in circulo signorum a longitudine propiori contentus sub angulo AHZ quia motus medius continetur sub angulo DAH, et arcus differentie idem qui prius AB qui subtenditur angulo ADB. Est autem angulus AZH equalis angulo AHZ. Quapropter motus medius a longitudine longiori sub angulo EAZ contentus maior est motu diverso sub angulo AZD contento quantitate anguli ADZ, et motus medius a longitudine propiore sub angulo DAH contentus minor est motu diverso sub angulo ZHA contento quantitate eiusdem anguli ADH, et hoc est.

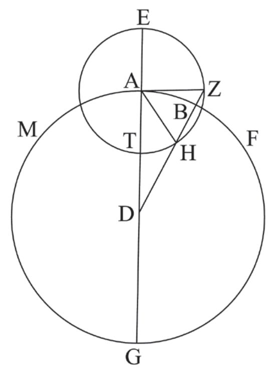


Ex premissis itaque colligitur quod stella unam solam causam diversi motus apparentis in circulo signorum habente – possibile est enim utrasque causas diversitatis simul in uno composito motu subesse – satis est secundum unum dictorum modorum diversum motum stelle assignare. Unicam autem causam diversi motus Solem compertum est habere.

11. Proportionem distantie inter centrum circuli signorum et centrum solaris ecentrici ad semidiametrum ecentrici necnon in cuius partis circuli signorum directo sit longitudo longior ecentrici deprehendere.

Hec proportio aliter deprehendi non potuit quam per notum tempus permeationis unius medietatis circuli signorum et per notum tempus permeationis unius quarte, et hec quidem tempora fere sunt ab equinoctio ad oppositum equinoctium et iterum ab equinoctio ad solstitium. Hiis ergo temporibus per instrumenta veridica ut in expositione prime propositionis presentis libri diximus deprehensis, ad propositi notitiam lineabimus circulum signorum ABGD

315 habere] *corr. ex* habente P_7 **317** contentus] contemptus *K* AZD] AZB *P* **318** ea] *corr. ex* AE *KM* **319** circulo signorum] signorum circulo *corr. ex* signo circulo *K* signorum circulo *M* **320** contentus] contemptus *K* **321** DAH] EAZ *N* **322** prius] *corr. in* primus *P* primus *N* **325** EAZ] AEZ *K* **327** a – propiore] a longitud†o† propior est *P* ad longitudinem propiorem P_7M (a longitudine propiore *Ba* ad longitudinem propiorem E_1) DAH] HAC *P* AHZ *K* HAT *N* (ABZ *Ba* HAT E_1) contentus] contemptus *K* **328** ZHA] ZAH *K* et – est] et hoc est quod voluimus *M om. N* **329** itaque] igitur *PN* colligitur] coll†igitur† *corr. ex* concl- *K* **330** apparentis – signorum] in circulo signorum apparentis P_7 est enim] enim est *N* **331** motu] modo *N* **332** Unicam] unam P_7 **333** compertum] compertus *K* **334** et] *corr. ex* in *K* **335** ad] et *M* **337** aliter – potuit] non potuit aliter deprehendi *PN* **339** quidem] *om. N* equinoctio] *corr. ex* equino (*other hand*) *K* **340** equinoctium] equinoctii *M* ergo] *om. N* **341** propositionis] *corr. ex* proportionis *K* presentis] *marg.* (*perhaps other hand*) *P* huius *N*

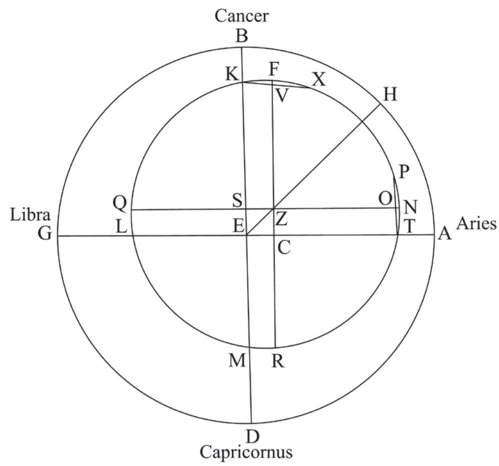


the ecliptic from the apogee and from perigee. For with it [i.e. the star] placed upon point Z, the apparent motion from the apogee will be contained under angle AZD and the arc of the difference will be AB. And with it placed upon point H, the apparent motion in the ecliptic from perigee will be contained under angle AHZ because the mean motion is contained under angle DAH, and the arc of the difference will be the same AB that was earlier, which subtends angle ADB. Moreover, angle AZH is equal to angle AHZ. For this reason the mean motion from apogee contained under angle EAZ is greater than the irregular motion contained under angle AZD by the quantity of angle ADZ, and the mean motion from perigee contained under angle DAH is less than the irregular motion contained under angle ZHA by the quantity of that same angle ADH, and this is ⟨what was proposed⟩.

From what has been set forth, therefore, it is gathered that with a star having one single cause of apparently irregular motion in the ecliptic – for it is possible that both causes of irregularity together are behind the one composite motion – it is enough to allot the star's irregular motion according to one of the said models. Moreover, it is found that the sun has a single cause of irregular motion.

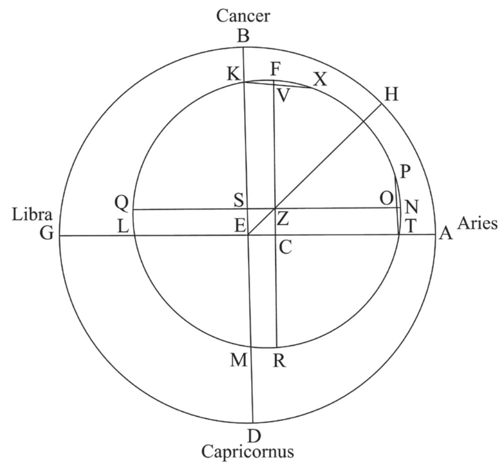
11. To discover the ratio of the distance between the ecliptic's center and the center of the solar eccentric to the eccentric's radius, as well as the direction of the part of the ecliptic in which the apogee of the eccentric may be.

This ratio could not be found in another way than through the known time of the traverse of one half of the ecliptic and through the known time of the traverse of one quarter, and indeed these times are generally from an equinox to the opposite equinox and again from the equinox to the solstice. Then, with these times having been discovered by means of trustworthy instruments, as we said in the exposition of the first proposition of the present book, for the knowledge of the proposition, we will draw ecliptic ABGD upon center E and



supra centrum E et duas eius diametros sese ortogonaliter secantes ad tropica et equinoctialia puncta procedentes AG BD. Et sit A vernale equinoctium, G autumpnale, et B tropicum estivum, D tropicum hiemale. Quia igitur maius tempus deprehenditur a vernali equinoctio ad autumpnale quam e converso et equabilis motus super ecentricum consistit, manifestum quod linea AG ecentricum Solis secat per inequalia et quod maior portio est versus punctum B. Quapropter ex hac parte est centrum ecentrici. Item quia maius tempus deprehenditur a vernali equinoctio ad estivum solstitium quam in altera quarta continua, palam quod iste due linee AE BE maioris portionis ecentrici maiorem partem comprehendunt. Est ergo centrum ecentrici alicubi inter has lineas. Et ponamus esse punctum Z interim. Describam ergo ecentricum Solis supra centrum Z quantumlibet occupando spatium, et sit TKLM. Et educam a puncto Z lineas QZN et RZV equidistantes premissis, et educam lineam per duo centra transeuntem ad circulum signorum EZH. Quantitatem ergo linee EZ respectu semidiametri ecentrici necnon arcum BH in circulo signorum querimus. Protraho itaque perpendicularem a puncto K super lineam RV et sit KVX, et aliam perpendicularem a puncto T super lineam QN et sit TOP. Quia ergo tempus permeationis semicirculi ABG est notum, erit arcus TKL respectu totius circuli super quem equabilis motus consistit notus. Dempta ergo medietate circuli NKQ erit uterque arcuum TN QL cum alter sit equalis alteri notus; quare corda PT et medietas corde TO respectu semidiametri eiusdem circuli nota. Ergo ZC nota sive linea ES. Rursum cum tempus ab A

343 duas] duos *K* **344** sese] Solem *P* **345** secantes] intersecantes *M* **349** tropicum] *om. M* **353** e converso] econtrario P_7 equabilis] *corr. in* equalis *M* equalis *N* **355/354** manifestum] manifestum est *N* **355/356** ecentricum – inequalia] per inequalia secat ecentricum (*the last word in marg.*) *N* **356** inequalia] *corr. ex* equalia P_7 **357** quod] quia P_7 *s.l. K* portio] *corr. ex* proportio *K* **359** estivum solstitium] solstitium estivum *N* **360** quarta] *corr. ex* quadra P_7 **361** alicubi] alicui *P corr. ex* alicui *K* **362** punctum] in puncto P_7 **364** QZN] *corr. ex* QSN *M* RZV] CZV *K* FCR *corr. in* FZR *corr. in* FCR *M* RZX *N* **366/367** respectu – querimus] querimus respectu ... signorum *N* **366** semidiametri] diametri P_7 **367** RV] XV *K* FER *corr. in* FCR *M* **368** QN] SN *K* QSN *M* TOP] *corr. ex* trop†icus† *K* **369** Quia] si *K* **370** respectu – notus] notus respectu ... consistit *M* equabilis] equalis *N* **371** NKQ] *corr. ex* NKC *K* **372** PT] POT *M*



its two diameters AG and BD cutting each other perpendicularly and proceeding to the tropics and equinoctial points. And let A be the vernal equinox, G the autumnal, B the summer tropic, and D the winter tropic. Therefore, because a greater time is found from the vernal equinox to the autumnal than conversely and motion remains uniform on the eccentric, it is manifest that line AG cuts the sun's eccentric into unequals and that the greater part is in the direction of point B. For this reason the center of the eccentric is on this side. Likewise, because a greater time is found from the vernal equinox to the summer solstice than in the other adjacent quarter, it is clear that those two lines AE and BE contain a greater part of the eccentric's greater part. Therefore, the eccentric's center is somewhere between these lines. And let us suppose it to be point Z for the present. I will describe, therefore, the sun's eccentric upon point Z by seizing whatever distance, and let it be TKLM. And I will draw from point Z the lines QZN and RZU parallel to the ones that have been set forth, and I will draw the line EZH passing through the two centers to the ecliptic. Therefore, we seek the quantity of line EZ with respect to the eccentric's radius, as well as arc BH in the ecliptic. Accordingly, I draw a perpendicular from point K upon line RU and let it be KUX, and another perpendicular from point T upon line QN and let it be TOP. Therefore, because the time of the traverse of semicircle ABG is known, arc TKL will be known with respect to the whole circle upon which the uniform motion exists. Therefore, with half circle NKQ subtracted, each of the arcs TN and QL will be known because the one is equal to the other; therefore, chord PT and half chord TO will be known with respect to the same circle's radius. Therefore, line ZC or ES is known. In turn, because in B quod est tempus quarte sit notum, erit arcus TK notus. Dempta ergo NF, cum arcus NT iam sit notus, erit residuus KF; quare corda dupli arcus KVX et medietas eius KV et equalis ei SZ sive EC nota. Et quia EZ recto angulo cuius latera ita sunt nota subtenditur, erit ipsa quoque nota, et hoc respectu partium semidiametri ecentrici, et hoc est pars propositi.

Rursum si ZE semidiametrum parvi circuli supra centrum Z descripti constituamus et lx partium generali more divisionis diametri ponamus, erit quoque hoc respectu corda dimidia EC nota, et arcus circuli parvi supra ipsam notus. Quare angulus CZE qui equalis est angulo SEZ notus. Et quia angulus SEZ notus consistit super centrum circuli signorum, erit arcus BH notus; quare et reliquus AH notus, quod intendebamus.

Nota tamen quod diversi consideratores hanc distantiam centrorum diversarum invenire quantitatum. Ptolomeus duarum partium et dimidie sicut Abrachaz. Albategni vero duarum partium et iiii minutorum et xlv secundorum. Arzachel vero licet variaverit motum medium, eandem tamen quam Albategni invenit centrorum differentiam. Rursum longitudo longior in diversis locis ab eis reperta est. Nam arcus inter tropicum Cancri et longitudinem longiorem sicut Ptolomeus posuit est xxiiii graduum et xxx minutorum, et sicut Albategni vii graduum et xiii minutorum, et sicut Arzacel xii graduum et x minutorum. Huius forsitan diversitatis causa ex parte esse potuit error in instrumento et ex parte motus octave spere ante et retro.

12. Maximam differentiam diversi motus Solis ad motum medium et in quanta elongatione a longitudine longiore in ecentrico ceciderit notificare.

Sit ergo circulus ecentricus ABG supra centrum D et diametrum a longitudine longiore ADG, et in ipso centrum circuli signorum punctum E, et a

374 quarte] *corr. ex* quadrate P_7 **375** KF] *corr. ex* FK *M* KF notus *N* arcus[2]] *marg. P* et] *om. KM* **377** ita] utraque *K* ista *M* **378** ecentrici] *s.l.* P_7 e†men†trici *K om. N* hoc] hec *M* propositi] propositum *K* **380** divisionis] dimidium *N* **381** hoc] *corr. ex* hic P_7 arcus] arcus illius *M* **382** Quare] quia *K* CZE] EZE *P* equalis est] est equalis *PN* **382/383** Et – notus[1]] qui quia P_7 **383** notus[1]] *s.l. P* et] et angulus *K* †angulus† *add. et del. M* **385** distantiam] differentiam *K* **386** invenire] invenere P_7 quantitatum] secundum *add. s.l. P corr. ex* quante *K* Ptolomeus] Ptholomeus P_7 Abrachaz] Abrachis *MN* **387** et[1]] *om. N* xlv] lxv *K* secundorum] secundarum *P* **388** Arzachel] Arzacel *P corr. ex* Arzacel *K* variaverit] narraverit *K* eandem – Albategni] tum eandem cum Albategni tum *N* **389** differentiam] distantiam P_7 *corr. in* distantiam *N* Rursum] rursus P_7 **390** longiorem] *om. N* **391** Ptolomeus posuit] Tholomeus posuit P_7 posuit Ptolomeus *N* xxiiii] *corr. ex* 34 *M* graduum] partium *N* Albategni] Albategni est *M* **392** xiii] 3 P_7 43 *N* Arzacel] Arzachel P_7M Arabes *corr. ex* Ar†...†es *K* Azarchel *N* x] ix *K corr. ex* 9 *M* minutorum[2]] minus P_7 **393** forsitan] forsan *PN* esse potuit] potuit esse *K* in instrumento] instrumenti P_7MN in instrumenta *K* (in instrumentis *Ba* instrumento E_1) ex[2]] *s.l. P* **395/396** motum – quanta] medium in qua *N* **396** quanta] quarta *P corr. ex* quanto *K* **398** diametrum] diameter *PMN* (dyametrum BaE_1) **399** ipso] ipsa *N* **400** E] *perhaps other hand K* a] *corr. in* ab eodem *M* the time from A to B, which is the time of the quarter circle, is known, arc TK will be known. With NF subtracted, therefore, because arc NT is already known, remainder KF will be ⟨known⟩; therefore, the chord KUX of ⟨its⟩ double arc ⟨will be known⟩, and its half KU and SZ or EC equal to it will be known. And because EZ subtends a right angle whose sides are thus known, it will also be known, and this with respect to the parts of the eccentric's radius, and this is a part of the proposition.

In turn, if we set up ZE as radius of a small circle described upon center Z and we suppose it to be 60^{p} in the general custom of the division of the diameter, half chord EC will also be known in this respect, and the arc of the small circle upon it will be known. Therefore, angle CZE, which is equal to angle SEZ, is known. And because angle SEZ stands known upon the ecliptic's center, arc BH will be known; therefore, also remainder AH will be known, which we intended.

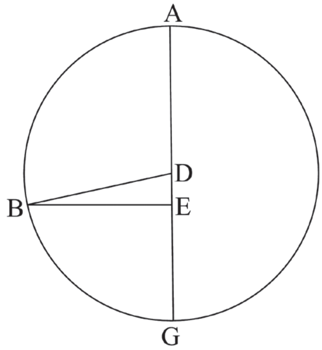
Note, nevertheless, that different observers found this eccentricity to be of different quantities. Ptolemy ⟨found it to be⟩ $2^{p}\,30'$ as did Hipparchus; however, Albategni $2^{p}\,4'\,45''$. And indeed Arzachel, although he changed the mean motion, found the same eccentricity as Albategni. In turn, the apogee was found in different places by them. For the arc between the tropic of Cancer and the apogee as Ptolemy posited is 24° 30′, and as Albategni 7° 13′,[12] and as Arzachel 12° 10′. The cause of this irregularity could perhaps have been partly error in the instrumentation and partly the eighth sphere's motion forwards and backwards.

12. To make known the greatest difference between the sun's irregular motion and the mean motion and in how great of an elongation on the eccentric from the apogee it may fall.

Then, let there be eccentric circle ABG upon center D and ADG, the diameter from the apogee, and on that the ecliptic's center point E, and from this

[12] This should be 7° 43′ to match Albategni's value (*De scientia astrorum*, 1537 ed., f. 29r). That *N* has the correct value suggests that Regiomontanus read *De scientia astrorum* alongside the *Almagesti minor*.

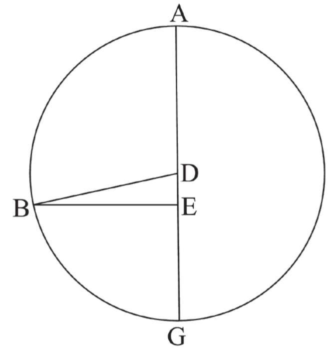
puncto perpendicularis super diametrum linea BE. Et iungo lineam BD. Palam ergo ex quinta presentis quod DBE maximam differentiam continet. Quia autem linea DE respectu partium semidiametri BD nota est, palam quod si centro B posito secundum spatium BD circulum intellexerimus, erit arcus super sinum DE notus; quare angulus DBE notus. Quare et angulus extrinsecus ADB notus; ergo arcus AB notus, et hoc est quod proposuimus.



13. Quamlibet differentiam motus Solis medii et motus diversi per notum arcum motus medii a longitudine longiore secundum ecentrici modum invenire. Unde etiam manifestum quod si notus fuerit unus trium angulorum, scilicet angulus differentie, angulus motus medii, angulus motus diversi, quicumque fuerit, reliqui duo erunt noti.

Pono concentricum circulo signorum ABG supra centrum D, et ecentricum EZH supra centrum T et diametrum transeuntem supra centrum eorum et punctum E quod est longitudo longior. Et excipio arcum EZ de ecentrico secundum libitum, verbi gratia xxx partium. Protractis ergo duabus lineis DZ TZ educam TZ donec a puncto D cadat super eam perpendicularis DK. Quia ergo arcus EZ est notus, erit angulus ETZ sed et equalis ei DTK notus. Sed et angulus TKD rectus. Per circulum ergo parvum centro T spatio DT intellectum, erit proportio linee DT – quotcumque partium ponatur – ad lineam TK sed et ad lineam KD nota. Quare cum linea TD respectu partium diametri ecentrici sit nota, erit similiter et KT et KD nota. Quare tota KZ nota; ergo et illa que subtenditur angulo recto DZ nota. Si ergo hec quoque ponatur lx partium et constituatur semidiameter circuli supra centrum Z et spatio DZ intellecti, erit et sinus DK et arcus super ipsum constitutus. Sed et angulus in arcum cadens notus respectu iiii rectorum, et hic est angulus differentie. Quare

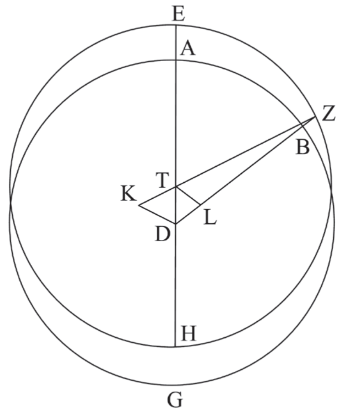
399 puncto] E *add.* (*s.l. K*) *KN* super] sub *K* **400** linea] *om. N* iungo] iunge *PN* lineam BD] BD lineam *N* **402** Quia] si *K* **408** ADB] *corr. ex* ABD *M* **411** arcum] *s.l. P* motus medii] medii motus *K* **413** diversi] *iter. et del. K* **414** fuerit – duo] fuerint reliqui duorum *M* **416** centrum[2]] *corr. ex* centra *P* centra P_7 (centrum *Ba*) **419** super – perpendicularis] perpendicularis super eam *M* **420** ETZ] notus *add. s.l. K* **421** T] D *N* **422** quotcumque] quodcumque *P* **422/423** lineam TK] TK lineam *N* **423** sed et] vel *K* sed] *s.l. P* ad] *s.l.* P_7 KD] *corr. in* KT *M* nota] *marg.* (*perhaps other hand*) *P* **424** erit] erunt *M* nota[3]] tota *P* **425** et] *om.* (*or del.*) *K* subtenditur] subtenduntur *M* **426** semidiameter circuli] circuli semidyameter *N* et[2]] *om.* (*or perhaps del.*) *K* **427** DK] *corr. ex* DZ *K* constitutus] notus *add. s.l. K* in] super P_7 **428** respectu iiii] iiii respectu *corr. ex* iiii vel respectus *K* Quare] *s.l. K*



point a perpendicular, line BE, upon the diameter. And I draw line BD. It is clear, therefore, from the fifth of the present, that DBE contains the greatest difference. Moreover, because line DE is known with respect to the parts of radius BD, it is clear that if we understand a circle with B placed as center and ⟨made⟩ according to distance BD, the arc upon the sine DE will be known; therefore, angle DBE will be known. Therefore, the extrinsic angle ADB is also known, so arc AB is known, and this is what we proposed.

13. To find whatever difference between the sun's mean motion and irregular motion through the known arc of the mean motion from the apogee according to the eccentric model. Whence it is also manifest that if one of the three angles is known, i.e. the angle of the difference, the angle of mean motion, or the angle of irregular motion, whichever it will be, the remaining two will be known.

I suppose ABG concentric to the ecliptic upon center D, and eccentric EZH upon center T and the diameter passing upon their centers and point E, which is the apogee. And I cut off arc EZ from the eccentric according to pleasure, for example 30°. With the two lines DZ and TZ drawn, therefore, I will extend TZ until perpendicular DK falls from point D upon it. Therefore, because arc EZ is known, angle ETZ, and also DTK equal to it, will be known. But also angle TKD is right. Through a little circle, therefore, understood with center T and distance DT, the ratio of line DT – however many parts it may be supposed to be – to line TK, and also to line KD, is known. Therefore, because line TD is known with respect to the parts of the eccentric's diameter, similarly both KT and KD will be known. Therefore, whole KZ is known, so, also that DZ, which subtends a right angle, is known. If, therefore, this also be posited as 60^{p} and is set up as a radius of the circle understood upon center Z and with distance DZ, both sine DK and the arc set up upon it will be ⟨known⟩. But also the angle falling on the arc will be known with respect to four right angles, and this is the angle of the difference. Therefore,



et reliquus BDA qui est angulus apparentis motus in circulo signorum, ac eius arcus AB notus, et hoc est.

Posito quoque arcu AB circuli signorum noto, dico quod et reliqui sunt noti. Lineetur ad hoc a puncto T perpendicularis super lineam DZ, et sit TL. Cum ergo angulus BDA sit notus, erit propter hoc proportio linee DT ad TL nota. Sed proportio DT ad TZ nota; ergo proportio TZ ad TL est nota. Quapropter cum angulus TLZ sit rectus, erit angulus TZD, qui est angulus differentie, notus, et angulus ETZ qui est motus medii et eius arcus EZ notus.

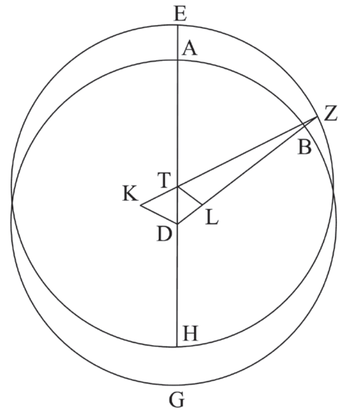
Et iterum si angulus differentie DZT est notus, erit proportio TZ ad TL nota, et propter hoc proportio DT ad TL nota. Quare angulus TDL notus cui equalis est angulus DTK, et huic angulus ETZ, qui est angulus motus medii. Et cum hoc erit notus angulus ADB, qui est angulus motus apparentis.

14. Quamlibet differentiam motus Solis medii et motus diversi per notum arcum motus medii a longitudine longiore secundum epicicli modum reperire. Unde liquidum erit quod si notus fuerit quilibet trium angulorum motus medii, motus diversi, aut differentie, reliqui duo quoque erunt noti.

Describo igitur concentricum ABG supra centrum D cuius diameter ADG, et epiciclum EZHT supra centrum A. Et excipiam arcum EZ xxx partium ut supra. Protractis ergo lineis ZBD et ZA, duco perpendicularem a puncto Z super lineam TE et sit KZ. Quia ergo angulus KAZ est notus, erit proportio AZ linee – quotcumque partium ponatur – ad unamquamque istarum KZ KA nota. Cum ergo AZ ad diametrum AD sit nota, erit DK nota et KZ similiter nota. Quapropter si DZ lx partium ponatur ut sit semidiameter, erit angulus KDZ notus, et hic est angulus differentie. Quare reliquus AZD qui est angulus

429 BDA] notus *add.* (*s.l. K*) *KM* **430** ac] *corr. in* at *K* **430/431** AB notus] notus scilicet AB P_7 notus *N* **431** est] proposuimus *M* est primum *N* **432** Posito quoque] positoque *K* **434** ad – a] ab hoc *K* **435** lineam DZ] DZ lineam *M* TL] TB *P* **436/437** erit – TL] et propter hoc proportio erit linee DT (*corr. ex* DET) ATL *K* **437** TL] TB *P* **437/438** ad TZ] ATL *corr. in* ATZ *K* ad TZ est *N* **438** ad TL] ATL *K* **439** TLZ] *corr. in* LTZ *M* **442** eius arcus] arcus eius *PN* **443** DZT] *corr. ex* TDZ *corr. ex* DEZ *M* TL] TB *K* **444** propter – proportio] proportio propter hoc *K* TDL] DTL P_7 **446** hoc] *s.l. N* erit notus] notus erit *N* ADB] ADG *K* est] ADB *add. et del. N* **448** reperire] invenire *K* **449** notus] *corr. ex* motus *K* **450** duo quoque] quoque duo P_7 quoque] *s.l. P* **451** diameter] diametri P_7 **452** et[1]] *iter. K* EZ] *om. N* **453** ZBD – ZA] ZBD et ZH *P* ZBD ZA *corr. ex* ZB DZ *K* ZA] *corr. ex* Z P_7 **454** Quia] si *K* **456** nota[3]] tota *P* **457** lx] ix *K* lx – ponatur] ponatur 60 partium *N* semidiameter] *corr. ex* diameter P_7

also remainder BDA, which is the angle of apparent motion in the ecliptic, and its arc AB are known, and this is ⟨what was proposed⟩.

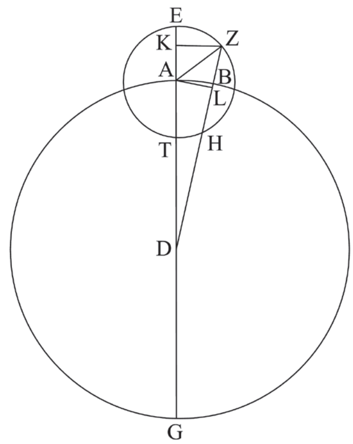


Also, with arc AB of the ecliptic supposed to be known, I say that also the others ⟨angles⟩ are known. For this let a perpendicular upon line DZ be drawn from point T, and let it be TL. Therefore, because angle BDA is known, because of this the ratio of line DT to TL will be known. But the ratio of DT to TZ is known; therefore, the ratio of TZ to TL is known. For this reason, because angle TLZ is right, angle TZD, which is the angle of the difference, will be known, and angle ETZ, which is ⟨the angle⟩ of the mean motion, and its arc EZ will be known.

And again, if angle of the difference DZT is known, the ratio of TZ to TL will be known, and because of this the ratio of DT to TL will be known. Therefore, angle TDL is known, to which angle DTK is equal, and angle ETZ, which is the angle of mean motion, ⟨is equal⟩ to this. And with this, angle ADB, which is the angle of apparent motion, will be known.

14. To find whatever difference between the sun's mean motion and the irregular motion through the known arc of the mean motion from the apogee according to the epicyclic model. Whence it will be certain that if any one of the three angles of the mean motion, the irregular motion, or the difference is known, the remaining two will also be known.

Accordingly, I describe upon center D concentric ABG, whose diameter is ADG, and epicycle EZHT upon center A. And I will cut off arc EZ 30° as above. Then, with lines ZBD and ZA drawn, I draw a perpendicular upon line TE from point Z, and let it be KZ. Therefore, because angle KAZ is known, the ratio of line AZ – however many parts it may be supposed to be – to both KZ and KA will be known. Therefore, because ⟨the ratio of⟩ AZ to diameter AD is known, DK will be known, and KZ will similarly be known. For this reason, if DZ is posited to be 60^p such that it is a radius, angle KDZ will be known, and this is the angle of difference. Therefore, remainder AZD, which



diversi motus est notus. Quod totum sicut iuxta modum ecentrici provenire ratio calculantis comperiet.

Ceterum si notus fuerit angulus motus apparentis, erunt reliqui noti. Ad hoc protrahimus perpendicularem AL super lineam DB. Cum ergo angulus apparentis motus AZL sit notus, erit propter hoc proportio ZA ad AL nota. Sed erat nota proportio ZA ad AD. Est ergo proportio DA ad AL nota. Quapropter erit angulus differentie ADL notus. Et cum hoc angulus extrinsecus EAZ qui est motus medii notus.

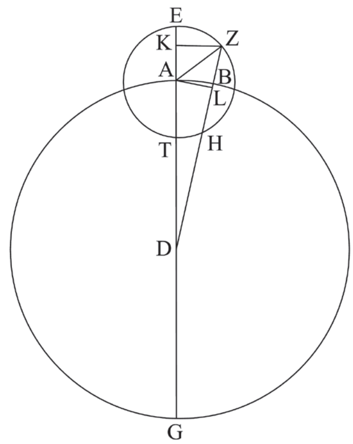
Pari modo sed e contrario, si posuerimus angulum differentie qui est ADB notum, erit propter hoc proportio AD ad AL nota. Sed erat nota proportio AD ad AZ. Est ergo proportio AZ ad AL nota. Quapropter erit angulus AZD notus et hic est angulus diversi motus in circulo signorum. Et cum hoc erit angulus extrinsecus EAZ notus, qui est angulus motus medii.

15. Quamlibet differentiam motus Solis medii et motus diversi per notum arcum motus medii a longitudine propiore secundum ecentrici modum scrutari. Cum quo etiam declarabitur quod si notus fuerit quilibet trium angulorum sive motus medii, sive diversi, sive differentie, reliqui duo quoque erunt noti.

Posito enim scemate orbis ecentrici separabo arcum HZ a puncto H quod est longitudo propior. Et protraham lineas DZB et TZ et perpendicularem DK super lineam TZ. Quia ergo arcus HZ est notus, erit angulus ad centrum ZTH eiusdem quantitatis notus. Erit ergo proportio DT ad utramque DK TK nota. Sed erat ex undecima proportio DT ad TZ nota; quare proportio TZ ad utramque est nota. Ergo et proportio KZ ad DK nota; ergo angulus DZK notus, et hic est angulus differentie. Cum quo etiam angulus extrinsecus GDB notus.

460 provenire] pervenire *N* calculantis] calculationis P_7 **462** angulus] angululus P_7 **464** AL] AG *K* **465** ergo] et *add. et del. K* **466** sit] est *K* **467** nota proportio] proportio nota *N* **470** hoc] hic *K* EAZ] *corr. ex* AEZ *K* **474** nota[1]] *corr. ex* notum *K* AL] *corr. ex* AB *K* **475** hic] hoc *PM* (hic *Ba* hoc E_1) angulus[2] – motus] diversi motus angulus *M* **476** EAZ] AEZ *K* qui] quia P_7 **478** notum] totum *P* **479** propiore] longiore *corr. ex* longiore propiore *P* **481** sive[2]] sive motus *M* duo] *om. K* **482** arcum] arcus *P* **484** HZ] AZ *K iter. et del. N* **485** Erit ergo] ergo erit *M* proportio] *om.* P_7 **486** undecima] ix[a] P_7 vi[a] *K* 11[a] presentis *M* TZ[1]] *corr. ex* DZ P_7 **487** nota[1]] *corr. ex* notam *K* Ergo[1]] *om. N* DZK] *corr. ex* DKZ P_7

is the angle of irregular motion, is known. The reasoning of one calculating will verify that everything results as according to the eccentric model.



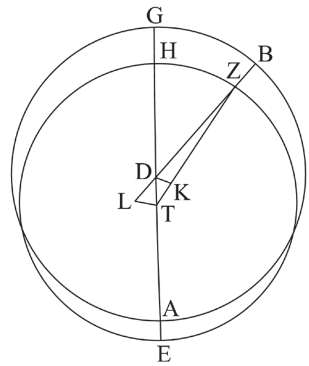
For the rest, if the angle of apparent motion is known, the others will be known. For this we draw perpendicular AL upon line DB. Therefore, because the angle of apparent motion AZL is known, the ratio of ZA to AL will be known because of this. But the ratio of ZA to AD was known. Therefore, the ratio of DA to AL is known. For this reason the angle of difference ADL will be known. And with this the extrinsic angle EAZ, which is the mean motion, will be known.

In a like way but in reverse, if we posit that the angle of difference, which is ADB, is known, the ratio of AD to AL will be known because of this. But the ratio of AD to AZ was known. Therefore, the ratio of AZ to AL is known. For which reason angle AZD will be known, and this is the angle of irregular motion in the ecliptic. And with this, extrinsic angle EAZ, which is the angle of mean motion, will be known.

15. To search for whatever difference between the sun's mean motion and irregular motion through the known arc of the mean motion from the perigee according to the eccentric model. With which it will also be declared that if any of the three angles, whether of the mean motion, the irregular, or the difference, is known, the remaining two will also be known.

For with the figure of the eccentric supposed, I will cut off arc HZ from point H, which is the perigee. And I will draw lines DZB and TZ and perpendicular DK upon line TZ. Therefore, because arc HZ is known, the angle ZTH at the center will be known of the same quantity. The ratio of DT, therefore, to both DK and TK will be known. But from the 11th the ratio of DT to TZ was known; therefore, the ratio of TZ to each is known. Therefore, also the ratio of KZ to DK is known; so angle DZK is known, and this is the angle of the difference. With which the extrinsic angle GDB is also known.

Quod si notus fuerit ex ypothesi angulus GDB, reliqui duo erunt noti. Producam enim lineam DB usque ad punctum L et super eam perpendicularem TL. Erit ergo angulus TDL notus. Quare proportio linee TD ad TL est nota, sed erat nota TD ad TZ. Quare erit ZT ad TL nota. Propter hoc ergo erit angulus TZL notus angulus differentie, et cum hoc angulus intrinsecus qui est motus medii HTZ.



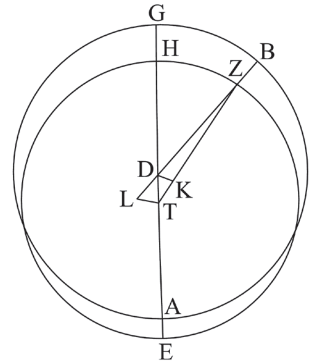
Quod si angulus differentie primum notus fuerit TZD, erit propter hoc e contrario proportio ZT ad TL nota. Sed erat proportio TZ ad TD nota, quare TD ad TL nota. Ob hoc ergo notus LDT qui est equalis angulo GDB, et ipse angulus motus diversi. Cum quo notificabitur angulus intrinsecus HTZ qui est motus medii.

16. Quamlibet differentiam motus Solis medii et motus diversi per notum arcum motus medii a longitudine propiore secundum epicicli modum perquirere. Cum quo demonstrabitur quod si notus fuerit unus trium angulorum motus medii, motus diversi, motus differentie, reliqui duo erunt noti.

Posito enim scemate epicicli et concentrici excipiam arcum quantumlibet a longitudine propiore et sit HT note quantitatis. Erit ergo angulus TAH notus. Quare cum angulus AKH sit rectus, erit AH ad AK note proportionis et ad KH quoque. Sed AH ad AD erat nota, ergo AD ad utramque notam habet proportionem. Est ergo DK ad KH nota. Quapropter notus est angulus differentie ADB, et cum hoc angulus extrinsecus apparentis motus AHB.

Quod si notus fuerit angulus primum angulus AHB, protracta perpendiculari AL erit ob hoc proportio AH ad AL nota. Et propter hoc angulus ADL et arcus differentie AB notus. Et cum hoc angulus relictus KAH et eius arcus TH qui est motus medii notus.

491 reliqui duo] reliqui *PN* reliquo duo *M* **498** hoc] hic *K* intrinsecus – est] intrinsecus *corr. ex* extrinsecus qu†are† *K* **499** HTZ] notus *add.* (*s.l. K marg. M*) *KM* **502** ZT] *corr. ex* †...† *K* **503** Ob] ad *K* qui est] *iter. et del. K* equalis] *s. l.* P_7 ipse] est *add.* (*s.l. K*) *KM* **504** diversi] diversi notus *N* est] est angulus *M* **505** medii] medii et cetera *N* **509** motus[3]] *om. P* et *N* **510** quantumlibet] quandolibet *K* **511** propiore – sit] propinquiore et sint *P* **513** AD[2]] *s.l.* P_7 **513/514** notam – proportionem] proportionem HT (habet *N*) notam *PN* **514** Est[1]] cum *K* KH] AH P_7 **515** ADB] ADH *PN* et] *om. N* hoc] *s.l.* P_7 hoc quoque *N* extrinsecus] *om. N* **516** angulus[1]] *om. MN* **517** ob] *corr. ex* ab *K* nota] nota et propter hoc HA ad AL nota *K* ADL] ad L *P* ADL notus *N* **518** notus] *om. K* relictus] reliquus P_7 **519** TH] TKH *P*



But if angle GDB is known by hypothesis, the other two will be known. For I will extend line DB to point L, and ⟨I will draw⟩ perpendicular TL upon it. Therefore, angle TDL will be known. Therefore, the ratio of line TD to TL is known, but TD to TZ was known. Therefore, ZT to TL will be known. Because of this, therefore, angle TZL will be the known angle of the difference, and with this the intrinsic angle HTZ, which is of the mean motion, ⟨will be known⟩.

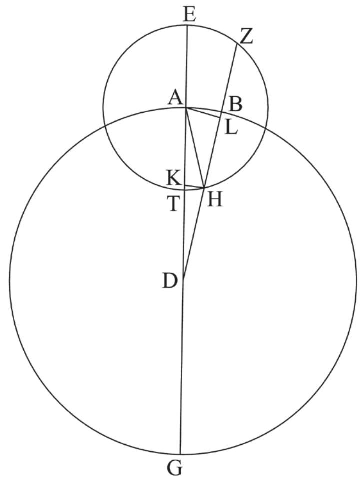
But if the angle of the difference TZD is known first, because of this conversely, the ratio of ZT to TL will be known. But the ratio of TZ to TD was known, so TD to TL is known. On account of this, therefore, LDT is known, which is equal to angle GDB, and this is the angle of the irregular motion. With which the intrinsic angle HTZ, which is of the mean motion, will be made known.

16. To search for whatever difference between the sun's mean motion and irregular motion through the known arc of the mean motion from the perigee according to the epicyclic model. With which it will be demonstrated that if one of the three angles of mean motion, irregular motion, or difference is known, the remaining two will be known.

For with the figure of the epicycle and concentric supposed, I will cut off an arc however large from the perigee, and let it be HT of a known size. Therefore angle TAH will be known. Therefore, because angle AKH is right, AH will be of a known ratio to AK, and also to KH. But AH to AD was known, so AD has a known ratio to each [i.e. AK and KH]. DK to KH, therefore, is known. For this reason the angle of the difference ADB is known, and with this the extrinsic angle of apparent motion AHB ⟨is known⟩.

But if the known angle is first angle AHB, with perpendicular AL drawn, the ratio of AH to AL will be known on account of this. And because of this, angle ADL and the arc of the difference AB are known. And with this, the remaining angle KAH and its arc TH, which is of the mean motion, are known.

Quod si constituatur angulus differentie ADB notus, sciemus etiam proportionem AD ad AL et ob hoc AH ad AL. Erit ergo angulus AHL qui est motus apparentis notus. Et cum hoc angulus intrinsecus oppositus KAH qui est motus medii notus, quod proposuimus.

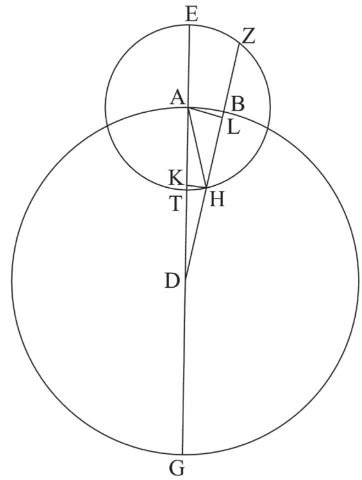


Cum itaque secundum premissas propositiones multiplices, possibile sit condere tabulas. Quocumque enim trium angulorum sumpto ut noto, reliquos notos esse oportet. Attamen commodius est et satius per medium cursum ceteros cognoscere eo quod hic regularis motus est et ordinatus, ceteri inequales.

17. Super fixam et certam radicem temporis locum Solis secundum cursum medium assignare in loco determinato, ut habita ad hoc relatione locus Solis verus ad omne deinceps tempus et in omni loco noto inveniatur.

Igitur secundum considerationem quoad fieri potest verissimam precipue aput autumpnale equinoctium iuxta id quod in expositione prime propositionis presentis libri diximus, locus Solis verus deprehendatur. Et sit gratia exempli in anteposita figura punctum B in circulo signorum punctum equalitatis autumpnalis et punctum G longitudo propior. Igitur ex undecima presentis arcus BG erit notus qui est motus diversi a longitudine propiore. Quare ex xv[a] arcus HZ qui est motus medii a longitudine propiori erit notus. Itaque locus Solis secundum cursum medium ad horam considerationis erit notus. Elige ergo annos alicuius viri noti vel rei note quos radicem velis constituere, ut Augusti vel Alexandri aut potissimum annos Christi qui est rex regum et dominus dominantium. Et quotquot preterierunt usque ad horam considerationis in unum collige, ac inde annos solares quotiens poteris proice. Deinde residuum cum

521 ADB] ABD *M* etiam proportionem] proportionem etiam P_7 **524** intrinsecus] *corr. ex* extrinsecus *M* **525** KAH – medii] qui est motus medii KAH *N* KAH] *corr. in* KAB *M* **527** Cum] quando *N* propositiones] proportiones *corr. in* portiones *P corr. ex* proportiones P_7N proportiones *K* (proportionis *Ba* propositiones E_1) **531** Attamen] ac tamen P_7 est – satius] et satius (*corr. ex* catius) est *P* est et facilius *M* et satis est *N* **532** ceteros] ceteras *K* **536** Solis] *corr. ex* solus P_7 Solis diversus sive *M* **538** quoad] quemadmodum *M* **539** id] hoc *K* illud *MN* propositionis] proportionis *K* **540** Solis] solus *K* Et – exempli] et sit exempli gratia P_7 ut exempli gratia *KM* **541** punctum[1]] in puncto P_7 **542** punctum G] punctum B *P* G *N* undecima] *corr. in* 13 P_7 **543** xva] *corr. in* 13 P_7 **545** ad – considerationis] *om. N* **546** velis constituere] constituere velis *N* **548** preterierunt] preterierint P_7 horam] *corr. ex* oram *K* considerationis] considerationis tunc *M* **549** ac] at *N* annos] annorum *P* poteris] potes *N*



But if the angle of difference ADB is set up as known, we will also know the ratio of AD to AL, and on account of this AH to AL. Therefore, angle AHL, which is of the apparent motion, will be known. And with this, the opposite intrinsic angle KAH, which is of the mean motion, is known, which we proposed.

Accordingly, when you repeat following the propositions set forth, it may be possible to build tables. For, with any of the three angles taken as known, it is necessary that the others are known. But yet it is more helpful and preferable to know the others through the mean course because this motion is regular and well-ordered, the others irregular.

17. To assign the sun's place according to mean course upon a fixed and known radix of time in a determined place so that with a reference had to this ⟨place⟩, the sun's true place may be found at any later time and at any known place ⟨on earth⟩.

Accordingly, let the true place of the sun be found by an observation as accurate as can be made, especially at the autumnal equinox, according to what we said in the exposition of the present book's first proposition. And for example in the figure given before [i.e. the figure of III.15], let point B on the ecliptic be the autumnal equinox and point G the perigee. From the 11th of the present, therefore, arc BG, which is of the irregular motion from the perigee, will be known. From the 15th, therefore, arc HZ, which is of the mean motion from the perigee, will be known. Accordingly, the sun's place according to mean course will be known at the hour of the observation. Then select the years of any famous man or famed event, which you want to establish as the radix, as the years of Augustus, Alexander, or especially Christ, who is king of kings and lord of lords. And add up however many ⟨years⟩ have passed by until the time of the observation, and from this cast out as many solar years as anno solari proportionando confer; et quantum de eo fuerit, tantum de ccclx minue. Et erit locus Solis secundum cursum medium ad principium annorum quos elegeris. Vide autem ut principium annorum illorum a media die vel a media nocte constituas. Super hoc ergo principium quod fundaveris ad singulas deinceps divisiones temporum ut in secunda presentis explanavimus, medium motum adiunge ut noto cursu medio ad omnia deinceps tempora verum locum Solis per viam operationis sumptam ex premissis propositionibus in loco considerationis cognoscas.

Via siquidem operandi est hec. Ad tempus quantum volueris a radice sumptum medium motum accipe. Et ex eo arcum motus medii a longitudine longiori, qui portio vel argumentum Solis dicitur, cognosce. Qui arcus, si minor semicirculo fuerit, per ipsum; si maior, per superfluum semicirculi ita operare.

Si arcus quem ita habueris minus quarta fuerit, eius sinum necnon et sinum illius qui ei ad perficiendum quartam deficit per quantitatem distantie duorum centrorum multiplica. Et utrumque productum per semidiametrum idest lx partire. Quodque exierit ex divisione sinus perfectionis semidiametro superadde, et totum in se multiplica. Et super quod fuerit, illud quod ex divisione sinus habiti arcus provenerat in se multiplicatum adde. Collectique radicem quere, et serva. Post hec ad id quod ex divisione sinus habiti arcus productum fuerat rediens, ipsum in diametri dimidium multiplica, et productum per servatam radicem partire.

Quod si arcus quem habueris quarta fuerit, tunc semidiametrum necnon et distantiam duorum centrorum in se multiplica et in unum collige. Collecti

551 minue] incepta computatione a loco noto hora considerationis contra ordinem signorum *add.* (*marg. K et del. M*) *KM* (*This addition is in Ba*) Et] et hoc *M* **552** elegeris] notus *add. s.l. K* ut] aut *K* **553** quod fundaveris] *corr. ex* profundaveris *M* **556** propositionibus] proportionibus *P corr. ex* proportionibus P_7 **558** volueris] voles *N* **559** medium motum] motum medium *PN* motum] *corr. ex* medium P_7 eo] ea *K* motus medii] medii motus *M* longitudine] *corr. ex* longe *K* **560** portio] proportio *P corr. ex* proportio *N* cognosce] *corr. ex* cognoscere *P* cognescere *K* Qui^{2}] igitur *K* **561** maior] *corr. ex* minor *KN* ita operare] operare ita *PN corr. ex* itaque operare *K* operare *M* **562** ita habueris] habueris ita *P* habueris *N* minus] minor P_7K (minor *Ba* minus E_1) quarta] quarta circuli *M* **563** qui ei] quod ei P_7 quod *M* perficiendum] perficiendam *N* quartam deficit] deficit quartam *M* duorum] et *N* **564** idest] *om. K* **565** partire] divide *N* perfectionis] *corr. in* complementi (*other hand*) *N* semidiametro] semidiametrum *K* **566/567** in – provenerat] *marg.* P_7 **566** quod^{1} – quod^{2}] quod fuerit illud *PK* illud quod *corr. ex* illud quod fuerit *N* (quod fert ita *Ba* quod fuerit †illud† E_1) ex] extra *K* **567** sinus] sinus perfectionis *M* habiti arcus] arcus habiti *K* provenerat] pervenerat *P* proveniant *K* **568** hec] hoc *MN* id] illud *K* sinus – arcus] arcus habiti *N* **569** fuerat] *corr. ex* fuerit P_7 fuerit *K* rediens] *corr. ex* redigens *P* rediges *M* redigens *N* **571** habueris] habueris plus *PN* **572** distantiam] differentiam *M* you can. Then compare the remainder with the solar year by making a ratio; and as much as it is of that, subtract so much from 360. And there will be the sun's place according to mean course at the beginning of the years that you chose. Moreover, see that you establish the beginning of these years from midday or midnight. Then, upon this beginning that you will have established, allot the mean motion for the individual divisions of time in succession, as we explained in the second of the present, so that with the mean course known at all times in succession, you may know the sun's true place through the way of operating taken from the preceding propositions instead of observation.

Accordingly, the way of operating is this. Take the mean motion for as much time taken from the radix as you want. And from that know the arc of the mean motion from the apogee, which is called the 'portion' or 'argument' of the sun.[13] If this arc is less than a semicircle, operate thus through it; if greater, ⟨operate⟩ through the excess of a semicircle.[14]

If the arc that you have thus is less than a quarter circle, multiply its sine as well as the sine of its complement by the quantity of the eccentricity. And divide each product by the radius, i.e. 60. And add what results from the division of the sine of the complement to the radius, and multiply the total by itself. And to what results add that which resulted from the division of the sine of the considered arc multiplied by itself. And seek the root of the result, and save it. Afterwards returning to that which was the result of the division of the sine of the considered arc, multiply that by the radius, and divide the product by the saved root.

But if the arc that you have is a quarter,[15] then multiply the radius and also the eccentricity by themselves, and combine them into one. Draw out the root

[13] Albategni is considering this in terms of the epicyclic model, so he writes, '… quod est portio nominata Soli et Lunae caeterisque stellis …' (*De scientia astrorum*, 1537 ed., f. 31r). The author here retains this term, but adds 'vel argumentum' to give the term usually used to refer to the mean motion in the eccentric model.

[14] This should say 'superfluum circuli' to be mathematically correct and to agree with Albategni's rules (*De scientia astrorum*, 1537 ed., f. 31r). The meaning is that for an arc of 350°, for example, one should work with an arc of 10° from apogee, and for an arc of 195°, with an arc of 165°.

[15] The added 'plus' in *P* and *N* would have made this difficult for readers to understand.

radicem elice et serva. Post hec distantiam duorum centrorum in lx multiplica, et quod provenerit per servatam radicem divide.

Quod si arcus quem habueris plus quarta fuerit, ipso a semicirculo subtracto residui sinum eiusque quod ipsi quoque ad perfectionem quarte deficit sinum per distantiam duorum centrorum multiplica, et per semidiametrum partire. Quodque ex sinu perfectionis provenerit a semidiametro minue, et reliquum in seipsum multiplica. Et ei quod ex sinu residui arcus provenerat in se multiplicato superadde, collectique radicem serva. Post hec ad id quod ex sinu arcus residui provenerat rediens, id in diametri dimidium multiplica, et per servatam radicem partire.

Et quodcumque exierit ex uno istorum trium modorum arcua. Nam arcus qui prodierit est differentia motus medii ad motum diversum, qui equatio Solis dicitur. Et si portio minus vi signis fuerit, a medio cursu minuitur. Et si plus vi signis fuerit, super medium cursum additur. Et erit cursus Solis diversus sive equatus, per quem verum Solis locum in circulo signorum cognosces.

Quod si in alio quam considerationis loco idem deprehendere volueris, oportet te distantiam inter meridianas lineas locorum scire et illam distantiam in tempora redigere. Quod si locus notus a loco considerationis orientalis fuerit, tempora distantie a tempore per quod motum medium sumpsisti minue. Si occidentalis eidem, adde. Et per tempus quod post additionem vel subtractionem fuerit, motum medium cognosce, ac deinceps ut prius operare.

18. Dies anni duabus de causis inequales esse invicem necessario comprobatur. Unde patet quosdam dies differentes dici, quosdam mediocres.

Dies hic dicitur spatium xxiiii horarum ut ab ortu ad ortum Solis vel ab occasu ad occasum aut a meridie ad meridiem aut a media nocte ad mediam noctem. Una ergo causa quare hii dies inequales sunt est diversus motus Solis

570 hec] hoc *MN* **571** provenerit] pervenerit *K* **572** ipso – subtracto] ab eo quarta subtracta *PN* **573** ipsi quoque] *om. N* quarte] *s.l.* P_7 **574** duorum] *om. N* **575** a] ex *M* reliquum] residuum P_7 **576** ei] *s.l. P* residui arcus] arcus residui *M* provenerat] provenerit *K* **577** collectique] collectique per *P* collectamque *K* radicem] radicem accipe et *N* hec] hoc *MN* ad id] *corr. ex* adde P_7 **577/578** arcus residui] residui arcus *N* **578** provenerat] pervenerat P_7 proveniant *K* rediens] *marg.* (*perhaps other hand*) *P* redigens *MN* **580** istorum] *om. PN* trium modorum] *marg.* (*perhaps other hand*) *P* **581** prodierit] prodier†um† *P* redierit P_7 prod†e†rit *K* (prodigerit *Ba* prodierit E_1) equatio Solis] Solis equatio P_7 **582** vi – fuerit] fuerit sex signis *M* **582/583** si[2] – vi] *corr. ex* simplus ex *K* **583/584** diversus – equatus] equatus sive diversus *K* diversus aut equatus *N* **584** Solis locum] locum Solis *MN* circulo signorum] signorum circulo *N* **585** quam] *s.l. PK* **586** distantiam[1]] distantias *N* scire] cognoscere *N* **587** tempora] tempus *N* Quod] *corr. ex* et *M* notus – loco] noto a loco *corr. ex* noto a noto *K* **588** quod] quem *M* minue] minue et *M* **589** eidem] id *K* post] per *PN* **590** fuerit] fuit *P* motum medium] medium motum *N* **591** comprobatur] comprobantur *KM* **592** dici] diei *P* **593** hic] igitur *K* ad – vel] Solis ad ortum vel *M* Solis ad ortum (ortum *corr. ex* occasum *other hand*) *N* of the result, and save it. Afterwards multiply the eccentricity by 60, and divide what results by the saved root.

But if the arc that you have is more than a quarter, with it having been subtracted from a semicircle,[16] multiply the sine of the remainder and the sine of its complement by the eccentricity, and divide by the radius. And from the radius, subtract what results from the sine of the complement, and multiply the remainder by itself. And add it to that which resulted from the sine of the remaining arc multiplied by itself, and save the root of the sum. Afterwards, returning to that which resulted from the sine of the remaining arc, multiply it by the radius, and divide it by the saved root.

And arc whatever results from one of those three ways. For the arc that results is the difference between the mean motion and the irregular motion, which is called the sun's 'equation.' And if the portion is less than 6 signs, it is subtracted from the mean course. And if it is more than 6 signs, it is added to the mean course. And there will be the sun's irregular or equated course, through which you will know the sun's true place in the ecliptic.

But if you want to find the same in a place other than that of the observation, it is necessary that you know the distance between the meridian lines of the places and convert that distance into time. And if the known place is east of the place of the observation, subtract the time of the distance from the time through which you took the mean motion. If west of the same, add. And through the time that there is after the addition or subtraction, know the mean motion, and operate hereafter as before.

18. It is confirmed that the days of the year are necessarily unequal to each other because of two causes. Whence it is clear that certain days are said to be diverse, others average.

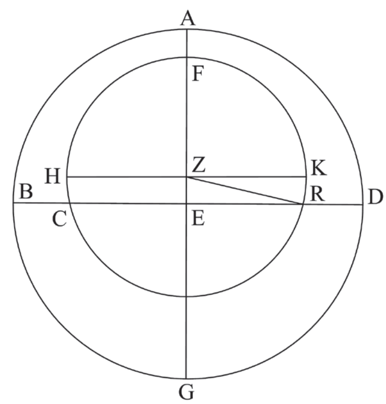
A day here means a duration of 24 hours as from the sun's rising to rising, from setting to setting, from noon to noon, or from midnight to midnight. One cause, then, why these days are unequal is the irregular motion of the sun

[16] The reading in *P* and *N* is perhaps original as it is closer to Albategni, but it could also be the result of a scribe attempting to fix what he saw to be a mistake as he copied out the *Almagesti minor* while consulting Albategni.

ad unam diem. Alia causa est equalium portiuncularum circuli declivis inequales ascensiones. Siquidem spatium talis diei est revolutio equinoctialis circuli et insuper elevatio eius quod Sol diverso vel medio motu ad unam diem percurrit. Est itaque dies mediocris revolutio equinoctialis circuli cum motu Solis medio ad unam diem addito, idest lix minutis et viii secundis. Dies differens est revolutio equinoctialis circuli cum elevatione maiori vel minori eius quod Sol ad illam diem perficit. Unius vero diei ad unum insensibilis est differentia, sed cum ex multis diebus collecta fuerit, sit manifesta.

19. Causa inequalitatis dierum ex diverso motu Solis proveniens ab alterutra longitudine media incipit et ad oppositam desinit, et differentia diei mediocris ad dies differentes maior, cum ex hoc collecta fuerit, ex duplo differentie maxime motus medii et motus diversi perficitur.

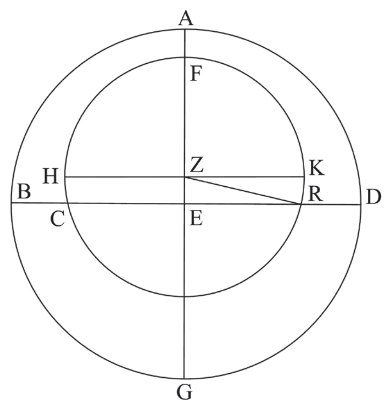
Siquidem aput utramque longitudinem mediam motus diversus ad unam diem equatur motui medio ad unam diem; ideo ad utramque hec causa inequalitatis incipit et ad oppositum desinit. Et ponemus ad demonstrandum quod sequitur figuram. Sit enim circulus signorum ABGD supra centrum E cuius duo diametri scilicet AG per longiorem et propiorem longitudinem transiens et BD perpendiculariter super illam per utramque longitudinem mediam transiens. Et sit ecentricus Solis HRK super centrum Z et diametrum communem quem alter eius diameter HK ad angulos rectos secat. Cum ergo aput puncta C et R sint longitudines medie, palam quod tempora que aggregantur inter has duas longitudines ex motu medio in medie-



599 causa] *om.* P_7 portiuncularum] portionumcularum P_7 portionuncularum *K* (p†a†rtiu†m† clarum *Ba* portiuncularum E_1) **600** Siquidem] si quod *corr. ex* si *P* spatium – diei] talis diei spatium P_7 **601** insuper] super *K* **601/602** ad – percurrit] percurrit ad unam diem *N* **602** itaque] ergo P_7 **603** addito] additis *N* idest] *om. KN* lix] *corr. ex* lx *K* secundis] secundis et cetera *M* **604** elevatione] e *add. et del. K* **605** illam] illum *N* **606** fuerit] fuerint *M* sit] fit P_7*M* erit *N* **607** inequalitatis] *corr. ex* inequalitas *K* **608** oppositam] oppositum *PN* **610** motus[1]] motus diei *PM* motus diei *corr. in* motus Solis *N* **611** aput] ad *M* utramque] *corr. ex* †...†amque *P* **612** motui medio] *corr. ex* motu dimidio P_7 ideo] ideoque P_7 utramque hec] utrumque hoc *N* inequalitatis] equalitatis *M* diversitatis *N* **613** oppositum] oppositam P_7*KM* desinit] *corr. ex* deficit *N* **614** quod] que *M* **616** duo] due *N* AG] et *add. et del.* P_7 per] per longitudinem *M* **617** et[1]] et per P_7 transiens] transeuntem *N* **618** BD] BA *M* perpendiculariter] perpendiculariter transiens P_7 **620** HRK] HFK *M* HTK *N* **622** quem – eius] quam altera *N* **623** puncta] punctum *P* C] T *MN* **625** aggregantur] *corr. ex* agantur *K* **626** medietate] mediate *P* for one day. Another cause is the unequal ascensions of equal small parts of the ecliptic. In fact, the duration of such a day is a revolution of the equator and additionally the elevation of that which the sun travels through by the irregular or mean motion in one day. Accordingly, an average day is a revolution of the equator with the sun's mean motion for one day added, that is 59′ 8″. A diverse day is a revolution of the equator with the greater or lesser elevation of that which the sun completes in one day. And indeed the difference between one day and the next is imperceptible, but when it has been gathered from many days, it is noticeable.

19. The cause of inequality of days resulting from the sun's irregular motion begins from one mean distance and ends at the opposite one, and the greatest difference between the average day and the diverse days when it is added up from this, is brought about from double the greatest difference between the mean motion and the irregular motion.

In fact, at either mean distance, the irregular motion for one day is equal to the mean motion for one day; for that reason, this cause of inequality begins at either ⟨mean distance⟩ and ends at the opposite one. And we will posit a figure for demonstrating what follows. Indeed, let there be upon center E the ecliptic ABGD, the two diameters of which, i.e. AG passing through the apogee and perigee and BD perpendicular to that, passing through each mean distance. And let there be the sun's eccentric HRK upon center Z and the common diameter that its other diameter HK cuts at right angles. Therefore, because the mean distances are at points C and R, it is clear that the times that are collected between these two distances from the mean motion in



tate longitudinis longioris sunt partes arcus RFC et tempora que aggregantur interim ex motu diverso sunt partes arcus DAB. Quia ergo KFH est medietas circuli, differentia illorum temporum ad hec est duplum arcus RK. Est enim arcus HC equalis arcui RK. Sed arcus RK subtenditur angulo RZK, qui equatur angulo ERZ, et hic angulus est maxima differentia duorum motuum. Ex duplo ergo istius anguli perficitur tota temporum differentia. Manifestum ergo quod dies mediocris superat dies differentes ex parte longitudinis longioris duplo differentie maioris duorum motuum, et dies differentes diem mediocrem superant ex parte longitudinis propioris duplo eiusdem differentie. Quare dies differentes maiores superant dies differentes minores quadruplo ipsius differentie, et hoc est quod intendebamus.

20. Differentiam ex diverso motu contingentem diei mediocris ad quamcumque iubearis diem differentem inquirere.

Cum enim ex posita radice temporis notum sit quo tempore Sol ad longitudinem longiorem veniat, sume ab hoc puncto totum tempus usque ad principium diei de qua queris. Et per ipsum arcum medii cursus a longitudine longiore addisce. Sume quoque ab eodem puncto omne tempus usque ad finem diei de qua queris, et per ipsum similiter arcum motus medii deprehende. Et per utrumque arcum motus medii arcum diversi motus cognosce. Cum ergo minorem arcum diversi motus a maiori arcu diversi motus dempseris, remanebit diversus motus ad illam diem differentem de qua queris notus. Cum ergo ab hoc motum medium si minor fuerit dempseris, vel ipsum si minor fuerit a motu medio, relinquetur differentia quam queris nota.

21. Causa inequalitatis dierum ex inequali ascensione aput orizonta declivem accidens a quo loco incipiat vel desinat, et differentia tota cum collecta ex hoc fuerit quanta sit depromere.

Locus qui queritur secundum climata variatur; in omni tamen climate ante punctum tropicum estivum et post tropicum punctum hiemale deprehenditur. Quere ergo secundum ascensiones signorum in climate in quo loco ante

627 arcus] DAB quia KFH est medietas circuli differentia illorum temporum *add. et del.* P_7 RFC] RFT *MN* **628** ergo] *s.l.* P_7 **629** illorum] istorum *N* hec] adhuc *M* ad hoc *N* (*om. Ba* adhoc E_1) arcus] *om. PN* **630** HC] HT *MN* angulo] *om. N* **631** ERZ] *corr. ex* EZ *P* maxima differentia] differentia maxima *M* **633** mediocris superat] *corr. in* mediocres superant *K corr. ex* mediocres superat *N* **634/635** diem – superant] superant diem mediocrem P_7 superant dies mediocres *corr. ex* superant diem mediocrem *K* dies mediocres superant *M* **635/637** Quare – intendebamus] *del. K* **636** quadruplo] duplo *N* ipsius] istius P_7 **639** iubearis] iubeatis *K* **640** enim] hoc *s.l. et del. K* in *N* **643** omne tempus] *om.* P_7 **644** qua] quo P_7 arcum] *corr. ex* arcus P_7 motus medii] medii motus *N* **645** diversi motus] motus diversi *N* **646** arcum] *om. N* **647** queris] quesieris *K* notus] *om. N* **648** ab] ad *P* minor[1]] maior *KM* **650** orizonta declivem] orisontem declivem *M* orisontem declivum *N* **651/652** ex – fuerit] fuerit ex hoc *corr. ex* fuerit hoc ex *K* **654** tropicum punctum] punctum tropicum P_7N **655** Quere] *corr. ex* quare *M* the apogee's half are the parts of arc RFC and the times that are meanwhile collected from the irregular motion are the parts of arc DAB. Therefore, because KFH is a semicircle, the difference of these times for it is double arc RK. For arc HC is equal to arc RK. But arc RK subtends angle RZK, which is equal to angle ERZ, and this angle is the maximum difference between the two motions. Therefore, the whole difference between the times is brought about from double that angle. Therefore, it is manifest that on the apogee's side, the average day exceeds the diverse days by double the greatest difference between the two motions, and on the perigee's side, the diverse days exceed the average day by double that same difference. Therefore, the greatest diverse days exceed the least diverse days by quadruple that same difference, and this is what we intended.

20. To seek the difference occurring from the irregular motion between an average day and whatever diverse day you are told ⟨to find⟩.

Indeed, because from the given radix of time it is known at what time the sun comes to the apogee, take from this point the whole time to the beginning of the day about which you seek. And through that learn the arc of the mean course from the apogee. Also take all the time from the same point to the end of the day about which you seek, and similarly through it find the arc of the mean motion. And through each arc of mean motion, know the arc of irregular motion. Therefore, when you subtract the lesser arc of irregular motion from the greater arc of irregular motion, the irregular motion for that diverse day about which you seek will remain known. Therefore, when you subtract from this the mean motion if it [i.e. the mean motion] is smaller, or ⟨subtract⟩ that [i.e. the day's irregular motion] from the mean motion if it is smaller, the difference that you seek will remain known.

21. To draw out the place from which the cause of the inequality of days occurring from the unequal ascension at a declined horizon begins or ends, and how great the whole difference is when it is added up from this.

The place that is sought varies according to climes; nevertheless, in every clime it is found before the summer tropic point and after the winter tropic point. Therefore, according to the ascension of signs in the clime, seek the tropicum estivum gradus unus circuli signorum cum uno gradu equinoctialis ascendat. Et simile post tropicum hiemale inquire. Et cum utrumque locum deprehenderis, ipse est a quo causa inequalitatis incipit vel desinit. Vide ergo portio circuli signorum inter hec duo loca quanta sit aut ex parte Libre aut ex parte Arietis, et cum quanta portione equinoctialis elevetur. Nam differentia portionis zodiaci ad suam elevationem ipsa est differentia quesita diei mediocris ad dies differentes cum aggregata fuerit. Et quia quantum dies mediocris addit super dies differentes ex parte Arietis tantum dies differentes addunt super diem mediocrem ex parte Libre, palam quod dies differentes maiores addunt super dies differentes minores duplum collecte differentie. Palam etiam quod differentia sic inventa augmentum maxime diei regionis super diem equinoctialem excedit eo quod causa inequalitatis a loco ante tropicum estivum incepta post tropicum hiemale terminetur. Tempora enim ascensionum huius portionis addunt super gradus suos plus cum sumpta fuit ex parte Libre quam tempora portionis inter caput Cancri et caput Capricorni deprehense addant super gradus suos. Sed hec augmenta elevationum que sunt capitis Cancri usque ad Capricornum sunt ea que addit dies maxima regionis super diem equinoctialem. Et illa elevationum tempora que plura esse necessario accidit sunt que differentiam quesitam perficiunt.

22. Causa inequalitatis dierum ex inequali transitu aput meridianum proveniens a iiii punctorum quolibet quartas inter solstitialia et equinoctialia puncta deprehensas mediante incipit et ad perfectionem quarte desinit, et differentia cum hinc collecta fuerit spatio quinque temporum extenditur.

Hec quoque causa a iiii punctorum quolibet incipit scilicet a medio Aquarii, Tauri, Leonis, Virginis quia penes unumquemque istorum locorum arcus motus medii ad unam diem equatur suo transitu per meridianum et non alibi sicut ex ascensionibus spere recte patet. Et quia quarta a medio Aquarii ad medium Tauri elevatur cum lxxxv gradibus equinoctialis, palam quod dies mediocres superant dies differentes, cum per hanc quartam collecte fuerint differentie, v graduum temporibus. Similiter accidit in quarta huic opposita propter hoc quod opposite portiones in spera recta equaliter oriuntur. Quarta vero a medio

657 ascendat] *corr. ex* accendat *K* simile] similiter *MN* **659** portio] portionem *M* que portio *N* **661** diei mediocris] *corr. in* dierum mediocrum *K* **662/665** Et – differentie] *del. K* **668** hiemale] hyemalem *K* **669** gradus suos] *corr. ex* gradus duos *K* suos (*corr. ex* duos) gradus *M* fuit] sint *K* fuerit *MN* (sumpserit *Ba* fuit E_1) **670** caput^1 – et] *marg. P* **671** Sed hec] secundum hoc *M* **672** diem] *s.l. P* **673** accidit] accidunt *M* **676** solstitialia] *corr. ex* solstitia *P* solticialia *K* **679** causa] *s.l. M* **680** Tauri] Thauri *MN* Leonis] et *add. s.l. M* Virginis] *corr. in* Scorpionis PP_7 Scorpionis *MN* (Virginis BaE_1) quia] *corr. ex* †...† (*other hand*) *K* unumquemque] *corr. ex* unumquodque *P* **681** transitu] transitui *KN* **683** Tauri] Thauri *MN* equinoctialis] equinoctialis circuli P_7 **683/684** mediocres superant] mediocris superat *PN corr. ex* mediocris superat *K* (mediocres superant *Ba* mediocris superat E_1) **684** fuerint] fiunt P_7K

place before the summer tropic where 1° of the ecliptic ascends with 1° of the equator. And seek the like ⟨place⟩ after the winter tropic. And when you have found each place, it is where the cause of the inequality begins or ends. Then see how great the part of the ecliptic between these two places is, either on Libra's side or on Aries' side, and with how large of a part of the equator it rises. For that difference between the part of the zodiac and its elevation is the sought difference between the average day and the diverse days when it is added up. And because the average day adds to the diverse days on Aries' side as much as the diverse days add upon the average day on Libra's side, it is clear that the greatest diverse days add upon the smallest diverse days double the gathered difference. It is also clear that the difference thus found exceeds the process of increasing of the region's longest day over the equinoctial day because the cause of inequality beginning from a place before the summer tropic ends after the winter tropic. For the times of ascensions of this part add upon their degrees more when they are taken on Libra's side than the times of the part caught between the beginning of Cancer and the beginning of Capricorn add upon their degrees. But these processes of increasing of the elevations that are of Cancer's beginning to Capricorn are those that the longest day of the region adds upon the equinoctial day. And those times of elevation, which necessarily happen to be more, are those that bring about the sought difference.

22. The cause of the inequality of days resulting from the unequal passage at the meridian begins from any of the four points halving the quarters caught between the solstice and equinox points and ends at the completion of a quarter circle, and when the difference is added up from this, it is increased to an interval of five time-degrees.

This cause also begins from any of the four points, namely from the middle of Aquarius, Taurus, Leo, or Virgo[17] because the arc of mean motion for one day belonging to each of those places is equal to its passage through the meridian and not otherwise, as is clear from the right sphere's ascensions. And because the quarter from the middle of Aquarius to the middle of Taurus is raised with 85° of the equator, it is clear that when the differences have been collected throughout this quarter, the average days exceed the diverse days by the times of 5°. Similarly it happens in the quarter opposite this because opposite parts rise equally in the right sphere. And indeed, the quarter from the

[17] As some scribes realized, this was a mistake for 'Scorpio.'

Tauri ad medium Leonis transit cum lxxxxv gradibus equinoctialis. Propter hoc ergo dies differentes superant diem mediocrem, cum collecte per hanc quartam fuerint differentie, quinque graduum temporibus. Similiter accidit in quarta huic opposita. Manifestum ergo quod dies differentes maiores superant dies differentes minores ob hanc causam x temporibus.

23. Differentiam ex inequali elevatione procedentem diei mediocris ad quamcumque iubearis diem differentem perquirere.

Elevationem ergo arcus medii motus Solis de illo gradu in quo Sol ea die de qua queris moratur accipe. Et si maior motu medio fuerit, ipsum de ea deme; et si minor, de ipso eam deme. Et relinquetur differentia quam queris. Sed si in spera recta quesieris, elevationem in spera recta; si in spera declivi, elevationem in spera declivi accipe.

Patet itaque ex predictis quod commodius est et satius dies a meridie vel media nocte incipere quam ab ortu vel occasu eo quod aput orizontem maior provenit dierum inequalitas; et quia in orizonte declivi principia ortus et occasus variantur eo quod modo maior modo minor arcus diei, in orizonte recto omnes arcus sunt secundum similitudinem partium equales; et ideo presertim quod inequalitas hec dierum secundum diversas regionum latitudines variatur aput orizontem, sed aput meridianum in omni regione est eadem.

24. Differentias ex causis ambabus prout contingit simul provenientes singulatim perpendere, et principium additionis super diem mediocrem et principium diminutionis a die mediocri adinvenire.

Singulas igitur ex utraque causa ad dies singulos differentias ut ex premissa et xx[a] habetur collige. Et ubi unaqueque causa suam differentiam super diem mediocrem addit vel minuit ex xix[a] et xxi[a] et xxii[a] attende. Cum ergo ambe cause simul addunt vel simul minuunt, differentias ad eandem diem attinentes in unum collige. Cum autem una causa minuit, alia addit, minorem a maiori minue, et habebis omnes ex duabus causis differentias. Cum vero quantum una causa minuit tantum alia addit, nulla provenit differentia, et fit dies medio-

687 Tauri] Thauri *MN* lxxxxv] *corr. ex* lxxxv *s.l. P* **688** ergo] *marg. P om. MN* diem mediocrem] *corr. in* dies mediocres *K* dies mediocres *M* **688/689** per – fuerint] fuerint per hanc quartam P_7*M* **689** Similiter] simile *N* **691** x] *corr. in* 4 *M* **694** in] *om. or del. K* ea] eadem *PN* illa P_7 **695** motu medio] medio motu P_7 **696** relinquetur] relinquitur *N* **697** in[1] – recta[2]] *s.l. K* si in] cum *N* **699** predictis] premissis P_7 est – satius] et facilius est *M* dies] diem *N* vel] vel a *M* **700** vel] vel ab P_7 orizontem] orientem *K* **701** provenit] est *s.l. N* principia] *marg. M* puncta *N* **702** maior – minor] minor modo maior est *N* **704** quod] quia P_7 dierum] dierum erit *M* regionum] *in another hand K* variatur] variantur *M* **705** est] sunt *M* **709** Singulas] angulos *P* differentias] differentia P_7 **710** suam differentiam] suam causam *P om. N* **712** simul[2]] *s.l. P* **713** una] *s.l. K* minuit] et *add.* (*s.l.* P_7) P_7*MN* a] de *N* **714** habebis omnes] omnes habebis P_7 duabus] ambabus *N* vero] ergo *M* quantum] quantitum P_7 **715** causa] *om. PMN* (causa *Ba om.* E_1) minuit – addit] addit tantum alia minuit *N* middle of Taurus to the middle of Leo passes with 95° of the equator. Because of this, therefore, the diverse days, when the differences have been gathered throughout this quarter, exceed the average day by 5 time-degrees. It happens similarly in the quarter opposite this. It is manifest, therefore, that on account of this cause the greatest diverse days exceed the smallest diverse days by 10 time-degrees.

23. To seek out the difference coming from the unequal elevation between the average day and whatever diverse day you are told ⟨to find⟩.

Then take the elevation of the arc of the sun's mean motion from that degree in which the sun stays on that day about which you seek. And if it is greater than the mean motion, subtract that [i.e. the mean motion] from it; and if less, subtract it from that. And the difference that you seek will remain. But if you sought in the right sphere, take the elevation in the right sphere; if in a declined sphere, take the elevation in the declined sphere.

Accordingly, it is clear from what has been said that it is more helpful and preferable that the day begins from noon or midnight than from rising or setting because at the horizon there results a greater inequality of days; and ⟨it is clear⟩ because in the declined horizon the beginnings of rising and setting vary because the arc of the day is at one time greater and at another time smaller, ⟨but⟩ in the right horizon all arcs are equal according to a likeness of parts; and particularly for the reason that the inequality of these days varies according to the different latitudes of regions at the horizon, but at the meridian in every region it is the same.

24. To assess the differences one by one resulting simultaneously from both causes together, as it occurs, and to find the beginning of addition to the average day and the beginning of the diminution from the average day.

Accordingly, collect the individual differences from each cause for the individual days as it is had from the preceding ⟨proposition⟩ and the 20th. And pay attention to where each cause adds or subtracts its own difference upon the average day from the 19th, 21st, and 22nd. Therefore, when both causes together add or together subtract, combine the differences pertaining to the same day into one. But when one cause subtracts and the other adds, subtract the smaller from the greater, and you will have all the differences from the two causes. And indeed, when one cause subtracts as much as the other adds, no difference results, and the average day comes about. And if thereafter both cris. Et si deinceps ambe cause simul addunt aut una plus addit quam alia minuit super diem mediocrem, tunc ibi est principium additionis; si vero ambe minuunt aut una plus minuit quam alia addit, tunc ibi est principium diminutionis, et hoc erat querendum.

25. Dies differentes in mediocres et mediocres in differentes vertere.

Super fixam igitur radicem temporis locum Solis secundum cursum medium et locum Solis secundum cursum apparentem cognosce. Deinde ad diem quam volueris utrumque similiter Solis locum scilicet secundum cursum medium et secundum cursum diversum considera. Et partes cursus medii que sunt inter duo loca secundum medium cursum deprehensas seorsum observa. Similiter partes cursus diversi que inter duo loca vera deprehenduntur observa, et istarum partium elevationes in spera declivi si dies ab orizonte incipiant aut in spera recta si dies a meridiano inchoent perpende. Et eas a motu medio si maior fuerit deme, et differentiam tempora horarum pone, et a diebus differentibus quos in mediocres convertere volueris minue. At si motus medius elevationibus minor fuerit, ipsum ab eis deme, et residuum tempora horarum pone, et diebus differentibus appone. Et fient dies mediocres. Huius conversionem facies si mediocres in differentes vertere volueris. Et nota quod hoc quoque modo facilius differentias ex duabus pariter causis provenientes ad dies singulos poteris colligere.

Hoc quoque animadvertendum quod si radix temporis posita fuerit super principium additionis ad diem mediocrem, differentiam que provenerit semper addendum est ut fiant dies mediocres ex differentibus, et semper minuendum a mediocribus ut ex eis fiant differentes; e converso fiat si radix temporis posita fuerit super principium diminutionis. Et hoc ideo quia quantum ex una parte additur super mediocres ex alia minuitur, et non equatur minutio additioni

716 plus addit] addit plus *N* **717** si] plus addit *add. et del. P* **719** hoc] hoc est quod *M* **720** et] et dies *N* **721** Solis] *om. PN* **722** Solis] *om. N* **723** Solis locum] locum Solis P_7 **724** secundum] *om.* P_7 sunt] sint *P om.* P_7 inter] *marg. P om. K* **725** loca] loco *N* medium cursum] cursum medium P_7N deprehensas] deprehensa sunt P_7K deprehensa *M* considera *add. et del. N* (deprehensa sunt *Ba* demp†ser†is E_1) **726** loca vera] vera loca *M* **726/727** istarum partium] istorum P_7K item partium *N* **727** incipiant] incipiantur *M* incipiatur *N* **728** inchoent] inchoantur *M* inchoetur *N* **729** differentiam] differentiam et *M* **730** At] aut *PN* elevationibus] *corr. ex* elongationi *N* **731** ipsum] vel temporum *P* deme] minue *N* **733** vertere] convertere *N* nota] notandum P_7 **736** quoque] quoque modo *M* animadvertendum] animadvertententendum *P* posita fuerit] posita fuit *P* fuerit posita *M* **737** differentiam] differentia *N* (differentiam BaE_1) provenerit] provenit *PN* proveniet *M* (provenerit *Ba* provenit E_1) **737/738** semper addendum] superaddenda *N* (superaddita est *Ba* semper addendum E_1) **738** mediocres – differentibus] differentes ex mediocribus *N* minuendum] minuendum est *M* minuenda *N* (medius *Ba* minuendum est E_1) **739** fiant] fiant dies *N* si] *s.l. P* **741** mediocres] mediocres et P_7 mediocres tantum *N* alia] alia parte *MN* minuitur] minuetur *K* equatur] adequatur P_7

causes together add or one adds more upon the average day than the other subtracts, then in that place is the beginning of addition; however, if both subtract or one subtracts more than the other adds, then in that place is the beginning of diminution, and this was what was to be sought.

25. To turn diverse days into average ⟨days⟩ and average into diverse.

Accordingly, upon a fixed radix of time, know the sun's place according to the mean course and the sun's place according to the apparent course. Then for the day that you want, consider similarly each place of the sun, i.e. according to the mean course and according to the irregular course. And note separately the degrees of the mean course that are found between the two places according to the mean course. Similarly, note the degrees of the irregular course that are found between the two true places, and assess those parts' elevations in the declined sphere if the days begin from the horizon, or in the right sphere if the days begin from the meridian. And subtract them from the mean motion if it it is greater, and place the difference as times of hours, and subtract from the diverse days that you want to convert into average ⟨days⟩. But if the mean motion is smaller than the elevations, subtract it from them, and place the remainder as times of hours and add them to the diverse days. And the average days will be made. You will do the reverse of this if you want to turn average days into diverse. And note that in this way also you will be able to combine the differences resulting from the two causes together for individual days more easily.

It must also be noticed that if the radix of time is placed upon the beginning of addition to the average day, the difference[18] that results must always be added so that the average days may be made from the diverse ⟨days⟩, and it must always be subtracted from the average ⟨days⟩ so that from them the diverse ⟨days⟩ are made; it would occur conversely if the radix of time is placed upon the beginning of diminution. And this ⟨is so⟩ for the reason that as much as is added upon the average days on one side is subtracted on the other, and the diminution does not equal the addition until it returns to the place

[18] The construction here of an impersonal gerundive with an accusative object is unusual, but it was used earlier in III.1.

donec ad locum additionis redeatur. Super dies itaque mediocres medius motus constitutus est, quorum equatio si neglecta fuerit in tardioribus quidem planetis non multum sentietur. Sed profecto in hiis que circa Lunam contingunt, manifesta apparebit in tempore considerationis tardior vel celerior diversitas.

Explicit liber tertius continens universam de motu Solis doctrinam.

744 que] qui *P* **745** considerationis] consideratoris *P* diversitas] diversitas et cetera *MN* **746** Explicit – doctrinam] *om.* P_7K explicit liber tertius *M* tertius explicit *N* (*om.* BaE_1)

of addition. Accordingly, the mean motion was set up upon average days, the correction of which, if it is ignored, will indeed not be perceived much in the slower planets. But certainly in those things that come to pass about the moon, a conspicuous difference slower or faster will appear at the time of observation.

The third book containing the whole doctrine concerning the sun's motion ends.

⟨Liber IV⟩ Incipit quartus de motu Lune.

Terram ad Lune distantiam sensibilem quantitatem habere. Ideoque ad speram Lune vicem centri non optinere.

Lunam ab orbe signorum et ad meridiem et ad septentrionem declinare et ad orbem signorum reverti.

Circuitiones Lune in longum tempore diversas esse.

Circuitiones Lune in latum tempore diversas esse.

Lunam in omni parte circuli signorum triplicem secundum visum motum habere, modo velociorem, modo mediocrem, modo tardiorem.

Umbram terre semper a Solis opposito Soli similiter et equaliter moveri.

Lunam a Sole menstruum lumen habere.

Faciem Lune Soli obversam semper a Sole illuminari.

Umbram terre causam lunaris eclipsis esse.

Lunam Soli et aspectui interpositam solaris defectus causam esse.

Equalis lunatio dicitur reditus Lune ad Solem secundum utriusque motum medium.

Mensis est equalis lunationis tempus.

Locus verus Lune in celo est punctum celi cui linea a centro terre per centrum Lune educta in celum occurrit.

Locus Lune verus in circulo signorum est communis sectio duorum orbium quorum unus est ipse orbis signorum et alius magnus orbis per polos circuli signorum et locum verum Lune in celo transiens.

Et latitudo Lune est arcus istius circuli inter verum locum Lune in circulo signorum et verum locum Lune in celo deprehensus.

Motus longitudinis est loci Lune in celo vel in circulo signorum progressio.

Motus latitudinis est a sectione communi circuli signorum et circuli declinantis Lune elongatio.

Motus diversitatis est Lune in epiciclo sive Lune in ecentrico cum propter alterutrum istorum modorum diversum motum habeat ambulatio.

1 Incipit – Lune] liber quartus *add. marg.* (*other hand*) *P* liber quartus P_7 quartus *marg. K* et incipit quartus *M* incipit quartus *marg. N* **3** optinere] obtinere *MN* **6** Circuitiones – esse] *marg.* P_7 Circuitiones] *corr. ex* circuitione *K* longum] *corr. ex* longo *KM* **7** Circuitiones – esse] *om. N* **8/9** motum habere] habere motum *N* **9** modo velociorem] velociorem modo *M* modo[2]] modo qualiter mov†er†i *P* **14** solaris defectus] defectus solaris *corr. ex* defectus (*perhaps other hand*) *P* defectus solaris *N* **19** in celum] *om. N* occurrit] *corr. ex* occurrunt *K* **21** est] *s.l. K* **22** verum Lune] Lune verum *K* **23** Et] *om.* P_7 locum Lune] Lune locum P_7 **24** Lune] *del. M* **25** vel] *corr. ex* et P_7 **26** Motus] *corr. ex* locus P_7 declinantis] *corr. ex* declinationis P_7 **29** modorum] *s.l. P*

Book IV

The fourth concerning moon's motion begins.

That the earth has a perceptible quantity to the distance to the moon. And for that reason it cannot occupy the place of a center for the moon's sphere.

That the moon turns aside from the ecliptic both to the south and to the north and returns to the ecliptic.

That the moon's revolutions in longitude are different in time.

That the moon's revolutions in latitude are different in time.

That the moon has a triple motion according to sight in every part of the ecliptic, sometimes faster, sometimes average, sometimes slower.

That the earth's shadow from the sun is always moved opposite the sun similarly and equally.

That the moon has monthly light from the sun.

That the face of the moon turned towards the sun is always lit up by the sun.

That the earth's shadow is the cause of a lunar eclipse.

That the moon placed between the sun and the gaze is the cause of a solar eclipse.

A mean lunation means the moon's return to the sun according to the mean motion of each.

A month is the time of a mean lunation.

The moon's true place in the heavens is the point of the heavens to which the line extended from the earth's center through the moon's center and into the heavens goes.

The moon's true place in the ecliptic is the intersection of two circles, one of which is the ecliptic itself and the other is the great circle passing through the ecliptic's poles and the moon's true place in the heavens.

And the moon's latitude is the arc of that circle caught between the moon's true place in the ecliptic and the moon's true place in the heavens.

The motion of longitude is the progression of the moon's place in the heavens or in the ecliptic.

The motion of latitude is the withdrawal from the intersection of the ecliptic and the moon's declined circle.

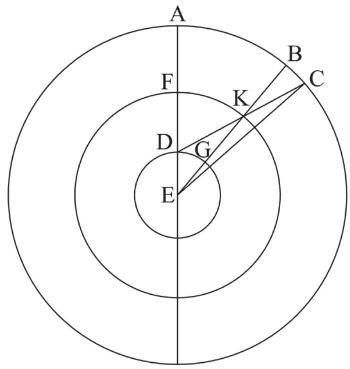
The motion of irregularity is the movement of the moon on an epicycle or of the moon on an eccentric when it has an irregular motion because of one or the other of those models.

Nodi sunt sectiones circuli signorum et circuli declinantis Lune.

Caput est nodus ille per quem transit Luna a meridie in septentrionem.

Cauda est nodus oppositus.

1. Verus Lune locus in celo vel in circulo signorum neque per considerationem instrumenti in loco obliquato neque per considerationem ex stellis fixis neque per solares eclipses deprehendi potest.



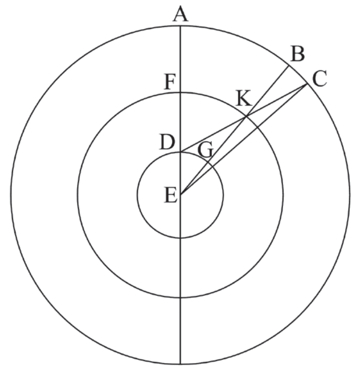
Cum enim terra sensibilem ad speram lunarem habeat quantitatem nec vice centri ad eam fungatur, sit spera terre DG cuius centrum E et spera Lune super idem centrum FK et spera celi ABC. Et sit D locus aspectus oculorum obliquatus, idest non in directo Lune, et punctum A cenit capitum, et punctum K in spera FK locus Lune. Palam ergo quod linea EKB educta a centro terre per corpus Lune assignat verum locum Lune in celo punctum B. Linea vero ab aspectu oculorum secundum considerationem producta est DKC secans aliam in puncto K, et protenditur ad punctum C. Non ergo per considerationem instrumenti ab hoc loco D invenitur verus locus Lune qui est in puncto B. Sed si aspectus oculorum esset a puncto G ut esset linea una EGK, tunc per considerationem verus locus Lune in celo qui est B posset deprehendi. Est etiam manifestum quod non est necesse lineam DKC pervenire ad verum locum Lune in circulo signorum quem circulus per polos zodiaci transiens et per corpus Lune invenit. Eadem est ratio quare per considerationem ex stellis fixis ab hoc loco locus Lune non possit inveniri.

Per solares vero eclipses ob eandem quoque causam non potest sciri eo quod Luna interposita aspectui et Soli causa est solaris eclipsis ut si Luna sit in puncto K et Sol in loco C super lineam EC. Palam ergo quod punctum C est apparens locus Lune a loco aspectus D per solarem eclipsim deprehensus, sed verus locus Lune est a centro terre super punctum B. Manifestum ex

32 oppositus] *corr. ex* appositus *K* **33** Lune locus] locus Lune P_7 considerationem] considerationes *N* **34** in – obliquato] in loco abliquato *del. K* **36/37** sensibilem – lunarem] *corr. ex* ad speram lunarem sensibilem *P* **37** vice] vicem *M* **38** spera terre] terre spera *M* **39/40** idem centrum] centrum idem *M* **40** FK] SK *P* **42** cenit] czenit *M* **45** assignat] designat *KM* **48** K] *corr. ex* Q *M* **49** ergo] *marg. P* **51** una] *s.l. M* in – est] qui est in celo *PN* **52** posset deprehendi] deprehendi posset *M* **53** verum locum] locum verum *MN* **54** quare] *om. P* quod *N* **55** ex stellis] extellis *K* **56** eandem – causam] hanc causam quoque *M* sciri] sciri ex *M* **57** interposita] *marg.* (*perhaps other hand*) *P* **58** Sol] Sol sit *M* C] T *N* **59** C] T *N* apparens] apparentis *P* eclipsim] eclipsem *PK* (eclipsis *Ba* eclipsem E_1) **60** Manifestum] manifestum est *M*

The nodes are the intersections of the ecliptic and the moon's declined circle. The head is that node through which the moon passes from south to north. The tail is the opposite node.

1. The moon's true place in the heavens or in the ecliptic can be discovered neither through the observation of an instrument in a oblique place nor through an observation from the fixed stars nor through solar eclipses.



Because indeed the earth has a perceptible quantity ⟨compared⟩ to the lunar sphere and does not serve as a center for it, let there be the earth's sphere DG whose center is E, the moon's sphere FK upon the same center, and the sphere of the heavens ABC. And let D be the oblique place of the eyes' gaze, that is, not in the direction of the moon, and point A the zenith, and point K the moon's place on sphere FK. It is clear, therefore, that line EKB extended from the earth's center through the moon's body designates point B, the moon's true place in the heavens. And indeed the line produced from the eyes' gaze according to observation is DKC, cutting the other at point K, and it is extended to point C. Therefore, from the observation with an instrument from this place D, the moon's true place, which is at point B, is not found. But if the eyes' gaze were from point G so that EGK would be one line, then the moon's true place in the heavens, which is B, would be able to be found through the observation. It is also manifest that it is not necessary that line DKC comes to the moon's true place in the ecliptic, which the circle passing through the poles of the zodiac and through the moon's body finds. The proof of why the moon's place is not able to be found through an observation from the fixed stars from this place is the same.

And indeed, on account of the same cause, it cannot be known through solar eclipses because the cause of a solar eclipse is the moon placed between the gaze and the sun, as if the moon were at point K and the sun at point C upon line EC. Therefore, it is clear that point C is the moon's apparent place from the place of vision D found through the solar eclipse,[1] but the moon's true place is from the earth's center upon point B. It is also manifest from

[1] While the accusative of 'eclipsis' is normally 'eclipsim', the witnesses suggest that the author spelled it 'eclipsem' here.

hiis quoque quod secundum diversa loca aspectus diversificatur apparens locus Lune in celo, sed qui a centro terre deprehenditur in eodem instanti temporis ubique unus est.

2. Verum locum Lune per lunaris eclipsis considerationem cognosci est possibile.

Quia enim lunaris eclipsis causa est umbra terre que dum eam Luna ingreditur prohibet radios Solis a Luna, et hec umbra necessario fertur ex opposito Solis, constat quod Luna in eclipsi sua Soli per diametrum opponitur. Nisi enim semicirculus maioris orbis spere comprehenderet Solem et Lunam, nullatenus umbra terre Lunam comprehenderet. Tempus ergo medie eclipsis ex consideratione principii et finis est perpendendum. Nam cum Luna est in medio umbre, centrum Lune est in puncto Solis opposito. Cum ergo ex precedenti libro verus locus Solis ad quodlibet tempus notus est, erit et locus Lune in medio eclipsis notus.

3. Tempus equalis lunationis verisimiliter investigare. Unde et tempus reducens integre diversitates Lune, et primum tempus reducens similem coniunctionem vel oppositionem similem Solis et Lune, necnon et medius motus diversitatis et medius motus longitudinis innotescent.

Adnotandum primum quod nec tempus equalis lunationis nec doctrina medii motus Lune aut diversi dari potuit nisi habita etiam notitia de tempore reversionis diversitatis Lune. Ad inveniendum autem tempus reversionis diversitatis, animadverterunt antiqui invenire intervallum temporis reducens semper equalem motum longitudinis, continens scilicet aut integras revolutiones in longum aut supra integras arcus equales. Hoc autem tempus non aliter quam per eclipses potuit deprehendi sicut ex premissis manifestum est. Sumamus ergo interim tempus tale deprehensum esse et sit AB. Dico quod tempus AB equat semper diversum motum cum medio. Multiplicetur enim tempus AB quantumlibet ut AB sit equale ei quod est BC. Erit itaque

61 diversificatur] diversificantur *P corr. ex* diversificantur *N* apparens] *corr. ex* apparentis *K* **62** Lune] *om. N* qui a] quia *M* **63** unus est] est unus *M* **66** eam Luna] Luna eam *M* **67** ex] *corr. ex* ad *P* **69** semicirculus] *corr. ex* circulus *P* comprehenderet] deprehenderet *M* **70** eclipsis] eclipsi *P* **73** verus – Solis] verus Solis locus *P* locus verus Solis P_7 quodlibet] quodlibet instans sive P_7 est] *s.l. P* **76** diversitates] diversitatis *N* primum tempus] tempus primum *K* **77** similem] *om.* P_7 **77/78** medius motus] motus medius P_7M **78** innotescent] innotescerent P_7 **79** Adnotandum primum] annotandum primum P_7K notandum primo *N* **80** equalis lunationis] lunationis equalis *K* **80/81** motus Lune] Lune motus *M* **81** habita etiam] etiam habita esset *N* **82** reversionis diversitatis] *corr. ex* diversitatis reversionis *M* **83** diversitatis] *marg. P* diversitatis Lune *MN* animadverterunt] animadvertunt *M* invenire] adinvenire *N* **84** semper] super *P* **85** integras] *corr. ex* integros *K* **87** interim] *om. K* tempus tale] tale tempus *N* **88** quod] sic *add. s.l. P* **89** AB sit] sit AB *PM* sit *N* these things that according to different places of the gaze, the moon's apparent place in the heavens varies, but that which is found from the earth's center in the same instant of time is one everywhere.

2. It is possible that the moon's true place be known through an observation of a lunar eclipse.

Because indeed the cause of a lunar eclipse is the earth's shadow that blocks the sun's rays from the moon while the moon goes into it, and this shadow is necessarily carried opposite the sun, it is established that in its eclipse the moon is placed diametrically opposite the sun. For unless a semicircle of a great circle of the sphere takes hold of the sun and moon, by no means would the earth's shadow take hold of the moon. Therefore, the time of the middle of the eclipse should be assessed from the observation of the beginning and end. For when the moon is in the middle of the shadow, the moon's center is in the point opposite the sun. Therefore, when the sun's true place at whatever time is known from the preceding book [i.e. III.17], the moon's place in the middle of the eclipse will also be known.

3. To find the time of a mean lunation approximately. Whence also the time returning the moon's irregularities wholly, the first time returning a similar conjunction or similar opposition of the sun and moon, and also the mean motion of irregularity and the mean motion of longitude will become known.

It should be noted first that neither the time of mean lunation nor the doctrine of the moon's mean or irregular motion could be given unless knowledge of the time of the return of the moon's irregularity also was had. Moreover, for finding the time of the return of the irregularity, the ancients took care to find an interval of time always returning an equal motion of longitude, i.e. containing either complete revolutions in longitude or equal arcs upon complete ⟨revolutions⟩. Moreover, this time could not be found in another way than through eclipses, as is manifest from what has been set forth. Let us suppose, therefore, for the present that such a time is found, and let it be AB. I say that the time AB always makes the irregular motion equal to the mean motion. For let time AB be multiplied however many times so that AB is equal to that which is BC. Accordingly, the mean motion of time AB motus medius temporis AB equalis motui medio temporis BC propter tempus equale, sed et motus diversus huius temporis motui diverso illius. Ergo aut utrimque motus medius equaliter addit super motum diversum, aut utrimque equaliter minuit, aut utrobique equatur. Sed palam quod impossibile est utrimque pariter addere aut pariter minuere continue. Sic enim in infinitum fieret equalis diminutio vel in infinitum equalis additio. Patet itaque quod tempus AB equat diversum motum medio. Sed hoc scilicet ut equetur diversus medio non contingit nisi in revolutione diversitatis. Si qua est ad hoc instantia, postea explicabitur. Itaque tempus AB continet integras revolutiones diversitatis secundum aliquem numerum ita ut nec plus nec minus. Est itaque opere pretium querere tempus AB, quod reducit motum in longum semper equalem.

Ad huius temporis notitiam querendum est primum tempus equale reducens eclipses, et hoc tum ex scriptis considerationibus in cronicis virorum doctrinalium in quarum veritate confidendum est, tum ex propriis considerationibus. Et est tempus illud, sicut referente Ptolomeo Abrachis ex Caldeorum et suis considerationibus per duo intervalla binarum et binarum eclipsium equalia deprehendit, centum milia et xxvi milia et vii dies et una hora equalis. Dico quod ipsum reducit motum Lune in longum semper equalem. Quia enim in omni eclipsi Luna est in opposito Solis, cum tantum tempus quod continet menses integros reducat motum Solis equalem in longum, reducet necessario motum Lune equalem. Si qua est instantia in motu Solis, postea demonstrabitur. In tempore igitur sic deprehenso numerus mensium eius cognoscendus est, qui facile per Lunam singulis mensibus crescentem et decrescentem sciri potest, et est in prescripto tempore iiii milia et cc et lxvii menses. Deprehendit etiam in tempore prefinito quod eclipses reducit quis sit numerus reversionum diversitatis. Nam tempus unius reversionis ad propinquum cognoscitur ex redeunte

90 temporis BC] BC temporis *M* **91** et] *om. K* illius] illius temporis *N* Ergo] *s.l. M* **92** utrimque[1]] utrique *M* utrimque[2]] uterque *M* **93** impossibile] *corr. ex* impossibilis *K* utrimque] utrumque *PM* (utrobique *Ba*E_1) **94** in] *om. P* **95** in] *om. P* equalis additio] additio equalis P_7 Patet] palam P_7K **96** motum] motum a *M* equetur] equatur *M* diversus] diversus motus P_7M **97** in] ex *K* instantia] *corr. ex* distantia P_7 **97/98** postea explicabitur] post hec explicatur *P* post hoc explicatur *N* **98** integras] integrales *P* diversitatis] *om. N* **99** ita] *om. P* **100** reducit] reducat *N* **103** tum] *iter. et del. M* **104** Et est] sed *N* referente Ptolomeo] Ptolomeo referente *PN* Ptolomeo] Ptholomeo P_7 Abrachis] ab Rachis *P* a brachis *K* ex] *om. P* **105** duo] dua *N* **106** xxvi] xx†...† *corr. ex* xx illi P_7 Dico] *corr. ex* dicit P_7 **109** equalem] *om. N* **110** est instantia] instantia est P_7K demonstrabitur] determinabitur P_7K (determinabitur *Ba* demonstrabitur E_1) **111** eius – est] comprehendendus (*del.*) est cognoscendus *N* **112** singulis] duobus *add. et del. N* sciri] *corr. in* sci†re† *M* **113** lxvii] *corr. ex* 26 *M* Deprehendit] *corr. in* deprehend†endum† P_7 **114** tempore prefinito] *corr. ex* tempore de prefinito P_7 prefinito tempore *N* reversionum] revolutionum P_7 **115** propinquum cognoscitur] propinquam cognoscetur *M* will be equal to the mean motion of time BC because of the equal time, but also the irregular motion of this time ⟨is equal⟩ to the irregular motion of that ⟨time⟩. Therefore, either on both sides the mean motion adds equally upon the irregular motion, or on both sides it subtracts equally, or in both cases it is equal. But it is clear that it is impossible that on both parts it adds equally or subtracts equally continuously. For thus a uniform diminution would be made *in infinitum*, or a uniform addition *in infinitum*.[2] Accordingly, it is clear that time AB makes the irregular motion equal to the mean ⟨motion⟩, but this, i.e. that the irregular ⟨motion⟩ equals the mean, does not occur except in a revolution [i.e. return] of the irregularity. If there is anything impending upon this, it will be explained afterwards [IV.5–6]. Accordingly, time AB contains complete revolutions [i.e. returns] of the irregularity according to some number such that it is neither more nor less. And so it is worth the effort to seek time AB, which always returns an equal motion in longitude.

For the knowledge of this time, first an equal time returning eclipses must be sought, and this from observations recorded in the chronicles of men, the truth of which doctrines must be trusted, and from one's own observations. And as, with Ptolemy reporting, Hipparchus discovered from the Chaldeans' and his own observations through two equal intervals of pairs of eclipses, that time is 126,007 days and one equal hour. I say that it always returns an equal motion of the moon in longitude. For, because the moon is opposite the sun in any eclipse, and because so great a time, which contains complete months, returns a motion of the sun equal in longitude, necessarily it will return an equal motion of the moon.[3] If there is anything impending ⟨upon this⟩ in the sun's motion, it will be demonstrated afterwards [IV.5–6]. Therefore, in the time thus found, the number of its months must be known, which is able to be known easily through the moon waxing and waning in each month, and there are 4,267 months in the time written before. In the determined time that returns eclipses, he also found what the number of returns of the irregularity is. For the time of one return to the next is known from the returning

[2] The impossibility involved here is not explained, but if the irregular motion continuously gained upon or fell behind the mean motion, the mean motion could not in fact be a mean motion.

[3] This sentence is confusing at best. It may mean that in such a time, the sun's motion in the ecliptic is equal to that of the moon (with whole revolutions cast out). It appears at first reading to mean that the sun and moon each return to the same places in the zodiac, but that is not the case.

velociori motu vel ex redeunte tardiori motu Lune, qui per considerationem loci Lune a stellis fixis videtur. Et est hic numerus in prefinito tempore iiii milia et quingente et lxxiii reversiones diversitatis. Hiis itaque cognitis numerus dierum et unius hore inter duas eclipses per numerum mensium dividendus, et exibit tempus equalis lunationis. Et est sicut ex premissis deprehenditur xxix dies et xxxi minuta et l secunda et viii tertia et ix quarta et xx quinta fere.

Rursum quia Luna singulis mensibus Solem consequitur et addit super circulum in motu longitudinis quantum Sol interim movetur, numerus revolutionum Solis in quesito intervallo temporis, et si quid supra integras revolutiones de medio cursu Solis restiterit, numero mensium addenda sunt. Et erit hic medius motus Lune ad quesitum temporis intervallum et est sicut ex premissis accidit secundum annum solarem Ptolomei iiii milia revolutiones longitudinis et sexcente et xi et insuper ex una revolutione imperfecta ccclii gradus et medietas unius gradus. Habes ergo certum numerum mensium inter alternatas eclipses qui reducit diversitates Lune.

Quod si scire velis tempus primum reducens similem oppositionem vel coniunctionem Solis et Lune, sume prescriptum numerum mensium quesiti intervalli et prescriptum numerum reversionum diversitatum, et quere numeros minimos in eorum proportione. Et secundum quod premisimus, quia xvii est maximus eos numerans, est numerus mensium primus reducens similem coniunctionem ccli menses et numerus reversionum diversitatis infra hos menses ita ut nec plus nec minus cclxix, et hoc est quod intendebamus.

4. Tempus reducens motum latitudinis inquirere.

Ad huius rei notitiam eligenda est eclipsis qua pars Lune et non tota obscuratur et pars obscurata an australis sit aut septentrionalis detinendum. Expectanda est itaque similium tenebrarum eclipsis et eiusdem magnitudinis et ex eadem parte et ut nichil diversitatis propter diversitatem Lune accidat et ut eclipsis secunda ad eundem nodum proveniat ad quem prima. Nam sic neces-

116 motu[1]] motu Lune *N* redeunte] recedente P_7 **117** est – numerus] hic numerus est P_7 **118** itaque] ita *P* **119** dividendus] dividendus est *MN* **121** et[1]] *om. MN* et xxxi] et 29 *s.l.* (*other hand*) *K* et[2]] *om. M* et[3]] *om. M* et[4]] *om. M* xx – fere] xxv sexte *P* 25 quinta *N* **122** Luna] Luna in *PN* **124** in quesito] inquisito *K* temporis] temporis inveniatur *N* **125** cursu Solis] Solis cursu *N* **126** medius motus] motus medius *K* **127** Ptolomei] Tholomei P_7 **127/128** iiii – xi] quatuor milia et sexingenta et undecim revolutiones longitudinis *M* 4611 revolutiones *N* **128/129** et[4] – gradus] *s.l.* (*other hand*) *K* **129** Habes] habemus P_7 alternatas] alternas P_7 **131** velis] volueris P_7 velles *M* reducens] *corr. ex* reducenti *K* reducens in *M* **131/132** oppositionem – coniunctionem] coniunctionem vel oppositionem *N* **135** numerans] numeratis *K* est[2]] est ergo *M* mensium primus] *iter. et del.* P_7 **136** ccli] *corr. ex* cccli *P corr. ex* et 51 P_7 **140** an] *s.l. P* aut *K* sit] fit *K* aut] an P_7 detinendum] determinandum est *M* **141** est itaque] itaque est *KN* et[2]] *om. M* **143** sic] si *P* necessario] *corr. ex* necessaria *K* fastest motion or from the returning slowest motion of the moon, which is seen through an observation of the moon's place from the fixed stars. And in the determined time, this number is 4,573 returns of the irregularity. Accordingly, with these things known, the number of days and of one hour between the two eclipses should be divided by the number of months, and the time of a mean lunation will result. And, as it is found from what has been set forth, it is approximately 29 days 31′ 50″ 8‴ 9^{iv} 20^{v}.[4]

In turn, because the moon in each month reaches the sun and adds upon a circle in the motion of longitude as much as the sun moves in the meantime, the number of the sun's revolutions in the sought interval of time and anything that might remain beyond the whole revolutions of the mean course of the sun,[5] should be added to the number of months. And this will be the moon's mean motion for the sought interval of time, and as it happens from what has been set forth, according to Ptolemy's solar year, it is 4,611 revolutions of longitude and additionally 352° 30′ of an incomplete revolution. You have, therefore, the known number of months between the eclipses succeeding each other that return the moon's diversities.

And if you want to know the first time returning a similar opposition or conjunction of the sun and moon, take the above-written number of months of the sought interval and the above-written number of the returns of the irregularity, and seek the smallest numbers in their ratio. And according to what we set forth, because 17 is the greatest numbering them, the first number of months returning a similar conjunction is 251 months and the number of returns of the irregularity in these months such that it is neither more nor less is 269, and this is what we intended.

4. To seek the time returning the motion of latitude.

For the knowledge of this matter, an eclipse must be selected in which part of the moon and not the whole is obscured, and whether the obscured part is south or north must be retained. Accordingly, one must wait for an eclipse of similar darkness and of the same size and on the same side and such that no difference occurs from the moon's irregularity and such that the second eclipse comes into being at the same node at which the first does. For thus necessarily

[4] This is the value reached by performing this division as Ptolemy describes, but it does not agree with the slightly larger value that Ptolemy provided in the *Almagest*, 29 days 31′, 50″, 8‴, 20^{iv} (Toomer, *Ptolemy's Almagest*, p. 176). While Neugebauer, *A History of Ancient Mathematical Astronomy*, p. 311, remarks that to the best of his knowledge, Copernicus was the first to give the value that results from following Ptolemy's procedure, this correction is found in many earlier Arabic and Latin sources, including Gerard of Cremona's translation of the *Almagest* (1515 ed., f. 36r) and Geber's *Liber super Almagesti* (Nuremberg: Johannes Petreius, 1534, f. 49). See Mancha, 'A Note on Copernicus' "Correction" of Ptolemy's Mean Synodic Month.'

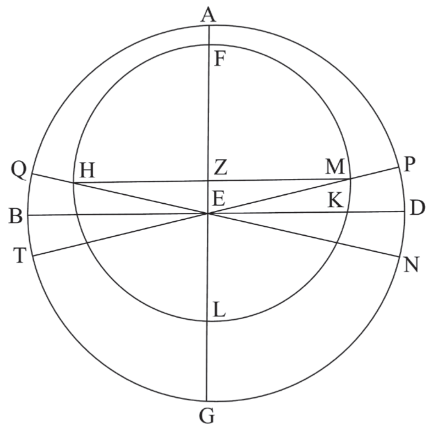
[5] The Latin syntax is difficult to replicate in English here.

sario eadem redibit latitudo. Tempus ergo inter huiusmodi duas eclipses deprehensum est illud quod querimus. Et secundum quod Abrachis invenit hoc tempus menses v milia et quadringenti et lviii menses et fiunt interim revolutiones latitudinis v milia et nongente et xxiii revolutiones. Nam una ad propinquum deprehendi potest per reversionem Lune ad stellam fixam.

5. Sumptam investigationem temporum per duo solum intervalla alternarum eclipsium equalia fallere duabus de causis est possibile.

Una causa est diversus motus Solis. Ad hoc enim ut tempus equale reducens omnes diversitates Lune recte sumptum sit, oportet ut in utroque intervallo quod est inter eclipses alternas post revolutiones Solis integras, aut nulla sit medii motus Solis ad diversum differentia, aut si aliqua, equalis. Alioquin error erit.

Et ponam ad hoc figuram circuli signorum ABGD supra centrum E et ecentricum Solis FHK supra centrum Z. Et diametri supra centra se ortogonaliter secent. Et transeat diameter BED super longitudines medias et AEG super longitudines alias. Sitque principium motus Solis in uno intervallo a puncto P cui Luna per diametrum opposita in puncto T. Et proveniat Sol in fine cursus primi intervalli ad punctum Q cui Luna per diametrum tunc opposita in puncto N. Proiectis ergo integris revolutionibus que sunt equalia annorum spatia, relinquitur arcus PAQ in tempore motus medii MFH. Sit iterum principium secundi cursus Solis in alio intervallo a puncto Q cui Luna per diametrum opposita in puncto N, et pervenerit Sol in fine cursus ad punctum P cui Luna per diametrum opposita in puncto T. Proiectis ergo integris ab hoc intervallo revolutionibus que sunt equalia annorum spatia et totidem quot prius cum equale sit intervallum, relinquitur arcus QGP in tempore motus medii HLM, quod

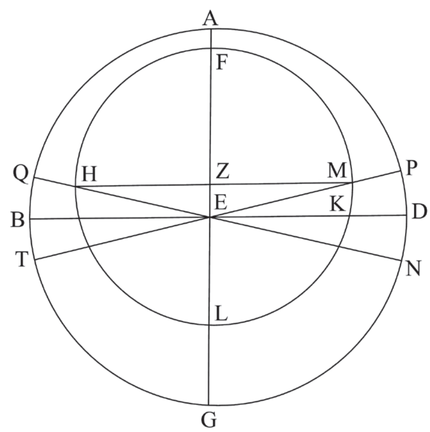


144 eadem redibit] redibit eadem *PN* huiusmodi] *corr. ex* huius valli modi *P* has *N* **145** Et] et est P_7K (et est *Ba* et E_1) **146** menses^2] *om. N* fiunt] fuerunt *N* **151** est] *om.* P_7 **152** sit] sic *corr. in* fit *M* **153** eclipses alternas] alternas eclipses P_7 Solis integras] integras Solis *N* aut] parva vel *M* **154** aut – aliqua] differentia autem si aliqua est est *M* **156** ABGD] AB et (transeat *add. et del.*) GD *P* **157** FHK] HFM *M* Z] *corr. ex* et *K* supra^2] *s.l. K* se ortogonaliter] sese ortogonaliter *K* Z E orthogonaliter se *M* **160** opposita] tunc *add. et del. P* proveniat] perveniat P_7 **161** per – tunc] tunc per diametrum *K* opposita] *om.* P_7 **164** MFH] in FH *P* Sit] sitque *N* **165** secundi cursus] secundum cursum *PM* **168** N] *corr. ex* T *N* **169** Luna] linea *P* **171** ab – intervallo] *marg. P* **175/176** quod tempus] quia tempus *corr. ex* quia totus *M* the same latitude will return. Therefore, the time caught between two eclipses of this kind is what we seek. And according to what Hipparchus found, this time is 5458 months, and meanwhile revolutions of latitude are made, 5923 revolutions. For one to the next can be discovered through the return of the moon to a fixed star.

5. It is possible that the investigation of time taken through only two equal intervals of eclipses succeeding each other be mistaken from two causes.

One cause is the sun's irregular motion. Indeed, for this, so that an equal time returning all the moon's diversities be taken correctly, it is necessary that in each interval that is between the successive eclipses after the sun's complete revolutions, there is no difference between the sun's mean and irregular motion, or if there is anything [i.e. any difference], it is equal. Otherwise there will be an error.

And for this I shall suppose a figure of the ecliptic ABGD upon center E, and the sun's eccentric FHK upon center Z. And let the diameters upon the centers intersect perpendicularly. And let diameter BED pass upon the mean distances, and AEG upon the other distances [i.e. the apogee and perigee]. And let the beginning of the sun's motion in one interval be from point P, to which the moon is diametrically opposite at point T. And let the sun come forth at the end of the first interval's course at point Q, to which the moon is then diametrically opposite at point N. Therefore, with complete revolutions, which are the equal durations of years, cast out, arc PAQ remains in the time of the mean motion MFH. Again, let the beginning of the sun's second course in the other interval be from point Q, to which the moon is diametrically opposite at point N, and at the end of the course, the sun reaches point P, to which the moon is diametrically opposite at point T. Therefore, with complete revolutions, which are the equal durations of years and are as numerous as before because the interval is equal, cast out from this interval, arc QGP remains in the time of



tempus motus medii equale est priori. Sed motus diversus in circulo signorum multum dissimilis. Quare nec motus Lune prioris intervalli similis est motui eius secundi intervalli; oportebat autem si utrumque intervallum esset reducens omnes diversitates Lune.

Alia causa est que impedire potest diversitas Lune non obstante etiam Solis diversitate. Ad hoc enim ut tempus reducens omnes diversitates Lune recte sumptum sit, oportet ut integre sint in duobus intervallis reversiones diversitatum, et non relinquantur imperfecte. Sed possunt intervalla inter alternas eclipses esse equalia duabus extremis reversionibus diversitatum manentibus imperfectis, ut si principium cursus Lune in uno intervallo incipiat a loco cursus minimi et in fine ipsius intervalli pervenerit ad locum cursus maximi, et principium cursus secundi in alio intervallo sit a loco cursus maximi et in fine istius intervalli perveniat ad locum cursus minimi. Sic enim utrobique nulla quidem erit differentia motus medii ad diversum. Et erunt proiectis integris revolutionibus tempora imperfectarum reversionum equalia. Aut rursum si principia et fines cursuum in duobus intervallis sint in locis equaliter distantibus a loco cursus maximi et loco cursus minimi. Sic enim equalis sed non eadem numero redibit motus medii ad motum diversum differentia in temporibus equalibus. Oportebat autem eandem redire si esset tempus continens omnes differentias motus medii ad motum diversum.

6. Tempus investigationis temporum quod fallere non possit eligere.

Primum igitur ne Solis diversitas impediat investigationem nostram, observanda sunt inter alternas eclipses equalia quidem intervalla temporum. Que sint huiusmodi equalia determinabo. Oportet etenim ut utrumque intervallum contineat integras Solis revolutiones et nichil supersit; vel ut post integras in uno intervallo Solis revolutiones superfluat medietas circuli que est a longitudine longiore ad longitudinem propiorem, et in alio intervallo superfluat alia medietas que est a longitudine propiore ad longitudinem longiorem; vel ut sit principium cursus in utroque intervallo ab uno et eodem loco circuli signo-

177 dissimilis] *perhaps corr. ex* di[†]...[†] *K* nec] non P_7 **178** esset reducens] reducens esset *N* **180/181** etiam – diversitate] diversitate Solis *N* **182** duobus] duabus *KM* reversiones] revolutiones *M* **184/185** manentibus imperfectis] imperfectis manentibus *M* **186** pervenerit] perveniat P_7 **186/188** et[2] – minimi] *marg.* P_7 **187** sit] fit *P* **188** perveniat] proveniat *P* minimi] minimi maximi *P* Sic enim] sicque P_7 **189** quidem] *om.* P_7M erit] *s.l.* (*perhaps other hand*) *P* **191** et] aut *K* cursuum] cursuum sint *N* sint] sicut *N* **192** loco[2]] *om. N* **193** numero redibit] redibit numero *P corr. ex* numquam redibit *K* redibit *N* motus] *iter. et del.* P_7 **194** Oportebat] oportebit P_7 autem] enim *M* redire] *corr. ex* reperire *N* continens] *corr. ex* conveniens *M* **196** eligere] *corr. ex* eligeret P_7 **197** diversitas impediat] impediat diversitas *K* **199** equalia] qualia P_7K **200** nichil] nil *M* **201** uno intervallo] intervallo uno *N* **202** propiorem] *corr. ex* longiorem *K* **203** a] *corr. ex* ad *K* sit] *corr. ex* si *K* **204** cursus] Solis *add.* (*s.l. K*) *KM* utroque] unoquoque *PN* (unoquoque *Ba* utroque E_1) ab uno] *s.l. P* mean motion HLM, which time of mean motion is equal to the earlier one. But the irregular motion in the ecliptic is very dissimilar. Therefore, neither is the moon's motion of the earlier interval similar to the motion of its second interval; however, it had to be ⟨similar⟩ if each interval were returning all the moon's diversities.

Another cause that is able to hinder is the moon's irregularity, even with the sun's irregularity not getting in the way. Indeed, for this that the time returning all the diversities of the moon may be taken correctly, it is necessary that there are complete returns of the irregularity in the two intervals and that incomplete ones do not remain. But intervals between successive eclipses are able to be equal with the two last returns of the irregularity remaining incomplete, as if the beginning of the moon's course in one interval begins from the place of least course and in the end of that interval it reaches the place of greatest course, and the beginning of the second course in the other interval is from the place of greatest course and in the end of that interval it reaches the place of least course. For thus in both instances there will indeed be no difference between the mean and irregular motion. And with complete revolutions cast out, the times of the incomplete returns will be equal. Or in turn, if the beginnings and ends of the courses in the two intervals were in places equally distant from the place of greatest course and the place of least course. For thus there will return an equal, but not the same in number, difference between the mean motion and irregular motion in equal times. It was necessary, however, that the same ⟨difference⟩ return if it would be a time containing all the differences between the mean motion and irregular motion.

6. To select a time of the investigation of times that cannot deceive.

First, accordingly, lest the sun's irregularity hinder our investigation, equal intervals of times indeed between successive eclipses must be observed. I will determine what equals may be of this sort. And indeed it is necessary that each interval contains complete revolutions of the sun and nothing is in excess; or that after the complete revolutions of the sun, in one interval the semicircle that is from the apogee to the perigee is in excess, and in the other interval, the other half that is from the perigee to the apogee is in excess; or that the beginning of the course in each interval is from one and the same place in the rum; aut ut sint principia et fines primi et secundi cursus in intervallis equalibus eiusdem distantie a longitudinibus duabus longiore et propiore. Sic enim aut nulla erit in duobus intervallis medii motus Solis ad diversum differentia, et erit diversus omnino equalis medio; aut erit eadem vel equalis in duobus intervallis equalibus medii motus ad diversum differentia, et erunt arcus superfluentes medii motus equales invicem, et arcus superfluentes motus diversi equales invicem.

Cum ergo propter motum Solis uno istorum iiii modorum electa fuerint duo intervalla, observandum etiam propter motum Lune diversum ut eadem intervalla sint sicut determinabo, scilicet ut in uno intervallo sit principium cursus Lune a loco cursus velocioris et non pervenerit ad locum cursus tardioris, et in alio intervallo sit principium cursus Lune a loco cursus tardioris et non pervenerit ad locum cursus velocioris; aut aliter ut in uno intervallo sit principium cursus eius a motu mediocri tendente ad velociorem, et principium secundi cursus sit a motu mediocri tendente ad motum tardiorem. Sic enim equalibus intervallis necesse est reversiones diversitatis fieri integras nec aliquid superfluere, quod querebamus.

Alioquin ponamus manentibus premissis post integras revolutiones in utroque intervallo arcus de imperfectis revolutionibus superfluere. Proiectis ergo integris revolutionibus cum equalibus earum de ambobus intervallis temporibus, necesse est equalia relinqui tempora de intervallis equalibus. Sed arcus necessario qui ex reversionibus diversitatis hinc inde superfluunt inequales faciunt arcus diversorum motuum Lune residuos propter cursus predicto modo sumptos, et tempora residua intervallorum esse equalia. At arcus residui diversorum cursuum Solis in eisdem temporibus aut nulli erant aut equales invicem. Necessario ergo Luna in fine alterius intervallorum non fit in puncto Soli opposito, sed constat quod fuerit propter hoc quod in fine utriusque intervalli eclipsis fuerit. Hanc igitur diligentiam in electione temporum, referente Ptolomeo, observavit Abrachis subtilissima consideratione ad deprehendendum prefinita revolutionum tempora. Fortassis tamen valde difficilis est huiuscemodi temporum electio.

7. Medium motum Lune in longitudine et medium motum diversitatis et medium motum latitudinis et mediam distantiam Solis et Lune ad quaslibet

205 in] *om. PM* **207** ad] *s.l. K* **208/209** et – differentia] *marg.* P_7 **209** medii] Solis *add. et del.* P_7 ad] *s.l. K* **210** medii motus] motus medii *M* **212** motum] *corr. ex* motus *M* uno] in uno P_7 **213** eadem] *om. N* **214** determinabo] determinando *P* **219** motum tardiorem] tardiorem motum *M* tardiorem *N* **220** equalibus] in equalibus P_7M inequalibus *K* est] *om.* P_7 **222** post] primo *P* **222/223** in – intervallo] *corr. ex* intervallo in utroque *P* **224** ambobus] *corr. ex* ambabus *K* temporibus] *corr. ex* partibus P_7 **226** inequales] *corr. ex* equales *M* **228** intervallorum] intervallorum necesse est *N* At] ac *M* **229** cursuum] *corr. ex* cursu (*perhaps other hand*) *P* temporibus] *corr. ex* poribus P_7 **230** fit] sit P_7M **232** Ptolomeo] Tholomeo P_7 **233** deprehendendum] deprehendum *N* **234/235** difficilis – electio] difficile est huiusmodi electio temporum P_7

ecliptic; or that the beginnings and ends of the first and second courses in the equal intervals are of the same distance from the two apsides, the apogee and perigee.[6] For thus, either there will be no difference between the sun's mean and irregular motion in these two intervals, and the irregular ⟨motion⟩ will be entirely equal to the mean; or there will be the same or an equal difference between the mean motion and the irregular in the two equal intervals, and the excess arcs of mean motion will be equal to each other, and the excess arcs of the irregular motion will be equal to each other.

Then, when two intervals have been selected by one of those four ways because of the sun's motion, because of the moon's irregular motion, it also should be heeded that the same intervals are as I will determine, i.e. that in one interval the beginning of the moon's course is from the place of fastest course and does not come to the place of slowest course, and in the other interval, the beginning of the moon's course is from the place of slowest course and does not come to the place of fastest course; or in another way, that in one interval the beginning of its course is from the average motion heading towards the fastest, and the beginning of the second course is from the average motion heading towards the slowest motion. For thus it is necessary that in equal intervals complete returns of the irregularity are made and that nothing is in excess, which we sought.

Otherwise, with what has been put before remaining, let us suppose that in each interval, arcs of incomplete revolutions are in excess beyond the whole revolutions. Then, with the complete revolutions along with their equal times from both intervals cast out, it is necessary that equal times remain from the equal intervals. But the arcs that are in excess from the returns of the irregularity on one side and the other necessarily make the remaining arcs of the moon's irregular motions unequal because of the courses taken in the said manner [i.e. in the ways listed in the preceding paragraph], and the remaining times of the intervals are equals. But the remaining arcs of the sun's irregular courses in the same times were either nothing or were equal to each other. Therefore, at the end of either of the intervals, the moon necessarily does not occur at the point opposite the sun, but it is evident that it would be ⟨opposite the sun⟩ because of this that there was an eclipse at the end of each interval. Therefore, with Ptolemy reporting, Hipparchus heeded this attentiveness in the selection of times with the most thorough observation in order to discover the determined times of revolutions. Nevertheless, the selection of such times is possibly very difficult.

7. To fit the moon's mean motion in longitude, mean motion of irregularity, mean motion of latitude, and the mean distance of the sun and moon to what-

[6] To make Ptolemy's fourth case clearer, Toomer, *Ptolemy's Almagest*, p. 177 n. 13 explains, 'That is, if the sun has an anomaly of α° at the beginning of the first interval, it must have an anomaly of (360- α)° at the end of the second interval.'

divisiones temporum, scilicet annos collectos, annos disgregatos, menses, dies, horas, minuta horarum adaptare.

Medium motum Solis ad unam diem in numerum dierum mensis unius qui est tempus equalis lunationis multiplica, et superadde revolutionem circuli. Et collectum erit motus Lune medius ad mensem huiusmodi. Divide ergo hunc motum medium per numerum dierum ipsius mensis, et exibit medius motus Lune in longitudine ad unum diem. Serva ut per eum motus medios longitudinis ad omnia cetera tempora invenias. Nam sicut tempus diei se habet ad quodlibet tempus quod elegeris sic se habet motus medius diei ad medium motum temporis quod elegeris. Duc ergo secundum in tertium et divide per primum.

Rursum numerum reversionum diversitatis qui similem coniunctionem reducit scilicet cclxix multiplica in circulum, et divide per numerum dierum mensium qui reducunt similem coniunctionem, et sunt ccli menses. Et proveniet motus medius diversitatis ad unam diem, cum quo ut superius ad cetera tempora operaberis.

Item numerum revolutionum latitudinum supra deprehensum in circulum multiplica, et productum per numerum dierum illorum mensium qui reducunt motum latitudinis, et sunt v milia et quadringenti et lviii menses, partire. Et exibit motus medius latitudinis Lune ad unam diem, cum quo ut supra operaberis.

Item medium motum Solis ad unam diem ex motu medio Lune ad unam diem minue, et reliquum erit media distantia Solis et Lune ad unam diem, cum quo similiter prioribus negotiare ad cetera tempora. Hec media distantia simplex longitudo vocatur.

Manifestum est itaque ex positis arcum medii motus diversitatis ad aliquod certum tempus arcu medii motus longitudinis ad idem tempus in proportione minorem esse.

8. Cum propter diversum motum positum fuerit Lunam habere concentricum cum epiciclo itemque ecentricum, fuerintque equalis magnitudinis concentricus et ecentricus, et distantia centrorum eorumdem fuerit equalis

240 numerum] numero *P* **241** equalis lunationis] lunationis equalis *M* **242** Lune] *corr. ex* linee *K* huiusmodi] huius *M* hunc *N* **243** medius motus] motus medius *M* **244** unum diem] diem unum P_7*K* ut – eum] eum ut per ipsum *N* **246** motus medius] medius motus P_7 medium motum] motum medium *MN* **248** reducit] *corr. ex* credunt *K* *corr. ex* reducet *N* **249** scilicet] *om. N* **250** ccli] 269 P_7*K* 250 *M* **251** motus medius] medius motus P_7*K* **253** Item] iterum *P* revolutionum latitudinum] reversionum latitudinis *M* **256** motus medius] medius motus P_7*K* **258** motu medio] medio motu *MN* **259** minue] *corr. ex* minime P_7 reliquum] reli†cum† P_7 Solis – Lune] inter Solem et Lunam *M* **261** simplex] duplex *N* **262** arcum] *corr. ex* arcuum *K* **262/263** diversitatis – motus] *s.l.* P_7 **263/264** in – esse] esse in proportione minorem *M* **267** concentricus – ecentricus] ecentricus et concentricus *N* ever divisions of time, namely collected years, separated years, months, days, hours, and minutes of hours.

Multiply the sun's mean motion for one day by the number of the days of one month, which is the time of a mean lunation, and add the revolution of a circle. And the sum will be the moon's mean motion for a month of this kind. Divide, therefore, this mean motion by the number of days of that month, and the moon's mean motion in longitude for one day will result. Save ⟨it⟩ so that through it you may find the mean motions of longitude for all the other times. For as the time of a day is disposed to whatever time that you select, thus the day's mean motion is disposed to the mean motion of the time that you selected. Lead, therefore, the second into the third and divide by the first.

In turn, multiply the number of the returns of the irregularity that restore a similar conjunction, i.e. 269, by a circle [i.e. 360°], and divide by the number of days of the months that restore a similar conjunction, and they are 251 months. And the mean motion of irregularity for one day will result, with which you will operate for the other times as above.

Likewise, multiply the number of revolutions of latitude [i.e. 5923] found above [i.e. in IV.4] by a circle [i.e. 360°], and divide the product by the number of days of those months that return the motion of latitude, and they are 5458 months. And the moon's mean motion of latitude for one day will result, with which you will operate as above.

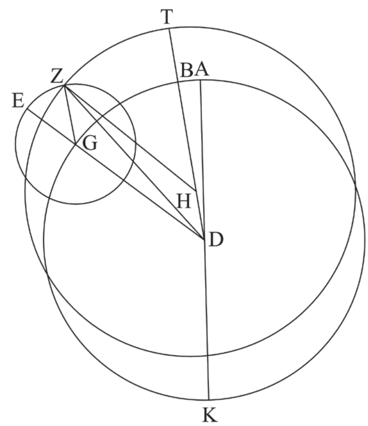
Likewise, subtract the sun's mean motion for one day from the moon's mean motion for one day, and the remainder will be the mean distance of the sun and moon for one day, with which carry on the business for the other times similarly to the previous ones. This mean distance is called the simple longitude.

Accordingly, it is manifest from what has been supposed that the arc of the mean motion of irregularity for any certain time is less in ratio than the arc of the mean motion of longitude for the same time.

8. When because of the irregular motion, it is supposed that the moon has a concentric with an epicycle, and likewise an eccentric, that the concentric and the eccentric are of equal size, and that the eccentricity is equal to the

semidiametro epicicli, positumque fuerit motum Lune in ecentrico similem motui ipsius in epiciclo, et ecentricum moveri in partem Lune secundum in proportione augmentum quod addit in eodem tempore medius motus longitudinis super medium motum diversitatis, omnia secundum utrumque modum similiter provenient.

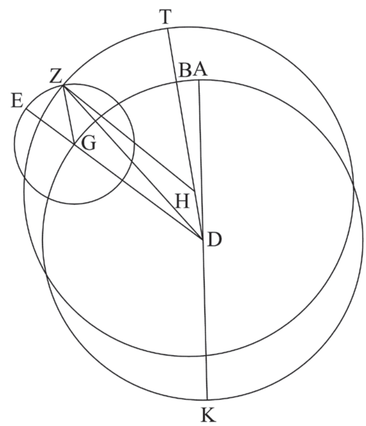
Describam ad hoc circulum concentricum ABGK supra centrum D et diametrum ADK et epiciclum EZ super centrum G. Sitque motus epicicli a puncto A ad punctum G et motus Lune in epiciclo interim a puncto E ad punctum Z. Et sit positum quod cum fuerat centrum epicicli in loco A, fuit Luna in longitudine longiore super punctum E. Quia igitur arcus AG maior est in proportione arcu EZ, sit arcus BG similis arcui EZ. Et protrahatur linea DB. Erit ergo motus ecentrici in eodem tempore secundum positionem angulus ADB qui est angulus differentie proportionum duorum motuum. Et erit centrum ecentrici in linea DB et eius longitudo longior similiter. Sumo itaque secundum quantitatem GZ semidiametri lineam DH, et ducta recta ZH secundum eius quantitatem centro H posito, describo circulum ZT. Producta deinceps linea DBT, dico quod linea HZ equalis est linee DG et arcus ZT similis arcui EZ. Siquidem arcus EZ similis est arcui GB, ergo angulus EGZ equus est angulo GDB, ergo linea GZ equidistat linee DH. Sed etiam est equalis ei; ergo linea ZH equidistans et equalis est linee GD. Quare angulus GDB equalis est angulo ZHT, et propter hoc erit arcus EZ similis arcui TZ. Quare secundum ambos motus Luna perveniet ad locum Z in circulo signorum vel in celo quem indicat linea DZ, quod intendebamus.



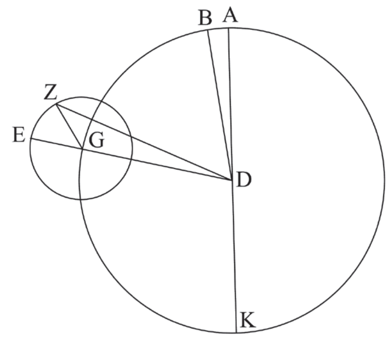
268 motum – similem] *marg. P* **269** et] *om. P s.l. K* ecentricum] *corr. ex* †econ†centricum *P* **269/270** in^{3} – augmentum] proportionem augmenti *N* **272** provenient] proveniunt *PN corr. ex* proveniunt *M* (provenire *Ba* provenient E_1) **274** Sitque] sicque *K* **276** fuerat] fuerit P_7 fuit] fuerit *N* **277** arcus AG] AG arcus P_7K **278** sit] fit *KM* **279** Erit ergo] eritque *PN* ecentrici] econcentrici *P* in] *corr. ex* T *K* **280** angulus] anguli *M* **281** proportionum] *corr. in* angulorum *N* **283** ecentrici] *corr. ex* exconcentrici *P* **286** eius] *s.l. K* **288** deinceps] deinde P_7 **289** HZ] *corr. ex* HT *K* equalis est] est equalis P_7 **290/291** EZ – arcui] *marg. P* **292** equus] equalis *N* GDB] *corr. ex* GBD *K* **293** est equalis] equalis est *MN* ZH] ZHE *P* **295** EZ] AZ *P* motus] modos *N* **296** signorum] *om.* P_7

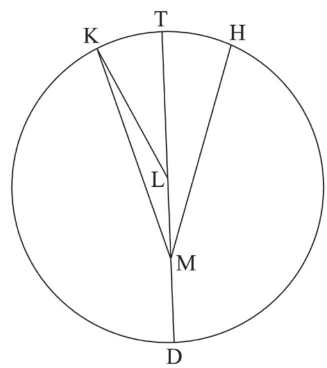
epicycle's radius, and it is supposed that the moon's motion on the eccentric is similar to its motion on the epicycle, and that the eccentric is moved in the direction of the moon according proportionally to the increase that the mean motion of longitude adds in the same time to the mean motion of irregularity, ⟨then⟩ all things will result similarly according to either mode.

For this I will describe concentric circle ABGK upon center D and diameter ADK, and epicycle EZ upon center G. And let the epicycle's motion be from point A to point G and the moon's motion on the epicycle meanwhile be from point E to point Z. And let it be supposed that that when the epicycle's center was at point A, the moon was at the apogee upon point E. Therefore, because arc AG is greater proportionally than arc EZ, let arc BG be similar to arc EZ. And let line DB be drawn. Therefore, according to the situation, the eccentric's motion in the same time will be angle ADB, which is the angle of the difference of the ratios of the two motions. And the eccentric's center will be on line DB, and its apogee similarly. Accordingly, I take line DH of the radius according to the size of GZ, and with straight line ZH drawn, according to its size and with H supposed as center, I describe circle ZT. Following this, with line DBT produced, I say that line HZ is equal to line DG and arc ZT is similar to arc EZ. Accordingly, arc EZ is similar to arc GB, so angle EGZ is equal to angle GDB; therefore, line GZ is parallel to line DH. But it is also equal to it; therefore, line ZH is parallel and equal to line GD. Therefore, angle GDB is equal to angle ZHT, and because of this arc EZ will be similar to arc TZ. Therefore, according to both motions, the moon will reach point Z in the ecliptic or in the heavens, which line DZ indicates, which we intended.



9. Et si inequalis magnitudinis fuerint ecentricus et concentricus dummodo proportionales fuerint eorum semidiametri ad distantiam centrorum ipsorum et semidiametrum epicicli, ceteris manentibus idem similiter secundum utrumque modum proveniet locus Lune in celo.

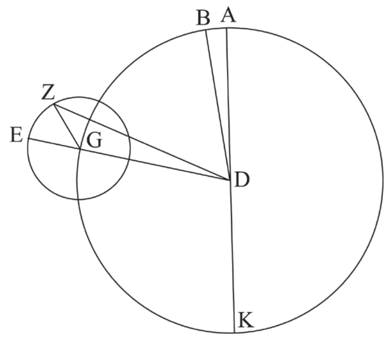


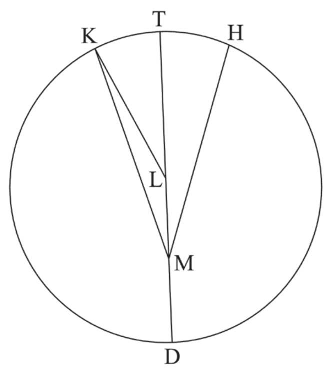


Describam unicuique duorum modorum figuram seorsum, concentricum quidem ABG supra centrum D et diametrum AK et epiciclum EZ supra centrum G. Et describam alibi ecentricum KTH supra centrum L et diametrum TD, et in ea diametro centrum circuli signorum punctum M. Et protraham in forma prima lineas DGE GZ DZ et in forma secunda lineas HM KM KL. Et ponam ut proportio DG ad GE sit sicut proportio TL ad LM. Et in uno tempore sit motus epicicli angulus ADG et motus Lune in epiciclo angulus EGZ equalis angulo TLK, et angulus ADG equalis duobus simul angulis TLK et HMT, et motus Lune in ecentrico arcus TK. Hiis itaque positis dico quod Luna secundum duos modos in uno et eodem tempore cernitur pertransire arcus equales in celo, scilicet quod angulus ADZ est equalis angulo HMK, hoc apposito quod Luna in principio motus fuerit in longitudine longiore et fuerit visa supra utramlibet lineam DA MH, et in fine motus fuerit super notas visa Z K scilicet secundum utramlibet lineam DZ MK. Et sit etiam arcus BG similis cuique duorum arcuum EZ KT. Protracta linea DB, quia ergo linea DG ad GZ est sicut propor-

298 inequalis] *corr. ex* equalis *M* fuerint] fuerit P_7 ecentricus] *corr. ex* econcentricus *P* **299** eorum] *om. PN* (eorum *Ba om.* E_1) semidiametri] semidiametrus P_7 *corr. ex* semidiametro *K* **302** duorum modorum] *corr. ex* modorum duorum *P* modorum duorum *N* **303** seorsum] deorsum P_7 **305** AK] AR *K* **306** G] *corr. ex* †...† *K* **308** circuli] orbis *N* **310** DGE – DZ] DGC GZ DZ P_7 *corr. ex* DGE GZ D†L† *K* DG EG Z DZ *N* **310/311** forma secunda] secunda forma *M* **311** HM KM] KM HM P_7 *corr. ex* LM KM *M* **313** in] etiam *PN* (in *Ba* etiam E_1) **319** itaque] ita P_7K (itaque BaE_1) **322** scilicet] secundum *MN* **323** HMK] HMT *corr. in* HMR *M* **324** supra utramlibet] secundum utramque *M* **325** super – visa] visa super nota P_7 visa super notas *K* visa secundum notas *M* (supra notas *Ba* super notas visa E_1) **325/326** scilicet – utramlibet] secundum scilicet utramlibet (*corr. ex* utramque) P_7 scilicet (*s.l.*) secundum utramque *M* **326** similis] *corr. ex* simili *P* cuique] *corr. ex* cuiusque *M* unicuique *N* duorum] *s.l.* P_7 **327** linea[2]] *om.* P_7K (linea *Ba om.* E_1)

9. And if the eccentric and the concentric are of unequal size provided that their radii are proportional to the eccentricity and the epicycle's radius, with the rest ⟨of the conditions⟩ remaining, the same place of the moon in the heavens will result similarly according to each model.





I will describe a figure separately for each of the two models, indeed a concentric ABG upon center D and diameter AK, and epicycle EZ upon center G. And I will describe in another place eccentric KTH upon center L and diameter TD, and on that diameter the center of the ecliptic point M. And I will draw in the first figure lines DGE, GZ, and DZ, and in the second figure the lines HM, KM, and KL. And I will posit that the ratio of DG to GE is as the ratio of TL to LM. And in one time let the epicycle's motion be angle ADG, the moon's motion on the epicycle be angle EGZ equal to angle TLK, angle ADG be equal to the two angles TLK and HMT together, and the moon's motion on the eccentric be arc TK. Accordingly, with these things supposed, I say that the moon according to the two models in one and the same time is seen to pass through equal arcs in the heavens, i.e. that angle ADZ is equal to angle HMK, with this assigned that the moon is at apogee at the beginning of the motion and is seen upon both line DA and MH, and it is seen at the end of the motion at point Z and K, i.e. according to both line DZ and MK. And let also arc BG be similar to each of the two arcs EZ and KT. Then, with line DB drawn, because line DG to GZ is as the ratio of KL to LM and angles L and G tio KL ad LM et anguli L G lateribus proportionalibus contenti sunt equales, erit triangulus GDZ equiangulus triangulo LKM. Quare angulus GZD equalis est angulo LMK. Sed et angulus BDZ equatur angulo GZD, propter hoc quod linee GZ et BD sunt equidistantes quoniam anguli ZGE BDG sunt equales propter arcus similes. Erit ergo angulus BDZ equalis angulo LMK. Et est angulus ADB qui est augmenti equalis angulo HMT qui est angulus motus ecentrici. Totus ergo angulus ADZ est equalis toti angulo HMK, quod intendimus.

Cum ergo idem secundum utrumque modum proveniat, contenti erimus deinceps quantum ad hanc primam diversitatem que simplex dicitur pertinet – nam et aliam habet Luna diversitatem ut postea ostendetur – unum tantum ponere modum ad demonstrationem sequentium scilicet modum per epiciclum. Et alium modumqui est ecentrici reservabimus alii diversitati.

10. Ad quantitatem diversitatis agnoscendam per tres eclipses notas pertingere.

Quantitas diversitatis est quantitas semidiametri epicicli vel quantitas linee que facit distantiam duorum centrorum ecentrici scilicet et circuli signorum, et attenditur hec quantitas respectu partium diametri concentrici supra quem est epiciclus. Imaginabimur itaque ad hoc in spera Lune circulum concentricum in superficie circuli signorum, et alium secantem ipsum per medium declinantem ab eo secundum quantitatem latitudinis Lune. Et imaginabimur epiciclum in superficie huius declinantis moveri secundum gradus ipsius qui sit motus longitudinis, et intelligatur moveri epiciclus motu medio secundum continuitatem signorum prout competit revolutioni longitudinis, et Luna in epiciclo contra continuitatem signorum a longitudine longiore prout competit revolutioni diversitatis.

Hiis memoriter retentis depingam epiciclum supra quem sint note ABG, et eligam tres eclipses notas ex scriptis considerationibus antiquorum. Et sit locus in quo fuit Luna in medio tempore eclipsis prime punctum A, et locus Lune in medio eclipsis secunde tempore punctum B, et locus Lune in medio tempore eclipsis tertie punctum G. Et sit motus Lune ab A ad G et deinde

328 contenti] contempti *K* **329** GDZ] DGZ P_7 LKM] LMK P_7 **330** LMK] *corr. ex* L†B† *K* angulo[2]] *om.* P_7 hoc] *s.l.* *K* quod] *om.* *N* **331** BDG] et BDG P_7N *corr. ex* BGD *M* **333** augmenti] *corr. ex* augmentum *K* est angulus] angulus est P_7 **334** intendimus] intendebamus *M* **337** habet Luna] Luna habet P_7 unum tantum] unde tantum unum sufficit *N* **338** scilicet] secundum *M* per epiciclum] *corr. ex* p†arvi† †cir†culum *P* per (*s.l.*) epiciclum *K* **340** agnoscendam] *om.* P_7 cognoscendam *K* **341** quantitas[2]] quantitas secunde *M* **344** epiciclus] epiciclum *N* Imaginabimur] ymaginemur *M* **345** circuli signorum] signorum circuli *N* secantem] sequantum *K* **346** Lune] *corr. ex* linee *P* epiciclum] *marg.* *P* **347** declinantis] declinationis *M* gradus] gradum *N* **349** revolutioni] revolutio *M* **352** quem] *s.l.* P_7 **353** sit] fit *K* **354** fuit Luna] Luna fuit *M* medio tempore] tempore medio *N* A] item *add. et del.* *K* **354/355** et – B] *om.* *P* *marg.* P_7 **355** eclipsis – tempore[1]] tempore eclipsis secundi P_7 tempore (*s.l.*) eclipsis secunde *M* eclipsis secunde *N* et locus] locus vero *N* **355/356** tempore[2] – tertie] tertie eclipsis *N* **356** ab] *corr. ex* ad P_7

[i.e. angles DZG and KLM] contained by proportional sides are equal, triangle GDZ will be equiangular to triangle LKM. Therefore, angle GZD is equal to angle LMK. But also angle BDZ is equal to angle GZD, because of this that lines GZ and BD are parallel because angles ZGE and BDG are equal because of similar arcs. Therefore, angle BDZ will be equal to angle LMK. And angle ADB, which is the augment ⟨of the epicycle's motion on the deferent over the moon's motion on the epicycle⟩, is equal to angle HMT, which is the angle of the eccentric's motion. The whole angle ADZ, therefore, is equal to whole angle HMK, which we intended.

Therefore, because the same thing results according to each model, we will be content hereafter as much as it pertains to this first irregularity, which is called 'simple' – for the moon also has another irregularity as will be shown afterwards – to suppose only one model for the demonstration of the following, i.e. the epicyclic model. And we will reserve the other model, which is the eccentric, for the other irregularity.

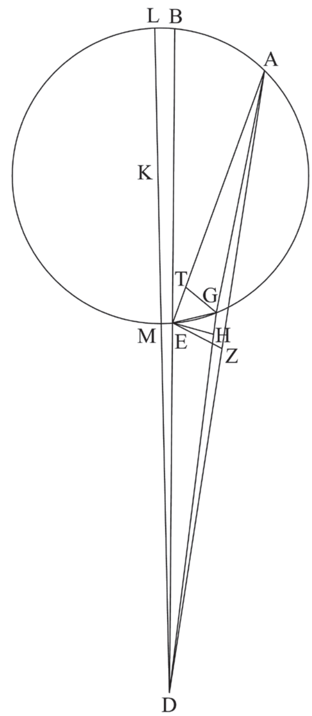
10. To attain knowledge of the irregularity's size through three known eclipses.

The size of the irregularity is the size of the epicycle's radius or the size of the eccentricity [*lit.*, the line that makes the distance of the two centers, i.e. of the eccentric and the ecliptic], and this size is considered with respect to the parts of the diameter of the concentric upon which the epicycle is. Accordingly, we will imagine for this a concentric circle in the moon's sphere in the plane of the ecliptic and another cutting it in half, declining from it according to the quantity of the moon's latitude. And we will imagine that the epicycle in the plane of this declined ⟨circle⟩ is moved according to the degrees of that which may be of the motion of longitude, and let it be understood that the epicycle[7] is moved by a mean motion according to the succession of the signs as agrees with a revolution of longitude, and the moon ⟨is moved⟩ on the epicycle from the apogee against the succession of the signs as agrees with the revolution [i.e. return] of the irregularity.

With these things preserved by the memory, I will depict an epicycle upon which are points A, B, and G, and I will select three known eclipses from the recorded observations of the ancients. And let the place in which the moon was in the middle time of the first eclipse be point A, and the moon's place in the middle time of the second eclipse point B, and the moon's place in the middle time of the third eclipse point G. And let the moon's motion be from A to G and then to B. Therefore, because the moon's true place in the

[7] Although an accusative is called for here, the witnesses all clearly have 'epiciclus.'

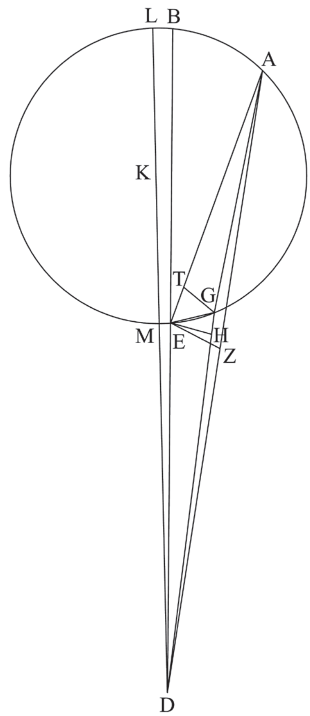
ad B. Quia ergo notus est locus Lune verus in circulo signorum in unaquaque trium notarum eclipsium scilicet propter locum Solis notum ex opposito, notus est etiam arcus circuli signorum inter alternas eclipses quem Luna interim perambulavit proiectis integris revolutionibus. Est enim equalis ei quem Sol perfecit. Rursum cum utrumque tempus inter alternas eclipses sit notum, erit ad utrumque tempus intermedium medius motus longitudinis notus et medius motus diversitatis notus; itaque et differentia medii motus longitudinis ad motum apparentem nota.



Et ponam ad hoc exemplum trium eclipsium in Babilonia observatarum quas refert Ptolomeus. Prima igitur eclipsis in primo anno Marduchei fuit in fine Virginis cum Sol teneret locum oppositum. Et secunda eclipsis que fuit in secundo anno Marduchei fuit in xlvº minuto quartidecimi gradus Virginis. Et tempus intermedium fuit cccliiii dies et due hore et medietas et xvª pars unius hore ex diebus mediocribus. Et tertia eclipsis in eodem anno Marduchei fuit cum Lune verus locus esset in xvº minuto quarti gradus Piscium. Et tempus intermedium secunde et tertie eclipsis clxx dies et xx hore et quinta hore ex diebus mediocribus.

Manifestum ergo quod Sol pertransivit a tempore medio eclipsis prime ad tempus medium eclipsis secunde et Luna similiter secundum motum apparentem in circulo signorum proiectis integris revolutionibus cccxlix gradus et

357 ad] *corr. ex* ab *K* est – verus] verus locus Lune P_7 **358** in] *s.l. K* **358/359** notarum eclipsium] eclipsium notarum *N* **360** etiam] *om. PN* **361** quem] quam *P* **363** perfecit] perficit *M* **365** intermedium] *s.l. K* **365/366** medius motus] motus medius *M* **368** nota] *corr. ex* notam *K om. N* **371** Ptolomeus] Tholomeus P_7 eclipsis] eclipsium *PN* **371/372** in – fuit] fuit in primo anno Marduchei P_7 **372** Marduchei fuit] fuit Mardochei *N* Marduchei] Mardochei *KM* **374** Marduchei] Mardochei *MN* fuit] *om. N* **375** quartidecimi gradus] gradus quartidecimi *PN* Virginis] Virginum *P*P_7 *corr. ex* Virginum *K* tempus] tunc P_7 **377** xva] 3ª *M* unius hore] hore unius P_7 ex] *corr. ex* in *M* **378** Marduchei] Mardochei *MN* (Mardothei *Ba* Mardochei E_1) **379** verus locus] locus verus *KM* **380** eclipsis] eclipsium *N* clxx] *corr. ex* clx *K* 176 *N* hore[2]] hore unius *N* ex] *corr. ex* et *K* **382** Manifestum] manifestum est *M* medio] medie *M* **382/385** eclipsis – medio] *om.* P_7 **383** medium] medie *M* Luna similiter] similiter Luna *M* motum] *s.l. P* **384** signorum] motum *add. et del. P*



ecliptic is known in each of the three known eclipses, i.e. because ⟨they are⟩ opposite the sun's known place, the arc of the ecliptic between successive eclipses that the moon meanwhile passed through with complete revolutions cast out is also known. For it is equal to that which the sun completed. In turn, because each time between successive eclipses is known, the mean motion of longitude and the mean motion of irregularity will be known for each intermediate time; accordingly, the difference between the mean motion of longitude and the apparent motion also will be known.

And I will suppose for this the example of three eclipses observed in Babylon that Ptolemy reports. Accordingly, the first eclipse was in the first year of Marducheus [i.e. Marduk-apla-iddina II] in the end of Virgo because the sun possessed the opposite point. And the second eclipse, which was in the second year of Marducheus was in the 45th minute of the 14th degree of Virgo [i.e. Virgo 13° 45′]. And the intermediate time was 354 days 2 34′ hours of average days. And the third eclipse was in the same year of Marducheus when the true place of the moon was in 15th minute of the fourth degree of Pisces [i.e. Pisces 3° 15′]. And the intermediate time of the second and the third eclipse was 170 days[8] 20 12′ hours of average days.

It is manifest, therefore, that the sun passed through from the middle time of the first eclipse to the middle time of the second eclipse and the moon similarly according to apparent motion in the ecliptic, 349° 15′ with complete

[8] Our author repeats a mistake found in Gerard's translation of the *Almagest* (1515 ed., f. 41r). This value should be 176.

xv minuta, et a tempore medio secunde eclipsis usque ad tempus medium eclipsis tertie clxix gradus et xxx minuta. Sed ad eadem tempora notus est motus medius in longitudine et motus medius diversitatis. Invenies ergo si inquiras ex superioribus arcum diversitatis quem transit Luna a prima eclipsi ad secundam proiectis integris revolutionibus cccvi gradus et xxv minuta, et quod propter ipsum adduntur super medium cursum gradus iii et xxiiii minuta; et arcum BAG quem transit Luna a secunda eclipsi ad tertiam cl gradus et xxvi minuta, et quod propter ipsum minuuntur a cursu medio xxxvi minuta. Propter hoc ergo erit arcus quem transit Luna BA liii gradus et xxxv minuta, et propter ipsum minuuntur a medio cursu iii gradus et xxiiii minuta. Et arcus quem pertransit Luna ab A ad G est xcvi gradus et li minuta, et propter eum adduntur supra cursum medium ii gradus et xviii minuta.

Hiis ita firmatis manifestum est quod in arcu BAG non cadit longitudo propior eo quod cum sit minor medietate circuli, propter eum non augetur motus sed minuitur; oporteret autem si in eo esset longitudo propior, quia tunc Luna in epiciclo secundum continuitatem signorum movetur. Ponam itaque punctum D centrum circuli declinantis quod et est centrum circuli signorum. Et ab eo ducam tres lineas ad puncta eclipsium trium DA DG DEB, deinde lineas EA et GA, et perpendicularem EZ super lineam AD, et EH perpendicularem super GD, et GT perpendicularem super EA.

Quia ergo differentia motus apparentis ad motum medium qui accidit propter arcum BA est nota, notus est etiam angulus BDA quia ipse est angulus differentie, et angulus Z rectus. Ergo proportio DE ad EZ nota facta scilicet DE semidiametro. Item arcus BA notus est, ergo angulus BEA notus. Quare reliquus angulus EAZ est notus, et angulus Z rectus. Est ergo proportio EA ad EZ nota facta scilicet EA semidiametro. Sed erat proportio EZ ad ED nota;

385/386 eclipsis tertie] tertie eclipsis *PMN* (eclipsis tertie *BaE_1*) **386** gradus] gradum *K* **387** motus] notus *P corr. ex* notus P_7 inquiras] inquiris *M* **388** transit] transivit *N* eclipsi] eclipsi usque P_7 **389** cccvi] *corr. in* 305 *M* **390** arcum] arcus *M* **391** transit Luna] Luna transit *M* **392** xxxvi] xxvi *PN* **393** gradus – minuta] graduum et 35 minutorum P_7 **394** medio cursu] curso medio *N* xxiiii minuta] 51 minuta P_7 **394/395** Et[2] – minuta] *marg.* P_7 **394** quem] quod *K* **395** pertransit] *corr. ex* transit *P* est] erit *M* xcvi] *corr. in* 106 *M* gradus – minuta] graduum et 51 minutorum *N* eum] ipsum *N* **396** xviii] 28 (vel 18 *add. s.l.*) P_7 *corr. in* xxviii *K* 27 *corr. in* 17 *corr. in* 28 *M* **397** ita] itaque *M* firmatis] *corr. ex* finitis *M* cadit] *corr. ex* eadem P_7 **398** eo quod] *om. N* circuli] circuli et *N* **399** quia] quod *M* **400** continuitatem signorum] signorum continuitatem *N* movetur] moveretur *M* **401** quod] *om. M* et[1] – signorum] est circuli signorum centrum (*the last word s.l.*) P_7 **402** eclipsium trium] trium eclipsium *N* DEB] et DB *N* **403** EA] *corr. ex* †esse† *K* EZ] et *P* super] *corr. ex* CR *K* **404** super[1]] super lineam *N* et] *s.l.* P_7 **406** notus est] est notus *P* etiam] *om. N* **407** differentie] DE *P* Z] Z est *N* Ergo proportio] *corr. ex* proportio ergo *K* **408** BEA] *corr. ex* BA *K* **409** est[1]] *om.* P_7N Z] Z est P_7 EA] *corr. ex* EZ *N* revolutions cast out, and from the middle time of the second eclipse to the middle time of the third eclipse 169° 30′. But for the same times the mean motion in longitude and the mean motion of irregularity are known. You will find, therefore, if you inquire from the things above [i.e. IV.7] the that the arc of irregularity that the moon passes from the first eclipse to the second, with complete revolutions cast out is 306° 25′, and that because of this, 3° 24′ are added beyond the mean course; and that arc BAG which the moon passes from the second eclipse to the third is 150° 26′, and that because of this 36′[9] are subtracted from the mean course. Because of this, therefore, the arc BA that the moon passes will be 53° 35′, and because of this, 3° 24′ are subtracted from the mean course. And the arc that the moon passes through from A to G is 96° 51′, and because of it, 2° 18′[10] are added upon the mean course.

With these things confirmed thus, it is manifest that the perigee does not fall on arc BAG because the motion on account of it does not grow but is diminished while it [i.e. arc BAG] is less than a semicircle; however, it would be necessary ⟨that it grow⟩ if the perigee were on it, because then [i.e. when at the perigee] the moon is moved on the epicycle according to the order of signs. Accordingly, I will suppose point D the center of the declined circle, which also is the center of the ecliptic. And from it I will draw three lines DA, DG, and DEB to the points of the three eclipses, then line EA and GA, and perpendicular EZ upon line AD, and perpendicular EH upon GD, and perpendicular GT upon EA.

Therefore, because the difference between the apparent motion and the mean motion that occurs because of arc BA is known, angle BDA is also known because it is the angle of difference, and angle Z is right. Therefore, the ratio of DE to EZ is known, with DE made a radius. Likewise, arc BA is known, so angle BEA is known. Therefore, the remaining angle EAZ is known, and angle Z is right. Therefore, the ratio of EA to EZ is known, with EA made a radius. But the ratio of EZ to ED was known; therefore, EA will be of known parts

[9] This should be 37′.

[10] This should be 2° 47′.

erit ergo EA ad semidiametrum ED notarum partium. Amplius quia differentia motus apparentis ad motum medium qui accidit propter arcum BAG est nota, erit angulus BDG. Et angulus qui est ad H est rectus. Facta ergo rursum DE semidiametro erit proportio DE ad EH nota. Sed et angulus BEG notus, reliquus ergo EGH notus. Facta ergo EG semidiametro, erit proportio EG ad EH nota. Sed EH ad ED nota; quare EG erit notarum partium ad semidiametrum DE. Amplius arcus AG notus est, ergo angulus GET notus. Et angulus ad T rectus. Facta ergo rursus EG semidiametro erit proportio ipsius ad utramque istarum ET GT nota. Sed erat proportio EG ad ED nota, ergo utraque istarum ET GT erit ad semidiametrum ED notarum partium.

Subtracta ergo ET ab EA quoniam et ipsa erat notarum partium ad ED, relinquitur AT nota. Cum TG ergo et AG que subtenditur angulo recto eodem respectu est nota. Ergo proportio AG ad EG est nota, sed recta AG ad diametrum epicicli cum fuerit cxx partium est notarum partium quia est corda arcus AG noti. Ergo et hoc respectu erit corda EG nota, ergo et arcus qui super eam est notus. Quare totus arcus BAGE notus est, et secundum premissa est clvii partes et xi minuta, minor scilicet semicirculo. Ergo et corda eius BE est nota et est cxvii partes et xxxvii minuta et xxxii secunda, secundum quod diameter epicicli est cxx partium. Si vero accideret hanc cordam BE esse equalem diametro, tunc esset in ea centrum epicicli et inquisitio nostra esset per ipsam tantum.

Et quia brevior est diametro et arcus BGE minor semicirculo, palam quod centrum cadit extra hanc portionem. Ponam ergo centrum K epicicli et protraham rectam DKL ut sit L longitudo longior et punctum M longitudo propior in epiciclo. Quia ergo nota est proportio BE ad EG et EG ad ED, erit ED notarum partium ad cordam EB; quare et tota BD ad diametrum epicicli est nota. Ergo et rectangulum quod continetur sub DB et ED notum, sed equale ei continetur sub DL et MD. Sed et quadratum quod describitur a semidiametro KM notum. At hoc quadratum et illud rectangulum ambo pariter equan-

411 erit ergo] ergo erit *M* **412** qui] que *N* propter] per P_7 **413** nota] notus P_7 BDG] notus *add.* (*s.l.* P_7) P_7*N* qui est] *om.* P_7 ad H] ADH *P* rectus] *om. P s.l.* (*placed before est* P_7) P_7*K* Facta] *perhaps corr. ex* †...† *P* ergo] *om. M* **414/416** Sed – nota] *marg.* P_7 **416** ED] *corr. ex* EB *M* **418** ad T] ADT *P* rursus] rusuus *P* rursum P_7 ipsius] istius P_7 **419** erat] *corr. ex* †...† *N* **420** ad] *s.l. K* **421** ET[1]] *corr. ex* erit *K* ab] ad *P* erat – partium] notarum partium erat *M* **421/423** ad – Ergo] *om.* P_7 **423** sed] scilicet *P* sed recta] sed *corr. ex* scilicet recta *N* **425** noti] *corr. ex* nota *K* arcus[2]] *corr. ex* angulus *N* **426** est notus] est est notus P_7 notus est *K* **427** eius] *corr. ex* arcus *K* **428** est nota] nota est *M* nota *N* partes] partium *M* xxxvii] xxxviii *P* xxxii] xxxii secundum *corr. ex* xxii secundum *K* **429** cxx] *corr. ex* ccx *K* partium] partes *N* BE – equalem] HE esse equalem *P* equalem esse *N* **430** esset[1] – ea] in ea esset *N* **432** BGE] *corr. ex* BEG *P* BAGE *N* **433** cadit] eadem *P* **436** tota] nota P_7 BD] AB *PM* **439** notum] *s.l.* (*perhaps other hand*) *P* At hoc] adhoc *M* ambo – equantur] pariter equantur ambo *N* to radius ED. Further, because the difference between the apparent motion and the mean motion that occurs because of arc BAG is known, angle BDG will be ⟨known⟩. And the angle that is at H is right. Therefore, with DE made a radius in turn, the ratio of DE to EH will be known. But also angle BEG is known, so the remainder EGH will be known. Therefore, with EG made a radius, the ratio of EG to EH will be known. But EH to ED is known; therefore, EG will be of known parts to radius DE. Further, arc AG is known, so angle GET is known. And the angle at T is right. Therefore, with EG in turn made a radius, the ratio of that to each of those ET and GT will be known. But the ratio of EG to ED was known, so each of those ET and GT will be of known parts to radius ED.

Therefore, with ET subtracted from EA, because that also was of known parts to ED, AT remains known. With TG, therefore, AG, which subtends a right angle, is also known in the same respect. Therefore, the ratio of AG to EG is known, but straight line AG is of known parts to the epicycle's diameter when it is 120^p, because it is the chord of known arc AG. Therefore, chord EG will also be known in this respect, so also the arc that is upon it is known. Therefore, whole arc BAGE is known, and according to what has been set forth, it is 157^p 11′,[11] i.e. less than a semicircle. Therefore, its chord BE is also known, and it is 117^p 37′ 32″ according to the terms by which the epicycle's diameter is 120^p. And indeed, if it should happen that this chord BE were equal to the diameter, then the epicycle's center would be on it, and our investigation would be through it only.

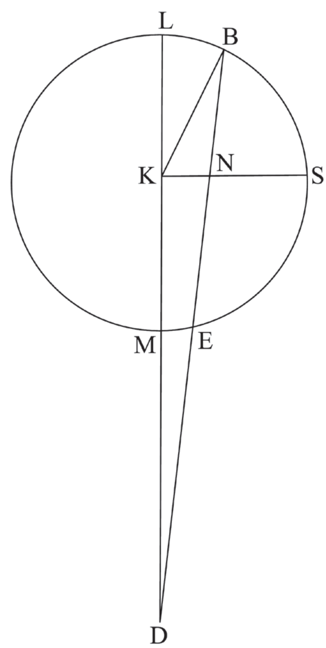
And because it [i.e. HE] is shorter than the diameter and arc BGE is smaller than a semicircle, it is clear that the center falls outside this part. I will posit, therefore, center K of the epicycle, and I will draw straight line DKL so that L is the apogee and point M is the perigee on the epicycle. Then, because the ratios of BE to EG and of EG to ED are known, ED will be of known parts to chord EB; therefore, whole BD to the epicycle's diameter is also known. Therefore, the rectangle that is contained under DB and ED is also known, but its equal is contained under DL and MD. But also the square that is described by radius KM is known. But this square and that rectangle together equal the

[11] This is 157° 10′ 1″ in Toomer, *Ptolemy's Almagest*, p. 196, but Gerard rounded upwards to '157 partes et 11 minuta fere' (*Almagest*, 1515 ed., f. 42r).

tur quadrato quod a DK describitur, ergo ipsum notum. Ergo et linea DK nota respectu partium diametri epicicli. Si ergo constituamus DK lx partium, est namque semidiameter circuli declinantis, erit semidiameter epicicli KL etiam hoc respectu notarum partium. Et accidit ex premissis partium v et xv minutorum, et hoc est quod querebamus.

Si quis vero idem velit inquirere per modum ecentrici, constituet punctum D scilicet centrum circuli signorum infra ecentricum et ab ipso protrahet tres lineas ad notas trium eclipsium. Et unam in directum producet ad punctum oppositum. Et ab hoc puncto protrahet perpendiculares super lineas eductas, et ad similitudinem premissorum procedet, et ad idem ad quod nunc perveniret scilicet ut distantia duorum centrorum sit partium v et minutorum xv.

11. Arcum epicicli inter longitudinem longiorem et locum cuiuslibet trium notarum eclipsium, necnon et differentiam duorum motuum que propter eundem arcum accidit, et locum Lune secundum medium cursum ad quem attinet eadem differentia notificare.

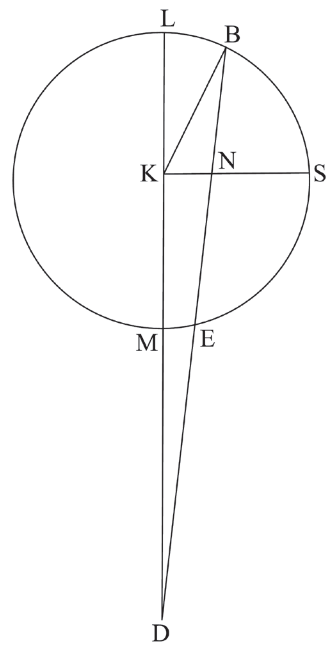


Repetatur similis figura superiori, et protrahatur a centro K perpendicularis super lineam BE KNS, et ducatur linea BK. Quoniam autem ostensum est quod DE ad DK est nota et BE similiter cuius medietas est EN, propter hoc erit DN ad DK nota. Facta igitur DK diametro cxx partium erit corda DN hoc quoque respectu nota, et arcus super eam consistens de circulo triangulum DKN circumscribente notus, quare angulus cui subtenditur DKN notus. Ergo et arcus epicicli SM notus, ergo et reliquus SL qui perficit semicirculum est notus. Sed et arcus BS cum sit medietas arcus BE est notus. Reliquus ergo LB qui est inter longitudinem longiorem et locum secunde eclipsis in epiciclo est notus, quod est unum ex propositis.

440 et] *om. N* **442** semidiameter[1]] semidiametrum P_7 **443** xv] xi P_7 **446** D] *om. N* circuli] orbis *N* protrahet] protrahat *N* **449** procedet] precedet *P* nunc] non *P corr. ex* non *M* perveniret] perveniet P_7K (proveniet *Ba* perveniet E_1) **451/452** trium notarum] trium *PN* notarum trium *M* (trium notarum BaE_1) **452** motuum] in *add. et del.* P_7 **455** Repetatur] *corr. ex* retatur *K* superiori] priori *M* **458** autem] ante *M* **461** cxx] xxx *P* 30 *corr. in* 60 *N* **463** circulo] circulo signorum *M* **464** notus] *corr. ex* notas P_7 **467** perficit] perfecit P_7 semicirculum] *corr. ex* semidiametrum *P* est notus] notus est *N* **469** LB] est notus *add. et del. P* **469/470** longitudinem longiorem] longiorem longitudinem *P* square that is described by DK [through *Elements* II.6], so it is known. Therefore, line DK is also known with respect to the parts of the epicycle's diameter. Therefore, if we set up DK as 60^p for it is the declined circle's radius, the epicycle's radius KL will also be of known parts in this respect. And it happens from what has been set forth, ⟨that it is⟩ of 5^p 15′,[12] and this is what we sought.

And indeed, if anyone should want to seek the same through the eccentric model, he will set up point D, i.e. the center of the ecliptic, in the eccentric, and from it he will draw three lines to the points of the three eclipses. And he will produce one directly to the opposite point. And from this point, he will draw perpendiculars upon the extended lines, and he will proceed to a likeness of what has been set forth, and to the same thing that he has reached, namely that the eccentricity is 5^p 15′.

11. To make known the arc of the epicycle between the apogee and the place of any of the three eclipses, as well as the difference between the two motions that occurs because of the same arc, and the moon's place according to the mean course to which the same difference pertains.



Let a figure similar to the above be repeated, and let perpendicular KNS be drawn from center K upon line BE, and let line BK be drawn. Because, moreover, it was shown that DE to DK is known and BE similarly, of which EN is half, because of this, DN to DK will be known. Therefore, with DK made a diameter of 120^p, chord DN will be known also with respect to these parts, and the arc standing upon it of the circle circumscribing triangle DKN will be known, and so angle DKN which subtends it will be known. Therefore, arc SM of the epicycle is also known, so also the remainder SL that completes a semicircle is known. But also arc BS is known because it is half of arc BE. The remainder LB, therefore, which is between the apogee and the second eclipse's place on the epicycle, is known, which is one of the proposed things.

12 This should be 5^p 13′ to follow the *Almagest*, but Ptolemy finds from his performance of this same procedure upon three more recent eclipses that the value is 5^p 14′ of the deferent's diameter, and he later uses the round value 5^p 15′ given here.

Rursum cum angulus DKN sit notus et angulus N sit rectus, ergo tertius KDN est notus, et ipse est angulus differentie que minuitur a medio cursu Lune propter arcum LB cum pervenerit Luna ab L in B. Et hec differentia est secundum quod ex premissis accidit lix minuta.

Itaque cum locus verus Lune in circulo signorum in medio secunde eclipsis sit xlv minutum quartidecimi gradus Virginis, addita hac differentia super verum locum, erit locus Lune secundum medium cursum xliiii minutum xvi gradus Virginis, quod querebamus.

12. Quantitatem diversitatis per alias tres eclipses notas et aliter cadentes experiri.

Hee tres alie eclipses quas assumemus secundum subtilem considerationem Ptolomei deprehense sunt. Et prima quidem eclipsium fuit in xviii anno Adriani cum verus locus Solis esset in Tauro gradus xiii et minuta xv, et hoc in medio tempore eclipsis, quare Luna erat in simili loco oppositi signi. Secunda eclipsis fuit in anno xix^{o} Adriani cum in medio tempore eclipsis esset Sol in Libra xxv gradus et x minuta. Tertia vero eclipsis in anno xx Adriani cum in medio tempore eclipsis esset Sol in Piscibus xiii gradus et xii minuta.

Patet igitur quod a tempore medio prime eclipsis usque ad tempus medium secunde eclipsis peragravit Sol secundum cursum diversum et Luna similiter post integras revolutiones clxi gradus et lv minuta, et a tempore medio secunde eclipsis usque ad tempus medium tertie eclipsis perambulavit Sol secundum cursum diversum et Luna similiter cxxxviii gradus et lv minuta. Fuit autem tempus intermedium prime et secunde eclipsis annus Egiptius et clxvi dies et xxiii hore et medietas et octava unius hore equalis secundum dies mediocres, et tempus intermedium secunde eclipsis et tertie annus Egiptius et cxxxvii dies et v hore equales et unius hore medietas secundum dies mediocres. Fuit ergo

472 N] non *P* *corr. ex* non *K* **473** KDN] *corr. ex* DKN *K* (DKN *Ba* DRN et RDN E_1) que] qui P_7 **476** locus verus] verus locus P_7 **477** xlv] et lv^{mum} *P* minutum] minuta *M* Virginis] Virginum *P* **477/479** addita – Virginis] *marg.* P_7 **477** super] secundum P_7 **478** verum locum] locum verum *M* medium cursum] cursum medium *N* minutum] minuta *M* **482** alie] alias *M* secundum] *om.* *P* per *N* subtilem considerationem] considerationem subtilem *M* **483** Ptolomei] Tholomei P_7 in] *om.* *N* **484** Tauro] Thauro *MN* gradus – xv] gradu 13° minuto 15° *M* xv] *corr. ex* xlv *K* **485** tempore] *om.* *M* **486** anno xixo] 19° anno *MN* **486/488** in^{3} – Sol] *om.* *P* *marg.* *N* **487** xxv – minuta] 25° gradu et decimo minuto *MN* **488** esset Sol] Sol esset *N* xiii] *corr. ex* 12 *M* gradus – minuta] gradu et 12 minuto *MN* **490** peragravit] perambulavit *M* cursum diversum] diversum cursum *PN* **491** lv] *corr. ex* 59 *M* **492** perambulavit] perambulat *N* secundum] per P_7 **493** et^{2}] *s.l.* *P* **494** intermedium] inter medium *K* eclipsis] eclipsium *N* Egiptius] Egiptiacus *M* **495** secundum] *corr. ex* secunde *K* **496/497** et^{1} – mediocres] *om.* *N* **496** intermedium] inter medium *PK* eclipsis – tertie] et tertie eclipsis P_7 tertie] tertie est *M* Egiptius] Egyptius P_7K cxxxvii] 237 *M* **497** hore^{2}] *corr. ex* diei *P*

In turn, because angle DKN is known and angle N is right, therefore, the third KDN[13] is known, and it is the angle of difference that is subtracted from the moon's mean course because of arc LB when the moon comes from L to B. And according to what occurs from what has been set forth, this difference is 59′.

Accordingly, because the moon's true place in the ecliptic in the middle of the second eclipse is the 45th minute of the 14th degree of Virgo [i.e. Virgo 13° 45′], with this difference added to the true place, the place of the moon according to mean course will be the 44th minute of the 16th degree of Virgo [i.e. Virgo 15° 44′],[14] which we sought.

12. To find the quantity of the irregularity through three other known and differently situated eclipses.

These three eclipses that we will take up were found according to Ptolemy's exact observation. And indeed the first of the eclipses was in the 18th year[15] of Hadrian when the sun's true place was in Taurus 13° 15′, and this in the middle time of the eclipse, so the moon was in the similar place of the opposite sign. The second eclipse was in the 19th year of Hadrian when in the middle time of the eclipse the sun was in Libra 25° 10′. And indeed, the third eclipse was in the 20th year of Hadrian when in the middle time of the eclipse the sun was in Pisces 13° 12′.[16]

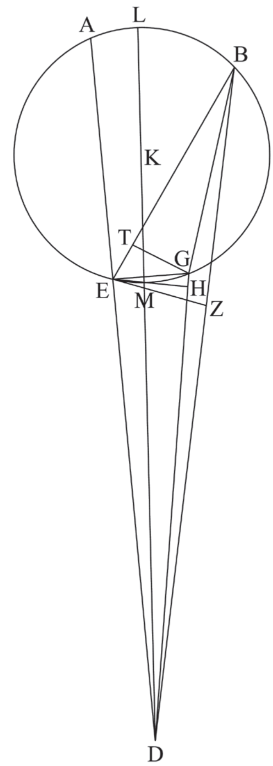
It is clear, therefore, that from the middle time of the first eclipse to the middle time of the second eclipse the sun traveled according to the irregular course, and the moon similarly, 161° 55′ after complete revolutions, and from the middle time of the second eclipse to the middle time of the third eclipse, the sun moved according to the irregular course, and the moon similarly, 138° 55′. Moreover, the intermediate time of the first and second eclipse was an Egyptian year and 166 days 23 37′ 30″ equal hours according to average days, and the intermediate time of the second eclipse and the third was an Egyptian year and 137 days and 5 30′ equal hours according to average days. There-

[13] That *K*, *Ba*, and E_1 all had the wrong text here suggests that the text may have originally had the mistaken 'DKN' or that it entered the transmission very early. The mistake was obvious, as the corrections in most of these witnesses show.

[14] This should be Virgo 14° 44′.

[15] This should be the 17th year of Hadrian.

[16] This should be 14° 5′ to match the *Almagest*. The mistake of 12 minutes for 1/12 of a degree would have been very easy to make and it was likely a corruption found in the manuscript of Gerard's translation that our author used when writing the *Almagesti minor* since this mistake is found in at least one *Almagest* manuscript (Paris, BnF, lat. 14738, f. 69r: '… in xiiia parte et xiio minuto Piscis fere').

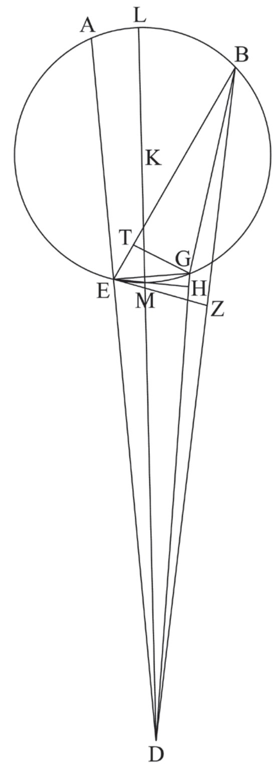


cursus medius longitudinis ad tempus intermedium precedens post integras revolutiones Lune clxix gradus et xxxvii minuta, maior cursu diverso qui preassignatus est vii gradibus et xlii minutis, et equalis cursus diversitatis ad idem tempus fuit cx gradus et xxi minuta, propter quem accidit nunc dicta differentia duorum motuum. Et fuit medius cursus longitudinis ad tempus intermedium subsequens cxxxvii partes vel gradus et xxxiiii minuta, minor cursu diverso qui preassignatus est gradu uno et xxi minutis; et equalis cursus diversitatis ad idem tempus propter quem etiam accidit hec differentia fuit lxxxi gradus et xxxvi minuta.

Hiis itaque declaratis lineabo epiciclum ABG. Sitque locus in tempore medio prime eclipsis punctum A in quo fuit Luna; et in tempore medio secunde eclipsis locus Lune punctum B; et in tempore medio tertie eclipsis locus Lune in epiciclo punctum G. Et imaginemur moveri Lunam ab A in B et deinde ad G. Erit ergo arcus AB cx partes et xxi minuta propter quem minuuntur a medio cursu longitudinis vii partes et xlii minuta. Et erit arcus BG lxxxi partes et xxxvi minuta propter quem adduntur super cursum medium in longitudine pars una et xxi minuta. Quare arcus GA residuus est clxxviii partes et iii minuta pro quo adduntur super cursum medium longitudinis partes residue scilicet vi partes et xxi minuta.

Manifestum quoque est quod oportet ut longitudo longior sit in arcu AB quoniam non est possibile ut sit in arcu BG vel in arcu GA eo quod uterque eorum est addens et minor semicirculo. Propter hoc ergo ponam centrum circuli signorum punctum D et protraham ab eo lineas ad puncta trium eclipsium DEA DG DB. Et producam lineam BG, et a puncto E duas EB et EG,

498 intermedium] inter medium *PK* **501** est] *om. P* xlii] lxii *P* 32 *N* **502** cx] 90 *M* **503** accidit] arcum *N* **504** dicta differentia] *corr. ex* differentia dicta *M* motuum] motuum accidit *N* **506** partes vel] *om. M* **507** est] *om. M* **510** xxxvi] 37 P_7 **511** itaque] ita P_7*K* (itaque *Ba*E_1) lineabo] lineabimus *M* **514** locus Lune] *om. N* **516** moveri Lunam] Lunam moveri *N* **517** ad] in *N* Erit ergo] et erit *M* cx] *corr. ex* 90 *M* **518** quem] quod *N* **519** medio cursu] cursu medio *N* **520** partes] *s.l. P* **522** clxxviii] *corr. ex* 168 *M* **525** ut] *corr. ex* quantum *K* **526** possibile] *corr. ex* p†possum† *K* in² – GA] GA arcu *N* **528** punctum D] D punctum P_7 **529** BG] *corr. ex* BD *K* EB – EG] EG (AG *add. et del.*) et EB *N*



fore, the course of mean longitude for the first intervening time after complete revolutions of the moon was 169° 37′, greater than the irregular course that was allotted earlier by 7° 42′, and the mean course of the irregularity for the same time was 110° 21′, because of which the said difference between the two motions occurs. And the mean course of longitude for the following intermediate time was 137° 34′, less than the irregular course which was allotted earlier by 1° 21′; and the mean course of irregularity for the same time because of which also this difference occurs, was 81° 36′.

Accordingly, with these declared, I will draw epicycle ABG. And let the place in which the moon was in the middle time of the first eclipse be point A; and the moon's place at the middle time of the second eclipse be point B; and the moon's place on the epicycle at the middle time of the third eclipse be point G. And let us imagine that the moon is moved from A to B and then to G. Therefore, arc AB will be 110° 21′, because of which 7° 42′ are subtracted from the mean course of longitude. And arc BG will be 81° 36′, because of which 1° 21′ are added upon the mean course in longitude. Therefore, remaining arc GA is 178° 3′,[17] for which the remaining degrees, i.e. 6° 21′, are added upon the mean course of longitude.

Also, it is manifest that it is necessary that the apogee be on arc AB because it is not possible that it be on arc BG or on GA because each of them is additive and less than a semicircle. Because of this, therefore, I will suppose point D the center of the ecliptic, and I will draw from it lines DEA, DG, and DB to the points of the three eclipses. And I will draw line BG, and the two ⟨lines⟩

[17] This should be 168° 3′, but the mistake is found in at least one manuscript of Gerard's translation (Paris, BnF, lat. 14738, f. 69r).

et duas perpendiculares EH super lineam DG et EZ super lineam DB, et a puncto G perpendicularem GT super lineam EB.

Quia ergo nota est differentia duorum motuum que accidit propter arcum BA, erit angulus ADB notus quia ipse est angulus differentie. Et cum angulus ad Z sit rectus, erit ergo proportio DE ad EZ nota. Item arcus BA notus est, ergo angulus AEB notus; quare reliquus angulus intrinsecus EBD notus. Et angulus qui est ad Z est rectus, ergo proportio BE ad EZ est nota. Sed erat proportio EZ ad ED nota, ergo EB ad semidiametrum ED est notarum partium. Amplius quia differentia duorum motuum que accidit propter arcum GEA nota est, erit angulus GDA notus. Sed angulus ad H est rectus, ergo proportio ED ad EH est nota. Item quia arcus ABG notus est, et angulus AEG notus, quare reliquus intrinsecus EGD notus. Cum ergo angulus ad H sit rectus, erit proportio EG ad EH, et mediante EH erit EG ad semidiametrum ED notarum partium. Amplius quia arcus BG notus est, est et angulus BEG notus. Et angulus qui est ad T est rectus, ergo proportio EG ad utramque istarum ET GT est nota. Mediante ergo EG erit utraque illarum ad semidiametrum ED notarum partium.

Subtracta ergo ET ab EB nota prius, erit TB nota eodem respectu sicut TG. Quare et BG que subtenditur angulo recto ad idem erit nota. Sed et AG cum sit corda arcus BG noti ad diametrum epicicli nota est. Ergo et hoc respectu erit corda EG nota, ergo arcus qui super eam est EG est notus. Quare totus arcus BGE notus, ergo et residuus arcus EA notus. Et est secundum premissa xcv gradus et xvi minuta et l secunda, minor scilicet semicirculo, et eius corda AE nota scilicet lxxxviii partes et xl minuta et xvii secunda secundum quod diameter epicicli est cxx partium.

Manifestum quod centrum epicicli cadit extra portionem circuli EA. Ponam itaque centrum eius punctum K et ducam lineam DMKL ut sit L punctum longitudo longior et M punctum longitudo propior. Quia ergo mediante

530/531 DG – lineam] *marg.* P_7 **532** ergo] *om. M* **534** ad Z] ADZ *P* EZD *corr. ex* †...† *K* ergo] *om. M* **535** quare] et (*s.l.*) quia *K* reliquus angulus] angulus reliquus P_7 intrinsecus] notus erit *add. marg. K* EBD] *corr. ex* EDB P_7 *corr. ex* ABD *K* notus[2]] notus est P_7 **536** ad Z] ADZ *P* **537** semidiametrum] *marg. M* **539** notus] *corr. ex* †...† *P* ad H] ADH *P* EHD *corr. ex* EDH *corr. ex* †..DH† *K* **540** est nota] nota est *PN* est[2]] est est *PK* **541** quare] *om. N* EGD] EDG *K* notus[2]] notus erit *N* ad H] ADH *P* **542** EH[1]] EB *M* EH[2]] *s.l. N* **543** quia] *corr. ex* quod *K* est et] et *M* erit *N* **544** Et angulus] *s.l. P* ad T] ADT *P* est rectus] rectus est *M* ET[2]] ET et *N* **545/546** ad – partium] notarum partium ad semidiametrum ED *M* **548** AG] *corr. in* BG P_7 BG *N* (AG BaE_1) **549** nota est] est nota P_7 **550** erit – nota] erit et corda EG nota P_7 EG erit nota *N* eam] EA *P* **551** BGE] BGE est *M corr. ex* BGG *M corr. in* ABGE *N* residuus] residuum P_7 **552** xcv] xxv *PN* **553** AE] *corr. ex* AG *K* **554** partium] partes *N* **555** Manifestum] manifestum itaque *N* **557** longitudo longior] longitudo largior *P* longitudinis longioris *M* longitudo propior] longitudinis propioris *M*

EB and EG from point E, and the two perpendiculars EH upon line DG and EZ upon line DB, and perpendicular GT from point G upon line EB.

Therefore, because the difference between the two motions that occurs because of arc BA is known, angle ADB will be known because it is the angle of difference. And because the angle at Z is right, the ratio of DE to EZ will be known. Likewise, arc BA is known, so angle AEB is known; therefore, the remaining intrinsic angle EBD is known. And the angle that is at Z is right, so the ratio of BE to EZ is known. But the ratio of EZ to ED was known, so EB is of known parts to radius ED. Further, because the difference between the two motions that occurs because of arc GEA is known, angle GDA will be known. But the angle at H is right, so the ratio of ED to EH is known. Likewise, because arc ABG is known, angle AEG is also known; therefore, the remaining intrinsic ⟨angle⟩ EGD is known. Therefore, because the angle at H is right, the ratio of EG to EH will be ⟨known⟩, and with EH mediating, EG will be of known parts to radius ED. Further, because arc BG is known, angle BEG is also known. And the angle that is at T is right, so the ratio of EG to each of those ET and GT is known. Therefore, with EG mediating, each of those will be of known parts to the radius ED.

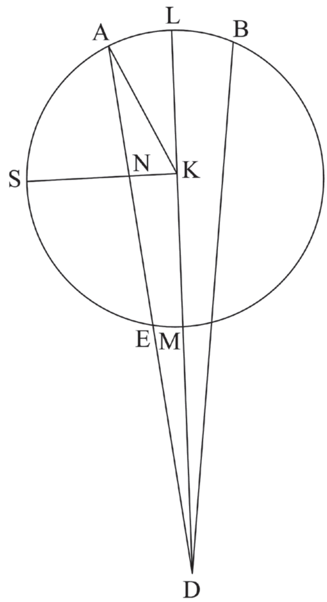
Therefore, with ET subtracted from EB known earlier, TB will be known in the same respect as TG. Therefore, BG, which subtends a right angle, will also be known to the same. But also AG[18] because it is the chord of known arc BG is known to the epicycle's diameter. Therefore, chord EG will also be known in this respect, so arc EG, which is upon it, is known. Therefore, the whole arc BGE is known, so the remaining arc EA is also known. And it is according to what was set forth, 95° 16′ 50″, i.e. less than a semicircle, and its chord AE is known, i.e. 88^p 40′ 17″, according to the terms by which the epicycle's diameter is 120^p.

It is manifest that the center of the epicycle falls outside the part EA of the circle ⟨because the apogee falls on arc AB⟩. Accordingly, I will suppose its center point K and I will draw line DMKL so that point L is the apogee and point M the perigee. Therefore, because, with chord EG mediating, the

[18] This should be BG, but the witnesses suggest that the mistake was original.

corda EG nota est proportio corde AE ad lineam ED, fiet et tota linea DEA ad diametrum epicicli cum sit cxx partium nota. Quare et rectangulum quod continetur sub tota DA et eius parte extrinseca ED notum, sed ipsum equatur ei quod continetur sub DL et MD. Sed et quadratum quod a semidiametro MK describitur notum est. At hoc quadratum et illud rectangulum ambo pariter equantur quadrato quod a DK describitur; ergo linea DK nota respectu partium diametri epicicli. Si ergo constituamus lineam DK lx partium, est enim semidiameter declinantis circuli Lune, erit KM semidiameter epicicli hoc quoque respectu notarum partium. Et accidit ex premissis v partium et xiiii minutorum et modicum amplius, et hoc concordat illi quantitati propinque que per tres antiquas eclipses inventa est. Et hoc est quod querebamus.

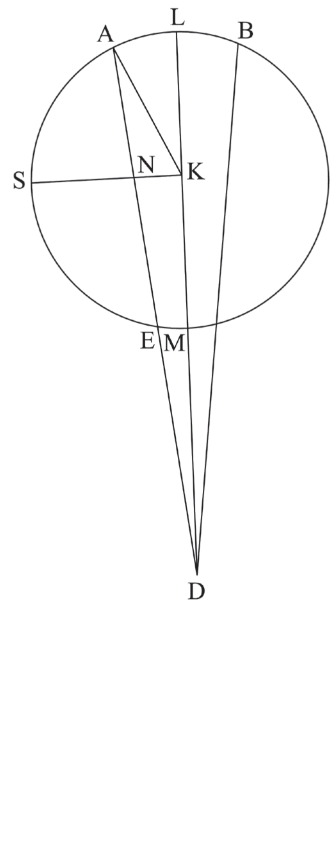
13. Arcum epicicli inter longitudinem longiorem et locum cuiuslibet trium notarum eclipsium, necnon et differentiam duorum motuum que propter eundem arcum accidit, et locum Lune secundum medium cursum ad quem attinet eadem differentia notificare.



Similem priori figuram resumo, et protraho a centro K perpendicularem super lineam DA que sit KNS, et produco KA. Ex antecedentibus autem DK notarum partium est respectu diametri epicicli et DE similiter. Sed et EN quoniam medietas est corde EN, quare tota DN notam habet proportionem ad DK. Ergo angulus DKN notus, ergo arcus qui ei subtenditur SEM notus, itaque residuus de semicirculo SAL notus. Subtracto itaque AS qui est medietas arcus AE, relinquitur arcus AL notus. Sed totus AB erat notus, reliquus ergo arcus LB scilicet a longitudine longiore ad punctum secunde eclipsis notus.

558 fiet] *corr. ex* fiat P_7 DEA] DEA nota P_7 **559** nota] *om.* P_7 **560** tota] *om.* P_7 extrinseca] *corr. ex* intrinseca P_7 **561/562** a – describitur] describitur a semidyametro MK *M* **562** MK] *om. N* et] *corr. ex* erit *P* ad *K* ambo] *corr. ex* ab P_7 **563** a DK] ADK *M* **565** declinantis circuli] circuli declinantis P_7 **566** quoque] ergo *P om. N* **567** minutorum] minuta *K* illi] alii P_7 **568** per tres] *corr. ex* partes *P* **570** et] *om. N* **571** medium cursum] cursum medium *N* **574/575** perpendicularem – DA] super lineam perpendicularem DA *corr. in* super lineam DA perpendicularem *K* **576/577** notarum – est] est notarum partium *N* **578** quoniam] qui *P* que *N* medietas est] est medietas *MN* **579** EN] EA P_7*MN* (enim *Ba* EN E_1) **580** DKN] DKN est *N* **581** notus] *om. N* **582** SAL] scilicet AL *N* **583** itaque] igitur *N* qui] que *M* **584** arcus AL] AL arcus *M* **585** LB scilicet] AB scilicet *P om. N* ratio of chord AE to line ED is known, the whole line DEA to the epicycle's diameter when it is 120^{p} will also be known. Therefore, the rectangle that is contained under whole DA and its extrinsic part ED will also be known, but what is contained under DL and MD is equal to it. But also the square that is described by radius MK is known. But this square and that rectangle together equal the square that is described by DK [through Euclid II.6] therefore, line DK is known with respect to the parts of the epicycle's diameter. If, therefore, we shall set up line DK to be 60^{p}, for it is the radius of the moon's declined circle, the epicycle's radius KM will also be of known parts in this respect. And it happens from what was set forth, ⟨that it is⟩ 5^{p} 14′ and a little more,[19] and this agrees closely with that quantity that was found through the three ancient eclipses. And this is what we sought.

13. To make known the arc of the epicycle between the apogee and the place of any of the three known eclipses, as well as the difference between the two motions that occurs because of the same arc, and the place of the moon according to mean course to which the same difference pertains.



I take up again a figure similar to the previous one, and I draw a perpendicular, which let be KNS, from center K upon line DA, and I produce KA. From the preceding things, moreover, DK is of known parts with respect to the epicycle's diameter, and DE similarly. But also EN ⟨is of known parts to DK⟩, because it is half of chord EN,[20] so also whole DN has a known ratio to DK. Therefore, angle DKN is known, so the arc SEM, which subtends it, and so SAL, the remainder of a semicircle, is known. Accordingly, with AS, which is half of arc AE, subtracted, the arc AL remains known. But the whole AB was known, so the remainder arc LB, i.e. from the apogee to the point of the second eclipse, is known.

[19] The value would actually be less than 5^{p} 14′.

[20] This should be EA, but the mistake appears to be original.

Rursum cum arcus DK notus iam sit, erit et angulus KDA qui ei ad perfectionem recti deest notus. Sed notus erat totus angulus BDA, reliquus ergo BDL notus. At ipse est angulus differentie qui minuitur a medio cursu Lune propter arcum LB cum Luna pervenerit ab L in B. Et est secundum quod ex premissis accidit gradus iiii et minuta xx.

Itaque cum verus locus Lune in circulo signorum tempore medio secunde eclipsis sit decimum minutum xxvi gradus Arietis, addita hac differentia super medium cursum Lune, erit locus Lune secundum medium cursum in medio secunde eclipsis xxx minutum tricesimi gradus Arietis. Et hoc intendebamus.

Et notandum quod Albategni quoque simili calle inquisitionis procedens eandem invenit quantitatem semidiametri epicicli. Unde easdem ponit omnino duorum motuum differentias que simplices equationes Lune dicuntur.

14. Medium motum longitudinis et medium motum diversitatis et mediam distantiam Solis et Lune per equationem ex considerationibus antiquarum et modernarum eclipsium certiorem facere.

Fuit ergo secunda trium eclipsium antiquarum sicut supra ostensum est Luna existente secundum cursum medium longitudinis in xliiii minuto xv^{ti} gradus Virginis, et secundum cursum diversitatis in epiciclo in xxiiii^{to} minuto xiii^{e} partis a longitudine longiore. Et fuit secunda trium modernarum eclipsium Luna existente secundum cursum medium longitudinis in xxx^{o} minuto xxx^{i} gradus Arietis, et secundum medium cursum diversitatis in epiciclo in xxxviii^{o} minuto lxi gradus a longitudine longiore. Palam igitur quod in tempore intermedio harum duarum eclipsium fuit medius motus longitudinis ccxiiii gradus et xlvi minuta post revolutiones integras et quod fuit medius cursus diversitatis post completas revolutiones lii gradus et xiiii minuta. Fuit autem tempus intermedium duarum eclipsium secundum veritatem equationis dierum mediocrium dccc et liiii anni Egyptii et lxxiii dies et xxiii hore et tertia hore, quod est cccxi milia et septingenti et lxxxiii dies et xxiii hore et tertia unius hore. Motus vero longitudinis in toto hoc tempore secundum quod supra inventum fuerat

588 arcus DK] C arcus DK *corr. in* angulus DKN P_7 angulus DKN *M corr. in* angulus DKN (*perhaps other hand*) *N* (arcus DK *Ba* arcus DR E_1) notus – sit] iam notus sit *M* iam sit notus *N* **589** recti deest] *corr. ex* deest recti P_7 recti deest est *M* recti deficit *N* recti] *marg. K* Sed] sed et *N* totus angulus] angulus totus *N* **590** qui] que P_7 propter] per P_7 **593** locus] in *add. et del. K* **594/595** sit – Lune^{1}] *marg.* P_7 **594** decimum minutum] decimo minuto $P_7$13^{m} minutum *M* xxvi] 16^{i} *N* **596** tricesimi] 20^{i} *N* **598** ponit omnino] omnino ponit P_7K (omnino ponit *Ba* ponit omnino E_1) **601** ex] *corr. ex* et *K* **603** trium] *om.* P_7 supra ostensum] preostensum P_7 **604** cursum medium] medium cursum P_7 minuto xvti] minuto (*corr. ex* gradu) tertii *M* **606** fuit] *s.l. K* trium modernarum] modernarum trium *M* **607** cursum medium] medium cursum P_7 minuto] minuto et *P* **610** motus longitudinis] longitudinis motus *M* **611** minuta] *om. P* cursus] motus *K* **613** equationis] *corr. ex* equatoris *K* **614** Egyptii] Egyptiaci *PN* tertia] tertia unius *M* quod est] *s.l.* P_7 **615** milia] *om. P* **616** vero] ergo *PN* autem *M* (vero BaE_1)

In turn, because arc DK[21] is already known, angle KDA, which is the complement, will also be known. But whole angle BDA was known, therefore, the remainder BDL is known. But it is the angle of the difference that is subtracted from the moon's mean course because of arc LB when the moon comes from L to B. And according to what occurs from what has been set forth, it is 4° 20′.

Accordingly, because the moon's true place in the ecliptic in the middle time of the second eclipse is the tenth minute of the 26th degree of Aries [i.e. Aries 25° 10′], with this difference added upon the mean course of the moon, the moon's place according to mean course in the middle of the second eclipse will be the 30th minute of the 30th degree of Aries [i.e. Aries 29° 30′]. And we intended this.

And it should be noted that Albategni, also proceeding by a similar path of inquiry, discovered the same quantity of the epicycle's radius. Whence he places differences between the two motions, which are called the 'simple equations of the moon', entirely the same ⟨as Ptolemy's⟩.

14. To make the mean motion of longitude, the mean motion of irregularity, and the mean distance of the sun and moon more certain through a correction from the observations of the ancient and modern eclipses.

Now, the second of the three ancient eclipses was, as was shown above, with the moon existing in the 44th minute of the 15th degree of Virgo [i.e. Virgo 14° 44′] according to the mean course of longitude, and in the 24th minute of the 13th degree [i.e. 12° 24′] on the epicycle from the apogee according to the course of irregularity. And the second of the three modern eclipses was with the moon existing in the 30th minute of the 30th degree of Aries [i.e. Aries 29° 30′] according to the mean course of longitude, and in the 38th minute of the 61st degree [i.e. 60° 38′][22] on the epicycle from the apogee according to the mean course of irregularity. It is clear, therefore, that in the intermediate time of these two eclipses, the mean motion of longitude was 214° 46′[23] beyond complete revolutions, and the mean course of irregularity was 52° 14′ beyond finished revolutions. Moreover, the intermediate time of the two eclipses according to the truth of the correction of average days was 854 Egyptian years 73 days and 23 20′ hours, which is 311,783 days, 23 20′ hours. And indeed, the motion of longitude in this whole time according to what had been

[21] Although this makes no sense here mathematically (angle DKN is what the mathematics require), it appears to be the original reading since it is found in *P*, P_7, *K*, *N*, *Ba* and E_1 (with the substitution of the letter R for K that is often found in this manuscript). Many witnesses have a corrected text.

[22] This should be 64° 38′.

[23] This should be 224° 46′.

per duo alternarum eclipsium intervalla, fuit post revolutiones integras ccxiiii gradus et xlvi minuta et motus medius diversitatis post revolutiones integras lii gradus et xxxi minuta. Itaque medius cursus longitudinis qui supra inventus fuerat non discordat a medio cursu longitudinis nunc invento, sed medius cursus diversitatis qui supra inventus fuerat maior est nunc invento xvii minutis. Dividantur itaque xvii minuta per numerum dierum positorum et provenient xi quarta et xlvi quinta et xxxix sexta. Et minuantur hec producta a motu medio diversitatis supra invento qui attinet ad unam diem, et habebis motum medium diversitatis ad unam diem per equationem huiusmodi correctum.

Et nota quod hec correctio secundum quod Ptolomeus invenit facta est, Albategni vero secutus eandem viam suo tempore invenit medium motum diversitatis a Ptolomeo positum addere super medium motum diversitatis quem predicta via suo tempore comprehendit medietatem unius et quartam. Et divisit hoc per numerum dierum qui fuerunt inter ipsum et Ptolomeum, et minuit a medio motu Ptolomei. Et ita est medius motus diversitatis in tabulis Toletanis. Motum vero longitudinis eundem invenit quem Ptolomeus nisi quod ei addidit id quod motui Solis addidit. Equalis enim lunationis tempus idem accepit. Et supradicto modo sicut in septima propositione presentis dicitur operatus cum via corrigendi uteretur, idem invenit. In tabulis vero Toletanis quia medius motus Solis ad certum tempus minor est medio motu Solis quem posuit Ptolomeus, idem quod a medio motu Solis subtrahitur a medio motu quoque Lune in longitudine subtrahendum est cum tempus equalis lunationis fuerit idem.

15. Super fixam et certam radicem temporis locum Lune in circulo signorum secundum medium cursum longitudinis et locum Lune in epiciclo certe distantie a longitudine longiori secundum medium cursum diversitatis assignare.

Elige ergo annos alicuius viri noti vel rei note sicut in Sole factum est quos velis radicem constituere. Totum quoque tempus quod fuerit inter radicem

617 duo] dua *M* **618** revolutiones] *corr. in* reversiones P_7 reversiones *KM* **620** nunc] non *P* **622** itaque] itaque hec P_7K igitur *N* positorum] *marg. M* **624** diversitatis] diversum PP_7K (diverso *Ba* diversitatis E_1) unam] unum *N* **624/625** motum – diversitatis] medium motum diversitatis P_7K motum diversitatis medium *N* **625** huiusmodi] huius *N* **626** Ptolomeus] Tholomeus P_7 **627** eandem – tempore] *s.l.* P_7 medium motum] motum medium *M* **628** Ptolomeo] Tholomeo P_7 **629** unius] gradus *add.* (*s.l. K*) *KM* **630** fuerunt] *corr. ex* fuerint *P* Ptolomeum] Tholomeum P_7 **631** motu] *om. N* Ptolomei] Tholomei P_7 ita] itaque *M* Toletanis] Tholetanis P_7 **632** vero] ergo P_7 eundem] *corr. ex* eandem *K* Ptolomeus] Tholomeus P_7 **633** quod] quod et P_7K motui – addidit[2]] addidit motui Solis *M* **634** propositione] *om. N* **635** tabulis] stabulis *K* Toletanis] Tholetanis P_7 **636** ad – est] est minor ad certum tempus *corr. in* ad certum tempus est minor *K* ad tempus certum minor est *N* medio] *om.* P_7 **637** Ptolomeus] Tholomeus P_7 idem] ideo est *M* medio[1] motu] motu medio *M* **637/638** motu quoque] quoque motu P_7K **639** idem] idem et cetera *N* **644** velis] voles *N* constituere] consumere *P* found above through the two intervals of successive eclipses, was 214° 46′[24] beyond complete revolutions, and the mean motion of irregularity was 52° 31′ beyond complete revolutions. Accordingly, the mean course of longitude that had been found above is not at variance with the mean course of longitude found now, but the mean course of irregularity that had been found above is greater than that found now by 17′. Accordingly, let the 17′ be divided by the number of posited days, and there will result 11^{iv} 46^{v} 39^{vi}. And let these results be subtracted from the mean motion of irregularity[25] found above that pertains to one day, and you will have the mean motion of irregularity for one day set improved through the correction of of this kind.

And note that this improvement was made according to what Ptolemy found, and indeed Albategni following the same way in his own time found that the mean motion of irregularity posited by Ptolemy adds 45′ upon the mean motion of irregularity that he [i.e. Ptolemy] grasped by the said way in his time. And he divided this by the number of days that were between himself and Ptolemy, and he subtracted it from Ptolemy's mean motion. And thus is the mean motion of irregularity in the Toledan Tables.[26] And indeed, he found the same motion of longitude as Ptolemy except that he added to it that which he added to the sun's motion. For he took the same time of a mean lunation. And operating in the abovesaid way as it is said in the seventh proposition of the present ⟨book⟩ when he used the way of improving, he found the same, and indeed, ⟨this is also⟩ in the Toledan Tables.[27] Because the sun's mean motion for a certain time is less than the sun's mean motion that Ptolemy posited, the same that is subtracted from the sun's mean motion must also be subtracted from the moon's mean motion in longitude because the time of mean lunation is the same.

15. To assign the moon's place in the ecliptic according to the mean course of longitude upon a fixed and certain radix of time and the moon's place on the epicycle a certain distance from the apogee according to the mean course of irregularity.

Now, select the years of any famous man or known event that you want to set up to be the radix, as was done with the sun. Heed attentively the whole

[24] Again, this should be 224° 46′.

[25] It appears that the mistaken reading 'diversum' must have entered the transmission early and is perhaps the author's mistake.

[26] This appears to be an error because the Toledan Tables have the same motion that Ptolemy has. See Pedersen, *The Toledan Tables*, Table CA21, pp. 1156–60.

[27] Pedersen, *The Toledan Tables*, Table CA11, pp. 1152–56.

positam et medium tempus eclipsis secunde trium notarum eclipsium diligenter observa, et equa secundum dies mediocres. Deinde ad illud tempus intermedium sume medium motum longitudinis. Et proiectis semper integris revolutionibus, si nichil superest, ipse locus Lune in medio secunde eclipsis secundum cursum medium qui per antepremissas propositiones inventus est est locus Lune secundum cursum medium super datam radicem. Si vero aliquid superfuerit de imperfecta revolutione, minue illud de loco Lune secundum cursum medium qui locus ad medium eclipsis sumpte inventus est, et remanebit locus Lune medius ad radicem positam. Simili modo ad tempus intermedium disce ex antedictis motum medium diversitatis. Et proiectis integris revolutionibus restat operandum ut ante, ut comprehendas ad positam radicem locum Lune in epiciclo certe distantie a longitudine longiore.

Hoc igitur ita fundato principio ad omnes deinceps divisiones temporum medius motus tam longitudinis quam diversitatis adaptandus est, ut verus locus Lune ad quodcumque velis tempus per viam operationis inveniatur quantum attinet ad simplicem equationem Lune. Via autem operandi eadem est quam de Solis equatione diximus.

16. Medium motum latitudinis Lune rectificare.

Quatuor sunt que propter hoc observanda sunt in duabus eclipsibus notis: primum ut par sit quantitas tenebrarum ex diametro Lune in duabus eclipsibus; secundo ut ambe eclipses sint aput eundem nodum Capitis vel Caude; tertio ut contingant ex eadem parte circuli signorum scilicet septentrionali vel meridiana; quarto ut distantia Lune in epiciclo a longitudine longiore sit una vel pene una in duabus eclipsibus. Sic enim distantia centri Lune a nodo uno – et ex parte una – in duabus eclipsibus erit equalis. Quapropter erit cursus Lune verus in latitudine – non dico medius – in tempore duarum eclipsium huiusmodi intermedio continens integras revolutiones latitudinis absque superfluitate. Et ponam ad hoc exemplum quod ponit Ptolomeus.

Fuit prima duarum eclipsium quas accepit propter hoc in anno xxxi° annorum Darii primi, et obscuratum est de diametro Lune ad quantitatem duorum digitorum ex parte meridiei. Et secunda eclipsis fuit in nono annorum Adriani, et obscurata est sexta pars diametri similiter et ex parte meridiei sicut

648 in – eclipsis] *corr. ex* †...† *P* **649** est²] *om. M* **650** Lune] l†iter†e *P* **651/652** cursum medium] medium cursum (*the second word s.l.*) P_7 **652** qui] igitur *N* **654** motum medium] medium motum P_7M **655** ut²] *s.l. K* ad] *om. M* **660** eadem est] est eadem sicut *N* **662** Lune] *om. N* **663** duabus – notis] *corr. ex* duobus eclipsibus notes *K* **664** primum] primum sit *M* **665** nodum] *corr. ex* modum P_7 **666** contingant] contingat P_7 **668** pene] *corr. ex* †...† *K* Sic] sit *M* Lune] et *add. s.l. M* **669** et] *om. PN* (*om. Ba* et E_1) eclipsibus erit] erit eclipsibus *P* **670** eclipsium] eclipsum *K* **671** huiusmodi] huius *N* **672** ponit Ptolomeus] ponit Tholomeus P_7 Ptolomeus ponit *M* **673** duarum] *om. N* xxxio] 21° *N* **675** eclipsis] eclipsi *K* nono] nono anno *M* **676** Adriani] *corr. ex* Drianii *P* obscurata] observata *P* time also that was between the supposed radix and the middle time of the second eclipse of the three known eclipses, and correct according to average days. Then take the mean motion of longitude for that intermediate time. And always with complete revolutions cast out, if nothing is in excess, that place of the moon according to mean course in the middle of the second eclipse that was found from the propositions before the preceding one [i.e. IV.11, IV.13] is the moon's place according to the mean course at the given radix. However, if anything of an imperfect revolution is in excess, subtract that from the moon's place according to the mean course that was found for the middle of the taken eclipse, and the moon's mean place at the given radix will remain. In a similar way, learn the mean motion of irregularity for the intermediate time from the things said before. And with complete revolutions cast out, it remains to be performed as before, so that you may grasp the moon's place on the epicycle of a certain distance from the apogee for the supposed radix.

Then, with this beginning established thus, the mean motion of both longitude and irregularity must be fitted to each of the following divisions of time in succession, so that the moon's true place at whatever time you wish may be found through the way of operation as much as it pertains to the moon's simple equation. Moreover, the way of operating is the same as that which we said about the sun's equation.

16. To correct the moon's mean motion of latitude.

There are four things that must be heeded for this in the two known eclipses: first, that the quantity of the darkness from the moon's diameter [i.e. measured as a fraction of the diameter] in the two eclipses is the same; second, that both eclipses are at the same node of the Head or the Tail; third, that they occur on the same side of the ecliptic, i.e. north or south; fourth, that the moon's distance on the epicycle from the apogee is one or nearly one in the two eclipses.[28] For thus, the distance of the moon's center from one node – also on one side – in the two eclipses will be equal. For this reason, the moon's true course in latitude – I do not mean the mean ⟨motion⟩ – in the intermediate time of the two eclipses of this kind, will contain complete revolutions of latitude without excess. And I will place for this the example that Ptolemy posits.

The first of the two eclipses that he took for this was in the 31st year of the years of Darius I, and the moon's diameter was obscured to the quantity of two digits on the south side. And the second eclipse was in the ninth year of Hadrian, and a sixth part of the diameter similarly was obscured, also on the

[28] While this fourth criterion matches that in Gerard's translation, it is clearer than Ptolemy's formulation, which is interpreted differently by Toomer, *Ptolemy's Almagest*, p. 206: '... the moon was at about the same distance [from the earth].' Both readings, i.e. the distance from apogee or from earth, amount to the same thing.

per considerationem comprehensum est. Et fuit transitus Lune aput Caudam quia cum pars Lune obscurata esset australis, necessario centrum Lune erat ex parte septentrionali a circulo signorum et erat tendens in meridiem. Distantia quoque Lune in epiciclo a longitudine longiore non erat multum diversa in duabus eclipsibus. Et ipsa quidem per premissa sciri potest, cum nota sit distantia Lune in epiciclo a longitudine longiore ad positam radicem temporis et totum tempus quod fuit inter positam radicem et eclipsim propositam notum sit. Distabat autem Luna in eclipsi prima referente Ptolomeo c gradibus et xix minutis. Fuit ergo cursus verus minuens de cursu medio v gradus. Et distabat Luna in eclipsi secunda a longitudine longiore ccli gradibus et liiii minutis. Fuit ergo cursus verus addens super cursum medium iiii gradus et liii minuta. Fuit ergo cursus verus Lune in tempore intermedio duarum eclipsium continens integras revolutiones latitudinis, et medius cursus latitudinis in eodem tempore minuens a perfectione integrarum revolutionum ipsas scilicet partes que aggregantur in utraque eclipsi ex ambabus diversitatibus, hoc est ix gradus et liii minuta. Fuit vero tempus intermedium duarum eclipsium sexcenti et lv anni Egyptii et cccxxxiii dies et xxi hore et medietas et tertia hore. In tanto igitur tempore secundum computationem inventionis Abrachis minuit medius cursus latitudinis a revolutionibus integris x gradus et duo minuta. Fit ergo medius cursus latitudinis in tanto tempore maior eo quem assignavit Abrachis ix minutis fere. Hec ergo ix minuta dividantur per numerum dierum qui fuerunt inter duas eclipses, et quod provenerit addatur super medium cursum latitudinis ad unam diem qui secundum Abrachis inventus est, et per eum similiter ad reliqua tempora cursus medii latitudinis corrigantur.

Sed nota quod Albategni eandem viam corrigendi vel experiendi secutus suo tempore habita revolutione ab eclipsi sue considerationis ad eclipsim Ptolomei invenit medium motum latitudinis xxvii minutis minorem eo qui in libro Pto-

677 comprehensum] deprehensum P_7M **679** septentrionali] septentrionis *M* a circulo] *om. N* tendens] *corr. ex* tentens *K* **681** quidem] secundum quod *M* **682** in] cum *PM* **683** quod fuit] *iter. P* inter positam] interpositam *K* inter predictam *M* eclipsim] eclipsem *P* propositam] per posita *N* **684** autem] *om. N* eclipsi] epiclipsi P_7 Ptolomeo] Tholomeo P_7 **685** minutis] minuta *P* verus] medius *P* **686** gradibus – minutis] gradus et 54 minuta *M* et] *s.l. P* **688** verus Lune] Lune verus P_7 verus *N* **689** revolutiones] *corr. ex* revolutione *K* **690** perfectione] perfectione ipsarum *N* **692** vero] ergo P_7M **693** cccxxxiii] 133 *N* hore[1]] *corr. ex* hora *P* **694** computationem] *corr. ex* compunctionem *K* **696** quem] quod P_7 **697** fuerunt] fuerant *corr. ex* †...† *P* fuerant *N* **699** unam] unum *M* Abrachis] Abrachem *N* **700** corrigantur] *om. PN* porigantur P_7 **701** eandem] *corr. ex* eam *K* corrigendi] corrigendo *P* corrigendi – experiendi] experiendi *corr. ex* experiendi vel experiendi *N* **702** revolutione] relatione P_7 *corr. in* relatione *K* vel renovatione *M* (revolutione BaE_1) Ptolomei] Tholomei P_7 **703** medium motum] motum medium *P* minutis] *marg.* P_7 libro] libris *N* Ptolomei] Tholomei P_7

south side as is grasped from observation. And the moon's passage was at the Tail because when the obscured part of the moon was south, the moon's center was necessarily on the northern side of the ecliptic and it was heading towards the south. Also, the distance of the moon on the epicycle from the apogee was not much different in the two eclipses ⟨albeit on opposite sides of the epicycle⟩. And that indeed is able to be known through what has been put forth [i.e. IV.15], because the distance of the moon on the epicycle from the apogee is known at the posited radix of time and the whole time that was between the posited radix and the proposed eclipse is known. Moreover, the moon stood away in the first eclipse, as Ptolemy reports, 100° 19′. Therefore, the true course subtracted 5° from the mean course. And the moon in the second eclipse stood 251° 54′[29] away from the apogee. Therefore, the true course added 4° 53′ upon the mean course. Therefore, the moon's true course in the intermediate time of the two eclipses contained complete revolutions of latitude, and the mean course of latitude in the same time subtracted from the completion of whole revolutions those parts that are gathered in each eclipse from both diversities, that is 9° 53′. And indeed, the intermediate time of the two eclipses was 655 Egyptian years, 333 days,[30] 21 50′ hours. Accordingly, in such a time according to the computation of the finding of Hipparchus, the mean course of latitude subtracted 10° 2′ from complete revolutions. Therefore, the mean course of latitude in such a time is greater than that which Hipparchus allotted by about 9′. Therefore, let these 9′ be divided by the number of days that were between the two eclipses, and let what results be added upon the mean course of latitude for one day that was found according to Hipparchus, and through it let them be corrected similarly for the remaining times of the mean course of latitude.

But note that Albategni, following the same way of correcting or testing in his own time with the revolution [i.e. the interval between similar eclipses] considered from the eclipse of his observation to the eclipse of Ptolemy, found the mean motion of latitude 27′ less than that which is posited in Ptolemy's

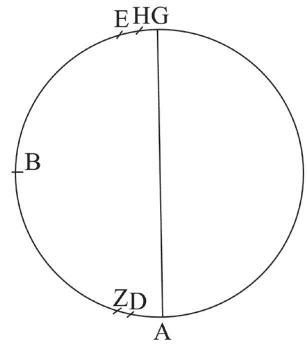
[29] This should be 251° 53′ to match the *Almagest*.

[30] This should be 615 Egyptian years, 133 days. The mistaken value of years is found in Paris, BnF, lat. 14738, f. 72r, but the mistake in days appears to have been made by the author of the *Almagesti minor*.

lomei ponitur, que per tempora que inter ipsum et illum fuerunt divisit, et de motu latitudinis qui est Ptolomei minuit. Et ita in tabulis scripsit.

17. Locus Lune secundum utrumque motum latitudinis in circulo declinante quantum distet a nodo tempore alicuius eclipsis note declarare.

Ad hoc declarandum observanda sunt in duabus eclipsibus notis tria eorum que supra determinavimus de pari quantitate tenebrarum et ut Luna utrobique sit meridiana vel utrobique septentrionalis a circulo signorum et distantia Lune in epiciclo a longitudine longiore sit una vel pene una. Quartum vero est ut una eclipsium contingat aput unum nodum, alia aput alium. Et ponam ad hoc exemplum Ptolomei.



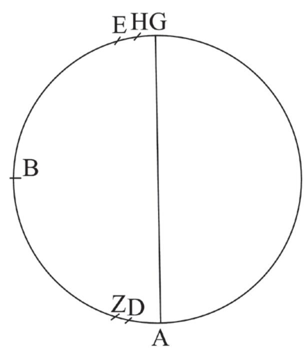
Prima harum duarum eclipsium est ea que supra nominata est que fuit in secundo anno Mardochei, et eclipsati sunt de Luna tres digiti ex parte meridiei. Et secunda eclipsis est ea per quam operatus est Abrachis que fuit in xx° annorum Darii qui regnavit post Philippum, et eclipsata est quarta diametri Lune similiter ex parte meridiei. Et tempus intermedium duarum eclipsium ccc et ix dies et xxiii hore equales et pars duodecima.

Et describam huius rei gratia circulum declinantem Lune ABG super diametrum AG et sit A nodus Capitis et G nodus Caude. Et punctum B sit maxima declinatio ad septentrionem. Et ponam propter supradicta duos arcus equales versus septentrionem AD et GE. Et sit centrum Lune in prima eclipsi supra punctum D et in eclipsi secunda supra punctum E. Fuit itaque elongatio Lune in epiciclo a longitudine longiore tempore medio eclipsis prime sicut per positam radicem cognosci potest xii gradus et xxiiii minuta. Et ob hoc medius cursus longitudinis maior vero lix minutis, que terminentur ad punctum Z. Et fuit elongatio Lune in epiciclo tempore medio secunde eclipsis a longitudine longiore ii gradus et xliiii minuta, et medius cursus longitudinis maior vero xiii

705 Ptolomei] Tholomei P_7 tabulis] *corr. ex* stabulis *K* scripsit] scripsit et cetera *N* **706** Locus] notus *P* motum] modum P_7 declinante] *corr. in* declinantis *M* **708** duabus] duobus P_7 eclipsibus] *corr. ex* eclipsis *K* **710** et] et ut P_7 **711** in – longiore] a longitudine longiore in epiciclo *N* **712** nodum] nodum et *N* alium] aliam *P* **713** Ptolomei] Tholomei P_7 **714** est ea] *om. N* **716** et] *om. N* **719** annorum] anno *KMN* (anno *Ba* annorum E_1) Philippum] Filippum P_7 **720** Lune similiter] similiter Lune *PN* **721** intermedium] *corr. ex* †...†ium *K* in medium *M* **722** ccc – ix] 319 *N* xxiii] xxxiii *P* **724** super] secundum *K* **729** a longitudine] ad longitudinem *K* **730** xii] 14 *N* **731** maior vero] vero maior *N* **731/733** lix – vero] *marg. N* **731/732** Z – Lune] *perhaps corr. ex* †...† *P* **731/733** Et – longiore] elongatio vero Lune a longitudine longiore epicicli in secunda eclipsi fuit *N* **732** in epiciclo] *om. P* secunde eclipsis] eclipsis secunde P_7 **733** et[1]] *om. M* medius – longitudinis] ob hoc medius cursus *N* maior] *s.l. K* book. Which ⟨27′⟩ he divided by the time that was between the one ⟨eclipse⟩ and the other, and subtracted it from Ptolemy's motion of latitude. And he wrote thus in tables.

17. To declare how far the moon's place on the declined circle according to either motion of latitude [i.e. true or mean motion] stands away from the node at the time of any known eclipse.

To show this, three of those ⟨criteria⟩ that we determined above must be heeded in the two known eclipses: about the equal quantity of darkness, that the moon in both instances is south or in both instances north of the ecliptic, and that the distance of the moon on the epicycle from the apogee is one or nearly one. However, the fourth ⟨criterion⟩ is that one of the eclipses occurs at one node, and the other at the other. And I will suppose for this Ptolemy's example.



The first of these two eclipses is that which was reported above that was in the second year of Marducheus [here spelled 'Mardocheus', i.e. Marduk-apla-iddina II], and three digits of the moon were eclipsed on the south side. And the second eclipse is that through which Hipparchus worked, which was in the 20th of the years of Darius, who ruled after Philippus,[31] and a fourth of the moon's diameter similarly was eclipsed on the south side. And the intermediate time of the two eclipses was 309 days and 23 5′ equal hours.[32]

And for the sake of this matter, I will describe the moon's declined circle ABG upon diameter AG, and let A be the node of the Head and G the node of the Tail. And let point B be the maximum declination to the north. And because of what was said above, I will posit two equal arcs AD and GE towards the north. And let the center of the moon in the first eclipse be upon point D, and in the second eclipse upon point E. Accordingly, the moon's elongation on the epicycle from the apogee in the middle time of the first eclipse was 12° 24′, as is able to be known through the supposed radix. And on account of this, the mean course of longitude is greater than the true by 59′, which let be ended at point Z. And the moon's elongation on the epicycle from the apogee in the middle time of the second eclipse was 2° 44′, and the mean course of longitude

[31] This is a mistake taken from Gerard's translation (*Almagest*, 1515 ed., f. 44v). Ptolemy wrote that this was in the 20th year of 'Darius who succeeded Kambyses' (Toomer, *Ptolemy's Almagest*, p. 208). This refers to Darius I. The name of the earlier king must have appeared to have been a mistake to somebody involved in the text's transmission (perhaps Gerard of Cremona, al-Ḥajjāj, Ishāq ibn Hunayn, or Thābit ibn Qurra), and they 'corrected' the text to refer to Darius III, who became king of Persia in 336 BC, when Philip II of Macedonia died.

[32] The number of years has been omitted. It should be 218 Egyptian years.

minutis, que terminentur ad punctum H. Est ergo locus medius centri Lune in prima eclipsi supra punctum Z et in secunda eclipsi supra punctum H. Et quia tempus inter duas eclipses est notum, erit motus medius latitudinis ad illud tempus notum ex premissa. Proiectis itaque integris revolutionibus erit arcus ZBH notus. Dempto ergo ab hoc arcu EH qui est xiii minuta, et addito ei arcu ZD qui est lix minuta, erit arcus EBD notus; residui ergo de semicirculo EG et DA simul noti. Et quoniam sunt equales, erit uterque eorum notus. Et est sicut ex dictis accidit uterque per se ix gradus et xxxv minuta. Et est arcus DA secundum cuius quantitatem distat verus locus centri Lune a nodo Capitis in prima eclipsi, et arcus EG secundum cuius quantitatem distat verus locus centri Lune a nodo Caude in secunda eclipsi. Et elongatio utriuslibet loci a puncto B, quod est maxima declinatio circuli ad septentrionem, nota, scilicet lxxx gradus et xxv minuta. Totus quoque arcus AZ notus est, et est x graduum et xxxiiii minutorum. Et arcus HG residuus notus, et est ix gradus et xxii minuta. Et arcus quidem AZ est secundum cuius quantitatem distat medius locus centri Lune a nodo Capitis in prima eclipsi, et eius elongatio a maxima declinatione que est punctum B est lxxix gradus et xxvi minuta. Et arcus HG est secundum cuius quantitatem distat medius locus centri Lune a nodo Caude, et eius elongatio a puncto B est lxxx gradus et xxxviii minuta, quod oportuit declarari.

18. Locus Lune in circulo declivi secundum medium motum latitudinis quantum distet a maxima declinatione septentrionali in fixa radice temporis indicare.

Sumatur ergo totum tempus quod effluxit a principio radicis usque ad medium tempus prioris eclipsis ex duabus de quibus novissime fuit sermo. Et ad illud tempus sumatur medius motus latitudinis, et proiectis integris revolutionibus reliquum observetur. Et quia distantia medii loci Lune secundum motum latitudinis a maxima declinatione que est punctum B in prima eclipsi nota est, ab ipsa distantia si sufficere potest – si minus, adiecta ei una revolutione – reliquum quod observasti minue. Et habebis quantum distat locus Lune secundum medium cursum latitudinis a maxima declinatione in radice temporis.

734 H] et quia tempus inter duas eclipses *add. et del.* P_7 locus medius] medius locus *K* **736** erit] *om. P* motus medius] medius motus P_7M notus medius *K* latitudinis] longitudinis *M marg. N* illud] idem *N* **737** itaque] *om. N* **739** lix] *corr. ex* 70 *M* EBD] EHD *P* residui] residuum P_7 **741** se] notus *add. et del. M* **742** centri Lune] *corr. ex* Lune centri *P* **742/744** Capitis – nodo] *om.* PP_7N **744** Caude] *corr. ex* Cauda *P* secunda] prima *N* **745** est] Z *N* lxxx] *corr. ex* 8 *M* **746** Totus quoque] totusque *M* **746/747** graduum – minutorum] gradus et 34 minuta *M* **747** minutorum] et arcus ZH est notus ergo totus AH est notus *add. marg. N* gradus – minuta] graduum et 22 minutorum P_7N **748** cuius] *om. P* **749** eius elongatio] elongatio eius P_7 **750** gradus – minuta] graduum et 26 minutorum P_7 **751** locus] *corr. ex* motus *K* motus *M* **752** gradus – minuta] graduum et 38 minutorum P_7 oportuit] oportet *P* **753** Locus] motus *P* motum] *corr. ex* cursum *M* **754** distet] distat P_7 indicare] iudicare *P* **755** totum tempus] tempus (*s.l. P*) totum *PN* **757** et] *corr. in* ut *M* **761** reliquum] *om. N* distat] distitit P_7K distiterit *M* (distiterit BaE_1)

was greater than the true by 13′, which let be ended at point H. Therefore, the mean place of the moon's center in the first eclipse is upon point Z, and in the second eclipse upon point H. And because the time between the two eclipses is known, the mean motion of latitude for that time will be known from the preceding ⟨proposition⟩. Accordingly, with complete revolutions cast out, arc ZBH will be known. Therefore, with arc EH, which is 13′, subtracted from this, and with arc ZD, which is 59′, added to it, arc EBD will be known; therefore, the remainders of the semicircle EG and DA will be known together. And because they are equal, each of them will be known. And as occurs from what was said, each by itself is 9° 35′. And arc DA is ⟨the arc⟩ according to the quantity of which the true place of the moon's center stands apart from the node of the Head in the first eclipse, and arc EG ⟨is that arc⟩ according to the quantity of which the true place of the moon's center stands apart from the node of the Tail in the second eclipse. And the elongation of each point from point B, which is the maximum declination of the circle to the north, is known, i.e. 80° 25′. Whole arc AZ is also known, and it is 10° 34′. And remaining arc HG is known, and it is 9° 22′. And arc AZ indeed is ⟨the arc⟩ by whose quantity the mean place of the moon's center stands apart from the node of the Head in the first eclipse, and its elongation from the maximum declination, which is point B, is 79° 26′. And arc HG is ⟨the arc⟩ according to the quantity of which the mean place of the moon's center stands apart from the node of the Tail, and its elongation from point B is 80° 38′, which it was necessary to show.

18. To show how far the moon's place on the declined circle according to the mean motion of latitude stands apart from the maximum northern declination at the fixed radix of time.

Now, let the whole time be taken that flowed from the beginning of the radix to the middle time of the earlier eclipse of the two about which the discussion most recently was. And let the mean motion of latitude be taken for that time, and with complete revolutions having been cast out, let the remainder be noted. And because the distance of the moon's mean place according to the motion of latitude from the maximum declination, which is point B, in the first eclipse is known, subtract the remainder that you noted from that distance if it is able to suffice – if it is insufficient, with one revolution added to it. And you will have how far the moon's place stands apart according to the mean course of latitude from the maximum declination at the time of the radix.

19. Medium motum Capitis Draconis elicere.

Quoniam medius motus longitudinis ad aliquod certum tempus minor est medio motu latitudinis ad idem tempus, manifestum est hanc differentiam accidere propter motum nodi. Refert enim motus nodi secundum quantitatem huius differentie contra ordinem signorum ipsum epiciclum cuius motus in circulo declinante est medius motus longitudinis. Ad quodcumque ergo tempus volueris medium motum Capitis, minue medium motum longitudinis ad ipsum tempus a medio motu latitudinis ad idem tempus. Et superfluum erit medius motus Capitis ad sumptum tempus, et erit motus iste contra ordinem signorum.

Explicit liber quartus.

764 medius motus] motus medius *PN* **766** motum] modum P_7 Refert] *corr. in* defert P_7 motus] *corr. ex* modus P_7 **767** huius differentie] differentie huius P_7 **768/769** tempus volueris] volueris tempus P_7K **772** Explicit – quartus] *om.* P_7K quartus liber explicit *N*

19. To draw forth the mean motion of the Dragon's Head.

Because the mean motion of longitude for some known time is less than the mean motion of latitude for the same time, it is manifest that this difference occurs because of the node's motion. For, according to the quantity of this difference and against the succession of signs, the node's motion carries back that epicycle, whose motion in the declined circle is the mean motion of longitude. Therefore, for whatever time you want the mean motion, subtract the mean motion of longitude for that time from the mean motion of latitude for the same time. And the remainder will be the mean motion of the Head for the taken time, and that motion will be against the succession of signs.

The fourth book ends.

⟨Liber V⟩ Incipit quintus.

Locus stelle secundum longitudinem est punctum circuli signorum super quod transit circulus magnus transiens super centrum corporis stelle et polos circuli signorum, qui etiam circulus longitudinis stelle dicitur.

Locus stelle secundum latitudinem est communis sectio duorum circulorum quorum unus transit super corpus stelle et polos zodiaci, alius similiter super corpus stelle transit et est equidistans zodiaco.

Diversitas aspectus Lune in circulo altitudinis est arcus circuli altitudinis inter verum locum Lune in celo et visum eius locum interceptus.

Diversitas aspectus Lune ad Solem in circulo altitudinis est, cum Sole et Luna in simili loco existentibus diversitas aspectus Solis a diversitate aspectus Lune subtracta fuerit, arcus circuli alter qui relinquitur.

Diversitas aspectus Lune in longitudine est, cum ipsa in circulo signorum fuerit, arcus circuli signorum deprehensus inter verum locum Lune et circulum transeuntem super polos circuli signorum et visum locum Lune in celo.

Diversitas aspectus in latitudine est ipsa in circulo signorum existente arcus circuli transeuntis super polos circuli signorum et visum locum Lune in celo inter circulum signorum et visum locum Lune deprehensus.

Media coniunctio Solis et Lune dicitur coniunctio secundum utriusque cursum medium.

Media oppositio sive preventio sive impletio vocatur oppositio secundum utriusque cursum medium.

Equalis longitudo longior in epiciclo nominatur punctum illud in summitate epicicli ex quo principium revolutionis Lune in epiciclo attenditur.

Longitudo longior vera in epiciclo dicitur punctum epicicli ad quod linea educta a centro mundi per centrum epicicli pervenit.

Equatio medie diversitatis vel portionis sive argumenti nominatur arcus epicicli inter longitudinem longiorem veram et longitudinem longiorem equalem deprehensus. Idem alias equatio puncti nominatur.

1 Incipit quintus] liber quintus *add. marg.* (*other hand*) *P om.* P_7 quintus *marg. corr. ex* sextus *K* et sequitur quintus *M* incipit quintus *N* **4** etiam] *om.* P_7 **5** stelle] *corr. ex* Lune P_7 est] *om. K* **9** Lune] eius *N* **12** circuli alter] circuli arcus alter *P corr. in* circuli altitudinis *K* circuli altitudinis *M* alter *N* (circuli †aliter† *Ba* circuli alter E_1) **13/15** Diversitas – celo] *om. P* **13** aspectus Lune] *corr. ex* Lune aspectus *M* Lune] *om.* P_7 longitudine] *corr. ex* longe *K* **13/14** in^2 – signorum] fuerit in zodiaco arcus zodiaci *N* **14** arcus] *corr. ex* aspectus *K* et] et inter *N* **15** circuli signorum] zodiaci *N* **16** aspectus] aspectus Lune *M* est – existente] ipsa in circulo signorum existente est *K* **17** super] per P_7 visum] *iter. et del. M* **18** locum] *om.* P_7 Lune] Lune in celo *corr. ex* Lune in Lune) *N* **21/22** Media – medium] *om.* P_7 **29** nominatur] vocatur P_7K

Book V

The fifth begins.

The place of a star according to longitude is the point of the ecliptic upon which the great circle passing through the center of the star's body and the ecliptic's poles, which ⟨great circle⟩ is also called the star's circle of longitude.

The place of a star according to latitude is the intersection of two circles, of which one passes upon the star's body and the poles of the zodiac, and the other passes similarly upon the star's body and is parallel to the zodiac.

The parallax of the moon on the circle of altitude is the arc of the circle of altitude cut off between the moon's true place in the heavens and its apparent place.

The parallax of the moon to the sun on the circle of altitude is the other[1] arc of the circle that remains when, with the sun and moon existing in a similar place,[2] the sun's parallax is subtracted from the moon's parallax.

The parallax of the moon in longitude is, when it is on the ecliptic, the arc of the ecliptic caught between the moon's true place and the circle passing upon the ecliptic's poles and the moon's apparent place in the heavens.

The parallax in latitude is, with it existing on the ecliptic, the arc of the circle passing upon the ecliptic's poles and the moon's apparent place in the heavens caught between the ecliptic and the moon's apparent place.

A mean conjunction of the sun and moon means a conjunction according to the mean course of each.

A mean opposition, anticipation,[3] or fulfillment means an opposition according to the mean motion of each.

The mean apogee on the epicycle means that point at the height of the epicycle from which the beginning of the moon's revolution on the epicycle is considered.

The true apogee on the epicycle means the point of the epicycle to which the line extended from the world's center through the epicycle's center reaches.

The equation of the mean irregularity, portion, or argument means the arc of the epicycle caught between the true apogee and the mean apogee. The same is called elsewhere the equation of point.

[1] While 'alter' appears to be original, perhaps it is a mistake for 'altitudinis.'

[2] I.e. the sun and moon are near to each other and thus have parallax in the same direction.

[3] 'Preventio' is also used in Plato's translation of Albategni, e.g. *De scientia astrorum* Ch. 42 (1537 ed., f. 58r).

Portio vel media diversitas equata sive argumentum equatum est arcus epicicli cum equatio portionis addita vel subtracta fuerit, Lune distantiam a longitudine longiore vera assignans.

1. Locum stelle secundum longitudinem et latitudinem artificio instrumenti deprehendere.

Queruntur primum due armille convenientis mensure orbiculares ambe similes et equales per omnia. Et sit utriusque tam interior que centrum respicit quam exterior superficies politissima et equalis per totum latitudinis, et utraque armilla eiusdem ubique spissitudinis. Et sic inseratur altera alteri ut sese secent ortogonaliter et per equalia. Imaginabimurque unam illarum habere vicem circuli signorum et alteram circuli meridiani in eo situ cum ipse transit per polos mundi et per polos zodiaci. In polis itaque zodiaci qui per quarte distantiam deprehenduntur duo claviculi rotundi et equalis grossitiei figantur ut exterius et interius promineant. Deinde aptabimus ad has tertiam armillam forinsecus circa positos claviculos quasi circa axem secundi motus leviter volubilem ita ut sua interiori superficie exteriores superficies duarum armillarum in omni loco et ex omni contactu vero superlabendo contingat. Pari modo aptetur intrinsecus quarta armilla eisdem claviculis innixa et circa eos leviter volubilis sic ut sua exteriori superficie interiores superficies duarum in omni loco et ex omni sublabendo tactu vero contingat. Et sit hec intrinseca armilla et que vicem zodiaci optinet utraque in ccclx partes divisa et unaqueque pars in quot particulas apte poterit subdivisa. Post hec armille intrinsece applicabimus regulam centro eius volubiliter affixam ut ipsa ad utrumque polum zodiaci ante et retro moveri possit et semper suis extremitatibus opposita armille cui affixa est attingat puncta. Eruntque iuxta extremitates due pinne super regulam erecte habentes duo foramina per diametrum opposita. Per hec enim tran-

30 argumentum] augmentum *P* **33** artificio instrumenti] instrumenti artificio *N* **35** Queruntur] querantur P_7 primum] *corr. ex* prime *K* convenientis] convenientes *P* **36** et^1] *s.l. M* que] *corr. ex* quam *N* **37** quam exterior] *iter. et del. P* **42** figantur] figurantur P_7 figuantur *corr. ex* figurantur *K* **43** ut] in *P* tertiam armillam] armillam tertiam *N* armillam] *corr. ex* marmillam *K* **44** forinsecus] forinsecas *corr. ex* forinsecos P_7 circa^1] citra *PN* (circa BaE_1) secundi] *corr. in* sui *M* leviter] liviter P_7 **45** superficie] *corr. ex* superficies *N* **46** ex] in *PN corr. in* in *M* (ex BaE_1) vero] vero suo (*corr. in* necnon) *M* superlabendo] superlambendo *PN corr. in* sublabendo P_7 (superlabendo BaE_1) **47** innixa] fixa P_7 **48** sic] sit *K corr. ex* sit *M* exteriori superficie] superficie exteriori *M* duarum] *corr. ex* duorum *K* armillarum *add.* (*s.l. K*) *KMN* (duarum armillarum *Ba* duarum E_1) **49** ex] in *PN* sublabendo – contingat] sublambendo tactu (*corr. ex* contactu) vero contingat *P* sublabendo contingat tactu vero P_7 superlabendo (*corr. in* sublabendo) tactu vero contingat *M* contactu vero superlambendo contingat *N* (contactu superlabendo tactu vero contingat *Ba* s†ub†labendo tactu vero contingat E_1) **50** optinet] obtinet *MN* utraque] *om. N* **51** subdivisa] dividatur *N* hec] hoc *MN* **52** zodiaci] zodiaci et *M* **53** semper] *corr. ex* super *M* suis] *marg. M* **54** Eruntque] *corr. ex* uterque P_7 **55** hec] hoc *N*

The equated portion, mean irregularity, or argument is the arc of the epicycle designating the moon's distance from the true apogee when the equation has been added or subtracted.

1. To discover the place of a star according to longitude and latitude by the craftsmanship of an instrument.

First, two round rings of an appropriate size are sought, both similar and equal in all ways. And of each, let the interior that looks to the center as well as the exterior surface be polished very well and uniform through the whole width, and let each ring be of the same thickness everywhere. And thus let one be inserted into the other so that they cut each other in half perpendicularly. And we will imagine that one of them has the place of the ecliptic, and the other the meridian circle in that place when it passes through the world's poles and the ecliptic's poles. Accordingly, let there be fixed two round and equally sized pivots on the zodiac's poles, which are discovered through the distance of a quarter circle, so that they jut out on the outside and inside. Then we will fit to these a third ring on the outside turning smoothly around the placed pivots as if around the axis of the second motion in such a way that it touches the outer surfaces of the two rings with its inner surface by gliding above in every place and from every true contact. In a like way, let a fourth ring be fitted on the inside resting upon the same pivots and turning smoothly around them in such a way that it touches the inner surfaces of the two ⟨first rings⟩ with its outer surface by gliding under in every place and from every true touch. And let this inner ring and that which holds the place of the zodiac each be divided into 360 parts and each part subdivided into as many small parts as are able to fit. Afterwards we will connect to this inner ring a rule fixed on its center turning so it is able to be moved forwards and backwards to each pole of the zodiac and so it always touches with its endpoints opposing points of the ring to which it is affixed. And near the endpoints there will be two fins set up on the rule having two apertures diametrically opposite. For the eyes' sibit aspectus oculorum. Subinde in circulo meridiano assumemus ab utroque polo zodiaci arcus equales secundum distantiam polorum mundi ab eis, et duas notas mundi polis inventis imprimemus.

Instrumento itaque sic constructo sedem ei in qua quasi super polos mundi secundum motum primum volvatur apparabimus, et ut secundum habitudinem loci inhabitati polus unus elevetur et alter deprimatur. Sedes igitur hec erit armilla quadrata immobilis erecta super superficiem orizontis et in superficie meridiani equedistanter collocata, sicut in libro primo de lamina diximus. Intra hanc igitur immobilem armillam instrumentum inseratur super polos fixos et secundum habitudinem loci habitati sitos rotabile.

Constructo tandem et secundum hunc modum collocato instrumento observandum quando Sol et Luna simul erunt super terram apparentes. Ut ergo locum Lune secundum longitudinem et latitudinem inveniamus, armilla extrinseca super polos zodiaci volubilis super gradum Solis vel minutum in ipsa hora considerationis ponenda est. Et locum sectionis duarum armillarum cum toto instrumento volvemus ad radium Solis donec utraque armilla, scilicet circuli signorum et circuli transeuntis super polos zodiaci et locum Solis, sese obumbret. Nam unum latus lateris oppositi eiusdem armille excipiet umbram. Tunc armillam intrinsecam per partes divisam volvemus ad Lunam, et regulam ei affixam tamdiu ante et retro torquebimus donec per duo foramina pinnarum Lunam in celo videamus. Cum pariter Solem in parte sua viderimus, arcus itaque armille intrinsece per partes divise inter summitatem regule et armillam circuli signorum deprehensus latitudinem Lune et locum secundum latitudinem indicat. Communis vero sectio huius circuli per quem latitudo cognoscitur et circuli signorum quem in partes quoque divisimus locum Lune secundum longitudinem demonstrat.

Quod si aliqua stellarum in linea cursus Solis fuerit vel alibi cuius locus scitur, per eam de nocte vice Solis operandum scilicet ut armilla extrinseca super locum longitudinis eius ponatur quasi in eo fixa et adherens, et oculus aspicientis super locum oppositum loco latitudinis. Et sic ad stellam notam donec videatur volvatur machina, et una armilla intrinseca ad stellam aliam

63 primo] *marg.* (*perhaps other hand*) *P* Intra] inter P_7 **64** hanc] hac *K* immobilem armillam] *corr. ex* immobilem marmillam *K* armillam immobilem *N* **65** habitati] *corr. ex* habitatu *K* sitos] scitos (*s.l. perhaps other hand P*) *PN* sitos in *M* rotabile] rotabilem P_7 **67** observandum] observandum est *N* erunt] sunt *N* **69** gradum] gradus *PN* minutum] minutum Solis *PN* **70** ponenda] *corr. ex* ponendum *N* **72** super] per P_7 **73** obumbret] obumbrent *M* **74** Tunc] *corr. ex* circa *M* **75** foramina] loca *N* pinnarum] primarum *P* **76** celo] celum P_7 parte sua] sua parte *MN* **82** aliqua] alia stellarum *corr. ex* alia stella *M* linea] *corr. ex* loco *N* alibi] alicubi *P* **83** operandum] operandum est *N* ut armilla] ut ar- *add. et del. P* **84** eo] ea *M* **86** donec – aliam] *marg. P* donec] *corr. ex* done *K* ad] a *P* gaze will pass through these. Immediately after, we will take equal arcs on the meridian circle from each pole of the zodiac according to the distance of the world's poles from them, and we will impress two points for the found poles of the world.

Accordingly, with the instrument thus made, we will prepare a seat for it in which it will be turned as if upon the poles of the world according to the first motion, and so that one pole is raised and the other depressed according to the disposition of the place inhabited. Then, this seat will be a squared ring [i.e. a band with squared edges] set up immobile upon the horizon's plane and set up parallelly in the meridian's plane, as we said about the plate in the first book [i.e. I.15]. Then, within this immobile ring, let the instrument be inserted, able to rotate upon poles fixed and positioned according to the disposition of the place inhabited.

Finally, with the instrument having been made and set up in this manner, it is to be observed when the sun and moon will be visible over the earth at the same time. Then, so that we may find the moon's place according to longitude and latitude, the outer ring turning upon the zodiac's poles should be placed upon the sun's degree or minute in that hour of observation. And we will turn the intersection of the two rings with the whole instrument to the sun's rays until each ring, i.e. of the ecliptic and of the circle passing through the zodiac's poles and the sun's place, casts a shadow upon itself. For one side will receive the shadow of the opposite side of the same ring. Then we will turn the inner ring divided into parts to the moon, and we will turn the rule affixed to it backwards and forwards until we may see the moon in the heavens through the two apertures of the fins. When we will have equally seen the sun in its direction, the arc, accordingly, of the inner ring divided into parts caught between the rule's highest point and the ring of the ecliptic shows the moon's latitude and the place according to latitude. And indeed, the intersection of this circle through which the latitude is known and the ecliptic, which we also divided into parts, shows the moon's place according to longitude.

But if any of the stars was in the line of the sun's course or elsewhere where the place is known, one should operate through it in the night in place of the sun, i.e. so that the outer ring be placed upon its place of longitude as if fixed and adhering in it, and the eye of the observer upon the place opposite the place of latitude. And thus let the machine be turned to the known star until it it is seen, and let one inner ring be turned to another star that we want to quam scire volumus donec per foramina regule conspici possit torqueatur. Et ita locum longitudinis et latitudinis ut prius cognosces.

Ratio est quod similes sunt circulorum arcus qui eidem angulo super centrum consistente subtenduntur. Sed attende quod hec consideratio ad modicam quantitatem fallit in Luna propter diversitatem aspectus in longitudine et latitudine; in superioribus vero stellis ubi diversitas aspectus non impedit, vicinior vero est consideratio.

2. Quod Luna secundam diversitatem habeat, et quod huius secunde diversitatis revolutio bis in mense lunari compleatur, semel scilicet tempore coniunctionis medie et secundo tempore impletionis medie manifestis indiciis demonstrare.

Quantitas prime diversitatis scilicet semidiametri epicicli sicut ostensum est est v partium et xv minutorum, et differentia duorum motuum, medii dico et diversi, que maxima propter ipsum accidere potest scilicet quando Luna est super punctum contactus in epiciclo educta linea a centro orbis circuli signorum, est v graduum fere. Tanti enim arcus sinus est. Cetere quoque differentie omnes que propter hanc diversitatem que singularis dicitur sunt note. Quotiens autem in mediis coniunctionibus vel oppositionibus per instrumenti considerationem cuius doctrina premissa est deprehensus est locus Lune secundum longitudinem, cognitus est concordare differentiis prius inventis que propter singularem diversitatem accidere debuerunt; aut si qua apparuit diversitas, tanta erat quantam accidere propter diversitatem aspectus Lune est possibile.

In aliis vero locis et in sectionum figuris extra mediam coniunctionem et oppositionem, manifesta apparuit diversitas quandoque maior quandoque minor, maior tamen semper ea que propter singularem diversitatem apparere debuit, ut videlicet in termino lateris decagoni, octogoni, exagoni, pentagoni, quadrati, trigoni a media oppositione. Maxima vero diversitas omnium in lateris quadrati termino ex utraque parte medie oppositionis apparuit, tunc quidem cum Luna a longitudine longiore epicicli distaret quarta vel modicum plus quarta. Et apparuerunt hee maxime diversitates equales semper ex utraque

87 regule] relique P_7 **89** Ratio] ratio huius *MN* quod] quem *P* quia *M* quoniam *N* (quod BaE_1) **90** hec] ista P_7 **91** quantitatem – Luna] fallit in Luna quantitatem *N* **93** consideratio] consideratio et cetera *N* **94** secundam] secundum *P corr. ex* secundum *K* huius secunde] huiusmodi secunde eius *M* **95** scilicet] scilicet in *N* **96** secundo] *corr. ex* semel *M* **98** semidiametri – est] diameter (dyametri *N*) epicicli est sicut ostensum *PN* **99** est – partium] 5 partium est *M* **100** ipsum] ipsam *N* **101** circuli] *om.* P_7K (*om. Ba* circuli E_1) **103** singularis] *corr. ex* singul- P_7 dicitur] dicitur accidunt *N* **104** vel] *om. P* et *N* **105/106** deprehensus – est] *marg. P* **107** debuerunt aut] debuerant vel *M* **108** erat] esset *PM* (erat BaE_1) quantam] quanta P_7 Lune] *om. N* **109** in²] *om. N* **110/111** maior – minor] minor quandoque maior *M* **110** quandoque²] quando *K* **111** ea] *corr. ex* †AE† *K* **113/114** lateris – termino] termino lateris quadrati *N* **114** termino] tunc *P* **115** Luna] *corr. ex* linea *K* **115/116** modicum – quarta] modico plus *N* **115** modicum] mediocri *corr. in* paulo *K* **116** Et] *corr. ex* cum P_7

know until it is able to be observed through the rule's apertures. And thus you will know the place of longitude and latitude as before.

The reasoning is that the arcs of circles that subtend the same angle standing upon the center are similar. But pay attention because this observation deceives a small amount in the moon because of the parallax in longitude and latitude; however, in the superior stars where parallax does not hinder, the observation is nearer to the truth.

2. To demonstrate by manifest pieces of evidence that the moon has a second irregularity and that this second irregularity's diversity is completed twice in a lunar month, i.e. once at the time of mean conjunction and second at the time of mean fulfillment [i.e. opposition].

The quantity of the first irregularity, i.e. the epicycle's diameter, is 5^p 15′, as was shown [in IV.10], and the greatest difference of the two motions, I mean of the mean and irregular, that is able to occur because of it, i.e. when the moon is upon the tangent point on the epicycle with a line having been extended from the center of the circle of the ecliptic, is about 5°. For it ⟨the epicycle's diameter⟩ is the sine of an arc of such a size. Also, all the other differences that ⟨occur⟩ because of this irregularity, which is called 'singular', are known. Moreover, whenever the moon's place according to longitude is found in mean conjunctions or oppositions through observation with an instrument, the instruction of which has been set forth, it is known to agree with the differences found earlier that ought to occur because of the singular irregularity; or if any irregularity appeared, it would be as much as is possible for there to be because of the moon's parallax.

However, in other places and in the figures of divisions except mean conjunction and opposition, a manifest irregularity appeared, sometimes greater, sometimes smaller, yet always greater than that which ought to appear because of the singular irregularity, as at the endpoints of the sides of a decagon, octagon, hexagon, pentagon, square, and triangle from mean opposition. And indeed, the greatest irregularity of all appeared at the endpoint of the square's side on either side of the mean opposition, at the time indeed when the moon stood a quarter or a little more than a quarter away from the epicycle's apogee. And these greatest irregularities always appeared equal on either side of the parte medie oppositionis in termino lateris quadrati. Quantum vero addebat apparens diversitas super debitam in processu Lune a coniunctione usque ad terminum lateris quadrati, tantum minuebat ab hoc termino lateris quadrati ordinate usque ad oppositionem, scilicet ut quantitatibus crementorum inde hinc responderent similes quantitates diminutionum. Quotiens autem Luna erat in longitudine longiore epicicli, non apparuit sensibilis diversitas nisi quantam propter diversitatem aspectus apparere est possibile. Palam ergo ex omnibus hiis indiciis quod Luna extra mediam coniunctionem vel oppositionem aliam diversitatem habet a prima singulari, et quod maxima que accidere potest est in termino lateris quadrati ex utraque parte medie oppositionis, et quod eius initium et perfectio est in mense lunari bis, semel scilicet in coniunctione media et semel in oppositione media.

3. Causam secunde diversitatis apparentibus convenientem assignare et eam in figuris visibiliter ostendere.

Causa huius secunde diversitatis rectissime ecentricus esse concipitur in superficie circuli declinantis qui est in spera Lune et ab eius circumferentia per suam longitudinem longiorem dependens. Ad cuius ecentrici longitudinem longiorem centrum epicicli bis in lunari mense pervenit, semel in oppositione media et semel in coniunctione media, et in quarta mensis ab utraque parte oppositionis fit in longitudine ecentrici propiore, et ab eius circumferentia centrum epicicli numquam recedit. Manentibus itaque superius assignatis motibus sicut sunt, scilicet medio motu longitudinis et medio motu latitudinis qui constat ex duobus scilicet motu longitudinis et motu nodi in diversam partem factis, et manente motu prime diversitatis, oportet intelligi ecentricum moveri in contrariam partem motus latitudinis secundum quantitatem motus que addita motui latitudinis compleat duplum distantie medie que est inter Solem et Lunam. Sic enim constitutis omnibus manent superiora omnia et accidunt convenientia apparentibus de secunda diversitate. Et representabo hoc in figuris.

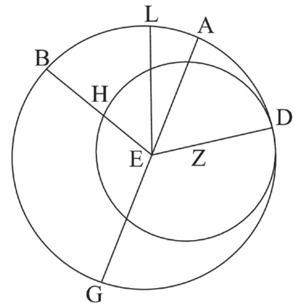
Imaginabimur itaque in spera Lune circulum declivem Lune ABGD super centrum E, quod etiam est centrum orbis signorum, et eius diameter AEG.

117 oppositionis] oppositis *P* **118** apparens] *corr. ex* media *P* Lune] Lune et *PN* **119** terminum] tertium *K* lateris^{2}] *om. N* **120** ordinate] ordinati P_7 **120/121** inde hinc] inde hic *K* hinc inde *N* **121/122** Luna erat] *s.l.* P_7 **122** erat] erit *P* epicicli] *om. N* sensibilis diversitas] diversitas sensibilis *M* **124** coniunctionem – oppositionem] oppositionem vel coniunctionem *N* **126** termino] extremo *N* **127** eius] est P_7 lunari – scilicet] lunaris bis scilicet semel *M* **130** visibiliter] verisimiliter *N* **131** huius] *corr. ex* eius P_7 secunde] *marg. P* **133** dependens] *corr. ex* †...† *P corr. ex* deprehendens P_7M deprehendens *N* (dependens BaE_1) **135** in^{2}] *s.l.* P_7 **136** fit – propiore] sit in longitudine propiore ecentrici *M* **137** assignatis] assignans *corr. ex* assigna†tis† *P* **140** factis] *corr. ex* factam *K* **142** inter] *s.l.* P_7 **143** accidunt] accidit P_7N (accidunt *Ba* accidit E_1) **144** convenientia] *corr. ex* convenienientia P_7 hoc] hec *N* **147** etiam est] est etiam *N* mean opposition at the endpoint of a square's side. And indeed, as much as the apparent irregularity added upon what ought to be there in the progress of the moon from conjunction to the endpoint of the square's side, so much was subtracted from this endpoint of the arranged side of a square to opposition, i.e. so that similar quantities of diminutions from this correspond to the quantities of the augmentations from that. Moreover, whenever the moon was at the epicycle's apogee, a perceptible difference did not appear except as much as is possible to appear because of parallax. Therefore, it is clear from all these pieces of evidence that the moon except at the mean conjunction or opposition has another irregularity besides the first singular, that the greatest that is able to occur is at the endpoint of a square's side on either side of the mean opposition, and that its beginning and completion are twice in a lunar month, i.e. once in the mean conjunction and once in the mean opposition.

3. To assign a cause of the second irregularity fitting the appearances and to show it visibly in figures.

The cause of this second irregularity is conceived most properly to be an eccentric in the declined circle's plane that is in the moon's sphere and hanging down from its circumference by its apogee. To which eccentric's apogee, the epicycle's center comes twice in a lunar month, once at mean opposition and once at mean conjunction, and in a quarter of a month it occurs at the eccentric's perigee on either side of the opposition, and the epicycle's center never recedes from its circumference. Accordingly, with the motions assigned above remaining as they are, i.e. the mean motion of longitude and the mean motion of latitude, which consists of two ⟨motions⟩ made in different directions, i.e. the motion of longitude and the node's motion, and with the motion of the first irregularity remaining, it is necessary that the eccentric be understood to be moved in the direction contrary to the motion of latitude according to the quantity of the motion that, added to the motion of latitude, completes double the mean distance that is between the sun and moon. For with everything thus disposed, all the above things remain and occur in conformity with the appearances concerning the second irregularity. And I will represent this in figures.

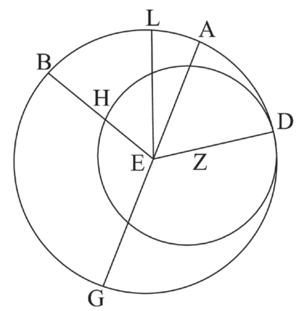
Accordingly, we will imagine in the moon's sphere the moon's declined circle ABGD upon center E, which is also the ecliptic's center, and its diameter



Et ponam longitudinem longiorem ecentrici et centrum epicicli et maximam declinationem circuli declinantis versus septentrionem et principium Arietis et locum medium Solis simul super punctum L quasi immobile. Et intelligantur tres linee simul EA ED EB super lineam EL quasi immobilem. Dico ergo quod in die una erit motus maxime declinationis secundum motum nodi tria minuta fere contra successionem signorum donec sit maxima declinatio in xxix partibus Piscium et lvii minuta fere, quem motum assignat linea EA separata a linea EL immobili. Et movetur centrum epicicli in die una motu medio longitudinis secundum successionem signorum a principio Arietis xiii gradus et xi minuta ex gradibus orbis signorum, quem motum assignat linea EHB separata a linea EL immobili secundum arcum BL ut sit centrum epicicli in puncto H. Est ergo motus latitudinis in eadem die super arcum AB coniunctum ex duobus arcu BL qui est longitudinis et arcu LA qui est sicut motus nodi xiii gradus et xiiii minuta. Et movetur longitudo longior ecentrici versus punctum D contra ordinem signorum a puncto quidem A xi gradibus et ix minutis in die una, quem motum assignat linea EZD quasi separata a linea EA per arcum AD. Et sit Z centrum ecentrici et ecentricus ipse HD. Elongatio ergo centri epicicli quod est H a longitudine longiore ecentrici que est punctum D est arcus BAD xxiiii graduum et xxiii minutorum coniunctus ex arcu latitudinis BA et arcu motus ecentrici AD. Atque hec quantitas aggregata ex duobus arcubus duplum est medie distantie Solis et Lune, et vocatur longitudo duplex.

Propter hoc ergo accidit quod in quarta mensis linea EHB fiet opposita linee EZD, et erit punctum H quod est centrum epicicli in longitudine propiore ecentrici sicut in subscripta figura apparet. Et in medietate mensis coniungentur iterum linea EHB et linea EZD quasi super lineam EG. Et hoc erit in oppositione media, et erit iterum centrum epicicli H in longitudine longiore ecentrici.

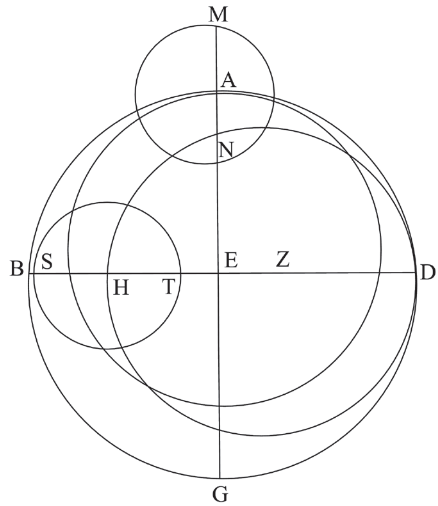
150 septentrionem] septentrionalem *K* **151** et] *om.* P_7K medium Solis] Solis medium *N* **152** punctum L] L punctum *corr. ex* locum L punctum *N* quasi] *corr. ex* quem *M* immobile] immobilem *MN* **153** super] sicut *K* **158** minuta] minutis *N* (minuta BaE_1) assignat] *corr. in* designat *M* **159** motu medio] medio motu *N* **160** minuta] minutis *N* **161** EHB] EB *N* **162** arcum] AB coniunctum *add. et del.* P_7 H] B *PN* **163** coniunctum] coniunctus *N* **164** arcu[1]] arcubus P_7M est[1]] motus *add. s.l.* P_7 **165** ecentrici] *om. N* **166** ordinem] successionem *N* gradibus – minutis] gradus et 9 minuta *M* **167** EZD] *corr. ex* AZD P_7 **168** sit Z] sic est *N* **169** ecentrici] *om. N* que] quod *M* **170** graduum – minutorum] gradus et 23 minuta *M* **173** EHB] *corr. ex* HB P_7 fiet] fit *N* opposita] oppositio *P* oppositione *N* **174** est centrum] centrum est P_7 **175** coniungentur] coniunguntur P_7M coniungitur *N* **176** iterum] *om.* P_7 EZD] EZB *P* **177** iterum] item P_7K H – longiore] ad longitudine†m† longiore (*the last word in marg.*) *P* ad longitudinem longiorem *N* **177/179** ecentrici – longitudinem] *marg. P*



AEG. And I will suppose that the eccentric's apogee, the epicycle's center, the declined circle's maximum declination towards the north, the beginning of Aries, and the sun's mean place are together at point L as if immobile. And let three lines EA, ED, and EB be understood to be together upon line EL as if immobile. I say, therefore, that in one day the motion of the maximum declination according to the node's motion will be about 3′ against the succession of signs until the maximum declination will be approximately in Pisces 29° 57′, which motion line EA separated from immobile line EL designates. And the epicycle's center is moved 13° 11′ of the degrees of the ecliptic from the beginning of Aries in one day by the mean motion of longitude according to the succession of signs, which motion line EHB designates, separated from immobile line EL according to arc BL so that the epicycle's center is at point H. Therefore, the motion of latitude in the same day is upon arc AB conjoined from the two, arc BL which is of the longitude and arc LA which is, as the node's motion, 13° 14′. And the eccentric's apogee is indeed moved 11° 9′ towards point D against the succession of signs from point A in one day, which motion line EZD designates, as separated from line EA by arc AD. And let Z be the eccentric's center and HD be that eccentric. Therefore, the elongation of the epicycle's center, which is H, from the eccentric's apogee, which is point D, is arc BAD of 24° 23′, conjoined from arc of latitude BA and arc AD of the eccentric's motion. And this quantity collected from the two arcs is double the mean distance of the sun and moon, and it is called the 'duplex longitude.'

Because of this, therefore, it happens that in a quarter month, line EHB will come to be opposite line EZD, and point H, which is the epicycle's center, will be at the eccentric's perigee as appears in the figure drawn below. And in a half month, line EHB and line EZD will be joined again as upon line EG. And this will be at mean opposition, and the epicycle's center H will again be

Et deinceps secundum similem ordinem sed conversum redibit centrum epicicli ad longitudinem propiorem ecentrici in tertia quarta mensis. Et in completione mensis ad longitudinem longiorem ecentrici perveniet in coniunctione media. Nichilominus tamen Luna in epiciclo movetur in die una xiii gradibus et iiii minutis fere secundum quod convenit motui diversitatis prime.

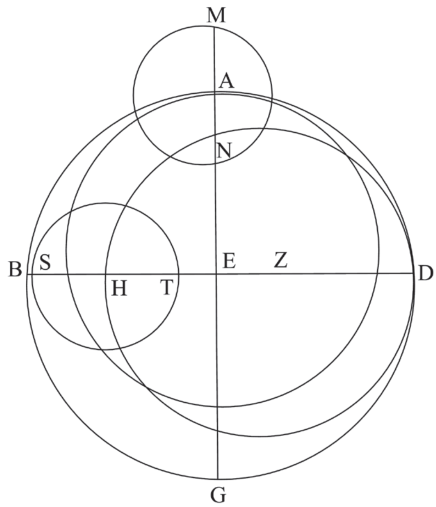


Hiis ita positis secundum huiuscemodi figuram motuum manifestum est accidere convenientia apparentibus, quoniam cum centrum epicicli est cum loco Solis medio vel cum est ex opposito, nulla est secunda diversitas eo quod centrum epicicli est in longitudine longiore ecentrici super circumferentiam declinantis circuli. Et si lineaverimus super punctum A epiciclum MN, erit proportio EA ad AM ea proportio quam declaravimus per tres eclipses, angulusque super E consistens continens epiciclum erit minimus omnium qui sequuntur deinceps. Procedente vero epiciclo ad longitudinem propiorem non cessat angulus augmentari, et secundum visum fit maior diversitas, et proportio semidiametri epicicli ad lineam interiacentem centro E et centro epicicli quod est H fit maior semper. Et cum fuerit centrum epicicli in longitudine propiore quod est in quarta mensis sive in termino lateris quadrati a coniunctione media, erit angulus continens epiciclum maximus qui esse poterit. Et ob hoc maxima secundum visum apparebit diversitas sicut ubi descriptus est epiciclus ST super punctum H, et maxima proportio omnium que precesserunt semidiametri epicicli ad lineam interiacentem centro E et puncto H est hic SH ad HE, quia cum SH sit equalis semper, linea EH hec est minima. Deinde redeunte centro

178 similem ordinem] ordinem similem *M* similem] *corr. ex* similitudinem *P* ordinem] *s.l.* P_7 **179** ad – ecentrici] *marg. K* quarta mensis] mensis quarta *M* **181** tamen] cum *N* Luna] *corr. ex* linea *K* in^1 – movetur] movetur in epiciclo *N* **182** motui] motu *P* **183** ita] itaque *M* huiuscemodi] huiusmodi *P* huiucemodi *K om. N* (huiusmodi *Ba* huiuscemodi E_1) **185** convenientia] convenientiam *N* **188** ex] *corr. ex* in *M* **192** lineaverimus] *corr. ex* lineverimus *K* **195** proportio] *om. N* **196/197** super E] *corr. ex* est *K* **198** sequuntur] secuntur *PM* (*om. Ba* sequuntur E_1) **199** deinceps] deinde P_7 **200** augmentari] agmentari *K* fit] sit *K* **203** est – quarta] in quarta est *P* **206** proportio omnium] proportionum *N* **207** hic] hoc PP_7 hec *M* hec scilicet *N* (hec *Ba* huius E_1) SH] *corr. ex* GH *K* **208** semper linea] super lineam *M* hec] hic P_7K *om. N* (hec *Ba* hoc E_1)

at the eccentric's apogee. And following this, according to a similar succession but conversely, the epicycle's center will return to the eccentric's perigee in the third quarter of the month. And in the completion of the month, it will return to the eccentric's apogee at mean conjunction. Nonetheless, the moon still is moved on the epicycle in one day about 13° 4′ according to what agrees with the first irregularity's motion.

With these things thus supposed according to the motions' figure of this kind, it is manifest that agreement with the appearances occurs, because when the epicycle's center is with the sun's mean place or when it is opposite, there is no second irregularity because the epicycle's center is at the eccentric's apogee on the circumference of the declined circle. And if we draw epicycle MN upon point A, the ratio of EA to AM will be that ratio that we declared through three eclipses, and the angle standing upon E containing the epicycle will be the smallest of all those that follow in succession. And indeed, with the epicycle proceeding to the perigee, the angle does not stop increasing, and the greatest irregularity according to sight comes about, and the ratio of the epicycle's radius to the line lying between center E and the epicycle's center, which is H, is always made greater. And when the epicycle's center is at the perigee, which is at a quarter of a month or at the endpoint of a square's side from mean conjunction, the angle containing the epicycle will be the greatest that can be. And on account of this, the greatest irregularity according to sight will appear, as when epicycle ST is described upon point H, and the greatest ratio of the epicycle's radius to the line lying between center E and point H, of all that precede is here SH to HE, because while SH is always equal, this line EH is the



epicicli ad longitudinem longiorem in oppositione media non cessant diminui angulus et proportio secundum quantitatem augmentorum sed conversis passibus. Quapropter minuitur secunda diversitas sicut apparebat. Hoc quoque palam quod propter ecentricum non accidit alia diversitas quam diximus, quoniam eius revolutio non est supra centrum Z, sed supra centrum E. Unde singuli motus preter motum diversitatis prime equabiliter fiunt supra circulos concentricos circulo signorum. Nam et centrum Z motu ecentrici circulum parvum describit circa E.

4. Maximam quantitatem secunde diversitatis pandere.

Tria ad hoc observanda sunt quantum vicinius vero fieri potest: scilicet ut media distantia Solis et Lune sit quarta circuli, quia tunc centrum epicicli est in longitudine propinquiore ecentrici; et ut Luna distet in epiciclo a longitudine longiore circiter quartam circuli, quia tunc maxima est diversitas que fieri potest unquam; et ut Luna distet ab orizonte per quartam zodiaci, quia tunc diversitas aspectus in sola latitudine est et non in longitudine eo quod circulus altitudinis tunc super polos zodiaci transeat. Hoc igitur minuto hore per considerationem instrumenti deprehensus est verus locus Lune, et cognoscendum quantum intersit inter verum locum Lune et locum Lune medium. Nam per hoc patebit maxima quantitas secunde diversitatis. Et ponam ad hoc exemplum observationis Ptolomei.

Observavit locum Solis et locum Lune in secundo anno annorum Antonii in Alexandria in xxvi[a] die mensis Camenut post ortum Solis et ante meridiem v horis et quarta hore equalibus. Et erat secundum quod apparuit per considerationem instrumenti Sol xviii gradibus et medietate et tertia gradus Aquarii sicut secundum computationem esse debuit. Et fuit medium celi in illa hora aput Alexandriam quarta pars Sagittarii. Et erat Luna secundum visum in ix gradibus et duabus tertiis gradus Scorpionis, qui erat verus eius locus. Fuit ergo eius elongatio a meridie in Alexandria versus occidentem circiter horam

209 oppositione] *corr. ex* longitudine P_7 cessant] cessat *N* diminui] minui P_7 **210** passibus] *corr. in* passionibus P_7 **211** minuitur] minuetur *K* secunda] secundum *M* sicut] sive *K* apparebat] apparebit *N* **212** alia] *om.* P_7 **213** est supra] *corr. ex* supra est *M* **214** equabiliter] *corr. ex* equaliter *P* fiunt] fuerit P_7 concentricos] *corr. ex* ecentricos P_7 **217** secunde diversitatis] diversitatis secunde *K* **218** observanda] conservanda P_7 **220** propinquiore] propiore P_7MN **221** maxima est] est maxima *M* est] *marg.* *P* **222** unquam] *corr. ex* numquam P_7 Luna distet] distet Luna *N* **223** est] *s.l.* (*perhaps other hand*) *P* **225** deprehensus] deprehendendus P_7M *corr. in* deprehendendus *K* deprehendendus *corr. ex* deprehendendum *N* (deprehensus *Ba* comprehensus E_1) **227** hoc[1]] hec P_7 patebit] *corr. ex* patebat *K* **228** Ptolomei] Tholomei P_7 **229** Observavit] observavit itaque *PN* locum[2]] *om.* *M* **230** xxvia] 26° *N* Camenut] Tamenut *N* **231** et[1] – equalibus] equalibus et quarta hore *N* **231/232** considerationem instrumenti] instrumenti considerationem *M* **232** Sol] Sol in *N* gradibus] gradu *M* **233** sicut – debuit] *om.* *N* **234** in] *om.* *M* ix] *corr. ex* 8 *N* **235** gradibus] gradu *K* tertiis] tertii *K* **236** meridie] medio celi (*the last word in marg.*) *N* versus occidentem] *om.* *N* smallest. Then with the epicycle's center returning towards the apogee at mean opposition, the angle and the ratio do not stop being diminished according to the size of the augmentations but with reversed steps. For this reason, the second irregularity is diminished as it appeared. Also, this is clear that because of the eccentric an irregularity different than what we described does not occur, because its [i.e. the moon's] revolution is not upon center Z, but upon center E. Whence each motion except for the motion of the first irregularity is made uniformly upon circles concentric to the ecliptic. For also the center Z describes a small circle around E with the motion of the eccentric.

4. To reveal the greatest quantity of the second irregularity.

Three things should be heeded for this so far as it is possible to be made closer to truth: i.e. that the mean distance between the sun and moon is a quarter circle, because then the epicycle's center is at the eccentric's perigee; that the moon stand about a quarter circle away from the apogee on the epicycle, because then there is the greatest irregularity that is ever able to occur; and that the moon stand away from the horizon by a quarter of the zodiac, because then there is parallax in latitude only and not in longitude because the circle of altitude then passes through the zodiac's poles. In this minute of an hour, therefore, the moon's true place is found through an observation with an instrument, and it should be known how much is between the moon's true place and the moon's mean place. For through this the greatest quantity of the second irregularity will be clear. And for this I will posit an example of Ptolemy's observation.

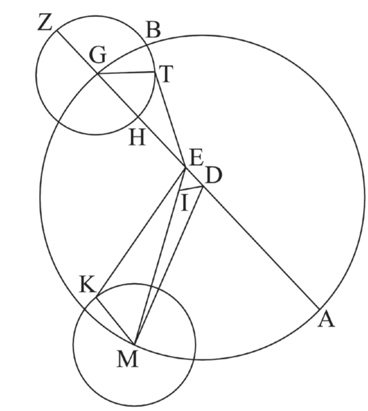
He observed the sun's place and the moon's place in the second year of the years of Antonius[4] in Alexandria on the 26th day of the month of Camenut after the sun's rising and 5 ¼ equal hours before noon. And, according to what appeared from the observation of an instrument, the sun was in Aquarius 18° 50′ as it ought to have been according to computation. And the middle heaven in that hour at Alexandria was in the fourth degree of Sagittarius.[5] And according to sight the moon was in Scorpio 9° 40′, which was its true place. Therefore, its elongation from the meridian in Alexandria was about

[4] This should be 'Antoninus.'

[5] Despite the use of the ordinal, this must mean Sagittarius 4° to match Ptolemy's value.

et medietatem hore, et ideo non fuit ei diversitas aspectus sensibilis in longitudine. Et fuit locus Lune secundum cursum medium longitudinis xviii gradus et x minuta Scorpionis, et fuit eius media distantia a Sole circiter quartam circuli. Et eius distantia in epiciclo a longitudine longiore lxxxvii gradus et xix minuta, et propter hoc diversitas maior. Fuit ergo cursus Lune verus minor medio vii gradibus et duabus tertiis unius gradus loco v graduum qui ex diversitate prima contingunt. Cuius arcus sinus est viii partes que sunt visa quantitas semidiametri epicicli que maxima accidere potest propter secundam diversitatem. Ponit quoque Ptolomeus considerationem Abrachis ex qua eadem quantitas secunde diversitatis prorsus deprehensa est, et fuit locus verus Lune maior medio secundum eandem quantitatem. Nam Luna a longitudine longiore epicicli ccvii gradibus et xlvii minutis distabat.

5. Quantitatem distantie duorum centrorum scilicet circuli signorum et ecentrici Lune cognitioni submittere.



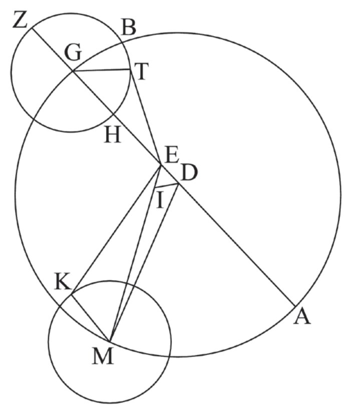
Lineabo ecentricum Lune ABG et in eo punctum E centrum orbis signorum, et sit punctum A longitudo longior ecentrici et punctum G longitudo propior. Quero ergo quantitatem linee ED, que quantitas attenditur respectu semidiametri AE. Et describo supra centrum G epiciclum Lune ZBT. Et protraho lineam ET contingentem epiciclum et semidiametrum GT. Manifestum ergo quod cum Luna fuerit in epiciclo supra punctum T, maior est diversitas que esse poterit. Et est nota ex premissis scilicet vii gradus et due tertie unius gradus, ergo angulus GET notus. Et angulus qui est ad T est rectus; ergo facta EG semidiametro erit proportio EG ad GT nota. Sed GT linea ut supra ostensum est est v partium et xv

237 medietatem] medietate *P* ei] *om. PN* eius P_7 (ei BaE_1) **238** medium] *om. PN* **239** x minuta] xi minuta *P* minutum secundum *N* (4 minutis *Ba* 10 minuta E_1) **240** lxxxvii] 88 *M* gradus] *om. PK s.l.* P_7 **243** arcus] *marg. P* sinus] *iter. K* **244** potest] potest scilicet P_7 **245** Ptolomeus] Tholomeus P_7 **245/246** secunde diversitatis] diversitatis secunde *M* **246** locus verus] verus locus *MN* **247** eandem quantitatem] quantitatem eandem *M* **248** distabat] *corr. ex* distabit P_7 distabat et cetera *N* **251** ABG] ABC *P* ABG super centrum D cuius diameter ADG P_7M ABG super dyametrum ADG *N* (ABG *Ba* ABG super centrum D cuius diameter ADG E_1) **253** punctum A] A punctum *N* longitudo longior] longitudine longiore *P* **254** punctum G] G punctum *N* Quero] quere PP_7 (quero *Ba* qu†oniam† E_1) **255** linee] line *K* **256** semidiametri] *corr. ex* diametri P_7 describo] *corr. ex* descebo P_7 **257** centrum G] G centrum *N* **258** ET] ETB *M* **260** Luna fuerit] fuerit Luna *M* Luna] linea *P corr. ex* linea *K* (Luna *Ba* linea E_1) **262** gradus] graduum *N* **264** T est] est *P* TE *K* **265** est[2]] *om. P*

1 1/2 hours towards the west, and for that reason there was not a perceptible parallax in longitude for it. And the moon's place according to the mean course of longitude was Scorpio 18° 10′,[6] and its mean distance from the sun was about a quarter circle. And its distance from apogee on the epicycle was 87° 19′, and because of this, ⟨there was⟩ the greatest irregularity. The moon's true course, therefore, was less than the mean by 7° 40′ instead of the 5° that occurs from the first irregularity. This arc's sine is 8^p, which is the greatest apparent quantity of the epicycle's radius that is able to occur because of the second irregularity. Also, Ptolemy posits an observation of Hipparchus from which the same quantity of the second irregularity is entirely found, and the moon's true place was greater than the mean according to the same quantity. For the moon stood 207° 47′[7] away from the epicycle's apogee.

5. To put forth for knowledge the quantity of the eccentricity.

I will draw the moon's eccentric ABG and in it point E the center of the ecliptic, and let point A be the eccentric's apogee and point G the perigee. I seek, therefore, the quantity of line ED, which quantity is considered with respect to radius AE. And I describe the moon's epicycle ZBT upon center G. And I draw line ET touching the epicycle and radius GT. It is manifest, therefore, that when the moon is at point T on the epicycle, there is the greatest irregularity that can exist. And from what has been set forth [i.e. in V.4], it is known, i.e. 7° 40′, so



angle GET is known. And the angle that is at T is right; therefore, with EG made a radius, the ratio of EG to GT will be known. But line GT, as was

[6] This should be 17° 20′.
[7] This should be 257° 47′ to match the *Almagest*.

minutorum respectu partium semidiametri EA. Ergo EG quoque hoc respectu est nota et est xxxix partes et xxii minuta. Ergo tota diametros AG est nota scilicet xcix partes et xxii minuta, et linea DA que est semidiameter ecentrici nota, et linea ED que est inter duo centra nota scilicet x partium et xix minutorum, quod erat demonstrandum.

6. Centro epicicli aput quodlibet punctum ecentrici secundum notam elongationem ab eius longitudine longiore constituto, visam quantitatem secunde diversitatis que in illo puncto maxima apparere potest notitie supponere.

In supposita figura item lineabimus epiciclum supra centrum M, sitque elongatio super arcum ecentrici que est longitudo duplex AM nota. Et duco contingentem EK et ad punctum contactus semidiametrum MK, et continuo duo puncta M E. Est ergo propositum ostendere quanta appareat MK sub angulo KEM. Nam hec est quantitas maxime diversitatis que aput punctum M contingit. Quia ergo nota est elongatio centri M a puncto A et ipsa consistit supra punctum E, notus est angulus AEM. Et angulus qui est ad I est rectus. Est ergo proportio ED ad utramque istarum IE ID nota. Sed ED est notarum partium respectu semidiametri EA; ergo utraque illarum hoc respectu nota est. Sed et DM eodem respectu est notarum partium; quare cum ipsa subtendatur angulo recto qui est ad I, erit IM. Cui si addatur IE, erit tota EM eodem respectu nota. Sed KM ad idem est notarum partium scilicet v partium et xv minutorum. Cum ergo angulus qui est ad K sit rectus, EM constituatur lx partium; erit hoc quoque respectu sinus MK et arcus super ipsum notus. Quare angulus KEM notus, quod intendebamus.

Pari modo colligi possunt aput quodlibet punctum inter longitudinem longiorem et longitudinem propiorem ecentrici maxime differentie secunde diversitatis que coniuncta est cum prima. Quare si differentiam maximam prime diversitatis – et est v partium – subtrahas sigillatim ab hiis differentiis, relin-

267 et[1] – nota[2]] *om.* P_7 partes] *s.l.* (*perhaps other hand*) *P* diametros] dyameter *MN* **268** xcix] xxviiii *PK* (99 *Ba* 10 *corr. in* 4 E_1) xxii] 12 P_7 ecentrici] ecentrici est *M* **269** x] *corr. in* 5 *M* minutorum] minuta *KM* **271** notam] notam eius *N* **273** supponere] subponere PP_7 **274** supposita] supraposita P_7 superposita *M* item] recte *N* centrum M] punctum M quod sit eius centrum P_7 **275** elongatio] elongatio centri *M* **277** propositum] propositam *P* **278** est – diversitatis] quantitas maxime diversitatis est *PN* **280** E] *corr. ex* A *N* I] L *M* est[3]] *om. K* **281** IE ID] ID IE P_7 LE LD *M* **282** semidiametri] dyametri *N* ergo utraque] utraque *P* utraque ergo *N* respectu[2]] quoque respectu *corr. ex* respectu quoque *M* **283** eodem] hoc *N* **284** I] L *M* IM] LM (*del.*) nota hoc respectu LM *M* IM nota hoc respectu *N* Cui] TM *P* IE] EL *corr. ex* ME *M* IE eodem respectu nota *N* tota] nota *PN* **284/285** eodem respectu] respectu eodem *corr. in* eodem modo *M* **285** nota – idem] KM *corr. ex* nota est sed KM ad id *N* **286** qui est] *s.l.* (*perhaps other hand*) *P* lx] xl *PK* (40 *Ba* 60 E_1) **289** aput quodlibet] apud quemlibet *corr. ex* ad quemlibet *M* **290** et longitudinem] *om. P* et *N* secunde] *om. K* **291/292** prime diversitatis] diversitatis prime *MN* **292** sigillatim] singulatim P_7 relinquuntur] relinquentur P_7KM (relinquuntur BaE_1)

shown above, is 5^p 15′ with respect to the parts of radius EA. Therefore, EG is also known in this respect and it is 39^p 22′. Therefore, whole diameter AG is known, i.e. 99^p 22′, and line DA, which is the eccentric's radius, is known, and line ED, the eccentricity, will be known, i.e. 10^p 19′, which was to be demonstrated.

6. With the epicycle's center set up at any point of the eccentric according to a known elongation from its apogee, to put forth for knowledge the greatest apparent quantity of the second irregularity that is able to appear at that point.

In the figure put forth, we will likewise draw an epicycle upon center M, and let the elongation be on the eccentric's arc AM, which is the known duplex longitude. And I draw tangent EK and radius MK to the point of contact, and I join the two points M and E. Therefore, it is proposed to show how great MK under angle KEM appears. For this is the quantity of the greatest irregularity that occurs at point M. Therefore, because the elongation of center M from point A is known and it stands upon point E, angle AEM is known. And the angle that is at I is right. Therefore, the ratio of ED to each of those IE and ID is known. But ED is of known parts with respect to radius EA; therefore, each of them is known in this respect. But also DM is of known parts in the same respect; therefore, because that one [i.e. DM] subtends the right angle that is at I [and because DI is known], IM will be ⟨of known parts⟩. If we add IE to which, the whole EM will be known in the same respect. But KM is of known parts to the same, i.e. 5^p 15′. Therefore, because the angle that is at K is right, let EM be set up as 60^p;[8] sine MK will also be known in this respect, and the arc upon it will be known. Therefore, angle KEM will be known, which we intended.

In a like way, the greatest differences of the second irregularity, which is conjoined with the first, are able to be obtained at whichever point between the eccentric's apogee and perigee. Therefore, if you subtract the greatest difference of the first irregularity – and it is 5° – one by one from these differences,

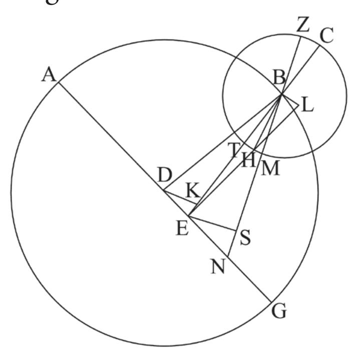
[8] The mistaken reading 'xl' must have entered the transmission early and is perhaps the author's own error.

quuntur differentie maxime aput puncta posita – differentie inquam secunde diversitatis separatim.

Habemus iam sufficientem doctrinam motuum Lune tunc quidem cum ipsa pervenerit ad coniunctionem vel preventionem mediam vel ad terminum quadrati ex utraque parte preventionis, scilicet tunc quidem cum est vel luminis orba vel plena aut semiplena. In aliis vero ipsius typis nondum sufficiunt que premissa sunt, scilicet cum est exesa vel corniculata et cum est protumida vel gibbosa.

7. Diameter epicicli ipsius longitudinem longiorem equalem indicans et tunc quidem veram cum centrum epicicli est in longitudine longiore vel longitudine propriore ecentrici, quod declinationem et reflexionem habeat, et quod eius declinatio et reflexio dirigatur neque ad centrum ecentrici neque ad centrum orbis signorum, sed ad punctum in diametro ecentrici quod tantumdem distat a centro orbis signorum versus longitudinem propiorem ecentrici quantum ex opposito centrum ecentrici distat ab eodem centro orbis signorum demonstrationibus manifestatur. Unde etiam manifestum quod procedente centro epicicli a longitudine longiore ecentrici ad longitudinem propiorem, longitudo longior epicicli vera precedit longitudinem longiorem equalem, et procedente centro epicicli a longitudine propiore ecentrici ad longitudinem longiorem, longitudo longior epicicli vera subsequitur longitudinem longiorem equalem.

Quod nunc proponitur ex multis considerationibus compertum est, sed excipiam duas in quarum tempore fuit epiciclus iuxta longitudines medias ecentrici et Luna prope longitudinem propiorem et prope longitudinem longiorem epicicli eo quod aput hec loca maxima sit declinatio vel reflexio diametri posita. Iam igitur scripsit Abrachis quod ipse consideravit instrumento in Rhodo Solem et Lunam in anno c^{o}lxvii^o post mortem Alexandri. Et invenit Solem per instrumentum in septimo gradu et medietate et quarta gradus in Tauro, et invenit Lunam secundum veritatem in xxi gradu Piscium

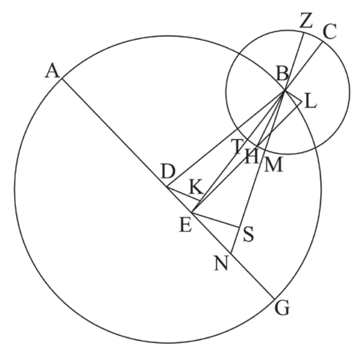


293 differentie maxime] maxime differentie *M* **295** Habemus] habes *N* **296** ad^2] *om. K* **297** quidem] *om. N* vel^1] *om.* P_7 orba] orbicularis *M* vel^2] aut *P* **298** aut] aut etiam P_7M typis] temporis *M* nondum] non *N* **299** est^1] *om.* P_7 exesa] exorsa *PMN* (ex esa BaE_1) **300** Diameter] dyametrum *M* epicicli] est in longitudine longiore *add. et del.* P_7 longitudinem longiorem] longiorem longitudinem *corr. ex* longituduni P_7 longitudinis longioris *M* **301** vel] vel in *N* **302** eius] eiusdem *M* **304** distat] distet *M* **306** ab eodem] a *N* **307** etiam manifestum] etiam est manifestum *M* est manifestum *N* **308** longior] *om.* P_7 **309** longiorem] *om.* P_7 **310** a – propiore] ad longitudinem propiorem *P* **311** longior] *om.* P_7 **314** propiorem] *corr. ex* longiorem *M* **316** sit] fit *PN* **318** Abrachis] *corr. ex* Aprachis *K* **319** Rhodo] Rodo P_7N *corr. ex* †m†odo *K* (†equande† *Ba* Rodho E_1) colxviio] 197^o *N* **322** Tauro] Thauro *KMN* the greatest differences at the posited points will remain – I mean the differences of the second irregularity separately.

We have now sufficient doctrine of the moon's motions for the times indeed when it comes to mean conjunction or opposition [*lit.*, anticipation] or to the endpoint of a square on either side of the opposition, i.e. at the times indeed when it is either bereft of light, full, or half full. However, in its other forms, what has been set forth is not yet sufficient, i.e. when it is hollowed out or crescent and when it is protruding or gibbous.

7. It is made manifest by proofs that the epicycle's diameter indicating the mean apogee – and ⟨indicating⟩ the true ⟨apogee⟩ indeed at those times when the epicycle's center is at the eccentric's apogee or perigee – has a turning aside and a bending back, and that its turning aside and bending back are directed neither to the eccentric's center nor to the ecliptic's center, but to the point on the eccentric's diameter that stands as far away from the ecliptic's center towards the eccentric's perigee as the eccentric's center stands away from the same center of the ecliptic on the opposite side. Whence it also is manifest that with the epicycle's center proceeding from the eccentric's apogee to perigee, the epicycle's true apogee precedes the mean apogee, and with the epicycle's center proceeding from the eccentric's perigee to apogee, the epicycle's true apogee follows the mean apogee.

What is now proposed has been verified by many observations, but I will extract two at whose times the epicycle was near the eccentric's mean distances and the moon near the epicycle's perigee and apogee because at these points the turning aside or bending back of the diameter is supposed the greatest. Accordingly, Hipparchus already wrote that he observed the sun and moon with an instrument at Rhodes in the 167th year[9] after Alexander's death. And he found the sun through the instrument in the 7th degree and ¾ of a degree in Taurus [i.e. Taurus 7° 45′],[10] and he found the moon according to truth in the 21st degree of Pisces and 11/24 of a degree [i.e.



[9] This should be the 197th.

[10] For the *Almagest*'s 'septem partibus et medietate et quarta partis Tauri', the author of the *Almagesti minor* wrote 'in septimo gradu et medietate et quarta gradus in Tauro', which could reasonably be taken to refer to Taurus 6° 45′.

et tertia et octava partis. Fuit ergo distantia vera Lune in illo tempore a vero loco Solis secundum successionem signorum cccxiii gradus et xlii minuta fere. Atque cum locus Solis secundum computationem a radice deprehensus est, fuit quidem secundum cursum medium vi gradus et xli minuta Tauri et secundum verificationem vii gradus et xlii minuta sicut apparuit per instrumentum. Et locus Lune secundum cursum medium longitudinis xxii gradus et xiii minuta Piscium, et locus Lune secundum cursum medium diversitatis a longitudine longiore equali in epiciclo clxxxv gradus et xxx minuta. Fuit itaque distantia Lune secundum cursum eius medium a vero loco Solis cccxiiii gradus et xxviii minuta.

Quibus ita positis describam ecentricum Lune ABG supra centrum D, sitque diameter ADG in quo centrum orbis signorum E. Et describam epiciclum ZHT supra centrum B cuius revolutio versus A longitudinem longiorem ecentrici secundum successionem signorum, et motus Lune a puncto Z ad H deinde ad T. Et ducam lineas DB ETB BZ. Quoniam ergo media distantia Solis et Lune secundum anteposita est cccxv gradus et xxii minuta, cum nos hoc duplicaverimus et inde integram revolutionem proiecimus, remanebit longitudo duplex nota cclxxi gradus et iiii minuta qui est motus centri epicicli a longitudine longiore ecentrici secundum continuitatem signorum. Quapropter angulus AEB notus est scilicet residuum iiii rectorum. Ducta ergo perpendiculari DK cum angulus ad K sit rectus, erit proportio DE que est distantia duorum centrorum ad utramque istarum DK EK nota, ergo utraque earum nota. Et quia DB semidiameter ecentrici etiam nota subtenditur angulo recto, erit etiam KB nota; quare et tota EB nota.

Rursusque medius cursus Lune est super lineam EB et distantia eius secundum medium cursum eius a vero loco Solis maior est vera distantia eius secundum considerationem xlvi minutis, sicut ex premissis patere potest. Si posueri-

324 octava] *corr. ex* 7 *M* **327** medium] fuit *add. et del. KM* Tauri] Thauri *MN* **330** Piscium] Piscis *K* diversitatis] *marg. M* **331** longiore] *om.* P_7 gradus] *om. M* **332** cursum – medium] medium cursum eius *M* xxviii] *corr. in* 38 *M* **334** Lune] *om. M* **335** quo] qua *N* E] est P_7 **336** ZHT] HT *N* centrum] punctum *N* revolutio] revolutio sit *N* **337** Z] *om. P s.l.* P_7K C *N* (*om.* BaE_1) **338** ETB] et TB *P* ET TB *N* distantia] differentia P_7 **339** secundum – est] sicut anteposita est est *N* **339/340** nos hoc] nos P_7 *corr. ex* nec hoc *K* hoc nos *N* **340** inde] *om. M* integram revolutionem] revolutionem integram *N* proiecimus] proiecerimus P_7MN (proiciemus *Ba* proiecerimus E_1) **341** iiii] *corr. ex* 44 *N* qui] quod P_7 que *KN* (et *Ba* quod E_1) **343** notus est] est notus *M* **345** earum] earum est *M* **346** Et quia] est etiam *N* **346/347** Et – nota[1]] *marg. P* **346** DB] *corr. ex* DT *corr. ex* †B† *M* etiam nota] etiam nota et P_7M est (*del.*) nota *K* nota est que *N* **346/347** erit etiam] quare erit *N* **347** quare] unde *N* **348** Rursusque] rursum quia P_7K (rursus quod *Ba* rursusque E_1) est] *om.* P_7K **349** vera – eius[2]] distantia eius vera *N* **350** considerationem – minutis] veram (*del.*) considerationem xlvi (*corr. ex* xlv) minuta *K*

Pisces 21° 27′ 30″].[11] Therefore, the true distance of the moon from the sun's true place at that time was approximately 313° 42′ according to the succession of signs. And when the sun's place was found according to computation from the radix, it was indeed Taurus 6° 41′ according to mean course, and according to correction ⟨of its anomaly, Taurus⟩ 7° 42′,[12] as appeared through the instrument. And the moon's place according to the mean course of longitude was Pisces 22° 13′, and the moon's place on the epicycle according to the mean course of irregularity was 185° 30′ from the mean apogee. And so the distance of the moon according to its mean course from the sun's true place was 314° 28′.

With these things thus supposed, I will describe the moon's eccentric ABG upon center D, and let there be diameter ADG, on which is the ecliptic's center E. And I will describe epicycle ZHT upon center B, whose revolution is towards the eccentric's apogee A according to the succession of signs, and the moon's motion is from point Z[13] to H and then to T. And I will draw lines DB, ETB, and BZ.[14] Therefore, because the mean distance between the sun and moon according to what has been posited before is 315° 22′,[15] when we double this and subtract a complete revolution from this, the duplex longitude will remain known 271° 4′, which is the motion of the epicycle's center from the eccentric's apogee according to the succession of signs. For this reason, angle AEB is known, i.e. the remainder from four right angles. With perpendicular DK drawn, therefore, because the angle at K is right, the ratio of DE, which is the eccentricity, to each of those DK and EK will be known, so each of them will be known. And because the eccentric's radius DB also known subtends a right angle, KB will also be known; therefore, whole EB will also be known.

And in turn, the moon's mean course is upon line EB and its distance according to its mean course from the sun's true place is greater than its true distance according to observation by 46′, as can be made clear from what has been set forth [i.e. in the first paragraph of the proposition]. If we suppose the moon's

[11] Again, the use of an ordinal for the degree makes this position ambiguous, but here the source of the wording is Gerard's translation.

[12] This should be 7° 45′.

[13] The 'Z' must have been omitted in the original or early in the text's transmission.

[14] The author does not yet specify what point Z represents, but it will be seen to be the mean apogee. Ptolemy produced lines DB and ETBZ, making Z the true apogee (Toomer, *Ptolemy's Almagest*, p. 228). The deviation from Ptolemy's proof appears in at least one manuscript of Gerard's translation, which has 'Et protraham lineas DB, ET, BZ' and which depicts BZ as an extension of line NB, not line EB (Paris, BnF, lat. 14738, f. 78v).

[15] This should be 315° 32′, but the mistake is already found in Gerard's translation (Paris, BnF, lat. 14738, f. 78v).

mus locum Lune in epiciclo punctum H eo quod iuxta longitudinem propiorem fuerit, et eduxerimus lineam EHL, erit angulus BEH continens illam diversitatem notus. Ducam ergo perpendicularem BL super lineam EHL et continuabo BH. Erit ergo proportio EB ad BL nota. Sed erat EB ad BH nota. Quare BH ad BL proportionem habet notam. Cum ergo angulus ad L sit rectus, facta HB semidiametro erit angulus BHL notus. Reliquus ergo intrinsecus TBH est notus; et ob hoc arcus TH notus qui est arcus epicicli, et est secundum quod accidit ex dictis vi gradus et xi minuta.

Rursum quia elongatio Lune in epiciclo a longitudine longiore equali fuit in hora considerationis clxxxv gradus et xxx minuta, manifestum quod Luna transiit iam longitudinem propiorem equalem v gradibus et xxx minutis. Et ob hoc constituemus eam in puncto M, et erit arcus HM v gradus et xxx minuta. Quare totus arcus TM factus est xi gradus et li minuta. Itaque angulus EBS eiusdem quantitatis est notus ducta scilicet recta ZBMSN. Quapropter educta super eam perpendiculari ES erit proportio EB ad ES nota. Rursum quia angulus AEB erat notus et nunc notus est angulus EBN, sequitur angulum ENS esse notum. Quare cum angulus ad S sit rectus, facta EN semidiametro erit proportio EN ad ES nota. Sed erat ES ad EB nota et EB ad ED; quare EN ad ED est nota. Et secundum operationem premissorum accidit quod EN sit x partium et xix minutorum fere. Est itaque EN equalis linee ED distantie duorum centrorum, et ad punctum N dirigitur diameter circuli brevis ZBM indicans longitudinem longiorem equalem in epiciclo. Et procedente centro epicicli a puncto G ad A longitudinem longiorem, subsequitur longitudo vera epicicli que videtur super punctum C educta recta EBG longitudinem longiorem equalem que est punctum Z, quod erat propositum.

Denuo scripsit Abrachis quod ipse consideravit in instrumento Solem et Lunam in eodem anno scilicet c^{o}xcvii^{o} post mortem Alexandri, et invenit Solem per instrumentum in undecimo gradu Cancri excepta decima unius gradus, et invenit Lunam secundum considerationem in xxix gradu Leonis. Et fuit ita secundum veritatem quia in hora considerationis non fuit diversitas aspectus in longitudine sensibilis. Fuit ergo vera elongatio Lune a Sole in illa hora secun-

352 et] *om. PKN* (*om. Ba* et *E_1*) **353** BL] *om. M* **353/354** et – BH^3] *marg. P* **355** habet] habebit *P_7* sit] *om. P_7* rectus] *corr. ex* notus *K* **356** semidiametro] *corr. ex* dyametro *M* **360** gradus] graduum *M* manifestum] manifestum est *N* **361** transiit iam] transit iam *M* iam transiit *N* **362** hoc] hec *P* constituemus] constituimus *N* **363** arcus] angulus *P* gradus – minuta] graduum et 51 minutorum *P_7* gradibus et 51 minuto *M* **365** perpendiculari] perpendicularem *K* **365/366** erit – EBN] *marg. P* **365** Rursum] rursus *P_7* **366** erat notus] notus erat *M* **368** erat] *corr. ex* quia *K* **370** linee] *om. N* **371** indicans] *corr. ex* indag- *M* **373** subsequitur] subsequetur *P* **374** C] T *N* EBG] ABG *corr. in* EBC *P_7* *corr. in* EBC *M* EBT *N* (EBG *BaE_1*) **376** in] *om. P_7* **377** scilicet] *s.l. K* coxcviio] *corr. in* 107 *M* 147^{o} *N* **381** a – hora] in illa hora a Sole *PN* place on the epicycle to be point H because it was near the perigee, and[16] we draw line EHL, angle BEH containing that irregularity will be known. Therefore, I will draw perpendicular BL upon line EHL and I will join BH. Therefore, the ratio of EB to BL will be known. But EB to BH was known. Therefore, BH has a known ratio to BL. Therefore, because the angle at L is right, with HB made a radius, angle BHL will be known. Therefore, the remainder, intrinsic ⟨angle⟩ TBH, is known; and from this arc TH is known, which is the epicycle's arc, and according to what occurs from what has been said, it is 6° 11′.[17]

In turn, because the moon's elongation from the mean apogee on the epicycle was 185° 30′ in the hour of the observation, it is manifest that the moon already passed the mean perigee by 5° 30′. And from this we will set it up at point M, and arc HM will be 5° 30′. Therefore, whole arc TM is made 11° 51′. Accordingly, angle EBS of the same size is known, i.e. with straight line ZBMSN drawn. For this reason, with perpendicular ES drawn upon it, the ratio of EB to ES will be known. In turn, because angle AEB was known and now angle EBN is known, it follows that angle ENS is known. Therefore, because the angle at S is right, with EN made a radius, the ratio of EN to ES will be known. But ES to EB was known and EB to ED; therefore, EN to ED is known. And according to the operation of what has been set forth, it happens that EN is approximately 10^p 19′.[18] Accordingly, EN is equal to line ED, the eccentricity, and the epicycle's [*lit.*, small circle's] diameter ZBM indicating the mean apogee on the epicycle is directed towards point N. And with the epicycle's center proceeding from point G to apogee A, the epicycle's true apogee[19] which is seen upon point C with straight line EBG[20] drawn, follows the mean apogee, which is point Z, which had been proposed.

Again, Hipparchus wrote that he observed the sun and the moon with an instrument in the same year, i.e. the 197th after Alexander's death, and through the instrument he found the sun in the 11th degree of Cancer minus 1/10 of a degree [i.e. Cancer 10° 54′], and he found the moon according to observation in the 29th degree of Leo.[21] And it was thus according to truth because there was not a perceptible parallax in longitude in the hour of observation. Therefore, the true elongation of the moon from the sun at that hour was 48° 6′

[16] The original may have not had this 'et' here, which would have made the following clause the apodosis.

[17] This should be 6° 21′.

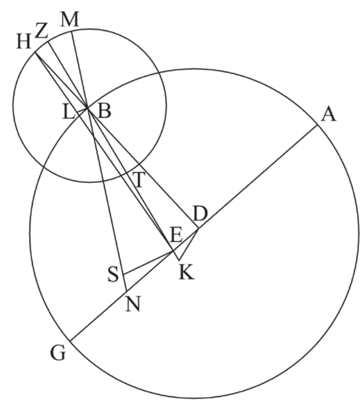
[18] Ptolemy finds it to be 10^p 18′.

[19] The author left out an understood 'longior.'

[20] This should be 'EBC.' Some witnesses have the correct reading, but they most likely reflect corrections to the original text.

[21] Despite the ordinal, this must mean Leo 29° to match the *Almagest*.

dum successionem signorum xlviii gradus et vi minuta. Atque cum locus Solis secundum computationem a radice deprehensus est, fuit quidem secundum cursum medium xii gradus et v minuta Cancri et secundum veritatem x gradus et liiii minuta. Et fuit locus Lune per medium cursum longitudinis xxvii gradus et xx minuta Leonis. Fit ergo distantia Lune secundum medium ipsius cursum a vero loco Solis xlvi gradus et xxvi minuta, et fuit elongatio Lune in epiciclo a longitudine longiore equali secundum medium motum diversitatis cccxxxiii gradus et xii minuta.

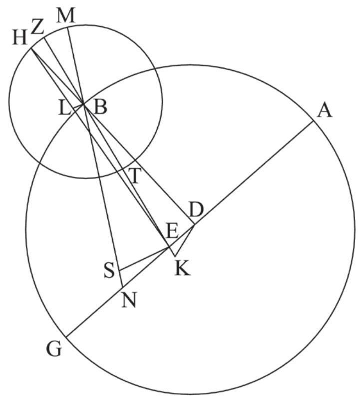


Quibus ita constitutis describam ecentricum lunarem sicut prius supra centrum D et epiciclum super centrum B ductis lineis DBH et ETBZ. Quoniam ergo longitudo duplex est xc gradus et xxx minuta, erit angulus AEB obtusus notus. Educta ergo DK perpendiculari super lineam EB fiet angulus residuus de duobus rectis DEK notus; et ob hoc utraque istarum DK EK nota ad ED, et propter hoc EB nota.

Item quia elongatio Lune secundum medium ipsius cursum a vero loco Solis minor est vera ipsius elongatione gradu uno et xl minutis, cum locum Lune secundum medium cursum assignet linea EBZ, si constituerimus punctum H locum Lune in epiciclo eo quod fuerit iuxta longitudinem longiorem epicicli et eduxerimus lineam EH, erit angulus BEH notus. Et ob hoc educta perpendiculari BL super lineam EH, erit BL ad EB nota, et propter hoc ad BH. Erit ergo angulus BHL notus. Reliquus ergo HBZ notus, quare arcus epicicli HZ notus, et ipse est elongatio Lune a longitudine longiore vera epicicli; et est xiv gradus et xlvii minuta.

382 successionem] *perhaps corr. ex* †...†ionem *K* gradus – minuta] gradibus et 6 minutis *M* **383** est] *om. N* **384** v minuta] *corr. in* 15 minuta *M* 15 minutum *N* (5 minuta *BaE*$_1$) **384/385** gradus[2] – minuta] gradibus et 54 (*corr. ex* 20) minutis *M* **386** Fit] fuit *N* medium – cursum] ipsius cursum medium *P*$_7$ medium eius cursum *N* **387** xxvi] 27 *M* (26 *Ba* 27 *E*$_1$) **389** gradus – minuta] graduum et 12 minutorum *M* **391** sicut] sic *N* **393** DBH] (BDH *Ba* DBH *E*$_1$) **394** gradus – minuta] graduum et 30 minutorum *M* xxx] 39 *N* **395** Educta] ducta *N* **396** fiet] fiet et *M* **397** de] *corr. ex* a *M* **398** DK EK] EK DK *P*$_7$*M* **399** EB] CB *K* **402** ipsius elongatione] elongatione ipsius *N* **403** medium cursum] cursum medium *M* **405** BEH] BFH *PK* (BHF *Ba* BEH *E*$_1$) **406** BL[2]] EL *P*$_7$ ad] *corr. ex* et *K* **407** hoc] *s.l. K* BHL] *corr. ex* BHK *M* Reliquus ergo] ergo reliquus *M* **407/408** notus quare] *s.l. P*$_7$ **407** notus[2]] *s.l. P* **409/410** vera – longiore] *om. PK* **409** vera epicicli] epicicli vera *N* **409/410** et[1] – quia] sed (longitudo Lune a loc- *add. et del.*) *N* **409** gradus – minuta] graduum et 47 minutis *M* according to the succession of signs. And when the sun's place is found according to computation from the radix, it was indeed Cancer 12° 5′ according to mean course and 10° 54′[22] according to truth. And the moon's place through the mean course of longitude was Leo 27° 20′. Therefore, the moon's distance according to its mean course from the sun's true place is 46° 26′,[23] and the moon's elongation on the epicycle from the mean apogee was according to the mean motion of irregularity 333° 12′.

With these things thus disposed, I will describe the lunar eccentric as before upon center D and an epicycle upon center B, with lines DBH[24] and ETBZ drawn. Therefore, because the duplex longitude is 90° 30′, obtuse angle AEB will be known. With perpendicular DK drawn upon line EB, therefore, the angle DEK, the supplement, will be known; and from this each of those DK and EK will be known to ED, and because of this, EB will be known.



Likewise, because the moon's elongation according to its mean course from the true place of the sun is less than its true elongation by 1° 40′,[25] when line EBZ designates the moon's place according to mean course, if we set up point H as the moon's place on the epicycle because it was near the epicycle's apogee and we draw line EH, angle BEH[26] will be known. And from this, with perpendicular BL drawn upon line EH, BL to EB will be known, and because of this, to BH. Therefore, angle BHL will be known. Therefore, the remainder HBZ is known.[27] Therefore, arc HZ of the epicycle is known, and that is the moon's elongation from the epicycle's true apogee; and it is 14° 47′.[28]

[22] This should be 10° 40′ to agree with the *Almagest*. The author apparently did not realize that Ptolemy is not giving here the true sun according to Hipparchus' observation, but the true sun according to his own computation, so our author replaces Ptolemy's value with the one observed by Hipparchus.

[23] This should be 46° 40′ to match the *Almagest*, but it is correct given the prior differing value.

[24] H is not on line DB. No witnesses have 'DB' which would be mathematically correct. The witnesses' figures have DBH drawn as one straight line.

[25] This should be 1° 26′ to match the *Almagest*, but it is correct given the other values given in the *Almagesti minor*.

[26] That *Ba*, *P*, and *K* have errors here shows that an error likely crept into the transmission early or that perhaps the original had a mistake here.

[27] This angle is known, not because it is a remainder, but because it is an external angle to triangle EBH that is equal to the two known interior angles.

[28] This should be 14° 43′ to match the *Almagest* V.5. The difference does not appear to be due to the changed value of the sun's true position because the result would be approximately

Item quia elongatio Lune a longitudine longiore equali secundum medium cursum diversitatis ccciii gradus et xii minuta, si nos posuerimus longitudinem longiorem equalem super punctum M, erit totus arcus MZH qui relinquitur ad perfectionem circuli xxvi gradus et xlviii minuta. Subtracto ergo arcu HZ erit ZM xii gradus et i minutum, ergo angulus ZBM atque etiam equalis ei EBS est notus, ducta videlicet recta MBSN. Quare ducta super eam perpendiculari ES erit proportio EB ad ES nota. Item quia angulus AEB erat notus et nunc notus est angulus EBN, erit propter hoc angulus SNE notus. Quapropter proportio EN ad SE et etiam ad EB et ad ED erit nota. Et secundum operationem premissorum fit EN x partium et xix minutorum fere, itaque ipsa est equalis ED. Palam ergo quod diameter epicicli transiens super longitudinem longiorem equalem que est punctum M dirigitur neque ad punctum E neque ad punctum D, sed ad punctum N quod est equalis distantie ab E cum puncto D. Manifestum quoque quod procedente centro epicicli ab A longitudine longiore ecentrici ad longitudinem propiorem, longitudo longior vera in epiciclo scilicet Z precedit longitudinem longiorem equalem. Ex pluribus quoque considerationibus similiter apparuit, nec inventa est fere ulla diversitas.

8. Centro epicicli aput quodlibet punctum ecentrici secundum notam elongationem ab eius longitudine longiore constituto, equationem portionis invenire et per eam portionem equatam reddere.

Describo ad hoc iterum ecentricum lunarem super centrum D et epiciclum note elongationis a puncto A quod est longitudo longior ecentrici super centrum B. Notus est ergo angulus AEB super quem fit elongatio ista. Quare et angulus reliquus de duobus rectis DEK notus, DK facta perpendiculari super EB. Similiter ergo premissis fiet EB nota respectu partium ED. Sumpta itaque EN equali linee ED et educta perpendiculari NS super BK, fient SE EK note. Erit ergo SB residua nota, et similiter SN nota cum sit equalis DK. Cum ergo angulus ad S sit rectus, erit angulus NBS notus cui equalis est angulus ZBM;

410 Item quia] itemque *M* Lune] Lune est P_7 **410/411** medium cursum] cursum medium P_7*M* **411** ccciii] fuit 333 *N* Si] si ergo *N* nos] *corr. ex* non *M* **414** ZM] *corr. ex* Z P_7 gradus] graduum *N* i minutum] l minuta *PM* i minuta *K* 50 minutorum *N* (i minutum *Ba* 50 minuta E_1) **415** est] *om. N* perpendiculari] *s.l. M* **416** ES^2] *corr. ex* †F† *M* Item quia] itemque *PN* nunc] modo *N* **417** notus est] est notus *PN* SNE] SEN P_7 **417/418** proportio – SE] BEG ad EN proportio *corr. in* EG ad EN proportio *corr. in* EN ad ES proportio *M* **418** erit nota] nota erit *N* **419** xix] *corr. ex* 30 *M* fere] fieri *P* **421/423** equalem – longiore] *om.* P_7 **422** ab] *corr. ex* AB *M* D^2] *corr. in* T *M* **423** centro] *corr. ex* dyametro *P* **424** longior] *om. N* **425** precedit] precedet *M* Ex] et P_7 **426** fere] *marg. P om. N* diversitas] diversitas et cetera *N* **427** Centro] *corr. ex* dentro P_7 **430** ad – iterum] etiam ad hoc P_7 ad hoc item *K* adhuc iterum circulum *M* centrum] punctum *M* **432** centrum] punctum *N* est ergo] ergo est P_7*N* elongatio] longatio *M* **433** DK] *s.l.* (*perhaps other hand*) *P* **434** ergo] ex *add.* (*s.l. M*) *MN* fiet] *corr. ex* fit *M* **435** fient] fit *P* fiunt *N* EK] *corr. ex* E†C† *K* **436** Erit] *om. N*

In turn, because the moon's elongation from mean apogee according to the mean course of irregularity is 303° 12′,[29] if we place the mean apogee upon point M, whole arc MZH that remains for the completion of a circle will be 26° 48′. Therefore, with arc HZ subtracted, ZM will be 12° 1′,[30] so angle ZBM, and also EBS equal to it, is known, i.e. with straight line MBSN drawn. Therefore, with perpendicular ES drawn upon it, the ratio of EB to ES will be known. Likewise, because angle AEB was known and now angle EBN is known, angle SNE will be known because of this. For this reason the ratio of EN to SE and also to EB and to ED will be known. And according to the operation of what has been set forth, EN will be approximately 10^p 19′,[31] and so it is equal to ED. Therefore, it is clear that the epicycle's diameter passing through the mean apogee, which is point M, is directed neither to point E nor to point D, but to point N, which is of equal distance from E as is point D. It is also manifest that with the epicycle's center proceeding from the eccentric's apogee A to the perigee, the true apogee on the epicycle, i.e. Z, precedes the mean apogee. It also appeared similarly from more observations, and scarcely any difference at all was found.

8. With the epicycle's center set up at any point of the eccentric according to a known elongation from its apogee, to find the equation of portion and to return the equated portion through it.

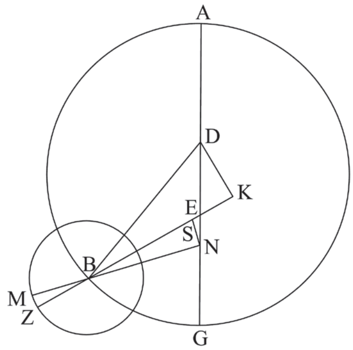
For this I describe again the lunar eccentric upon center D and upon center B the epicycle of known elongation from point A, which is the eccentric's apogee. Therefore, angle AEB, upon which that elongation is made, is known. Therefore, also the supplement angle DEK is known, with DK made perpendicular upon EB. Similarly to what has been set forth, therefore, EB will be known with respect to the parts of ED. Accordingly, with EN taken equal to line ED and with perpendicular NS drawn upon BK, SE and EK will be known. Therefore, remainder SB will be known, and similarly SN will be known because it is equal to DK. Therefore, because the angle at S is right, angle NBS will be known, to which angle ZBM is equal; therefore, arc ZM of the epicycle is

17° if the author carried out the operations with his different values. The source of the value given here is *Almagest* V.6 (1515 ed., f. 50v), in which Ptolemy computes the moon's position for the time of this example from Hipparchus.

29 This should be 333° 12′ to match the *Almagest*.

30 This should be 12° 5′ to match *Almagest* V.5. This mistaken value is taken from *Almagest* V.6 (1515 ed., f. 50v). In many witnesses, 50 is found instead, which is probably due to the Roman numeral 'i' being mistaken for 'l.'

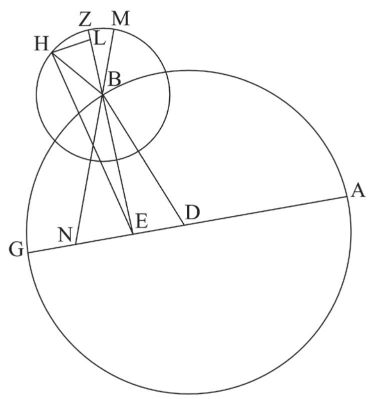
31 Ptolemy finds it to be 10^p 20′.



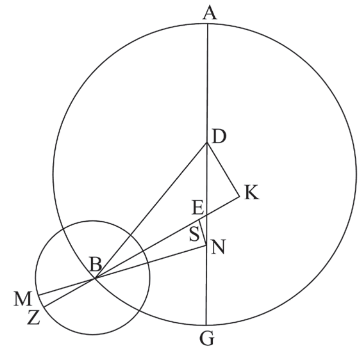
quare arcus epicicli ZM notus. Sed punctum M est longitudo longior equalis respiciens punctum N, et Z est longitudo longior vera respiciens ad punctum E. Et quia Z precedit M procedente epiciclo a longitudine longiore ecentrici ad propiorem a qua longitudine longiore incipit numeratio, quotiens longitudo duplex minor est semicirculo, addenda est hec equatio portionis vel puncti super portionem Lune cum motus Lune in superiori parte epicicli sit contra motum ecentrici. Et cum longitudo duplex est maior semicirculo, minuenda est ab ea. Et erit portio equata, et hoc erat propositum.

9. Verum locum Lune in circulo signorum ex mediis motibus positis in omni tempore presto est cognoscere.

Describam evidentie gratia ad hoc ecentricum Lune iterum super diametrum ADG ut prius. Sumptis itaque ad datum tempus motibus mediis scilicet motu longitudinis, motu diversitatis, media distantia Solis et Lune duplicata, equabimus Lunam sic. Sit enim longitudo duplex secundum elongationem DB linee ab A longitudine longiore ecentrici nota. Per hanc ergo fiat equatio portionis et portio equata nota. Et ponamus locum Lune in epiciclo ubilibet secundum medium motum diversitatis a longitudine longiore equali, que est punctum M, et sit locus Lune H. Erit ergo arcus ZH notus quia est portio equata; ergo et sinus eius HL notus, et propter hoc linea LB nota. Constituta itaque HB que est semidiameter epicicli v partium et xv minutorum, erit hoc quoque respectu utraque HL BL nota. Quapropter addita BL super BE eodem respectu nota erit tota EL sicut HL nota; quare et HE que subtenditur angulo recto erit nota. Facta igitur HE semidiametro fiet angulus HEL notus, et hic est angu-

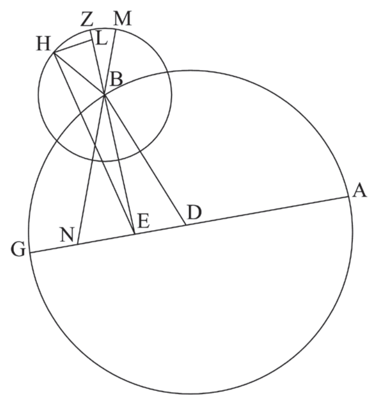


438 ZM] et M *P* **439** longior] ergo *add. et del. M* **442** procedente] precedente *P* **446** super] secunde *K* **449** maior semicirculo] semicirculo maior *N* erit] proveniet *N* portio] proportio P_7 **450** erat] est *N* **453** iterum] verum *M* **454** motibus mediis] mediis motibus *N* **455** longitudinis] *corr. ex* longiore P_7 **456** Lunam] lineam *P perhaps corr. ex* lineam *K* DB] EB P_7 **458** et[1]] qua *PN* portio] *corr. ex* portione *N* ubilibet] ubibet *M* **459** medium motum] motum medium P_7 medium cursum *N* **466** quoque] *om. N* HL] HL et *M* **468** nota[1] – tota] erit nota *PN* notam erit tota *K* **469** et HE] ZHE *M* **470** nota] *s.l. N* **471** HEL] *corr. ex* HEB *M* known. But point M is the mean apogee facing point N, and Z is the true apogee facing point E. And because Z precedes M with the epicycle proceeding from the eccentric's apogee to the perigee, from which apogee the numbering begins, whenever the duplex longitude is less than a semicircle, this equation of portion or point must be added to the moon's portion because the moon's motion on the upper part of the epicycle is against the eccentric's motion. And when the duplex longitude is greater than a semicircle, it must be subtracted from it. And there will be the equated portion, and this had been proposed.



9. To know the moon's true place in the ecliptic from the posited mean motions at any time is ⟨the task⟩ at hand.

For the sake of clarity for this, I will describe the moon's eccentric again upon diameter ADG as before. Accordingly, with the mean motions taken for this time, i.e. the motion of longitude, the motion of the irregularity, and the doubled mean distance between the sun and moon, we will correct the moon thus. Indeed, let the duplex longitude be known according to the elongation of line DB from the eccentric's apogee A. Through this, therefore, let the equation of portion and the equated portion be made known [through V.8]. And let us suppose the moon's place anywhere on the epicycle according to the mean motion of irregularity from the mean apogee, which is point M, and let the moon's place be H. Therefore, arc ZH will be known because it is the equated portion; therefore, its sine HL is also known, and line LB is known because of this. Accordingly, with HB, which is the epicycle's radius, set up as 5^{p} 15′, both HL and BL will be known also in this respect. For this reason, with BL added to BE, whole EL will be known in the same respect as HL is known; therefore, HE, which subtends a right angle, will also be known. Therefore, with HE made a radius, angle HEL will be known, and this is the angle of the dif-



lus differentie medii motus longitudinis ad diversum in situ provenientis. Hic itaque si portio equata scilicet HZ minor semicirculo, minui debet a medio motu longitudinis, et quod quo pervenerit numeratio ibi est verus locus Lune in circulo signorum.

Via vero operationis est hec. Ad tempus quantum volueris a radice sumptum, primum medium motum longitudinis quem seorsum scribes, et medium motum diversitatis similiter seorsum scribens, et mediam distantiam duplicans eam accipe, quam, si in tabulis non habueris, minue medium motum Solis de medio motu Lune et reliquum duplica. Quod duplicatum si minus semicirculo, per ipsum, si plus, per superfluum semicirculi ita operare.

Si arcus quem ita habueris minus quarta fuerit, sinum eius necnon sinum illius quod ei ad perfectionem quarte deficit accipe. Et utrumque per quantitatem distantie duorum centrorum scilicet x partes et xix minuta multiplica, et per lx partire; et quod ex utroque provenerit serva. Deinde semidiametrum ecentrici idest xlix partes et xli minuta in se multiplica, et ex eo quod provenerat ex sinu arcus quem ita habuisti in se multiplicatum deme, et super residui radicem quod provenerat ex sinu perfectionis adde. Et aggregatum serva. Nam ipsum est linea inter centrum orbis signorum et centrum epicicli EB.

Quod si arcus quem habueris plus quarta fuerit, sinum eius quod ei deest ad complementum duorum rectorum necnon et sinum perfectionis huius. Et utrumque ut prius in distantiam duorum centrorum multiplica, et per lx partire, et serva. Deinde ex semidiametro ecentrici in se ducto quod ex sinu complementi duorum rectorum provenerat in se ductum deme, et ex radice residui quod ex sinu perfectionis provenerat subtrahe. Et reliquum serva. Nam ipsum est linea EB.

Quod si arcus quem habueris quarta fuerit, ex semidiametro ecentrici in se multiplicato distantiam duorum centrorum in se ductam minue, quia radix residui erit linea EB, quam diligenter serva.

472 situ] hoc situ P_7M situ tali *N* provenientis] *corr. ex* provenientes *K* Hic] hec *MN* **473** itaque] *corr. ex* ita qua *K* semicirculo] est semicirculo P_7M semicirculo fuerit *N* **473/474** medio – longitudinis] motu longitudinis medio *N* **474** quod] *del. K om. MN* (quod BaE_1) pervenerit] nunc perveniet *N* **476** est hec] hec est P_7 **478** seorsum scribens] seorsum scribes *PM* deorsum scribens P_7 seorsum *N* (seorsum scribes *Ba* seorsum scribens E_1) duplicans] duplans *M* **479** de] a *N* **480** si] si est *N* **481** plus] autem plus semicirculo *N* ita] *om. N* **482** minus quarta] quarta minus P_7 minor quarta *N* necnon] necnon et P_7K sinum[2]] finitus *M* **483** quod] qui *PN* deficit] defecit *M* **485** per] hoc *add. et del. P* provenerit] proveniet *N* **486** idest] in *M perhaps del. N* xli] 40 *N* in – multiplica] multiplica in se *corr. ex* multiplica *N* provenerat] provenerit *M* **488** radicem] illud *add.* (*s.l. K*) *KM* provenerat] provenerit *M* ex sinu] *s.l. K* **489** EB] scilicet EB *M* **490** plus] *corr. ex* erit *P* **491** rectorum] *s.l. K* huius] eius P_7 huius accipe *N* utrumque] tempus *add. et del. P* **492** distantiam – centrorum] duorum centrorum distantiam *N* **493** ecentrici] *om. N* **499** quam] quod P_7

ference of the mean motion of longitude from the irregular ⟨longitude⟩ resulting at the location. Accordingly, if the equated portion, i.e. HZ,[32] is less than a semicircle, this ought to be subtracted from the mean motion of longitude, and that place which the calculation reaches is the moon's true place in the ecliptic.

And indeed, the way of operation is this. For a time taken as far from the radix as you want, take first the mean motion of longitude, which you write separately, ⟨secondly⟩ the mean motion of irregularity, likewise writing separately, and ⟨thirdly⟩ the mean distance, doubling it. If you do not have this in tables,[33] subtract the sun's mean motion from the moon's mean motion and double the remainder. If this doubled ⟨quantity⟩ is less than a semicircle, operate thus through it; if more, through the excess of a semicircle.[34]

If the arc that you thus have is less than a quarter circle, take its sine as well as the sine of its complement. And multiply each by the quantity of the eccentricity, i.e. 10^p 19′, and divide by 60; and save what results from each. Then multiply the eccentric's radius, i.e. 49^p 41′, by itself, and from it subtract that which resulted from the sine of the arc that you had thus multiplied by itself, and to the root of the remainder, add that which resulted from the sine of the complement. And save the sum. For it is EB, the line between the ecliptic's center and the epicycle's center.

But if the arc that you have is more than a quarter circle, ⟨take⟩[35] the sine of its supplement as well as the sine of the complement of this. And multiply each as before by the eccentricity, divide by 60, and save. Then from the eccentric's radius multiplied by itself, subtract what resulted from the sine of the supplement multiplied by itself, and from the root of the remainder, subtract that which resulted from the sine of the complement. And save the remainder. For it is line EB.

But if the arc that you have is a quarter circle, from the eccentric's radius multiplied by itself, subtract the eccentricity multiplied by itself, because the root of the remainder will be line EB, which you carefully save.

32 This refers to the arc taken clockwise from Z to H.

33 Ptolemy does not include a table of lunar elongations. Such a table is found among the Toledan Tables in a small number of manuscripts (Pedersen, *The Toledan Tables*, Tables CH, pp. 1219–21).

34 As in III.17, the 'excess of a semicircle' refers not to the supplement, but to 360° minus the arc.

35 A verb such as 'sume' or 'accipe' is understood here. The verb in *N* is surely the scribe's emendation.

Quod si arcus quem habuisti minus quarta fuerit, quod ex reductione utriusque sinus provenerat scilicet ipsius quarta minoris arcus et eius quod ei ad perfectionem deerat accipe, et unum scilicet perfectionis super lineam EB pone. Et quadrati totius cum quadrato reliqui radicem elice. Cumque ipsum reliquum in lx duxeris, quod exierit per hanc radicem divide. Et quod tandem provenerit arcua. Nam iste arcus est equatio portionis vel puncti.

Quod si plus quarta fuerit, per id quod ex sinu complementi duorum rectorum et sinu perfectionis eius provenerat, cum id quod ex sinu perfectionis erat a linea EB subtraxeris, similiter operare.

Quod si quarta fuerit, distantiam duorum centrorum in se ductam linee EB in se ducte superpone, et radicem elice. Cumque distantiam in lx multiplicaveris, per hanc radicem divide et arcua.

Habita itaque equatione portionis, si longitudo minor semicirculo fuerit, adde, si maior, minue a medio motu diversitatis. Et erit portio equata. Hec igitur portio si minor semicirculo, per ipsam, si maior, per superfluum semicirculi ita operare. Si arcus quem ita habueris minor quarta fuerit, sinum eius necnon et sinum illius qui ei ad perfectionem quarte deficit per quantitatem semidiametri epicicli scilicet v partes et xv minuta multiplica, et utrumque productum per lx partire. Quodque exierit ex divisione sinus perfectionis quantitati linee EB superadde. Et totum in se multiplica, et super quod fuerit illud quod ex divisione sinus habiti arcus provenerat in se multiplicatum adde. Collectique radicem quere, et serva. Post hec ad id quod ex divisione sinus habiti arcus productum fuerat rediens, ipsum in lx multiplica, et productum per servatam radicem partire.

Quod si arcus quem habueris quarta fuerit, tunc lineam EB in se multiplicatam semidiametro epicicli qui est v partium et xv minutorum in se ducto

500 minus] minor *N* **501** quod] qui *PN* **503** elice] elicere *P corr. ex* elicere *K* **504/505** tandem provenerit] tandem provenerat *P* provenerit tandem *M* tandem proveniet *N* **506** id] illud *M* **507** cum] eum *P* id] illud *M* **509** fuerit] fuerit per *M* linee] *corr. ex* lineam P_7 EB] iunge *add. et del. N* **510** superpone] suppone *P* multiplicaveris] *corr. ex* duxeris *M* **511** et] et exiens *N* **512** equatione portionis] portionis (*corr. ex* portiones *P*) equatione *PN* **513** maior] *corr. ex* minor *K* medio motu] motu medio *P* **514** semicirculo] semicirculo fuerit *N* **515** sinum eius] eius sinum *KM* **516** qui] quod *N* quarte deficit] deficit quarte *P* **517** epicicli] *s.l. M* utrumque] utrimque P_7 **518** per] in *M* divisione] ductu *M* **519** super – quod^2] et quod superfuerit illud quod (*om. P*) *PM* et quod superfuerat scilicet *N* (et quod superfuerit illud quod *Ba text confirmed by* E_1) **520** divisione] ductu *corr. in* ductione *M* habiti arcus] arcus accepti in semidyametrum epicicli multiplicati per 60 *N* provenerat] *corr. ex* provenerit *M* **521** radicem] radice *P* hec] hoc *MN* quod] *om. N* divisione] *corr. ex* ductione *M* **522** productum fuerat] provenit *N* fuerat] *corr. ex* fuerit P_7 rediens] redigens *M* **524** multiplicatam] *corr. ex* multiplicam P_7 **525** qui] que *N* partium – minutorum] partes et 15 minuta *N* ducto] ducte *N*

And if the arc that you had is less than a quarter circle, take what resulted from the reduction of each sine, i.e. of that arc less than a quarter circle and its complement [i.e. what resulted when we multiplied these sines by the eccentricity and divided by 60], and add one, i.e. of the complement to line EB. And extract the root ⟨of the sum⟩ of the square of the whole with the square of the remaining one [i.e. the 'reduction' of the sine of the duplex longitude]. And when you multiply that remainder [i.e. the 'reduction' of the sine of the duplex longitude] by 60, divide what results by this root. And arc what finally results. For that arc is the equation of portion or point.

But if it is more than a quarter circle, operate similarly through those that resulted from the sine of the supplement and the sine of its complement, when you have subtracted that which was from the sine of the complement from line EB.

But if it is a quarter circle, add the eccentricity multiplied by itself to line EB multiplied by itself, and extract the root. And when you have multiplied the distance [i.e. the eccentricity] by 60, divide by this root, and arc ⟨the result⟩.

Accordingly, with the equation of portion held, if the longitude is less than a semicircle, add it, and if greater, subtract it from the mean motion of irregularity. And there will be the equated portion. Therefore, if this portion is less than a semicircle, operate thus through itself, and if greater, through the excess of a semicircle.[36] If the arc that you have thus is less than a quarter circle, multiply its sine as well as the sine of its complement by the quantity of the epicycle's radius, i.e. $5^p\ 15'$, and divide each product by 60. And add what results from the division of the sine of the complement to the quantity of line EB. And multiply the whole by itself, and to what that is,[37] add what resulted from the division of the sine of the considered arc multiplied by itself. And seek the root of the sum, and save it. Afterwards, returning to that which had been produced from the division of the sine of the considered arc, multiply it by 60, and divide the product by the saved root.

But if the arc that you have is a quarter circle, then add line EB multiplied by itself to the epicycle's radius, which is $5^p\ 15'$, multiplied by itself, and extract

[36] I.e. 360 – equated portion.

[37] The awkward wording here, 'et super quod fuerit illud quod', is copied from Albategni's 'et super quod fuerit id quod' (*De scientia astrorum*, 1537 ed., f. 48v).

superadde, et collecti radicem elice et serva. Post hec v partes et xv minuta in lx multiplica, et per servatam radicem divide.

Quod si arcus quem habueris plus quarta fuerit, ab eo quarta subtracta. Residui sinum eiusque quod ei ad perfectionem quarte deficit per v partes et xv minuta multiplica, et per semidiametrum idest lx partire. Quodque ex sinu perfectionis provenerit a quantitate linee EB minue, et reliquum in se ipsum multiplica. Et ei quod ex sinu residui arcus provenerat in se multiplicato superadde, collectique radicem serva. Post hec ad id quod ex sinu arcus residui provenerat rediens, id in lx multiplica et per servatam radicem divide.

Et quodcumque ex uno istorum trium modorum exierit arcua. Nam arcus qui prodierit est differentia motus medii ad motum apparentem. Et si portio equata minus sex signis fuerit, minuitur a medio. Si plus, additur super medium cursum Lune. Et quo pervenerit numeratio ibi erit verus locus Lune.

Artificium vero tabularum equationis Lune sic disponitur. Primum in tabula prima disponuntur numeri communes mediorum motuum ut portionis equate, longitudinis duplicis, motus latitudinis, per quos intratur in tabulas equationum. Deinde in secunda quia portio primum equanda est per longitudinem duplicem, recte ordinatur tabula continens equationem portionis que alias equatio puncti nominatur sicut ex octava presentis elicitur. Iuxta hanc bene ponitur tabula minutorum proportionalium quia in eam quoque per longitudinem duplicem intratur. Et hec minuta proportionalia sunt superfluitates maximarum differentiarum secunde diversitatis super maximam prime diversitatis gradatim collecte centro epicicli a longitudine longiore usque ad longitudinem propiorem procedente sicut in sexta presentis habetur. Nam superfluitas maxime differentie aput longitudinem propiorem proveniens lx minutorum ponitur. Et relique superfluitates in longitudinem longiorem et propiorem accidentes – de maximis semper dico – ad lx sub proportione conferuntur, et quod provenerit in hac tabula minutorum ordinatur. Post has due tabule propioris et longioris lon-

526 hec] hoc *MN* **527** et] et productum *N* **528** plus] maior *N* eo] ea *M* subtracta] fuerit *add. et del. P* **529** Residui] *om. M* perfectionem] perfectionem seu completionem *M* completionem *N* **530** semidiametrum idest] diametrum in P_7 semidyametrum idest per *M* **531** provenerit] provenerit et *M* **532** residui arcus] arcus residui *M* **533** hec] hoc *MN* ad id] *om. P corr. ex* ad idem *M s.l. N* arcus residui] residui arcus *N* **534** rediens] redigens *M* id] idem *N* et] et productum *N* **536** prodierit] prodibit *N* est] erit P_7 **537** equata minus] equato minor *N* sex] ex *P* **538** cursum Lune] Lune cursum *PN* pervenerit] provenerit *M* perveniet *N* **539** tabularum] *corr. ex* stabularum *K* **541** duplicis] duplices *P* equationum] *corr. ex* equationis P_7 **542** equanda est] *corr. ex* equant†...† *K* **544** bene] *om. PN* **545** eam] ea *K* **548** collecte] collecte a *M* **551** in] inter P_7*M* (in *Ba* inter E_1) longitudinem – propiorem] longitudine longiori *N* accidentes] accidentes et *M* **552** provenerit] provenit *N* **553** ordinatur] ordinantur P_7 ponitur *N* propioris – longitudinis] longitudinis longioris et propioris longitudinis *N* propioris] *corr. ex* prioris P_7

the root of the sum and save it. Afterwards, multiply 5^p 15′ by 60, and divide by the saved root.

But if the arc that you have is more than a quarter circle, subtract a quarter circle from it. Multiply the sine of the remainder and its complement by 5^p 15′, and divide by the radius, i.e. 60. And subtract what results from the sine of the complement [i.e. BM] from the quantity of line EB, and multiply the remainder by itself. And add to it that which resulted from the sine of the remaining arc multiplied by itself, and save the root of the sum. Afterwards, returning to that which resulted from the sine of the remaining arc, multiply it by 60 and divide by the saved root.

And arc whatever results from one of those three ways. For the arc that results is the difference between the mean motion and the apparent motion. And if the equated portion is less than six signs, it is subtracted from the mean ⟨motion⟩. If more, it is added upon the moon's mean course. And the place to which the calculation comes will be the moon's true place.

And indeed, the crafting of the tables of the moon's equation is set out thus. First, the common numbers of the mean motions, i.e. the equated portion, the duplex longitude, and the motion of latitude, are set out in the first column, through which the columns of equations are entered. Then in the second ⟨column⟩, because the portion must first be equated through the duplex longitude, a table is rightly put in order containing the equation of portion, which elsewhere is called the equation of point, as is extracted from the eighth of the present ⟨book⟩. Next to this, the column of proportional minutes is well placed because it is also entered with the duplex longitude. And these proportional minutes are the excesses of the greatest differences of the second irregularity over the greatest of the first irregularity, collected degree by degree, with the epicycle's center proceeding from the apogee to the perigee, as is had in the 6th of the present. For the excess of the greatest difference resulting at perigee is supposed 60′. And the remaining excesses occurring between the apogee and perigee – I always speak about the greatest ⟨differences⟩ – are compared under a ratio to 60, and what results is put in order in this column of minutes. After these, the two columns of the perigee and apogee are added. Of these the one gitudinis iunguntur, quarum illa que est longitudinis longioris continet omnes differentias integraliter prime diversitatis gradatim sicut in equatione Solis collectas. Et inscribitur simplex equatio vel singularis, alias coequatio partis Lune. In illa vero que longitudinis propioris est tabula, ponuntur superfluitates sicut sunt omnium differentiarum secunde diversitatis in longitudine propiore super singulas differentias prime diversitatis, cum utrobique differentie de gradu in gradum collecte fuerint et ille ab hiis subtracte. Et intitulatur hec tabula superfluitates longitudinis propioris vel longitudo propior, alias equatio diversitatis. In septima vero tabula digeruntur latitudines Lune eo modo quo declinationes Solis cum maxima latitudo per instrumentum deprehensa sit sicut ostendetur.

Cum ergo centrum epicicli fuerit in longitudine longiore ecentrici quod contingit in mediis coniunctionibus Solis et Lune vel mediis oppositionibus, tunc quidem portio equanda non est, nam ipsa longitudo longior equalis epicicli est longitudo longior vera, sed utendum simplici equatione tantum sicut in Sole. Cum autem centrum epicicli fuerit in longitudine propiore ecentrici, tunc quoque portio equanda non est propter eandem rationem. Sed intrandum cum ipsa portione sicuti est in duas tabulas longioris et propioris longitudinis, et quod in propiori inventum fuerit integre addendum est super id quod in longiori occurrit, eo quod hec coniuncta faciunt differentiam propioris longitudinis. Cum vero centrum epicicli in aliis locis ab hiis fuerit, quod totum cognoscitur per longitudinem duplicem, tunc quidem portio equanda est per longitudinem duplicem. Et cum eadem intrandum in minuta proportionalia, et servandum quod in directo inventum fuerit. Nam ipsum est maxime differentie que ibi contingere potest superfluitas. Deinde cum portione equata intrandum in tabula propioris longitudinis, et id quod ibi inventum fuerit non totum sumendum est, sed de eo tantum quantum minuta proportionalia que tibi occurrerunt sunt de lx. Et id addendum est super id quod in tabula longitudinis longioris in directo portionis equate invenietur, quia prope verum sicut superfluitas maxime differentie alterius loci ad superfluitatem maxime differen-

554 illa] illa est *P* longitudinis longioris] longitudo longior *M* **555** gradatim] graduatim *N* sicut] fient *P* equatione] equatore *P corr. ex* equatore *N* (equatione *Ba*E_1) **556** singularis – coequatio] singulis alias equatio *M* **557** ponuntur] ponitur P_7 **561** propioris] *corr. ex* propiore *K* **562** digeruntur] diriguntur *M* **564** longiore] *corr. ex* propiore *M* **566** portio] proportio *M* **566/567** equalis – longior] *om.* P_7 **566** equalis epicicli] epicicli equalis *M* **567** utendum] utendum est *MN* **569** portio] *corr. ex* proportio *M* intrandum] intrandum est *N* **570** sicuti] sicut ipsa *N* **571** et] *om. P* id] illud *MN* **572** longiori] longitudine *M* longitudine longiori *N* **573** vero] ergo P_7 **575** intrandum] intrandum est *M* etiam intrandum *N* proportionalia] proposita P_7 **576** inventum fuerit] invenitur *N* **578** intrandum] intrandum est *N* in tabula] in tabulam P_7 est in tabulam *N* id] illud *N* **580** occurrerunt] occurrunt *KN* occurrent *M* (occurrunt *Ba* occurrent E_1) de lx] 560 P_7 dlx *K* (de 60 *Ba* dlx E_1) Et] *om. N* **581** invenietur] invenientur *P* invenitur *N* sicut] sit *N* **582** alterius – differentie[2]] *om. M* differentie[2]] alterius *add. et del. K* of the apogee contains all the differences completely of the first irregularity collected degree by degree as for the sun's equation. And it is entitled 'the simple or singular equation', or elsewhere 'the coequation of the part of the moon.' And indeed, in that column of the perigee, there are placed the excesses, as they are, of all the differences at the perigee of the second irregularity upon the individual differences of the first irregularity, because in both instances the differences are obtained from degree to degree ⟨on the epicycle⟩ and those are subtracted from these [i.e. the difference from mean motion found at apogee is subtracted from that found at perigee]. And this column is entitled 'the excesses of perigee', 'perigee', or elsewhere 'the equation of irregularity.' And indeed, the moon's latitudes are laid out in the seventh column[38] in the way in which the sun's declinations were, when the greatest latitude has been found through an instrument as will be shown [in V.11].

Therefore, when the epicycle's center is at the eccentric's apogee, which occurs at mean conjunctions or mean oppositions of the sun and moon, then indeed the portion does not need to be equated – for that mean apogee of the epicycle is the true apogee, and also only the simple equation needs to be used, as with the sun. Moreover, when the epicycle's center is in the eccentric's perigee, then also the portion does not need to be equated because of the same reason. But the two columns of apogee and perigee must be entered with this portion as it is, and what is found in the nearer should be completely added upon that which occurs in the further, because these conjoined make the perigee's difference [i.e. equation of anomaly]. However, when the epicycle's center is in places other than these, which all is known through the duplex longitude, then indeed the portion must be equated through the duplex longitude. And the proportional minutes must be entered with the same, and what is found in the line must be saved. For that is the excess of the greatest difference that is able to occur there. Then the column of the perigee must be entered with the equated portion, and that which is found there is not taken whole, but only as much of it as the proportional minutes that occurred for you are of 60. And that must be added upon that which will be found in the table of the apogee in line with the equated portion, because approximately as is the excess of the greatest difference of some other place to the excess of the greatest difference

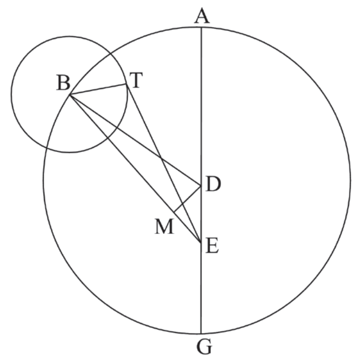
[38] He does not list a sixth column. The same strange numbering of the columns is found in Albategni, *De scientia astrorum* Ch. 30 and Ch. 36 (1537 ed., ff. 33v–35v and 47r). The cause may be that Albategni includes the sun's equation in the same table after the common numbers. Thus when he talks about columns 2–5 regarding the moon, he is actually referring to the columns 3–6 of his tables.

tie longitudinis propioris ita relique superfluitates illius alterius loci ad reliquas longitudinis propioris ordine eodem sumpte.

Sub hoc autem compendio tabule iste ita constitute sunt ne si ad singulos gradus inter longitudinem longiorem et longitudinem propiorem ecentrici differentias omnes quis velit colligere que singule ad singulos gradus variantur, nimis in immensum tenderentur tabule. Nam c et lxxx oporteret constitui tabulas singulas c et lxxx scalas continentes.

10. Superfluitatem secunde diversitatis que maxima accidere potest ab applicationibus Solis et Lune media ad veram modice quantitatis esse, verum equationis portionis non semper postponendam esse convincitur.

Quoniam non est necesse ut media coniunctio vel oppositio sit etiam vera, in mediis autem necessario nulla est secunda diversitas, nichil impedit quin in veris aliqua etsi modica proveniat secunda diversitas. Nam ad plus duorum minutorum erit. Et ponam ad hoc ecentricum Lune ABG supra centrum D et E centrum orbis signorum, et separabo arcum AB a longitudine longiore A. Et lineabo super centrum B epiciclum, et ducam lineas BE BD. Et ponam veram applicationem esse Solis et Lune. Maxima itaque diversitas secunda que sic provenire potest Luna existente super contingentem sui epicicli et Sole similiter super lineam contingentem sui epicicli. Et alterius equatio addetur super medium cursum, alterius minuetur; et erit media distantia quod aggregabitur ex duabus equationibus. Sit enim locus Lune super contingentem in puncto T, et longitudo duplex sit ex duabus equationibus Solis et Lune maximis aggregatis et duplicatis. Et est xiiii gradus et xlvii minuta secundum opus Ptolomei. Erit ergo angulus AEB notus. Via ergo sexte propositionis presentis erit angulus BET notus, et provenit secundum operationem v

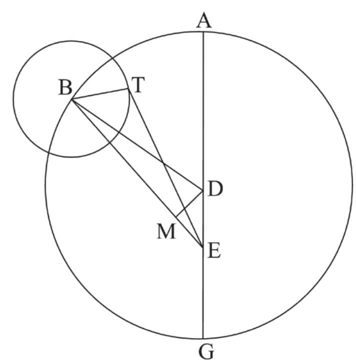


584 sumpte] *corr. ex* sumpto *K* **585** ita] *s.l.* (*perhaps other hand*) *P* si] quis *add. marg.* *N* **586** gradus] qui sunt *add.* P_7M **587** quis – que] quiveris colligere quod si *M* quis] *om.* *N* **588** immensum] intensum P_7 tenderentur] reddeuntur *M* tendantur *N* c] centrum *P* **590** Superfluitatem] *corr. ex* quantitatis esse verum superfluitatem P_7 maxima] maxime *N* applicationibus] applicatione *M* **591** quantitatis – verum] *om.* P_7 equationis] equationem P_7MN (equationis *Ba* equationem E_1) **592** convincitur] *corr. ex* convincimur *K* **596** E] est *P* **599** B] D *P corr. ex* D *K* (D BaE_1) **600** BD] HD *P* **601** itaque] *om.* P_7 **602** potest] potest erit *M* **604** super lineam] super super *N* **605** super] secundum P_7 **606** alterius] alterius vero *N* **607** aggregabitur] aggregatur *N* **609** longitudo] *corr. ex* longitudu- *K* **610** xiiii] 15 P_7 gradus – minuta] graduum et 47 minutis *M* **611** Ptolomei] Tholomei P_7 Tolomei *K* of the perigee, so are the remaining excesses of that other place to the remaining ⟨excesses⟩ of the perigee taken in the same order.

Moreover, these columns are thus set up under this abridgement so that the tables would not be extended much too far if someone wanted to gather all the differences, which each change for each degree, for each degree between the apogee and perigee of the eccentric. For it would be necessary to set up 180 columns, each containing 180 rungs.

10. It is established that from a mean syzygy[39] of the sun and moon to the true, the greatest excess of the second irregularity that is able to occur is of a modest quantity, but ⟨the excess⟩ of the equation of portion should never be disregarded.

Because it is not necessary that a mean conjunction or opposition always be the true one, while there is necessarily no second irregularity at the mean ⟨syzygies⟩, in fact nothing prevents some second irregularity from resulting at the true ⟨syzygies⟩, albeit small. For it will be 2′ at most. And I will suppose for this the moon's eccentric ABG upon center D and E the center of the ecliptic, and I will cut off arc AB from apogee A. And I will draw the epicycle upon center B,[40] and I will draw lines BE and BD. And I will posit that it is a true syzygy of the sun and moon. Accordingly, there is the greatest second irregularity that is able to come forth thus with the moon existing upon the tangent to its epicycle and with the sun similarly upon the line tangent to its epicycle.[41] And the equation of the one will be added upon the mean course, and ⟨the equation⟩ of the other will be subtracted; and the mean distance will be what is combined from the two equations. For let the moon's place be upon the tangent at point T, and let the duplex longitude be combined from the greatest two equations of the sun and moon and doubled. And it is 14° 47′[42] according to the work of Ptolemy. Therefore, angle AEB will be known. By the way of the sixth proposition of the present, therefore, angle BET will be known, and according to operation it comes forth as 5° 3′



[39] The author follows Gerard of Cremona (*Almagest*, 1515 ed., f. 52r) in using 'applicatio' to mean syzygy.

[40] The mistaken reading 'D' must have entered the transmission early or often.

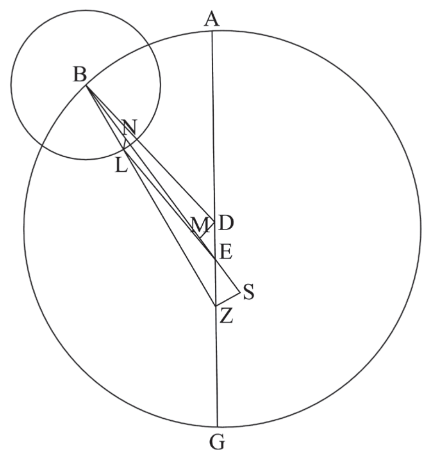
[41] It is interesting that our author preferred here to refer to the sun's epicyclic model, according to Albategni's practice, rather than to the eccentric model, as is Ptolemy's practice.

[42] This should be 14° 48′ to match the *Almagest*.

graduum et trium minutorum loco v graduum et unius minuti, que est maxima equatio in puncto A. Fit ergo superfluitas duo minuta tantum, quod non pervenit ad hoc ut sit medietas octave partis unius hore in motu Lune.

Item in applicationibus mediis nulla est equatio portionis, at in applicationibus veris nichil prohibet esse et eam non esse postponendam in investigatione vere applicationis per mediam. Nam pretermissa inducere potest errorem circa motum Lune ad applicationem veram in octava parte unius hore quandoque, quandoque etiam in quarta parte unius hore fere.

Et tunc quidem in octava cum Luna quidem fuerit aput longitudinem longiorem vel propiorem epicicli equalem, et tunc quidem nulla erit diversitas prima et media distantia Solis et Lune erit equatio Solis tantum. Reposito itaque ecentrico cum epiciclo et ductis lineis BD BS BZ, pono locum Lune coniuncte aput propiorem longitudinem equalem punctum L, et ducam lineam EL et perpendicularem LN super EB. Est ergo angulus LBN equatio quam querimus portionis, et angulus LEN differentia motuum propter ipsam eveniens. Quia ergo media distantia est equatio Solis que cum maxima sumpta fuerit et duplicata, erit AEB notus; et propter hoc linea SB nota; et propter hoc angulus SBZ notus;

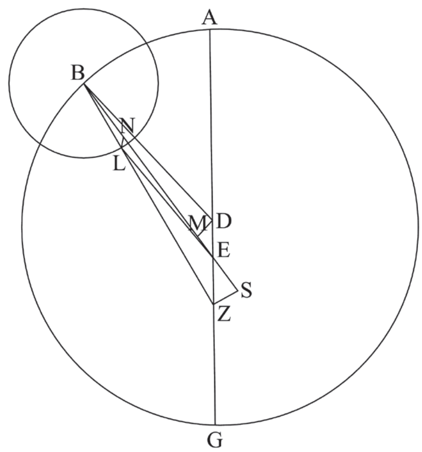


et propter hoc quoque proportio BL ad LN et ad BN nota; et propter hoc quoque proportio LE ad LN nota. Quapropter angulus LEN notus. Et accidit secundum operationem predictorum iiii minutorum fere, et illud cuius pretermissio in motu Lune potest inducere errorem in octava parte unius hore donec comprehendat Solem.

613 loco – graduum[2]] 5 graduum loco P_7 **614** Fit] sit *PKM corr. ex* fuit P_7 sic *N* (sic *Ba* fit E_1) ergo] enim *K* tantum] tantum erit *N* quod] qui *P* que *N* (quod BaE_1) pervenit] provenit P_7 **615** ad hoc] adhuc *M* **616** Item in] iterum *M* applicationibus[1]] mediis verus nihil prohibet esse *add. et del. N* at] AT *P* **617** eam] tamen *N* in investigatione] in vestigatione *P* investigatione *K* **618** Nam] non *M* **619** quandoque] *om. PN* **621** tunc quidem] quidem tunc P_7 cum] est *P* quidem[2]] *om. N* **622** epicicli equalem] equalem epicicli *N* erit] *iter. P* **623** Lune] Lune et *N* tantum] iterum *P* **625** BS] DS *P* pono] pone *M* **628** ducam] perducam *M* **629** LN] LI *N* **630** quam – portionis] *corr. in* portionis quam querimus *N* **632** motuum propter] motum per P_7 ipsam] ipsum *K* **634/635** sumpta fuerit] fuerit sumpta *M* **635** AEB] angulus AEB P_7M AEB angulus *N* (AEB *Ba* angulus AEB E_1) **636** linea] *s.l. P* **637** SBZ] *corr. in* SBN *N* **638** et[1]] *om. N* nota] nota est *M* **640** illud] est *add. s.l. N* pretermissio] premissio *P* omissio *N* instead of 5° 1′, which is the greatest equation at point A. Therefore, the excess is only 2′, which does not come to this that it is 1/16 of an hour in the moon's motion.

Likewise, there is no equation of portion at the mean syzygies, but nothing prevents it from existing at the true syzygies and it should not be disregarded in the investigation of a true syzygy through the mean ⟨syzygy⟩. For its neglect is able to introduce an error in the moon's motion for the true syzygy sometimes of 1/8 hour and sometimes even of about 1/4 hour.

And indeed, it is 1/8 ⟨hour⟩ at that time when the moon indeed is at the epicycle's mean apogee or perigee, and indeed at that time there will be no first irregularity and the mean distance between the sun and moon will be the sun's equation only. Accordingly, with the eccentric and the epicycle supposed again and with lines BD, BS, and BZ drawn, I suppose the place of the moon conjoined ⟨with the sun⟩ to be at point L, the mean perigee, and I will draw line EL and perpendicular LN upon EB. Therefore, angle LBN is the equation of portion that we seek, and angle LEN is the difference of motions that results because of it. Therefore, because the mean distance ⟨between the sun and moon⟩ is double the sun's equation taken when it is greatest, AEB will be known; and because of this, line SB is known; and because of this, angle SBZ is known; and because of this, the ratios of BL to LN and to BN are also known; and because of this, the ratio of LE to LN is also known. For this reason, angle LEN is known. And according to the operation of what has been said, it happens to be about 4′, and the neglect of this[43] in the motion of the moon is able to introduce an error of 1/8 hour while it catches up to the sun.[44]



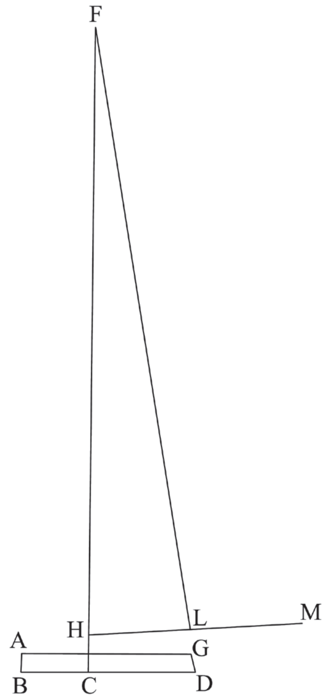
[43] The Latin is awkward here. 'Illud' appears to refer to the 4′, but it is singular. Also, the author probably intends the reader to understand an implicit 'est.'

[44] The value is slightly less than 1/8 hour, and Ptolemy's conclusion is that the difference does not even amount to 1/8 hour and can thus be ignored (*Almagest*, 1515 ed., f. 53r).

At in quarta hore errorem inducere potest quando equatio Lune trium graduum esse debet et equatio Solis ii ut sit media distantia v graduum. Nam tunc simplex portio Lune erit xl gradus et equatio portionis unus gradus et dimidius, quare portio equata xli gradus et dimidius. At si cum portione simplici rectifices Lune locum et deinde cum portione equata, occurret tibi in differentia duorum locorum octava unius gradus quod in motu Lune quartam partem hore fere continet.

11. Latitudo Lune maxima qualiter per instrumentum deprehendi potuit patefacere.

Queruntur ergo tres regule recte et planissime quadrilaterarum superficierum. Et habeant in longitudine circiter cubitos iiii; eius vero grossitiei sint ut fortes et rigide permanere possint. Et in dimidio latitudinis cuiusque recta ducitur linea quas hic in figura representant lineas scilicet FH FL HM. Una itaque trium regularum que fortior est basi quam hic representat ABGD firmissime infigatur, cuius basis una superficies sit plana ut linea HF in ipsa produci possit usque ad C. In alia vero regula due pinne equales et omnino similes aptentur ita ut earum linee medie erecte super lineam mediam FL – una quidem iuxta unam extremitatem et altera iuxta alteram. In duabus autem pinnis duo sunt orbicularia foramina parva super lineas medias ad eandem distantiam facta. Et sit quod oculo aspicientis apponetur minus, alterum aliquantulum maius, ut per ipsum tota Luna fere apparere possit aspicienti per utrumque foramen. Deinde has duas regulas axe rotundo et equali firmiter connectes ita ut

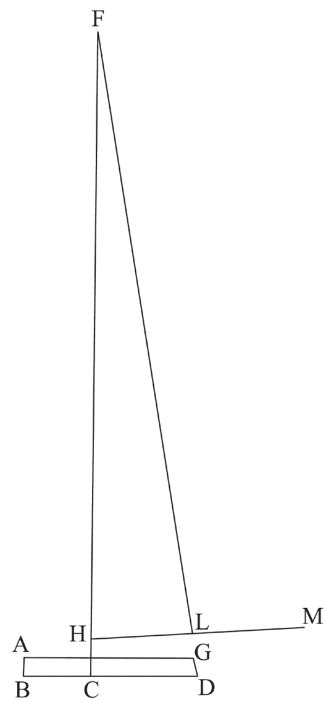


643 errorem – potest] inducere potest errorem *N* **644** sit] si *M* **645** gradus^1] graduum *K* unus] unius *KM* dimidius] *corr. ex* dimidium *K corr. in* dimidii *M* **647** in] etiam *PN* (in BaE_1) **648** quod] qui *PM* que *N* (quod BaE_1) **648/649** partem hore] unius hore partem *M* hore unius *N* **652** regule recte] recte linee (*del.*) regule *P* et] *om. N* superficierum] *corr. ex* figurarum *N* **654** cubitos iiii] 4 cubitos *M* vero] quoque *M* **656** ducitur] *iter. P* ducatur *N* **657** figura] figuras *K* lineas scilicet] linee *N* **659** hic] *corr. ex* habet *M* **662/663** et omnino] omnino *corr. ex* adeo *M* **665** altera] alia *N* **666** alteram] aliam *M* **667** sunt] sint *K* fiant *N* foramina] foramia *P* **670** aliquantulum] aliquantum P_7 maius] visus *P* maius ita P_7 enim maius *M corr. ex* visus *N* **671** aspicienti] fere *add. et del. K* **673** connectes] connectos *P* ut] quod *PN*

But it is able to introduce an error of 1/4 hour when the moon's equation ought to be 3° and the sun's equation is 2°, so that the mean distance is 5°. For then the moon's simple portion will be 40° and the equation of portion 1° 30′, so the equated portion will be 41° 30′. But if you correct the moon's place with the simple portion, and then with the equated portion, there will occur for you 1/8 of a degree of difference between the two places, which comprises about 1/4 hour in the moon's motion.

11. To reveal how the moon's greatest latitude could be found through an instrument.

Now, three straight and very even rules with rectangular surfaces are sought. And let them be about 4 cubits in length, but let them be of such a thickness that they are able to remain strong and rigid. And in the half of the width of each, a straight line is drawn, which lines are depicted here in the figure by FH, FL, and HM. Accordingly, let the one of the three rules that is the strongest be fastened very securely to the base, which ABGD represents here, and let one surface of this base be flat so that line HF can be produced on it to C. And indeed, let two fins equal and similar in all ways be fitted on another rule thus that their middle lines are set up upon middle line FL – one indeed near one end and the other near the other. On the two fins, moreover, there are two small, round apertures made on the middle lines at the same distance. And let the one designated for the observer's eye be smaller, the other a little larger, so that through it almost the whole moon is able to be visible to one looking through both apertures. Then you will firmly connect these two rules by a round and even axis such that the rule on which



regula in qua sunt due pinne circa axem leviter volvi possit sursum et deorsum absque inclinatione ad dextram vel sinistram. Sint etiam due regule ita per axem constricte invicem et illaqueate ut superficies earum plane apparentes in una permaneant plana superficie. Deinde a medio puncto axis quod sit F in linea FC lineam FH ad equalitatem verissimam linee alterius regule FL abscinde. Post hec tertiam regulam cum prima mediante scilicet axe similiter omnino consues, et sit medium punctum huius axis H circa quod tertia regula sursum et deorsum leviter sit volubilis absque inflexione ad dextram vel sinistram. Et sit in ea linea HM equalis premissis HF FL. Incidetur autem hec regula tertia et ad angulum rectum cavabitur per totum usque ad mediam lineam que relinquetur intacta. Secunda quoque regula in qua sunt pinne incidetur versus extremitatem ut in cavatura alterius superduci possit sic ut linea FL media et linea HM in una sint plana superficie apparente. Deinde linea HM in xxx partes equaliter dividitur, et unaqueque pars in sua minuta quot capere poterit.

Hiis ita constitutis regula prima super basim suam erigitur in loco plano orizontis, et angulus superior ad meridiem convertitur donec triangulus FHL sit in superficie meridiani cum laterali superficie basis ABGD, que etiam superficies basis sit orienti obversa. Et sit linea FH perpendiculariter descendens super orizontem, quod per plumbeum perpendiculum a summitate puncti F suspensum perpendetur.

Hoc itaque vel simili parato instrumento et collocato observatum est a Ptolomeo in Alexandria, cuius latitudo ab equinoctiali est xxx gradus et lviii minuta, quando locus verus Lune erat in principio Cancri eo quod tunc Luna ad meridianam lineam veniente eius altitudinis circulus qui transit super polos orizontis et centrum Lune est vere meridianus et transit etiam super polos circuli signorum. Cum hoc quoque observavit per motum latitudinis in maxima

674 sunt] sint *P* possit] possint *P* **675** inclinatione] declinatione P_7 Sint] *corr. ex* sunt *K* **677** plana superficie] superficie plana *M* quod] qui *M* linea] lineam P_7 **679** hec] hoc *MN* mediante] medietate *M* scilicet] simili P_7 axe] axis *P* omnino] *corr. ex* omn†es† *M* **681** absque] *corr. ex* a†t†que P_7 dextram – sinistram] dextrum vel ad sinistrum *M* **682** Incidetur] *corr. ex* incidente *P* hec] huiusmodi *M* regula tertia] tertia regula *P* **683** lineam] lineam HM P_7M **684** relinquetur] relinquitur *N* quoque] vero *M* **685** extremitatem] extremitatem linea *PN* linea *add. et del. M* (extremitatem linea BaE_1) cavatura] curvatura P_7 superduci] si perduci *PK* (si perduci *Ba* super duas E_1) sic ut] sit ut *P* ut sit *N* **686** sint] sicut *N* HM[2]] *corr. in* FH *N* xxx] 60 *N* **687** dividitur] dividatur *N* quot] quod *M* poterit] poterat *P corr. ex* possit *M* **688** ita] itaque P_7M plano] *corr. ex* primo *P* **689** triangulus] *marg. P* **690** meridiani] *corr. ex* meridiei *M* ABGD] ABCD *N* superficies] superficiem P_7 **691** orienti] orisonti *M* **692** puncti] *om.* P_7 **692/693** suspensum perpendetur] suspendetur *M* **694** simili] vel *add.* P_7 et collocato] *om. N* **695** Ptolomeo] Tholomeo P_7 Tolomeo *K* Ptholomeo *N* **696** locus – Lune] verus locus P_7 Luna] linea P_7 **697** altitudinis] latitudinis *P corr. ex* latitudinis *N* **698** transit etiam] etiam transit *N* **699** latitudinis] *om.* P_7

the two fins are can be turned smoothly on the axis upwards and downwards without any inclination to the right or left. Also, let the two rules be bound to each other and entangled by the axis in such a way that their visible flat surfaces remain in one plane surface. Then from the axis' middle point, which let be F, cut off line FH on line FC very accurately equal to line FL of the other rule. Afterwards, in a very similar way, you will join the third rule to the first by means of an axis, and let the middle point of this axis be H, around which let the third rule be rotatable upwards and downwards smoothly without any bending to the right or left. And let there be line HM on it equal to HF and FL set forth. Moreover, this third rule will be cut into, and throughout the whole, it will be hollowed out at a right angle to the middle line, which let remain intact. Also, the second rule, on which the fins are, will be cut into towards the endpoint so that it is able to be drawn[45] in the other's hollow thus that the middle line FL and line HM are in one, visible plane surface. Then line HM is divided equally into 30 parts, and each part into its minutes, as many as it is able to hold.

With these things thus disposed, the first rule is set up upon its base in a place level with the horizon, and the upper angle is turned towards the meridian until triangle FHL is in the meridian's plane with the base's lateral surface ABGD, which also let be the base's surface facing the east. And let there be line FH descending perpendicularly upon the horizon, which shall be assessed through a lead plumb hanging from the top of point F.

Accordingly, with this instrument or a similar having been prepared and positioned, it was observed by Ptolemy in Alexandria, where the latitude from the equator is 30° 58′, when the moon's true place was in the beginning of Cancer because then, with the moon coming to the meridian line, its circle of altitude that passes through the horizon's poles and the moon's center is truly the meridian and passes also through the ecliptic's poles. He also observed with this [i.e. the instrument] when the moon was at the maximum declination

[45] The reading 'si perduci' must have entered the transmission early, but it seems unlikely to be the original reading.

declinatione ab orbe signorum versus septemtrionem ut esset Luna. In ipso itaque meridie elevata linea HM et revoluta linea FL tamdiu donec per utrumque foramen Luna comparuit oculo aspicientis, continuate sunt linee iste super aliquam partium HM, ut verbi gratia ad punctum L. Ergo corda HL, sicut sinus arcuari solent, arcuatur. Et arcus qui provenerit duplicatur. Nam ipse duplicatus necessario est similis arcui circuli altitudinis qui deprehenditur inter cenit capitum et locum Lune visum. Et fuit arcus iste secundum quod Ptolomeus deprehendit in Alexandria duo gradus et octava unius gradus fere. Quia ergo in tam parva latitudine regionis diversitas aspectus in latitudine insensibilis est, si hanc quantitatem a latitudine regionis que est xxx partes et lviii minuta minuas, relinquitur distantia Lune tunc ab equinoctiali. A qua distantia si item minuas maximam declinationem orbis signorum que secundum Ptolomeum inventum est xxiii graduum et li minutorum, relinquitur distantia Lune ab orbe signorum nota que est maxima latitudo Lune versus septentrionem, et accidit secundum premissa v graduum. Et similiter erit ex altera parte orbis signorum. Cognita itaque maxima latitudine ceteras latitudines sicut declinationes Solis per xvi^{am} primi poteris cognoscere per motum videlicet latitudinis equatum. Nota quod Albategni quoque eandem ponit maximam latitudinem Lune.

12. Diversitatem aspectus Lune in latitudine per instrumentum accipere.

Observandum itaque quando Luna ex loco suo vero qui sit caput Capricorni vel iuxta aut caput Cancri vel iuxta in remotis climatibus ab orizonte climatis xc gradibus circuli signorum destiterit. Et in illa hora per instrumentum accipienda est visa elongatio Lune a cenit capitum in circulo altitudinis qui tunc necessario est circulus transiens per polos zodiaci. Dehinc considerandum est que sit latitudo Lune sive meridiana sive septentrionalis, et que sit declinatio

700 septemtrionem] septentrionalem *K* ut] ubi *N* **700/701** ipso itaque] ipsa quoque P_7 **701** HM] *corr. ex* HOI *K* linea FL] Lune FH P_7 **702** comparuit] apparuit *M* continuate] conterminate *N* iste] ille *N* **703** HM] *corr. ex* HL *M* L] *corr. ex* B *M* **704** solent] solet *N* provenerit] provenit *N* **705** est similis] similis est *PN* altitudinis] similitudinis P_7 qui] que *M* **706** inter] *corr. ex* a *P* cenit] czenit *M* capitum] capitis P_7 iste] ille *M* **707** Ptolomeus] Tholomeus P_7 duo] duos P_7 et] et una *N* **708** latitudine regionis] regionis latitudine P_7 **709** partes] gradus *N* **711** item] tantum *P corr. ex* tantum *K* iterum *MN* maximam] *om. N* **712** Ptolomeum] Tholomei P_7 Ptholomeum *N* inventum] inventa *MN* graduum – minutorum] gradus li minuta *corr. ex* linee minuta *K* distantia] *s.l. P* **713** ab – signorum] *om. PN* versus septentrionem] in septemtrione *PN* **714** accidit] accidet *M* **715** ceteras latitudines] *om.* P_7 **716** xviam] sextam *PN* 16^{a} P_7K **717** equatum] *om.* P_7 quod – quoque] quoque quod Albategni *N* **718** Lune] Lune et cetera *N* **720** itaque] itaque est *M* est (*s.l.*) itaque *N* **720/721** Capricorni – iuxta^{1}] vel iuxta capud Capricorni P_7 **721** aut] vel *MN* remotis] *corr. ex* remoti†bus† *M* **722** destiterit] distiterit *MN* **723** visa] visu *P corr. ex* visu *K om. N* capitum] capitis *M* qui] *om. M* **724** circulus] *om. N* Dehinc] deinde P_7N est^{2}] *om.* P_7 **725** latitudo Lune] Lune latitudo *N* from the ecliptic towards the north through the motion of latitude. Accordingly, at that noon, with line HM raised and line FL turned until the moon appeared to the observer's eye, those lines are joined upon one of the parts of HM, as for example at point L. Therefore, let the chord HL be arced as sines are accustomed to be arced. And let the arc that results be doubled.[46] For its double is necessarily similar to the circle of altitude's arc that is caught between the zenith and the moon's apparent place. And according to what Ptolemy found in Alexandria, that arc was about 2° 7′ 30″. Therefore, because in such a small latitude of the region, the parallax in latitude is imperceptible, if you subtract this quantity from the region's latitude, which is 30° 58′, there remains the moon's distance from the equator at that time. If you also subtract from this distance the ecliptic's maximum declination, which according to Ptolemy was found to be 23° 51′, the moon's distance from the ecliptic remains known, which is the moon's greatest latitude towards the north, and according to what has been set forth, it happens to be 5°. And on the other side of the ecliptic, it will be similarly. Accordingly, with the greatest latitude known, you will be able to know the remaining latitudes through the equated motion of latitude, in the same way as the sun's declinations through the 16th of the first. Note that Albategni also posits the same greatest latitude of the moon.[47]

12. To take the moon's parallax in latitude with an instrument.

Accordingly, it must be observed when the moon, off of its true place, which should be at or near the beginning of Capricorn or at or near the beginning of Cancer, stands 90° along the ecliptic from the clime's horizon in the distant climes.[48] And in that hour the apparent elongation of the moon from the zenith on the circle of altitude, which then necessarily is the circle passing through the zodiac's poles, must be taken with the instrument. Then what the moon's latitude is, whether south or north, and what the declination of the moon's degree

[46] Since the units of this third rule are twice as large as the sixtieths of the others, the arc corresponding to them in a sine table will be half of the arc that would be found if this third rule's units were sixtieths of the radius and one used a chord table.

[47] Albategni, *De scientia astrorum* Ch. 30 (1537 ed., f. 35v).

[48] I.e. in an oblique sphere.

partis Lune. Et si Luna in septentrionalibus signis fuerit et latitudo eius septentrionalis, addenda est latitudo super declinationem partis; et si fuerit meridiana, minuenda. Quod si Luna in signis australibus fuerit, e converso faciendum. Et quod post additionem vel diminutionem provenerit erit elongatio Lune ab equinoctiali eo quod circulus transiens super polos zodiaci pene sit iuxta quod positum est transiens super polos equinoctialis. Que elongatio, si Luna ex parte equinoctialis versus austrum fuerit, super latitudinem regionis addenda est; et si versus septentrionem, minuenda a latitudine regionis cum latitudo regionis maior sit maxima declinatione Cancri. Et quod provenerit erit vera elongatio Lune a cenit capitum. Hanc igitur veram elongationem que necessario minor est visa elongatione per instrumentum in hiis climatibus a visa elongatione subtrahe. Nam quod relinquitur est diversitas aspectus in latitudine. Est enim arcus circuli transeuntis super polos zodiaci et visum locum Lune – arcus inquam deprehensus inter locum latitudinis Lune et visum locum Lune.

13. Distantia centri Lune a centro terre quanta sit ad diametrum terre et qualiter per unum diversitatis aspectum per instrumentum accepte inventa propalare.

Sole quidem existente secundum cursum medium in vii gradibus et xxxi minutis Libre et secundum verificationem in v gradibus et xxviii minutis Libre, et Luna per cursum medium in xxv gradibus et xliiii minutis Sagittarii. Quapropter media distantia inter eos lxxviii gradus et xiii minuta extitere, elongatione quoque Lune in epiciclo a longitudine longiore equali secundum cursum medium diversitatis existente in gradibus cclxii et xx minutis, motu vero latitudinis medio a maxima declinatione septemtrionale cccliiii gradibus et xl minutis. Quapropter fuit Luna secundum verificationem tunc in tribus gradibus et x minutis Capricorni, et elongatio Lune a maxima declinatione septentrionale secundum verum motum latitudinis duo gradus et vi minuta. Fuit

726 septentrionalibus signis] signis septentrionalibus *N* **728** australibus] *corr. ex* australibris *K* faciendum] faciendum est *N* **729** quod] quia P_7 diminutionem] fuerit *add. et del. M* **730** iuxta] secundum *N* **733** latitudine] longitudine P_7 **734** provenerit] proveniet *N* vera] *om. PN* elongatio] declinatio P_7 **736** climatibus] *corr. ex* regionibus *N* **737** quod] qui *P* **738** super] per P_7 **739** Lune[2]] Lune et cetera *N* **740** diametrum] semidiametrum P_7M (dyametrum *Ba* semidiametrum E_1) **741** diversitatis aspectum] diversitatis aspectus *P* aspectum diversitatis *N* inventa] alie inveniantur *N* **743** quidem] *om. N* **743/744** vii – minutis[1]] 7 gradibus et 30° minuto P_7 septimo gradu et 20 minutis *M* septimo gradu et 31° minuto *N* **744** v – minutis[2]] quinto gradu et 28° minuto P_7MN xxviii] *corr. ex* xxvii *K* **745** per] secundum P_7 cursum medium] medium cursum *N* xxv – minutis] 25° gradu et 44° minuto P_7N **746** distantia – eos] inter eos distantia *N* gradus – minuta] gradibus et 13 minutis *M* extitere] existente *PN corr. ex* existere *K* existere *corr. in* existente *M* (extitere *Ba* ex†tutere† E_1) **747** Lune] *om.* P_7 **748** existente] *om. N* gradibus] gradu *MN* xx] 30 P_7 **751** Capricorni] *corr. ex* caprus *M* elongatio] elongatione *PK* **752** verum] vero *M* vi] vii *PN corr. ex* 7 *M* (*om. Ba* 6 E_1)

⟨on the ecliptic⟩ is must be considered. And if the moon is in the northern signs and its latitude is northern, the latitude must be added to the declination of the ⟨moon's⟩ degree; and if it is south, it must be subtracted. And if the moon is in the southern signs, it must be done conversely. And what results after the addition or subtraction will be the elongation of the moon from the equator because according to what was supposed, the circle passing upon the zodiac's poles is, nearly passing upon the equator's poles. If the moon is on the side of the equator towards the south, this elongation must be added to the region's latitude; and if towards the north, it must be subtracted from the region's latitude when the region's latitude is greater than Cancer's maximum declination.[49] And what results will be the true elongation of the moon from the zenith. Therefore, subtract this true elongation, which is necessarily less than the elongation seen through the instrument in these climates, from the apparent elongation. For what remains is the parallax in latitude. For it is the arc of the circle passing upon the zodiac's poles and the moon's apparent place – I mean the arc caught between the place of the moon's latitude and the moon's apparent place.

13. To make manifest how great the distance of the moon's center from the earth's center is ⟨compared⟩ to the earth's diameter and how it is found through one parallax taken with an instrument.

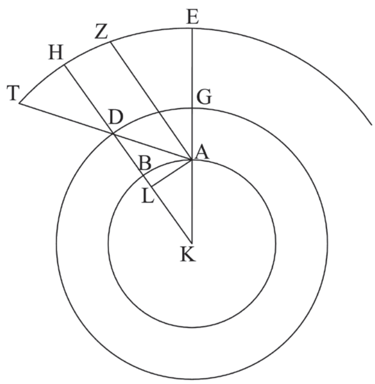
Indeed, with the sun existing according to mean course in Libra 7° 31′ and according to correction ⟨of its anomaly⟩ in Libra 5° 28′, and the moon was through mean course in Sagittarius 25° 44′. For this reason, the mean distance between them proved to be[50] 78° 13′, with also the moon's elongation on the epicycle from the mean apogee according to the mean motion of irregularity proving to be 262° 20′, and indeed, with the mean motion of latitude 354° 40′ from the greatest northern declination. For this reason, the moon was according to correction ⟨for its anomalies⟩ in Capricorn 3° 10′ then, and the moon's elongation from the greatest northern declination according to the true motion

[49] It must also be greater than the ecliptic's elongation plus the moon's latitude if the moon is near the summer solstice and at its northernmost latitude. The author does not treat the case when the moon appears north of the zenith.

[50] 'Extitere' is a form of 'extiterunt.'

enim equatio Lune vii gradus et xxvi minuta. Et propter hoc fuit latitudo Lune vera iiii gradus et lix minuta ex orbe descripto super polos circuli signorum qui tunc fere fuit meridianus. Hiis inquam ita existentibus – fuit visa elongatio Lune a cenit Alexandrie l gradus et lv minuta sicut per instrumentum didicit philosophus, et fuit elongatio vera xlix gradus et xlviii minuta. Ergo fuit diversitas aspectus Lune in latitudine pars una et vii minuta, et hoc quoque in circulo altitudinis.

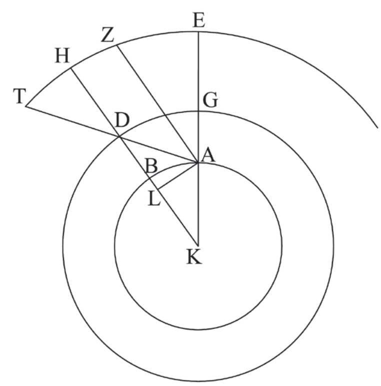
Quibus omnibus sic constitutis lineabo in superficie circuli altitudinis orbem terre AB, et in spera Lune in eadem superficie secundum distantiam Lune a centro terre orbem alium GD, et iterum orbem alium in celo aput quem sit terra sicut punctum EZHT. Et omnium commune centrum sit punctum K, et linea a centro ad cenit capitum transiens KAGE. Et sit Luna in puncto D cuius vera elongatio a cenit capitum quod est G est partes posite, xlix partes et xlviii minuta. Protraham ergo duas lineas KDH ADT et a puncto A quod est locus aspectus perpendicularem AL super HK. Et sit linea AZ equidistans linee HK. Palam ergo quod aspicienti a puncto A sit diversitas aspectus arcus TH qui est notus scilicet pars una et vii minuta secundum quod constitutum est ante. Et quia tota terra aput orbem EZH est quasi centrum, erit AZ sicut linea HK a centro educta huius respectu.



Et ob hoc arcus TZ non maior arcu TH circiter partem unam et vii minuta. Quapropter quoniam cum punctum A positum fuerit centrum orbis ZHT, non est in illo diversitas computata, erit angulus ZAT notus scilicet pars una et vii minuta. Quapropter erit angulus ADK ei equalis notus. Erit ergo proportio DA ea facta semidiametro ad utramque istarum AL DL nota, cum DL secundum quod computatur diversitas sit minor linea DA. Item quia arcus GD est notus

753 gradus – minuta] graduum et 26 minutorum *M* xxvi] 20 P_7 hoc] hec P_7 Lune[2]] *corr. ex* linee *K* **754** gradus – minuta] gradibus et 59 minutis *M* **755** fere] vere P_7 inquam ita] ita inquam P_7 **756** cenit] czenit *M* gradus – minuta] 50 gradibus et 55 minutis *M* **757** philosophus] Tholomeus P_7 Ptolomeus *M* gradus – minuta] gradibus et 48 minutis *M* **760** constitutis] *corr. ex* omnibus P_7 **761** Lune[1]] *corr. ex* linee *K* **761/762** Lune[2] – terre] *corr. ex* terre a centro Lune *M* **762** orbem[1] alium] alium orbem *PN* orbem[2]] *s.l. P* **764** cenit] zenit *M* sit] *om. N* **766** est G] GD est *corr. ex* G est *corr. ex* DG est *N* partes[2]] gradus *del. N* **768** quod] quidem *P* qui *N* **770** linea] *om. N* **771** aspicienti] *corr. ex* aspecienti P_7 **772** sit] fit P_7N **777** huius] hoc *M* **778** ob] DB *PN* **779** orbis] orbis signorum *M* ZHT] ZHZ *P* **780** ZAT] *corr. ex* ZHT P_7 **782** ea] EA *M corr. ex* ad ea *N* semidiametro] dyametro *N* of latitude was 2° 6′. For the moon's equation was 7° 26′. And because of this, the moon's true latitude was 4° 59′ of the circle described upon the ecliptic's poles, which was then nearly the meridian. With these things existing thus – I say, the moon's apparent elongation from the zenith of Alexandria was 50° 55′ as the philosopher learned from the instrument, and the true elongation was 49° 48′. Therefore, the moon's parallax in latitude was 1° 7′, and this also was on the circle of altitude.

With all of these things thus established, I will draw the earth's circle AB in the circle of altitude's plane, and in the same plane, another circle GD in the moon's sphere according to the distance of the moon from the earth's center, and again another circle EZHT in the heavens, to which let the earth be as a point. And let the common center of all be point K, and let the line passing from the center to the zenith be KAGE. And let the moon be at point D, whose true elongation from the zenith, which is G, is the posited degrees, 49° 48′. Then I will draw the two lines KDH and ADT and perpendicular AL from point A, which is the place of gazing, upon HK. And let line AZ be parallel to line HK. It is clear, therefore, that for one observing from point A, the parallax is arc TH, which is known, i.e. 1° 7′ according to what was established previously. And because the whole earth is as a center to circle EZH, AZ will be in this respect as line HK drawn from



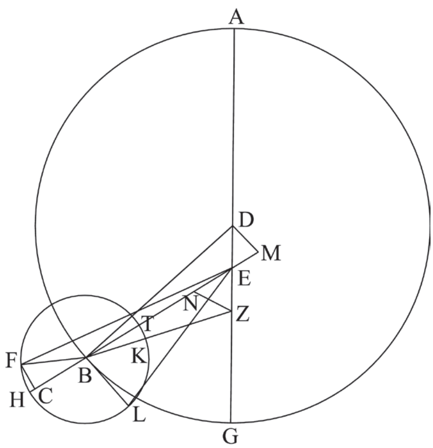
the center. And on account of this, arc TZ, not greater than arc TH, will be approximately 1° 7′. For this reason, because when point A was supposed the center of circle ZHT, a difference in it was not reckoned, angle ZAT will be known, i.e. 1° 7′. For this reason, angle ADK equal to it will be known. Therefore, the ratio of DA, with it made radius, to each of those AL and DL will be known, because DL, according to which the difference is reckoned, is less than line DA.[51] Likewise, because arc GD is known, i.e. 49° 48′, angle AKL will be

[51] In the *Almagest*, Ptolemy states that DL is less than DA by a negligible amount and thus can be seen as being the same number of parts, but the author of the *Almagesti minor* included only a faulty clause 'cum DL secundum quod diversitas sit minor linea DA', which lacks an essential 'non', and serves no clear purpose since the statement that it justifies is not included.

scilicet xlix partes et xlviii minuta, erit propter hoc angulus AKL notus; facta ergo KA semidiametro erit proportio KA ad utramque istarum AL KL nota. Posito ergo quod linea KA que est semidiameter terre sit pars una tantum, erit secundum hoc quoque utraque istarum AL KL nota, et mediante AL erit LD ad KA cum sit pars una nota. Quapropter tota DK ad eandem cum sit pars una nota, et provenit DK secundum operationem premissorum xxxix partes et xlv minuta prout KA est pars una. Atque hec est distantia centri Lune in hoc situ a centro terre, quod erat propositum.

14. Linea educta a centro terre ad longitudinem longiorem ecentrici lunaris atque linea educta ex opposito ad longitudinem propiorem necnon et semidiameter epicicli, unaqueque istarum linearum quanta sit ad semidiametrum terre edocere. Unde etiam manifesta erit in omni loco centri Lune a centro terre distantia.

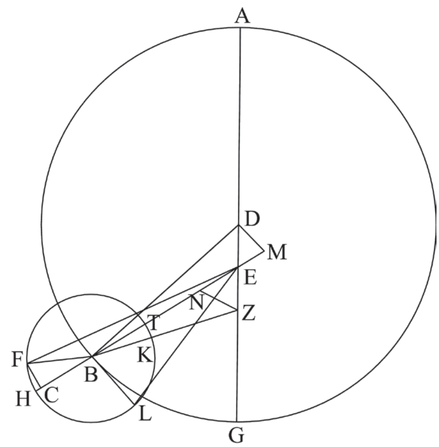
Depono ecentricum Lune ABG supra centrum D et in diametro eius ADG centrum terre E et nota declinationis diametri epicicli punctum Z. Manentibus ergo omnibus in premissa propositione constitutis, describam epiciclum Lune HL supra centrum B. Et protraham lineas ETBH et DB et BK, sitque locus Lune in consideratione proposita punctum L. Et protraham duas lineas EL BL, et super lineam EBH producam perpendicularem unam DM et aliam ZN. Quia ergo media distantia Solis et Lune lxxviii gradus et xiii minuta, erit cum hoc duplicatum fuerit angulus AEB notus. Quare et residuus duorum rectorum angulus DEM et angulus ZEN notus. Et propter hoc via superius posita fiet linea EB nota ad distantiam duorum centrorum que est x partes et xix minuta. A qua cum subtracta fuerit EN nota,



785 AL KL] KL AL P_7 **786** Posito – pars] posita ergo linea que est KA semidyameter terre parte *M* Posito] *corr. ex* posita *K* quod] *s.l.* P_7 **787** quoque] *om. M* istarum] istarum linearum *N* **787/788** erit – ad[1]] nota erit LD et *N* **788** tota] *om.* P_7 **790** hec] hoc *MN* **791** propositum] propositum et cetera *N* **792** educta – terre] a centro terre educta *N* lunaris] lunari *P corr. ex* lunari *K* Lune *N* **793** educta] ducta P_7 propiorem] *corr. ex* longiorem P_7 **794** linearum] *om. N* semidiametrum] *corr. ex* diametrum *P* **797** Depono] prepono P_7 describo *N* **799** premissa] simili *N* propositione] proportione *PN corr. ex* proportione P_7 **800** ETBH] *corr. ex* BH *M* **803** EL] EB *M* et] DB et BK *add. et del. P* **804** producam] *corr. ex* productam *K* **805/806** media distantia] distantia media *M* **806** distantia – Lune] Solis et Lune distantia est *N* lxxviii] *corr. ex* lxxvii *K* **811** via] una *PK corr. ex* una P_7N (via BaE_1) **814** qua] quibus *M* cum] *s.l.* (*other hand*) *K* known because of this; therefore, with KA made a radius, the ratio of KA to each of those AL and KL will be known. Therefore, with it supposed that line KA, which is the earth's radius, is only one part, each of those AL and KL will also be known according to this, and with AL mediating, LD will be known to KA, when it is one part. For this reason, whole DK is known to the same, when it is one part, and according to the operation of the things set forth, DK results as 39^p 45′ when KA is 1^p. And this is the distance of the moon's center in this place from the earth's center, which was what was proposed.

14. With a line drawn from the earth's center to the apogee of the moon's eccentric, with the line drawn on the opposite side to the perigee, and also with the epicycle's radius ⟨drawn⟩, to teach how great each of those lines is in relation to the earth's radius. Whence the distance of the moon's center from the earth's center will also be manifest in every place.

I lay down the moon's eccentric ABG upon center D, the center of the earth E on its diameter ADG, and point Z the epicycle's diameter's point of turning aside. Then, with everything set up in the preceding proposition remaining, I will describe the moon's epicycle HL upon center B. And I will draw lines ETBH, DB, and BK,[52] and let the moon's place at the proposed observation be point L. And I will draw the two lines EL and BL, and upon line EBH I will produce one perpendicular DM and another ZN. Therefore, because the mean distance of the sun and moon is 78° 13′, when this is doubled, angle AEB will be known. Therefore, also the supplement, angle DEM and angle ZEN, will be known. And because of this, by the way posited above [i.e. V.7 and V.9], line EB will become known to the eccentricity, which is 10^p 19′. When known EN is subtracted from this, BN will remain



[52] The author does not specify clearly, but point K is the mean perigee, which must be on line BZ. Perhaps the text here originally was 'BKZ' and the 'Z' was omitted by a scribe misreading it as a superfluous 'et.'

relinquetur BN nota. Et propter hoc fiet arcus epicicli TK notus qui scilicet est inter longitudinem propiorem veram et longitudinem propiorem equalem. Et quia elongatio Lune in epiciclo a longitudine longiore equali fuit in hora considerationis cclxii gradus et xx minuta, cum subtraxerimus inde medietatem circuli clxxx, relinquitur a puncto K quod est longitudo propior media arcus KL notus scilicet lxxii gradus et xx minuta. Et quia accidit quod arcus TK est vii gradus et xl minuta, erit totus TKL xc gradus; ergo angulus EBL est rectus. Quocirca cum linea BL que est semidiameter epicicli nota sit sicut BE, est enim hoc respectu v partium et xv minutorum, erit propter hoc EL similiter nota. Et accidit xl partium et xxv minutorum iuxta hanc quantitatem qua BL est v partium et xv minutorum. Et iuxta eandem quantitatem erat EA nota scilicet lx partium et GE nota scilicet xxxix partium et xxii minutorum. Fuit autem ostensum in premissa quod linea EL est xxxix partes et xlv minuta iuxta quod semidiameter terre est pars una. Est enim distantia centri Lune a centro terre in hora considerationis. Ergo cum proportionaverimus quantitates harum linearum, erit linea EA quidem lix partes et EG xxxviii partes et xliii minuta, et linea BL que est semidiameter epicicli v partium et x minutorum videlicet iuxta quantitatem qua semidiameter terre est pars una. Et ita quoque ponit Albategni.

Hiis cognitis in quocumque loco epicicli Luna fuerit, epiciclo etiam in quolibet loco ecentrici posito, erit distantia Lune a centro terre nota. Nam linea EB ad omnem distantiam a longitudine longiore secundum hunc quoque modum erit nota, cum distantia duorum centrorum hoc quoque respectu nota fuerit scilicet x partium et ix minutorum fere. Ponamus ergo Lunam in epiciclo super punctum F secundum notam elongationem a puncto H et ducamus perpendicularem FC super EH. Erit ergo proportio BF ad BC et ad CF nota; quare tota EC sicut CF nota; quare et EF que subtenditur angulo recto nota iuxta id secundum quod semidiameter terre est pars una.

815 scilicet] scilicet notus *M* **816** veram] notam *K* **818** minuta] minuta et *M* subtraxerimus inde] inde subtraximus *N* **819** clxxx] clxix *K* **820** KL] FL P_7 quia] *om. PN* **821** est] *om. M* totus] totus arcus *M* arcus *N* **822** semidiameter] *corr. ex* diameter *P* **823** minutorum] minutis *M* **823/825** erit – minutorum] *om. PKN* **825** xv] 19 *M* erat] *om. M* **826** scilicet^2] videlicet P_7M xxxix] *corr. ex* xxix *P* et xxii] 22 *corr. ex* 47 *M* **827** autem] ante *P* partes – minuta] partium et 45 minutorum *M* **828** semidiameter] *corr. ex* diameter *P* enim] *om. N* **829** cum] premissis *add. et del.* P_7 **829/830** proportionaverimus – linearum] quantitates harum linearum proportionaveris P_7 **830** partes^2] *om. PN* **831** x] *corr. ex* 19 *M* **834** fuerit] fuerit inventa *KM* quolibet] quocumque *M* **836** distantiam] nota *add. et del. K* modum] *corr. ex* lodum P_7 **837** duorum] *om. N* nota fuerit] fuerit nota *M* **838** ix] *corr. ex* 19 *M* 19 *N* Lunam] lineam P_7 *corr. ex* lineam *K* **840** super] super lineam *MN* ad^2] *om.* P_7 CF] BF *PN corr. ex* BF *K* EF *M* (BF *Ba* †E†F E_1) **841** CF] EF *PM* (EF *Ba* †E†F E_1) EF] *corr. ex* F P_7 id] illud *N* **842** secundum] scilicet P_7M semidiameter] diameter *PKMN* (semydyameter BaE_1)

known. And because of this, the epicycle's arc TK, i.e. that which is between the true perigee and the mean perigee, will be known. And because the moon's elongation on the epicycle from the mean apogee was 262° 20′ at the time of observation, when we subtract a semicircle, i.e. 180°, from this, arc KL from point K, which is the mean perigee, remains known, i.e. 72° 20′.[53] And because it happens that arc TK is 7° 40′, whole TKL will be 90°; therefore, angle EBL is right. On account of this, because line BL, which is the epicycle's radius, is known as is BE, for it [i.e. BL] is 5^p 15′ in this respect, EL will similarly be known because of this. And it happens to be 40^p 25′ according to this quantity by which BL is 5^p 15′. And according to the same quantity, EA was known, i.e. 60^p, and GE was known, i.e. 39^p 22′. It was shown, moreover, in the preceding ⟨proposition⟩ that line EL is 39^p 45′ according to which the earth's radius is 1^p. For it is the distance of the moon's center from the earth's center at the time of the observation. Therefore, when we make the quantities of these lines proportional, line EA will indeed be 59^p, EG 38^p 43′, and line BL, which is the epicycle's radius, 5^p 10′, that is according to the quantity by which the earth's radius is one part. And thus also Albategni posits.

With these things known in whatever place on the epicycle the moon is, with the epicycle also supposed in any place on the eccentric, the moon's distance from the earth's center will be known. For line EB will be known at every distance from the apogee according to this way also, when the eccentricity is also known in this respect, i.e. approximately 10^p 9′.[54] Therefore, let us place the moon on the epicycle upon point F according to the known elongation from point H and let us draw perpendicular FC upon EH. Therefore, the ratio of BF to BC and to CF[55] will be known; therefore, whole EC, as CF,[56] will be known; and so also EF, which subtends a right angle, is known in terms in which [*lit*., according to that according to which] the earth's radius[57] is 1^p.

[53] This should be 82° 20′ to match the *Almagest*. The error could have been easily seen from other values given in this passage.

[54] This is the value of the distance between the two centers in terms of the size of the earth's radius. Regiomontanus and *M*'s scribe apparently did not realize this, and at least initially wrote the eccentricity in the terms in which the eccentric's diameter is 120.

[55] The mistaken reading 'BF' must have entered the transmission of the text early and was perhaps original.

[56] The mistaken reading 'EF' must have entered the transmission of the text early and was perhaps original.

[57] The mistaken reading 'diameter' must have entered the transmission of the text early and was perhaps original.

Cum vero centrum epicicli fuerit in longitudine longiore ecentrici et Luna in longitudine longiore epicicli, addita quantitate semidiametri epicicli que hoc respectu est v partes et x minuta super lix partes que sunt linea EA, erit maxima distantia centri Lune a terre centro que esse potest lxiiii partes et x minuta. Et si tunc Luna fuerit in longitudine propiore epicicli, subtracta quantitate semidiametri epicicli a linea EA, remanebit distantia centri Lune a centro terre liii partes et l minuta. Cum vero centrum epicicli fuerit in longitudine propiore ecentrici et Luna in longitudine longiore epicicli, addita hac quantitate semidiametri epicicli super lineam EG, erit distantia centri Lune a centro terre xliii partes et liii minuta. Et si Luna tunc fuerit in longitudine propiore epicicli, erit minima distantia centri Lune a centro terre que esse potest xxxiii partes et xxxiii minuta, subtracta videlicet quantitate semidiametri epicicli dicta a linea EG.

15. Diameter Lune in maxima centri eius a centro terre distantia quantum arcum maioris circuli cordet invenire. Unde etiam manifestum erit de semidiametro umbre in hoc Lune transitu quanto arcui maioris circuli subtendatur, et que ipsius ad semidiametrum Lune proportio.

Neque per clepsedras aquarum neque per elevationes circuli equinoctialis hoc etiam accedendo ad prope verum deprehendi est possibile propter multas erroris incidentias. Sed elegit philosophus duas lunares eclipses in quarum utraque Luna aput longitudinem longiorem epicicli fuit. Et fuit prima earum in anno cxxvii° annorum Nabugodis, et eclipsatum est de diametro Lune ex parte meridiei ad quartam diametri eius. Et fuit locus Lune in medio tempore eclipsis per medium cursum longitudinis xxv gradus et xxii minuta Libre, et locus eius verus xxvii gradus et v minuta Libre. Et fuit elongatio Lune a longitudine longiore in epiciclo cccxl gradus et v minuta. Fuit elongatio Lune vera a nodo ix gradus et xx minuta, et ob hoc fuit latitudo Lune xlviii minuta et medietas minuti quod est arcus circuli magni cadentis super ipsam et centrum umbre in orbe signorum ad angulos rectos. Erat ergo quarta diametri Lune cadens tunc in umbra.

845 partes^1 – minuta] partium et 10 minutorum *N* x] *corr. in* 19 *M* **846** terre centro] centro terre P_7N **848** semidiametri epicicli] semidiametri P_7 epicicli semidyametri *M* epicicli *N* **851** epicicli] *s.l. M* **852** tunc] etiam *N* **854** minuta] minutis *M* subtracta] *om. N* **855** dicta] dempta *N* EG] EG et cetera *N* **857** cordet] correspondet *M* **858** quanto arcui] *corr. ex* quantum arcum *M* **860** elevationes] *corr. ex* elongationes *N* **862** philosophus] Tholomeus P_7 Ptolomeus *M* **864** cxxviio] 197 *M* Nabugodis] Nobuchodonosor *M* Nabuchodonosor *N* **866** gradus – minuta] gradibus 22 (*corr. ex* 21) minutis *M* **867** xxvii] 27^{us} *N* v] 27 P_7 minuta] minutum *N* **868** minuta] minutum *M* Fuit] *corr. in* fuitque P_7 et fuit *N* Lune vera] vera Lune *P* vera *N* **869** minuta^2] *corr. ex* gradus *N* **870** quod] *corr. ex* que *M* et *N* **872** umbra] umbram P_7

And indeed when the epicycle's center is at the eccentric's apogee and the moon is at the epicycle's apogee, with the quantity of the epicycle's radius, which is 5^p 10′ in this respect, added to the 59^p that are line EA, the greatest distance of the moon's center from the earth's center that can be will be 64^p 10′. And if then the moon is at the epicycle's perigee, with the quantity of the epicycle's radius subtracted from line EA, there will remain the distance of the moon's center from the earth's center 53^p 50′. And indeed when the epicycle's center is at the eccentric's perigee and the moon is at the epicycle's apogee, with this quantity of the epicycle's radius added to line EG, the distance of the moon's center from the earth's center will be 43^p 53′. And if the moon then is at the epicycle's perigee, the least distance of the moon's center from the earth's center that can be will be 33^p 33′, that is with said quantity of the epicycle's radius subtracted from line EG.

15. To find how great an arc of a great circle the moon's diameter subtends [*lit*., is a chord for] at the moon's greatest distance from the earth's center. Whence it will also be manifest how great of an arc of a great circle the shadow's radius subtends in this passage of the moon, and what is the ratio of that to the moon's radius.

It is possible for this to be found, even approximately [*lit*., by approaching near to truth], neither by water clocks[58] nor by the elevations of the equator on account of many incidents of error. But the philosopher chose two lunar eclipses in each of which the moon was at the epicycle's apogee. And the first of them was in the 127th of the years of Nabugodis [i.e. Nabonassar], and a fourth of the moon's diameter was eclipsed on the south side [*lit*., there was eclipsed of the moon's diameter from the south side to a fourth of its diameter]. And the place of the moon through the mean course of longitude in the middle time of the eclipse was Libra 25° 22′,[59] and its true place was Libra 27° 5′. And the moon's elongation from the apogee on the epicycle was 340° 5′.[60] The moon's true elongation from the node was 9° 20′, and on account of this the moon's latitude was 48′ 30″, which is an arc of the great circle falling upon it [i.e. the moon] and the shadow's center in the ecliptic at right angles. Therefore, a quarter of the moon's diameter fell in the shadow then.

[58] The more usual spelling of 'clepsedra' is 'clepsydra.'

[59] This should be 25° 32′.

[60] This should be 340° 7′.

Secunda vero eclipsis fuit in anno ccxxv annorum Nabugodis, et eclipsatum est de Luna ad medietatem diametri eius. Et fuit locus Lune in medio tempore eclipsis per cursum medium longitudinis xx gradus et xiiii minuta Capricorni, et secundum cursum equatum xviii gradus et xii minuta. Et fuit elongatio Lune a longitudine longiore in epiciclo xxviii gradus et v minuta, et elongatio Lune vera in circulo declinante a nodo vii gradus et iiii quinte unius. Quapropter fuit latitudo Lune xl minuta et due tertie unius minuti quod est arcus circuli magni cadentis super centrum Lune et centrum umbre in orbe signorum ad angulos rectos. Eratque tunc dimidium diametri Lune cadens in umbram.

Palam ergo expositis quod superfluum duarum latitudinum Lune in duabus eclipsibus fuit 7 minuta et medietas et tertia unius minuti, et hoc ex eodem circulo magno quia distantia centri Lune a centro terre pene fuit eadem. Superfluum vero partium obscuratarum de diametro in duabus eclipsibus non fuit nisi quarta diametri; igitur quarta diametri applicatur vii minutis circuli magni et medietati et tertie unius minuti. Patet ergo quod cum hoc quater ductum fuerit, quod totus diameter Lune in hac distantia subtenditur arcui xxxi minutorum et tertie unius minuti.

Patet etiam quod medietas diametri umbre in hoc transitu Lune subtenditur arcui xl minutorum et duarum tertiarum unius minuti. Nam tanta erat latitudo Lune in secunda eclipsi in qua medietas diametri Lune erat cadens in umbram tantum, et ob hoc centrum Lune erat contingens circulum umbre. Et ob hoc eius distantia a centro umbre erat arcus latitudinis Lune cui subtenditur semidiameter umbre. Cum itaque proportionaverimus adinvicem quantitatem diametri Lune dimidiam et quantitatem semidiametri umbre, inveniemus semidiametrum umbre continere semidiametrum Lune bis et eius tres quintas fere. Et nota quod diameter Lune eiusdem quantitatis reputatur cum arcu cui subtenditur. Nam arcus circuli magni per centrum Lune transiens et ad terminos semidiametrorum Lune hinc inde terminatus pene recte linee subtense propter magnitudinem circuli et brevitatem arcus equatur. Eodem modo de semidiametro umbre intellige.

873 Nabugodis] Nabuchodus *M* Nabuchodonosor *N* (Nabugodis *Ba* Nabuchod†is† E_1) **874** est – Luna] de Luna est *M* diametri] *iter. P* **875** eclipsis] eclipsis et *M* cursum medium] medium cursum *N* et] *om. K* minuta] minutum *MN* **876** gradus] graduum *M* xii] 10 P_7 **879** quod] et *N* **881** Lune cadens] *corr. ex* cadens Lune P_7 cadens] cadentis *K* **882** expositis] ex positis P_7N superfluum] *corr. ex* fluum *P* **883** hoc] hic *P* **885** diametro] dyametro Lune *N* **886** igitur] ergo si *M* minutis] minuti P_7 **887** quod] *s.l. P del. N* quater] *corr. ex* quantum *K* **888** quod] et P_7 totus] tota *N* minutorum] *om. P* **890** medietas diametri] *corr. ex* diametri medietas *K* **893** tantum] *om. N* **895** Cum itaque] cumque *PN* cum ergo P_7 (cum vera *Ba* cum itaque E_1) **896** diametri – dimidiam] dimidiam diametri Lune P_7K dimidiam] dividiam *P* **899** arcus] cui subtenditur *add. et del.* P_7 transiens] tendens *PN* transens *corr. ex* transeas *K* (transiens BaE_1)

And indeed, the second eclipse was in the 225th year of the years of Nabugodis, and half of the moon's diameter was eclipsed [*lit*., there was eclipsed from the moon to the half of its diameter]. And the place of the moon in the middle time of the eclipse was Capricorn 20° 14′[61] through the mean course of longitude, and 18° 12′[62] according to corrected course. And the moon's elongation from the apogee on the epicycle was 28° 5′, and the moon's true elongation from the node on the declined circle was 7° 48′. For this reason, the moon's latitude was 40′ 40″, which is an arc of the great circle falling upon the moon's center and the shadow's center on the ecliptic at right angles. And half of the moon's diameter fell into the shadow then.

Therefore, it is clear from what has been set forth that the excess of the two latitudes of the moon in the two eclipses was 7′ 50″, and this from the same great circle because the distance of the moon's center from the earth's center was nearly the same. However, the excess of the obscured parts of the diameter in the two eclipses was nothing other than a quarter of the diameter; therefore, a quarter of the diameter is connected with 7′ 50″ of a great circle. Therefore, it is clear that when this is multiplied by four, the moon's whole diameter at this distance subtends an arc of 31′ 20″.

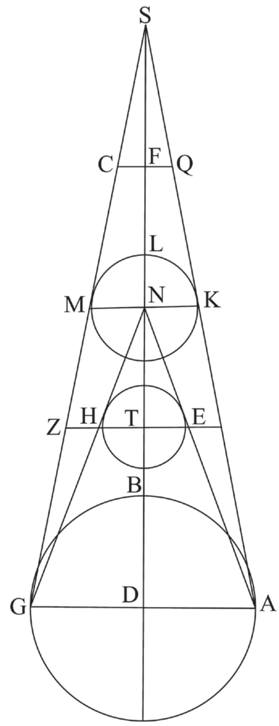
It is also clear that the shadow's radius in this passage of the moon subtends an arc of 40′ 40″. For such was the moon's latitude in the second eclipse in which only half of the moon's diameter fell in the shadow, and on account of this, the moon's center was touching the shadow's circle. And on account of this, its distance from the shadow's center was the arc of the moon's latitude, which the shadow's radius subtends. Accordingly, when we take the ratio of [*lit*., will have made proportional to each other] half the quantity of the moon's diameter and the quantity of the shadow's radius, we find that the shadow's radius contains the moon's radius approximately 2 3/5 times. And note that the moon's diameter is considered to be the same quantity as the arc which it subtends. For the arc of a great circle passing through the moon's center bounded by the endpoints of the moon's radii on one side and the other nearly equals the subtending straight line because of the magnitude of the circle and the shortness of the arc. Understand in the same way about the shadow's radius.

[61] This should be 20° 22′ to match Toomer, *Ptolemy's Almagest*, p. 254, or 20° 20′ to match Gerard's translation (*Almagest*, 1515 ed., f. 55v; and Paris, BnF, lat. 14738, f. 87r).

[62] This should be 18° 14′.

Neque enim veri diametri Lune vel umbre accipiuntur quia nec visus aspicientis comprehendere potest eo quod linee ab oculo aspicientis exeuntes et Lunam ex oppositis punctis contingentes necessario minus diametro Lune includunt quamvis modicum et insensibile propter magnam Lune distantiam. Eodem modo in Sole accipiendum erit.

16. Quantitatem diametri Lune ad semidiametrum terre commensurare. Unde etiam manifesta erit quantitas diametri umbre in Lune transitu ad semidiametrum terre.



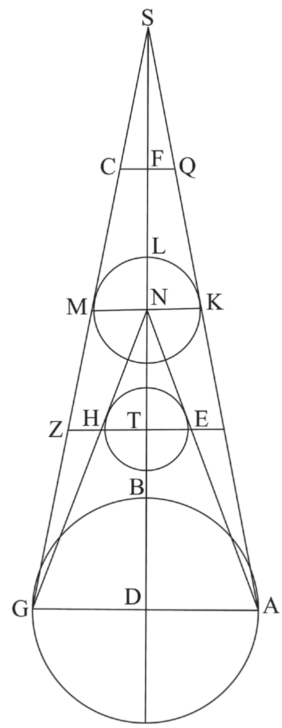
Sit ergo centrum terre punctum N et centrum Lune in maiori sua distantia punctum T et magnus circulus corporis Lune cuius diameter HTE. Et educantur contingentes NH EN. Quia ergo diameter Lune subtenditur arcui circuli magni concentrici xxxi minutorum et xx secundorum cum fuerit Luna in maxima sua distantia, erit angulus HNE huius quantitatis note; quare et medietas eius scilicet angulus HNT notus. Facta ergo NH semidiametro erit proportio NT ad TH nota. Sed nota est proportio NT ad semidiametrum terre. Est enim lxiiii partium et x minutorum cum fuerit semidiameter terre pars una. Quare proportio HT ad semidiametrum terre est nota. Posito ergo quod semidiameter terre sit pars una, erit semidiameter Lune xvii minuta et xxxiii secunda. Et quia semidiameter umbre continet semidiametrum Lune bis et eius tres quintas, erit secundum hanc quantitatem qua semidiameter terre est pars una umbre semidiameter xlv minuta et xxxviii secunda, quod intendimus.

17. Quantitatem diametri Solis necnon et centri eius a centro terre distantiam patefacere, cum quo etiam patens erit quantitas axis umbre.

903 veri diametri] vere (*corr. ex* vel P_7) diametri P_7N vera dyameter (*corr. ex* semidyameter) *M* accipiuntur] accipitur *M* aspicientis] *corr. ex* apparentis P_7 **904** oculo] oculis P_7 **906** insensibile] insensibilem *M* **907** accipiendum] ascipiendum *corr. ex* aspiciendum *K* erit] erit et cetera *N* **908** diametri Lune] Lune diametri *K* **911** centrum[1] – N] punctum N centrum terre *M* N] *corr. ex* H P_7 **912** sua] *om.* P_7 **914** HTE] *corr. ex* HET P_7 **916** magni] *om. N* secundorum] secundum ac *P* secundarum *K* **917** sua] sui P_7 *om. N* **918** HNE] *corr. ex* HEN P_7 note] notus *N* **919** notus] nota *N* **920** NT] EN *M* **921** nota est] est nota *PN* NT] NC *M* **922** partium] *corr. ex* partes *M* **924** Quare – HT] quapropter proportio HC *M* semidiametrum terre] terre semidyametrum *N* **925** ergo] *om. M* semidiameter] *corr. ex* diameter *P* **928** erit] *corr. ex* et *M* **930** pars] *corr. ex* par *K*

Indeed, neither the moon's nor shadow's true diameters are taken because the sight of the observer is not able to grasp it because the lines going from the observer's eye and touching the moon at opposite points necessarily enclose less of the moon's diameter although a very small and insensible amount because of the moon's great distance. It will have to be taken in the same way for the sun.

16. To compare the quantity of the moon's diameter to the earth's radius. Whence the quantity of the shadow's diameter in the moon's passage ⟨compared⟩ to the earth's radius will also be manifest.



Now, let there be the earth's center point N, the center of the moon at its greatest distance point T, and the great circle of the moon's body, of which the diameter is HTE. And let the tangents NH and EN be drawn. Therefore, because the moon's diameter subtends an arc of a concentric great circle of 31′ 20″ when it is at its maximum distance, angle HNE will be of this known quantity; therefore, its half, i.e. angle HNT, is also known. With NH made a radius, therefore, the ratio of NT to TH will be known. But the ratio of NT to the earth's radius is known. For it is 64^{p} 10′ when the earth's radius is 1^{p}. Therefore, the ratio of HT to the earth's radius is known. Therefore, with it supposed that the earth's radius is 1^{p}, the moon's radius will be 17′ 33″. And because the shadow's radius contains the moon's radius 2 3/5 times, the shadow's radius will be 45′ 38″ according to this quantity by which the earth's radius is 1^{p}, which we intended.

17. To reveal the quantity of the sun's diameter as well as the distance of its center from the earth's center, with which the quantity of the shadow's axis will also be clear.

Compertum est per aspectum et instrumentum quod cum Luna fuerit in sua maxima distantia, Solem totum tegit nec moram habet integendo. Unde diametri eorum – Solis et Lune dico – eidem angulo vel arcui magni circuli tunc subtenduntur. Quo statuto describam magnum circulum corporis solaris ABG supra centrum D, et circulum magnum corporis terre MLK supra centrum N, et circulum magnum corporis Lune supra centrum T ut prius, et hoc in sua maxima distantia. Et educam lineas contingentes tam Solem quam terram GMS AKS in piramidali figura cuius axis DNS, et contingentes Lunam ad centrum terre N que sint GHN AEN. Et educam rectam TH usque ad Z et sit FN linea equalis linee NT. Et ducam per punctum F diametrum umbre CFQ. Quia ergo FN equatur linee NT, erunt due linee FC et TZ pariter accepte duplum linee MN. Fiunt ergo due partes integre. Subtractis ergo inde FC et HT notis, remanet ZH nota et est lvi minuta et xlix secunda. Est ergo MN ad HZ nota, sed eadem est NG ad HG sive ND ad DT propter similitudinem triangulorum. Secundum quantitatem ergo qua erit ND pars una, erit DT lvi minuta et xlix secunda, et linea TN residua de parte una erit tria minuta et xi secunda. Ergo secundum quantitatem qua erit linea TN lxiiii partes et x minuta et semidiameter terre pars una, erit linea ND m cc et x partes fere, et hec est distantia Solis a centro terre. Et quia proportio GD ad DN est sicut proportio HT ad TN nota, cum sit DN nota, erit quoque DG nota et est v partium et xxx minutorum fere. Continet ergo diameter Solis diametrum terre quinquies et eius medietatem et diametrum Lune decies et occies et insuper iiii quintas eius fere.

Rursum quia proportio MN ad CF cum sit nota est ea proportio quam habet NS ad FS, si constituamus NS partem unam, erit FS sicut FC xlv minutorum et xxxviii secundorum. Ergo FN residuum de una parte erit xiiii minu-

934 aspectum – instrumentum] instrumentum et per aspectum *M* **935** sua] *s.l. P* distantia] ad *add. et del. N* **936** magni circuli] circuli magni *N* **937** circulum] circulum vel *M* solaris] soloris *K* **938** magnum] *om.* P_7 MLK] *corr. ex* MLH *P* LMK P_7 MLR *M* **939** N] *corr. ex* M *K* magnum] *om.* P_7 **940** maxima] magna *PN* **941** terram] *corr. ex* Lunam *N* AKS] ARS *M* piramidali] pitamidali *K* et] *om. P* contingentes Lunam] lineas contingentes Solem et Lunam concurrentes *N* **942** N] *corr. ex* non *K* TH] EH *M* **943** FN] SN *P* umbre] *marg. P* **944** CFQ] EFQ *P* linee[1]] *corr. ex* †Lune† *K* FC] FE *P corr. ex* FZ *K corr. ex* FE *N* **945** Subtractis] subtraxtis *P* **946** FC] FE *P* minuta – secunda] minutorum et 49 secundorum *M* **947** sive] sive ad *P* ND] *corr. ex* AD P_7 **948** erit[1]] erat *K* ND] NO *P* **949** TN] *corr. ex* tamen P_7 erit] *corr. ex* erint *P* **950** TN] *corr. ex* TM *P* **951** ND] *corr. ex* MD *M* m – partes] mille ducente et 10 scilicet 1210 partes *M* cc] *s.l. P* partes] *om. N* fere] fece *K* **952** hec] hoc *M* DN] DM *P corr. ex* DM *K* GN *M* (DM *Ba* GN E_1) **953** est] *om. M* **954** minutorum] minuta *P* diameter – terre] diametrum terre diameter Solis (*the last word corr. ex* terre) P_7 **956** quintas eius] eius quintas P_7 fere] *marg. P* **957** Rursum] rursus P_7 CF] EF *P* **958** constituamus] continuamus *M* erit – sicut] FS erit *M* **959** xxxviii] 48 (vel 38 *add. marg.*) P_7 secundorum] secundarum *PK* **959/960** Ergo – secundorum] *marg.* P_7 **959** una – erit] parte una et *N*

It is found by sight and an instrument that when the moon is at its maximum distance, it covers the whole sun and does not have a delay for the covering. Whence their diameters – I mean of the sun and the moon – subtend the same angle or arc of a great circle at that time. With this established, I will describe the great circle of the solar body ABG upon center D, the great circle of the earth's body MLK upon center N, and the great circle of the moon's body upon center T as before, and this at its maximum distance. And I will draw lines GMS and AKS tangent as much to the sun as to the earth in a pyramidal figure, whose the axis is DNS, and ⟨I will draw⟩ tangents to the moon to the earth's center N, which let be GHN and AEN. And I will draw straight line TH to Z, and let there be line FN equal to line NT. And I will draw the shadow's diameter CFQ through point F. Therefore, because FN equals line NT, the two lines FC and TZ taken together will be double line MN. Therefore, they become wholly 2^p. With known FC and HT subtracted from this, there remains ZH known, and it is 56′ 49″. Therefore, MN to HZ is known, but NG to HG or ND to DT is the same because of the similarity of triangles. Therefore, according to the quantity by which ND will be 1^p, DT will be 56′ 49″ and line TN, the remainder from 1^p, will be 3′ 11″. Therefore, according to the quantity by which line TN will be 64^p 10′ and the earth's radius 1^p, line ND will be approximately 1210^p, and this is the distance of the sun from the earth's center. And because the ratio of GD to DN[63] is as the known ratio of HT to TN, and because DN is known, DG will also be known, and it is about 5^p 30′. Therefore, the sun's diameter contains the earth's diameter 5 1/2 times and the moon's diameter approximately 18 4/5 times.

In turn, because the ratio of MN to CF, because it is known, is that ratio that NS has to FS, if we set up NS as 1^p, FS, as FC, will be 45′ 38″.[64] There-

[63] The reading 'DM' appears to have entered the transmission early and is perhaps an original mistake on the author's part.

[64] FC and FS have the same value here, but the units are different. MN is 1^p for the first, and NS is 1^p for the second.

torum et xxii secundorum. Ergo secundum quantitatem qua erit linea FN lxiiii partes et x minuta et semidiameter terre pars una, erit linea SF cciii partes et l minuta fere, et tota linea NS que est axis totius umbre cclxviii partes iuxta quod semidiameter terre est pars una.

18. Magnitudinem Solis et magnitudinem Lune metiri, et trium corporum Solis, Lune, et terre proportiones adinvicem assignare.

Quoniam enim et Solis et Lune diameter notus est ad positam lineam rationalem semidiametrum terre, et circumferentia magni circuli utriusque nota erit, eo quod pene continet triplum diametri cum adiectione septime partis. Et propter hoc superficies magni circuli utriusque nota, scilicet cum semidiametrum duxeris in semicircumferentiam. Et propter hoc cum diametrum duxeris in aream circuli magni, fiet columpna nota, que sexqualtera est ad speram propositam, ideoque soliditas spere erit nota.

Proportiones vero eorum adinvicem sunt ita. Quia diameter Solis continet diametrum terre quinquies et eius medietatem, diametri vero ad diametrum est proportio que spere ad speram triplicata, que etiam est cubi ad cubum, si diametrum terre ponas partem unam, cum cubus unitatis non sit nisi unum, cubus vero quinque et dimidii est clxvi et quarta et octava unius, manifestum quod magnitudo Solis continet magnitudinem terre centies et sexagies sexcies et insuper eius quartam et octavam. Rursum quia diameter Solis continet diametrum Lune decies et occies et iiii quintas eius, si diametrum Lune constituas partem unam cuius cubus est unum, cum cubus xviii et iiii quintarum sit vi milia et dc et xliiii et dimidium fere, palam quod magnitudo Solis continet magnitudinem Lune sexcies milies et sexcenties et quadragies quater et insuper

960 xxii] *corr. ex* 12 *M* secundorum] secundarum *PK* quantitatem] *om. N* erit] erat P_7 **961/962** et^2 – minuta] *marg.* P_7 **961** cciii] *corr. ex* †...† (*perhaps other hand*) *P* **962** cclxviii] 258 (vel 268 *add. marg.*) P_7 **965** Solis] Solis et *M* proportiones] proportionales *P corr. ex* proportionales *K* **966** enim et] autem *PN corr. ex* autem et *M* (et *Ba* enim et E_1) notus] nota *MN* **967** semidiametrum] semidyameter *M* circumferentia] *corr. ex* circumferentiam *K* magni] *s.l.* (*perhaps other hand*) *P* **968** adiectione] additione P_7 *corr. ex* additione *M* **969** magni – utriusque] utriusque magni circuli P_7 nota] nota erit *PN* (nota BaE_1) semidiametrum] diametrum Lune (*the last word del.*) *P* diametrum *KN* (semydiametrum *Ba om.* E_1) **970** in – duxeris^2] *marg. K* in] aream *add. et del. M* semicircumferentiam] *corr. ex* semidiametrum *P* **971** circuli magni] magni circuli *PN* sexqualtera est] est sesquialtera *N* **972** erit nota] nota erit *MN* **973** ita] *om.* P_7 **975** que etiam] scilicet (*s.l.*) que *N* **975/976** si diametrum] *corr. ex* semidiametrum *P* si ergo dyametrum *MN* **976** partem unam] parte unam erit cubus ipsius unum *N* cubus] *corr. ex* cubis P_7 cubus alias *M* unitatis] unius *N* sit] sit ibi *M* unum] *corr. ex* unus *K* **977** vero] enim *P* autem *N* manifestum] manifestum est *MN* **978** sexagies sexcies] *corr. ex* sexagies septies *K* sexagesies sexies *MN* **979** insuper] *om.* P_7 Rursum quia] rursus quia P_7N rursumque *KM* (rursum quia BaE_1) **980** quintas eius] eius quintas P_7 constituas] constitues *P* **981** unam] *marg. P* **983** sexcies – quadragies] sexies millesies sexcenties et quadragesies *N* sexcies] sexies *K* sexies *corr. ex* sex *M* quater – insuper] *corr. ex* quarta et super *M* fore, FN, the remainder from 1^p, will be 14′ 22″. Therefore, according to the quantity by which line FN will be 64^p 10′ and the earth's radius 1^p, line SF will be approximately 203^p 50′, and the whole line NS, which is the axis of the whole shadow, will be 268^p according to earth's radius being 1^p.

18. To measure the volume of the sun and the volume of the moon, and to designate the ratios of the three bodies of the sun, moon, and earth to each other.

Indeed, because the sun and moon's diameters are known in relation to a posited rational line, the earth's radius, the circumference of a great circle of each will also be known, because it nearly contains the triple of the diameter with the addition of a seventh part. And because of this, the surface area of a great circle of each will be known, i.e. when you multiply the radius by the semicircumference. And because of this, when you multiply the diameter by the area of the great circle, a column will be made known, which is sesquialter [i.e. 3/2 times] the proposed sphere, and for that reason the volume of the sphere will be known.

Indeed, their ratios to each other are thus. Because the sun's diameter contains the earth's diameter 5 1/2 times, and the tripled ratio of the diameter to the diameter is the ratio of the sphere to the sphere,[65] which is also ⟨the ratio⟩ of the cube to the cube, if you suppose the earth's diameter to be 1^p, because the cube of unity is nothing other than 1, and the cube of 5 1/2 is 166 3/8, it is manifest that the sun's volume contains the earth's volume 166 3/8 times.[66] In turn, because the sun's diameter contains the moon's diameter 18 4/5 times, if you set up the moon's diameter as 1^p, whose cube is 1, because the cube of 18 4/5 is approximately 6644 1/2, it is clear that the sun's volume contains the

[65] A reader who did not already know this fact would likely read this as '...but [the ratio] of the diameter to diameter is the tripled ratio that is of sphere to sphere...'

[66] The author may have relied upon Albategni, *De scientia astrorum* Ch. 30 (1537 ed., f. 38v) for the ratio of the sun and earth's volumes, which is more precise than that in the *Almagest*. The author could have easily carried out this calculation himself.

eius medietatem. Rursum quia diameter terre continet diametrum Lune ter et insuper eius duas quintas, si cubum ex diametro Lune ponas unum, erit cubus ex diametro terre surgens xxxix et quarta et xx^{a}. Itaque magnitudo terre continet magnitudinem Lune trigies novies et quartam et xx^{am} eius. Et hee sunt proportiones eorum secundum quod Ptolomeus invenit.

Et constituit Solis et Lune diametros eidem angulo vel arcui subtendi cum et Sol et Luna esset in sua longitudine longiori. Et diametro quidem Solis nullam in quantitate variationem ponit pro diversa eius a terra distantia; Lune autem ponit sicut ostendetur. Et hoc ideo quia Solis multipliciter maior est a terra elongatio quam Lune, et centrum solaris ecentrici parum distat a centro terre; et propter hanc unam causam variatur eius elongatio a terra. At centrum ecentrici lunaris pluribus gradibus distat a centro terre, et preter hoc habet aliam causam remotionis a terra, semidiametrum epicicli; ideoque manifeste sensibilis est varietas remotionis eius et varietas diametri.

Porro Albategni eclipses tam solares quam lunares a se visas et cognitas invenit multum diversificari tam quantitate quam tempore ab hiis eclipsibus sicut per constitutiones et opus Ptolomei accidere debuerunt ut ait. Causam ergo perscrutans de variatione quantitatis eclipsium, dixit minorem esse diametrum Lune in sua longitudine longiore quam qui a Ptolomeo inventus est. Secutus enim viam Ptolomei in huius investigatione per duas eclipses lunares in quibus Luna a longitudine longiore equata in epiciclo pene secundum eundem arcum distabat. Nam in una erat portio equata cxiiii gradus et ix minuta; in alia fuit portio equata cxi gradus et v minuta. Et superfluum de diametro alterius eclipsis ad alteram fuit octava et medietas octave et quarta, et superfluum latitudinum fuit iii minuta et l secunda. Per has inquam eclipses, invenit diametrum Lune tunc esse xxxiii minutorum et xx secundorum fere et medietatem umbre in transitu Lune xliii minuta et xxx secunda fere.

984 Rursum] rursus P_7 **986** et xxa] *marg.* (*perhaps other hand*) *P* terre^{2}] Lune *M* **987** Lune] terre *M* trigies] trigesies *PN* tricesies *M* (trigies BaE_1) xxam] xx^{a} *PK* proportiones] propositiones *P* **988** Ptolomeus] Tholomeus P_7 Tolomeus *K* **989** vel] et P_7 **990** et^{1}] *om. M* diametro] dyameter *M* dyametri *N* **991** in – variationem] *corr.* ex †variationem† *P* pro – distantia] propter diversam eius a terra distantiam *N* diversa] *corr. ex* diametro P_7 **992** ostendetur] ostenditur *PN* **994/995** et – terre] *marg.* P_7 **994** eius] *om. K* **995** preter] propter *KM* hoc] *om.* P_7 **996** semidiametrum] *corr. in* semidyameter *M* **997** sensibilis est] est sensibilis P_7 et] *s.l. N* **1000** Ptolomei] Tholomei P_7 Ptholomei *corr. ex* Tomei *K* ergo] *corr. ex* †...† *N* **1002** qui] que *N* Ptolomeo] Tholomeo P_7 Tolomeo *K* inventus] inventa *N* **1003** Ptolomei] Tholomei P_7 Tolomei *K* **1004** Luna] *om. N* equata] iii gradus et 5 minuta *add. et del.* P_7 **1004/1006** in – minuta] *marg.* P_7 **1004** pene] *corr. ex* Lune *K* **1005** una] uno *P* portio] *corr. ex* proportio *P* proportio *M* ix] xi P_7 **1006** equata] equatata *K* de diametro] diametrorum *corr. ex* diametro *K* **1007** latitudinum] latitudinis *MN* **1008** iii] *corr. ex* tertia *K* secunda] secunde *PK* **1009** xxxiii] 30 (vel 33 *add. s.l.*) P_7 secundorum] secundarum *PK* secunda *M* medietatem] medietatem diametri P_7MN (medietatem *Ba* medietatem dyametri E_1) **1010** minuta] puncta *N* moon's volume 6644 1/2 times. In turn, because the earth's diameter contains the moon's diameter 3 2/5 times, if you suppose the cube of the moon's diameter to be 1, the cube arising from the earth's diameter will be 39 3/10. Accordingly, the earth's volume contains the moon's volume 39 3/10 times. And these are their ratios according to what Ptolemy found.

And he established that the diameters of the sun and moon subtend the same angle or arc when both the sun and the moon are each at their apogee. And indeed he posits no variation in quantity for the sun's diameter for its varying distance from the earth; however, he posits ⟨a variation⟩ for the moon as will be shown. And this for the reason that the elongation of the sun from the earth is many times greater than that of the moon, and the center of the solar eccentric stands only very little away from the earth's center; and because of this single cause, its elongation from the earth varies. On the other hand, the center of the lunar eccentric stands more parts away from the earth's center, and in addition to this, it has another cause of withdrawal from the earth, the epicycle's radius; and for that reason, the change of its withdrawal and the change of its ⟨apparent⟩ diameter are manifestly perceptible.

On the other hand, Albategni found that both solar and lunar eclipses seen and known by him were made very different both in quantity and time from those eclipses as they ought to have happened through Ptolemy's arrangements and work, as he said. Therefore, searching for the cause of the variation of the quantity of the eclipses, he said that the moon's diameter was less in its apogee than that which was found by Ptolemy. For, he followed the way of Ptolemy in this investigation through two lunar eclipses in which the moon stood away from the corrected apogee on the epicycle by nearly the same arc. For in one the equated portion was 114° 9′; in the other, the equated portion was 111° 5′.[67] And the excess of the diameter of the one eclipse to the other was an eighth and half an eighth and a quarter,[68] and the excess of the latitudes was 3′ 50″. I say, through these eclipses, he found that the moon's diameter then was approximately 33′ 20″[69] and that half of the shadow in the moon's passage was about 43′ 30″.

[67] These two values are incorrectly given as 94° 10′ and 91° 5′ in Albategni, *De scientia astrorum*, 1537 ed., f. 37r, but they are also given as here in another part of Albategni's text in *P*, f. 35v.

[68] This phrase would probably be read as meaning 1/8 + (3/4)(1/8), which is 7/32, but for the mathematics to work properly, the meaning must be 1/8.75. The Latin version of Albategni seems to have been confusing or in error as well: 'Alterius vero eclypsis ad alteram superfluum de 8 medietatis et quartae lunaris diametri ...' (*De scientia astrorum* Ch. 30, 1537 ed., f. 37r). Nallino makes more sense of the passage in his Latin translation (Nallino, *al-Battānī*, vol. I, pp. 57–58). Also see Nallino, *al-Battānī*, vol. I, p. 234 for a possible explanation of the derivation of this value.

[69] Nallino, *al-Battānī*, vol. I, p. 58, claims that this last number should be 33′ 30″, but the mistake is found in the Latin translation of Albategni (*De scientia astrorum*, 1537 ed., f. 37r).

Et proportionando hunc diametrum Lune cum motu Lune diverso in una hora tunc, itemque ex motu Lune diverso in longitudine longiore accipiens eandem proportionem, invenit sic diametrum Lune in longitudine longiore esse xxix minutorum et dimidii vice xxxi minutorum et tertie unius minuti que Ptolomeus invenerat. Quare et diametrum umbre dimidium xxxviii minutorum et xx secundorum fere deprehendit, servata scilicet eadem Ptolomei proportione qua semidiameter umbre continet semidiametrum Lune bis et eius tres quintas. Pari modo in omni longitudine Lune quantitatem diametri eius per motum diversum in una hora invenit, scilicet multiplicando eum in sex octava minus et deinde dividendo per vi. Nam huiusmodi proportionem in uno loco primum invenerat. Quare diameter Lune in longitudine propiori erit xxxv minuta et tertia unius minuti. Et per diametrum Lune semidiametrum umbre quem in longitudine longiore Lune fere duobus minutis et tertia minorem ita invenit eo quem Ptolomeus invenerat.

Diametro quoque Solis variationem ponit. Nam cum in sua longitudine longiore, sit xxxi minuta et xx secunde sicut etiam Ptolomeus ponit. Unde totus Sol a Luna numquam occultari potest cum uterque sit in sua longitudine longiore. Proportionatus est etiam hanc quantitatem diametri Solis cum motu diverso Solis in ipsa longitudine longiore ad unam horam, et per hanc proportionem quantitatem diametri eius in omni longitudine sumit, scilicet motum diversum ad unam horam multiplicando in duo et quintam unius, deinde dividendo quod exit per x. Erit ergo cum in sua longitudine propiori Sol fuerit, diameter eius xxxiii minuta et due tertie. Solis igitur diameter respectu diame-

1011 proportionando] *corr. ex* proponendo *K* hunc] hanc *N* **1012** itemque] quia *PN* motu – diverso] motu diverso Lune P_7*K* diverso motu Lune *M* **1013** sic] sed *M* sic diametrum] semidyametrum *N* in] *s.l. K* longitudine] *iter. N* **1014** xxix] *corr. ex* xxx *K* vice] loco *corr. ex* vide *N* tertie] tertia *M* **1015** Ptolomeus invenerat] invenerat Tholomeus P_7 Tolomeus invenerat *K* umbre] umbre et *P corr. ex* umbre †...† *N* dimidium] dimidii P_7 dimidiam *N* **1016** xx] 30 (vel 20 *add. marg.*) P_7 *corr. ex* 30 *M* secundorum fere] secundarum fere *PK* fere secundorum *M* deprehendit] deprehenderat *M* servata] servat *corr. in* servans *K* Ptolomei] Tholomei P_7 Tolomei *K* **1017** semidiameter] *corr. ex* semidiametrum *P* Lune] *om. P* tres] ter *P* **1019** hora] *corr. ex* ora *K* sex] et *add. s.l. N* octava minus] octav†a† unius *M* octavam unius *N* **1020** huiusmodi] huius *N* proportionem] portionem *M* **1020/1021** primum invenerat] invenerat primum *N* **1021** propiori] *corr. ex* longiore *N* **1022** diametrum] semidiametrum P_7 *corr. ex* semidyametrum *M* Lune] *marg. N* quem] *om. N* in] *om. P* **1023** ita] *om. N* **1024** quem Ptolomeus] quod Tholomeus P_7 quod Tolomeus *K* **1025** Diametro] *corr. in* dyametri *M* sua] *om. M* **1026** secunde] secunda P_7 *perhaps other hand K* secunda fere *N* Ptolomeus] Tholomeus P_7 Tolomeus *K* Ptholomeus *N* **1027** occultari] eclipsari *M* **1028** Proportionatus est] proportionatus *M* proportionavit *N* cum] cum ipso *N* **1029** per – proportionem] propter hanc propositionem *K* **1030** sumit] scivit *N* **1030/1032** scilicet – Erit] *marg.* P_7 **1032** exit – x] erit per 10 (*corr. in* 20) *M* Erit] *corr. ex* exit *M* propiori – fuerit] Sol fuerit propiori P_7 propiori] *corr. ex* longiori *M* longiori *N* **1033** xxxiii] *corr. ex* xxxiiii *K*

And by taking the ratio of this diameter of the moon with the moon's irregular motion for one hour at that time, and also taking the same ratio from the moon's irregular motion at apogee, he thus found the moon's diameter at apogee to be 29′ 30″ instead of the 31′ 20″ that Ptolemy had found. Therefore, he also found the half diameter of the shadow to be about 38′ 20″, with Ptolemy's same ratio preserved by which the shadow's radius contains the moon's radius 2 3/5 times. In a like way, he found at every distance of the moon the quantity of its diameter through the irregular motion for one hour, by multiplying it by 5 7/8 and then by dividing through 6.[70] For first he had found a ratio of this kind in one place. Therefore, the moon's diameter at perigee will be 35′ 20″. And through the moon's diameter, ⟨he found⟩ the shadow's radius, which he thus found at the moon's apogee to be about 2′ 20″ less than that which Ptolemy had found.

He also posited a variation for the sun's diameter. For when at its apogee, it is 31′ 20″ as Ptolemy also posits. Whence the whole sun is never able to be concealed by the moon when each is at its apogee. Also, he took the ratio of[71] this quantity of the sun's diameter with the sun's irregular motion at that apogee for one hour, and through this ratio, he took the quantity of its diameter at every distance, by multiplying the irregular motion for one hour by 2 1/5, then by dividing what results by 10.[72] Therefore, when the sun is at its perigee, its diameter will be 33′ 40″. The sun's diameter, therefore, is thus found to vary

[70] This rule is found in the three sets of canons to the Toledan Tables that Pedersen edited: Pedersen, *The Toledan Tables*, Ca186, Cb194, and Cc290, pp. 304–05, 462–63, and 710–11. The wording is more similar to the rules in Ca and Cc than the one in Cb. The rule is equivalent to taking 47/48 (or 58′ 45″ in sexagesimal notation) of the moon's hourly speed to find the moon's apparent diameter. This operation is not quite consistent with the values for the speeds and diameters provided by Albategni (*De scientia astrorum*, 1537 ed., ff. 37r-v). His examples of the diameters of the moon are approximately 58′ 37″ of the corresponding hourly speeds.

[71] Because 'proportionatus est' has a direct object, it appears to be a form of the deponent verb 'proportionor, proportionari.' The word, also used earlier in this proposition, may have been the author's own invention.

[72] This rule is also in the canons to the Toledan Tables: Pedersen, *The Toledan Tables*, Ca185, Cb193, and Cc289, pp. 304–05, 460–61, and 708–11. As with the other rule, the wording is closer to Ca and Cc than to Cb. As it stands, the rule in the *Almagesti minor* produces a result that is 1/60 of the size expected. The canons direct the reader to make an adjustment from seconds to minutes, but even without explicit directions, attentive readers of the *Almagesti minor* would easily see that the need for shifting a place in the sexagesimal system. The ratio implied by this rule (with this correction) is close to one of the ratios that Albategni reports, i.e. 33′ 40″ to 2′ 33″, but not to the other, i.e. 31′ 20 to 2′ 22″ (*De scientia astrorum*, 1537 ed., f. 37v).

tri Lune inter duas longitudines suas duobus minutis et tertia unius minuti diversificari sic invenitur. Item convenit ex hoc ut semidiameter umbre inter utrasque longitudines Solis l fere secundas differentiam habeat. Namque semidiametrum umbre in longitudine Solis propiore minorem quam in longitudine Solis longiore per hanc quantitatem oportet existere.

Secundum hec ergo distantiam centri Solis a centro terre et quantitatem axis umbre Albategni ita invenit. Secundum antedicta cum et Sol et Luna in sua maxima distantia a terra fuerint, Lune diameter in aspectu minor est diametro Solis uno minuto et dimidio et tertia minuti. Huius itaque differentie proportionem ad v minuta et dimidium et tertiam que per diametrum Lune inter longitudinem longiorem et propiorem variatur accepit, et est proportio tertia pars quinta decime minus. Secundum hanc ergo proportionem dempsit de x partibus et tertia partis que sunt diameter epicicli Lune ut ostensum est per que Lune distantia a terra in coniunctionibus et oppositionibus variatur, et quod provenit est tres partes et sexta quinte partis fere. Hoc ergo cum diminutum fuerit de maxima distantia Lune a terra que est lxiiii partes et x minuta ut ostensum est, relinquitur distantia Lune a terra in eo loco ubi diameter Lune est sicut diameter Solis xxxi minuta et tertia unius minuti. Tunc enim totum Solem occultare aspectui potest. Et est lx partes et lviii minuta hec distantia centri Lune a centro terre. Quare tunc erit iuxta assignatam proportionem semidiameter umbre xl minuta et xl secunda. Erit ergo linea ZH ut prius lvi minuta et xlix secunda cuius differentia ad semidiametrum terre MN qui est pars una est tria minuta et xi secunda. Erit ergo proportio DN ad TN sicut partis unius ad tria minuta et xi secundas. Quare longitudo centri Solis a centro terre secundum hec in longitudine Solis longiore est mcxlvi vicibus fere continens semidiametrum terre. Item quia semidiameter terre est ad eam differentiam que est inter

1034 suas] *marg. M* duobus minutis] duo minuta *M* **1035** sic] *corr. ex* non P_7 **1036** Solis] Sol *P* Solis longiorem scilicet et propiorem *M* secundas] secunda *N* habeat] habeant *M* **1037** umbre] *iter. et del. N* **1038** Solis longiore] *corr. ex* Solis propiore *M* longiore Solis *N* **1039** hec] hanc P_7 hoc *MN* **1040** Albategni] *corr. ex* Albatungni *P* et[1]] *om. N* **1040/1041** sua maxima] maxima sua *N* **1041** fuerint] fuerit P_7 est] quam *add. et del. N* **1042** Solis] Solis in *M* differentie] distantie *M* **1043** que – diametrum] per que diameter P_7 **1045** quinta decime] quintadecima P_7N quintadecime *M* (quinta decime BaE_1) minus] unius *P corr. ex* unius *MN* ergo] quoque *KM* proportionem] propositionem *P* dempsit] sumpsit P_7 *corr. ex* sumpsit *M* **1048** sexta quinte] *corr. in* quinta *N* (sexta quinte BaE_1) Hoc] hic P_7 diminutum] diameter P_7 *corr. ex* diameter *K* **1049** distantia – terra] Lune a terra distantia *N* a terra] *marg. P* x] *om. P* **1050** a terra] *om. M* **1051** Solis] *s.l. P* **1052** est lx] est 66 *corr. ex* secunda 60 *M* **1053/1055** Quare – MN] *iter. et del.* P_7 **1053** proportionem] *corr. ex* proportio *K* **1055** secunda] *corr. ex* secundum *K* differentia] *corr. ex* differentias *P* differentiam *K* qui] que *MN* est[2]] et P_7M **1057** secundas] secunda *MN* terre] Lune *M* **1058** hec] hoc *MN* Solis longiore] longiore Solis *N* with respect to the moon's diameter[73] by 2′ 20″ between its two apsides. Also, from this it is agreed that the shadow's radius has a difference of about 50″ between the sun's apsides. For it is necessary that the shadow's radius at the sun's perigee be less than at the sun's apogee by this quantity.[74]

Then, according to these, Albategni thus found the distance of the sun's center from the earth's center and the quantity of the shadow's axis. According to what was said before, when both the sun and moon are at their greatest distances from the earth, the moon's diameter appears smaller than the sun's diameter by 1′ 50″. Accordingly, he took the ratio of this difference to the 5′ 50″ by which the moon's diameter varies between the apogee and perigee, and the ratio is 47/150. Therefore, according to this ratio, he took ⟨a part⟩ away from 10^{p} 20′, which is the diameter of the moon's epicycle as was shown, through which the moon's distance from the earth in conjunctions and oppositions varies, and what results is approximately 3^{p} 2′.[75] Therefore, when this is subtracted from the maximum distance of the moon from the earth, which is 64^{p} 10′ as was shown, there remains the distance of the moon from the earth in that place where the moon's diameter is as the sun's diameter, 31′ 20″. For then it is possible that it conceal the whole sun from sight. And this distance of the moon's center from the earth's center is 60^{p} 58′. Therefore, according to the designated ratio, the shadow's radius at this time will be 40′ 40″. Therefore, line ZH will be as before 56′ 49″, the difference of which to the earth's radius MN, which is 1^{p}, is 3′ 11″. Therefore, the ratio of DN to TN will be as 1^{p} to 3′ 11″. According to these things, therefore, at the sun's apogee, the distance of the sun's center from the earth's center contains the earth's radius approximately 1146 times.[76] Likewise, because the earth's radius is to that dif-

[73] The mention of the moon here adds nothing to the meaning.

[74] The reasoning behind Albategni's 50″ is not made clear here or in his own work (*De scientia astrorum*, 1537 ed., f. 37v).

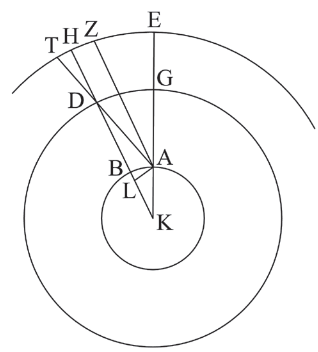
[75] This value should be 3^{p} 12′ to match Albategni (*De scientia astrorum*, 1537 ed., f. 39r) and to work mathematically since it is a better approximation of the 3 107/450 that one reaches with the given values. The incorrect value is likely the author's fault and is perhaps due to his misreading of his copy of Albategni's text, which perhaps had a correction (appearing as '3 partium et "~~sexta~~ quinte"').

[76] Here and below, Albategni, *De scientia astrorum*, 1537 ed., f. 39r has the incorrect value 1156, but the value is given correctly in *P*, f. 37v.

ipsum et semidiametrum umbre sicut NS ad NF que est equalis NT, palam quod axis umbre secundum hoc ccliiii vicibus duabus insuper tertiis superadditis continet semidiametrum umbre. Item cum semidiametrum epicicli Solis, qui est distantia duorum centrorum secundum alium modum, addiderimus super lx idest semidiametrum concentrici, excrescent lxii partes et v minuta secundum inventum Albategni. At hec linea est maxima distantia centri Solis a centro terre scilicet cum fuerit in sua longitudine longiore. At hec linea continet semidiametrum terre ut dictum est mcxlvi vicibus; ergo semidiameter epicicli continet semidiametrum terre xxxviii vicibus, qui duplicatus facit lxxvi. Solis itaque distantia terre propior continet semidiametrum terre mlxx vicibus, eiusque distantia media mcviii, longitudo vero longior mcxlvi. Et Luna quidem totum Solem occultat cum eius a Sole distantia semidiametrum terre mlxxxv fere vicibus amplectitur. Atque hee proportiones quantitatum diametrorum et distantiarum solaribus eclipsibus visis ut ait Albategni respondent. Manifestum ex hiis sicut in Luna ad quamlibet notam elongationem Solis in epiciclo a longitudine longiore, notam quoque esse centri eius a centro terre sicut in Luna distantiam.

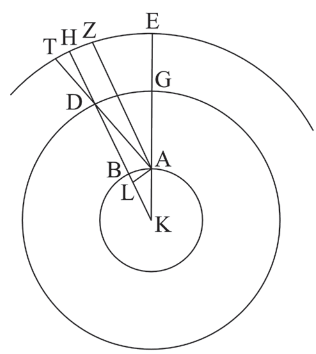
19. Diversitatem aspectus Lune et Solis in circulo altitudinis – quamvis Solis modica sit – ad omnem a centro terre distantiam notam et ad quamlibet a cenit capitum elongationem certam demonstrare.

Resumpta paulo ante premisse simili figura cum notis et habitudinibus suis, ponemus arcum GD qui est elongatio sive Solis sive Lune a cenit capitum notum scilicet xxx vel plurium vel pauciorum pro libito partium. Et sumemus lineam KD notam pro libito quotlibet partium sicut contingit. Nam est distantia sive Solis sive Lune a centro terre. Et investigabimus quantitatem arcus TH. Itaque quia notus est arcus DG, notus est angulus AKL.



1060 ipsum] ipsam P_7N semidiametrum umbre] umbre semidyametrum *N* **1062** semidiametrum[1]] semidyametrum terre alias *M* semidiametrum[2]] semidyameter *M* **1063** addiderimus] addidimus *N* **1064** lx idest] lxi P_7MN (61 BaE_1) **1065** inventum] *corr. in* inventionem *M* inventionem *N* **1066** cum fuerit] *s.l. P* hec] hec illa *N* **1067** semidiameter] semidiametrum *P* **1068** qui duplicatus] que duplicata *N* **1070** mcviii] 1108 vicibus *MN* mcxlvi] mclxv (vel xlvi *add. marg.*) P_7 *corr. ex* mxlvi *K* **1071** distantia] distantia per *M* mlxxxv] *corr. ex* mcxxxv *K* **1072** fere] *om. N* **1073** visis] *corr. ex* visit P_7 **1074** epiciclo] epicicli *M* **1078** ad[2]] *corr. ex* †...† *M* **1080** Resumpta] *corr. ex* de sumpta P_7 **1081** ponemus] ponamus *N* **1082** qui] que *M* **1083** a] ad *M* **1084** pauciorum] paucorum P_7 *corr. ex* paucorum *K* paucarum *M* (pauciorum BaE_1) libito] libitu *N* **1085** sumemus] summemus *M* sumamus *N* libito] libitu *N* **1086** distantia] *corr. ex* differentia P_7 **1087** investigabimus] investigaverimus *M* **1089** AKL] *corr. ex* A†...†L *M* ference that is between it and the shadow's radius as NS to NF, which is equal to NT, it is clear that the shadow's axis contains the shadow's[77] radius according to this 254 2/3 times. Also, when we add the radius of the sun's epicycle, which is eccentricity according to the other model, to 60^{p},[78] i.e. the concentric's radius, there grows out 62^{p} 5′, according to the findings of Albategni. But this line is the greatest distance of the sun's center from the earth's center, i.e. when it is at its apogee. But this line contains the earth's radius, as was said, 1146 times; therefore, the epicycle's radius contains the earth's radius 38 times, which doubled makes 76. Accordingly, the sun's nearest distance to the earth contains the earth's radius 1070 times, and its mean distance 1108, and indeed its apogee 1146. And the moon indeed covers the whole sun when its distance from the sun includes the earth's radius about 1085 times. And these ratios of the quantities of the diameters and distances answer to the observed solar eclipses, as Albategni said. It is manifest from these things as with the moon, that for whatever known elongation of the sun from the apogee on its epicycle, the distance of its center from the earth's center is also known as with the moon.

19. To show the moon and sun's parallax on the circle of altitude – although the sun's may be modest – for every known distance from the earth's center and for whatever known elongation from the zenith.



With a figure taken up again similar to the one given a short while before [i.e. identical to V.13's figure] with its points and dispositions, we will suppose arc GD, which is the elongation of either the sun or the moon from the zenith, to be known, i.e. 30° or more or fewer degrees as you wish. And let us take line KD known of however many parts you may wish as it happens. For it is the distance of either the sun or the moon from the earth's center. And we will search for the quantity of arc TH. Accordingly, because arc DG is known, angle AKL is known. Therefore,

[77] This should be 'earth's.' The value reached here agrees with that of Albategni. Note that the shadow's radius in earth radii is not 40′ 40″, but 45′ 38″. Albategni reaches his result by using a different proportionality that involves the ratio of the radii of the earth and the sun, not the shadow's radius (*De scientia astrorum*, 1537 ed., f. 39r; the printed version has errors in values, so also see *P*, f. 37v).

[78] The mistaken reading 'lxi' for 'lx idest' that is found in many of the witnesses would have been easy to have been made and must have entered the transmission early.

Cum ergo angulus ad L sit rectus, nota est proportio linee AK que est pars una ad AL KL. Subtracta ergo KL a KD nota erit reliqua DL nota sicut AL. Propter hoc etiam erit angulus ADL notus, et ipse est equalis angulo DAZ; quare arcus TZ est notus. Sed arcus THZ non est maior arcu TH secundum sensibilem quantitatem quoniam tota terra aput orbem EZHT fuit sicut punctum. Est ergo TH qui est diversitas aspectus in altitudinis circulo notus.

Est itaque diversitas aspectus Solis in maxima elongatione eius a terra secundum quod ponit Ptolomeus et in elongatione Solis a cenit capitum xxx graduum, diversitas hec inquam est unum minutum et xxv secunda. Et in distantia Lune a terra maxima que est terminus primus a Ptolomeo positus cum arcus GD sit xxx graduum, est diversitas aspectus xxv minuta et ix secunda. Et cum fuerit Lune distantia a centro terre liii partes et l minuta que est terminus secundus, erit diversitas xxxii minuta et xxvii secunda. Et cum fuerit Lune distantia a centro terre xliii partes et liii minuta que est terminus tertius, erit diversitas aspectus xl minuta. Et cum fuerit Lune a centro terre distantia xxxiii partes et xxxiii minuta que est terminus quartus, erit diversitas aspectus Lune in circulo altitudinis lii minuta et xxx secunda. Ideo vero hos terminos distantiarum Lune et Solis excepi quia secundum eos ponit Ptolomeus tabulas diversitatum aspectus.

Cum autem per operationis methodum diversitatem aspectus Lune in altitudinis circulo scire volueris – et hoc quidem cum Luna in circulo signorum fuerit sine latitudine, nondum enim scimus cum latitudinem habet remotionem eius a cenit capitum, primum disce distantiam eius a centro terre, cuius facilis est notitia cum via operationis equandi Lunam quam post expositionem none propositionis diximus, lineam EH semper cognoveris que est distantia Lune a terra iuxta quantitatem partium qua semidiameter epicicli est v partes et xv

1090 proportio] propior *P* **1091** AL[1]] AL et *N* nota[2]] *om. N* **1093** TH] TK *P corr. ex* TK *K* **1094** EZHT] ZEHT *M* **1095** qui] que *M* **1096** elongatione eius] elongatione *M* eius elongatione *N* **1097** Ptolomeus] Tholomeus P_7 Tolomeus *K* et] *om. M* cenit] czenit *M* xxx] 90 *N* **1098** inquam] unquam *K* xxv] *corr. in* 29 *M* **1099** a terra] ad terram *M* terra] terre P_7 terminus] *corr. ex* tres *M* Ptolomeo] Tholomeo P_7 Tolomeo *K* **1100** xxx] 90 *N* graduum] gradus *M* xxv] 26 *M* 27 *N* ix] 20 *M* **1101** centro terre] terre P_7*K* terra *M* **1102** diversitas] *corr. ex* distantia *N* xxxii] *corr. ex* xxxiii *K* **1103** terminus tertius] tertius terminus *N* **1104** a – distantia] distantia a centro terre *M* **1105** xxxiii[1]] xxx (vel xxxiii *add. marg.*) P_7 xxxiii[2]] *corr. ex* 13 *M* terminus quartus] quartus terminus *N* **1106** et] *om. P* xxx secunda] *corr. ex* xxxii P_7 Ideo] *corr. ex* iam *M* **1107** Lune – Solis] Solis et Lune *N* Ptolomeus] Tholomeus P_7 Tolomeus *K* **1108** diversitatum] diversitatis *MN* **1110** et – quidem] *iter. et del. N* **1111** cum] dum *N* **1111/1112** remotionem eius] eius remotionem *N* **1113** notitia] notia *P* cognitio *N* cum] in *M* operationis] *om. N* post expositionem] prius expositioni *M* none] nove *P* **1114** propositionis] proportionis *P corr. ex* proportionis P_7 EH] EB *MN* (EH *Ba* EB E_1) **1115** qua] quam PP_7 *corr. ex* quam *K* quarum *N* (qua BaE_1) semidiameter] semidiametrum P_7

because the angle at L is right, the ratio of line AK, which is 1^p, to AL and KL is known. With KL subtracted from KD, therefore, the remainder DL will be known, as will AL. Because of this, angle ADL will also be known, and it is equal to angle DAZ; therefore, arc TZ is known.[79] But arc THZ is not greater than arc TH by a perceptible quantity because the whole earth was as a point compared to circle EZHT. Therefore, TH, which is the parallax on the circle of altitude, is known.

Accordingly, the sun's parallax at its greatest elongation from the earth according to what Ptolemy posited and at the sun's elongation of 30° from the zenith – this parallax, I say, is 1′ 25″. And at the moon's greatest distance from the earth, which is the first term given by Ptolemy, when arc GD is 30°, the parallax is 25′ 9″.[80] And when the moon's distance from the earth's center is 53^p 50′, which is the second term, the parallax will be 32′ 27″. And when the moon's distance from the earth's center is 43^p 53′, which is the third term, the parallax will be 40′. And when the moon's distance from the earth's center is 33^p 33′, which is the fourth term, the moon's parallax on the circle of altitude will be 52′ 30″. Indeed I follow these terms of the distances of the moon and sun because Ptolemy lays down the tables of parallax according to them.

Moreover, when you want to know the moon's parallax on the circle of altitude through the method of operation – and this indeed when the moon is on the ecliptic without latitude, for we do not yet know its distance from the zenith when it has latitude[81] – first, learn its distance from the earth's center, the knowledge of which is easy because by the way of operation of correcting the moon that we declared after the exposition of the ninth proposition [i.e. V.9], you always know line EH [in V.9's figure], which is the moon's distance from the earth according to the quantity of parts by which the epicycle's radius is 5^p 15′. For, when you have this line in the way said there, subtract 1′ from whatever the degree and 1″ from whatever the minute,[82] and there will be the

79 Point A can be treated as the center of circle EHT.

80 This number should be 27′ 9″ to match the *Almagest*.

81 This is treated near the end of Book V in V.22–25.

82 This means to take 59/60 of the distance according to the first units to get the distance according to the second units. The reason for taking such a ratio and conversions for key distances according to this ratio (or ones approximately equal) are found at the end of *Almagest* V.13 and in *Almagesti minor* V.14. The reasoning for this conversion is incorrect or at best unclear in Plato of Tivoli's translation (Albategni, *De scientia astrorum* Ch. 39, 1537 ed., f. 48v), pp. 'In premissis autem longiorem egressi circuli lunaris a centro terrae longitudine 60 partium fore iam depraehensum est. Cumque diametri terre medietas unius partius fuerit, erit Lunae a terrae superficie longitudo 59 partium. Eruntque ex illa quantitate, illae 5 partes et quarta quae sunt diametri circumvolubilis circuli medietas, 5 partes et 6.' The inclusion of the word 'superficie' makes it appear as if a mere subtraction of the earth's radius were taking place, which would not require a conversion into different units. Nallino's Latin translation of this passage is even more unclear or wrong (Nallino, *al-Battānī*, vol. I, p. 78).

minuta. Cum enim hanc lineam modo ibi dicto habueris, a quolibet gradu unum minutum subtrahe et a quolibet minuto secundum unum, eritque distantia centri Lune a centro terre secundum quantitatem qua semidiameter terre est pars una. Talis est enim proportio istarum partium ad illas. Deinde elongationem gradus Lune in quo est a polo orizontis ex opere xxxv^{e} secundi libri vel ex tabulis ad hoc in climate constitutis addisce. Huius ergo arcus cordam mediatam et cordam mediatam illius arcus qui ei ad perfectionem quarte deficit que est corda altitudinis Lune sume, et per lx utramque divide – hoc est redigere ad posteriorem differentiam sumendo per unum gradum unum minutum. Quodque ex corda altitudinis provenerit de distantia Lune a centro terre minue. Et reliquum in se ductum super id quod ex corda elongationis exiit in se etiam ductum adde, et aggregati radicem extrahe. Post hoc ad minuta corde elongationis rediens, in lx multiplica et per radicem aggregati divide. Et exibunt minuta et secunda que arcuabis. Nam arcus qui provenerit erit diversitas aspectus in circulo altitudinis.

Quod si diversitatem Solis velis in circulo altitudinis, similiter distantiam centri Solis a centro terre accipe, cuius facilis est cognitio cum via operationis equandi Solem quam in opere xvii^{e} propositionis libri tertii diximus, lineam que tociens servata radix dicitur cognoveris. Nam ipsa est distantia Solis a centro terre iuxta quantitatem partium qua id quod est inter duo centra est due partes et v minuta fere. Cum enim hanc lineam modo in dicto habueris, in xviii partes et xlvi minuta et xx secunda multiplica, eritque distantia centri Solis a centro terre iuxta quantitatem qua semidiameter terre est pars una. Nam talis est proportio istarum partium ad illas.

Si vero tabulare volueris has diversitates aspectus subtili compendio Ptolomei, ix lateraliter ordinabis tabulas et in unaquaque xc scalas. Atque in prima

1117 secundum unum] unum secundum *MN* **1118** qua semidiameter] *corr. ex* †quase† diameter *K* **1119** est enim] enim est *M* istarum partium] illarum partium *corr. ex* talium partium *N* **1121** in climate] inclinate *P* **1122** mediatam[2] – arcus] illius arcus mediatam *N* mediatam[2]] *om. M* **1123** utramque] utrumque *M* **1124** ad] *corr. ex* a *M* per – gradum] pro uno gradu *M* per uno gradu *N* **1125** provenerit] proveniet *N* centro terre] terre centro *N* **1127** etiam] *om. N* hoc] hec P_7K (hec *Ba* hoc E_1) **1128** rediens] redigens *P* lx] xl P_7K (60 BaE_1) et[1]] et productum *N* aggregati] inventam *N* **1129** arcuabis] *in other hand where scribe left space* P_7 provenerit] proveniet *N* **1131** diversitatem] diversitatem aspectus *N* **1133** opere] expositione *N* xviie] x^{o}viii^{e} P_7 propositionis] proportionis *P corr. ex* proportionis *K* libri tertii] tertii libri *PN* **1134** servata] *corr. ex* servuta *K* cognoveris] *corr. ex* cognitionis *K* **1135** terre] terre similis Lune *M* id] illud *N* **1136** in] ibi *M om. N* habueris] habueris et *M* **1137** et[1]] *om. M* centri] *om.* P_7 **1138** qua] quam *K* **1139** istarum] illarum *M* illas] reliqua fac sicut in Luna *add. M* **1140** compendio] *corr. ex* compendi P_7 *corr. ex* compoto *K* Ptolomei] Tholomei P_7 Tolomei *K* **1141** lateraliter] literaliter *corr. ex* †t†aliter *P* literarum P_7 *corr. ex* †taliter† *K* (latera *Ba* lateraliter E_1) ordinabis tabulas] *corr. ex* ordinabulas P_7 xc] xx P_7 **1141/1142** prima tabula] tabula prima *M* distance of the moon's center from the earth's center according to the quantity by which the earth's radius is 1^p. For such is the ratio of these parts to those. Then learn the elongation of the degree in which the moon is from the horizon's pole from the work of the 35^{th} ⟨proposition⟩ of the second book or from the tables set up for this in the clime.[83] Therefore, take the sine [*lit*, half chord] of this arc and the sine of its complement, which is the chord of the moon's altitude, and divide each by 60 – i.e. return to the next 'difference' [i.e. sexagesimal place] by taking 1′ for 1^p. And from the moon's distance from the earth's center, subtract what results from the sine [*lit.*, chord] of the altitude. And add the remainder multiplied by itself to that which resulted from the sine [*lit*, chord] of elongation multiplied by itself also, and extract the root of the sum. After this, returning to the minutes of the sine [*lit.*, chord] of elongation, multiply them by 60 and divide by the root of the sum. And there result minutes and seconds, which you will arc. For the arc that results will be the parallax on the circle of altitude.

But if you want the sun's parallax on the circle of altitude, similarly take the distance of the sun's center from the earth's center, the knowledge of which is easy because by the way of operation of correcting the sun that we declared in the work of the 17^{th} proposition of the third book, you know the line that is called so many times 'the saved root.' For that is the distance of the sun from the earth's center according to the quantity of parts by which the eccentricity is about 2^p 5′.[84] For when you have this line in the said manner, multiply it by 18^p 46′ 20″,[85] and there will be the distance of the sun's center from the earth's center according to the quantity by which the earth's radius is 1^p. For such is the ratio of these parts to those.[86]

And indeed, if you want to make a table of these parallaxes for a precise compendium of Ptolemy, you will arrange columns nine across and 90 rungs in

83 *Almagest* II.13.

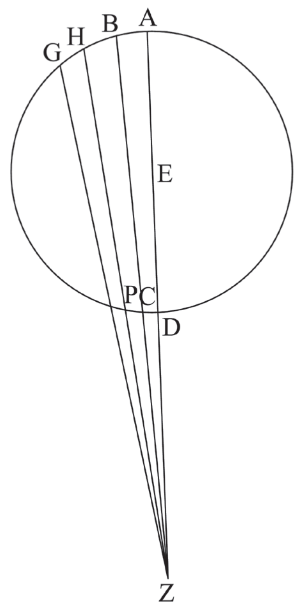
84 This value of the eccentricity is an approximation of Albategni's, which our author reported in III.11. Ptolemy's is 2^p 30′.

85 Albategni, *De scientia astrorum*, 1537 ed., f. 49v, has 18 46′ 50″; however, the value is found as here in *P*, f. 47r. While this is number is close to the ratios of Albategni's values of the sun's sphere in its own right and in terms of the earth's radius, reported in *Almagesti minor* V.18, it is not clear exactly how Albategni derived it. Nallino, *al-Battānī*, vol. I, p. 256, attempted to explain it.

86 There seems to be an error in the numbers or reasoning here. According to Albategni's numbers reported here in V.18, the eccentricity is 38 times the earth's radius, so one of the parts by which the eccentricity is 2^p 5′ should be 18 6/25 earth radii.

tabula ponuntur numeri communes per quos intratur in tabulas alias, numeri scilicet partium elongationis Solis vel Lune a cenit capitum, portionis equate Lune, medie distantie Solis et Lune cum a coniunctione vel oppositione quecumque propior fuerit accepta sit. In secunda tabula ponuntur ex ordine omnes diversitates Solis cum in longitudine longiore Sol fuerit et hoc secundum opus Ptolomei. In tertia vero ordinantur omnes diversitates Lune cum fuerit in termino primo. In quarta diversitatum superfluitates termini secundi super diversitates termini primi. Porro in quinta statuuntur diversitates omnes termini tertii, et in sexta superfluitates ab hiis termini quarti.

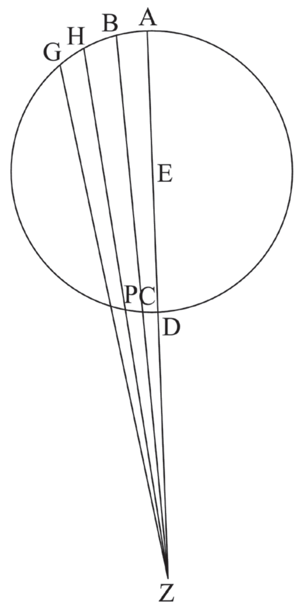
Centro itaque epicicli Lune in longitudine longiore ecentrici constituto et Luna in longitudine longiore epicicli, sufficit per numerum partium elongationis a cenit capitum intrare in tabulam tertiam. Nam quod ibi inventum fuerit est diversitas aspectus quesita. Si vero Luna in longitudine propiore epicicli fuerit, intrandum in tabulam quartam et tertiam et quod inventum in eis fuerit est tunc diversitas aspectus cum simul aggregatum fuerit. Eodem modo concipe de tabula quinta et sexta cum centrum epicicli in longitudine propiore ecentrici fuerit, Luna quidem in longitudine longiore epicicli et in longitudine propiore.



Quid in tabula septima ponatur et octava ex figura cognosces. Sit epiciclus Lune ABG super centrum E quod sit longitudo longior ecentrici, et Z centrum terre. Cum ergo Luna fuerit super punctum B vel H vel G, minuitur linea ZB vel ZH vel ZG que est tunc distantia Lune a centro terre a linea ZA que est maxima distantia. Et differentia que remanet confertur cum linea DA secundum quantitatem quam hic videtur habere.

1143 vel] *corr. ex* et *P* et *K* capitum] capitis *M* **1144** a coniunctione] coniunctione *corr. ex* adiunctione *M* **1146** hoc] hec P_7 **1147** Ptolomei] Tholomei P_7 Tolomei *K* fuerit] fiunt *K* termino] *corr. ex* tertio P_7 **1148** diversitatum superfluitates] superfluitates diversitatum P_7K *corr. in* diversitates superfluitatum *M* **1151** Lune] *om. N* **1155** intrandum] intrandum est *N* **1156** tertiam] *corr. ex* quartam *M* **1156/1157** inventum – tunc] in eis tunc inventum fuerit est *N* inventum – eis] in eis inventum P_7 **1157** in eis] *corr. ex* †...† *P* **1158** concipe] *iter.* P_7 **1160** in – fuerit] fuerit in longitudine propiore ecentrici *M* **1163** Quid] quod *N* qu†i†t *M* tabula septima] septima tabula *M* ponatur – octava] et octava ponitur *N* **1164** Sit] sit enim *M* **1165** sit] †...† *add. et del. P* **1166** Z] etiam *P* fuerit] *om. PN* **1167** G] G est *N* **1168** est tunc] tunc est *N* **1169** distantia] distantia Lune a centro terre a linea ZA que est maxima distantia *P* distantia Lune a centro *M* distantia Lune a centro terre *N* (distantia BaE_1) **1171** videtur] videntur *PN* each.[87] And in the first table are placed the common numbers through which the other columns are entered, i.e. the numbers of degrees of the sun or moon's elongation from the zenith, the moon's equated portion, the mean distance between the sun and moon when it is taken from a conjunction or opposition, whichever is closer. In the second column are placed in order all the sun's parallaxes when the sun is at apogee, and this is according to the work of Ptolemy. And indeed in the third are arranged all the moon's parallaxes when it is in the first term. In the fourth are the excesses of the parallaxes of the second term over the parallaxes of the first term. In turn, in the fifth are set up all the parallaxes of the third term, and in the sixth, the excesses of the fourth term from these.

Accordingly, with the center of the moon's epicycle set up at the eccentric's apogee and the moon at the epicycle's apogee, it is sufficient to enter the third column through the number of degrees of elongation from the zenith. For what is found there is the sought parallax. However, if the moon is at the epicycle's perigee, the fourth and third columns must be entered, and when what is found in them is collected together, it is the parallax at that time. In the same way, think about the fifth and sixth columns when the epicycle's center is at the eccentric's perigee and indeed the moon is at the epicycle's apogee and perigee.



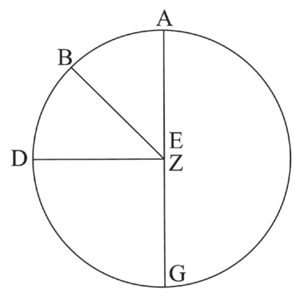
You may learn what is placed in the seventh and eighth columns from a figure. Let the moon's epicycle be ABG upon center E, which is the eccentric's apogee, and Z the earth's center. Therefore, when the moon is at point B, H, or G, line ZB, ZH, or ZG, which is then the distance of the moon from the earth's center, is subtracted from line ZA, which is the greatest distance. And the difference that remains is compared with line DA according to the quantity that is seen to hold here. And according to this ratio,

[87] Ptolemy's table has 45 rows increasing by steps of two from 2 to 90.

Atque secundum hanc proportionem sumitur numerus minutorum de lx, et hec sunt que per progressum graduum portionis per binarium crescentium collecta in septima tabula disponuntur. Item sit centrum E longitudo propior ecentrici. Differentia ergo ZB vel alterius linee sequentis ad distantiam duorum graduum epicicli ad lineam ZA sumitur semper, et cum linea DA confertur secundum quantitatem cuius hic apparet. Et secundum huius collationis proportionem minuta de lx sumuntur, et hec sunt que in octava tabula digeruntur.

Quotiens itaque centrum epicicli in longitudine longiore ecentrici fuerit et Luna a longitudine longiore epicicli distiterit, cum portione equata intrandum in septimam tabulam, et minuta inventa quantum de lx fuerit observandum. Et tantumdem de hoc quod in quarta tabula cum elongatione a cenit inventum fuerit accipiendum, et super id quod in tertia est addendum. Quotiens vero centrum epicicli in longitudine propiore ecentrici fuerit et Luna a longitudine longiore epicicli distiterit, similiter per octavam, sextam et quintam tabulam operandum, eo quod sicut differentia distantiarum se habet ad diametrum epicicli, que est maxima differentia distantiarum, sic prope verum se habet superfluitas diversitatis aspectus illius distantie ad hanc superfluitatem diversitatis aspectus que est minime distantie. Nota quod cum portionis equate dimidio intrandum est eo quod numerus non crescit nisi ad xc vice graduum clxxx.

Quid deinceps in nona tabula conscribatur in figura videbis. Sit ecentricus Lune ABD supra centrum E, et sit A longitudo longior, G propior, et Z centrum terre, a quo linee plurime ZB ZD secundum distantiam semper duorum graduum orbis signorum. Igitur centro epicicli existente aput punctum B vel D minuitur linea ZB vel ZD nota a linea ZA. Et hec differentia inventa confertur cum maxima differentia que est ZA ad ZG, et secundum proportionem huius collationis accipiuntur minuta de lx. Et hec sunt que ponuntur in nona tabula.

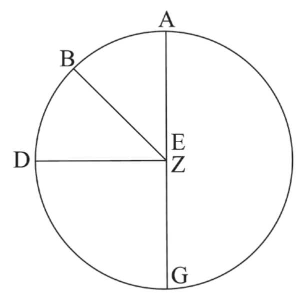


1172 lx] xl (vel lx *add. marg.*) P_7 hec] hoc *M*
1173 per binarium] *om. N* crescentium] tres centium *P* **1174** disponuntur] ponuntur *N* **1175** ergo] *corr. ex* †vero† *K* **1178** digeruntur] diriguntur *M* disponuntur *N* **1179** fuerit] *om. N* **1180** epicicli distiterit] epicicli (*marg.*) destiterit *P* intrandum] intrandum est *MN* **1181** septimam] octavam P_7 inventa] inventa et *M* fuerit] fuerint *KN* (fuerit BaE_1) **1182** tantumdem] *corr. ex* tandem *M* tabula] ponitur *add. et del. N* **1183** in] tabula *add. et del. M* **1184** propiore] *corr. ex* longiore *K* **1185** longiore] *s.l. K* distiterit] destiterit *P* sextam] *corr. ex* septimam *M* **1185/1186** tabulam operandum] tabulas operandum est *N* **1187** differentia] *corr. ex* distantia *M* **1188** diversitatis[1]] *corr. ex* diversitas P_7 **1189** distantie] distantie est *M* **1190** xc] *corr. ex* 60 *N* **1191** Quid] quod *MN* conscribatur] *om.* P_7 videbis] videbitur *M* **1195** signorum] signorum educantur *N* **1197** ZA] ea *K* inventa] *om. N* **1200** hec] hoc *M* sunt] servant (vel sunt *add. s.l.*) P_7 **1201** tabula] *om.* P_7

a number of minutes is taken from 60, and these, collected by an increase of 2° [*lit.*, of a portion of degrees growing by 2], are what are laid out in the seventh table. Likewise, let center E be the eccentric's perigee. Therefore, the difference of ZB or another of the following lines at intervals of 2° on the epicycle are always taken to line ZA, and it is compared with line DA according to the quantity of which it appears here. And minutes are taken from 60 according to the ratio of this comparison, and these are what are distributed in the eighth column.

Accordingly, when the epicycle's center is at the eccentric's apogee and the moon stands away from the epicycle's apogee, the seventh column must be entered with the equated portion, and it must be seen how much the found minutes are of 60. And as much must be taken from what was found in the fourth column ⟨entered⟩ with the elongation from the zenith, and to that must be added what is in the third. And indeed, when the epicycle's center is at the eccentric's perigee and the moon stands away from the epicycle's apogee, one should operate similarly through the eighth, sixth, and fifth columns, because as the difference of the distances are to the epicycle's diameter, which is the greatest difference of the distances, thus approximately is the excess of the parallax of that distance to that excess of parallax which is of the least distance. Note that it must be entered with the half of the equated portion because the number grows only to 90 instead of 180°.

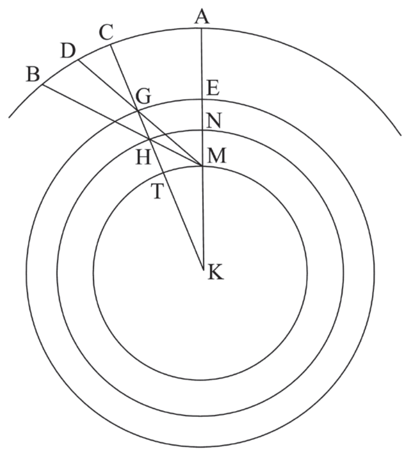
You will see what is written following this in the ninth column in a figure. Let the moon's eccentric be ABD upon center E, and let A be the apogee, G the perigee, and Z the earth's center, from which there are several lines ZB and ZD, always according to a distance of 2° of the ecliptic. Accordingly, with the epicycle's center being at point B or D, known line ZB or ZD is subtracted from line ZA. And this found difference is compared with the greatest difference, which is between ZA and ZG, and according to the ratio of this comparison, minutes are taken from 60. And these are what are placed in the ninth column.



Quotiens itaque centrum epicicli fuerit inter longitudinem longiorem et longitudinem propiorem ecentrici, intrandum cum longitudine duplici dimidiata idest cum distantia media in tabulam nonam que circuli egressi intitulatur. Et accipiendum quantum minuta ibi inventa fuerint de lx, et secundum eorum proportionem de superfluo quod inter quintam et tertiam tabulam fuerit cum predicto modo equate fuerint accipiendum. Et quod de superfluo exierit tertie tabule equate, sicut dixi, addendum. Et erit diversitas aspectus in circulo altitudinis eo quod sicut differentia aliarum distantiarum epicicli a centro terre ad differentiam maximam sic superfluitas diversitatis aspectus propter illam distantiam accidens ad superfluitatem per differentiam maximam eveniens prope verum se habet. Et ista quidem acceptio diversitatis aspectus inter assignatos terminos cadens non excedit verum, sed in minimo potest deficere a vero.

20. Diversitatem aspectus Lune ad Solem in circulo altitudinis presto sumere.

Evidentie causa describo circulum terre MT, et circulum Lune NH, et circulum Solis EG, et circulum in celo ADB ad quem terra est sicut punctum. Et sit KA vadens ad cenit capitum, et Sol in puncto G, et Luna in puncto H super eandem lineam KHGC. Palam ergo quod diversitas aspectus Solis est arcus DC, diversitas aspectus Lune in circulo altitudinis est arcus BC. Cum ergo subtractus arcus DC ab arcu BC, relinquitur arcus BD qui est diversitas aspectus Lune ad Solem. Et hac quidem diversitate opus est nobis in eclipsibus solaribus.



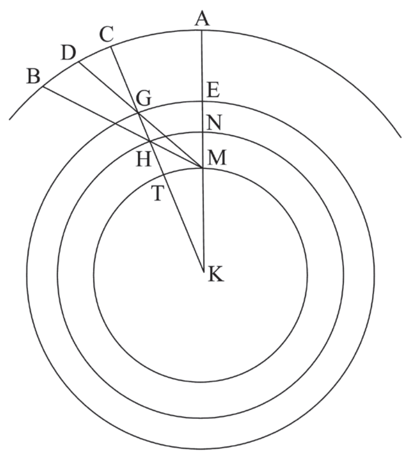
Diversitatem vero aspectus Solis in circulo altitudinis, si ex tabula Ptolomei ad hoc constituta scire velis secundum opus Albategni, cum elongatione Solis a

1202 longitudinem[2]] longiorem *KM* **1205** quantum] quot *corr. ex* quam *M* **1205/1206** eorum proportionem] proportionem eorumdem *N* **1206** quod] quod est *N* fuerit] *corr. ex* fuerint *K* **1207** tertie] tunc P_7 **1209** aliarum distantiarum] distantiarum (*s.l*) aliarum *P* distantiarum duarum aliarum *N* **1210** differentiam] *corr. ex* distantiam *N* **1210/1211** sic – maximam] *marg.* (*perhaps other hand*) *P* **1211** accidens] *corr. in* a cenit *M* per] propter *M* prope] se *add. et del. K* **1212** aspectus] *om. N* **1213** deficere – vero] †discendere† (*del.*) a vero deficere et cetera *N* **1215** causa] cause *N* **1216** Lune] *s.l. N* **1217** EG] *corr. ex* EH *N* **1218** ADB] ABD P_7 ACDB *corr. ex* ABGD *M* **1219** sit] linea *add. marg. M* **1221** KHGC] KHGE *PN* **1223** DC] DE *N* diversitas] diversitas autem *N* Lune] *marg.* P_7 **1224** BC] BE *N* **1225** subtractus arcus] subtractus fuerit (*s.l.* P_7) arcus P_7N arcus subtractus *M* DC] DE *N* BC] BE *corr. ex* DBE *N* **1230** Ptolomei] Tholomei P_7 Tolomei *K* **1231** Solis] *om. N*

Accordingly, when the epicycle's center is between the eccentric's apogee and perigee, the ninth column, which is entitled 'of the eccentric circle', is to be entered with the half of the duplex longitude, i.e. with the mean distance ⟨between the sun and moon⟩. And how many minutes are found there are to be taken from 60, and according to their ratio there must be taken from the excess that is between the fifth and third columns when they have been corrected according to the said way. And what results from the excess must be added to the third column corrected, as I said. And it will be the parallax on the circle of altitude, because as the difference of the other distances of the epicycle from the earth's center is to the greatest difference, thus approximately is the excess of the parallax occurring because of that distance to the excess resulting from the greatest difference. And indeed, that taking of the parallax falling between the designated terms does not exceed the truth, but it is able to fall short of the truth by the smallest amount.

20. To obtain at hand the parallax of the moon to the sun on the circle of altitude.

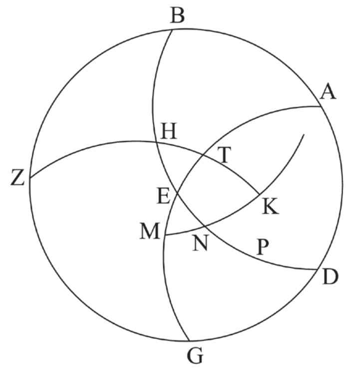
For the sake of clarity, I describe the earth's circle MT, the moon's circle NH, the sun's circle EG, and the circle in the heavens ADB, to which the earth is as a point. And let there be KA going to the zenith, let the Sun be at point G, and let the moon be at point H upon the same line KHGC. Therefore, it is clear that the sun's parallax is arc DC, and the moon's parallax on the circle of altitude is arc BC. Therefore, when arc DC is subtracted from arc BC, there remains arc BD, which is the parallax of the moon to the sun. And indeed, we need this parallax in solar eclipses.



And if you want to know the sun's parallax on the circle of altitude from Ptolemy's column made for this, according to the work of Albategni, you will cenit capitum intrabis in secundam tabulam. Et ei quod inveneris xviii^{am} partem ipsius superadicies, quia differentia maxime distantie Solis a terra quam Albategni invenit ad distantiam Ptolomei hanc habet proportionem ad ipsam Ptolomei distantiam. Itaque cum argumento Solis in tabulam equationis Lune intra. Et secundum proportionem ibi inventi in minutis partium ad lx minuta, sume de xiii secundis per que diversitas aspectus Solis inter longitudinem longiorem et longitudinem propiorem variatur, et quod exierit collecto prius adicies. Quod per hec duo opera provenerit erit diversitas aspectus Solis in circulo altitudinis equata super distantiam Solis a terra, et hoc quidem prope verum.

21. Diversitatem aspectus Lune in longitudine et in latitudine cum Luna latitudinem ab orbe signorum non habuerit colligere.

Sit enim medietas circuli signorum AEG et medietas circuli altitudinis BED sese intersecantes ad punctum E. Et sit circulus descriptus super polos utriusque ABGD, et polus circuli signorum nota Z, et cenit capitum punctum P. Et Luna sit in puncto circuli signorum E, et diversitas aspectus in circulo altitudinis arcus HE. Duco itaque a polo Z arcum circuli magni ZHT. Est ergo diversitas aspectus in latitudine arcus HT cum H sit visus locus Lune, et arcus ET diversitas aspectus in longitudine. Palam ergo expositis quemlibet istorum arcuum ZA ZT EB EA esse quartam circuli quia super polos suos invicem transeunt. Patet etiam ex ultima secundi libri quod angulus BEA est notus ad iiii rectos, ergo arcus BA similis scilicet quantitatis

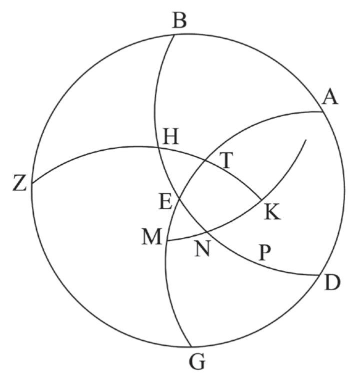


est notus eo quod angulus super polum huius circuli consistat aput E. Itaque cum a puncto A duo arcus magnorum circulorum descendant, manifestum per kata coniunctam quod sinus arcus AB ad sinum AZ est sicut sinus HT ad

1232 cenit] zenit *M* inveneris] invenies *N* **1233** ipsius superadicies] adicies *N* **1234** Ptolomei] Tholomei P_7 Tolomei *K* **1235** Ptolomei] Tpolomei P_7 Tolomei *K* tabulam] tabula PP_7K **1236** ibi] *corr. ex* ubi *K* inventi] inventam *MN* **1237** de – per] dxiii secundum *PN* dxiii secunda per P_7 dxiii secundum per *K* (de 13 secundum *Ba* de 13 secundis per E_1) **1238** longitudinem] longiorem *M* exierit] exibit *N* adicies] adicies et *N* **1239** opera] *corr. ex* corpora P_7 provenerit] proveniet *N* **1240** altitudinis] *corr. in* latitudinis *M* Solis] *om.* P_7 prope] proportio *M* **1242** habuerit] habuit PP_7K (habuerit *Ba* habuit E_1) **1245** Z] etiam *P corr. ex* etiam *K* capitum] capitis *P* **1246** E] est *P* **1248/1249** arcum – magni] *corr. ex* circuli magni arcum *P* **1251** et[1]] erit *N* ET[2]] *corr. ex* HT *M* **1252** ergo expositis] *corr. in* sicut ex anteposito *M* **1253** istorum] illorum *M* **1255** etiam] *om. M* **1256** iiii] *corr. ex* †...† *P* **1257** BA] *corr. ex* BN *M* scilicet] secundum *P om. N corr. ex* secundum *K* **1258** consistat] *corr. ex* constat *M* **1260** kata] cata *K* coniunctam] *corr. ex* quintam P_7 sinum] sinum arcus *N* enter the second column with the elongation of the sun from the zenith. And to what you find, add 1/18 of it,[88] because the difference of the sun's greatest distance from the earth that Albategni found to Ptolemy's distance has this ratio to that distance of Ptolemy. Accordingly, enter the column of the moon's equation [i.e. the ninth column] with the sun's argument. And according to the ratio of what is found there in minutes of parts to 60′, take from the 13″ through which[89] the sun's parallax varies between the apogee and perigee, and you will add what results to the earlier result. What results from these two labors will be the sun's parallax on the circle of altitude corrected for the sun's distance from the earth, and this indeed is approximate.

21. To obtain the moon's parallax in longitude and in latitude when the moon does not have[90] latitude from the ecliptic.

Indeed, let there be half of the ecliptic AEG and half of the circle of altitude BED intersecting at point E. And let the circle described upon the poles of each be ABGD, the pole of the ecliptic be point Z, and the zenith be point P. And let the moon be at point E of the ecliptic, and the parallax on the circle of altitude be arc HE. Accordingly, I draw ZHT, an arc of a great circle, from pole Z. Therefore, the parallax in latitude is arc HT when H is the apparent place of the moon, and arc ET is the parallax in longitude. Therefore, it is clear from what has been shown that each of those arcs ZA, ZT, EB, and EA are quarter circles because they pass upon the poles of each other. Also, it is clear from the last of the second book [i.e. II.36] that angle BEA is known to 4 right angles, so arc BA similar, namely in size, is known because the angle upon this circle's pole stands on E. Accordingly, because two arcs of great circles descend from point A, it is manifest through the conjunct kata that the sine of arc AB to the sine of AZ is as the sine of HT to the sine



[88] This is the correct value, and it is also found in *De scientia astrorum*, 1537 ed., f. 50v; however, Nallino, *al-Battānī*, vol. I, p. 80, and *P*, f. 48r have the incorrect value 1/8.

[89] The phrase 'de xiii secundis per que' must have been corrupted early in the transmission since garbled readings are found in *P*, P_7, and *K*.

[90] Many of the early witnesses have 'habuit', but this is most likely an easily made misreading of the abbreviated 'habuerit.' The mistake must have entered the transmission of the text early.

sinum HE. Sed tria nota sunt; ergo quartum notum scilicet sinus HT, et ita arcus HT qui est diversitas aspectus in latitudine notus.

Rursum super polum H ad distantiam quarte HK vel HN lineo circulum magnum KNM. Dico quod MT est quarta circuli. Quia enim ZTK transit super polos circuli signorum AEG, et circulus AEG necessario transit super polos circuli ZTK; quare in arcu AEG, cum sit medietas circuli, est polus circuli ZTK. Item ZTK transit super polos KNM, ergo et ille mutuo transit super polos ZTK. Est ergo punctus M polus circuli ZTK, quare MT est quarta circuli. Et dico quod arcus KN qui subtenditur THE angulo longitudinis est notus. Nam per kata disiunctam proportio sinus HT ad TK componitur ex duabus, una scilicet HE ad EN et alia MN ad MK. Cum ergo reliqua sint nota, erit arcus MN notus. Ergo et arcus NK qui deest ad perfectionem quarte est notus. Et nota quod si dempseris arcum BA sive angulum BEA de quantitate unius recti, invenies reliquum fere equale arcui KN sive angulo KHN. Cum ergo a puncto K duo arcus magnorum orbium descendant per kata coniunctam, proportio sinus NK ad sinum MK est sicut sinus ET ad sinum EH. Cum ergo reliqua tria sint nota, erit arcus ET notus, et ipse est diversitas aspectus Lune in longitudine.

Operationis modus est ut ex opere ultime secundi libri vel ex tabulis ad hoc constitutis scias angulum ex cursu circuli altitudinis et orbis signorum, et ex antepremissa vel ex tabulis ad hoc scias diversitatem aspectus in circulo altitudinis. Et addiscas cordam eius et cordam dicti anguli qui est angulus latitudinis et cordam mediatam eius quod deest ei ad completionem xc. Deinde multiplices sinum anguli latitudinis in sinum arcus altitudinis, et productum dividas per lx. Et quod exit arcues. Nam iste arcus est diversitas aspectus in latitudine. Sinum vero anguli longitudinis multiplices similiter in sinum arcus altitudinis, et productum dividas per lx. Nam arcus illius sinus qui exierit est diversitas aspectus in longitudine.

1261 sinus HT] HT sinus *PN* **1263** Rursum] rursus *N* **1264** KNM] *corr. ex* KMN *M* **1265** super[1]] *corr. ex* per *M* per *N* **1267** polos] circuli *add. et del. K* polos circuli *M* **1268** M] *s.l. P* MT] †MIT† *P* **1269** est] erit *N* **1269/1270** est notus] notus est *M* **1270** Nam] cum *N* kata] cata *K* **1271** ergo] *om.* P_7 **1272** deest] *corr. ex* est (*perhaps other hand*) *P* deest ei *N* **1274** equale] *corr. ex* equalem *PN* equalem *M* **1275** orbium] circulorum *N* coniunctam] *corr. ex* disiunctam *K* **1277** sint] sunt *PM* **1280** cursu] concursu *MN* (cursu *Ba* concursu E_1) et orbis] orbisque *N* et[2]] *om. M* **1281** antepremissa] ante premissa *PK* (antepremissa BaE_1) **1282** addiscas] addiscas mediatam *M* cordam[2]] cordam mediatam *M* **1283** completionem xc] perfectionem quarte *N* **1284** sinum[1]] sinum arcus *N* arcus] arcus diversitatis in circulo *N* **1285** exit] erit *P* exierit *M* exibit *N* (exit *Ba* †erit† E_1) iste arcus] arcus iste *N* aspectus] aspectus Lune *M* **1286** arcus] arcus diversitatis aspectus in circulo *N* **1287** dividas] divide *N* illius sinus] sinus illius *M* exierit] exerit *P* exit P_7 exibit *N* of HE.[91] But three are known;[92] therefore, the fourth, i.e. the sine of HT, is known, and thus arc HT, which is the parallax in latitude, is known.

In turn, I draw a great circle KNM upon pole H with the distance of quarter circle HK or HN. I say that MT is a quarter circle. Indeed, because ZTK passes upon the poles of the ecliptic AEG, circle AEG also necessarily passes upon circle ZTK's poles; therefore, a pole of circle ZTK is on arc AEG because it is a semicircle Likewise, ZTK passes upon the poles of KNM, so it in return also passes upon ZTK's poles. Therefore, point M is circle ZTK's pole, so MT is a quarter circle. And I say that arc KN, which subtends the angle of longitude THE, is known. For through the disjunct kata, the ratio of the sine of HT to TK is composed of two ⟨ratios⟩, i.e. the one of HE to EN and the other of MN to MK. Therefore, because the others are known, arc MN will be known. Therefore, arc NK, the complement, is also known. And note that if you subtract arc BA or angle BEA from the quantity of one right angle, you will find the remainder to be approximately equal to arc KN or angle KHN.[93] Therefore, because two arcs of great circles descend from point K, through the conjunct kata, the ratio of the sine of NK to the sine of MK is as of the sine of ET to the sine of EH. Therefore, because the remaining three are known, arc ET will be known, and it is the moon's parallax in longitude.

The way of operation is that from the work of the last of the second book [i.e. II.36] or from the tables set up for this,[94] you know the angle from the course of the circle of altitude and the ecliptic, and from the proposition preceding the last [i.e. V.19] or from the table for this,[95] you know the parallax on the circle of altitude. And you learn its sine [*lit.*, chord], the sine [*lit.*, chord] of the said angle, which is the angle of latitude, and the sine [*lit.*, the half chord] of the complement.[96] Then you multiply the sine of the angle of latitude by the sine of the arc of ⟨parallax on the circle of⟩ altitude, and divide the product by 60. And you arc what results. For that arc is the parallax in latitude. And indeed, you multiply the sine of the angle of longitude similarly by the sine of the arc of altitude, and divide the product by 60. For the arc of that sine that results is the parallax in longitude.

[91] From the conjunct kata, one finds that (sin AZ : sin AB) comp. of (sin ZT : sin HT) & (sin HE : sin BE). But, because the sines of ZT and BE are equal, it is true that (sin HE : sin HT) comp. of (sin HE : sin BE) & (sin ZT : sin HT). Therefore, (sin AZ : sin AB) :: (sin HE : sin HT).

[92] HE is known from V.19 above.

[93] One way in which this additional approximative way of calculating the size of NK can be seen to work is that the spherical triangle EHT is nearly rectilinear because of its small size. Angle HTE is right, so the remaining two angles, THE and HET, are approximately equal to a right angle.

[94] *Almagest* II.13.

[95] *Almagest* V.18.

[96] This complement is called the 'angle of longitude' later in the paragraph. Note that this rule of operation does not include steps corresponding to the strict method of finding the length of NK, but uses the approximative shortcut of treating triangle HET as if its angles equaled 180 degrees.

Theum vero Alexandrinus tabulas diversitatum aspectus in longitudine et latitudine composuit quarum opus non ita verum est ut illud quod per angulos et arcus sumitur. Tabularum vero artificium hoc est. Fecit nempe has super vii climata ac si Luna esset in signorum principiis. Et constituit ingressum in tabulas per horas ipsius diei equales antemeridianas vel postmeridianas, minuitque primum diversitatem aspectus Solis in circulo altitudinis sicut in libro Ptolomei invenit a diversitate aspectus Lune in termino primo, hoc est in maxima distantia Lune a terra. Et collegit ad singulas horas per opus angulorum quod premisimus, diversitates aspectus in longitudine et latitudine sicut in termino primo evenire possunt. Post hec propter ceteros terminos et que inter eos accidere possunt, fecit tabulas equationis que v sunt lateraliter iuncte. Et in prima et secunda posuit numeros communes portionis equate et longitudinis duplicis qui numeri per vi crescunt. In quarta vero cum maximam distantiam Lune a terra constituisset lx minuta esse et diametrum epicicli xii minuta quia minorem proportionem quam veram sumere voluit, posuit differentias distantiarum que sunt inter terminum primum et terminum secundum scilicet propter loca Lune in epiciclo accidentes – posuit inquam sub proportione ad lx. In quinta autem tabula cum differentiam maxime et minime distantie constituisset xxxii minuta et ipsam lx – minorem enim quam veram sumere voluit proportionem – posuit differentias distantiarum propter egressum circulum accidentes, et hoc sub proportione ad lx. Hinc est quod cum portione equata intratur in tabulam quartam, et secundum proportionem ibi inventi ad lx sumitur ex minutis longitudinis et latitudinis sigillatim, et additur super ea. Et cum longitudine duplici intratur in tabulam quintam. Et sit premisso modo. Hoc opus autem minus distat a vero cum Luna iuxta orbem signorum fuerit. Tertia vero tabula in quam non intratur continet differentias distantiarum inter terminum primum et terminum secundum cum diameter epicicli lx minuta – sic in opere Ptolomei positus fuerit.

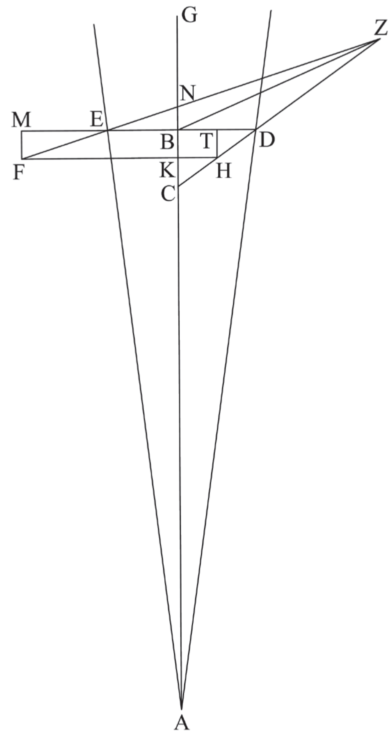
1289 Theum] Thum *K* Thebit *M* Theon *N* diversitatum] diversitatis *MN* **1290** quarum – non] quare non opus *corr. in* quale opus non *M* **1290/1291** angulos – arcus] arcus et angulos *N* **1291** nempe has] nempheas P_7 **1292** ac] at *P* signorum principiis] principiis signorum *N* constituit] constuit *P* **1293** tabulas] tabulis P_7 antemeridianas] ante meridianas *K* vel] et *PN* postmeridianas] post meridianas *PK* **1295** Ptolomei] Tholomei P_7 Tolomei *K* in[2]] de *N* **1298** hec] hoc *MN* **1300** prima et] primam PP_7N et[1]] et in *M* secunda] *om.* *N* **1301** numeri] termini *M* vero] *om.* *N* **1302** constituisset – minuta[1]] 60 minuta constituisset *N* diametrum] semidyametrum *M* **1304** secundum] sextum PP_7K (sextum *Ba* secundum E_1) **1305** accidentes] accidentis *M* **1306** distantie] distantiam *M* **1307** ipsam] (*om.* *Ba* ipsam maximam E_1) **1307/1308** sumere voluit] voluit sumere *M* **1309** cum] *s.l.* *K* **1310** intratur] *iter.* *N* **1312** sit premisso] fit simili *N* **1313** autem] non P_7 **1314** in quam] unquam *K* **1315** inter] in *M* **1316** sic] sicut *M* sit *N* Ptolomei] Tolomei P_7K positus fuerit] positum fuerit *M* posita *N*

And indeed, Theum Alexandrinus [i.e. Theon of Alexandria] made tables of the parallaxes in longitude and latitude, the work of which is not as true as that which is obtained through angles and arcs. And indeed, this is the crafting of the tables. Truly, he made these upon 7 climes as if the moon were at the beginnings of the signs. And he set up the entrance into the tables through equal hours of that day before or after noon, and he first subtracted the parallax of the sun on the circle of altitude as he found in Ptolemy's book from the moon's parallax in the first term, that is at the moon's greatest distance from the earth. And, through the work of angles that we set out before [i.e. earlier in this proposition], he collected for the individual hours the parallaxes in longitude and latitude as they are able to come about in the first term. Afterwards, for the other terms and what is able to happen between them, he made tables of correction, which are 5 ⟨columns⟩ joined across. And in the first and second, he placed the common numbers of the equated portion and of the duplex longitude, which numbers grow by 6. And indeed, in the fourth, when he had set up that the greatest distance of the moon from the earth was 60′ and the epicycle's diameter 12′ because he wanted to take a smaller ratio than the true one, he placed the differences of the distances that are between the first term and the second[97] term, namely because of the moon's places happening on the epicycle – I say he placed ⟨them⟩ under a ratio to 60. Moreover, in the fifth column, because he set up that the difference between the greatest and least distance was 32′ and that was 60′ – for he wanted to take a smaller ratio than the true one, he placed the differences of the distances occurring because of the eccentric circle, and this under a ratio to 60. From here, it is that the fourth table is entered with the equated portion, and according to the ratio of what is found there to 60, a part is taken separately from the minutes of the longitude and the latitude, and is added upon that. And the fifth table is entered with the duplex longitude. And let it be in the preceding way. Moreover, this work differs less from the truth when the moon is near the ecliptic. And indeed, the third column, which is not entered, contains the differences of the distances between the first term and the second term when the epicycle's diameter is 60′ – as it had been posited in Ptolemy's work.

[97] Many of the witnesses have the incorrect 'sextum', but this is probably due to a misreading of the abbreviation 's^{m}.'

22. Cum Luna latitudinem habuerit, cuius rei investigationem oporteat precedere ad cognitionem omnium diversitatum aspectus declarare.

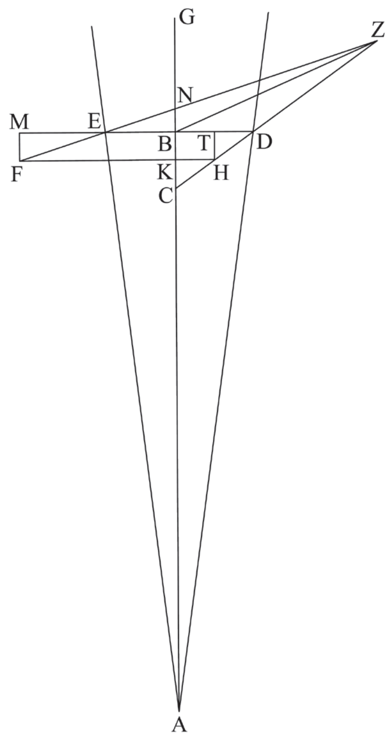
Describam vice arcus orbis signorum lineam ABG, et vice circuli declinantis Lune cum ad septemtrionem declinaverit lineam AD et cum ad meridiem AE. Et ponam locum Lune D vel E, et ponam circulum longitudinis Lune DBE qui etiam semper erectus est super circulum signorum. Et ponam alibi punctum Z cenit capitum, et lineabo super ipsum et punctum Lune D circulum altitudinis ZDC, et iterum alium circulum altitudinis super locum Lune in orbe signorum ZB. Et sit diversitas aspectus Lune in circulo altitudinis DH arcus, et a puncto H quod est visus locus Lune in celo portionem circuli transeuntis super polos circuli signorum HK et portionem circuli equidistantis circulo signorum HT. Est ergo vera elongatio loci Lune a nodo in circulo signorum arcus AB et visa elongatio arcus AK. Est itaque diversitas aspectus in longitudine arcus BK qui similis est arcui TH. Et vera latitudo Lune arcus DB et visa latitudo arcus HK qui est equalis arcui BT. Unde diversitas aspectus in latitudine est arcus DT. Quilibet ergo istorum arcuum DH DT TH querendus est. Palam autem ex premissa scilicet ex xviiiª quod si notus sit arcus ZD scilicet elongatio Lune a cenit capitum, notus erit arcus DH. Nunc autem non habemus nisi notitiam arcus ZB qui est elongatio a cenit capitum ad gradum Lune. Oportet ergo investigari arcum ZD propter habendam notitiam arcus DH. Ad sciendum vero utrumque istorum



1317 habuerit] habuit *PK* (habuerit *Ba*E_1) **1319** arcus] *s.l. P* orbis signorum] signorum orbis *K* **1320** cum^{1}] *om. P s.l.* P_7*K* AE] AC *K* **1321** E] C *K* DBE] DHE *P* DBC *K* DTB *corr. ex* DBT *N* **1321/1322** qui etiam] que est *M* **1322** est] est perpendiculariter *M* **1323** alibi] alicubi P_7*K* **1326** ZDC] et DE *P* ZDE *K corr. ex* ZDE *N* **1327** altitudinis] *corr. ex* latitudinis *K* **1330** quod – locus] qui est locus visus *M* **1331** circuli] *om. M* **1332** super] *corr. in* per *M* circuli] transeuntis *add. et del.* P_7 **1336** elongatio] Lune *add. et del. K* **1337** Est] et *N* **1342** DT] *corr. ex* BT *M* istorum] illorum *M* **1345** xviiia] xiiiiª P_7 *corr. in* xviiiiª *K* (18 *Ba* xviiiª E_1) si] *corr. ex* sit *P* **1347** gradum] graduus *P* Lune] Lune notum *M*

22. When the moon has latitude, to declare which thing's search must precede for the knowledge of all the parallaxes.

I will describe line ABG in place of the arc of the ecliptic, and line AD in place of the moon's declined circle when it inclines to the north and AE when to the south. And I will suppose the moon's place to be D or E, and I will place the moon's circle of longitude DBE, which is also always set up perpendicularly upon the ecliptic. And in another place, I will suppose point Z to be the zenith, and I will draw circle of altitude ZDC upon it and the moon's point D, and again another circle of altitude ZB upon the moon's place on the ecliptic. And let the moon's parallax on the circle of altitude be arc DH, and from point H, which is the moon's apparent place in the heavens, ⟨I will draw⟩[98] a part HK of the circle passing upon the ecliptic's poles, and a part HT of a circle parallel to the ecliptic. Therefore, the true elongation of the moon's place from the node on the ecliptic is arc AB, and the apparent elongation is arc AK. Accordingly, the parallax in longitude is arc BK, which is similar to arc TH. And the moon's true latitude is arc DB, and the apparent latitude is arc HK, which is equal to arc BT. Whence the parallax in latitude is arc DT. Therefore, each of those arcs DH, DT, and TH must be sought. Moreover, it is clear from what has been set forth, i.e. from the 18th,[99] that if arc ZD, i.e. the moon's elongation from the zenith, is known, arc DH will be known. Moreover, now we do not have anything except the knowledge of arc ZB, which is the elongation from the zenith to the moon's degree. Therefore, it is necessary that arc ZD be found in order to have knowledge of arc DH. And indeed, to know

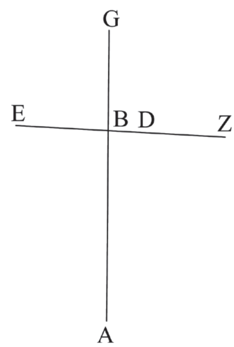


[98] A verb such as 'lineabo' must be understood here.

[99] This should refer to the 19th proposition of Book V. Perhaps the original numbering of the propositions was different.

arcuum DT TH sive BK sufficit scire angulum ZCG cui in potentia equalis est angulus THD. Ipse autem scietur, si cognitus fuerit angulus TDH vel e converso. Nam est cum illo completio unius recti. Nunc autem non habemus notum nisi angulum ZBG. Oportet ad notitiam diversitatum aspectus in longitudine et in latitudine investigari angulum ZCG. Quo habito operandum uti per alios angulos incidentes super circulum signorum.

Item sit locus Lune in celo super E, et erit latitudo Lune vera EB. Et ducamus circulum altitudinis ZEF, sitque diversitas aspectus in circulo altitudinis arcus EF. Et ducamus a puncto F equidistantem circulo signorum MF et alium erectum super circulum signorum qui est circulus magnus FK. Patet ergo quod AB est vera elongatio a nodo, et AK est visa elongatio. Unde BK hec est diversitas aspectus in longitudine. Item EB est vera latitudo Lune. FK est visa latitudo cui equalis est MB, ergo EM est diversitas aspectus in latitudine. Ad cognoscendum igitur EF oportet investigari quantitatem arcus EZ. Et ad sciendum utrumque istorum arcuum EM MF sive BK sufficit investigare angulum ZNG cui in potentia est equalis angulus EFG. Nam tunc reliquus MEF completio unius recti erit notus, per quos operandum ut per superiores incidentes aput orbem signorum. Vides ergo quod semper oportet inquirere arcus circuli altitudinis a cenit capitum ad ipsam Lunam et angulos qui ex hoc circulo altitudinis aput orbem signorum proveniunt, quod intendimus.

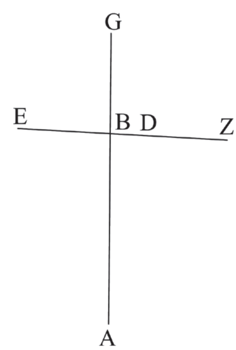


23. Cum fuerit circulus altitudinis circulo signorum ad angulos rectos incidens, et arcus et angulos propositos investigare.

Ponemus circulum signorum ABG ut prius et circulum altitudinis erectum super circulum signorum ZDBE qui erit tunc coniunctus cum circulo longitudinis Lune. Et sint D vel E locus Lune. Tunc manifestum quod diversitas aspectus erit in latitudine tan-

1349 arcuum] arcuum scilicet P_7 *om.* *N* DT TH] *corr. ex* †...† (*perhaps other hand*) *K* DT] *corr. ex* DH *M* cui] *corr. ex* cum P_7 **1349/1350** equalis est] est equalis P_7 **1350** TDH] *corr. ex* ETDH *M* **1351** habemus] habueris *M* **1352** notum nisi] *corr. ex* nisi rectum *K* ZBG] ZBD *P* Oportet] oportet autem *N* ad] unam *add. et del.* *P* notitiam diversitatum] *corr. ex* nonam diversitatem P_7 nonam diversitatem *M* diversitatum] *corr. in* diversitatis *K* **1353** in] *om.* *M* ZCG] ZEHG *M* **1355** celo] circulo *N* **1359** vera elongatio] elongatio vera *N* hec] hoc *K* hic *corr. in* non *M* *om.* *N* **1361/1362** Ad – igitur] igitur ad cognoscendum *PN* **1362** quantitatem] eius *add. et del.* P_7 Et] *s.l.* *P* **1363** EM] EM et *M* **1364** EFG] EFM P_7N (EFG BaE_1) **1367** capitum] capitis P_7 **1368** orbem] orbem circuli *N* signorum] *corr. ex* signorem P_7 **1369** proveniunt] *corr. ex* provenit *P* **1373** Ponemus] ponamus *N* **1376** sint] situm P_7 sit *MN* D] B *PM* *corr. ex* B P_7 vel – Lune[2]] locus Lune vel E *M* **1377** aspectus erit] erit aspectus P_7 aspectus Lune erit *N* each of those arcs DT and TH or BK, it is sufficient to know angle ZCG, to which angle THD is equal in power. Moreover, it will be known if angle TDH is known, or conversely. For with it there is the completion of one right angle. Moreover, now we have nothing known except angle ZBG. For knowledge of the parallaxes in longitude and in latitude, it is necessary that angle ZCG be found. With this had, one should operate in the same way as through the other angles falling upon the ecliptic.

Likewise, let the moon's place in the heavens be upon E, and the moon's true latitude will be EB. And let us draw circle of altitude ZEF, and let the parallax on the circle of altitude be arc EF. And let us draw MF from point F parallel to the ecliptic and another ⟨circle⟩ set up perpendicularly upon the ecliptic, which is great circle FK. Therefore, it is clear that AB is the true elongation from the node, and AK is the apparent elongation. Whence this BK is the parallax in longitude. Likewise, EB is the moon's true latitude. FK is the apparent latitude, to which MB is equal, so EM is the parallax in latitude. Therefore, to know EF, it is necessary that the quantity of arc EZ be found. And to know of each of those arcs EM and MF or BK, it is sufficient to find angle ZNG, to which angle EFG[100] is equal in power. For then the remainder MEF, the complement, will be known, through which one should operate as through those upper ones falling on the ecliptic. Therefore, you see that it is always necessary to seek the arc of the circle of altitude from the zenith to the moon itself and the angles that come from this circle of altitude at the ecliptic, which we intended.



23. To find the proposed arcs and angles when the circle of altitude falls on the ecliptic at right angles.

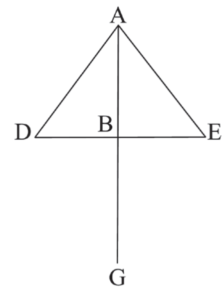
We will suppose the ecliptic to be ABG as before, and the circle of altitude set up perpendicularly upon the ecliptic to be ZDBE,[101] which will then be joined together with the moon's circle of longitude. And let D or E be the moon's place. Then it is manifest that the parallax will be in latitude only. And arc ZD

[100] This should be EFM.
[101] As in the previous proposition, point Z is the zenith.

tum. Et arcus ZD erit notus cum vera latitudo Lune BD subtracta fuerit arcu ZB pridem noto. Et cum latitudo Lune EB addita fuerit super ZB, erit arcus altitudinis EZ notus. Palam etiam quod anguli aput puncta D et E ex circulo altitudinis et circulo declinante Lune provenientes non sunt sensibiliter diversi a rectis, quia fere sunt equales angulis qui aput B proveniunt propter modicam declinationem. Et hoc erat propositum.

24. Cum fuerit circulus altitudinis coniunctus in eadem superficie cum circulo signorum, et arcus et angulos propositos invenire.

Ponemus iterum circulum signorum ABG et polum orizontis punctum A et circulum longitudinis Lune DBE atque locum Lune D vel E. Et ducemus duos arcus circulorum altitudinis AD AE et tertium coniunctum cum circulo signorum AB. Querimus ergo utrumlibet istorum arcuum AD AE et utrumlibet istorum angulorum DAB EAB. Et possumus uti proportione arcuum sicut rectarum propter parvitatem diversitatis. Itaque cum anguli ad B hinc inde sint recti et ambe AB EB sint note, erit quoque AE que subtenditur recto nota, et similiter eius equalis AD. Item cum proportio AE ad EB sit nota, si constituamus AE semidiametrum, erit secundum hoc corda EB nota; ergo angulus EAB cui subtenditur notus. Et ipse quoque est equalis angulo DAB. Et hoc oportuit demonstrari.

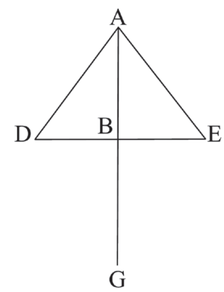


25. Cum fuerit circulus altitudinis circulo signorum ad angulos obliquos incidens, et arcus et angulos propositos determinare.

Ponemus iterum circulum signorum AGB, et circulum altitudinis ZBK ad obliquos angulos ei incidentem, et locum Lune D vel E, et Z polum orizontis. Querimus ergo duos arcus ZE ZD et duos angulos AGZ ATZ. Protrahemus ergo duas perpendiculares DK EL super ZB. Et quia angulus ABZ est datus et angulus ABE est rectus, erit propter hoc uterque angulorum orthogoniorum BEL BDK et angulis et arcubus datus cum arcus BD vel BE equalis

1378 BD] *corr. ex* EBD *K* fuerit] fuerit ab *M* **1379** ZB[1]] ZD *P* **1383** erat] erit *K corr. ex* erit P_7 propositum] propositum et cetera *N* **1388** D] *corr. ex* B *M* ducemus] ducamus *M* **1389** et] utrumlibet istorum angulorum *add. et del. N* **1390** utrumlibet] utrumque *M* **1392** uti] *marg. P* **1393** sicut] sicut linearum *MN* rectarum] *corr. ex* rectorum P_7 **1394** ad B] ADB *P* sint] *corr. in* sunt *M* ambe] *s.l. P* **1395** subtenditur] subtenditur angulo *N* **1396** eius equalis] *corr. ex* equalis eius *K* eius] ei *N* proportio] *corr. ex* proportione P_7 **1398** EAB] *corr. ex* ACB *K* **1399** demonstrari] demonstrare et cetera *N* **1401** incidens] incidiens P_7 determinare] declarare *N* **1402** ZBK] *corr. ex* Z BZ *K* GBK *N* **1404** ZE] *corr. ex* †...† *K* AGZ ATZ] AGZ *corr. in* ATZ *M* Protrahemus] protrahimus *N* **1406** uterque] utrique *PP*$_7$*K* (uterque *Ba*E_1) angulorum] triangulorum *N* (angulorum *Ba* triangulorum E_1) **1407** BDK] *corr. ex* BDH *K* et[1]] *corr. ex* vel *M* datus[1]] datus et *M* BE] GE *PN* will be known when the moon's true latitude BD is subtracted from arc ZB previously known. And when the moon's latitude EB is added upon ZB, the arc of altitude EZ will be known. It is also clear that the angles at points D and E resulting from the circle of altitude and the moon's declined circle are not sensibly different from right angles, because they are almost equal to the angles that come forth at B because of the modest declination. And this had been proposed.

24. To find the proposed arcs and angles when the circle of altitude is joined together with the ecliptic in the same plane.



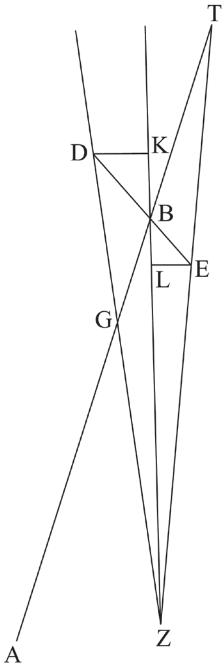
We will place again the ecliptic ABG, the pole of the horizon point A, the moon's circle of longitude DBE, and the moon's place D or E. And we will draw two arcs of the circle of altitude AD and AE, and a third AB joined together with the ecliptic. Therefore, we seek each of those arcs AD and AE and each of those angles DAB and EAB. And we are able to use the ratio of arcs as ⟨the ratio⟩ of straight lines because of the smallness of the difference. Accordingly, because the angles at B on one side and the other are right angles and both AB and EB are known, also AE, which subtends the right angle, will be known, and similarly its equal AD. Likewise, because the ratio of AE to EB is known, if we set up AE as a radius, the sine [*lit*., chord] EB will be known according to this; therefore, angle EAB, which it subtends, will be known. And that also is equal to angle DAB. And it was necessary that this be shown.

25. To determine the proposed arcs and angles when the circle of altitude falls upon the ecliptic at an oblique angle.

Again, we will place the ecliptic AGB, the circle of altitude ZBK falling upon it at oblique angles, the moon's place D or E, and the horizon's pole Z. Therefore, we seek the two arcs ZE and ZD and the two angles AGZ and ATZ. Then, we will draw the two perpendiculars DK and EL upon ZB. And because angle ABZ is given and angle ABE is right, each of the right angles[102] BEL and BDK will given in both angles and arcs because arc BD or equal BE

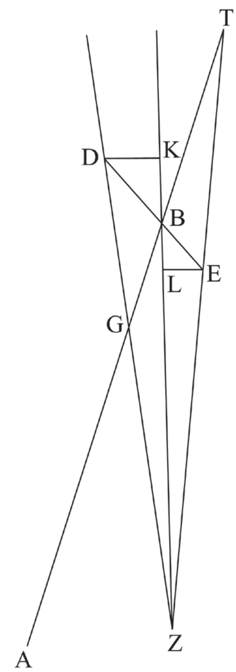
[102] This should say 'each right triangle.' While most of the witnesses that I used have the reading 'utrique angulorum', this is nonsensical grammatically and mathematically. The 'utrique' is most likely an easily made scribal error. The 'angulorum' appears to be the author's mistake.

sit datus scilicet sicut in undecima propositione presentis ostenditur. Quapropter erit et ZL et ZK nota quia ZB est nota. Et propter hoc utraque istarum ZE ZD cum subtendatur angulo recto nota. Et propter proportiones linearum notas, erit uterque angulorum DZK EZL datus. Et quia duo anguli ABZ BZG iam noti equantur pariter accepti angulo extrinseco AGZ, erit et ipse notus. Et quia angulus ABZ notus pridem superat angulum ATZ angulo BZE iam noto, erit et angulus ATZ datus, quod oportebat ostendi.



Tenor operandi ad notitiam istorum arcuum et angulorum is est. Queremus ut supra primum arcum ZB et angulum ABZ qui est angulus latitudinis. Et minuemus xc et remanebit angulus ZBE quasi longitudinis. Et utriusque anguli sinum in sinum latitudinis Lune scilicet BE multiplicabimus, per lx idest semidiametrum dividemus, et arcuabimus. Quodque exierit ex angulo latitudinis erit arcus BL sive BK. Si ergo Luna fuerit aput E scilicet a circulo signorum versus cenit capitum, minuemus BL ab arcu BZ, et remanebit arcus LZ notus. Si vero fuerit Luna super punctum D, addemus BL super BZ, et erit arcus KZ notus. Quod autem provenerit ex angulo longitudinis erit arcus EL sive DK. Eam ergo lineam in se multiplicatam adde super ZL vel ZK sicut evenerit in se multiplicatam, et collecti radicem accipe, et erit arcus ZE vel ZD sicut evenerit. Et ipsi sunt arcus quesiti. Deinde multiplicabo EL sive DK in semidiametrum, et dividam per ZL primum, et dividam secundo per ZK, et utrimque productum arcuabo. Quodque

1408 undecima] xxª *PKM corr. ex* 20ª *N* (xxª *BaE*$_1$) propositione] proportione *P corr. ex* proportione *P*$_7$ *om. N* **1409** et ZL] *corr. ex* EZL *N* ZL] ZB *M* **1410** istarum] illarum *M* **1411** subtendatur] subtenditur *PN* propter] propter hoc *M* **1414** AGZ] *corr. ex* AEZ *M* **1416** BZE] BEZ *P* BZT *N* noto] nota *P* **1416/1417** erit – ATZ] et angulus ATZ erit *M* **1418** istorum] illorum *M* **1419** is] *om. M* **1421** Et[1]] quem *marg. N* minuemus] *corr. ex* †...† *P* minuemus de *M* (minuemus *Ba* minuemus de *E*$_1$) **1423/1424** per – dividemus] et productum dividemus per 60 idest semidyametrum *N* **1425** Quodque exierit] quod exibit *N* arcus] *corr. ex* angulus (*perhaps other hand*) *K* **1426** fuerit] *s.l.* (*perhaps other hand*) *P* E] C *P* **1427** capitum] *s.l.* (*perhaps other hand*) *P* **1428/1429** et – BZ] *om. P*$_7$ **1428** fuerit Luna] Luna fuerit (*the last word marg. P*) *PN* **1429** BL] BK *N* BZ] LZ *P* KZ] LZ *P*$_7$ provenerit] proveniet *N* **1430** ergo] *corr. ex* vero *M* **1431/1433** et – multiplicabo] *marg.* (*perhaps other hand*) *P* **1433** multiplicabo] multiplicando *PP*$_7$*K* (multiplicabo *BaE*$_1$) ZL] KL *PK corr. ex* KL *P*$_7$ *corr. in* KL *M* ZE *corr. ex* KL *N* (KL *Ba* ZL *E*$_1$) **1434** secundo] secundum *N* (secundo *BaE*$_1$) ZK] *corr. in* ZD *N* utrimque] utrumque *MN* arcuabo] *corr. ex* arcua *P* Quodque] *iter. et del. M*



is given, as is shown in the 11th proposition[103] of the present ⟨book⟩. For this reason, both ZL and ZK will be known because ZB is known. And because of this, each of those ZE and ZD is known because it subtends a right angle. And because of the known ratios of the lines, each of the angles DZK and EZL will be given. And because the two angles ABZ and BZG already known taken together are equal to extrinsic angle AGZ, it will also be known. And because angle ABZ previously known exceeds angle ATZ by angle BZE already known, angle ATZ will also be given, which was necessary to be shown.

The method of operating for the knowledge of these arcs and angles is this. As above [i.e. II.35–36], we will first seek arc ZB and angle ABZ, which is the angle of latitude. And we will subtract 90,[104] and there will remain angle ZBE as if ⟨the angle⟩ of the longitude. And we will multiply the sine of each angle by the sine of the moon's latitude, i.e. BE,[105] we will divide by 60, i.e. the radius, and we will arc ⟨the result⟩. And what results from the angle of latitude will be arc BL or BK. Therefore, if the moon is at E, i.e. from the ecliptic towards the zenith, we will subtract BL from arc BZ, and arc LZ will remain known. However, if the moon is at point D, we will add BL to BZ, and arc KZ will be known. Moreover, what results from the angle of longitude will be arc EL or DK. Therefore, add this line multiplied by itself to ZL or ZK, as it comes about, multiplied by itself, and take the root of the sum, and it will be arc ZE or ZD, as it comes about. And they are the sought arcs. Then I will multiply EL or DK by the radius, and I will divide first ⟨the product of EL and the radius⟩ by ZL,[106] and I will divide secondly ⟨the product of DK and the radius⟩ by ZK,[107] and I

[103] Most witnesses refer to the 20th proposition, but this is most likely due to the misreading of 'xi^a^' as 'xx^a^' early in the text's transmission.

[104] This should say to subtract angle ABZ from 90°. This mistake could be the result of the omission of a preposition early in the text's transmission.

[105] It is unclear why the author would call BE the 'sine of the moon's latitude' and not merely the 'moon's latitude' or the 'chord of the moon's latitude.'

[106] Both this reading and KL, which appears in multiple witnesses, are mathematically incorrect. The correct line is ZE.

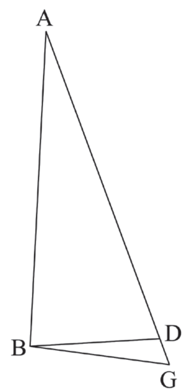
[107] This line should be ZD.

exierit ex divisione per ZL erit angulus EZL, et quod ex divisione per ZK erit angulus DZK. Si itaque Luna fuerit super punctum D scilicet ex altera parte circuli signorum a cenit capitum, addam angulum BZG super angulum GBZ, et erit angulus AGZ quem minuam a recto. Quod si Luna fuerit super punctum E, minuam angulum EZB ab angulo ZBG, et remanebit angulus ZBT quem minuam a recto. Et ita habebo angulos latitudinis et longitudinis equatos, quibus utar vice aliorum angulorum longitudinis et latitudinis.

Et nota quod cum fuerit latitudo Lune v graduum, maxima differentia diversitatum aspectus que propter hoc accidere potest est x minutorum fere. Et cum fuerit latitudo maxima que accidere potest in solaribus eclipsibus que latitudo est gradus et medietas unius gradus fere, erit differentia diversitatum aspectus propter hoc tantum minutum unum et medietas minuti scilicet secundum quantitatem graduum latitudinis Lune, et illud quoque rarissime eveniet.

26. Motum Lune in circulo declinante et in circulo signorum arcus differentis longitudinis efficere necesse est, sed differentia admodum parve quantitatis esse convincitur.

A nodo A etenim sumamus duos arcus equales arcum AB circuli declivis et arcum AG circuli signorum. Et sit B punctum in quo sit Luna. Et quia locum Lune in circulo signorum assignat circulus magnus transiens per polos circuli signorum et punctum B, palam quod si educamus a puncto B perpendicularem super lineam AG, ipsa invenit locum Lune in linea AG. Et quia AB et AG sunt equales, necesse est perpendicularem a puncto B cadere inter A et G. Sit ergo BD. Erit ergo differentia longitudinis DG. Et ipsa quidem cum maxima latitudo sit v graduum, non potest amplius esse quam v minuta. Et hoc declaratur per kata disiunctam

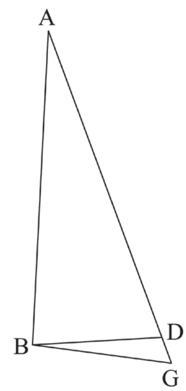


1435 exierit] *corr. ex* exerit *P* exibit *N* ZL] *corr. in* ZE *N* ex[2]] *s.l.* (*perhaps other hand*) *P* ZK] ZK (*corr. in* ZD) exibit *N* **1436** Luna fuerit] fuerit Luna *N* Luna] *corr. ex* linea *M* fuerit] *s.l. P* fuit *K* **1437/1438** super – GBZ] *marg.* P_7 **1438** GBZ] *corr. ex* G†Z† *K* **1439** EZB] *corr. ex* EB *P* **1440** ZBT] ZGB *M corr. in* ZTB *N* (ZBC *Ba* AGZ E_1) latitudinis – longitudinis] longitudinis et latitudinis *N* **1441** vice] *s.l. P* **1443** minutorum fere] fere minutorum *M* **1444** solaribus eclipsibus] eclipsibus solaribus *PN* **1445** unius gradus] gradus unius *N* **1446** tantum – unum] unum minutum *M* et] *s.l.* P_7 **1447** eveniet] eveniet et cetera *N* **1448** declinante] inclinante P_7 differentis] *corr. ex* differentes *P* **1449** sed] si *M* admodum] ad motum *P* **1450** esse] *om. M* **1451** A[1]] *om. P s.l.* P_7 etenim] *corr. ex* et P_7 **1452** AB] EB *K* **1453** punctum] punctus P_7 quo] qua P_7K **1455** et] et per *M* **1458** AB – AG] AG et AB *N* **1459** BD] BD differentia *K* Erit] *del. K* **1460** ergo] *s.l. N* differentia – DG] DG differentia longitudinis P_7K quidem] *om. N* **1461** potest – esse] potest esse amplius *M* amplius esse potest *N* **1462** declaratur] declarabitur *M* kata] cata *K* will arc the product in each case. And what results from the division by ZL[108] will be angle EZL, and what ⟨results⟩ from the division by ZK[109] will be angle DZK. Accordingly, if the moon is at point D, i.e. on the other side of the ecliptic from the zenith, I will add angle BZG to angle GBZ, and there will be angle AGZ, which I will subtract from a right angle.[110] But if the moon is at point E, I will subtract angle EZB from angle ZBG, and there will remain angle ZBT,[111] which I will subtract from a right. And thus I will have the corrected angles of latitude and longitude, which I will use in place of the other angles of longitude and latitude.

And note that when the moon's latitude is 5°, the greatest difference of parallaxes that is able to occur because of this [i.e. by using the incorrect elongation from the zenith and the wrong angle at the ecliptic] is approximately 10′. And when there is the greatest latitude that is able to occur in solar eclipses, which latitude is about 1° 30′, the difference of parallaxes ⟨that occurs⟩ because of this will be only 1′ 30″, i.e. according to the quantity of the degrees of the moon's latitude,[112] and that also will occur very infrequently.

26. It is necessary that the moon's motion on the declined circle and on the ecliptic make arcs of different length, but it is established that the difference is of an exceedingly small quantity.

And indeed, from node A let us take two equal arcs, arc AB of the declined circle and arc AG of the ecliptic. And let B be the point at which the moon is. And because the great circle passing through the poles of the ecliptic and point B determines the moon's place on the ecliptic, it is clear that if we draw a perpendicular from point B upon line AG, it will find the moon's place on line AG. And because AB and AG are equal, it is necessary that the perpendicular from point B fall between A and G. Then, let it be BD. Therefore, the difference of longitude will be DG. And when the maximum latitude is 5°, it is indeed not able to be more than 5′. And this is declared by the disjunct kata and a figure similar



[108] Again, this line should be ZE.

[109] Again, this line should be ZD.

[110] AGZ is the corrected angle of latitude and its complement is the corrected angle of longitude.

[111] Another mathematical mistake is made. The remaining internal angle is ZTB, not ZBT.

[112] I.e. the number of degrees of latitude is the same as the number of minutes of parallax.

et figuram similem ei quam posuimus ad declinationes Solis cognoscendas. In eclipsibus vero non nisi multo minor potest esse hec differentia quia prope nodum semper sunt eclipses. Non ergo erit error magnus si in cursu Lune accipiamus arcum AG loco arcus AD. At si quis huius rei scientiam vellet prosequi et hoc equare, multo maior esset difficultas operis quam utilitas impendii.

27. Visum locum Lune in circulo signorum ex vero Lune loco cognito comprehendere.

Diversitatem aspectus Lune in longitudine accipe ex supradictis. Et cum Luna orientali orizonti propior fuerit, idest cum ab ascendente minus xc gradibus circuli signorum distiterit, hanc diversitatem aspectus in longitudine loco Lune vero superaddes. Cum vero occidentali orizonti Luna propior fuerit, a loco Lune vero in circulo signorum eam minues. Quodque post augmentum vel diminutionem fuerit erit visus locus Lune in circulo signorum.

28. Visam Lune latitudinem perpendere.

Diversitatem aspectus Lune in latitudine predicto modo colligere. Et si cum Lune gradus in medio celi erit, Luna a cenit capitum meridiana fuerit, diversitas aspectus Lune — in latitudine dicetur — et erit meridiana. Et si versus septemptrionem, diversitas aspectus — in latitudine dicetur — et erit septemtrionalis. Et fere semper erit meridiana in hiis climatibus quorum latitudo maior est maxima declinatione Solis et Lune latitudine. Cumque vera visi loci Lune in longitudine latitudo et hec diversitas aspectus in eandem partem fuerit, eas in unum collige. Si vero diverse fuerint, minorem de maiori deme. Et quod post augmentum vel diminutionem fuerit erit latitudo Lune visa, quam propter solares eclipses querimus.

1463 posuimus] possumus P_7 ad declinationes] *corr. ex* ad declarationes *M* **1464** multo] minuto *corr. ex* †muto† *N* **1465** erit error] error erit *N* **1466** prosequi] persequi *N* **1467** hoc] *corr. in* hec *P* hec *N* maior] iustior *PN* mitior *K* (maior *Ba*E_1) impendii] impendii et cetera *N* **1468** comprehendere] deprehendere *M* **1470** ex] *corr. ex* ut *K* **1471** ab] *om. K* xc] xx P_7 **1472** circuli – distiterit] distiterit de circulo signorum *N* distiterit] destiterit *P* aspectus] *corr. ex* aspic- *K* **1473** superaddes] superadde *N* Luna] *om. N* **1474** minues] minue *N* post] per P_7 *corr. ex* †...† *K* augmentum] argumentum *P*P_7 *corr. ex* argumentum *K* (augmentum *Ba*E_1) **1475** vel] vel post *N* **1476** perpendere] comprehendere *N* **1477** colligere] collige *MN* **1477/1478** Et – fuerit] *corr. in* et si gradus medii celi a cenith capitum meridianus fuerit *N* **1477** si cum] sicut *P* sicuti P_7*K* sit *M* (†sicudi† *Ba* si cum E_1) **1478** erit] *corr. in* et si *M* cenit] zenit *M* **1479** versus septemptrionem] *corr. ex* verso septembri P_7 **1480** septemtrionalis] *corr. ex* septembri P_7 **1481** quorum] *corr. ex* quo P_7 **1482** visi] *om. N* **1483** in longitudine] *om. N* eandem – fuerit] eadem parte fuerint *N* **1485** augmentum] agmentum *K* quam propter] quapropter P_7 **1486** querimus] querimus et cetera *N*; explicit liber quintus *add. M* finit quintus *add. N* to that which we posited for knowing the declinations of the sun. And indeed, in eclipses this difference is not able to be anything except much less because eclipses are always near the node. Therefore, the error will not be great if in the moon's passage we take arc AG in place of arc AD. But if anyone wanted to describe the science of this matter in detail and to correct this, the difficulty of the work would be much greater than the usefulness of the expenditure.

27. To grasp the moon's apparent place on the ecliptic from the moon's known true place.

Take the moon's parallax in longitude from what has been said above [i.e. from the method shown in V.21]. And when the moon is near the eastern horizon, i.e. when it stands less than 90° of the ecliptic away from the ascendant, add this parallax in longitude to the moon's true place. And indeed, when the moon is near the western horizon, subtract it from the moon's true place on the ecliptic. And what results after the addition or subtraction will be the moon's apparent place on the ecliptic.

28. To assess the moon's apparent latitude.

Let the moon's parallax in latitude be obtained in the aforesaid way [i.e. from the method shown in V.21]. And if when[113] the moon's degree will be at the middle heaven, the moon is south of the zenith, the moon's parallax – let it be said, in latitude – will also be south. And if towards the north, the parallax – let it be said, in latitude – also will be north. And it will almost always be south in these climes whose latitude is greater than the sun's maximum declination and the moon's latitude. And when the true latitude of the moon's apparent place in longitude and this parallax are on the same side, combine them into one. However, if they are different, subtract the smaller from the greater. And what results after the addition or subtraction will be the moon's apparent latitude, which we seek for solar eclipses.

[113] While of my main witnesses, only *N* and E_1 have 'si cum', the text requires it to make sense, and the other variants are misreadings that entered the text early.

⟨Liber VI⟩

Superlatio Lune ad datum tempus est id quod relinquitur cum diversus motus Solis ad ipsum tempus subtractus fuerit a diverso motu Lune ad ipsum tempus.

Media superlatio Lune ad datum tempus est id quod relinquitur cum medius motus Solis ad ipsum tempus diminutus fuerit a medio motu Lune ad idem tempus.

Visus motus Lune est visi loci Lune per diversitatem aspectus in longitudinem progressio.

Visa superlatio Lune ad aliquod tempus est, cum diversus motus Solis ad ipsum tempus a viso motu Lune ad idem tempus subductus fuerit, id quod relinquitur.

Termini ecliptici lunares sunt termini arcuum circuli declinantis Lune ex utralibet parte nodi recisorum infra quos terminos versus nodum Luna existente secundum cursum medium possibile est Lunam eclipsari, ultra vero est impossibile. Et ibi sunt termini isti ubi primum contactum Lune et umbre esse contingit post mediam in vera oppositione.

Termini ecliptici solares sunt termini arcuum circuli declinantis Lune ex utralibet parte nodi recisorum infra quos versus nodum Luna secundum medium cursum existente possibile est Solem in aliquo vii climatum eclipsari; ultra est impossibile. Et hii quidem termini ibi sunt ubi primum contactum Solis et Lune esse contingit post mediam coniunctionem in coniunctione visa. Sunt et alii termini infra quos cum Luna fuerit applicata Soli, necesse est Lunam vel Solem pati eclipsim.

Quinque sunt tempora lunaris eclipsis: principium obscurationis, principium more, medium eclipsis, finis more sive principium detectionis, finis detectionis.

Tria solaris: principium, medium, et finis.

Digitus eclipsis est pars duodecima diametri sive Solis sive Lune obscurata.

1 Liber VI] liber sextus *marg.* (*other hand*) *P* quintus *marg. corr. in* sextus (*other hand*) *K* incipit liber sextus *M* incipit sextus liber *N* **2** diversus] *corr. ex* diversis *P* **3** ipsum[1]] *corr. ex* primum P_7 subtractus] *corr. ex* subtractum *KM* ipsum[2]] idem *MN* **5** Lune] *s.l.* *P* **7** longitudinem] longitudine *N* **9** aliquod] *om.* *N* **10** subductus] subtractus *MN* (subductus BaE_1) **13** utralibet] utraque *M* **14** cursum medium] medium cursum *N* eclipsari] eclipsimari *PK corr. ex* eclipsimari P_7 (eclipsari BaE_1) **19** medium cursum] cursum medium *P* climatum] *corr. ex* c†rem†atum *K* eclipsari] eclipsimari *PK corr. ex* eclipsimari P_7 (eclipsari BaE_1) **20** ultra] ultra vero *N* **22** termini] et *add. et del.* P_7 **23** Lunam – Solem] Solem vel Lunam P_7M pati] pati per *M* **25** detectionis[1]] directionis *M* detectionis[2]] directionis *M* **26** Tria] tria sunt *N* principium] principium et P_7 **27** diametri] mihi *add. et del.* P_7

Book VI

The carrying beyond of the moon for a given time is that which remains when the sun's irregular motion for that time is subtracted from the moon's irregular motion for that time.

The mean carrying beyond of the moon for a given time is that which remains when the sun's mean motion for that time is subtracted from the moon's mean motion for the same time.

The moon's apparent motion is the progression in longitude of the moon's apparent place through parallax.

The moon's apparent carrying beyond for any time is that which remains when the sun's irregular motion for that time is removed from the moon's apparent motion for the same time.

The limits of lunar eclipses are the endpoints of the arcs of the moon's declined circle cut off on both sides of the node such that when the moon is below those endpoints toward the node according to mean course, it is possible that the moon be eclipsed, but beyond ⟨them⟩ it is impossible. And these limits are in that place where the first contact of the moon and the shadow happens to be at the true opposition beyond the mean ⟨opposition⟩.

The limits of solar eclipses are the endpoints of the arcs of the moon's declined circle cut off on both sides of the node such that when the moon is below them towards the node according to mean course, it is possible that the sun be eclipsed in any of the 7 climes; beyond it is impossible. And indeed these limits are in that place where the first contact of the sun and moon happens to be at the apparent conjunction beyond the mean conjunction. There are also other limits such that when the moon is joined to the sun [i.e. at syzygies] below them, it is necessary that the moon or sun undergo an eclipse.

There are five times of a lunar eclipse: the beginning of obscuring, the beginning of the delay [i.e. of the totality], the middle of the eclipse, the end of the delay or the beginning of the uncovering, and the end of the uncovering.

⟨There are⟩ three of a solar ⟨eclipse⟩: the beginning, middle, and end.

A digit of an eclipse is an obscured twelfth part of either the sun or moon's diameter.

Minuta casus sunt minuta in linea transitus Lune per que incidit in suam vel Solis eclipsim, et per que excidit a sua vel Solis eclipsi.

Minuta more sunt minuta in linea transitus Lune per que Luna tota moratur sub umbra.

Petitiones due sunt.

Tenebras in circulo lunari ad eam partem orizontis declinare in quam vergit arcus circuli magni transeuntis per duo centra Lune et umbre.

Tenebras in solari circulo ad eam partem orizontis declinare in quam cadit arcus circuli magni transeuntis per centrum Solis et visum locum Lune.

Atque hee declinationes flexus tenebrarum in utraque eclipsi dicuntur.

1. Tempus et locum medie applicationis Solis et Lune quam volueris prefinire.

Sume itaque in puncto temporis a quo computationem medie coniunctionis vel oppositionis queris medium locum Solis et medium locum Lune. Et disce distantiam inter Solem et Lunam, et serva. Sume iterum medium motum Solis ad unam diem et medium motum Lune ad unum diem, et disce per hoc mediam superlationem Lune ad unam diem. Atque per hanc mediam superlationem divide servatam distantiam Solis et Lune, et exibit tempus quesitum scilicet quod est a puncto temporis a quo computationem incipis usque ad primam coniunctionem mediam. Ratio. Nam sicut superlatio unius diei ad inventam distantiam que est superlatio quesiti temporis ita est dies unus ad ipsum quesitum tempus. Huic vero tempori si dimidium tempus equalis lunationis superaddas, habebis tempus medie oppositionis sequentis. Et si integrum tempus equalis lunationis addideris, erit tempus secunde coniunctionis et ita deinceps. Per tempus autem inventum ad notitiam loci pervenies, sumendo scilicet medium cursum Solis semper ad ipsum tempus inventum et superponendo loco primo Solis.

28 transitus] *corr. ex* tranea *K* incidit] *corr. ex* accidit P_7 **30** tota] *om. N* **32** due sunt] sunt due *M om. N* **38** Solis – Lune] *om. N* **40** in] a *N* **42** Solem – Lunam] Lunam et Solem *K* **43** unum] unam *N* **44** diem] Atque per hanc mediam superlationem Lune ad unam diem *add. et del. M* **45** quesitum] *corr. ex* quesitam P_7 **46** computationem] *corr. ex* compunctionem P_7 **49** ipsum – tempus[1]] tempus quesitum *N* vero] ergo *KM* **49/50** tempus[2] – superaddas] equalis lunationis tempus addideris *N* **50** superaddas habebis] *corr. ex* addas super habebis *K* **51** ita] sic *N* **52** pervenies] perveniens *P* **53** medium] *corr. ex* medio *P* ipsum] idem *N*

The minutes of immersion are the minutes on the line of the moon's passage through which it falls into its own or the sun's eclipse and through which it falls out of its own or the sun's eclipse.

The minutes of delay are the minutes on the line of the moon's passage through which the whole moon remains under the shadow.[1]

There are two postulates.

That the darkness in the lunar circle declines towards that part of the horizon to which the arc of the great circle passing through the two centers of the moon and the shadow tends.

That the darkness in the solar circle declines towards that part of the horizon onto which the arc of the great circle passing through the center of the sun and the moon's apparent place falls.

And these declinations are called the directions[2] of the darkness in either eclipse.

1. To determine the time and place of a mean syzygy of the sun and moon that you want.

Accordingly, take the sun's mean place and the moon's mean place at the point of time from which you seek the calculation of the mean conjunction or opposition. And learn the distance between the sun and moon, and save it. Again, take the sun's mean motion for one day and the moon's mean motion for one day, and learn through this the mean carrying beyond of the moon for one day. And by this mean carrying beyond, divide the saved distance of the sun and moon, and the sought time will result, i.e. what is from the point of time from which you begin the calculation to the first mean conjunction. Reasoning. For as the carrying beyond of one day is to the found distance, which is the carrying beyond of the sought time, thus is one day to that sought time. And indeed, if you add half the time of a mean lunation to this time, you will have the time of the next mean opposition. And if you add the complete time of a mean lunation, there will be the time of the second conjunction and thus so on. Moreover, you will reach the knowledge of the place through the found time, namely by taking the sun's mean course always for that found time and adding it to the sun's first place.

[1] I translate 'minuta casus' and 'minuta more' in the way that is found in other modern translations, such as Pedersen, *The Toledan Tables*, p. 277 and Swerdlow and Neugebauer, *Mathematical Astronomy in Copernicus's De Revolutionibus*, p. 283.

[2] In Plato of Tivoli's translation of Albategni, the word 'zenith' (also spelled 'cent' or 'cenit') is used to refer to certain intersections of circles with the horizon (*e.g.* Ch. 7 and Ch. 43, 1537 ed., ff. 13r and 63v), including the points on the horizon towards which the darkened parts of the sun or moon are directed at the significant times of eclipses (*De scientia astrorum* Ch. 43–44, *P*, ff. 59v–60r, 61r–62r, 65v, 67r, and 67v). The *Almagesti minor*'s author uses 'cenit capitum' or 'cenit' to refer to the zenith throughout the book. When paraphrasing the passages in Albategni's Ch. 43–44, the *Almagesti minor*'s author deviates from his source and avoids the use of 'cenit' for other astronomical concepts. Instead, he uses 'flexus' when discussing the directions of shadows in eclipses.

Tenor vero tabularum ad hoc constitutarum est ita. In primo mense primi anni annorum collectorum queritur prima coniunctio media modo quo diximus, et tempus illius coniunctionis pro radice figitur. Et v lateraliter iunguntur tabule tot gradus quotus est numerus annorum collectorum habentes, et in primo gradu prime tabule radix dicta figitur. In secunda tabula dies et hore et minuta horarum a principio mensis usque ad punctum medie coniunctionis. In tertia medius motus Solis vel Lune attinens ad illud tempus. In quarta motus medius diversitatis Lune qui portio nomen habet. In quinta medius motus latitudinis prout tempori sumpto convenit. Fundatis igitur sic principiis omnium tabularum deinceps in gradibus prime tabule ordinantur secundum crementum suum numeri annorum collectorum. Deinde consideratur quantitas temporis quod inter duos numeros vicinos cadit, et de quot mensibus lunaribus proici potest attenditur. Et superfluum de uno mense lunari super positam radicem temporis debet adici. Et si plus quam mensis excrescat, datus mensis abiciatur, et reliquum scribendum servatur. Vel si placet, de quantitate temporis inter duos numeros cadentis menses lunares quotquot possunt abiciuntur, et superfluum temporis deposita radice temporis si potest abiciatur. Si minus, additur super radicem mensis date quantitatis et sic inde proicitur. Reliquum vero in secunda tabula suo annorum numero opponitur. Et sic secundum eandem quantitatem secunda tabula continue crescit vel decrescit. In tertia deinde tabula et quarta et quinta medii motus sicut sumptis temporibus convenit collocantur.

Post hec in alia pagina ordinantur tabule annorum expansorum, et in prima quidem numeri ipsorum. Et primum quidem quantitas unius anni de quot mensibus lunaribus minui possit ad minus attenditur, et quod superfluit de uno mense lunari in prima area collocatur. Et si anni expansi omnes equales fuerint, secundum additionem huius superflui relique aree secunde tabule formantur. Et si plus quam mensis date quantitatis excreverit, abiecto mense reliquum scribitur. Si vero inequales fuerint, quantitas annorum sumptorum de mensibus lunaribus proicitur, et superfluum mensis suo numero annorum expansorum opponitur. In tertia rursum pagina sunt tabule mensium lunarium. Et in prima quidem numeri mensium ponuntur ex ordine, et in secunda primum quantitas

55 est ita] erit ista *M* primo mense] principio *N* **55/56** primi anni] anni primi *M* **59** et[1]] *om. N* **60** horarum] hararum *P* **61** medius – vel] locus secundum medium motum Solis et *M* **62** qui] que *M* medius[2] – latitudinis] motus medius latitudinis a nodo *M* **65** Deinde] deinceps P_7 temporis] *marg. P* **68** adici] addisci *P corr. ex* addici P_7 addisci *corr. in* addi *N* excrescat datus] lunaris excrescat *M* **69** servatur] servaturi PP_7 (scribatur *Ba* servatur E_1) **70** abiciuntur] abiciantur *N* **71** abiciatur] abicitur *KM* **72** mensis] mensis lunaris *M* **73** suo] suorum *M* opponitur] apponitur *N* eandem] eam *M* **76** hec] hoc *MN* in alia] *corr. ex* mala *K* pagina] *corr. ex* tabula *K* **78** uno] *om. N* **80** formantur] sumantur *N* **81** mensis] mensis lunaris *M* date quantitatis] date (*corr. in* dati) quantitas *N* **82** scribitur] scribatur *N* inequales] equales P_7 fuerint] fiunt *P* fuerit *M* quantitas] *corr. ex* quantitatis *N* annorum sumptorum] sumptorum annorum *M* **83** et] *om.* P_7 **84** rursum] rursus *M* **85** primum] primum quidem P_7

And indeed, the way of proceeding of the tables set up for this is thus. In the first month of the first year of the collected years, the first mean conjunction is sought in the way by which we said, and the time of that conjunction is established as a radix. And five columns are joined across, having as many rungs as is the number of collected years, and in the first rung of the first column, the said radix is fixed.[3] In the second column the days, hours, and minutes of hours from the beginning of the month to the point of the mean conjunction. In the third, the sun or moon's mean motion pertaining to that time. In the fourth, the mean motion of the moon's irregularity that has the name 'portion.' In the fifth, the mean motion of latitude as agrees with the taken time. Accordingly, with the beginnings of all the columns established in this way, the numbers of the collected years are arranged in succession in the rungs of the first column according to their increase. Then the quantity of time that falls between two adjacent numbers is considered, and it is noticed from how many lunar months it is able to be subtracted. And the excess from one lunar month should be added to the posited radix of time [i.e. the first entry of the second column]. And if it grows to more than a month, the given month is subtracted, and the remainder to be written down is saved. Or if it pleases, as many lunar months as possible are subtracted from the quantity of time falling between two numbers, and let the excess of time be subtracted from the posited radix of time, if it is possible. If not, a month of the given quantity is added upon the radix, and thus it is subtracted from this. And indeed, the remainder is placed in the second column opposite its number of years. And thus the second column continuously increases or decreases according to the same quantity.[4] Then in the third, fourth, and fifth columns, the mean motions as they agree with the taken times are set out.

Afterwards, the tables of expanded years are arranged on another page, and in the first ⟨column⟩ indeed are numbers of them [i.e. the expanded years]. And indeed first it is seen how many lunar months are the least from which the quantity of one year is able to be subtracted, and what is in excess from one lunar month is set out in the first area. And if all the expanded years are equal, the remaining areas of the second column are fashioned according to the addition of this excess. And if it grows to more than a month of the given quantity, the remainder with a month having been subtracted is written. However, if they [i.e. the expanded years] are unequal, the quantity of taken years is subtracted from lunar months, and the excess of a month is placed opposite its number of expanded years. In turn, the tables of lunar months are on the third page. And indeed, in the first ⟨column⟩, the numbers of months are

[3] The author confusedly uses the word 'root' to refer both to the first entry of the first column, which is 1 in Ptolemy's table, and the first entry of the second column, which is 24^{d} 44′ 17″ (*Almagest*, 1515 ed., f. 61v).

[4] Our author gives two equivalent methods here for finding this amount. Only the latter is given by Ptolemy (*Almagest* V.3, 1515 ed., f. 61r). The former set fits better with the table found in the Toledan Tables (Pedersen, *The Toledan Tables*, Table GA11, pp. 1328–32).

unius lunationis equalis, deinde eadem quantitas duplicata, post hec triplicata, et sic deinceps. In reliquis vero tabulis medii motus sicut tempori sumpto competit statuuntur.

Quotiens ergo in aliquo mense proposito mediam coniunctionem queris, si ille mensis primus est anni, sufficit intrare in tabulam annorum tantum, dummodo cum anno nondum completo intres. Si vero alius mensis fuerit, quot coniunctiones post primam illius anni tunc sint observandum. Et cum hoc numero intrandum in tabulam mensium, et quod in secunda tabula inventum fuerit est tempus a prima coniunctione anni. Cui si addideris quod in directo ipsius in secunda tabula scriptum est, erit tempus a principio anni usque ad quesitam coniunctionem.

Propter oppositiones vero in alia pagina statuuntur denuo numeri annorum collectorum. Deinde a tempore coniunctionis – primum eo quod pro radice ponitur – medietas lunaris mensis subtrahitur, et reliquum in secunda tabula preventionis scribitur. Nam ipsum est tempus oppositionis medie quod pro radice figendum. Deinde medii motus secundum quod huic tempori convenit, quod est medietas mensis lunaris sumpti, a mediis motibus coniunctionis minuantur, et quod reliquum est in tabulis preventionis scribitur. In reliquis vero scalis tabularum preventionis sicut in coniunctione fit faciendum. Unde non oportet mutari tabulas annorum expansorum vel mensium quia utrisque deservire possunt.

Si vero super annos et menses Arabum applicationes medias tabulare volueris, sicut in annis collectis prius factum est non dissimiliter fiat. Et quia hii anni vel menses equalibus lunationibus fere respondent, et alternatim in modica quantitate habundat, conferendi sunt anni primum sigillatim cum lunationibus equalibus. Et si lunationes habundaverint, superhabundantia, quia in hiis horis

86 hec] hoc *N* **87** sumpto competit] competit sumpta *P* convenit sumpta *N* **89/93** Quotiens – numero] Quotiens ergo mediam coniunctionem alicuius anni queris, sufficit intrare in tabulam annorum tantum, dummodo cum anno nondum completo intres. Et quod inventum fuerit super illud quod in annis collectis inventum est pone. Et si plus quam mensis lunaris excrescat, abicitur mensis. Si vero *M There is also a correction adding in the margin*: Et si ille mensis est primus anni, sufficit. Si vero alius mensis fuerit, quot coniunctiones post primam illius anni tunc sint observandum. Et cum hoc numero intrandum in tabulam mensium. Et quod in secunda tabula scriptum est erit tempus a principio anni usque ad coniunctionem quesitam. *M* **90** intrare – tabulam] in tabulam intrare *PN* intrare tabulam P_7 **92** sint] situm P_7 **95** secunda tabula] tabula secunda *K* **96** quesitam coniunctionem] coniunctionem quesitam *M* **98** primum] primo *K* enim *add. et del.* P_7 prime *M* (primum BaE_1) eo] *om. K* **100** oppositionis] *corr. ex* operationis *K* **101** figendum] figendum est *PMN* (figitur *Ba* figendum E_1) **103** minuantur] minuatur *M* minuuntur *N* **104** faciendum] faciendum est *N* **105** mutari] *perhaps corr. ex* imitari *K* vel] *om. P* et *N* utrisque] *corr. ex* utriusque *P* **107** applicationes] *corr. ex* applicas *M* **108** quia] *om. PN* **109** lunationibus fere] fere lunationibus *PN* **110** anni primum] primum anni *PN* **111** habundaverint] superhabundaverint *PN* superhabundantia quia] super habundantiam que *M* hiis] in *add. marg. K om. M del. N* (hiis *Ba om.* E_1)

placed in order, and in the second ⟨column⟩ is first the quantity of one mean lunation, then the same quantity doubled, afterwards tripled, and thus so on. And indeed, in the remaining columns, the mean motions are set up as agrees with the taken time.

Therefore, whenever you seek a mean conjunction in any proposed month, if that month is the first of the year, it is sufficient to enter into the table of years only, provided that you enter with the year not yet completed. But, if it is another month, it must be noted how many conjunctions after the first of that year there are then. And the column of months must be entered with this number, and what is found in the second column is the time from the year's first conjunction. If you add to this what is written in line with it in the second table [i.e. the table of years],[5] it will be the time from the year's beginning to the sought conjunction.

And indeed, for the oppositions the numbers of the collected years are set up a second time on another page. Then half a lunar month is subtracted from the time of the conjunction – first from that which is supposed as a radix, and the remainder is written in the second column of oppositions. For that is the time of mean opposition that is to be established as the radix. Then the mean motions according to what agrees with this time, which is half the taken lunar month, are subtracted from the mean motions of the conjunction, and what is the remainder is written in the tables of opposition. And indeed, in the remaining rungs of the tables of opposition, it ought to be done as it is done for conjunctions. Whence it is not necessary that the tables of expanded years or of months be changed because they are able to be of use for either.[6]

But if you want to make tables for the mean syzygies upon the years and months of the Arabs, let it not be done dissimilarly than was done for collected years earlier. And because these years or months almost correspond to mean lunations and they alternately exceed by a small quantity, first the years are to be compared one by one with mean lunations. And if the lunations exceed, the

[5] The quantity taken from the table of months should be added to the quantity that is found for the first conjunction of the year, which is found using the tables of collected years and single years.

[6] I.e. the tables of expanded years and of months can be used for both conjunctions and oppositions.

et minutis horarum tantum consistit, scribenda est in areis horarum et minutorum et nichil in diebus. Si vero quantitas annorum sumptorum habundaverit, quia in horis et minutis consistit habundantia, tantum unum in diebus numero istorum annorum opponendum est, scilicet ut unus dies de diebus annorum collectorum minuatur, et hore cum minutis que ad completionem lunationis supererunt in areis horarum et minutorum subscribantur. In pagina vero mensium in directo quidem Almuharam qui primus est ubique nichil titulatur eo quod determinata sunt tempora coniunctionis vel oppositionis ipsius in annis collectis et expansis. Ceteri vero singuli cum equalibus lunationibus conferuntur. Et si lunationes habundaverint, quoniam in horis et minutis horarum tantum est hec habundantia, nichil in diebus sed in areis horarum et minutorum quod eis debetur suo mensi opponitur. Si vero quantitas sumptorum mensium habundaverit, unum in area dierum scribitur, scilicet ut unus dies de diebus annorum collectorum minuatur, et hore cum minutis que ad completionem equalis lunationis supersunt suo mensi opponuntur. Et hec quidem tabularum est ratio. Et nota quod dies qui colliguntur ex tabulis mediocres sunt, non differentes, et ad meridiem illius civitatis supra quam constitute sunt tabule.

2. Diversum motum Solis sive Lune ad datam horam excipere.

Huius rei vera notitia est ut eques locum stelle ad principium hore date idest terminum precedentis; eques etiam ad finem ipsius hore date; deinde minorem locum a maiori demas. Nam quod relinquitur est diversus motus stelle ad horam datam. Quod si facilius ad idem et prope verum vis pervenire, sume portionem equatam usque ad horam datam, et per eam disce equationem Lune simplicem. Deinde sume medium motum portionis unius hore, et multiplica hunc motum medium in acceptam equationem, et divide quod provenit per portionem usque ad datam horam si ipsa minor fuerit xcv gradibus, ubi scilicet est media longitudo. Et si maior fuerit, minue a clxxx. Quod si etiam maior fuerit clxxx et minor cclxv, minue ab ea clxxx. Quod si etiam maior fuerit cclxv ubi iterum est longitudo media epicicli, minue eam de ccclx donec

112 minutis horarum] *corr. ex* momentis horum *P* **113** habundaverit] superabundaverit *N* **114** unum] *om. N* **114/115** numero – annorum[1]] istorum annorum numero *PN* illorum annorum *M* **115** opponendum est] opponendum *P* appondendum est P_7 apponendum *N* **117** supererunt] superaverunt *PN* (supererunt *Ba*E_1) **118** Almuharam] Alunaram *PN* Alumaram P_7 Almuarum *corr. ex* Al†...† *K* (Abmandram *Ba* Almuaram E_1) nichil] non *N* titulatur] intitulatur P_7*N* **119** determinata] deinde mutata *PN* (determinata *Ba*E_1) **120** singuli] singulis *M* **122** hec] *om. PN* **126** suo – opponuntur] opponuntur suo mensi *M* **126/127** tabularum est] est tabularum P_7*K* **127** mediocres sunt] sunt mediocres et *M* mediocres sunt et *N* **128** meridiem] unum diem P_7 meridianum *M* constitute] statute *K* **129** ad datam] *corr. ex* adda *K* **130** stelle] *corr. ex* stelles *K* **131** terminum] *corr. ex* †ter...† *M* **132** diversus] *corr. ex* diversitas P_7 **136/137** per portionem] *corr. ex* proportionem *M* **137/138** xcv – fuerit] *marg.* (*perhaps other hand*) *P* **137** scilicet] *om. PN* **139/140** clxxx[1] – fuerit] *marg.* (*perhaps other hand P*) *P* **140** iterum] verum *P* excess should be written in the areas of hours and minutes and nothing in the days because it consists only in these hours and minutes of hours. However, if the quantity of the taken years exceeds, because the excess consists in hours and minutes, only one should be placed in the days opposite the number of those years, namely so that one day is subtracted from the days of the collected years, and let the hours with minutes that are superfluous for the completion of a lunation be written in the areas of hours and minutes. And indeed, on the page of months in the line of Almuharam, which is the first, nothing is inscribed anywhere because its times of conjunction or opposition were designated in ⟨the tables of⟩ the collected and expanded years. However, the remaining individuals [i.e. the remaining months] are compared with mean lunations. And if the lunations exceed, because this excess is only in hours and minutes of hours, nothing is placed in the days but in the areas of hours and minutes, what is owed by them to their month is placed opposite its month. But if the quantity of the taken months exceeds, '1' is written in the area of days, namely so that one day is subtracted from the days of the collected years, and the hours with minutes that are superfluous for the completion of a mean lunation are placed opposite their month. And this indeed is the reasoning of the tables. And note that the days that are collected from the tables are average ones, not diverse ones, and are for the meridian of that city upon which the tables were set up.

2. To extract the sun or moon's irregular motion for a given hour.

The true knowledge of this matter is that you correct the star's place at the beginning of the given hour, i.e. the endpoint of the preceding ⟨time⟩; you also correct ⟨it⟩ at the end of that given hour; and then you subtract the smaller place from the larger. For what remains is the star's irregular motion for the given hour. But if you want to reach the same more easily and approximately, take the equated portion up to the given hour, and through it learn the moon's simple equation. Then take the mean movement of the portion in one hour, and multiply this mean movement by the taken equation, and divide what results by the portion up to the given hour if that [i.e. the portion] is less than 95°, i.e. where there is the mean distance. And if it is greater, subtract ⟨it⟩ from 180°. And if it is also greater than 180° and less than 265°, subtract 180° from it. And if it is also greater than 265°, where again there is the epicycle's mean distance, subtract it from 360°, namely until you have the arc of videlicet habeas arcum epicicli ab alterutra longitudine longiore vel propiore. Et per hoc divide id quod ex multiplicatione provenerat. Et quod exierit erit equatio ad unam datam horam pertinens, eo quod sicut portio unius hore ad arcum predicto modo sumptum ita pene se habet equatio quesita ad equationem acceptam. Inventam itaque equationem, si portio usque ad datam horam ceciderit inter duas longitudines medias versus longitudinem longiorem, minue de medio cursu unius hore; et si ceciderit versus longitudinem propiorem, adde. Et habebis motum diversum stelle ad datam horam. Et hoc quidem opus Ptolomei est et est propinquius vero quando portio usque ad datam horam citra vel ultra medias longitudines longius terminabitur.

Aliter cum portione usque ad datam horam sume equationem simplicem, deinde ab ipsa portione minue portionem unius hore, que est xxxii minuta et xl secunda. Et cum residua portione sume iterum equationem simplicem quanto verius poteris, et minue minorem equationem de maiori. Et residuum erit equatio pertinens ad datam horam, quam addes vel minues predicta via de medio cursu unius hore.

Quod si hoc tabulare volueris ut sit ingressus in tabulas per portionem per vi et vi augmentatam, ita affinius vero operaberis. Sumes primum equationem simplicem que debetur portioni vi graduum, et tempus huius motus diligenter attendes, et est quidem xi hore et unum minutum fere. Ad hoc deinde tempus sumes medium motum longitudinis et minues equationem ab eo. Et reliquum cum ipsum alibi servaveris divides per horas accepti temporis que sunt xi et unum minutum, et quod exierit est motus diversus ad unam horam cum portio fuerit vi graduum. Deinde duplicabis portionem ut sit scilicet xii graduum, et cum ea sumes equationem simplicem. Et minues eam a motu medio duplicato, et erit motus equatus, a quo cum ipsum alibi servaveris minues motum equatum quem prius servasti. Et reliquum divides ut prius per horas accepti temporis que sunt xi et unum minutum. In tanto enim tempore parum variatur diversus motus. Et quod exierit est diversus motus ad unam horam cum portio

141 alterutra] alterutra parte *PN* longitudine – propiore] longitudinis longioris vel prope *N* **142** per] *corr. ex* propter *K* divide id] idem dividendus *M* id] *s.l. P* exierit] exibit *N* **143** datam horam] horam datam *M* **146/148** ceciderit – horam] *marg.* P_7 **146** ceciderit] cecidit P_7 **148** Ptolomei] Tholomei P_7 **149** propinquius] propinquus *P* **150** medias longitudines] longitudines medias *N* **153** xl secunda] xl *corr. ex* xlii *K* **154** verius poteris] *corr. ex* numerus positis P_7 poteris] potes *N* minorem equationem] equationem minorem *P* equationem *N* **155** quam – predicta] *marg.* (*perhaps other hand*) *M* **157** ut sit] insit *P* tabulas] tabulam *N* **158** augmentatam] aumentatam *K* ita] illa *M* affinius] vicinius *N* primum] primo *MN* primum equationem] *iter. et del. P* **160** xi] ix P_7 deinde] autem *N* **162** ipsum] *corr. in* ipso *M* servaveris] servaveris et reliquum *M* accepti temporis] temporis accepti *N* xi] 11 hore *M* **163** exierit] exibit *N* motus] *corr. ex* medius *M* horam] et *add. et del.* P_7 **164** duplicabis] duplabis P_7 scilicet] *om. N* **165** motu medio] medio motu *N* **166** alibi] *corr. ex* alibibi *M* **168** xi] 11 hore *M* **169** diversus[1] motus] motus diversus *N* exierit] exibit *N* est] erit *PMN* (est BaE_1) diversus[2] motus] motus diversus *N* the epicycle from either the apogee or perigee. And divide what resulted from the multiplication by this. And what results will be the equation pertaining to the one given hour, because as the portion of one hour is to the arc taken in the said way thus almost does the sought equation have itself to the taken equation. Accordingly, if between the two mean distances the portion up to the given hour falls towards the apogee, subtract the found equation from the mean course of one hour; and if it falls towards the perigee, add. And you will have the star's irregular motion in the given hour. And this indeed is the work of Ptolemy, and it is nearer to the truth when the portion up to the given hour will be bounded by a greater length on this side of the mean distances or beyond them.

In another way, with the portion up to the given hour, take the simple equation, then subtract from it the portion of one hour, which is 32′ 40″. And with the remaining portion, take again the simple equation as truly as you can, and subtract the smaller equation from the greater. And the remainder will be the equation pertaining to the given hour, which you add or subtract in the said way from the mean course of one hour.

And if you want to make a table of this so that the entrance into the tables is through the portion increased by 6° and 6°, you will operate closer to the truth in this way. First you will take the simple equation that is owed to a portion of 6°, and carefully consider the time of this motion, and indeed it is approximately 11 hours 1′. Then you will take the mean motion of longitude for this time, and you will subtract the equation from it. And after you have saved it in another place, you will divide the remainder by the hours of the taken time, which are 11 1′, and what results is the irregular motion for one hour when the portion is 6°. Then you will double the portion, namely so that it is 12°, and you will take the simple equation with it. And you will subtract it from the doubled mean motion, and there will be the corrected motion, from which, after you have saved it elsewhere, you will subtract the corrected motion that you saved earlier. And as before you will divide the remainder by the hours of the taken time, which are 11 1′. For in such a time the irregular motion changes very little. And what results is the irregular motion for one fuerit xii graduum. Deinde triplicabis portionem et sumes cum ea equationem. Triplicabis etiam motum medium ac tempus acceptum, et operaberis ad instar dicti modi donec portio semicirculum compleverit.

Palam autem quod hec equatio diversi motus non accipit id quod in motu Lune ex secunda diversitate accrescere potest, eo quod propter coniunctiones et oppositiones veras sit eius investigatio in quibus, ut decima propositio precedentis libri ostendit, non nisi modica potest esse superfluitas secunde diversitatis. Nam maxima distantia Solis et Lune media non nisi vii graduum fere esse potest.

3. Tempus et locum vere applicationis Solis et Lune prope verum preoccupare.

Hoc siquidem ad verum doctrina omnino non comprehendit eo quod diversi motus Solis et Lune singulis momentis variantur, et hoc neque proportionaliter sibimetipsis neque invicem. Attamen duo sunt huius propositi opera, unum Ptolomei et aliquantulum differentius a vero sed facilius, alterum Albategni laboriosius quidem sed vero cognatius ut ostendemus.

Et opus quidem Ptolomei est ut primum queras et scias tempus et locum applicationis medie. Deinde verifices locum Solis et locum Lune, et hoc per equationem simplicem que per portionem sicut ex tabulis extrahitur invenitur. Quod si tunc in eodem gradu vel in oppositis Sol et Luna convenerint, habes quod quesisti. Si vero inter eos distantia fuerit, illa quidem propter equationes accidet, et erit vera distantia Solis et Lune que ad plus vii graduum esse potest cum ex utrisque equationibus maximis collecta fuerit. Hanc itaque divide per veram superlationem Lune ad unam horam. Et exibit tempus a media coniunctione usque ad veram, quod debes addere super tempus medie coniunctionis si media precedit veram idest si Luna nondum vere consecuta est Solem; minues autem si vera coniunctio precessit mediam. Per tempus autem inventum ad locum vere applicationis pertinges scilicet multiplicando tempus per diversum motum Lune ad unam horam, et quod exierit superponendo loco Lune verifi-

170 xii] vii *P* cum ea] *corr. ex* eam *P* **171** motum medium] medium motum *N* ac] ad *MN* ad] *del. P s.l.* P_7 *om. K* **172** portio] *corr. ex* portam P_7 semicirculum] semicirculi *KM* compleverit] complebitur *M* **173** quod hec] *iter. P* in motu] *corr. ex* initio P_7 **175** oppositiones] (*perhaps written in another hand*) *K* sit] fit P_7 decima – precedentis] x proportio precedentis *P* decima proportio (*corr. in* propositio) precedendis *K* in decima propositione precedentis *N* **176** ostendit] ostenditur *N* potest esse] esse potest *K* **178** potest] potest et cetera *N* **181** Hoc – omnino] hec siquidem doctrina omnino ad verum *M* **183** invicem] adinvicem *PN* huius] huiusmodi *M* unum] *corr. ex* unde P_7 **184** Ptolomei] Tholomei P_7 differentius – vero] *corr. ex* differentibus †meo† *K* Albategni] †scilicet† *add. et del. N* **185** ut] *s.l. P* **186** quidem] *om. N* Ptolomei] Tholomei P_7 Tolomei *K* **188** portionem] *corr. ex* potionem *K* invenitur] invenietur *K* **189** tunc] *corr. ex* †...† *K* **190** quesisti] quesivisti *N* quidem] inquam que *N* **191** accidet] accidere potest *N* vera] *s.l. P* **196** vera] vero *N* **197** pertinges] pertingens *P corr. ex* pertingens *N* hour when the portion is 12°. Then you will triple the portion and with it take the equation. You will also triple the mean motion and the taken time, and you will operate according to the likeness of the said way until the portion will have completed a semicircle.

Moreover, it is clear that this correction of the irregular motion does not admit that which is able to grow in the moon's motion from the second irregularity, because its investigation is for true conjunctions and oppositions, in which the excess from the second irregularity is only able to be modest, as the tenth proposition of the preceding book showed. For the greatest mean distance of the sun and moon can only be about 7°.

3. To anticipate the time and place of a true syzygy of the sun and moon approximately.

The doctrine indeed does not grasp this entirely truthfully because the sun and moon's irregular motions vary at each moment, and this neither proportionally to each other nor reciprocally. But yet there are two works of this proposition: one of Ptolemy, slightly more unlike the truth but easier, and another of Albategni, more laborious indeed but more similar to the truth, as we will show.

And indeed Ptolemy's work is that first you seek and know the time and place of the mean syzygy. Then you correct the sun's place and the moon's place, and this through the simple equation that is found through the portion as it is extracted from the tables [i.e. *Almagest* IV.10 or V.8]. And if at this time the sun and moon meet at the same degree or are at opposites, you have what you sought. But if there is a distance between them, indeed that will occur because of the equations, and there will be the true distance of the sun and moon, which is able to be at most 7° when it is combined from both greatest equations. Accordingly, divide this by the moon's true carrying beyond for one hour. And there will result the time from the mean conjunction to the true, which you ought to add to the time of the mean conjunction if the mean ⟨conjunction⟩ precedes the true, i.e. if the moon has not yet reached the sun; however, you will subtract if the true conjunction preceded the mean. Moreover, through the found time, you will reach the place of true syzygy, i.e. by multiplying the time by the moon's irregular motion for one hour, and by adding what results to the moon's corrected place if the moon has not yet cato si Luna nondum consecuta est Solem in media coniunctione; et si pridem est consecuta, subtrahendo. Aut multiplicabis tempus intermedium per diversum motum Solis ad unam horam, et quod exierit loco Solis vero superpones vel subtrahes.

Quod si velis per locum intermedium tempus cognoscere, inventam distantiam Solis et Lune accipe, et ei duodecimam partem ipsius superpone semper. Nam tantum fere interim perambulat Sol donec Luna coniuncta sit vere Soli. Et collectum divide per diversum motum Lune ad unam horam, et erit tempus intermedium applicationis medie et vere. Loco autem Lune equato totum cum duodecima superpone, et loco Solis duodecimam – ita dico si Solis est superfluum; et si Lune, minue. Et videbis Solem et Lunam in eodem loco convenisse. Quicquid autem loco Lune addis vel subtrahis addendum vel subtrahendum similiter medio motui latitudinis ut ipse quoque fere equatur. Nam per ipsum equatum eclipses querende sunt.

Opus vero Albategni est ut si non convenerint Sol et Luna in eodem minuto post equationes premisso modo factas, distantia que inter eos reperta fuerit sumatur. Et per eam portio equetur videlicet duplicando distantiam et per eam accipiendo equationem portionis que et puncti equatio dicitur, et addendo eam super portionem si coniunctio vera futura est post mediam vel subtrahendo si post. Quod si velis, distantie reperte sextam et octavam partem accipe. Nam hec est fere equatio addenda vel subtrahenda portioni sicut experientia temptatum est. Per hanc ergo equatam portionem simplicem equationem Lune sumens, locum Lune ut prius verifices addendo scilicet vel subtrahendo simplicem equationem medio cursui Lune. Et loco Lune sic verificato uteris vice prioris verificationis, verificationem vero Solis non mutabis. Distantiam itaque Solis et Lune hoc modo repertam divides per veram superlationem Lune, et operaberis per cetera ut prius.

200 Aut] autem *K* intermedium] medium *N* **201** exierit – superpones] exibit loco vero Solis suprapones *N* vero] *corr. ex* non P_7 **204** ipsius] *om. N* superpone] suppone *P* **205** interim – Sol] perambulat Sol interim P_7 perambulat] *corr. ex* perambulabat *K* **207** medie – vere] vere et medie *N* autem] *corr. ex* ante *K* **208** superpone] superponere *P* duodecimam] duodecima *PM* **209** Solem – Lunam] Lunam et Solem *M* **210** addendum] addendum sit *M* subtrahendum] subtrahendum est *N* **211** medio motui] medio motu *PM* motui medio P_7 equatur] *corr. in* equetur *K* equetur *N* (equetur *Ba* equatur E_1) **212** equatum] equatum ipse *M* **213** convenerint] conveniunt *N* **214** premisso] predicto *MN* **215** equetur] equatur *P* **218** post] *corr. in* ante *M* (post BaE_1) distantie reperte] reperire distantie *N* et] vel *M* accipe] accipere *P* **219** hec] *s.l.* P_7 hoc *KM* fere equatio] equatio fere *PN* vel] *marg.* P_7 experientia] experitia *M* **220** ergo] quoque *K* **221/222** ut – Lune[1]] *marg. P* **221** addendo – subtrahendo] subtrahendo vel addendo *N* scilicet] *om. P* (*om. Ba* scilicet E_1) **222** cursui] *corr. ex* cursu P_7 cursu *KM* uteris] utaris *N* **225** per – prius] *corr. ex* ut prius per cetera *P* reached the sun in the mean conjunction; and by subtracting if it has reached ⟨it⟩ previously. Or, you will multiply the intermediate time by the sun's irregular motion for one hour, and you will add or subtract what results to the sun's true place.

And if you wish to know the intermediate time from the place ⟨of the mean conjunction in another way⟩, take the found distance of the sun and moon, and always add its twelfth part to itself. For the sun moves approximately this much in the meantime until the moon is conjoined to the true sun. And divide the sum by the moon's irregular motion for one hour, and there will be the intermediate time between the mean and true syzygy. Moreover, add the whole with a twelfth to the moon's corrected place, and the twelfth to the sun's place – thus I say if the excess is the sun's; and if the moon's, subtract. And you will see that the sun and moon meet in the same place. Moreover, whatever you add or subtract to the moon's place must similarly be added or subtracted to ⟨or from⟩ the mean motion of latitude so that it also is corrected approximately. For the eclipses must be sought through it [i.e. the motion of latitude] corrected.

And indeed, Albategni's work is that if the sun and moon do not meet at the same minute after the corrections made in the way set forth, the distance that was found between them is taken. And through it the portion is equated, i.e. by doubling the distance and by taking through it the equation of portion, which is also called the equation of point, and by adding it to the portion if the true conjunction is going to be after the mean, or by subtracting if after.[7] And if you wish, take 7/24 of the found distance. For this is approximately the equation to be added or subtracted to the portion, as was tested by experience.[8] Therefore, taking the moon's simple equation through this equated portion, you will correct the moon's place as before, i.e. by adding or subtracting the simple equation to the moon's mean course. And you will use the moon's place thus corrected instead of the earlier correction, but you will not change the sun's correction. Accordingly, you will divide the distance of the sun and moon found in this way by the moon's true carrying beyond, and you will operate through the rest as before.

[7] This should say 'before', but the mistake is found in all the witnesses.

[8] Nallino, *al-Battānī*, vol. I, pp. 273–74 offers a detailed explanation of how al-Battānī may have derived this value, but al-Battānī may have merely taken it as an approximation of the values from the tables for the moon's anomalies. He gives (*De scientia astrorum* Ch. 42, 1537 ed., f. 60v) his own explanation through values apparently taken from his tables: a distance between the sun and moon of 5° causes an equation of portion of 89′ (taken from his table while the text says approximately 90′), resulting in the quotient 0.29666. This is fairly close to 7/24, which in decimal notation is 0.29166, and an even closer correspondence can be gained by taking a distance of 3° between the sun and moon, which results in an equated portion of 53′, according to both Ptolemy's and Albategni's tables (Toomer, *Ptolemy's Almagest*, p. 238 and Nallino, *al-Battānī*, vol. II, p. 78); and this results in the quotient 0.29444.

Opus vero istud ideo vero affinius est quam illud superius, quia ut primo ponamus coniunctionem veram post mediam futuram esse, distantia que per opus Ptolomei reperitur vera quidem distantia est Solis et Lune in coniunctione media, et est portio circuli signorum percurrenda a Luna cum eo etiam quod Sol interim perficiet antequam comprehendat Solem, sed preterea accrescet interim huic portioni signorum circuli aliquid ex secunda diversitate et aliquid ex reflexione diametri epicicli etiam percurrendum a Luna antequam comprehendat Solem. Sed quod augeri potest ex secunda diversitate, quia modicum est nec multum detinet interim motum Lune, postponitur. Illud vero augmenti quod ex reflexione diametri circuli brevis accidere potest nequaquam est postponendum. Nam potest detinere motum Lune donec ipsa comprehendat Solem per quartam unius hore. Et hoc quidem tunc accidet cum equatio Lune est trium graduum minuenda et Solis duorum fere addenda. Nam tunc distantia fit si duplicatur x graduum, et equatio portionis que ei debetur est unus gradus et dimidius fere, que faciunt sextam et octavam distantie fere. Si itaque portionem absque equatione sua sumas et cum ea simplicem equationem Lune invenias, deinde si portionem equatam sumas et cum ea simplicem equationem invenias, videbis equationes simplices differre in octava parte unius gradus fere, quod est in Lune motu quarta unius hore ad minus. Et nota quod minor erit error si negligatur hec equatio portionis cum distantia Solis et Lune vii erit graduum quam cum erit v. Nam ubi vii est graduum, Lune simplex equatio est v graduum et portio Lune equata xcv graduum. At si portioni equate demas equationem suam que hic longitudini duplici debetur scilicet duos gradus fere, et ita cum simplici portione sumas equationem Lune simplicem, videbis equationes differre in tribus secundis tantum. Quare respectu trium graduum maior erit differentia quam respectu v, et hoc est quod volebamus. Pari modo accidet in minuendo si vera coniunctio precedit mediam.

226 ideo vero] *marg. P* vero[2]] *om. N* primo] primum *M* **228** Ptolomei] Tholomei P_7 reperitur] invenitur *N* **229** cum – etiam] etiam cum eo *M* cum etiam illo *N* **230** perficiet] proficiet *P* describit *N* accrescet] adcrescet *marg. P* accrescent *N* **231** interim] *om. N* signorum circuli] circuli signorum *MN* aliquid[1] ex] interim de *N* et] *corr. ex* vel *K* **232** diametri] *marg. N* **232/235** epicicli – diametri] *om. PN* **233** potest] *corr. ex* prius P_7K **234** interim] *corr. ex* iterum *M* **235** potest] ideo *add. s.l. N* **237** Et – quidem] et hec que P_7 et hoc (*these last two words s.l.*) que *K* **238** Solis] Sol *M* **239** fit – duplicatur] si duplicatur fit *N* fit] sit *P* portionis] potionis P_7 **240** distantie] *iter. et del. K* **241** absque] sine *PN corr. ex* abque *K* sua] *om. N* sumas] *corr. ex* asumas *P* **242** si] equationem *add. et del. M* **243** invenias] invenias et *P* videbis] *corr. ex* videlicet *K* differre] differentie *K* **244** quarta] *om. P* hore] fere *add. et del. N* **246** vii est] est 7 *M* **247** graduum[1]] *om. N* equata] equata est *MN* xcv] xxv *P corr. ex* 25 *M* **248** hic] huic *M* **249** equationes] equationem *M* **250** differre] differentie *K* in tribus] *iter. et del. M* **251** erit] est P_7 **252** in] *om. PN* precedit] precedet *N*

And indeed, that work is closer to the truth than that above, because, as soon as we suppose that the true conjunction will occur after the mean, the distance that is found by Ptolemy's work is indeed the true distance of the sun and moon at the mean conjunction and it is the portion of the ecliptic that must be traveled through by the moon, also with that which the sun accomplishes in the meantime before it [i.e. the moon] catches up to the sun, but in addition, something is added to this portion of the ecliptic in the meantime from the second irregularity, and also something from the bending back of the epicycle's diameter must be traveled through by the moon before it catches up to the sun. But what is able to be added from the second irregularity is disregarded, because it is modest and does not protract the moon's motion much in the meantime. However, that of the augment that is able to occur from the bending back of the epicycle's diameter is by no means to be disregarded. For it is able to protract the moon's motion by a quarter of one hour until it catches up to the sun. And this indeed will happen at that time when the moon's equation is 3° to be subtracted and the sun's is about 2° is to be added. For then a distance of 10° is made if it is doubled, and the equation of portion that is owed to it is approximately 1° 30′, which makes about 7/24 of the distance. Accordingly, if you take the portion without its equation and you find the moon's simple equation with it, then if you take the equated portion and you find the simple equation with it, you will see that the simple equations differ by about 1/8°, which is at least a quarter of one hour in the moon's motion.[9] And note that if this equation of portion is disregarded, the error will be smaller when the distance of the sun and moon is 7° than when it is 5°. For when it is 7°, the moon's simple equation is 5° and the moon's equated portion 95°. But if for this equated portion you subtract its equation that is owed here to the duplex longitude, i.e. about 2°, and thus you take the moon's simple equation with the simple portion, you will see that the equations differ by only 3″.[10] Therefore, the difference with respect to 3° will be greater than with respect to 5°, and this is what we wanted. It will happen in a like way in the subtraction if the true conjunction precedes the mean.

[9] The author is following Albategni here, but using either Ptolemy or Albategni's tables, the difference between the simple equations should be approximately 6′, which would result in about 1/5 hour in the moon's motion.

[10] This is a mistake that appears to have been introduced by the author, as neither Ptolemy nor Albategni give a number for this difference. Using the tables of the moon's anomalies (*Almagest* V.8 or Nallino, *al-Battānī*, vol. II, pp. 78–81), one finds that the difference caused by ignoring the equation of portion is about 40″.

Etiam nota quod distantia Solis et Lune que per huiusmodi equationes colligitur in coniunctione media quasi media distantia erit in coniunctione vera, quia ipsa quoque ex paribus equationibus tunc colligitur eo quod Solis equatio infra tantum tempus vix mutatur. Hoc quoque attendendum quod supradicte portioni circuli signorum que est vera distantia Solis et Lune in coniunctione media aliquid etiam accrescere potest vel decidere propter motum Lune interim in epiciclo. Sed propter hoc recompensandum iubemur repertam distantiam Solis et Lune dividere per superlationem diversi motus Lune, non medii. Unde etiam manifestum quod si illam portionem Lune accipias que est in dimidio temporis interiacentis vere coniunctioni et equali, et per eam diversum motum Lune ad unam horam addiscas, atque per huius superlationem dividas distantiam repertam Solis et Lune, verior erit operatio. Et illius quidem portionis scientia est ut dimidium distantie reperte sumens, ei duodecimam partem eius superaddas, quantum scilicet Sol interim movetur. Et inde collectum equate prius portioni superaddas si Luna nondum vere consecuta est Solem. Ratio huius est quod motus Lune in epiciclo pene est sicut Lune motus in longitudine.

Sunt etiam qui equare velint in coniunctionibus vel oppositionibus quod Lune accidere potest propter secundam diversitatem, et ad hoc reperies parvam tabellam in tabulis Toletanis que nichil omittunt. In quam tabellam intratur per longitudinem que est inter Solem et Lunam tempore applicationis, et crescit usque ad vii et opponuntur ei secunda tantum minuenda vel addenda diverso motui Lune ad horam. Tunc quidem minuenda cum portio ceciderit versus longitudinem longiorem inter duas longitudines medias epicicli; addenda vero si versus longitudinem propiorem. Sane etiam animadvertendum quod quicquid de motu longitudinis loco Lune addendum est etiam motui latitudinis cum motu Capitis in ipso tempore addi debet; et quicquid de motu longitudinis loco Lune demendum mandatur etiam motui latitudinis cum motu Capitis in

255 ipsa quoque] *corr. ex* qua ipsa quod *M* paribus] partibus *P corr. ex* partibus *KN* **256** attendendum] atendendum est *K* **257** vera] *s.l. P* in] *s.l.* P_7 **258** media] *s.l. P* aliquid etiam] etiam aliquid *N* accrescere potest] potest accrescere *PN* decidere] decrescere P_7 **259** Sed] si *P* **260** superlationem] *corr. ex* superfluitatem *P corr. ex* superlationes *K* Lune[2]] *om. N* **261** manifestum] manifectum *K* manifestum est *M* si] *om. K* **262** vere] veri *M* coniunctioni] coniunctione *PK* (coniunctioni BaE_1) et[2]] *om. PN* **263** huius] huiusmodi *M* **264** portionis] operationis *M* **265** eius] *om. N* **266** superaddas] *corr. ex* super eius *K* **266/267** equate prius] prius equate *M* **267** nondum vere] vere nondum *M* **268** quod] quia *N* Lune motus] motus Lune *MN* **270** velint] *corr. ex* voluit P_7 volunt *MN* quod] *s.l.* P_7 **272** tabellam[1]] tabulam P_7KMN (tabulam *Ba* tabellam E_1) Toletanis] Tholetanis P_7 *corr. ex* Ptolemei *K* omittunt] obmittunt *M* omittit *N* tabellam[2]] tabulam P_7MN **274** opponuntur] apponuntur *N* ei – tantum] *s.l. K* **275** motui] motu *M* ceciderit] *corr. in* acciderit *M* **277** animadvertendum] *corr. ex* addendum *P* **279** de] de medio *N* **280** loco Lune] *om. N* **280/281** in tanto] iterato *PN*

Also note that the distance of the sun and moon that is collected from equations of this kind in a mean conjunction will be as though the mean distance in a true conjunction, because that also is collected from similar equations at that time because the sun's equation under such a time is hardly changed. It also must be noticed that something can also be added to or cut off from the above said portion of the ecliptic, which is the true distance of the sun and moon in the mean conjunction, because of the moon's motion on the epicycle in the meantime. But to compensate for this, we are commanded to divide the found distance of the sun and moon by the carrying beyond of the moon's irregular motion, not of the mean. Whence it is also manifest that if you take that portion of the moon that is at the middle of the time lying between the true and equal conjunctions, and you learn the moon's irregular motion for one hour through it,[11] and you divide the found distance of the sun and moon by this carrying beyond, the operation will be truer. And indeed the knowledge[12] of that portion is that, taking half of the found distance, you add to itself its twelfth, i.e. as much as the sun moves in the meantime. And you add the sum from this to the portion equated earlier if the moon truly has not yet reached the sun. The reasoning for this is that the moon's motion on the epicycle is almost as the moon's motion in longitude.

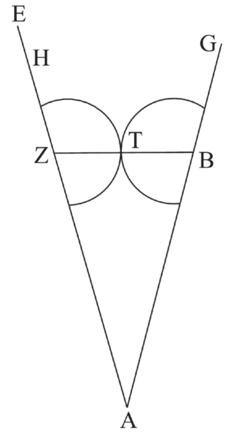
There are also those who would want to correct in conjunctions or oppositions what is able to happen to the moon because of the second irregularity, and for this you will find a small table in the Toledan Tables that leaves out nothing. This table is entered by the distance that is between the sun and moon at the time of the syzygy, and it grows up to 7 and only seconds that are to be subtracted or added to the moon's irregular motion for an hour are placed opposite it. Indeed, they are to be subtracted at that time when the portion falls towards the apogee between the two mean distances of the epicycle; however, they are to be added if towards the perigee. Also, it is to be soberly noticed that whatever of the motion of longitude must be added to the moon's place also ought to be added to the motion of latitude with the motion of the ⟨Dragon's⟩ Head in that time; and whatever of the motion of longitude is ordered to be subtracted from the moon's place also ought to be subtracted from the motion of latitude with the motion of the Head in such

[11] Through VI.2.

[12] 'Scientia' is not taken in a strict sense here since this is an approximation of the portion at the time halfway between the conjunctions.

tanto tempore demi debet, eo quod motus latitudinis constat ex motu longitudinis et motu Capitis.

4. Terminos eclipticos lunares sub certo numero consignare.



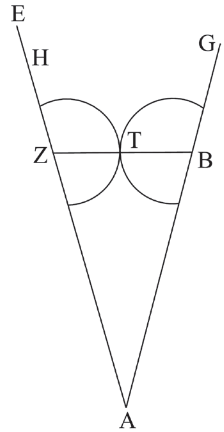
Oportet propter hoc prius scire de diametro Lune cum ipsa fuerit in longitudine propiore epicicli quantum arcum maioris circuli cordet, etiam de semidiametro umbre. Et propter hoc observavit Ptolomeus duas eclipses lunares cum Luna quidem esset iuxta longitudinem propiorem, et observavit in eis differentiam tenebrarum ex diametro Lune et differentiam latitudinum in mediis eclipsibus. Et deprehendit per hoc eo modo quem diximus in xv^{a} precedentis quod Lune diameter in longitudine propiori cordat arcum maioris circuli qui est xxxv minutorum et tertie unius minuti et quod semidiameter umbre cordat arcum maioris circuli qui est xlvi minutorum. Neque hoc discordat a proportione assignata quin semidiameter umbre contineat semidiametrum Lune bis et eius tres quintas. Iste quoque quantitates diametrorum in longitudine propiore quantitatibus eorum positis ab Albategni conveniunt. Cum itaque semidiameter Lune sit hic xvii minutorum et xl secundorum, cum hoc duplicaveris cum tribus eius quintis appositis, fiet medietas duorum diametrorum pars una et tria minuta et xxxvi secunda. Et ponam vice circuli signorum lineam ABG, et vice circuli declinantis Lune AZE, et centrum Lune Z, et centrum circuli umbre B, et Lunam contingentem circulum umbre. Et continuabo medietatem duorum diametrorum ZTB quasi perpendicularem super lineam AZE eo quod ad sensum sit sicut equidistans linee ABG. Erit ergo ZTB nota scilicet pars una et tria minuta et xxxvi secunda. Et hoc quidem cum Luna erit contingens circulum umbre, quod tunc quidem erit cum latitudo Lune vera erit ut medietas duorum diametrorum. Nam indifferenter arcus ut rectas hic ponimus eo quod

281 demi] minui *N* **285** propiore epicicli] epicicli *P* epicicli propiore (*this last word corr. ex* longiore epi-) *N* **290** differentiam tenebrarum] tenebrarum differentiam *PN* diametro] *corr. in* motu *M* Lune] *om. N* **291** latitudinum] latitudinis P_7 **291/292** deprehendit per] comprehendit propter *N* **292** quem] quo *MN* **293** propiori] *om. N* **294** cordat] *corr. ex* cadat *P* maioris circuli] circuli maioris *PN* **295** tertie] tertia *M* semidiameter] diameter P_7 **296** maioris circuli] circuli maioris *M* **297** proportione] propositione *PM* (proportione *Ba* propositione E_1) **298** semidiameter] *corr. ex* diameter *P* umbre] Lune *N* **300** eorum] *om. N* sit] fit *M* **301** secundorum] secundarum *P* sol†ut†orum *K* eius] *om. PN* **302** fiet] fiat *PK corr. ex* fiat P_7 duorum] duarum *N* **303** secunda] secunda et cetera *M* **305** duorum] duarum *N* **306** perpendicularem] perpendiculariter *K* **307** ergo] ergo linea *M* pars una] una pars *PN* **309** Lune vera] vera *N* **310** rectas] rectos *M* ponimus] *corr. ex* ponamus P_7

a time, because the motion of latitude consists of the motion of longitude and the motion of the Head.

4. To establish the lunar eclipse limits under a certain number.

For this it is necessary to know first how great of an arc of a great circle the moon's diameter subtends [*lit*, serves as a chord for] when it is at the epicycle's perigee, and also ⟨to know⟩ the radius of the shadow. And for this Ptolemy observed two lunar eclipses when the moon indeed was near perigee, and he observed in them the difference between the darkness and the moon's diameter, and the difference between the latitudes at the middle of the eclipses. And he discovered through this in that way which we said in the 15th of the preceding ⟨book⟩ that the moon's diameter at perigee subtends an arc of a great circle that is 35′ 20″ and that the shadow's radius subtends an arc of a great circle that is 46′. And this is not at variance with the designated proportion that the shadow's radius contains the moon's radius 2 3/5 times. Also, those quantities of the diameters at perigee agree with their quantities posited by Albategni.[13] Accordingly, because the moon's radius is here 17′ 40″, when you multiply this by 2 3/5, half of the two diameters will be made 1^p 3′ 36″. And I will suppose line ABG in the place of the ecliptic, AZE in the place of the moon's declined circle, Z the moon's center, B the center of the shadow's circle, and the moon touching the shadow's circle. And I will draw the half of the two diameters ZTB as if perpendicular upon line AZE because to the senses it is as parallel to line ABG. Therefore, ZTB will be known, namely 1^p 3′ 36″. And indeed this ⟨is⟩ when the moon will be touching the shadow's circle, because it will indeed be at that time that the moon's true latitude will be as the half of the two diameters. For we place the arcs as straight lines without



13 The values in agreement are found in Albategni, *De scientia astrorum* Ch. 30 (1537 ed., ff. 37r-v).

non sit sensibilis differentia eorum in tam modica quantitate. Quia ergo nota est latitudo Lune ZB, erit arcus a nodo ZA sive BA notus, eo quod sit proportio maxime sinus declinationis ad sinum huius latitudinis ZB sicut sinus quadrantis ad sinum ZA vel BA. Accidit autem ex dictis ut sit arcus ZA qui est distantia veri loci Lune a nodo xii graduum et xii minutorum fere.

Et quia querimus maximam distantiam oppositionis medie a nodo, post quam in vera oppositione primum contactum esse contingit Lune et umbre, sit HZ quod plurimum interesse potest medie oppositioni et vere. Ipsum autem sic invenietur. Sumatur in oppositione media maxima distantia Solis et Lune que esse potest, et ipsa quidem colligitur ex equationibus Solis et Lune maximis dummodo simplicem equationem Lune sumas. Erit itaque hec distantia secundum quod Ptolomeus invenit vii graduum et xxiiii minutorum. Nam Solis equatio secundum ipsum plurima est ii graduum et xxiii minutorum, et Lune v graduum et unius minuti. Secundum Albategni vero est vii gradus fere hec distantia eo quod plurima equatio Solis secundum ipsum sit unus gradus et lix minuta et x secunda, et Lune eadem que dicta. Quia vero hec distantia percurrenda est a Luna cum eo quod Sol interim perficiet donec ipsa comprehendat Solem, dividimus primum hanc distantiam per xiii, quia dum Luna illam longitudinem percurrit, Sol tertiamdecimam eius fere perficit. Hanc iterum xiii subdividimus per xiii propter hoc quod dum Luna illud spatium percurrit, Sol interim eius xiii[am] perficit. Et quia huius tandem modica quantitas est nec sensibilis est eius xiii, non ultra fit divisio. Reperies ergo si aggregaveris totum quod Sol interim perficit esse duodecimam prime distantie, et hec est ratio dividendi distantiam per xii. Inventam itaque duodecimam Solis equationi superpone, et collectum est id quod plurimum medie et vere applicationi

312 notus] *corr. ex* notas P_7 **313** maxime sinus] sinus maxime *MN* sinus[1]] proportionis *add. et del. P* sinum] sinus *M* **315** xii[2]] *corr. ex* 22 *M* (12 *Ba* xxii E_1) **317** contingit] contigit P_7 **318** HZ] *corr. ex* HT *K* **319** invenietur] invenitur *MN* maxima distantia] *corr. ex* distantia maxima *K* **320** esse potest] potest esse *N* **321** Lune] *om. PN* **322** Ptolomeus] Tholomeus P_7 **322/324** Nam – minuti] *marg. P* **323** graduum – minutorum] gradus et 23 minuta *M* **324** graduum] gradus P_7 minuti] minuta et P_7 minutis *M* est – fere] 7 gradus fere est *N* est] *om. M* **325** unus gradus] gradus unus *P* unius gradus *MN* **326** lix] *corr. ex* †...† *P* minuta – secunda] minutorum et 10 secundorum *N* dicta] dicta est *M* vero – distantia] ergo distantia hec *PN* **328** primum] primo *N* **328/329** illam longitudinem] longitudinem illam *M* **329** tertiamdecimam – fere] fere 13[am] eius *N* fere perficit] perficit (*corr. ex* perficiet) fere *M* **330** xiii[1]] xiii[a] *K* quod] *om. M* illud spatium] spatium illud *M* **331** interim] iterum P_7K xiiiam] partem *add. s.l.* (*perhaps other hand*) *P* quia] quod *PN* **332** fit] sit P_7 **333** totum] *s.l. P* perficit] perficiet *M* prime distantie] distantie prime est *N* **334** dividendi] dividendo *PN* distantiam] *om. M* equationi] equationem *M* **335** est] *corr. ex* dividere *P* medie – vere] vere et medie *N* difference here because in such a modest quantity, there is not a perceptible difference between them. Therefore, because the moon's latitude ZB is known, arc ZA or BA from the node will be known, because the ratio of the sine of the maximum declination to the sine of this latitude ZB is as the sine of a quadrant to the sine of ZA or BA.[14] Moreover, it happens from what was said that arc ZA, which is the distance of the moon's true place from the node, is approximately 12° 12′.

And because we seek the greatest distance of the mean opposition from the node after which it happens that there is first contact of the moon and shadow at the true opposition, let HZ be the most that is able to lie between the mean and true opposition. Moreover, it will be found in this way. Let the greatest distance of the sun and moon that can exist in a mean opposition be taken, and indeed that is combined from the greatest equations of the sun and moon, provided that you take the moon's simple equation. And so, according to what Ptolemy found, this distance will be 7° 24′. For according to him the greatest equation of the sun is 2° 23′, and of the moon 5° 1′. However, according to Albategni this distance is about 7° because according to him the greatest equation of the sun is 1° 59′ 10″,[15] and of the moon the same that was said. And indeed, because this distance must be traveled through by the moon along with that which the sun will accomplish in the meantime until it [i.e. the moon] catches up to the sun, we first divide this distance by 13, because while the moon travels through that distance, the sun accomplishes its 13th. Again, we divided this 13th by 13 because of this that while the moon travels through that distance, the sun almost accomplishes its 13th in the meantime. And because the quantity of this is after all modest and its 13th is not perceptible, the division is not made further. Therefore, if you add, you will find that the whole that the sun accomplishes in the meantime is a twelfth of the first distance, and this is the reason for dividing the distance by 12. Accordingly, add the found twelfth to the sun's equation, and the sum is the greatest that is

[14] This can be derived from the Menelaus Theorem in the manner of I.16.

[15] This value is not found in Albategni's text, but it is found in one of his tables (Nallino, *al-Battānī*, vol. II, p. 81), which was included among the Toledan Tables (Pedersen, *The Toledan Tables*, Table EA01, pp. 1245–49).

interiacere potest, eo quod Solis equatio sit id per quod Sol distat a loco medie applicationis et quod Sol perambulat usque dum a Luna comprehendatur est id quod Soli et loco vere applicationis interest. Est igitur totum collectum secundum inventa Ptolomei scilicet HZ iii gradus, secundum Albategni vero ii gradus et xxxv minuta fere. Quare arcus HZ est notus secundum Ptolomeum quidem xv graduum et xii minutorum, secundum Albategni xiiii graduum et xlvii minutorum. Si itaque motum latitudinis incohes a maxima declinatione in septentrionem ut Ptolomeus facit, erunt hii secundum Ptolomeum termini ecliptici sub numero certo consignati. Si vero motum latitudinis a nodo septentrionali incohes ut Albategni facit, erunt hii secundum Albategni termini ecliptici lunares. Et ita habemus quod proposuimus.

Lunares termini Ptolomei			Albategni termini		
	gradus	minuta		gradus	minuta
tempus ex quo	74	48	tempus ad quem	14	35
tempus ad quem	105	12	tempus ex quo	165	25
tempus ex quo	254	18	tempus ad quem	194	35
tempus ad quem	285	12	tempus ex quo	345	25

5. Terminos eclipticos solares signanter exprimere.

Cum itaque semidiameter Lune in longitudine propiore sit notus scilicet xvii minuta et xl secunda et semidiameter Solis sit xv minutorum et xl secundorum sicut ostensum est in longitudine longiore Sol cum fuerit, si nullam ei ponamus variationem modo Ptolomei, erit medietas duorum diametrorum Solis et Lune xxxiii minutorum et xx secundorum. At si modo Albategni diametro Solis quoque variationem ponamus — cum in longitudine propiori sit totus diameter xxxiii minutorum et due tertie — erit secundum hoc quoque medietas duorum diametrorum nota scilicet xxxiiii minutorum et dimidii.

336 sit] fit P_7 sit per *N* **338** id] *om.* P_7 illud *N* **340** xxxv] xxx *PN* HZ] *corr. in* AZ *N* (HZ *Ba* HA E_1) **341** xii] xix *P* minutorum] minuta *K* Albategni] Albategni vero *M* Albategni autem *N* xiiii] xiii *PN* (9 *Ba* xiiii E_1) **342** xlvii] xli *PN* xliii P_7K (43 *Ba* 47 E_1) **344** ecliptici] *corr. ex* epicicli *P* **345** ut] in *P* Albategni[2]] Albegni *P* **346** habemus] habebis *M* quod proposuimus] quod supra posuimus P_7 propositum et cetera *N* **347/352** Lunares – 25] *marg. PKM om.* P_7N (*om.* BaE_1) **347** Ptolomei] *corr. in* Ptholomei *K* secundum Ptolomeum *M* Albategni termini] lunares termini secundum Albategni *M* **348** gradus[1] minuta] *om. P* gradus[2] minuta] *om. P* **349** 48] 44 *P corr. in* 44 *M* **354** semidiameter] semidiametrum P_7 notus] nota *N* **355** et[1]] *om.* P_7 semidiameter] *corr. ex* semidiametrum *K* **356** Sol cum] Sol tamen *K* cum Sol *N* **356/357** ei ponamus] ei ponam P_7 ponamus ei *M* **357** erit] et *N* **358/359** diametro Solis] Solis dyameter *M* dyametri Solis *N* **359** variationem ponamus] ponamus variationem P_7 ponamus – sit] *corr. ex* †...† *P* sit] fit P_7 totus] tota *corr. in* nota *M* nota *N* **360** xxxiii] 23 *M* **361** duorum] *om. N* minutorum] minuta *M* able to lie between the mean and true syzygy, because the sun's equation is that by which the sun stands apart from the place of the mean syzygy, and what the sun moves until it is caught by the moon is that which is between the sun and the place of the true syzygy. Therefore, according to what was found by Ptolemy, the whole sum, i.e. HZ, is 3°, but according to Albategni it is approximately 2° 35′. Therefore, arc HZ[16] is known according to Ptolemy, indeed 15° 12′, and according to Albategni 14° 47′.[17] Accordingly, if you begin the motion of latitude from the maximum declination in the north as Ptolemy does, there will be those eclipse limits established under a certain number according to Ptolemy. However, if you start the motion of latitude from the northern node as Albategni does, there will be those lunar eclipse limits according to Albategni. And thus we have what we proposed.

Ptolemy's Lunar Limits		Albategni's Limits	
Time from which:	74° 48′	Time to which:	14° 35′
Time to which:	105° 12′	Time from which:	165° 25′
Time from which:	254° 18′[18]	Time to which:	194° 35′
Time to which:	285° 12′	Time from which:	345° 25′

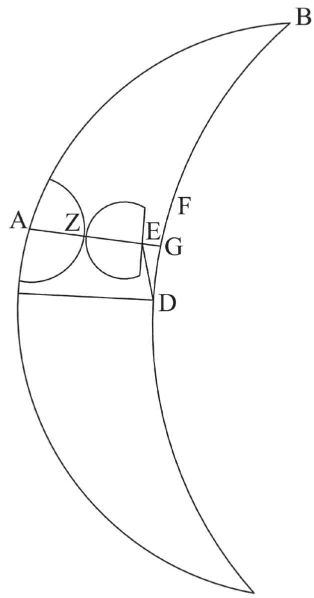
5. To distinctly portray the solar eclipse limits.

Accordingly, because the moon's radius at perigee is known, i.e. 17′ 40″, and the sun's radius is 15′ 40″, as was shown when the sun was at apogee if we suppose no variation for it in the manner of Ptolemy, half of the two diameters of the sun and of the moon will be 33′ 20″. But, if in the manner of Albategni we suppose a variation also for the sun's diameter – when it is at perigee, the whole diameter is 33′ 40″ – according to this also half of the two diameters will be known, i.e. 34′ 30″.

16 This arc should be AH.

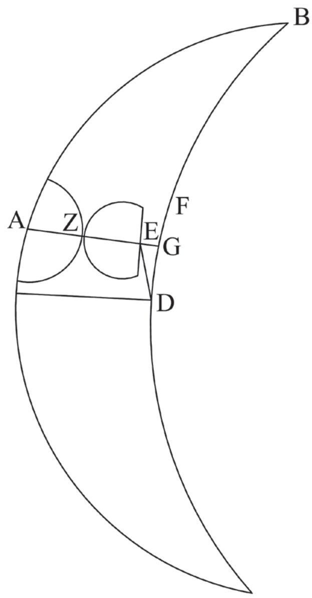
17 This is the correct value, but the manuscript evidence does not make it clear which reading was original.

18 This should be 254° 48′.



Lineabimus igitur circulum signorum AB et orbem declivem DGB. Et ponemus primum contactum Solis et Lune secundum visum ut sit centrum Solis super A, et centrum Lune secundum visum super E, secundum verum super D, et punctus contactus Z. Et continuabo AZE rectam super lineam DGB quasi perpendiculariter eo quod DG ad sensum sit equidistans orbi signorum. Est ergo DE diversitas aspectus Lune in circulo altitudinis, et GD diversitas aspectus in longitudine fere, et GE diversitas aspectus in latitudine que maxima accidere potest in applicatione propter primum hoc fieri contactum. Et est EA visa latitudo scilicet nota quia est tanquam medietas duorum diametrorum. Ex premissis autem que de diversitate aspectus dicta sunt, manifestum esse potest quod cum diversitas aspectus Solis subtracta fuerit de diversitate aspectus Lune, quod maxima diversitas Lune in latitudine que esse potest a climate primo scilicet in locis ubi maximus dies est xiii horarum equalium usque ad clima septimum scilicet in locis ubi maximus dies est xvi horarum equalium — et hoc cum erit in longitudine propiori in horis applicationum Luna — est versus septentrionem quidem a cenit capitum viii minuta fere et versus meridiem lviii minuta fere. Est ergo linea GE si ad partem septentrionis, est diversitas aspectus viii minuta, et si ad partem meridiei, lviii minuta. Nota autem maiore diversitate aspectus in latitudine nota etiam est que cum ea sit

362 Lineabimus] leneabimus P_7 linebimus *K* **363** orbem declivem] orbem declinem *P* circulum declinantem *N* primum] primo *N* **366** secundum – super^2] sed verum sit *N* **368** rectam – lineam] *corr. ex* super rectam lineam *K corr. ex* super lineam rectam *M* **373/376** in – Ex] *marg.* (*other hand*) *K* **374** hoc] *corr. in* hunc *N* **375** quia] quod *P* que *MN* (quia *Ba* que E_1) **376** duorum diametrorum] duarum dyametrorum (*last word corr. ex* semidyametrorum) *N* premissis] *corr. ex* missis *K* predictis *M* **377/378** de – sunt] dicta sunt de diversitate aspectus *N* **379** fuerit] fuerit in circulo altitudinis *KM* (fuerit in circulo altitudinis BaE_1) **380** quod] quia *KM* quod – latitudine] *corr. ex* †...† *P* diversitas] diversitas aspectus *N* **380/381** esse – a] potest esse a (*corr. ex* in) P_7 **381** maximus] maxima *M* dies est] est dies *PN* **382** ad] *marg. P* maximus] maxima *M* dies est] est dies *P* **383** horis] *corr. ex* his *K* applicationum] cum *add. s.l. N* **384** quidem] quod *M om. N* **385** partem septentrionis] septentrionem *N* **386** et] *om. N* minuta^2] fere *add. et del.* P_7 minuta fere *M* **387** autem] autem quod cum *M* etiam – que] est etiam quem (*last word corr. in* quam) *M* que] †quod† *del. N*



Accordingly, we will draw the ecliptic AB and the declined circle DGB. And we will place the first contact of the sun and moon according to sight so that the sun's center may be upon A, the moon's center according to sight upon E and according to truth upon D, and the point of contact Z. And I will draw straight line AZE upon line DGB as if perpendicularly because to the senses DG is parallel to the ecliptic. Therefore, DE is the moon's parallax on the circle of altitude, GD is approximately the parallax in longitude, and GE is the greatest parallax in latitude that is able to occur at a syzygy because this is made the first contact. And EA is the apparent latitude, known because it is as the half of the two diameters. Moreover, from those things set forth that were said about the parallax, it can be manifest that that when the sun's parallax is subtracted from the moon's parallax, the moon's greatest parallax in latitude that can exist from the first clime, i.e. in places where the greatest day is 13 equal hours, to the seventh clime, i.e. in places where the greatest day is 16 equal hours – and this is when the moon will be at perigee at the times of the syzygies – is indeed approximately 8′ towards the north from the zenith and approximately 58′ towards the south. Therefore, if line GE is in the direction of the north, the parallax is 8′, and if in the direction of the south, 58′. Moreover, with the greatest parallax in latitude known, also there is known what the greatest parallax in maior diversitas aspectus in longitudine, et ipsa quidem cum diversitas aspectus in latitudine ad partem septentrionis fuerit est xxx minuta fere. Et cum ad partem meridiei fuerit, est xv minuta fere. Itaque DG arcus aut xxx minutorum est aut xv. Et quia arcus GE notus est et arcus EA, erit coniunctus ex illis arcus GA notus, et ipse est vera latitudo Lune a loco longitudinis in quo esse videtur. Et est quidem hic arcus secundum Ptolomeum gradus unus et xxxi minuta et xx secunda, et secundum Albategni gradus unus et xxxii minuta et xxx secunda. Et quia latitudo hec nota est, erit etiam arcus BG notus; quare et totus arcus DGB notus. Et ipse est distantia veri loci Lune a nodo, et est secundum opus Ptolomei cum Luna fuerit meridiana a Sole et ipsa septentrionalis a cenit capitum in maiore diversitate aspectus in latitudine viii gradus et xxii minuta. Et cum fuerit Luna septentrionalis a Sole sed meridiana a cenit capitum, erit arcus DB xvii partes et xli minuta. Cum ergo fuerit elongatio centri Lune verificati a quolibet duorum nodorum in orbe declivi ad partem meridiei quidem a cenit capitum sed septentrionalis a Sole, xvii gradus et xli minuta; et ad partem septentrionalem a cenit capitum sed meridiani a Sole, viii gradus et xxxii minuta. Tunc possibile est ut accidat primus contactus secundum visum Solis et Lune qui esse potest in locis habitatis. Si ergo arcui DB addiderimus id quod plurimum interiacere potest applicationi medie et applicationi vere, habebimus locum in quo Luna existente secundum cursum medium, possibile est in locis habitatis ut accidat locus in quo primum Luna secundum visum Solem contingere potest. Sunt itaque secundum operationes Ptolomei termini solares ecliptici huiusmodi. Nam ipse inchoat motum latitudinis a maxima latitudine in septentrione.

Quod si secundum Albategni inventa subtiliare voluerimus, cum secundum hunc auctorem quoque arcus GA sit notus, erit etiam arcus GB notus sive septentrionalis sive meridiana fuerit Luna a Sole. Nam est proportio arcus GA ad arcum GB fere sicut proportio unius ad xi et dimidium. Et quia visa coniunctio est aput puncta G A, verus autem locus Lune est in puncto D, cum arcus

388 quidem] quod *K*   **389** septentrionis] septentrionalem *M*   minuta fere] fere minuta P_7 minutorum fere *N*   Et cum] cum *corr. ex* et quod (*perhaps other hand*) *P* cum *N*   **390** est – minuta] 15 minutorum est *N*   minuta] minutorum *M*   DG arcus] arcus DG *M*   **390/391** xxx – est[1]] est 30 minutorum *N*   **391** est[1]] erit *M*   **392** ipse] ipsa *M*   **395** latitudo hec] hec latitudo *KN*   **396** DGB] DGH *K*   notus] *s.l.* (*perhaps other hand*) *P*   **400** DB] DK *P*   partes] partium *N*   xli] xii *K*   minuta] minutorum *N*   **402** xli] iii *PN* xli unum P_7 *corr. in* 3 *M* (41 BaE_1)   minuta] *om.* *K*   **403** septentrionalem] septentrionis *MN*   meridiani] *corr. in* meridiana P_7 meridiana *N*   viii] *corr. ex* 13 *M*   **404** xxxii] (32 BaE_1)   **404/405** secundum – Lune] Solis et Lune secundum visum *N*   **405** arcui] *corr. ex* arcu P_7 id] *om.* *N*   **407** cursum medium] medium cursum *N*   **408** Luna – visum] secundum visum Luna *N*   Solem] *corr. ex* locum P_7 **409** operationes] aparationes *K* operationem *N*   **412** inventa] *s.l.* (*perhaps other hand*) *P* *om.* *N*   **414** fuerit Luna] Luna fuerit *P*   **415** et dimidium] cum dimidio *M*   **415/416** coniunctio est] est coniunctio *PN* longitude with it is, and indeed when the parallax in latitude is in the direction of the north, that [i.e. the greatest parallax in longitude] is approximately 30′. And when it is in the direction of the south, it is approximately 15′. Accordingly, arc DG is either 30′ or 15′. And because arc GE and arc EA are known, arc GA conjoined from them will be known, and it is the moon's true latitude from the place of longitude in which it appears to be. And indeed this arc is 1° 31′ 20″ according to Ptolemy, and 1° 32′ 30″ according to Albategni.[19] And because this latitude is known, arc BG will also be known; therefore also the whole arc DGB will be known. And it is the distance of the moon's true place from the node, and according to Ptolemy's work, it is 8° 22′ when the moon is south of the sun and it is north of the zenith in the greatest parallax in latitude. And when the moon is north of the sun but south of the zenith, arc DB will be 17° 41′. Therefore, when the elongation of the moon's corrected center from whichever of the two nodes on the declined circle is in the direction of the south from the zenith indeed but north of the sun, ⟨it is⟩ 17° 41′; and ⟨when⟩ towards the northern side of the zenith but south of the sun, ⟨it is⟩ 8° 32′.[20] At that time it is possible that the sun and moon's first contact occurs according to sight that can be in inhabited places. Therefore, if we add to arc DB the greatest amount that can lie between the mean syzygy and the true syzygy, we will have the place which with the moon existing in it according to mean course, it is possible in inhabited places that there occurs the place where the moon first is able to touch the sun according to sight. And so, according to the operations of Ptolemy, the solar eclipse limits are of this kind. For he begins the motion of latitude from the greatest latitude in the north.

But, if you will want to make it more exact according to Albategni's findings, because according to this authority also arc GA is known, arc GB will also be known whether the moon is north or south of the sun. For the ratio of arc GA to arc GB is approximately as the ratio of 1 to 11 1/2. And because the apparent conjunction is at points G and A, but the moon's true place is at

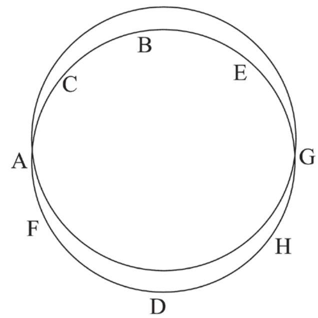
[19] Our author is taking the case here in which parallax is to the south.

[20] This should be 22′, but the mistake is also found in at least one manuscript of Gerard's translation of the *Almagest*, Paris, BnF, lat. 14738, f. 98v.

DG sit notus quia est diversitas aspectus in longitudine, si ei duodecimam eius partem superponamus qua sit GF, palam quod aput punctum F erit vera coniunctio et erit arcus GF notus; quare et reliquus FB notus. Itaque si arcui FB id quod plurimum secundum Albategni interiacere potest medie coniunctioni et vere addiderimus, habebimus elongationem a nodo notam in qua cum Luna secundum medium cursum fuerit, possibile est ut accidat primus contactus Solis et Lune secundum visum qui esse potest in locis habitatis. Et est quidem hec elongatio cum Luna australis fuerit x gradus et xl minuta fere. Et si Luna septentrionalis fuerit a circulo signorum, erit xx gradus et xii minuta fere. Quare si motum latitudinis a nodo Capitis inchoemus, erunt termini ecliptici solares huiusmodi secundum considerationes Albategni.

6. Solis vel Lune eclipsim in sexto mense lunari iterari est possibile.

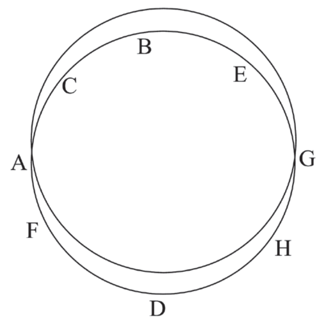
Sit propter hoc demonstrandum circulus declinans Lune ABG et nodus Capitis A, et nodus Caude G, et medietas septentrionalis ABG. Et sint termini ecliptici ex parte septentrionis C et E et ex parte meridiei F et H. Quoniam vero arcus AC vel GE quantum ad solares terminos Ptolomei continet xx gradus et xli minuta vel secundum Albategni xx gradus et xii minuta, palam quod arcus CBE continet gradus cxxxviii et xxxviii minuta vel gradus cxxxix et xxxvi minuta. Porro



417 est] *om. K* **418** superponamus] supponamus P_7 qua] que *MN* (que *Ba*E_1) **419** et[1] – arcus] *iter. et del. P* et[2]] *om. N* **420** FB] *corr. ex* FG *K* **420/421** medie coniunctioni] coniunctim P_7 **422** medium cursum] cursum medium *M* medium] *marg.* (*perhaps other hand*) *P* **424/425** gradus – fere] graduum et xl fere minuta (minutorum *N*) *PN* **425/426** gradus – minuta] graduum et 12 minutorum *N* **426** Capitis] *om. N* inchoemus] inchoamus *marg.* (*perhaps other hand*) *P* inchoaverimus *N* **427** ecliptici solares] epicicli solares P_7 solares ecliptici *N* huiusmodi] *marg. M* Albategni] *K and M have the following in the form of a table*: Solares termini (termini solares *M*) Ptolomei: tempus ex quo, 69 (109 *M*) gradus 19 minuta; tempus ad quem, 101 [gradus] 22 [minuta]; tempus ex quo, 258 (*corr. ex* 259 *K*) [gradus] 38 [minuta]; tempus ad quem, 290 [gradus] 41 [minuta]. Solares termini Albategni: tempus ad quem, 20 [gradus] 12 [minuta]; tempus ex quo, 159 [gradus] 48 [minuta]; tempus ad quem, 190 [gradus] 40 [minuta]; tempus ex quo, 349 [gradus] 20 [minuta]. **428** lunari] *om. N* **429** propter] *iter.* P_7 **430** declinans] declinationis *PMN* (declinans *Ba*E_1) **432** termini] *corr. ex* non P_7 septentrionis] *corr. ex* septemtrionalis P_7 **433** et[2]] *s.l. P* F – H] *corr. ex* FEH *N* **434** AC] *corr. ex* AD *M* AT *N* GE] et sunt termini ecliptici ex parte septemtrionalis *add. et del.* P_7 **435** Ptolomei] Tolomei *K* **435/438** xx – continet] *marg.* (*perhaps other hand*) *P* **436** xli] *corr. ex* 40 *M* vel] et *PN om. K* **437** CBE] *corr. ex* BCE *K corr. ex* CDE *M* TBE *N* (CBE *Ba* CDE E_1) **439** cxxxix] *corr. ex* cxxxi (*perhaps other hand*) *P* point D, and because arc DG is known because it is the parallax in longitude, if we add to it its twelfth part, by which let there be GF, it is clear that the true conjunction will be at point F, and arc GF will be known; therefore, the remainder FB is also known. Accordingly, if we add to arc FB the greatest quantity according to Albategni that can lie between the mean conjunction and the true, we will have the known elongation from the node which with the moon existing in it according to mean course, it is possible that the first contact of the sun and moon occurs according to sight that can be in inhabited places. And indeed, when the moon is south, this elongation is approximately 10° 40′. And if the moon is north of the ecliptic, it will be about 20° 12′. Therefore, if we begin the motion of latitude from the node of the Head, there will be the solar eclipse limits of this kind according to Albategni's observations.

6. It is possible that an eclipse of the sun or moon be repeated in the sixth lunar month.

For demonstrating this, let there be the moon's declined circle ABG, the node of the Head A, the node of the Tail G, and the northern half ABG. And let the eclipse limits on the northern side be C and E and on the southern side F and H. And indeed, because arc AC or GE of the size for Ptolemy's solar limits contain 20° 41′ or according to Albategni 20° 12′, it is clear that arc CBE contains 138° 38′ or 139° 36′. On the



motus latitudinis in vi mensibus lunaribus equalibus continet gradus clxxxiiii et i minutum proiectis integris revolutionibus; maior est ergo arcus qui relinquitur de motu latitudinis arcu CBE. Dico etiam quod ipse minor est arcu FBH. Nam arcus AF sive GH quantum ad solares terminos ex parte meridiei continet xi gradus et xxii minuta ut invenit Ptolomeus vel secundum Albategni x gradus et xl minuta. Palam ergo quod motus latitudinis in sexto mense lunari iterum cadit infra terminos eclipticos solares versus nodum. Quare ex premissa possibile est iterum Solem eclipsari. Par est demonstratio si sumas arcum FDH. Nam ipse necessario minor est arcu qui relinquitur de motu latitudinis in vi mensibus proiectis integris revolutionibus, et arcus CDE maior est eodem.

Consimilis est demonstratio circa Lunam. Nam arcus AC vel GE itemque arcus AF vel GH unusquisque quantum ad lunares terminos continet xv gradus et xii minuta vel xiiii gradus et xlvii minuta. Palam ergo quod arcus CBE minor est arcu motus latitudinis sex mensium, et arcus FBH maior. Cadit ergo motus latitudinis in sexto mense iterum infra terminos lunares eclipticos. Unde possibile est iterum obscurari Lunam, quod proposuimus.

7. Quantitatem diametri Lune et quantitatem semidiametri umbre ad omnem distantiam Lune inter longitudinem longiorem epicicli et longitudinem eius propiorem cum epiciclus in longitudine longiore ecentrici fuerit affiniter comprehendere.

Primum sumatur differentia diametri Lune cum fuerit in longitudine longiore et cum fuerit in longitudine propiore. Et est hec differentia secundum Albategni v minuta et dimidium et tertia unius minuti et secundum Ptolomeum iiii minuta tantum. Deinde sumatur centri Lune a centro terre per arcum portionis distantia, nam et ipsa est nota. Et minuatur hec distantia a maxima distantia Lune a terra, scilicet que est in termino primo, et differentia conferatur cum quantitate diametri epicicli. Et secundum hanc proportionem sume de differentia quantitatum diametri Lune quam prediximus. Et quod inveneris

441 minutum] interim P_7 proiectis] reiectis *M* est ergo] ergo est *N* **442** arcu[1]] arcus *M* CBE] TBE *N* quod] quia *K corr. ex* quia *M* minor est] est minor P_7 minor *N* **443** FBH] *corr. ex* †FEH† *P corr. in* FDH *M* **444** vel] et *PN* **445** ergo] *om.* P_7 **446** cadit] cadet *M* premissa] premissis *N* **447** iterum Solem] tunc Solem iterum *M* iterum] item P_7K **448** minor est] est minor *M* in] *s.l. K* **449** CDE] TDE *N* **450** est demonstratio] demonstratio est P_7K Nam] namque *N* itemque] *corr. ex* itaque *K* **452** vel – minuta[2]] *marg.* (*perhaps other hand*) *P* xlvii] *corr. in* 45 *M* 45 *N* CBE] TBE *N* **453** motus] motu *P* Cadit] eodem *N* **454** mense] *corr. ex* mensibus *N* iterum] *corr. in* interim P_7 lunares eclipticos] eclipticos lunares P_7K eclipticos] eclipticos cadet *N* **455** iterum] *corr. in* interim P_7 obscurari Lunam] Lunam obscurari *N* **457/458** longitudinem eius] eius longitudinem *N* **458** fuerit] fiunt P_7 affiniter] affinitum *P* affinite *corr. ex* diffinite *M* affinitum *corr. in* affinitam *N* (affiniter BaE_1) **459** comprehendere] apprehendere *M* **460** Primum] iterum *PN* **462** et[3]] *om. M* Ptolomeum] Tholomeum P_7 **463/464** per – distantia[1]] distantia per arcum portionis *N* **466** cum] tum *K* proportionem] *corr. ex* quantitatem P_7

other hand, the motion of latitude in 6 mean lunar months contains 184° 1′ with complete revolutions cast out; therefore, the arc that remains from the motion of latitude is greater than arc CBE. I say also that it is less than arc FBH. For arc AF or GH of the size for the solar limits on the southern side contains 11° 22′ as Ptolemy found or 10° 40′ according to Albategni. Therefore, it is clear that the motion of latitude in the sixth lunar month falls again below the solar eclipse limits towards the node.[21] Therefore, from what has been set forth, it is possible that the sun is eclipsed again. The proof is the same if you take arc FDH. For it is necessarily less than the arc that remains from the motion of latitude in six months with complete revolutions cast out, and arc CDE is greater than the same.

The proof concerning the moon is very similar. For arc AC or GE and likewise AF or GH each of the size for the lunar limits contains 15° 12′ or 14° 47′. Therefore, it is clear that arc CBE is less than the arc of the motion of latitude of six months, and arc FBH is greater. Therefore, the motion of latitude in the sixth month falls again below the lunar eclipse limits. Whence it is possible that the moon is obscured again, which we proposed.

7. To grasp approximately the size of the moon's diameter and the size of the shadow's radius at each distance of the moon between the epicycle's apogee and its perigee when the epicycle is at the eccentric's apogee.

First, let there be taken the difference between the moon's diameter when it is at apogee and when it is at perigee. And this difference is 5′ 50″[22] according to Albategni and only 4′ according to Ptolemy. Then let the distance of the moon's center from the center of the earth be taken through the arc of the portion, for it also is known. And let this distance be subtracted from the greatest distance of the moon from the earth, i.e. what is in the first term, and let the difference be compared with the size of the epicycle's diameter. And according to this ratio, take ⟨a part⟩ from the difference of the sizes of the moon's diameter that we spoke of before. And add what you find to the moon's diameter

[21] Of course, this will not always occur. If the first eclipse is near C, six months later the sun and moon will be at a point beyond H.

[22] This is correct according to other values given by Albategni earlier, but Albategni gives a slightly different value here. The value 5′ 45″ is found not only in *De scientia astrorum*, 1537 ed., ff. 61r-v, but also in Nallino, *al-Battānī*, vol. I, p. 97, and *P*, ff. 57v–58r. The author could have easily made the correction himself as Albategni's largest and smallest sizes for the moon's diameter are given above in V.18.

adde super diametrum Lune in longitudine longiore, quia sicut differentia distantiarum se habet ad diametrum epicicli que est maxima distantiarum differentia sic affiniter se habet differentia quesita diametri Lune ad maximam distantiam diametri Lune. Et recole quod hee differentie distantiarum centri Lune a centro terre sunt minuta in septima tabula diversitatis aspectus collocata sub proportione ad lx minuta. Unde illinc promptius assumi possunt. Nota autem quantitate diametri Lune nota est quantitas diametri umbre aut predicta via aut quia continet diametrum Lune bis et eius tres quintas secundum assignatam proportionem. Albategni vero secundum motum diversum Lune in una hora has quantitates diametrorum investigat eo modo quo supra diximus.

8. Eclipsim Lune in quinto mense lunari iterari aliquando est contingens. Unde manifestum quod in ambabus huiusmodi eclipsibus si contingant, Luna erit septentrionalis a duobus nodis aut in ambabus erit meridiana.

Ponamus enim quod Sol in hiis v mensibus sit cursu velox procedens scilicet a longitudine media ad aliam longitudinem mediam per longitudinem propiorem, et quod Luna sit cursu tarda. Invenimus siquidem medium cursum Solis vel Lune in v mensibus equalibus cxlv gradus et xxxii minuta fere, qui cum divisi fuerint equaliter per medium ut sumantur arcus equales ex utraque parte longitudinis propioris, addent hii gradus super medium cursum Solis per equationem quidem Ptolomei iiii gradus et xxxviii minuta et per equationem Albategni iii gradus et xlviiii minuta. Et invenimus motum Lune in epiciclo in v mensibus equalibus cxxix gradus et v minuta, qui cum equaliter per medium divisi fuerint ut sumantur arcus equales ex utraque parte longitudinis longioris epicicli, minuent hii gradus ex medio cursu Lune viii gradus et xl minuta ex opere Ptolomei et ex opere Albategni viii gradus et liiii minuta cum portio equata fuerit. In tempore ergo intermedio quod est v mensium, cum fuerit Sol quidem velox cursu et Luna tarda cursu, erit Sol precedens secundum partes aggregatas ex ambabus differentiis, nam secundum medium cursum non distabunt. Et sunt partes aggregate xiii et xviii minuta ex opere Ptolomei et ex Albategni partes xii et xliii minuta. Si ergo huius spatii sumpserimus par-

469 que] non *add. et del. K* maxima] duarum *add. et del. N* **470** affiniter] affinite *M* **471** quod] quia *K* **473** lx] xl P_7 illinc] illic *MN* **474** aut] Lune (*del.*) a *M* **475** aut] autem *M* tres quintas] 2/5 *M* **477** diametrorum] *om. N* eo] *corr. ex* eodem *P* **478** iterari] *om.* P_7 aliquando est] est aliquando *K* **480** aut] vel *N* erit meridiana] meridiana erit *N* **481** enim] *om. KM* **483** cursu tarda] tarda cursu *N* siquidem] quidem *M* **484** vel] et *M* **485** arcus equales] equales arcus *M* equales *N* **486** hii] *corr. ex* hic *K* **487** et[2]] *om. N* **488** et[1]] *om.* PP_7M **489** gradus] *corr. ex* gradibus *N* **491** minuta] minuta et P_7 secunda *N* **492** Ptolomei] Tolomei *K* et[2]] *om. M* **493** ergo] vero P_7 **496** sunt] *corr. ex* secundum P_7 xiii] 13 gradus *M* et[2]] vel PP_7K (et BaE_1) **497** partes xii] 12 partes *M* huius] huiusmodi *N* **497/498** sumpserimus – duodecimam] partem duodecimam sumpsimus *P* partem sumpserimus duodecimam *M* partem duodecimam sumpserimus *N* at apogee, because as the difference of the distances is disposed to the epicycle's diameter, which is the greatest difference of the distances, thus approximately is the sought difference of the moon's diameter disposed to the greatest length [*lit.*, distance] of the moon's diameter. And recall that these differences of the distances of the moon's center from the earth's center are the minutes set out under a ratio to 60′ in the seventh column of parallax.[23] Whence they are able to be taken more readily from there. Moreover, with the size of the moon's diameter known, the size of the shadow's diameter is known either by the said way or because it contains the moon's diameter two and three fifths times according to the designated ratio. And indeed, Albategni searches for these sizes of diameters according to the moon's irregular motion in one hour in that way by which we spoke above [i.e. in V.18].

8. It happens sometimes that an eclipse of the moon is repeated in the fifth lunar month. Whence it is manifest that in both eclipses of this sort if they occur, the moon will be north from the two nodes or it will be south in both.

Indeed, let us suppose that the sun in these 5 months is fast of passage, i.e. proceeding from the mean distance through the perigee to the other mean distance, and that the moon is slow of passage. Accordingly, we find the mean course of the sun and moon in 5 mean months to be approximately 145° 32′; and when they are divided equally in half so that equal arcs are taken on both sides of the perigee, these degrees add 4° 38′ to the sun's mean course indeed through Ptolemy's equation, and 3° 49′ through Albategni's equation. And we find the moon's motion on the epicycle in 5 mean months to be 129° 5′; and when they are divided equally in half so that equal arcs are taken on both sides of the epicycle's apogee, these degrees subtract from the moon's mean course 8° 40′ from Ptolemy's work and 8° 54′ from Albategni's work when the portion is equated.[24] Therefore, in the intermediate time, which is 5 months, when the sun is indeed fast of passage and the moon slow of passage, the sun will be preceding according to the degrees collected from both differences, for according to the mean course they do not stand apart. And the collected degrees are 13° 18′[25] from Ptolemy's work and 12° 43′ from Albategni's. If, therefore,

[23] *Almagest* V.18 (1515 ed., f. 58r). Explained in *Almagesti minor* V.19.

[24] Our author only equated the portion of Albategni, and he appears to have rounded upwards slightly since I arrive at a value of 8° 52′ using al-Battānī's tables (Nallino, *al-Battānī*, vol. II, pp. 78–83).

[25] The mistaken reading 'xiii vel xviii minuta' must have entered the text's transmission early and is perhaps the author's mistake.

tem duodecimam et addiderimus super equationem Solis, collectum erit id quod est inter mediam applicationem et veram ubi Luna consequetur Solem. Et illud quidem secundum Ptolomeum collectum est v gradus et xliiii minuta et secundum Albategni iiii gradus et liii minuta. Et hoc est quod addunt v lunationes tarde super medium cursum longitudinis, sed et idem addunt fere super medium motum latitudinis. Porro medius motus latitudinis in spatio v mensium equalium continet post integras revolutiones cliii partes et xxi minuta fere. Erit ergo quod aggregatur ex cursu vero latitudinis in v tardis lunationibus clix gradus et v minuta et hoc quidem secundum Ptolomeum, secundum Albategni vero clviii et xiiii minuta.

Rursum cum nota sit portio Lune, erit propter hoc diversus motus Lune ad unam horam notus, et propter hoc diameter Lune notus et semidiameter umbre. Et secundum opus quidem Ptolomei fit hic medietas duorum diametrorum scilicet Lune et umbre pars una fere, et secundum opus Albategni lvii minuta. In tanta igitur latitudine Luna constituta erit Luna contingens circulum umbre. Nota autem hac latitudine notus est arcus a nodo usque ad terminos eclipticos qui tunc sunt cum Luna fuerit prope longitudinem mediam epicicli. Et est hic arcus secundum Ptolomeum xi gradus et xxx minuta et secundum Albategni x gradus et lvii minuta. Sit ergo tante quantitatis quilibet istorum arcuum AC GE sive AF GH. Erit ergo arcus CBE clvii gradus tantum vel clviii et vi minuta. Sed erat arcus veri motus latitudinis in v tardis lunationibus maior secundum Ptolomeum quidem duobus gradibus et v minutis et secundum Albategni viii minutis solummodo. Si ergo ceperit hic motus latitudinis infra C versus A, possibile est ut terminetur infra E versus G in quinque tardis lunationibus. Et ita aput utrumque nodum aliquid de Luna obscurabitur sed ex eadem parte nodorum tantum, cum hic motus latitudinis sit minor semicirculo et in modica quantitate extendi possit hinc et inde ultra arcum

498 addiderimus] add†id†imus *P* **499** consequetur] consequitur P_7*M* **501** liii] *corr. ex* 43 *M* **502** lunationes] *corr. ex* lunatione *K* **504** post] *s.l. P* xxi] *corr. ex* 22 *M* **505** cursu vero] *corr. ex* †...† (*perhaps other hand*) *K* **506/507** et[2] – minuta] *marg.* (*perhaps other hand*) *P* **506** hoc] hic *P* secundum[1]] *s.l.* P_7 **506/507** Albategni vero] vero Albategni *M* **507** clviii] 158 gradus *MN* **508** portio] proportio *P corr. ex* proportio *N* Lune[1]] *om. N* **509** notus[2]] nota *N* **510** opus quidem] quidem opus *P* opus P_7 hic] hec *MN* duorum] duarum *N* **511** pars] manifeste *add. et del.* P_7 opus Albategni] Albategni opus P_7 lvii] lviii P_7 **512** latitudine – constituta] Luna constituta latitudine *N* **513** est arcus] *iter. P* **516** Sit] sic *N* **517** AC] AT *N* sive] *s.l. P* vel *N* CBE – gradus] TBE 157 graduum *N* **518** clviii] 158 gradus *M* 15 graduum *N* minuta] minutorum *N* arcus] *s.l. P* **519** secundum – quidem] quidem secundum Ptolomeum *M* **520** minutis] minuta *M* **521** C] T *N* possibile est] *corr. ex* posset (*other hand*) *K* possibile *M* **523** parte nodorum] nodorum parte *PN* motus] *corr. ex* modus *M* minor] maior P_7 **524** extendi possit] extenditur *N* et[2]] *om. M* we take the twelfth of this distance and add it to the sun's equation, the sum will be that which is between the mean syzygy and the true where the moon will reach the sun. And indeed that sum is 5° 44′ according to Ptolemy and 4° 53′ according to Albategni. And this is what 5 slow lunations add to the mean course of longitude, but also they add approximately the same to the mean motion of latitude. In turn, the mean motion of latitude in the space of 5 mean months contains approximately 153° 21′ after complete revolutions. Therefore, what is collected from the true course of latitude in 5 slow lunations will be 159° 5′ and this indeed according to Ptolemy, but according to Albategni 158° 14′.

In turn, because the moon's portion is known, the moon's irregular motion for one hour will be known because of this,[26] and the moon's diameter and the shadow's radius will be known because of this.[27] And the half of the two diameters, i.e. of the moon and shadow, is here made approximately 1° according to Ptolemy's work indeed, and 57′ according to Albategni's work.[28] Therefore, with the moon set up at such a latitude, the moon will be touching the shadow's circle. Moreover, with this latitude known, the arc from the node to the eclipse limits that are at the times when the moon is near the epicycle's mean distance is known. And this arc is 11° 30′ according to Ptolemy and 10° 57′ according to Albategni. Therefore, let each of those arcs AC, GE, AF, or GH be of such a size. Arc CBE, therefore, will be only 157° or 158° 6′. But the arc of the true motion of latitude in 5 slow lunations was greater indeed by 2° 5′ according to Ptolemy and only 8′ according to Albategni. Therefore, if this motion of latitude took hold below C towards A, it is possible that it is ended below E towards G in five slow lunations. And thus some part of the moon will be obscured at each node but only on the same side of the nodes because this motion of latitude is less than a semicircle and it is able to be extended beyond arc CBE by a modest quantity on one side and the other. And accord-

[26] Through VI.2 or a table of hourly motion (e.g. Nallino, *al-Battānī*, vol. II, p. 88; or Pedersen, *The Toledan Tables*, Table JA11, pp. 1410–12).

[27] Through V.18.

[28] Again, the manner that the author reaches this value is not certain. Using al-Battānī's tables for the moon's hourly motion (Nallino, *al-Battānī*, vol. II, p. 88) and the method outlined above in V.18, I reach approximately 55′ 46″ for the combined radii of the moon and shadow.

CBE. Et secundum inventa Albategni pene insensibiles erunt obscurationes si contingant.

9. Eclipsim Lune in septimo mense lunari iterari omnino est impossibile.

Sumamus enim vii lunationes minimas sicut continue accidere possunt, hoc est cum Sol erit tardus cursu et Luna velox. Invenimus siquidem in vii mensibus equalibus medium motum Solis et Lune cciii gradus et xlv minuta fere, qui cum divisi fuerint equaliter per medium ut sumantur arcus equales ex utraque parte longitudinis longioris Solis, minuent hii gradus de medio motu Solis per equationem quidem Ptolomei iiii gradus et xlii minuta et per equationem Albategni 3 gradus et liiii minuta. Et invenimus motum Lune in epiciclo in vii mensibus equalibus clxxx gradus et xliii minuta, qui cum equaliter divisi fuerint per medium ut sumantur duo equales arcus ex utraque parte longitudinis propioris epicicli, addent hii gradus super medium cursum Lune ex opere quidem Ptolomei ix gradus et lviii minuta et ex opere Albategni cum portio equata fuerit x gradus et i minutum. In tempore ergo intermedio quod est vii mensium equalium cum fuerit Sol quidem tardus cursu et Luna velox, erit Luna precedens Solem secundum partes aggregatas ex ambabus differentiis. Si ergo harum partium aggregatarum sumpserimus partem duodecimam et addiderimus super equationem Solis, collectum erit idem quod est inter mediam coniunctionem et veram. Et illud quidem collectum secundum Ptolomeum est v gradus et lv minuta fere et secundum Albategni v gradus tantum. Et hoc est quod minuunt vii lunationes parve de medio cursu longitudinis, sed et idem minuunt de medio cursu latitudinis. Porro medius cursus latitudinis in spatio vii mensium equalium continet post integras revolutiones ccxiiii gradus et xlii minuta. Erit ergo quod relinquitur verus motus latitudinis ccviii gradus et xlvii minuta, et hoc quidem secundum Ptolomeum, secundum Albategni vero ccix gradus et xlii minuta. Sicut autem ostensum est prius cum Luna fuerit aput longitudines medias epicicli, erit arcus a nodo usque ad terminos eclipticos qui tunc sunt secundum Ptolomeum quidem xi gradus et xxx minuta et secundum Albategni x gradus et lvii minuta. Ponamus itaque utrumque istorum arcuum AF GH huius quantitatis. Erit ergo arcus FBH cciii gradus tantum,

525 CBE] *corr. ex* †...† *P* TBE *N* obscurationes] obscuritates *K* **527** lunari] *om. N* **528** continue – possunt] possunt accidere *N* **529** Invenimus] inveniemus *M* siquidem] quidem *N* **530** et[1]] vel P_7K gradus – minuta] graduum et 45 minutorum *N* **531** ut] *corr. ex* non *K* **532** Solis[1]] *om. N* **534** invenimus] *corr. in* inveniemus *M* **535** xliii] *corr. in* 42 *M* 42 *N* **536** equales arcus] arcus equales P_7N **537** addent] addunt *N* **543** equationem] equationes *M* idem] *corr. in* id P_7 id *M* inter] *om. PK s.l.* P_7 (inter *Ba om.* E_1) **545** gradus[1] – minuta] graduum et 55 minutorum *N* **546** minuunt] minuit PP_7K (minuunt BaE_1) **546/547** sed – latitudinis[1]] *marg. M* **548** ccxiiii] *corr. ex* 244 *M* **549** minuta] fere *add. et del. M* **551** ccix] *corr. ex* 99 *N* autem] *corr. ex* alias *N* **553** Ptolomeum] vel *add. et del. K* quidem] *om. M* gradus] *om. PK* et[2]] *om. M* **554** lvii] 55 *M* **555** cciii gradus] 203 graduum *N* ing to the findings of Albategni, the obscurations will be almost imperceptible if they do occur.

9. It is entirely impossible that an eclipse of the moon be repeated in the seventh lunar month.

Indeed, let us take the 7 smallest lunations that can occur continuously, that is when the sun will be slow of course and the moon fast. Accordingly, we find in 7 mean months the mean motion of the sun and moon is approximately 203° 45′, and when they are divided equally in half so that equal arcs are taken on both sides of the sun's apogee, these degrees will subtract 4° 42′ from the sun's mean motion indeed through Ptolemy's equation and 3° 54′[29] through Albategni's equation. And we find the moon's motion on the epicycle in 7 mean months to be 180° 43′, and when they have been equally divided in half so that two equal arcs are taken on both sides of the epicycle's perigee, these degrees add upon the moon's mean course 9° 58′ indeed from Ptolemy's work and 10° 1′ from Albategni's work when the portion has been equated. Therefore, in the intermediate time, which is 7 mean months,[30] when the sun indeed is slow of course and the moon fast, the moon will be preceding the sun according to the degrees collected from both differences. If, therefore, we take the twelfth of these collected degrees and we add them to the sun's equation, the sum will be the same that is between the mean conjunction and the true. And indeed, that sum is approximately 5° 55′ according to Ptolemy and only 5°[31] according to Albategni. And this is what 7 small lunations subtract[32] from the mean course of longitude, but they also subtract the same from the mean course of latitude. In turn, the mean course of latitude in the space of 7 mean months contains 214° 42′ after complete revolutions. Therefore, what remains, the true motion of latitude, will be 208° 47′, and this indeed according to Ptolemy, but according to Albategni 209° 42′. Moreover, as was shown earlier,[33] when the moon is at the epicycle's mean distances, the arc from the node to the eclipse limits that are at that time will be 11° 30′ indeed according to Ptolemy and 10° 57′ according to Albategni. Accordingly, let us suppose each of those arcs AF and GH to be of this quantity. Arc FBH, therefore, will only be

[29] Note that the '3' is one of the rare cases where an Arabic numeral is found even in *P* and *K*, so it appears to be original.

[30] This is an improvement over Gerard's translation, which has 'In tempore ergo septem mensium minorum …' (*Almagest*, 1515 ed., f. 65r), 'In the time of seven shortest months' implies that the sun and moon are together.

[31] This is rounded down to whole degrees from the 5° 3′ 35″ that is reached by the steps described here.

[32] The mistaken singular reading found in *P*, P_7, and *K* must have crept into the text early in its transmission.

[33] I.e. in VI.8.

et hoc secundum Ptolomeum, at vero secundum Albategni cci gradus et liiii minuta. Quare arcus veri motus latitudinis eum excedit v gradibus et amplius, at uterque maior est semicirculo. Si ergo contigerit eclipsim esse in una oppositione inter F et A, in septima lunatione non continget eclipsis versus alterum nodum quia motus latitudinis verus excedit arcum FBH plus quam v gradibus et minus quam vii. Ultra autem non fit eclipsis scilicet in arcu HDF.

10. Solis eclipsim iterari in mense quinto in pluribus plagis habitatis aliquando nullatenus erit impossibile.

Ostensum siquidem est in premissis quod in v tardis lunationibus erit verus motus latitudinis clix gradus et v minuta, et hoc secundum Ptolomeum, secundum Albategni vero clviii gradus et xiiii minuta. Et quoniam medietas duorum diametrorum Solis et Lune dum fuerint in longitudinibus mediis continet secundum utrumque auctorem pene xxxii minuta, si Lune nulla fuerit diversitas aspectus in latitudine, et fuerit latitudo Lune secundum hanc quantitatem medietatis duorum diametrorum, erit arcus a nodo usque ad terminos eclipticos vi gradus et xii minuta. Et sit eius quantitas AC et similiter GE. Erit ergo arcus CBE in quo non erit eclipsis clxvii gradus et xxxvi minuta. Manifestum ergo quod cum non fuerit Lune diversitas aspectus, non est possibile ut sit bis eclipsis Solis in quinque tardis lunationibus propter hoc quod arcus CBE erit maior arcu veri motus Lune per orbem declinantem Lune — maior inquam viii gradibus et xxxi minutis, et hoc secundum Ptolomeum, secundum Albategni vero ix gradibus et xxii minutis. Et cum hunc arcum veri motus Lune per orbem declinantem dempserimus de semicirculo et reliqui sumpserimus medietatem que erit quidam arcus a nodo et quesierimus termini ipsius latitudinem ab orbe signorum, inveniemus quidem secundum Ptolomeum liiii minuta et xxxi secunda, secundum Albategni vero lvi minuta et xlvi secunda. Cumque

556 at – Albategni] secundum Albategni vero *M* cci] ccii P_7 **557** minuta Quare] minuta quasi P_7 eum] cum *P* **558** maior est] est maior *M* esse – oppositione] in una esse oppositione P_7 esse in oppositione una (*the last word corr. ex* vera) *M* una] parte *add. et del. P* **559** continget] contingit *M* **561** quam] quod *M* HDF] HDF et cetera *N* **563** erit] *s.l. P* est *M* **564** siquidem est] est siquidem *M* in[1]] *corr. ex* ex *P* **566** duorum] duarum *N* **567** Solis] Solis scilicet *M* dum] cum *N* fuerint] fuerit *M* **568** utrumque] *om.* P_7 pene] pene et *P* fuerit] fuit *K* **570** medietatis] medietas *MN* duorum] duo *K* duarum *N* **571** AC] AT *N* **572** arcus] *s.l.* (*other hand*) *K* CBE] TBE *N* clxvii] *corr. ex* clviii P_7 **574** quod] *om. PN* CBE] TBE *N* **575** maior arcu] minor arcu *P* veri] cum *P* cum *del. N* orbem – Lune[2]] declinantem Lune orbem *N* **576/577** secundum[2] – minutis] sed secundum Albategni 9 gradus et 22 minuta *M* **577** xxii] *corr. ex* xxxii *P* **578** dempserimus] dempseris *N* sumpserimus] sumpseris *N* **579** quidam] quidem *MN* termini] tunc *N* **580** inveniemus] invenerimus *P* inveniremus P_7 (invenimus *Ba* inveniemus E_1) quidem] *om.* P_7 liiii] 59 *N* **580/581** minuta – secunda[1]] gradus et 31 minuta et *M* **580** et] *om.* P_7 **581** vero] *om.* P_7KM lvi – secunda[2]] *corr. ex* 56 gradus (*these two words iter. et del.*) et 46 minuta *M* secunda[2]] *s.l.* (*perhaps other hand*) *P*

203°, and this according to Ptolemy, and indeed according to Albategni 201° 54′. Therefore, the arc of the true motion of latitude exceeds it by 5° and more, but each is greater than a semicircle. Therefore, if it happens that there is an eclipse in one opposition between F and A, an eclipse will not occur in the seventh lunation towards the other node because the true motion of latitude exceeds arc FBH by more than 5° and less than 7°.[34] Moreover, an eclipse will not occur beyond, i.e. in arc HDF.

10. It will by no means be impossible that an eclipse of the sun sometime be repeated in the fifth month in several inhabited regions.

Accordingly, it was shown in what has been set forth[35] that the true motion of latitude in 5 slow lunations will be 159° 5′, and this according to Ptolemy, but according to Albategni 158° 14′. And because the half of the two diameters of the sun and moon while they are at the mean distances contains about 32′ according to both authorities,[36] if there is no parallax of the moon in latitude, and ⟨because⟩ the moon's latitude is according to this quantity of the half of the two diameters, the arc from the node to the eclipse limits will be 6° 12′. And let AC and likewise GE be its quantity. Therefore, arc CBE, in which there will not be an eclipse, will be 167° 36′. It is manifest, therefore, that when there is no parallax of the moon, it is not possible that there be an eclipse of the sun twice in five slow lunations because of this that arc CBE will be greater than the arc of the moon's true motion on the declined circle of the moon – I say, greater by 8° 31′, and this according to Ptolemy, but according to Albategni by 9° 22′. And when we subtract this arc of the moon's true motion on the declined circle from a semicircle and we take the remainder's half, which will be a certain arc from the node and we seek its endpoint's latitude from the ecliptic, we find indeed 54′ 31″ according to Ptolemy, but 56′ 46″ according

[34] This should say that it is less than 8° since according to the values determined for Albategni, the motion of latitude is 209° 42′ and arc FBH is 201° 54′.

[35] VI.8.

[36] To confirm that Albategni would have reached approximately 32′, the *Almagesti minor*'s author seems to have used tables of hourly motion (Nallino, *al-Battānī*, vol. II, p. 88; or Pedersen, *The Toledan Tables*, Table JA11, pp. 1410–12) and the methods outlined above in V.18.

a quantitate huius latitudinis minuerimus medietatem duorum diametrorum et reliquum duplicaverimus, superfluent pro Ptolomeo xlv minuta fere, pro Albategni xlix minuta. In quibuscumque itaque climatibus accidere poterit ut diversitas aspectus in una duarum coniunctionum extremarum aut in ambabus simul sit maior xlv minutis Ptolomei vel xlix Albategni, tunc ille coniunctiones extreme v tardarum lunationum procul dubio possunt esse ecliptice quia tunc contingere potest quod in utraque coniunctione latitudo Lune visa minor sit posita quantitate medietatis duorum diametrorum.

Videamus ergo in quibus locis circuli signorum et in quibus horis hee coniunctiones ecliptice possint cadere. Quoniam autem tempus v mensium equalium continet cxvii dies et xv horas et medietatem et quartam hore fere, Sol autem quia est in cursu velociore et Luna in suo cursu tardiore, Lune post mediam coniunctionem restat peragrandum antequam Solem comprehendat secundum Ptolomeum quidem xiii gradus et xviii minuta cum duodecima ipsorum, secundum Albategni autem xii gradus et xliii minuta cum duodecima eorum. Hoc autem Luna spatium percurrit cursu medio in die una et horis duabus et quarta unius hore, et hoc quidem secundum Ptolomeum, secundum alios vero in die una et hora una et decima unius hore. Ex hiis omnibus palam quod tempus v tardarum lunationum continet dies cxlviii et xviii horas aut cxlviii dies et horas xvi et minuta hore xxi. Liquet igitur quod si prima duarum coniunctionum de quibus sermo est fuerit iuxta occasum Solis, secunda erit secundum Ptolomeum quidem vi horis ante occasum, secundum Albategni vero vii horis et xxxix minutis ante occasum. Rursum quia verus cursus Solis inter has duas coniunctiones est circiter cl gradus et ipse est cursus maior Solis in v lunationibus tardis, ex utraque parte longitudinis propioris resecentur arcus equales continentes simul cl gradus. Cum itaque longitudo propior fuerit in xviii^{o} gradu Sagittarii, palam quod coniunctio prima duarum de quibus sermo

582 duorum] duarum *N* **583** duplicaverimus] duplicabimus *N* pro Ptolomeo] per Ptolomeum *MN* pro[2]] per *MN* **584** quibuscumque] *corr. ex* quibus cuique P_7 quibus cuique *K* itaque] *s.l. K* **586** minutis] minuta *M* xlix] 49 minuta *M* **586/587** coniunctiones extreme] extreme coniunctiones *K* **587** tardarum lunationum] *corr. ex* cordarum coniunctionum P_7 lunationum] *corr. ex* Lunarum *M* possunt] poterunt *N* **588** latitudo Lune] Lune latitudo *M* sit] vise *add. et del. N* **589** duorum] duarum *N* **591** possint] possunt *N* **592** continet] *iter. et del. P* cxvii] *corr. in* cxlvii *PM* cxviii *corr. in* clxvii P_7 147 *N* (cxviii *Ba* 117 E_1) hore] *corr. ex* horam *K* **593** est] *om. PN* **594** mediam coniunctionem] *corr. ex* medias coniunctiones *K* peragrandum] peragendum P_7 **596** xliii] *corr. ex* liii *K* **597** Luna spatium] spatium Luna *M* spatio Luna *N* cursu] *corr. ex* suo P_7 die una] una die *M* **599** unius hore] hore unius *N* **600** aut] secundum Ptolomeum secundum Albategni autem (autem Albategni *N*) *MN* **601** cxlviii] *corr. in* 158 *M* xxi] (21 BaE_1) igitur] *s.l.* P_7 **604** vero] autem *N* horis] *corr. ex* horas P_7 minutis] minutiis *K* occasum] occasum Solis *M* **605** has duas] duas has *M* est[1]] idest P_7 cursus – Solis] maior cursus Solis *PN* cursus Solis maior *M* **608** coniunctio prima] prima coniunctio *N* to Albategni.[37] And when we subtract the half of the two diameters from the quantity of this latitude and double the remainder, there will be approximately 45′ in excess for Ptolemy, 49′ for Albategni. Accordingly, in whatever climes it will be possible to happen that the parallax is greater than the 45′ of Ptolemy or the 49′ of Albategni in one of the two extreme conjunctions or in both together, those extreme conjunctions of 5 slow lunations then are doubtlessly able to have eclipses because then it is possible to happen that in each conjunction the moon's apparent latitude is less than the posited quantity of the half of the two diameters.

Let us see, therefore, in which places of the ecliptic and in which hours these conjunctions that have eclipses are able to fall. Moreover, because the time of 5 mean months contains approximately 117[38] days, 15 hours, and 45 minutes, and also because the sun is in its faster course and the moon in its slower course, it remains that a traveling of the moon after the mean conjunction before it catches up to the sun must occur, indeed according to Ptolemy 13° 18′ with their twelfth, but according to Albategni 12° 43′ with their twelfth.[39] Moreover, the moon travels through this distance by mean course in 1 day, 2 hours, and 15 minutes, and this indeed according to Ptolemy, but according to others[40] in 1 day, 1 hour, 6 minutes. From all of these things, it is clear that the time of 5 slow lunations contains 148 days and 18 hours or 148 days, 16 hours, and 21 minutes.[41] Therefore, it is certain that if the first of the two conjunctions about which the discussion is was near the sun's setting, the second will be 6 hours before the setting indeed according to Ptolemy, but 7 hours and 39 minutes before the setting according to Albategni. In turn, because the sun's true course between these two conjunctions is around 150° and it is the sun's greatest course in 5 slow months, let equal arcs containing together 150° be cut off on both sides of the perigee. Accordingly, because the perigee is in the 18th degree of Sagittarius,[42] it is clear that the first conjunction of the two about which the discussion is will be around Libra 3°, and the

[37] The value for Ptolemy is not found in the *Almagest*, so the *Almagesti minor*'s author must have calculated both of these values using the corollary of I.16.

[38] This should be 147, but our author appears to have copied this mistaken value from an *Almagest* manuscript such as Paris, BnF, lat. 14738, f. 100r.

[39] As shown in VI.8.

[40] It is not clear whom our author has in mind besides Albategni.

[41] This should be 51 minutes and thus in the following sentence, the amount of time that the second eclipse occurs before sunset should be 7 hours and 9 minutes for Albategni. That the subsequent calculations are built upon this value shows that it is original.

[42] He probably has in mind the value of the position of the apogee at Gemini 17° 50′ that was attributed to Arzachel above in III.11.

est erit circiter quartum gradum Libre, et coniunctio secunda erit circiter quartum gradum Piscium. Habemus ergo horas et loca in signorum orbe in quibus hee due coniunctiones esse possunt. Et quoniam a secundo climate deinceps in plagis septentrionalibus diversitates aspectus latitudinis ad dictas horas diei et in locis circuli signorum determinatis ambe – inquam diversitates – simul sunt plus quam xlv minuta vel etiam quam xlix minuta cum Luna fuerit in longitudine media, manifestum est quod in illis plagis habitantes possibile est videre eclipsim Solis duabus vicibus in v mensibus tardis, neque hoc continget nisi cum Luna erit septentrionalis tantum ab orbe signorum, scilicet cum fuerit in eclipsi prima recedens a nodo Capitis et in eclipsi secunda accedens ad nodum Caude, et hoc est quod proposuimus.

11. Solis eclipsim in septimo mense iterari in quibusdam plagis septentrionalibus non est omnino impossibile.

Ostensum siquidem est prius quod in vii brevioribus lunationibus erit verus motus latitudinis ccviii gradus et xlvii minuta, et hoc quidem secundum Ptolomeum, sed secundum Albategni ccix gradus et xlii minuta. Et quoniam cum nulla fuerit diversitas aspectus Lune in latitudine et medietas duorum diametrorum fuerit secundum quantitatem assignatam pene xxxii minuta sicut ad longitudines medias contingit, erit arcus a nodo usque ad terminos eclipticos sicut prius vi gradus et xii minuta, sit ergo uterque istorum arcuum AF GH huius quantitatis. Erit ergo arcus FBH cxcii gradus et xxiiii minuta. Palam ergo quod arcus motus latitudinis maior est arcu FBC xvi vel xvii gradibus ad minus. Cadet ergo motus latitudinis etiam si ab F inchoaverit in arcu HDF in quo non fit eclipsis. Igitur manifestum quod cum non fuerit Lune diversitas aspectus in latitudine, non est possibile ut sit quod diximus. Porro cum de arcu veri motus latitudinis dempserimus semicirculum, et residui sumpserimus medietatem que erit quidam arcus a nodo, et quesierimus termini ipsius latitudinem ab orbe signorum, inveniemus quidem secundum Ptolomeum gradum i et xv minuta et secundum Albategni gradum unum et xvii minuta. Cumque a quantitate huius latitudinis minuerimus medietatem duorum diametrorum et

610 loca] *corr. ex* loc†os† *K* signorum orbe] orbe signorum *M* **613** determinatis] determinantis *PN* **614** etiam] etiam plus *M* **615/616** videre – Solis] eclipsim Solis videre *N* **616** continget] contingit *PN* nisi] *corr. ex* †nichil† *P* **617** scilicet] si *M* **621** est omnino] omnino est *M* **622** siquidem est] est siquidem *M* **623** xlvii minuta] minuta xlvii P_7 **624** sed – Albategni] secundum Albategni autem *N* **625** duorum] duarum *N* **626** quantitatem] fere *add. et del. P* minuta] *s.l.* (*perhaps other hand*) *P* minutorum *N* **627** arcus – nodo] a nodo arcus *M* **628** sit] fit *N* istorum] illorum duorum *M* **630** maior est] est maior *M* FBC] *corr. in* FBH P_7*N* (FBC *Ba*E_1) **631** Cadet] cadit *M* **632** fit] sit *K* eclipsis] eclipsim *P* **634** residui] *corr. ex* residuum *K* **635** arcus] *iter. et del. P* **636** quidem] *om.* P_7 **637** secundum] per *PN* xvii] 7 *N* **638** duorum] duarum *N* second conjunction will be around Pisces 3°. We have, therefore, the hours and the places in the ecliptic in which these two conjunctions can be. And because from the second clime[43] onward in the northern regions the parallaxes of latitude at the said hours of the day and in the determined places of the ecliptic both – I mean the parallaxes – together are more than 45′ or also more than 49′ when the moon is at the mean distance, it is manifest that in these regions it is possible for the inhabitants to see an eclipse of the sun twice in 5 slow months, and this will not occur except when the moon will be north of the ecliptic, i.e. when it is receding from the node of the Head in the first eclipse and approaching the node of the Tail in the second eclipse, and this is what we proposed.

11. It is not entirely impossible that an eclipse of the sun be repeated in the seventh month in certain northern regions.

Accordingly, it was shown earlier that in 7 shorter lunations the true motion of latitude will be 208° 47′, and this indeed according to Ptolemy, but according to Albategni 209° 42′.[44] And because when there is no parallax of the moon in latitude and according to the assigned quantity, the half of the two diameters is about 32′ as happens at the mean distances, the arc from the node to the eclipse limits will be 6° 12′ as before, therefore, let each of those arcs AF and GH be of this quantity. Arc FBH, therefore, will be 192° 24′. It is clear, therefore, that the arc of the motion of latitude is greater than arc FBC[45] by at least 16° or 17°. Therefore, even if the motion of latitude begins from F, it will fall on arc HDF, in which an eclipse does not occur. Therefore, it is manifest that when there is not a parallax of the moon in latitude, it is not possible that what we said exists [i.e. that a solar eclipse repeats in 7 months]. On the other hand, when we subtract a semicircle from the arc of the true motion of latitude, take the remainder's half, which will be a certain arc from the node, and seek its endpoint's latitude from the ecliptic, we will find 1° 15′ indeed according to Ptolemy and 1° 17′ according to Albategni. And when we subtract the half of the two diameters from the quantity of this latitude and double the

[43] Ptolemy does not mention a specific clime, merely stating that the necessary amount of parallax occurs for places north of the equator; however, the *Almagesti minor*'s author seems to be using Ptolemy's numbering of climes here. Albategni's first clime has a longest day of 13 hours and it is clear from his tables of parallax that the necessary amount of southward parallax of latitude occurs for an eclipse to be repeated (see Nallino, *al-Battānī*, vol. II, p. 95; or Pedersen, *The Toledan Tables*, Table HC11, pp. 1182–84).

[44] As was found in VI.9.

[45] This should be FBH, but the mistake appears to be original.

reliquum duplicaverimus, superfluent pro Ptolomeo quidem gradus unus et xxv minuta et pro Albategni gradus unus et xxx minuta. In quibuscumque ergo climatibus accidere poterit ut diversitas aspectus in una duarum coniunctionum extremarum aut in ambabus simul sit maior gradu uno et xxv minutis Ptolomei vel gradu uno et xxx minutis Albategni, tunc ille coniunctiones extreme vii brevium mensium procul dubio possunt esse ecliptice, quia tunc contingere potest quod in utraque coniunctione latitudo Lune visa minor sit quantitate medietatis duorum diametrorum.

Videamus ergo in quibus locis circuli signorum et in quibus horis hee coniunctiones ecliptice possunt cadere. Continet enim tempus vii mensium mediorum ccvi dies et xvii horas fere. Et quia Sol est in cursu tardiore et Luna in cursu velociore, erit Luna iam preteriens Solem secundum Ptolomeum quidem xiiii gradibus et xl minutis, et secundum Albategni xiii gradibus et lv minutis. Sed has quantitates cum duodecima percurrit Luna secundum Ptolomeum quidem in die una et v horis et hoc per cursum medium, sed secundum Albategni in die una et duabus horis fere. Cum ergo hoc tempus minuerimus de tempore vii mensium mediorum, palam quod tempus vii mensium breviorum continet secundum Ptolomeum ccv dies et xii horas fere et secundum Albategni ccv dies et xv horas. Quapropter tempus coniunctionis extreme erit secundum Ptolomeum post xii horas temporis coniunctionis prime, et ita si prior coniunctio fuerit prope ortum Solis, altera erit prope occasum Solis. At secundum Albategni tempus postreme coniunctionis erit post xv horas temporis prime coniunctionis, et ita si prior coniunctio fuerit paulo ante occasum, altera poterit paulo post ortum Solis. Aliter enim non essent ambe super terram.

Rursum quia verus motus Solis inter has duas coniunctiones est circiter cxcviii gradus, et ipse est motus tardior Solis in vii mensibus brevibus, resecabimus ex utraque parte longitudinis longioris arcus equales continentes simul cxcviii gradus. Cum itaque longitudo longior fuerit in xviii° gradu Geminorum, palam quod coniunctio prima duarum de quibus sermo est erit circiter

639 pro Ptolomeo] pro Tolomeo *K* per Ptolomeum *M* quidem] *om.* P_7M **640** pro] per *MN* gradus] *corr. ex* gradum *M* quibuscumque] quibusdam *M* **641** aspectus] *om. PN* **642** simul] *om. N* minutis] minuta *M* **643** minutis] minuta *M* **644** contingere] accidere *M* **645** minor sit] sit minor *N* **646** medietatis] *s.l.* (*perhaps other hand*) *P* duorum] duarum *N* **647** et] *om. P* **648** enim tempus] tantum PP_7K enim spatium *N* (tempus *Ba* tantum E_1) mediorum] *corr. ex* equalium *N* **649** ccvi] *corr. ex* ccvii *P* xvii] *corr. ex* 22 *M* **651** gradibus[1] – minutis[1]] gradus et 55 minuta *M* gradibus[2]] gradus *PM* minutis[2]] minuta *M* Sed] *corr. ex* secundum P_7 **653** v] *corr. in* 2 *M* **654** die una] una hora *N* **655/656** continet – Ptolomeum] secundum Ptolomeum continet *M* **656** xii] *corr. ex* 22 *M* **657** horas] horas fere *M* extreme] *corr. in* postreme *M* **658** prior coniunctio] coniunctio prima *M* prima coniunctio *N* **659** altera – Solis[2]] *marg.* (*other hand*) *K* **661** poterit] poterit esse *MN* **663** Rursum] rursus *M* has] *s.l.* (*perhaps other hand*) *P* **664** cxcviii] *corr. ex* 128 *M* motus tardior] tardior motus *N* **666** cxcviii] clxcviii *P* **667** sermo est] est sermo *M* remainder, there are 1° 25′ in excess indeed for Ptolemy and 1° 30′ for Albategni. Therefore, in whatever climes it can happen that the parallax in one of the two extreme conjunctions or in both together is greater than the 1° 25′ of Ptolemy or the 1° 30′ of Albategni, those extreme conjunctions of 7 short months are then doubtlessly able to have eclipses, because then it can happen that in each conjunction the moon's apparent latitude is less than the quantity of the half of the two diameters.

Let us see, therefore, in which places of the ecliptic and in which hours these conjunctions having eclipses can occur. Indeed, the time[46] of seven mean months contains approximately 206 days 17 hours. And because the sun is in ⟨its⟩ slower course and the moon in ⟨its⟩ faster course, the moon will already be going 14° 40′ beyond the sun indeed according to Ptolemy and 13° 55′ according to Albategni.[47] But the moon travels through these quantities with a twelfth in 1 day and 5 hours indeed according to Ptolemy and this by mean course, but according to Albategni in approximately 1 day and 2 hours.[48] Therefore, when we subtract this time from the time of 7 mean months, it is clear that the time of 7 shorter months contains approximately 205 days and 12 hours according to Ptolemy and 205 days and 15 hours according to Albategni. For this reason the time of the last conjunction will be 12 hours after the time of the first conjunction according to Ptolemy, and thus if the earlier conjunction was near the sun's rising, the other will be near the sun's setting. But, according to Albategni, the time of the last conjunction will be 15 hours after the time of the first conjunction, and thus if the earlier conjunction is a little before the setting, the other can be a little after the sun's rising. For otherwise both would not be above the earth.

In turn, because the sun's true motion between these two conjunctions is around 198°,[49] and that is the sun's slower motion in 7 short months, we will cut equal arcs containing together 198° off from both sides of the apogee. Accordingly, because the apogee is in the 18th degree of Gemini, it is clear that the first conjunction of the two about which the discussion is will be

[46] The mistaken reading 'tantum' must have entered the text's transmission early. The readings in *M* and *N* are likely later corrections, and the original text appears to be lost.

[47] These are the sums of the anomalies of the sun and moon that are found in VI.9.

[48] This number is approximately 1 1/2 hours too low. It is unclear exactly how the author made this error. The subsequent values rely upon this value, so it is clear that the mistake is original.

[49] This can be gathered from VI.9. The value is closer to 199° if one takes the anomalies according to Albategni's parameters.

decimum gradum Piscium; coniunctio postrema erit circiter xxvii gradum Virginis. Habemus itaque loca in orbe signorum et horas in quibus hee coniunctiones esse possunt. Et quoniam a quarto climate deinceps in climatibus septentrionalibus cum Luna fuerit iuxta longitudinem mediam, diversitates aspectus latitudinis ad dictas horas Ptolomei in locis circuli signorum determinatis ambe – inquam diversitates – simul sunt plus quam gradus unus minuta xxv, possibile est secundum opus Ptolomei ut hii qui sunt in hiis plagis videant eclipsim Solis duabus vicibus in vii mensibus brevibus, neque hoc continget nisi cum Luna erit septentrionalis ab orbe signorum tantum scilicet in eclipsi prima apropinquans nodo Caude et in eclipsi secunda recedens a nodo Capitis. At vero secundum Albategni quoniam non contingit in aliquo climate ut diversitates aspectus ad dictas horas Albategni in locis circuli signorum determinatis sint plus simul quam gradus unus minuta xxx vel xxv, etiam non est possibile secundum opus Albategni ut aliqui homines videant eclipsim Solis bis in vii mensibus. Sed ad alias horas ut una sit ante meridiem, alia post mediam noctem, nichil prohibet in determinatis etiam locis eo quod tunc ambe diversitates aspectus maiores esse contingat posita quantitate, sed non erunt ambe eclipses super terram.

12. Solis eclipsim in uno mense lunari bis contingere aput homines unius habitabilis omnimodis est impossibile.

Et si enim aliquis aggregaverit causas eclipsium omnes simul, quarum quidem actu ipso impossibile est coniunctio et convenientia – possibile tamen ut propria voluntate eas quis imaginetur et congreget – neque sic esse possibile quod dicitur. Causas autem intelligo ut Luna sit in longitudine propiore ad habendum maiorem diversitatem aspectus in latitudine, ut sit lunatio minima que esse potest ad habendum minorem cursum latitudinis, et ut non constituamus diversitatem aspectus variari pro horis et locis signorum, sed sumatur maxima que esse potest in zona habitabili. Cum investigaverimus modo suprapposito, inveniemus motum latitudinis verum ad mensem minimum pene xxx graduum

668 decimum gradum] x graduum *P* Piscium] Piscium et *MN* erit] *s.l. K* circiter – gradum[2]] circa 27 *N* **670** deinceps] *iter. N* **672** latitudinis – dictas] ad has *N* Ptolomei] *corr. in* dies *M* **673** unus] unus et *N* **673/674** possibile est] possunt *P* **674** videant] *iter. N* **675** continget] contingit *N* **676** erit] fuerit *MN* scilicet] *perhaps corr. ex* sed *P* **678** aliquo] alio P_7*KM* diversitates] diversitas *PN* **679** ad] *om. M* **680** sint] *corr. ex* sunt *P* sunt *N* xxv] xxi *P corr. in* 21 *M* 31 *N* (25 *Ba*E_1) **681** aliqui] *corr. ex* alii *M* **682** ad] *corr. ex* et *K* ut] cum P_7 meridiem] meridiem et *N* **683** in determinatis] indeterminatis *P* etiam] *om.* P_7 **687** habitabilis] *corr. ex* habitationis *M* habitationis *N* omnimodis] omnimodum *K* omnimode *N* **688** Et] *om. N* aggregaverit – omnes] causas eclipsium omnes aggregaverit *PN* **689** ipso] *om. N* impossibile] impossibilis P_7 possibile] possibile est *PN* **690** et] *om. K* esse] erit P_7*M* esset *K* (esse *Ba* erit E_1) **692** latitudine] et *add. s.l.* P_7 **694** diversitatem – variari] variari diversitatem aspectus *M* variari] *om. N* **695** esse potest] potest esse P_7 **696** verum] unum P_7 ad] *corr. ex* et *K* around the tenth degree of Pisces; the last conjunction will be around the 27th degree of Virgo.[50] Accordingly, we have the places in the ecliptic and the hours in which these conjunctions are able to exist. And because from the fourth clime[51] continuously to the northern climes when the moon is near the mean distance, the parallaxes of latitude at the said hours of Ptolemy in the determined places of the ecliptic are both – I mean the parallaxes – together more than 1° 25′, it is possible according to the work of Ptolemy for those who are in these regions to see an eclipse of the sun twice in 7 short months, and this will not occur except when the moon will be north of the ecliptic, i.e. only when it is approaching the node of the Tail in the first eclipse and receding from the node of the Head in the second eclipse. But indeed, according to Albategni, because it does not happen in any clime that the parallaxes at the said hours of Albategni in the determined places of the ecliptic are together more than 1° 30′ or 25′, it is not even possible according to the work of Albategni for some men to see an eclipse of the sun twice in 7 months. But at other hours, as one before noon and the other after midnight, nothing prevents ⟨this⟩ also in the determined places ⟨of the ecliptic⟩ because it would happen then that both parallaxes are greater than the posited quantity, but both eclipses will not be above the earth.

12. It is wholly impossible that an eclipse of the sun occur twice in one lunar month in the view of the men of one inhabitable zone.[52]

And indeed, if anyone added all the causes of eclipses together, the combination and arrangement of which indeed are impossible in reality – it is possible, nevertheless, that by extraordinary will someone may imagine and bring them together – and thus what is said would not be possible. Moreover, I understand the causes to be that the moon is at perigee in order to have the greatest parallax in latitude, that there is the smallest lunation that can be in order to have a smaller course of latitude, and that we do not establish that the parallax varies for the hours and places of the ecliptic, but that the greatest that there can be in an inhabitable zone is taken. When we investigate in the way posited above, we will find that the true motion of latitude for the

[50] When 99° is added and subtracted from the 18th degree of Gemini, one would expect the results to be the 27th degree of Virgo and the ninth degree of Pisces, not the tenth degree of Pisces; however, this discrepancy is just a result of rounding. The author probably has in mind that the apogee is at Gemini 17° 50′ (see III.11 above), and thus the two positions 99° on either side are Virgo 26° 50′ and Pisces 8° 50′, the latter of which rounds to Pisces 9°, i.e. the tenth degree of Pisces.

[51] For this to agree with Ptolemy's claim that the required parallax is found from around the latitude of Rhodes northwards, the numbering of the climes here must be that of Albategni's, in which Rhodes is at the fourth clime (Nallino, *al-Battānī*, vol. II, p. 98; or Pedersen, *The Toledan Tables*, Table HC41, pp. 1391–93). This appears to be inconsistent with VI.10 above, in which he seems to use Ptolemy's numbering of climes.

[52] By 'habitabilis' our author seems to mean one of the two inhabitable sections of the earth, one north and one south that are separated by an uninhabitable zone around the earth's equator.

proiecta una revolutione. Et cum medietatis eius latitudinem acceperimus et ab ea medietatem duorum diametrorum proiecerimus reliquumque duplicaverimus, inveniemus gradum unum et xxvii minuta fere. Oportet ergo si Sol eclipsari debeat duabus vicibus in mense uno, quod si Lune non fuerit diversitas aspectus in coniunctione una, sit ei diversitas aspectus in coniunctione altera maior gradu uno et xxvii minutis; aut si fuerit Lune diversitas aspectus in utraque coniunctione et hoc versus partem eandem, quod altera diversitas alteram superet maiore augmento quam gradu uno et xxvii minutis; aut si fuerit Lune diversitas aspectus in utraque coniunctione et in una quidem versus septentrionem, in altera versus meridiem, quod ambe diversitates simul sint plus quam gradus unus et minuta xxvii. Sed non contingit alicui terrarum ut diversitas aspectus in latitudine in applicatione sit maior gradu uno. Non ergo possibile est ut in mense uno eclipsetur eis Sol bis cum aut sic fuerit Luna ut in una coniunctionum non sit ei diversitas aspectus, aut sic ut in utraque coniunctione sit ei diversitas aspectus versus eandem partem. Restat ergo si debeat fieri bis eclipsis in mense uno ut diversitates aspectus in duabus coniunctionibus sint versus partes oppositas, et ambe simul sint maiores gradu uno et minutis xxvii. Sed habitantibus sub equinoctiali maxima diversitas aspectus in latitudine non est plus quam xxv minutis in quamcumque partem. Itaque nullis habitantibus citra equinoctialem usque sub capite Cancri diversitas aspectus in partem septentrionis maior est xxv minutis. Sed neque eis neque aliquibus magis septentrionalibus diversitas aspectus in partem meridiei maior est parte una. Non ergo unius habitabilis homines in uno vel in pluribus climatibus possunt videre eclipsim Solis bis in mense uno. Nichil autem prohibet homines unius habitabilis et homines alterius habitabilis qui obliqui nobis dicuntur videre duas eclipses Solis in mense uno, eo quod ambe diversitates aspectus in partes oppo-

697 una] integra *N* cum] cuius *M* medietatis] *corr. ex* medietatem P_7 **698** duorum] duarum *N* **701** una] *om. N* coniunctione altera] altera coniunctione *N* **702** gradu – minutis] 1 gradus et 27 (*corr. ex* 25) minuta *M* fuerit] fuit P_7 **703** utraque coniunctione] coniunctione utraque P_7N partem eandem] eandem partem *M* **704** minutis] minuta *M* **706** altera] alia *N* **707** minuta xxvii] 27 minuta *N* alicui] alicubi *MN* terrarum] *corr. ex* †...† (*other hand*) *K* **709** eis] ei *M* aut] *del.* P_7 autem *KMN* sic – Luna] sic Luna fuerit *P* Luna sic fuerit *N* ut[2]] *s.l. M* **710** coniunctionum] coniunctione *M* non] nisi *P* aut] vel *M* ut] *om. P* quod *N* **711** fieri bis] bis fieri *N* **712** mense uno] uno mense *M* diversitates] diversitas PP_7N coniunctionibus] *iter. et del. P* sint] sit *MN* **713** simul] *om. K* sint] *corr. ex* sunt *P* gradu uno] uno gradu *P* minutis xxvii] minuta 27 *M* 27 minutis *N* **715** quam] *om. N* minutis] minuta *M* (minuta *Ba* minutorum E_1) **717** neque[2]] in *PN* **718** aspectus] *marg. P* parte una] una parte *PN* **719** ergo] enim *M* habitabilis] habitationis *MN* in[2]] *om. M* **720** mense uno] uno mense *N* habitabilis] habitationis *MN* **721** habitabilis] habitationis *MN* nobis dicuntur] dicuntur (*s.l.*) nobis *K* **722** mense uno] uno mense *M* smallest month is about 30°[53] with one revolution cast out. And when we take the latitude of its half, subtract the half of the two diameters from it, and double the remainder, we will find approximately 1° 27′. It is necessary, therefore, if the sun should be eclipsed twice in one month: that if there were not a parallax of the moon in one conjunction, the parallax for it in the other conjunction would be greater than 1° 27′; or if there were a parallax of the moon in each conjunction and this in the same direction, that one parallax would exceed the other by a greater increase than 1° 27′; or if there were a parallax of the moon at each conjunction, in one towards the north and in the other towards the south, that both parallaxes together would be more than 1° 27′. But it does not happen for any of the regions that the parallax in latitude in a syzygy is more than 1°. Therefore, it is not possible that the sun be eclipsed twice in one month for them [i.e. for any region] when either the moon would be thus that in one of the conjunctions there would not be a parallax for it, or thus that in each conjunction there would be a parallax in the same direction for it. Therefore, if an eclipse ought to be occur twice in one month, it remains that the parallaxes in the two conjunctions are in opposite directions and both together are greater than 1° 27′. But for the inhabitants under the equator, the greatest parallax in latitude is not more than 25′[54] in whichever part ⟨of the ecliptic⟩. Accordingly, for no inhabitants on this side of the equator to under the beginning of Cancer, is the parallax in the direction of the north greater than 25′. But neither for those nor for any more northern ones, is the parallax in the direction of the south greater than 1°. Therefore, the men of one inhabitable zone in one or in more climes are not able to see an eclipse of the sun twice in one month. Moreover, nothing hinders the men of one inhabitable zone and the men of the other inhabitable zone [i.e. in the southern hemisphere], who are called 'slanted ones' by us, from seeing two eclipses of the sun in one month, because both parallaxes occurring for them in opposite directions can

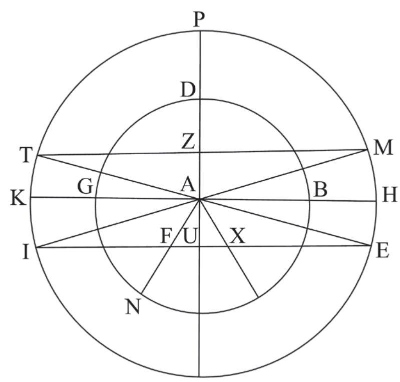
[53] Our author has rounded from the 29° 14′ that is found in the *Almagest*.

[54] The comparative with 'quam' should be followed by a nominative to match 'diversitas', but the mistaken ablative 'minutis' appears to be original.

sitas eis contingentes maiores esse possunt posita quantitate scilicet gradu uno et minutis xxvii. Et hoc est quod volebamus.

13. Digitos lunaris eclipsis ad quamcumque latitudinem Lune ab orbe signorum et ad quamcumque distantiam centri Lune a centro terre ostensive declarare. Unde patebit quando particularis et quando universalis erit eclipsis, et quando moram habebit Luna sub umbra et quando non, et quando erit maxima eclipsis que unquam esse potest.

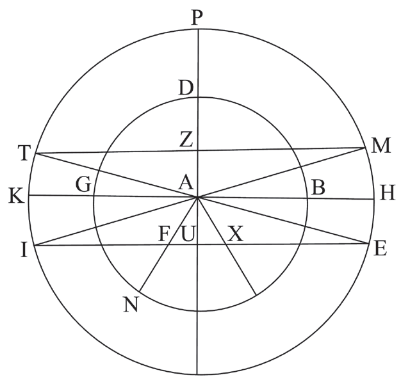
Quando siquidem eclipsis Lune possit vel debeat accidere et propter quid ex premissis innotuit. Quod autem nunc proponitur ad quantitatem eclipsis pertinet. Inveniatur itaque cum vera coniunctione Solis et Lune ecliptica motus latitudinis equatus, et per motum latitudinis vera Lune latitudo ad ipsum tempus coniunctionis vere. Nam illud tempus est tempus eclipsis medie. Deinde queratur ex octava presentis que sit quantitas medietatis duorum diametrorum scilicet Lune et umbre. Quibus habitis evidentie causa describam circulum umbre in transitu Lune supra centrum A et supra circulum umbre notas B D G; item super idem centrum alium circulum secundum quantitatem medietatis duorum diametrorum et supra circulum notas H P K. Palam ergo quod A punctum oppositum est semper in circulo signorum loco Solis, et cum Luna in directo huius puncti fuerit, ipsa est in medio eclipsis, et tunc maxime sunt eius in presenti eclipsi tenebre. Ponamus itaque vice arcus circuli signorum lineam HPK, et super eam perpendicularem PA vice circuli transeuntis super centrum Lune in medio eclipsis et polos circuli signorum. Si ergo transitus Lune fuerit super punctum P ut eius latitudo sit PA scilicet sicut medietas duorum diametrorum, tunc manifestum est quod



724 minutis xxvii] minuta 27 *M* 27 minutis *N* **727** declarare] *corr. ex* declarere *K* et quando] aut P_7 quando *KM* **729** esse potest] potest esse P_7 **730** siquidem] quidem P_7 et – quid] *corr. ex* †...† (*perhaps other hand*) *K* quid] *corr. ex* quidem P_7 **731** premissis innotuit] predictis non est ignotum *M* **732** coniunctione] oppositione *N* **733** Lune latitudo] latitudo Lune *N* **734** tempus[1]] *om.* P_7 tempus est] *iter. et del. K* eclipsis medie] medie eclipsis *M* **735** octava] xiii[a] *P* septima *N* (octava *Ba*E_1) **736** medietatis] *om. N* duorum] duarum *N* **737** habitis] *corr. ex* habitatis P_7 **743** circulum] circulum hunc *M* **745** semper] *corr. ex* super P_7 **748** maxime – eclipsi] sunt maxime in presenti eclipsi eius *M* **749** HPK] HBK P_7 HBK *corr. in* HAK *M corr. in* HAK *N* (HPK *Ba* BPR E_1) **750** PA] *corr. ex* AK *N* super] *corr. ex* per *N* **752** duorum] *corr. ex* duarum P_7 duarum *N* manifestum] manifactum *K* be greater than the posited quantity, i.e. 1° 27′. And this is what we wanted.

13. To clearly declare the digits of a lunar eclipse for whatever latitude of the moon from the ecliptic and for whatever distance of the moon's center from the earth's center. Whence it will be clear when the eclipse will be partial and when total, and when the moon will have a delay under the shadow and when not, and when there will be the greatest eclipse that can ever be.

Accordingly, from what has been set out, it has become known when an eclipse of the moon is possible or ought to occur and because of what. What is proposed now, however, pertains to the quantity of the eclipse. Accordingly, with a true conjunction[55] of the sun and moon having an eclipse, let the corrected motion of latitude be found, and through the motion of latitude, the moon's true latitude at that time of the true conjunction ⟨is found⟩. For that time is the time of the middle of the eclipse. Then, from the eighth[56] of the present ⟨book⟩, let there be sought what the quantity of the half of the two diameters, i.e. of the moon and shadow, is. With these things had, for the sake of clarity, I shall describe the circle of the shadow in the moon's passage upon center A and points B, D, and G upon the shadow's circle, and likewise another circle upon the same center according to the quantity of the half of the two diameters and points H, P, and K upon the circle. It is clear, therefore, that point A is always opposite the sun's place in the ecliptic, and because the moon is in the direction of this point, it is at the middle of the eclipse, and its darkness in the present eclipse is the greatest at this time. Accordingly, let us place line HPK[57] in the place of an arc of the ecliptic, and perpendicular PA upon it in place of the circle passing through the moon's center in the middle of the eclipse and the poles of the ecliptic. Therefore, if the moon's passage is upon point P so that its latitude is PA, i.e. just as the half of the two diameters, then it



[55] Here and later in the sentence, there should be 'opposition' instead of 'conjunction' since the proposition is about lunar eclipses.

[56] This should refer to VI.7, but the mistake appears to be original.

[57] This line should be HAK, but the mistake appears to be original.

Luna contingit circulum umbre exterius et nichil eius obscuratur quia PD est medietas diametri Lune et P centrum Lune et DA medietas diametri umbre.

Sit iterum DZ equalis linee PD, et sit centrum Lune in transitu umbre super punctum Z. Palam quod tunc tota Luna obscurabitur scilicet cum latitudo Lune ZA minor fuerit medietate duorum diametrorum quantitate linee PZ que est sicut diameter Lune. Et nulla erit ei mora sub umbra eo quod Luna contingit circulum umbre intrinsecus cum centrum Lune sit in puncto Z et eius semidiameter sit linea DZ.

Ex hiis itaque patet quod quotiens medietas duorum diametrorum superabit latitudinem Lune minori augmento quam sit diameter totus Lune, qui est PZ, non obscurabitur Luna tota sed in parte tantum quia centrum Lune cadet in transitu inter punctum P et Z. Et quotiens medietas duorum diametrorum superabit latitudinem Lune maiori augmento quam sit linea PZ scilicet diameter Lune, tunc et tota Luna obscurabitur et erit ei mora. Quod si Luna omnino latitudine caruerit, tunc erit maxima eclipsis que esse potest quia centrum Lune in transitu erit super punctum A, maxime si Luna fuerit in longitudine propiore epicicli. Cum itaque digitos eclipsis volueris, deme latitudinem Lune de medietate duorum diametrorum; reliquum est id quod obscurabitur de diametro Lune. Ipsum ergo reliquum multiplica in xii et divide per quantitatem diametri Lune inventam, et exibunt digiti eclipsis et minuta digitorum si ulterius diviseris. Quod si hi digiti plures xii fuerint, Luna moram habebit.

14. Minuta casus et minuta more si Luna moram habuerit diffinire.

Ponemus primum Lune non esse moram sub umbra et lineam umbre in transitu Lune quasi equidistantem linee circuli signorum licet sit arcus circuli declivis, ad sensum enim fere equidistat. Et sit hec linea exempli causa MZT in figura premissa, et quia Luna moram non habet, linea ZM sive linea ZT eius equalis continet minuta casus que querimus. Nam a puncto M incidit in eclipsim usque ad punctum Z, et a puncto Z excidit ab eclipsi usque ad punctum T. Quia autem nota est linea AM que subtenditur angulo recto et nota est

753 contingit] continget *K* obscuratur] obscurabitur *M* **754** et[1] – Lun[2]e] *marg.* (*other hand*) *K* **756** Palam] palam ergo *PN* **757** duorum] duarum *N* **758** erit] *om. P* ei] *om. N* **759** intrinsecus] *corr. in* in transitu *M* Lune] *om. PN* **761** duorum] duarum *N* **762** Lune[1]] Lune in *M* totus] tota *N* qui] quoniam *PK* que *N* **763** Luna tota] tota Luna *N* cadet] cadit *M* **764** duorum] duarum *N* **766** tunc – Luna[1]] et tota Luna tunc *M* ei] ibi P_7 **767** quia] quia cum *M* **768** in[1] – erit] erit in transitu *M* **769** deme] *corr. ex* demere P_7 **770** duorum] duarum *N* reliquum] et reliquum autem *N* est] *om. M* **772** inventam] inventa *M* **773** digiti] *corr. ex* diti *K* xii fuerint] xii fuerint et *K* fuerint 12 *N* habebit] habebit et cetera *N* **774** habuerit] habuit P_7 diffinire] definire *PM corr. ex* deficere P_7 **775** Ponemus] ponamus *M* primum] primo *N* **776** linee – signorum] circulo signorum †primi† *N* **777** declivis] signorum *M* declinationis *N* equidistat] *corr. ex* equidistant *M* **778** linea[1]] *om. PN* **779** eius] ei *M* **781** autem] enim *N* nota[1] – AM] linea est nota AM *P* linea est nota AM *corr. in* linea AM est nota *N* nota[2] est] est nota *PN* is manifest that the moon touches the shadow's circle externally and nothing of it is obscured because PD is half of the moon's diameter and P the moon's center and DA half of the shadow's diameter.

Again, let DZ be equal to line PD, and let the moon's center be upon point Z in the shadow's passage. It is clear that the whole moon will then be obscured, namely when the moon's latitude ZA is less than the half of the two diameters by the quantity of line PZ, which is as the moon's diameter. And there will be no delay under the shadow for it because the moon touches the shadow's circle interiorly because the moon's center is at point Z and its radius is line DZ.

Accordingly, it is clear from these things that whenever the half of the two diameters exceeds the moon's latitude by a smaller increase than the whole diameter of the moon, which is PZ, the whole moon will not be obscured but only partially because the moon's center in the passage will fall between point P and Z. And whenever the half of the two diameters exceeds the moon's latitude by an increase greater than line PZ, i.e. the moon's diameter, then the whole moon will be obscured and there will be a delay for it. But if the moon is entirely devoid of latitude, then there will be the greatest eclipse that can be because the moon's center in the passage will be upon point A, especially if the moon is at the epicycle's perigee. Accordingly, when you want an eclipse's digits, subtract the moon's latitude from the half of the two diameters; the remainder is that which will be obscured of the moon's diameter. Therefore, multiply that remainder by 12 and divide by the found quantity of the moon's diameter, and there result the digits of the eclipse and the minutes of digits if you divide further. And if these digits are more than 12, the moon will have a delay.

14. To specify the minutes of immersion and, if the moon has a delay, the minutes of delay.

We will suppose first that there is not a delay of the moon under the shadow and that the line of the shadow in the moon's passage [i.e. the moon's path during the eclipse] is as if parallel to the line of the ecliptic although it is an arc of a declined circle,[58] for to the senses it is almost parallel. And for example let this line be MZT in the preceding figure, and because the moon does not have a delay, line ZM or its equal, line ZT, contain the minutes of immersion that we seek. For it falls into the eclipse from point M to point Z, and it falls out of the eclipse from point Z to point T. Moreover, because line AM, which subtends a right angle, is known and the latitude AZ at the middle

[58] It is unsure whether our author intends this to mean 'the declined circle' or 'a declined circle.' In this first, simplified method of finding the minutes of immersion and of delay, latitude from the ecliptic is treated as if constant during the eclipse, so either reading makes sense; however, as will be seen in the more certain method given later in this proposition, the passage of the moon through the shadow is not identical with the moon's declined circle.

secundum eandem quantitatem minutorum AZ latitudo ad medium eclipsis, erit ZM nota, scilicet cum a quadrato linee AM dempseris quadratum linee ZA, et reliqui radicem sumpseris. Utimur enim hiis lineis tanquam rectis propter insensibilem fallaciam.

Deinde ponemus Lune moram et transitum eius per lineam EXI, et hoc exempli gratia. Erit ergo AU latitudo ad medium eclipsis minor medietate duorum diametrorum quantitate superante diametrum Lune. Ponamus itaque principium more in puncto F et educamus rectam AFN. Erit ergo FN ut semidiameter Lune quia cum a puncto I pervenerit ad F, tunc primum tota latebit sub umbra. Est itaque AF nota quia est minor medietate duorum diametrorum quantitate diametri Lune. Et similiter eius equalis AX nota que dirigitur per finem more. Et quia AF sive AX subtenditur angulo recto et AU perpendicularis est nota, erit tota FX nota. Et ipsa continet minuta totius more, et FU minuta dimidii more. Relinquitur ut FI sive EX contineat minuta casus, et utralibet nota est quia tota IU est nota modo quo prius. Dempta ergo FU erit IF nota. Quotiens ergo minuta more et casus simul volueris, de medietate duorum diametrorum in se ducta latitudinem Lune in se ductam minue, et reliqui radicem accipe. Nam proveniunt minuta casus et dimidii more simul. Et si dimidium more per se volueris, de medietate duorum diametrorum diametrum Lune deme, et ex reliquo in se ducto Lune latitudinem in se ductam abice, et residui radicem accipe. Nam ipsa est minuta more dimidie. Que subtrahe a minutis casus et dimidii more, et erunt minuta casus per se.

Secundum hanc doctrinam duplices tabule composite sunt de eclipsibus lunaribus, una quidem cum Luna fuerit in longitudine longiore tantum et alia ad longitudinem propiorem tantum. Et intratur in illas tabulas vel per motum latitudinis equatum aut alias per longitudinem Lune a nodo vel per Lune latitudinem. Sed omnium eadem est ratio, que ex antedictis colligi potest. Quo-

782 eandem] *om. PN* minutorum – latitudo] AZ que est latitudo Lune *N* **784** hiis lineis] lineis his *K* **786** EXI] EXL *M* **787** AU] AY *corr. ex* AN *M* AS *N* latitudo] latitudo Lune *M* **787/788** medietate – diametrorum] quantitate duarum semidyametrorum *N* **789** AFN] AFM *P* AFU *corr. ex* AUF *N* FN] FU *N* semidiameter] *corr. ex* diameter *P* **790** I] L *M* tunc – latebit] *corr. ex* †...† (*perhaps other hand*) *P* **791** medietate duorum] quantitate duarum *N* **792** eius] ei *M* **793** AU] AY *P* AY *corr. ex* AF *M* AS *N* (AN BaE_1) **794** tota] nota *PKN* nota *del.* P_7 (*om. Ba* tota E_1) FX] SX *P* nota[2]] *om. N* FU] FY *PM perhaps corr. ex* FY *K* FS *N* **795** dimidii] dimidie *MN* FI] FY *P corr. ex* FY P_7 FL *M* **795/796** sive – FU] *marg.* (*perhaps other hand*) *P* **796** tota IU] nota XY *P corr. ex* UY P_7 tota LY *M* SI nota *N* nota[2]] *om. N* FU] FY *PKM corr. ex* FY P_7 FS *N* **797** IF] YF *P* LF *M* simul] similiter *P corr. ex* similiter *N* **797/800** de – volueris] *marg.* P_7 **797** duorum] *om. N* **799** dimidii – simul] more dimidii insimul P_7 dimidii] dimidie *N* **800** duorum] duarum *N* **803** dimidii] dimidie *MN* more] more simul *N* **804/805** composite – lunaribus] de eclipsibus lunaribus sunt composite *N* **806** longitudinem] *om. N* of the eclipse is known according to the same quantity of minutes, ZM will be known, namely when you subtract the square of line ZA from the square of line AM, and you take the root of the remainder. For we use these lines as if straight because of the imperceptible falsity.

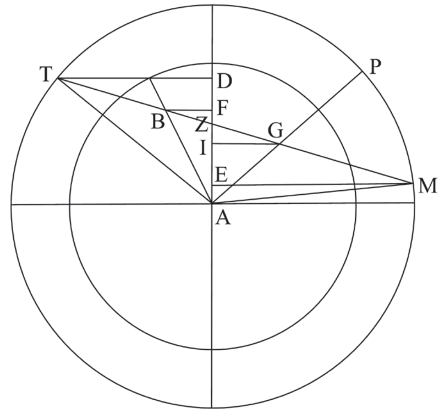
Then we will suppose a delay of the moon and its transit through line EXI, and this is for the sake of an example. Therefore, latitude AU at the middle of the eclipse will be less than the half of the two diameters by a quantity exceeding the moon's diameter. Accordingly, let us suppose the beginning of the delay at point F[59] and let us extend straight line AFN. Therefore, FN will be as the moon's radius because when from point I it comes to F, then the whole will first be hidden under the shadow. Accordingly, AF is known because it is less than half of the two diameters by the quantity of the moon's diameter. And similarly its equal AX is known, which is directed through the end of the delay. And because AF or AX subtends a right angle and perpendicular AU is known, whole FX will be known. And it contains the minutes of the whole delay, and FU the minutes of half of the delay. It remains that FI or EX contains the minutes of immersion, and each is known because the whole IU is known in the way which ⟨we used⟩ earlier. Therefore, with FU subtracted, IF will be known. Therefore, whenever you want the minutes of delay and of immersion together, subtract the moon's latitude multiplied by itself from the half of the two diameters multiplied by itself, and take the root of the remainder. For there results the minutes of immersion and half of the delay together. And if you want half of the delay by itself, subtract the moon's diameter from the half of the two diameters, subtract the moon's latitude multiplied by itself from the remainder multiplied by itself, and take the root of the remainder. For it is the minutes of half the delay. Subtract these from the minutes of immersion and half of the delay, and there will be the minutes of immersion by themselves.

Double tables concerning lunar eclipses have been made according to this teaching, one indeed when the moon is only at apogee and another only at perigee. And these tables are entered either through the corrected motion of latitude, or elsewhere through the distance of the moon from the node or through the moon's latitude.[60] But of all, there is the same reasoning, which can be obtained from what has been said before. And indeed, whenever the moon

[59] While earlier the author spoke of the moon passing through the shadow from M to T, i.e. from right not left, now he speaks of the moon moving from left to right.

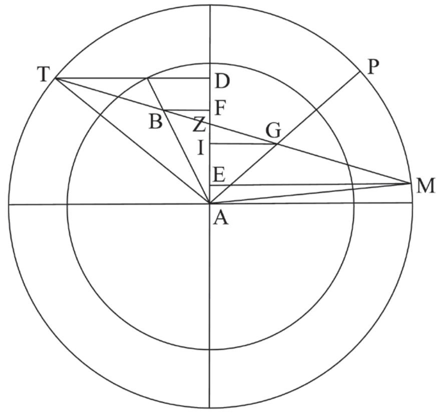
[60] The *Almagest* VI.8 has tables entered with the motion of latitude (1515 ed., f. 69r); al-Battānī has tables entered with the moon's latitude from the ecliptic (Nallino, *al-Battānī*, vol. II, p. 90); and while the Toledan Tables include tables entered by the motion of latitude, they also have tables taken from al-Zarqālī's *Almanac* that could be entered with either the distance from the node or the moon's latitude (Pedersen, *The Toledan Tables*, Tables JD21 and JE21, pp. 1463–71 and 1475–78).

tiens vero Luna inter utramque longitudinem ceciderit, intratur in utrasque, et minuuntur minora a maioribus, et de superfluo sumitur secundum proportionem supradictam scilicet minutorum affinitatis in quorum tabulam intratur per Lune portionem. Et quod provenerit minoribus superponitur. Ratio operandi ex dictis patet.



Verum quia arcus circuli declivis obliquus est ad circulum signorum, et ob hoc minuta casus et more ante eclipsim diversa sunt a minutis casus et more post eclipsim, si hec diffinitius scire queris, resumemus similem priori figuram, et in ea lineam transitus Lune MZT quolibet modo obliquatam. Et erit MZ linea per quam transit a principio eclipsis ad medium, et ZT per quam transit a medio ad finem eclipsis, et AZ latitudo Lune ad medium eclipsis que per motum latitudinis equatum est nota, et EA latitudo Lune ad principium eclipsis, et AD latitudo ad finem eclipsis. Itaque minuta casus et dimidii more supra inventa vel minuta casus tantum secundum quod evenerit accipe. Et eis propter motum Solis interim duodecimam partem eorum superpone, et quod provenerit motui latitudinis equato ad medium eclipsis primum subtrahe deinde superpone, et habebis motum latitudinis indefinitum et ad principium eclipsis et ad finem eclipsis. Et per utrumque latitudinem Lune disce. Et sic utraque latitudo AE AD scilicet ad finem et ad principium eclipsis erit nota. Et quoniam AM que est medietas duorum diametrorum est nota et ipsa subtenditur angulo recto, erit propter hoc EM nota. Et quia AZ nota est, subtracta EA fiet EZ nota, que cum EM continet angulum rectum. Quare ZM est nota, et ipsa continet minuta casus et more a principio eclipsis ad medium eclipsis vel minuta casus solum secundum quod evenerit. Rursum cum AD sit nota, que cum DT conti-

809 Luna] *corr. ex* Lune P_7 inter] *corr. ex* in *M* utramque] utram P_7 intratur] intra *M* in] *om. N* **811** tabulam] tabulas P_7 **812** provenerit] proveniet *N* superponitur] supponitur PP_7 **819** hec] ergo hoc *M* hoc *N* **820** similem priori] superiori similem *M* **821** lineam] linea *M* **824** eclipsis] eclipsis usque *M* ad] *corr. ex* per P_7 **826** eclipsis] *corr. ex* eclipsim *K* **826/829** et – eclipsis] *om. N* **826** AZ] *corr. ex* ZI *K corr. ex* DZ *M* **827** eclipsis] AZ *add. et del. M* **829** latitudo] latitudo Lune *M* dimidii] dimidie *MN* **830** eis] ei *M* **831** provenerit] proveniet *N* **832** medium] *corr. ex* dimidium *P* **833** indefinitum] indefiniter *P* **834** per] *s.l. M* latitudinem Lune] Lune latitudinem *M* latitudo] latitudo Lune *M* **835** ad[3]] *om.* P_7 que] qui *P* **836** duorum] duarum *N* **837** EZ] *s.l.* P_7 **838** nota] *s.l.* P_7 **839** et] dimidie *add. et del. N* eclipsis[2]] *om. MN* **840** evenerit] eveniet *N* falls between each apsis, both ⟨tables⟩ are entered, and the smaller ⟨values⟩ are subtracted from the greater, and ⟨a part⟩ is taken from the difference according to the aforesaid ratio, i.e. of the minutes of affinity, the table of which is entered through the moon's portion. And what results is added to the smaller ⟨values⟩. The rule of operation is clear from what has been said.



However, because an arc of the declined circle is oblique to the ecliptic, and on account of this the minutes of immersion and of delay before the eclipse are different from the minutes of immersion and of delay after the eclipse, if you seek to know these more precisely, we will take up again a figure similar to the previous one and the line MZT of the moon's passage tilted in any way in it.[61] And MZ will be the line through which it passes from the beginning of the eclipse to the middle;[62] ZT through which it passes from the middle to the end of the eclipse; AZ the moon's latitude at the middle of the eclipse, which is known through the corrected motion of latitude; EA the moon's latitude at the beginning of the eclipse; and AD the latitude at the end of the eclipse. Accordingly, take the minutes of immersion and of half of the delay found above or the minutes of immersion only, according to what comes about. And add to them their twelfth because of the sun's motion in the meantime, and first subtract and then add what results from and to the corrected motion of latitude at the middle of the eclipse, and you will have the imprecise motion of latitude both for the beginning of the eclipse and for the end of the eclipse. And through each learn the moon's latitude. And thus each latitude AE and AD, i.e. at the end and at the beginning of the eclipse, will be known. And because AM, which is the half of the two diameters, is known and subtends a right angle, EM will be known because of this. And because AZ is known, with EA subtracted, EZ, which with EM contains a right angle, will be known. Therefore, ZM is known, and it contains the minutes of immersion and delay from the beginning of the eclipse to the middle of the eclipse or the minutes of immersion alone, according to what comes about. In turn, because AD is known,

[61] Note that this line is not the moon's declined circle. The shadow is moving while the moon passes through it, so the moon's path across the shadow will be more tilted than the moon's declined circle.

[62] The author now has the moon moving from right to left through the shadow.

net angulum rectum cui subtenditur AT nota, erit DT nota. Et quia cum AZ subtracta fuerit ab AD, relinquitur ZD nota, erit propter hoc ZT nota, et ipsa continet minuta casus et more vel minuta casus solum secundum quod evenerit a medio eclipsis ad finem. Et manifestum quod linea TZ minor est quam linea ZM in hoc situ.

Quod si minuta more per se volueris diffinitius, pari modo operaberis, scilicet minutis more supra inventis duodecimam partem eorum adicies, et motui latitudinis superpones et subtrahes. Cum utroque latitudinem ad principium et ad finem more addisces scilicet AI et AF. Et sit in puncto G principium more et in puncto B finis. Et quia AG nota est quia est augmentum medietatis duorum diametrorum super diametrum Lune GP, erit IG nota, et propter hoc GZ nota que continet minuta more ante medium eclipsis. Pari modo fiet BF nota, et propter BF BZ nota que continet minuta more post medium eclipsis. Et hoc est quod volebamus.

15. Quinque vel tria tempora lunaris eclipsis cum evenerint et loca Lune ad hec tempora determinare.

Cum Luna moram habuerit, quinque sunt tempora lunaris eclipsis; cum moram non habuerit, tria tantum. Quorum semper unum est medium eclipsis, et ipsum iam notum est quia est tempus vere oppositionis. Quod si principium eclipsis velis indefinite, sume minuta casus et dimidii more vel casus tantum secundum quod evenerit, et divide per superlationem Lune ad unam horam. Et horas cum minutis que provenerint deme ab horis medie eclipsis, et habebis tempus principii eclipsis. Easdem horas cum suis minutis adde super horas medie eclipsis, et habebis tempus finis eclipsis. Similiter minuta more dimidie divide per superlationem Lune, et quod exierit deme ab horis medie eclipsis vel adde, et habebis horas ad initium more vel ad finem. Quod si diffinitius hec scire volueris tempora, operaberis cum diffinitis minutis casus et more ante eclipsim mediam et post eclipsim mediam. Et habebis diffinite horas quesitas, quas si volueris, in horas temporales vertes.

841 AT] sit *add. et del. K* **842** ab] ad *P* ZD] ZT *P corr. ex* AZB *P_7 corr. ex* ZT *N* **843** vel] et *P* hec *P_7 corr. ex* hec *K* aut *N* (hec *BaE_1*) **844** Et] *om. PN* manifestum] manifactum *K* **846** diffinitius] definitius *P_7* operaberis] preparabis *P_7* **848** Cum] et (*s.l.*) cum *P_7 corr. in* et *N* **849** ad] *om. N* AI] AL *MN* et AF] et F *P corr. ex* et F AF *P_7* ZF *N* in] de *N* **850** medietatis] *s.l. P_7* duorum] duarum *N* **851** GP – IG] GB erit LG *M* **853** propter] propter hoc *M* **853/854** post – volebamus] *corr. ex* †...† (*perhaps other hand*) *P* **853** medium] medium ipsius *N* **855** Quinque] *corr. ex* cumque (*perhaps other hand*) *P* **858** unum – medium] est medium unum *P* eclipsis] eclipsis scilicet *M* **859** est[2]] *om. N* **860** indefinite] indefinire *P* indiffinite *M* invenire *N* dimidii] dimidie *MN* vel] vel minuta *N* **862** cum – provenerint] et minuta que provenient *N* **863** super] *iter. et del. P* **864** eclipsis[1]] *corr. ex* oppositionis *N* more dimidie] dimidie more *MN* **865** Lune] *om. N* exierit] exibit *N* **866** ad[2]] *om. MN* diffinitius] definitius *P_7K* **867** diffinitis] definitis *P_7K* distinctis *M* **868** diffinite] diffinire *P* definite *P_7 corr. ex* diffinita *K* **869** vertes] verte *P* verte *corr. ex* verte†re† *N* which with DT contains the right angle that known AT subtends, DT will be known. And because when AZ is subtracted from AD, ZD remains known, ZT will be known because of this, and it contains the minutes of immersion and delay or[63] the minutes of immersion alone, according to what comes about, from the middle of the eclipse to the end. And it is manifest that line TZ is less than line ZM in this situation.

And if you want the minutes of delay by themselves more precisely, you will operate in a like way, namely you will add to the minutes of delay found above their twelfth, and you will add to and subtract from the motion of latitude ⟨at the middle of the eclipse⟩. With each you will learn the latitude at the beginning and at the end of the delay, i.e. at AI and AF. And let the beginning of the delay be at point G and the end at point B. And because AG is known because it is the excess of the half of the two diameters over the moon's diameter GP, IG will be known, and GZ, which contains the minutes of delay before the middle of the eclipse, will be known because of this. In a like way, BF will be known, and because of BF, BZ, which contains the minutes of delay after the middle of the eclipse, will be known. And this is what we wanted.

15. To determine when the five or three times of a lunar eclipse will occur and the places of the moon at these times.

When the moon has a delay, there are five times of the lunar eclipse; when it does not have a delay, only three. One of which is always the middle of the eclipse, and that is already known because it is the time of true opposition. And if you want the beginning of the eclipse imprecisely, take the minutes of immersion and of half of the delay or of immersion only, according to what comes about, and divide ⟨them⟩ by the moon's carrying beyond in one hour. And subtract the hours with minutes that result from the hours of the middle of the eclipse, and you will have the time of the beginning of the eclipse. Add the same hours with their minutes to the time of the middle of the eclipse, and you will have the time of the end of the eclipse. Similarly, divide the minutes of half of the delay by the moon's carrying beyond, and subtract or add what results from ⟨or to⟩ the time of the middle of the eclipse, and you will have the times at the beginning or at the end of the delay. But if you want to know these times more precisely, you will operate with the precise minutes of immersion and delay before the middle of the eclipse and after the middle of the eclipse. And you will have the sought precise times, which you will convert into temporal times if you want.

[63] Although this is what the meaning of the passage calls for, the mistaken 'hec' must have entered the text's transmission early and is perhaps the author's own mistake.

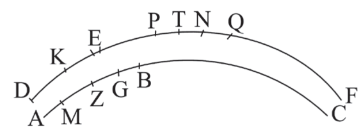
Quod si etiam loca Lune in hiis temporibus volueris, tempus intermedium medio eclipsis et principio eclipsis sive medio eclipsis et initio more vel cuicumque volueris multiplica per locum diversum Lune ad unam horam. Et quod provenerit subtrahes vel superpones loco Lune invento ad medium eclipsis.

16. Visum motum Lune ad assignatam horam accipere.

Visum locum Lune ad principium assignate hore sicut in penultima quinti dicitur addisce, et similiter ad finem ipsius hore. Deinde minue minorem locum de maiore. Nam quod relinquitur est visus motus Lune ad horam assignatam. Aliter sume diversitatem aspectus in longitudine ad principium date hore et similiter sume ad finem date hore. Et differentiam inter diversitates accipe. Et si diversitas ad principium hore maior fuerit, deme differentiam de diverso motu Lune ad ipsam horam; et si minor, adde. Nam quod provenerit post diminutionem vel additionem est visus motus Lune ad eam horam.

17. Visam Lune coniunctionem cum Sole ex vera comprehendere.

Ponam propter hoc declarandum lineam temporis ABC, et sit B meridies, A ortus, C occasus. Et ponam lineam transitus Lune DEF, et tempus vere coniunctionis primum propinquius ortui aput G, et locum vere coniunctionis E. Palam est autem quod si fuerit diversitas aspectus in latitudine tantum et nulla in longitudine, quod contingit cum Luna distiterit ab orizonte xc gradibus circuli signorum, tunc quidem vera coniunctio est ipsa visa coniunctio. Si vero Luna fuerit propinquior ortui, visa coniunctio precedit veram, et si fuerit propinquior occasui, visa coniunctio erit post veram. Si ergo motus proprius Lune ab E versus D secundum successionem scilicet signorum, sumo itaque a puncto temporis G quod sit principium hore tertie et a loco E diversitatem aspectus in longitudine, et manifestum quod dirigitur versus D. Sit ergo hec diversitas EK. Divido eam per diversum motum Lune ad horam, atque tempus quod exierit sit equale GZ. Et minuo diversitatem EK ab EF ut sit ei equalis ET. Palam ergo quod Luna in puncto temporis Z nota fuerit in loco T noto. Sumo iterum a puncto Z temporis et a loco T diversitatem



871 eclipsis[1]] eclipsi *M* cuicumque] *corr. ex* cuique *K* **872** locum] motum *MN* (locum *Ba* motum E_1) **873** provenerit – superpones] proveniet subtrahas vel superponas *N* provenerit] *corr. ex* proveneris *K* **876** ipsius] illius *N* **877** Nam] namque *M* ad] unam *add. et del. N* **879** sume] *om. N* **881/882** provenerit – additionem] proveniet post additionem vel diminutionem *N* **883** comprehendere] deprehendere *M* **886** aput] aput punctum *M* **887** est autem] autem est *PN* **888** nulla – longitudine] in longitudine nulla *M* distiterit] destiterit *P* **893** ergo] *corr. ex* †vero† *K* ab E] AB (*corr. in* ab E *N*) est *PN* **894** secundum – scilicet] scilicet secundum successionem *M* scilicet] *om. N* **895** a – temporis] in puncto ipsius P_7 **896** manifestum] manifestum est *N* **898** quod] *s.l. M* **899** ET] et *P* nota] noto P_7 *corr. in* noto *N* **900** a[1]] in P_7 loco[2] T] *corr. ex* puncto T *N*

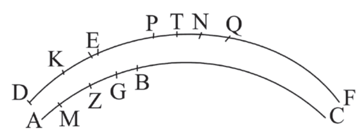
And if you also want the moon's places at these times, multiply the intermediate time between the middle of the eclipse and the beginning of the eclipse or the middle of the eclipse and the beginning of the delay or whatever ⟨time⟩ you want by the moon's irregular place[64] for one hour. And you will subtract or add what results to ⟨or from⟩ the found place of the moon at the middle of the eclipse.

16. To take the moon's apparent motion for an assigned hour.

Learn the moon's apparent place at the beginning of the assigned hour, as is declared in the penultimate of the fifth [i.e. V.27], and similarly at the end of that hour. Then subtract the smaller place from the greater. For what remains is the moon's apparent motion for the assigned hour. In another way, take the parallax in longitude at the beginning of the given hour, and similarly take it at the end of the given hour. And take the difference between the parallaxes. And if the parallax at the beginning of the hour is greater, subtract the difference from the moon's irregular motion for that hour; and if smaller, add ⟨it⟩. For what results after the subtraction or addition is the moon's apparent motion for that hour.

17. To grasp the moon's apparent conjunction with the sun from the true ⟨conjunction⟩.

For declaring this, I shall posit the line of time ABC, and let B be noon, A rising, and C setting. And I shall place the line of the moon's transit DEF, and the time of the true conjunction first closer to the rising at G, and the place of the true conjunction E. Moreover, it is clear that if there is parallax only in latitude and none in longitude, which occurs when the moon stands 90° of the ecliptic away from the horizon, then indeed the true conjunction is that apparent conjunction. But if the moon is nearer the rising, the apparent conjunction precedes the true, and if it is nearer the setting, the apparent conjunction will be after the true. Therefore, if the moon's proper motion is from E towards D, i.e. according to the succession of signs, I take accordingly the parallax in longitude at the point of time G, which is the beginning of the third hour, and at place E, and it is manifest that it is directed towards D. Therefore, let this parallax be EK. I divide it by the moon's irregular motion for the hour, and let the time that results be equal to GZ. And I subtract the parallax EK from EF so that ET may be equal to it [i.e. EK]. It is clear, therefore, that the moon at the point of time Z will be known at known place T. At point of time Z and at place T, I take again the



[64] This should be 'motum', but the false reading 'locum' appears to be the author's own mistake.

aspectus in longitudine que ultra E fortassis extenditur. Minuo eam iterum a loco E ut sit equalis EN. Et divido ut inveniam tempus per superlationem veram Lune ad horam, et exeat tempus equale ei quod sit GM. Et sumo ad tempus GM diversum motum Lune, et minuo a loco E, et proveniat EQ. Erit ergo EN superlatio Lune in ipso tempore, et NQ diversus motus Solis cui sit equalis EP. Itaque a puncto Q et a puncto temporis M sumo tertia vice diversitatem aspectus in longitudine.

Que si equalis fuerit secunde diversitati scilicet EN, habemus quod querimus. Dico enim quod in puncto temporis M noto et in loco Q noto est visa coniunctio. Palam enim est quod in puncto temporis M Luna sit in puncto Q et Sol in puncto P eo quod EQ sit diversus motus Lune in tempore GM et EP diversus motus Solis in eodem tempore. Atque diversitas aspectus a loco Q extenditur usque ad locum P cum sit equalis EN. Ergo in loco P et in tempore M est visa coniunctio.

Quod si tertia diversitas maior secunda fuerit, tunc diversitas aspectus in puncto temporis M superat simili augmento quantitatem quam tunc temporis inter Solem et Lunam esse contingit, et erit augmentum illud visa superlatio Lune ad tempus ignotum. Quod si comprehensum fuerit et superpositum tempori GM, habebis tempus in cuius principio sumpta diversitas aspectus in longitudine equatur quantitati que tunc temporis erit inter Solem et Lunam. Et hoc proxime vero.

Comprehendetur autem illud tempus sic. Sume visum motum Lune ad horam qui dum tertia diversitas maior est secunda, necessario minor est diverso motu ad horam quia diversitas aspectus in longitudine decrescit secundum successionem signorum. Et per hunc visum motum disce visam superlationem Lune ad unam horam, per quam divides augmentum diversitatis tertie super primam. Et exibit tempus quesitum quia sicut superlatio hore ad illam superla-

901 E] est *P corr. ex* est *K corr. ex* Z *M*   fortassis] fortasse *N*   **902** E] *corr. ex* et *M*   sit] sit ei *N*   **903** veram Lune] Lune veram *PN*   exeat] exit *M*   **904** diversum] divertum *P*   proveniat] perveniat *P* proveniet *M*   **905** diversus] diversitas *N*   **906** Itaque] *perhaps corr. ex* †...† *P*   **908** Que si] quasi *P*   si] si *s.l. K*   **909** visa] est *add. et del. K*   **910** enim est] ergo *PN* est enim *M*   **911** EQ] *corr. ex* eque P_7 diversus] *corr. ex* diversitas *P*   Lune in] Solis in eodem *P*   GM] *s.l.* (*perhaps other hand*) *P*   **911/912** et^2 – tempore] *marg.* (*perhaps other hand*) *P*   **913** extenditur] *corr. ex* extendatur P_7 sit equalis] equalis sit *PN*   **915** diversitas^1] diversitas aspectus *MN*   secunda fuerit] secunda (*corr. ex* tertia) fuerit *M* fuerit secunda *N*   **917** contingit] contigerit *PN* (contingerit *Ba* contingit E_1)   **918** ad] at P_7K **919** habebis] habebitur *M*   tempus] *marg.* (*perhaps other hand*) *P*   cuius] quo *M*   **922** Comprehendetur] comprehendatur *P* comprehenditur *N*   autem] *corr. ex* ante *M*   **923** qui dum] qua *M*   **924** aspectus] *om. N*   **926** divides] dividas *N*   diversitatis tertie] tertie diversitatis *M*   **927** primam] idest secundam *add. s.l.* P_7 *corr. in* secundam *N* (positam *Ba* primam E_1)   quia] nam *N* parallax in longitude, which is perhaps extended beyond E. I subtract it again from place E so that EN may be equal. And I divide by the moon's true carrying beyond for the hour so that I may find the time, and the time equal to it, which let be GM, may result. And I take the moon's irregular motion for the time GM, and I subtract ⟨it⟩ from place E, and let EQ result. Therefore, EN will be the moon's carrying beyond in that time, and NQ the sun's irregular motion, to which let EP be equal. Accordingly, at point Q and at the point of time M, I take for a third time the parallax in longitude.

If this [i.e. the third parallax] is equal to the second parallax, i.e. EN, we have what we seek. For I say that the apparent conjunction is at the known point of time M and at known place Q.[65] It is clear, indeed, that at the point of time M, the moon is at point Q and the sun at point P because EQ is the moon's irregular motion in the time GM and EP is the sun's irregular motion in the same time. And, the parallax is extended from place Q to place P because it is equal to EN. Therefore, the apparent conjunction is at place P and at time M.

But, if the third parallax is greater than the second, then the parallax at point of time M exceeds the quantity that there happens to be at that moment between the sun and moon by a similar increase, and that increase will be the moon's apparent carrying beyond for an unknown time. But if it is grasped and added to time GM, you will have the time at the beginning of which the taken parallax in longitude is made equal to the quantity that will be between the sun and moon at that moment. And this is near the truth.

Moreover, that time will be grasped thus. Take the moon's apparent motion for an hour, which while the third parallax is greater than the second, is necessarily less than the irregular motion for the hour because the parallax in longitude decreases according to the succession of signs. And through this apparent motion, learn the moon's apparent carrying beyond for one hour, by which you will divide the increase of the third parallax over the first.[66] And the sought time will result because as the carrying beyond of an hour is to that carrying

[65] This should read 'P.' At the time of the apparent conjunction, the moon is at Q, but it appears to be with the sun at P.

[66] This should say 'the second.'

tionem sic affiniter se habet hora ad tempus quesitum. Quod inventum superpones tempori GM, et collecti principium est tempus vise coniunctionis.

Quod si tertia diversitas minor secunda fuerit, tunc diversitas aspectus in puncto temporis M superatur a quantitate que tunc temporis inter Solem et Lunam esse contingit simili augmento. Atque ideo visa coniunctio erit post punctum M tanto tempore fere quantum attinet ad illud augmentum, quod est visa superlatio ad ipsum tempus ignotum. Disce ergo visam superlationem Lune ad horam, que necessario maior est vera superlatione quia diversitas aspectus crescit. Ac per hoc disce ut prius tempus quesitum, quod inventum minues a tempore GM. Et residui principium erit tempus vise coniunctionis. Sume etiam post additionem vel diminutionem ad tempus inventum diversum motum Solis et diversum motum Lune, et minue a loco E, et occurrent loca Solis et Lune in tempore vise coniunctionis.

Ponemus iterum veram coniunctionem propinquiorem occasui, et propter hoc visa coniunctio subsequitur tempus vere coniunctionis. Sit ergo A occasus et C ortus et motus proprius Lune ab E in F et Solis similiter. Sitque in puncto temporis G ubi est vera coniunctio diversitas aspectus in longitudine sicut EK. Nam dirigitur hic in contrarium successionis signorum. Et divido EK ut prius per diversum motum Lune ad unam horam, et exeat tempus cui sit equale GZ. Et addo diversitatem aspectus EK super locum E ut sit ET quia visa coniunctio subsequitur veram. Erit itaque Luna in puncto temporis Z in loco T. Sumo itaque a puncto Z et a loco T diversitatem aspectus in longitudine. Et addo iterum super locum E ut sit equalis EN, et divido eam per superlationem veram Lune ad horam. Et exeat tempus quod sit equale GM, et sumo ad tempus GM diversum motum Lune. Et addo super locum E, et proveniat EQ. A puncto itaque Q et puncto temporis M sumo tertia vice diversitatem aspectus in longitudine. Que si equalis fuerit secunde diversitati, palam ut prius quod in puncto temporis M et in loco Q erit visa coniunctio. Nam similis erit demonstratio superiori. Quod si tertia diversitas maior vel minor fuerit secunda, eodem modo ut supra per omnia est operandum ut habeas et locum et tempus vise coniunctionis.

928 affiniter] affinite *M* superpones] superponas *N* **930** secunda fuerit] fuerit secunda *N* **931** que] *corr. ex* quam *M* quam *N* **932** ideo] ita *add. s.l.* (*perhaps other hand*) *P* **934** ad] *corr. in* at P_7 **936** Ac] at *MN* minues] minuas *N* **939** E – loca] et occurret locus *N* et[4]] et locus *N* **940** tempore – coniunctionis] vise coniunctionis tempore *PN* **942** vere] *corr. ex* vise P_7 **943** et[1]] *om. M* proprius] propius *M* Solis – Sitque] Sol similiter sit *M* **946** exeat] exiet *M* **947** E] Lune E *M* Lune *N* **948** Z] *om. M* **949** loco[1]] puncto *N* **950** locum] Lune *add. et del. M* eam] *om. M* per] *om. P* **951** exeat] exiet *M* **952** proveniat] proveniet *MN* **953** vice] die *P* **956** demonstratio] *perhaps other hand K* **957** et] *om. PN* beyond, thus approximately is an hour disposed to the sought time. You will add that found ⟨time⟩ to time GM, and the beginning of the sum is the time of the apparent conjunction.

But if the third parallax is less than the second, then the parallax at point of time M is exceeded by the quantity that there happens to be at that moment between the sun and moon by a similar increase. And, for that reason the apparent conjunction will be after point M by about as much time as pertains to that increase, which is the apparent carrying beyond for that unknown time. Therefore, learn the apparent carrying beyond of the moon for the hour, which is necessarily greater than the true carrying beyond, because the parallax increases. And as before also learn through this the sought time, which when found, you will subtract from time GM. And the beginning of the remainder will be the time of the apparent conjunction. Also, after the addition or subtraction, take the sun's irregular motion and the moon's irregular motion for the found time, and subtract from place E, and the places of the sun and moon at the time of the apparent conjunction will present themselves.

Again, we will suppose the true conjunction nearer the setting, and because of this the apparent conjunction follows the time of the true conjunction. Therefore, let A be the setting, C the rising, and the moon's proper motion from E to F and similarly of the sun. And at the point of time G when there is the true conjunction, let the parallax in longitude be as EK. For it is directed here against the succession of signs. And I divide EK as before by the moon's irregular motion for one hour, and let there result a time to which let GZ be equal. And I add the parallax EK to place E so that there may be ET because the apparent conjunction follows the true. Accordingly, at the point of time Z, the moon will be in place T. Accordingly, I take the parallax in longitude at point Z and at place T. And I add ⟨it⟩ again upon place E so that there may be equal EN, and I divide it by the moon's true carrying beyond for an hour. And let there result a time which let be equal to GM, and I take the moon's irregular motion for the time GM. And I add it to place E, and let there result EQ. Accordingly, at point Q and point of time M, I take the parallax in longitude for a third time. If this is equal to the second parallax, it is clear as before that the apparent conjunction will be at point of time M and at place Q.[67] For the proof is similar to the one above. But if the third parallax is greater or smaller than the second, everything must be done in the same way as above so that you may have both the place and time of the apparent conjunction.

[67] Again, while the moon is at Q at the time of the apparent conjunction, the moon appears then along with the sun at point P, which could be found as before.

Et nota quod in omnibus hiis diversitatibus aspectuum querendis, diversitas aspectus Solis in circulo altitudinis subtrahendus est a diversitate aspectus Lune in circulo altitudinis. Item quia portio necessaria est inquerendis diversitatibus, ad habendam portionem quicquid loco Lune in vera coniunctione additur vel demitur loco portionis in vera coniunctione etiam similiter addendum vel demendum quia in tam brevi tempore non sunt sensibiliter dissimiles motus Lune in epiciclo et motus longitudinis. Item quia motus latitudinis etiam necessarius est ad querendam latitudinem Lune tempore vise coniunctionis, quod extremum additur vel demitur loco Lune ad habendum visum eius locum tempore vise coniunctionis, similiter addendum vel demendum cum motu Capitis in eodem tempore motui latitudinis equato ad veram coniunctionem.

18. Digitos solaris eclipsis ostensive invenire. Unde etiam liquidum erit quando Luna totum Solem teget et quando non totum.

Inveniemus primum visam coniunctionem Solis et Lune, et portionem Lune ad idem tempus, et argumentum Solis, et motum latitudinis sicut predictum est, per motum latitudinis verum visi loci Lune latitudinem, preterea quantitatem semidiametri Lune ad ipsum tempus, et quantitatem semidiametri Solis – et hoc secundum opus Albategni quia secundum opus Ptolomei non variatur. Variatur autem secundum opus Albategni inter longitudinem longiorem et longitudinem propiorem duobus minutis et tertia unius minuti. Quibus prehabitis sumemus etiam diversitatem aspectus Lune in latitudine ad tempus vise coniunctionis, et per hoc inveniemus visam Lune latitudinem.

Iungam igitur medietates duorum diametrorum Solis et Lune, et secundum hanc quantitatem describam circulum ABG super centrum E, et circulum Solis MZN super idem centrum, et vice circuli signorum lineam AEG, et lineam transitus Lune KHT sicut equidistantem arcui signorum, et super ambas perpendicularem BHE vice circuli transeuntis super Lunam et polos circuli signorum. Palam ergo est quod si transitus centri Lune secundum visum fuerit super punctum B vel C, nichil de Sole eclipsimabitur eo quod Luna secundum visum

960 subtrahendus est] subtrahendum *M* subtrahenda est *N* (subtrahendus est *Ba* subtrahendum E_1) Lune] *om. N* **961** necessaria est] necessariis *corr. in* vera *M* **963** demitur] *corr. ex* d†iminui†tur P_7M in – etiam] etiam in vera coniunctione et *M* etiam in vera coniunctione *N* addendum] addendum est vel (*the last word marg.*) *N* **964** dissimiles] *corr. ex* differentias *N* **965/966** etiam – est] est necessarius etiam P_7 necessarius etiam est *N* **966** tempore] *s.l. K* **967** visum] †sen†sum *P* verum *N* eius locum] locum eius *N* **968** demendum] minuendum *N* **969** equato] equata *P* **970** liquidum] siquidem *M* **972** primum] primus *N* et^2 – Lune^2] *om. N* portionem] locum *M* **973** argumentum] augmentum *P om. M corr. marg. ex* augmentum *N* **974** est] et *add.* (*s.l.* P_7) P_7M verum] vere *PN* veram *M* visi] *s.l.* P_7 preterea] postea *N* **976** hoc secundum] secundum hoc *PK corr. ex* secundum hoc P_7 **978** duobus minutis] *marg.* (*perhaps other hand*) *P* **979** etiam] etiam et P_7 **980** per] *corr. ex* super *M* **981** duorum] duarum *N* **984** sicut] sui *M* arcui] arcui circuli *MN* **985** BHE] BHC P_7 circuli^1] *om. N* et] et super *N* **986** est] *om. PN* **987** eclipsimabitur] eclipsabitur *MN*

And note that in all these parallaxes to be found, the sun's parallax on the circle of altitude is to be subtracted[68] from the moon's parallax on the circle of altitude. Also, because the portion is necessary for obtaining the parallaxes, in order to have the portion, whatever is added to or subtracted from the moon's place at the true conjunction must also be similarly added to or subtracted from the place of the portion in the true conjunction because in such a short time, the moon's motion on the epicycle and the motion of longitude are not perceptibly different. Likewise, because the motion of latitude is also necessary for seeking the moon's latitude at the time of the apparent conjunction, whatever outward part is added to or subtracted from the moon's place in order to have its apparent place at the time of the apparent conjunction, similarly is, along with the motion of the Head in the same time, to be added to or subtracted from the corrected motion of latitude at the true conjunction.

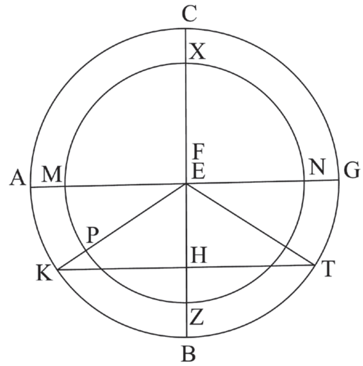
18. To find clearly the digits of a solar eclipse. Whence it will also certain when the moon will cover the whole sun and when it will not cover the whole.

We will find first the apparent conjunction of the sun and moon, the moon's portion at the same time, the sun's argument, the motion of latitude as has been spoken about before [i.e. at the time of the apparent conjunction as in VI.17], the latitude of the moon's apparent place through the true motion of latitude, and in addition the quantity of the moon's radius at that time and the quantity of the sun's radius – and this according to Albategni's work because it does not vary according to the work of Ptolemy. According to the work of Albategni, moreover, it varies 2′ 20″ between the apogee and perigee.[69] With these things had, we will also take the moon's parallax in latitude at the time of the apparent conjunction, and through this we will find the moon's apparent latitude.

Therefore, I will add the halves of the two diameters of the sun and moon, and according to this quantity I will draw circle ABG upon center E, the sun's circle MZN upon the same center, line AEG in place of the ecliptic, line KHT of the moon's passage as if parallel to the ecliptic [*lit*., the arc of the signs], and BHE the perpendicular upon both in the place of the circle passing upon the moon and the poles of the ecliptic. Therefore, it is clear that if the passage of the moon's center according to sight is at point B or C, no part of the sun will be eclipsed because the moon according to sight will touch the sun at point Z

[68] The grammatically incorrect 'subtrahendus' appears to be original.

[69] See V.18 above. In the passage of *De scientia astrorum* corresponding to *Almagesti minor* VI.18 (1537 ed., ff. 67v–68r; and *P*, f. 63v), Albategni incorrectly states that the difference is 2′ 15″, but the mistake is apparent from the value he gives in *De scientia astrorum* Ch. 30 (1537 ed., f. 37v) for the variation in the sun's diameter.

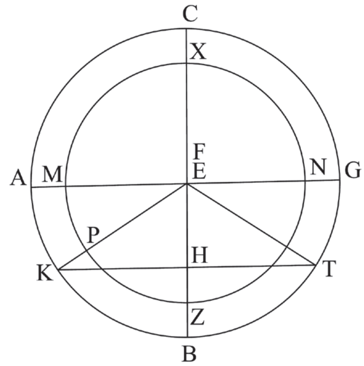


continget Solem super punctum Z vel X. Et hoc quidem tunc erit cum dimidium duorum diametrorum erit velud visa Lune latitudo. Et si transitus centri Lune secundum visum fuerit intra B vel C versus E quod tunc contingit cum visa latitudo est minor medietate duorum diametrorum, tunc aliquid de diametro Solis obscurabitur. Quod si visa latitudo nulla fuerit, tunc transitus Lune secundum visum erit super punctum E. Et si tunc quidem diameter Lune in aspectu fuerit sicut diameter Solis vel maior, Luna totum teget Solem. Et si sit maior, erit eclipsi Solis morula. Ut itaque digitos eclipsis solaris inveniamus, sit evidentie causa visa latitudo Lune in medio eclipsis EH scilicet centrum Lune secundum visum super punctum H ut extendatur semidiameter Lune usque ad punctum F. Tunc demo visam Lune latitudinem EH de medietate duorum diametrorum, et relinquitur HB. Dico quod tantum de diametro Solis obscurabitur. Nam constat quod de diametro Solis FZ obscurabitur, sed FH est equalis BZ quia utraque est sicut semidiameter Lune. Cum itaque visam Lune latitudinem a dimidio duorum diametrorum dempseris, reliquum in xii multiplica, et quod exierit per inventum Solis diametrum partire. Et provenient digiti solaris eclipsis.

19. Minuta casus in solari eclipsi terminare.

Sit exempli causa in premissa figura visus transitus Lune KHT quasi equidistans linee circuli signorum. Liquet igitur quod cum centrum Lune secundum visum fiet super punctum K, erit eclipsis principium quia Luna secundum visum tunc contingit Solem. Et propter hoc linea KH continet minuta casus

988 continget] contingit *N* **989** quidem] quod *P* duorum] duarum *N* **990** erit] EX *M* velud] *s.l.* (*other hand*) *P* Lune latitudo] latitudo Lune *M* **991** Et] hoc quidem tunc erit *add. et del.* P_7 **991/992** secundum – fuerit] quod fuerit secundum visum *M* **992** intra] iuxta *PN* B] *marg.* (*perhaps other hand*) *P* versus] *corr. ex* visum *K* **994** medietate] *corr. ex* mediante *K* duorum] *om. N* **995** de – obscurabitur] obscurabitur de dyametro Solis *M* **998** quidem] quod *P* quod quod *both del. N* **999** totum teget] tegit totum *M* Solem] *om. N* **1000** erit] *iter. et del. K* erit in *M* eclipsi] *corr. in* eclipsis P_7 eclipsis solaris] eclipsis (*corr. ex* eclipsi) Solis *K corr. ex* eclipsis lunaris *M* solaris eclipsis *N* **1001** visa] *om. N* eclipsis – scilicet] eclipsi EH sed *M* **1002** punctum H] H punctum *M* extendatur] extenditur *PN corr. ex* extendendatur P_7 semidiameter] *marg.* (*perhaps other hand*) *P* semidiametrum P_7 **1003** demo] denuo *K* Lune latitudinem] latitudinem Lune P_7 EH] EB *K* duorum] duarum *N* **1005** est equalis] *iter. K* **1007** duorum] duarum *N* dempseris] depresseris *K* **1008** exierit – inventum] exibit per inventam *N* **1009** eclipsis] eclipsis et cetera *M* **1010** solari eclipsi] eclipsi solari *N* terminare] determinare *KN corr. ex* determinare *M* (determinare *Ba* terminare E_1) **1013** fiet] fuerit *M* fuit *N* **1014** continget] contingit *MN*



or X. And indeed, this will be at that time when the half of the two diameters will be as the moon's apparent latitude. And if the passage of the moon's center according to sight is within B or C towards E, which occurs at that time when the apparent latitude is less than the half of the two diameters, then something of the sun's diameter will be obscured. But if there is no apparent latitude, then the moon's passage according to sight will be upon point E. And if indeed at that time the moon's diameter in sight will be as the sun's diameter or greater, the moon will cover the whole sun. And if it is greater, there will be a brief delay to the sun's eclipse. Accordingly, so that we may find the digits of the solar eclipse, for the sake of clarity, let EH be the moon's apparent latitude at the middle of the eclipse, i.e. ⟨let there be⟩ the moon's center according to sight upon point H so that the moon's radius may be extended to point F. Then I subtract the moon's apparent latitude EH from the half of the two diameters, and HB is left. I say that so much of the sun's diameter will be obscured. For it is evident that FZ of the sun's diameter will be obscured, but FH is equal to BZ because each is as the moon's radius. Accordingly, when you have subtracted the moon's apparent latitude from the half of the two diameters, multiply the remainder by 12 and divide what results by the sun's found diameter. And there will result the digits of the solar eclipse.

19. To demarcate the minutes of immersion in a solar eclipse.

For the sake of an example, in the preceding figure, let KHT be the moon's apparent passage as if parallel to the line of the ecliptic. Therefore, it is certain that when the center of the moon according to sight will be upon point K, there will be the beginning of the eclipse because the moon according to sight will touch the sun then.[70] And because of this, line KH contains the minutes

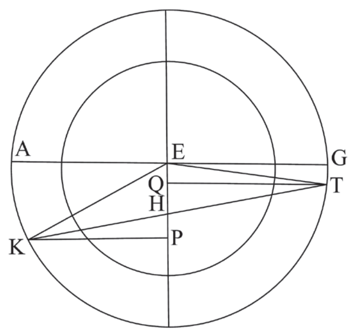
[70] Against convention, the moon's transit is here portrayed as advancing from left to right.

que querimus. Sed cum EK nota sit, est enim medietas duorum diametrorum, et ipsa subtenditur angulo recto qui est aput H, et EH visa latitudo, nota erit propter hoc KH. Patet ergo quod si visam Lune latitudinem in se ductam dempseris de dimidio duorum diametrorum in se ducto, radix residui continet minuta casus.

Secundum hanc siquidem doctrinam constitute sunt duplices tabule eclipsis solaris — una ad longitudinem longiorem Lune et alia ad longitudinem propiorem eius, sed utraque ad longitudinem Solis mediam tantum, scilicet cum diameter Solis secundum Albategni xxxii et xxx secunda. Et ideo de superfluo alterius tabule ad alteram secundum proportionem sumitur minutorum affinitatis in quam intratur per Lune portionem equatam.

20. Tria tempora solaris eclipsis indefinita et per hec minuta casus definitiora reperire.

Siquidem minuta casus prius reperta per Lune veram superationem ad horam divide, et horas cum minutis que provenerint a tempore medie eclipsis – nam ipsum omnino certum est – deme. Et habebis tempus indefinitum principii eclipsis. Easdem horas cum minutis super tempus medie eclipsis pone, et habebis tempus indefinitum finis eclipsis.



Post hec cum horis casus et eius minutis motum Solis diversum et motum Lune diversum disce, scilicet multiplicando eas in motum diversum unius hore, et quod ex Sole fuerit loco Solis ad medium eclipsis deme. Et erit locus Solis in principio eclipsis. Item adde et erit locus Solis ad finem eclipsis. Quod autem ex Luna fuerit vero loco Lune ad medium eclipsis deme, et portioni Lune idem et motui latitudinis cum motu nodi, et habebis locum Lune et portionem et motum latitudinis ad principium eclipsis. Item adde et habebis ad finem. Dehinc veram Lune latitudinem in utroque

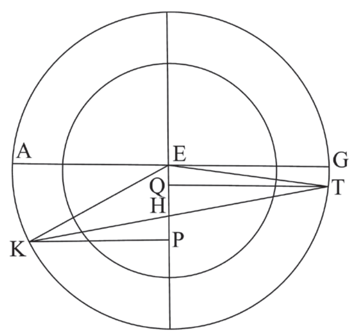
1015 medietas – diametrorum] duorum (duarum *N*) diametrorum medietas *PN* **1016** latitudo] latitudo est *M* **1017** KH] KH nota *M* **1018** duorum] duarum *N* residui] *s.l.* (*perhaps other hand*) *P* **1020** constitute sunt] sunt constitute *M* **1021** Lune] *s.l.* P_7 **1022** Solis mediam] mediam Solis *N* **1023** diameter] dyametro *N* secundum – xxxii] 32 minutorum secundum Albategni *N* xxxii] minuta *add. s.l.* P_7 est 32 gradus minuta (*the last word corr. ex* gradus) *M* **1024** secundum – sumitur] sumitur secundum proportionem *M* **1025** Lune portionem] portionem Lune *N* **1026** hec] hoc *KM* definitiora] diffinitiora *M* definita *N* **1028** veram] tertiam *P* visam *corr. ex* tertiam *N* superationem] superlationem P_7M **1029** divide] *corr. ex* dimidie *M* provenerint] proveniunt *N* **1030** indefinitum] infiniter *P* **1032** finis eclipsis] *corr. ex* eclipsis finis *P* **1033** hec] hoc *MN* casus] *om. N* **1034/1035** et – diversum] *om. PN marg. M* **1035** eas] ea *M* **1037** fuerit] a *add. s.l.* P_7 **1038** Item] idem *M* **1041** portioni] portionem *N* **1044** Item] idem *M* ad] *om.* P_7 **1044/1045** in – cum] cum utroque *N* of immersion that we seek. But because EK is known, for it is the half of the two diameters, and it subtends the right angle that is at H, and EH is the apparent latitude, KH will be known because of this. It is clear, therefore, that if you subtract the moon's apparent latitude multiplied by itself from the half of the two diameters multiplied by itself, the root of the remainder contains the minutes of immersion.

Accordingly, double tables of solar eclipse have been made according to this teaching[71] – one for the moon's apogee and the other for its perigee, but each only for the sun's mean distance, i.e. when the sun's diameter according to Albategni is 32′ 30″. And for that reason ⟨a part⟩ is taken from the difference between one table and the other according to the ratio of the minutes of affinity, which is entered through the moon's equated portion.

20. To find the three imprecise times of a solar eclipse and through these the more precise minutes of immersion.

Accordingly, divide the minutes of immersion found earlier by the moon's true surpassing for an hour, and subtract the hours and minutes that result from the time of the middle of the eclipse, for that is entirely certain. And you will have the imprecise time of the beginning of the eclipse. Add the same hours and minutes to the time of the middle of the eclipse, and you will have the imprecise time of the end of the eclipse.

Afterwards, with the immersion's hours and its minutes, learn the sun's irregular motion and the moon's irregular motion, namely by multiplying them by the irregular motion of one hour, and from the sun's place at the middle of the eclipse, subtract what there is from the sun. And there will be the sun's place at the beginning of the eclipse. Likewise, add, and there will be the sun's place at the end of the eclipse. Moreover, from the moon's true place at the middle of the eclipse, subtract what there is from the moon, and ⟨subtract⟩ the same from the moon's portion and the motion of latitude with the motion of the node, and you will have the moon's place, the portion, and the motion of latitude at the beginning of the eclipse. Likewise, add, and you will have ⟨them⟩ at the end. Then take the moon's true latitude at each of the times with the



[71] This table is both for the eclipse's digits and minutes of immersion.

temporum cum motu latitudinis equato sume. Deinde in utroque temporum scilicet finis et principii diversitatem aspectus in latitudine addisce, et per hoc visam Lune latitudinem in utroque tempore. Sitque in principio eclipsis visa latitudo EP in figura simili priori, et in fine eclipsis visa latitudo EQ, et linea KHT obliqua super transitum Lune visum. Itaque linea KH continet minuta casus definita ante medium eclipsis et TH post medium eclipsis. Sed quoniam ut ostendimus in xxiiii[a] quinti admodum parve differentie sunt motus in circulo declinanti et motus in circulo signorum, sufficit querere lineam KP loco KH et lineam TQ loco TH. Sed KP nota est propter KE et EP notas, et TQ nota propter ET et EQ notas. Unde patet quod si visam Lune latitudinem in principio eclipsis indefinito in se ductam demas de dimidio duorum diametrorum in se ducto, radix reliqui continet minuta casus definita que sunt ante medium eclipsis; et si visam Lune latitudinem in fine eclipsis indefinito in se ductam demas de dimidio duorum diametrorum in se ducto, reliqui radix continet minuta casus que sunt post medium eclipsis. Et hec propter distinctionem dicantur definita minuta detectionis.

21. Tria tempora solaris eclipsis definita investigare.

Medium quidem iam definitum est quia ipsum est tempus vise coniunctionis. Sed propter reliqua definienda queratur per diversitatem aspectus in longitudine visus locus Lune ad principium indefinitum et verus locus Solis ad idem tempus quod iam prehabitum est. Deinde consideretur si quantitas que tunc est inter visum locum Lune et locum Solis sit veluti definita minuta casus, quia si hoc est, ipsum indefinitum principium est illud definitum principium quod querimus. Quippe tunc Luna secundum visum continget Solem.

Quod si quantitas que tunc erit inter Solem et visum locum Lune minor fuerit ipsis minutis casus, a Luna Solem ante principium indefinitum occultari

1045 temporum[1]] tempore *M* **1046** latitudine] latitudinem P_7 **1048** latitudo EP] latitudo Lune EP (*last word corr. ex* EQ) *N* EQ] *corr. ex* EA *M* **1049** KHT] KHC *N* **1050** definita] diffinita *K* eclipsis[1]] eclipsim *K* **1051** admodum] ad motum *M* **1052** declinanti] declivi *N* KP] *corr. ex* KQ *M* **1053** notas] nota *M* **1055** indefinito] indefinita *P* infinito *K* indiffinite *M* (infinito *Ba* indefinito E_1) duorum] duarum *N* **1056** radix reliqui] reliqui radix *M* reliqui] residui *N* definita] *corr. ex* diffinita *K corr. ex* indefinita *N* **1057** eclipsis[1]] eclipsim *K* indefinito] indiffinite *M* **1058** de] a *N* duorum] duarum *PN* **1060** definita minuta] minuta definita *N* detectionis] *corr. ex* detestationis P_7 **1061** investigare] vestigare *PK corr. ex* vestigare P_7 (investigare *Ba* reperire E_1) **1062** ipsum] *om. N* vise coniunctionis] coniunctionis vise *N* **1064** principium] tempus *N* **1065** quod – prehabitum] qui iam prehabitus *M* **1066** Lune – Solis] *corr. ex* Solis et locum Lune *P* et] et visum *N* veluti] sicut *N* definita] diffinita *P* **1067** principium est] *corr. ex* est principium *P* illud] id *N* definitum] diffinitum *P* **1068** tunc] *s.l. K* cum *M* continget] contingit *N* **1069** et] Lunam *add. et del. N* **1069/1070** minor fuerit] fuerit minor *M* **1070** ipsis] temporis *K* Solem] *s.l.* (*perhaps other hand*) *P* ante] autem *M* corrected motion of latitude. Then at each of the times, i.e. of the end and the beginning, learn the parallax in latitude, and through this the moon's apparent latitude at each time. And let EP be the apparent latitude in the beginning of the eclipse in a figure similar to the earlier one, EQ the apparent latitude in the end of the eclipse, and oblique line KHT upon the moon's apparent passage.[72] Accordingly, line KH contains the precise minutes of immersion before the middle of the eclipse and TH after the middle of the eclipse. But because, as we showed in the 24th of the fifth ⟨book⟩,[73] the motion on the declined circle and the motion on the ecliptic are of an exceedingly small difference, it is sufficient to seek line KP instead of KH and line TQ instead of TH. But KP is known because of known KE and EP, and TQ is known because of known ET and EQ. Whence it is clear that if you subtract the moon's apparent latitude at the eclipse's imprecise beginning multiplied by itself from the half of the two diameters multiplied by itself, the root of the remainder contains the precise minutes of immersion that are before the middle of the eclipse; and if you subtract the moon's apparent latitude at the imprecise end of the eclipse multiplied by itself from the half of the two diameters multiplied by itself, the root of the remainder contains the minutes of immersion that are after the middle of the eclipse. And for the sake of distinction, let these be called the precise minutes of uncovering.

21. To find the three precise times of a solar eclipse.

Indeed the middle is already precise because it is the time of the apparent conjunction. But for the sake of specifying the others let the moon's apparent place at the imprecise beginning be sought through the parallax in longitude, and the sun's true place at the same time, which was already had. Then let it be considered if the quantity that is then between the moon's apparent place and the sun's place is as the precise minutes of immersion, because if this is, that imprecise beginning is that precise beginning which we seek. Surely the moon according to sight will touch the sun then.

But if the quantity that will be then between the sun and the moon's apparent place is less than those minutes of immersion, there is no doubt that the

[72] Once again, the moon moves from left to right during the eclipse.

[73] This refers to V.26.

non est dubitatio. Sume itaque superlationem visam Lune ad horam, et vide ut intra terminos ipsius hore quasi in medio sit indefinitum principium eclipsis. Et per hanc visam superlationem divide superfluum quod est inter dictam quantitatem Solis et Lune et definita minuta casus, et tempus quod exierit scilicet pars hore erit cuius initium est definitum principium eclipsis. Quod si minuta que sunt inter Solem et visum locum Lune fuerint plura definitis minutis casus, ad locum in quo aliquid Solis occultari possit nondum Lunam pervenisse certum est. Inveni igitur superlationem Lune et per eam superfluum partire, et tempus quod exierit erit cuius finis est definitum principium eclipsis. Simili modo quere per diversitatem aspectus longitudinis visum locum Lune in fine eclipsis indefinito et verum locum Solis. Et si quantitas que tunc erit inter visum locum Lune et Solem maior fuerit definitis minutis detectionis, constat Lunam preteriisse locum in quo primo nichil de Sole occultare debuit. Inveni itaque predicto modo visam superlationem Lune ad horam, et superfluum quod occurrerit per eam divide. Atque tempus quod inde exierit erit cuius initium est definitum tempus finis eclipsis. Quod si quantitas que tunc est inter Solem et visum locum Lune minor est definitis minutis casus, Lunam nondum pervenisse ad locum in quo sic a Sole separatur quod nichil eius occultare possit manifestum est. Inveni itaque visam superlationem Lune ad horam cuius indefinitus finis quasi medium sit, et per eam superfluum divide. Nam tempus quod exierit erit cuius finis est definitus finis eclipsis.

Quod si aliter facilius quidem et iuxta verum definita tempora scire volueris scilicet via Ptolomei, scias quod nisi Luna iuxta orizontem fuerit, diversitas aspectus in longitudine ascendente Luna ad medium celi paulatim non cessat decrescere. Ideoque visus motus Lune tardior est vero eius motu, et ideo tempus equatum per diversitatem aspectus quod est inter initium eclipsis et medium prolixius est horis indefiniti casus absolute inventis. Et similiter Luna paulatim a medio celi descendente diversitas aspectus in longitudine non cessat crescere

1071 dubitatio] dubium *N* superlationem visam] visam superlationem *M* **1072** intra] inter *K* infra *N* medio] media *K* eclipsis] *om. N* **1074** definita] diffinita *P* exierit] exibit *N* **1075** erit] tempus *add. s.l.* P_7 scilicet erit tempus *M* est definitum] definitum est *M* **1076** visum – fuerint] *corr. ex* †...† (*other hand*) *P* fuerint plura] fuerint plurima *K* plura fuerint *N* definitis] difinitis *P* **1076/1077** definitis minutis] minutis definitis *M* **1079** exierit] exiet *N* definitum] diffinitum *K* **1080/1082** in – Lune] *marg. K* **1081** indefinito] indefinite *M* **1082** detectionis] deiectionis P_7 **1083** nichil] *corr. ex* nil *M* occultare] occultari *N* **1085** occurrerit] occurret *corr. ex* occurreret P_7 **1086** finis] *om. N* **1088** eius] *om. PN* **1091** exierit] exibit *N* **1092** facilius – verum] quidem et iuxta verum facilius *M* facilius quidem] facimus quod *P* facimus quod (*this last word del.*) *N* definita] diffinita *K* **1093** nisi] *perhaps* ubi P_7 *corr. ex* nichil *K* Luna] *corr. ex* †...† (*perhaps other hand*) *P* iuxta] prope *N* **1095** decrescere] *corr. ex* crescere *MN* visus motus] motus visus *N* **1096** eclipsis] eclipsim *K* **1097** indefiniti] indefinita *P* indefinitis *N* **1098** diversitas] diversitatis *P corr. ex* diversita†tis† *N* sun is concealed by the moon before the imprecise beginning. Accordingly, take the moon's apparent carrying beyond for an hour, and see to it that the imprecise beginning of the eclipse is within the limits of that hour as if in the middle. And divide the excess that is between the said quantity between the sun and moon and the precise minutes of immersion by this apparent carrying beyond, and the time that results, i.e. a part of an hour, will be that whose beginning is the precise beginning of the eclipse. But if the minutes that are between the sun and the moon's apparent place are more than the precise minutes of immersion, it is certain that the moon has not yet reached the place in which something of the sun is able to be concealed. Therefore, find the moon's carrying beyond and divide the excess by it, and the time that results will be that whose end is the precise beginning of the eclipse. In a similar way, through the parallax in longitude, seek the moon's apparent place at the imprecise end of the eclipse and the sun's true place. And if the quantity that will then be between the moon's apparent place and the sun is greater than the precise minutes of uncovering, it is evident that the moon has gone beyond the place in which it first ought to obscure no part of the sun. Therefore, find in the said way the moon's apparent carrying beyond for an hour, and divide the excess that will have resulted by it. And the time that results from this will be that whose beginning is the precise time of the end of the eclipse. But if the quantity that is then between the sun and the moon's apparent place is less than the precise minutes of immersion, it is manifest that the moon has not yet reached the place in which it is separated from the sun thus that no part of it can conceal ⟨the sun⟩. Accordingly, find the moon's apparent carrying beyond for the hour of which the imprecise end is as a middle, and divide the excess by it. For the time that results will be that whose end is the precise end of the eclipse.

But if you want to know the precise times in another way, indeed easier and approximatively, i.e. by Ptolemy's way, know that unless the moon is near the horizon, with the moon gradually ascending to the middle heaven, the parallax in longitude does not cease to decrease. And for that reason, the moon's apparent motion is slower than its true motion, and for that reason, the time corrected by the parallax that is between the beginning of the eclipse and the middle is longer than the hours of imprecise immersion found without qualification. And similarly, with the moon gradually descending from the middle heaven, the parallax in longitude does not cease to increase, except when nisi cum iuxta orizontem Luna fuerit. Et ob hoc etiam visus motus Lune semper tardior est vero motu, ideoque tempus quod est inter medium eclipsis et finem prolixius est horis prenominatis que sunt indefiniti casus absque diversitate aspectus invente. Hinc patet quod si tempora invicem conferas quorum unum a principio eclipsis ad medium, alterum a medio ad finem, illud quod meridiei propius est maius est.

Sume igitur diversitatem aspectus longitudinis in medio eclipsis et in dictis temporibus indefinitis principii et finis. Post hec superflua que inter diversitatem aspectus medii eclipsis et diversitatem utriusque duorum temporum fuerint addiscens, eorum unumquodque per Lune veram superlationem ad horam partire. Et quod utrinque exierit erunt partes hore. Horas igitur casus indefiniti absolute inventas in duobus locis servans, alteri locorum alteram partem divisionum ex superfluo diversitatum inventam superadde, et alteri locorum alteram. Cum ergo horas casus sic equatas in duobus locis habueris, eas que minus sunt tempori medie eclipsis deme et eas que plus temporis sunt super medium eclipsis adde. Ita dico si longitudo medie eclipsis ab ascendente minus xc gradibus fuerit. Quod si longitudo medie eclipsis ab ascendente plus xc gradibus fuerit, conversam facies, scilicet quod maius est a tempore medie eclipsis demes et quod minus est addes propter hoc scilicet quod duorum terminorum longior iuxta medium celi semper esse debet. Et ita habebis propinque vero definita tempora que querimus.

22. Quantitatem lunaris circuli obscuratam ex digitis diametri demonstrare.

Sit itaque circulus Lune ABGD et circulus umbre AZGH, et quod eclipsatur ex diametro lunari notum ZD, et diameter Lune notus, et diameter umbre cum contineat diametrum Lune bis et eius tres quintas. Querimus ergo scire aream de lunari circulo obscuratam contentam duobus arcubus AZG et GDA. Nam ipsa est que obscuratur de circulo Lune. Quoniam autem circumferentia circuli minus continet quam triplum diametri et eius x septuagesimas, plus autem

1099 Luna fuerit] fuerit Luna *KN* **1100** est^{1}] *om.* P_7 **1103** unum] unum est *N* medio] medio eclipsis *M* **1104** maius] prolixius *N* **1106** hec] hoc *MN* **1107** diversitatem] diversitate *P* **1108** Lune veram] veram Lune *M* **1109** utrinque] *perhaps corr. ex* utrumque P_7 erunt] erit *KN* (erunt *Ba* erit E_1) **1110** servans] servatis P_7 **1111** diversitatum] diversitatis *M* **1112** habueris] *corr. ex* habemus *P* eas que] easque *P* easque que *N* **1113** tempori] tempore *M* **1113/1114** deme – clipsis^{2}] *marg.* P_7 **1114** adde] *corr. ex* deme *PK* xc] *corr. ex* †...† (*perhaps other hand*) *P* **1115** medie eclipsis] Lune *N* **1116** conversam facies] conversum facias *N* maius] *corr. ex* minus *M* tempore] *corr. ex* temporis P_7 medie eclipsis] eclipsis medie *M* **1116/1117** demes – addes] demas et quod minus est (*last two words corr.* ex interest) addas *N* **1117** minus] *corr. in* maius *M* **1118** celi] *corr. ex* eclipsis *P* **1118/1119** vero – tempora] duo tempora definita *M* **1119** querimus] querimus et cetera *N* **1121** AZGH] AZBH P_7 *corr. in* AHGZ *N* **1122** notum] *corr. ex* †tantum† *K* notus] nota *N* **1123** cum] et P_7 **1126** eius x] x eius P_7KM **1126/1127** plus – septuagesimas] *marg.* (*other hand*) *P* the moon is near the horizon. And on account of this, the moon's apparent motion is also always[74] slower than the true motion, and for that reason, the time that is between the middle of the eclipse and the end is longer than the hours already specified which are of the imprecise immersion found without parallax. From this it is clear[75] that if you compare the times to each other, of which one is from the beginning of the eclipse to the middle, the other from the middle to the end, that which is nearer the meridian is greater.

Therefore, take the parallax of longitude at the middle of the eclipse and at the said imprecise times of the beginning and end. Afterwards, learning the excesses that are between the parallax of the middle of the eclipse and the parallax of each of the two times, divide each of them by the moon's true carrying beyond for an hour. And what results on each side will be parts of an hour. Therefore, saving in two places the hours [i.e. the duration of time] of the imprecise immersion found without qualification, add one quotient [*lit.*, part of the divisions] found from the excess of the parallaxes to one of the places [i.e. to the time written down at one place], and the second ⟨quotient⟩ to the second of the places [i.e. to the time written down at the other place].[76] Therefore, when you have the hours of immersion thus corrected in the two places, subtract the ones that are less from the time of the middle of the eclipse and add those that are of more time to the middle of the eclipse. Thus I declare if the longitude of the middle of the eclipse from the ascendant is less than 90°. But if the longitude of the middle of the eclipse from the ascendant is more than 90°, you will do the converse, i.e. you will subtract what is greater from the time of the middle of the eclipse and you will add what is less, namely because of this that the further of the two extremities should always be near the middle heaven. And thus you will approximately have the precise times that we seek.

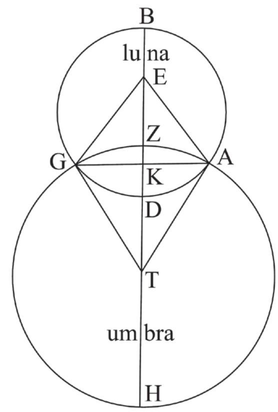
22. To show the obscured quantity of the lunar circle from the digits of the diameter.

Accordingly, let ABGD be the moon's circle, AZGH the shadow's circle, ZD the known amount that is eclipsed of the moon's diameter, the moon's diameter known, and the shadow's diameter ⟨known⟩ because it contains the moon's diameter 2 3/5 times. Therefore, we seek to know the obscured area of the lunar circle contained by the two arcs AZG and GDA. For that is what is obscured of the moon's circle. Moreover, because the circle's circumference contains the diameter less than 3 10/70 times, but more than 3 10/71 times,

[74] Although he has just added a qualification to Ptolemy's statement that the parallax in longitude is always greater nearer the horizon, the author here speaks as if what Ptolemy says were universally true.

[75] Actually, our author does not include the reason for this fact. Ptolemy gives it: 'Et quia fuit semper superfluitas additionis inter duas diversitates aspectus maior in cursibus qui sunt propinquiores medietati diei ...' (*Almagest*, 1515 ed., f. 70v).

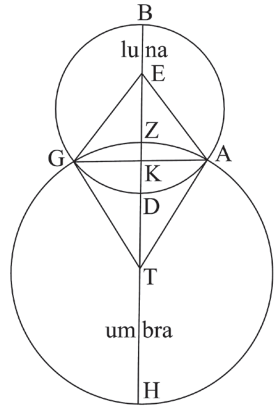
[76] Here the author puts his instructions in terms of the physical act of calculating.



quam triplum diametri et eius x septuagesimas primas sicut ostendit Assamides, inter utrumque sicut mos est astrologis ponemus proportionem circuli ad diametrum sicut proportionem trium partium et viii minutorum et xxx secundorum ad partem unam. Cum itaque diameter Lune sit notus, et ponemus eum xii partium, erit propter hoc circumferentia circuli eius nota. Et ob hoc ducto semidiametro in semicircumferentiam area circuli lunaris nota. Et est circumferentia quidem xxxvii partes et xlii minuta, et area circuli cxiii partes et vi minuta. Pari modo cum diameter umbre sit notus scilicet xxxi partes et xii minuta, erit propter hoc circumferentia nota, et est xcviii partes et unum minutum. Et ob hoc area circuli eius nota scilicet dcclxiii partes et xxii minuta.

Rursum cum ZD sit nota, si subtrahatur a DE, relinquitur EZ nota. Cui cum addita fuerit ZT semidiameter umbre, erit ET que continetur inter duo centra nota. Item quia TA nota est sed et EA nota, erit utriusque quadratum notum. Si ergo differentia quadratorum dividatur per lineam ET, exibit differentia linee EK ad lineam KT nota. Et propter hoc utralibet EK KT erit nota. Et quia GKA stat super utramlibet perpendiculariter, erit et ipsa nota et hec est communis corda duorum circulorum. Aliter quoque possumus pervenire ad eius notitiam leviori opere. Palam enim quod ZB et DT pariter duplum sunt eius quod continetur inter duo centra et ideo notum. Sed illa duo simul se habent ad unum eorum scilicet ZB notum sicut DZ notum ad KZ, eo quod disiunctim una est proportio. Ergo linea ZK que est sagitta circuli umbre est nota, et ob hoc KD lunaris circuli sagitta nota. Inter quam et KD medio loco proportionalis est KA; ergo ipsa nota.

1127 eius] *om. N* **1128** primas] *om. M* Assamides] Asamides *corr. in* Asanides *M* Archimedes *N* (Assamides *Ba*E_1) **1130** sicut] *s.l.* (*perhaps other hand*) *P* **1133** notus] nota *N* eum] eam *MN* **1134** circuli eius] eius circuli *M* **1135** semicircumferentiam] *corr. ex* circumferentiam *N* **1136** Et est] est et *N* **1138** vi] 7 P_7 **1139** umbre] *s.l.* (*perhaps other hand*) *P* notus] nota *N* **1140** est] *corr. ex* etiam P_7 *s.l. K* xcviii] *corr. ex* xxviii (*perhaps other hand*) *P corr. ex* cxviii P_7 **1141** dcclxiii] *corr. ex* 263 *M* **1142** si] *om. M* a DE] *corr. ex* †ABDE† *P* ABDE *K corr. ex* ABDE *N* (AB de- *Ba* ab ED E_1) EZ nota] DE (*del.*) nota EZ *N* **1143** cum] *s.l. K* addita] *om. N* umbre] umbre adiuncta *N* **1144** TA] *corr. ex* TE *K* nota est] est nota P_7*M* et] quia *M* **1145/1146** ET – lineam] *marg.* (*perhaps other hand*) *P* **1146** erit] *om. N* **1147** perpendiculariter] *corr. ex* particularem *K* perpendicularem *M* hec] ob hoc *M* **1148/1149** ad – notitiam] *om. N* **1149** leviori] *corr. ex* leviore *P* DT] *corr. in* DH *N* **1152** una] vera *P*P_7 *corr. ex* vera *N* (una *Ba*E_1) est[1]] *om.* P_7 **1153** KD[2]] KB *M corr. in* KB *N* (ZK *Ba* RB E_1) **1154** KA] RA *K corr. ex* KD *N*



as Assamides [i.e. Archimedes] showed, we will suppose the ratio of the circle to the diameter between them, as is the custom for the astronomers, just as the ratio of $3^p\ 8'\ 30''$ to 1^p. Accordingly, because the moon's diameter is known, and we will suppose it to be 12^p, the circumference of its circle will be known because of this. And on account of this, with the radius multiplied by the semicircumference, the area of the lunar circle will be known. And the circumference is indeed $37^p\ 42'$, and the circle's area $113^p\ 6'$. In a like way, because the shadow's diameter is known, i.e. $31^p\ 12'$, the circumference will be known because of this, and it is $98^p\ 1'$. And on account of this, the area of the its circle is known, i.e. $763^p\ 22'$.[77]

In turn, because ZD is known, if it is subtracted from DE, EZ remains known. When the shadow's radius ZT is added to this, ET, which is contained between the two centers, will be known. Also, because TA is known but also EA is known, the square of each will be known. Therefore, if the difference of the squares be divided by line ET, the known difference between line EK and line KT will result.[78] And because of this, EK and KT will each be known. And because GKA stands upon each perpendicularly, it will also be known and this is the common chord of the two circles. We are also able to come to the knowledge of it [i.e. GKA] in another way with easier work. For it is clear that ZB and DT[79] together are double that which is contained between the two centers and thus <their sum is> known. But those two together are disposed to one of them, i.e. known ZB, as known DZ is to KZ because *disiunctim* the ratio is one [i.e. the same].[80] Therefore, line ZK, which is the sagitta of the shadow's circle, is known, and on account of this, KD, the sagitta of the lunar circle, is known. Between which [i.e. KD] and KD,[81] KA is the mean proportional [*lit.*, proportional in the middle place]; therefore, it is known.

77 This number should be $764^p\ 32'$ to match the *Almagest*.

78 Paraphrasing Toomer, *Ptolemy's Almagest*, p. 303 n. 62: $AT^2 - EA^2 = (KT^2 + AK^2) - (EK^2 + AK^2) = KT^2 - EK^2 = (KT + EK)(KT - EK) = ET(KT - EK)$. Therefore, $(AT^2 - EA^2) \div ET = KT - EK$.

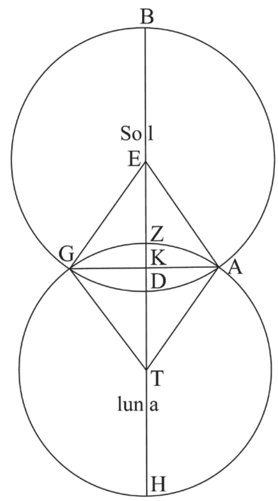
79 This should be line DH.

80 The truth of the proportion HD : BZ :: KD : KZ can be proved using Euclid's *Elements* III.35.

81 This should refer to line KB. The mistake may have been the author's or it could have entered the transmission early due to easily confused forms of the letters 'b' and 'd'.

Si ergo EK ducatur in lineam KA, erit superficies trianguli EGA nota. Pari modo fiet superficies trianguli umbre scilicet TGA nota. Amplius quia linea GA respectu partium semidiametri ED est nota, si semidiameter constituatur lx partium more cordarum, erit et hoc respectu corda GA nota; et ob hoc arcus GDA notus secundum quod circumferentia continet ccclx partes. Et quia proportio circumferentie ad arcum est sicut proportio aree circuli ad sectorem, erit area sectoris contenti sub arcu GDA et duabus lineis AE EG nota. A qua si dempseris aream trianguli EAG notam, relinquitur portio circuli contenta sub arcu GDA et corda GA nota. Simili modo fiet sector circuli contentus sub arcu GZA et duabus lineis GT ZA notus. Abiecto ergo triangulo relinquitur portio circuli contenta sub linea GA et arcu GZA nota. Quare tota superficies contenta sub duobus arcubus GZA GDA est nota secundum quod area lunaris circuli continet cxiii partes et vi minuta. Quare si ponas eandem aream circuli xii partium, erit hoc quoque respectu proposita superficies nota, et hoc est quod volebamus.

23. Quantitatem solaris circuli obscuratam ex digitis diametri eius ostendere.

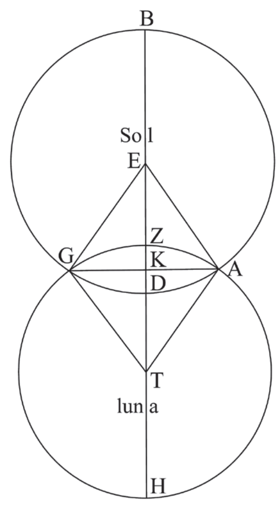


Sit enim circulus Solis ABGD super centrum E, AHZ super centrum T circulus Lune secans circulum Solis super duo puncta G A. Et quod obscuratur de diametro Solis ZD notum, et diameter Solis notus, et diameter Lune eodem respectu notus. Propterea erit linea inter duo centra ET nota, et ob hoc corda communis GA nota. Atque predicta via utriusque trianguli et utriusque sectoris superficies nota, et propter hoc superficies de circulo Solis obscurata scilicet que continetur sub duobus arcubus GDA GZA nota. Propter has quoque quantitates presto habendas, composita est parva tabula, in

1156 fiet] fit P_7 quia] quod *K* **1157** si] *s.l.* P_7 semidiameter] *corr. ex* diameter *P* semidiameter constituatur] constituatur semidyameter *M* **1160** ad[1] – est] est ad arcum *M* **1161** sectoris] *corr. ex* sectionis *K* **1162** trianguli] *s.l.* (*other hand*) *K* portio] *corr. ex* proportio *P* **1164** lineis] lineiis P_7 GT ZA] *corr. ex* GT TA *M* AT GT *N* (GT ZA *Ba* GT et TA E_1) triangulo] *corr. ex* angulo *K* **1166** GZA] EGZA *P* secundum quod] sed quia *K* secundum] *corr. ex* set P_7 area] *corr. ex* areas *M* **1167** cxiii – vi] xciii partes et iii P_7 ponas] ponamus *M* eandem aream] aream eandem *PN* **1169** volebamus] volebamus et cetera *N* **1173** AHZ] AHG *M* super – Lune] circulus Lune super centrum T *N* **1176** notus] nota *N* eodem] *marg.* *P* **1177** notus] nota *N* Propterea] propter hoc (*the second word s.l.*) *K* **1179** trianguli] *corr. ex* anguli *M* **1180** sectoris] *s.l.* (*other hand*) *K* **1181** circulo] *corr. ex* semicirculi P_7 **1182** sub] in *corr. ex* †...† *P* in *N* arcubus] *marg.* (*perhaps other hand*) *P* GDA] GDA et *N* **1184** parva tabula] tabula parva *K*

Therefore, if EK be multiplied by line KA, the surface area of triangle EGA will be known. In a like way, the surface area of the shadow's triangle, i.e. TGA, will be known. Further, because line GA is known with respect to the parts of radius ED, if the radius be set up as 60^p in the custom of chords, chord GA will also be known in this respect; and on account of this, arc GDA will be known according to the terms by which the circumference contains 360^p. And because the ratio of the circumference to an arc is as the ratio of the circle's area to the sector, the area of the sector contained under arc GDA and the two lines AE and EG will be known. If you subtract the known area of triangle EAG from this [i.e. the sector], there is left known the portion of the circle contained under arc GDA and chord GA. In a similar way, the sector of the circle contained under arc GZA and the two lines GT and ZA[82] will be made known. With the triangle subtracted, therefore, there is left known the portion of the circle contained under line GA and arc GZA. Therefore, the whole surface area contained under the two arcs GZA and GDA is known according to the terms by which the area of the lunar circle contains 113^p 6′. Therefore, if you suppose the same area of the circle to be 12^p, the proposed surface area will be known also in this respect, and this is what we wanted.

23. To show the obscured quantity of the solar circle from the digits of its diameter.

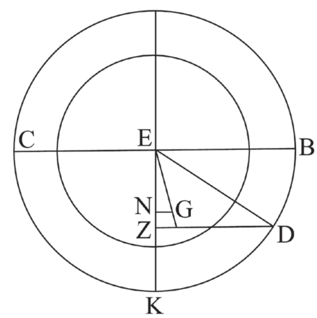


Indeed, let there be the sun's circle ABGD upon center E, the moon's circle AHZ upon center T cutting the sun's circle at the two points G and A. And ZD, what is obscured of the sun's diameter, is known, and the sun's diameter is known, and the moon's diameter is known in the same respect. Therefore, the line ET between the two centers will be known, and on account of this the common chord GA will be known. And in the said way [i.e. as in VI.22] the surface area of each triangle and of each sector is known, and because of this, the obscured surface area of the sun's circle, i.e. what is contained between the two arcs GDA and GZA, is known. In order to also have these quantities at hand, a small table

[82] This should be TA.

cuius prima proselide continentur numeri digitorum diametri, et in secunda quod attinet eis de solari circulo, et in tertia de lunari secundum quod Luna in longitudine media constituitur. Proportio enim diametrorum Lune et umbre vel Lune et Solis non multum variatur ab hac in ceteris longitudinibus.

24. Quantitates angulorum in notis temporibus eclipsis Lune provenientium ex concursu circuli signorum et circuli super duo centra Lune et umbre transeuntis, sive etiam in notis temporibus defectus Solis ex conventu orbis signorum et circuli transeuntis super centrum Solis et visum locum Lune notas efficere.

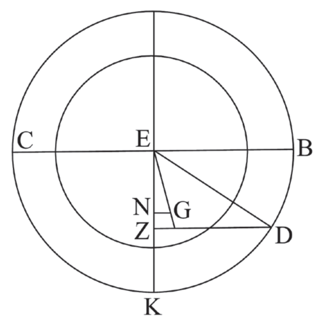


Sit igitur primum circulus medietatis duorum diametrorum Lune et umbre et circulus umbre super centrum E, et linea circuli signorum BEC, et latitudo Lune in principio eclipsis EZ cum centrum Lune super punctum D. Palam ergo quod linea ZD que sit equidistans linee BE continet minuta casus fere, etiam definita, et est DE arcus circuli magni transeuntis super duo centra Lune et umbre in principio eclipsis. Querimus ergo primum quantitatem anguli DEB qui est in principio eclipsis. Et quia linea ED nota, notam habet proportionem ad EZ notam. Palam quod si ED constituamus semidiametrum lx partium, erit secundum EZ corda mediata notarum partium; quare et arcus qui super eam notus, et ob hoc angulus EDZ notus. At ipse equalis est angulo BED, quem querimus.

Rursum sit EN latitudo Lune in principio more, et centrum Lune super punctum G. Erit ergo EG linea circuli magni transeuntis super duo centra Lune et umbre in principio more. Itaque angulum GEB in principio more querimus. Et quia linea EG nota est, est enim minor medietate duorum diametrorum quantitate diametri Lune, et ipsa notam habet ad EN proportionem. Ergo EG facta semidiametro circuli erit arcus super EN notus, quare et angulus EGN notus, et ipse est equalis angulo BEG. Eisdem modis noti erunt anguli

1185 proselide] parte *N* **1187** et] *s.l. M* **1188** longitudinibus] longitudinibus et cetera *N* **1194** igitur] *om. N* medietatis] medietas *P* medietatis duorum] secundum quantitatem medietatis duarum *N* **1195** Lune] *iter. K* **1197** BEC] BET *N* **1198** D] D fuerit *N* **1199** ergo] *om. N* **1201** magni] *corr. ex* signorum P_7 **1202** principio] *corr. ex* medio (*perhaps other hand*) *P* **1203** primum] *om. N* **1204** quia] *om. N* habet] habebit *M* **1206** secundum] secundum hoc *M om. N* notarum] *corr. ex* duarum *K* arcus] *corr. ex* angulus *N* **1207** eam] eam est erit *N* et – notus[2]] *marg.* (*perhaps other hand*) *P* **1211** angulum] angulus *K* GEB] *corr. ex* GBE *N* **1212** nota est] est nota *M* medietate] *corr. ex* mediante *K* duorum] duarum *N* **1213** habet – EN] ad EN habet *M* **1214** semidiametro] *corr. ex* dyametro *M* et] *om. N* angulus] *corr. ex* arcus P_7 **1215** notus] *om. N* est equalis] equalis est *N* has been made. In the first column of this, the numbers of the digits of the diameter are contained; in the second what pertains to them of the solar circle; and in the third ⟨what pertains to them⟩ of the lunar ⟨circle⟩ according to the fact that the moon is set up at mean distance. For the ratio of the diameters of the moon and shadow or of the moon and sun do not vary much from this in the other distances.

24. To make known at the known times of the moon's eclipse, the quantities of the angles resulting from the meeting of the ecliptic and the circle passing upon the two centers of the moon and the shadow, or also at the known times of the sun's eclipse, ⟨the quantities of the angles resulting⟩ from the meeting of the ecliptic and the circle passing upon the sun's center and the moon's apparent place.



Therefore, first let there be the circle of the half the two diameters of the moon and of the shadow and the circle of the shadow upon center E, the line of the ecliptic BEC, and the moon's latitude EZ at the beginning of the eclipse when the moon's center is upon D. It is clear, therefore, that line ZD, which is parallel to line BE, contains approximately the minutes of immersion – also precise, and DE is the arc of the great circle passing upon the two centers of the moon and shadow at the beginning of the eclipse. Therefore, we seek first the quantity of angle DEB, which is at the beginning of the eclipse. And because line ED is known, it has a known ratio to known EZ. It is clear that if we set up ED as a radius of 60^p, afterwards the half chord EZ will be of known parts; therefore, also the arc that is upon it will be known, and on account of this, angle EDZ will be known. But this is equal to angle BED, which we seek.

In turn, let EN be the moon's latitude at the beginning of the delay, and let the moon's center be upon point G. Therefore, EG will be the line of the great circle passing upon the two centers of the moon and the shadow at the beginning of the delay. Accordingly, we seek angle GEB at the beginning of the delay. And because line EG is known, for it is less than the half of the two diameters by the quantity of the moon's diameter, it also has a known ratio to EN. Therefore, with EG made a radius of a circle, the arc upon EN will be known, so angle EGN will also be known, and it is equal to angle BEG. In in fine eclipsis et in principio detectionis posito quod G sit locus detectionis et D locus Lune in fine eclipsis. Nam in medio eclipsis palam quod angulus qui queritur rectus est scilicet cum centrum Lune fuerit super lineam EZK.

Rursum propter solares eclipses sit visa latitudo Lune ZE in principio, et punctum D visus locus Lune, atque ED medietas duorum diametrorum. Palam ergo quod querimus angulum DEB. Atque huius investigatio non est dissimilis priori sive D sit visus locus Lune in principio eclipsis sive in fine. Nam in medio eclipsis qui queritur rectus est.

Propter hos autem angulos ad manum habendos reperies tabulam que intitulatur reflexio sive inclinatio tenebrarum in utraque eclipsi. Et intratur in eam per numerum digitorum eclipsis sive solaris sive lunaris. Nam cum digiti eclipsis noti sunt, et latitudo nota est, et per eam anguli noti. Et sunt in eadem tabula iiii proselides. Una continens numerum digitorum eclipsis quo usque etiam extendi in Luna potest. In secunda vero numeri qui opponuntur continent quantitates angulorum in principio et in fine solaris eclipsis acsi minuta casus eadem essent ante et retro et Luna in longitudine media. In tertia vero quantitates angulorum in principio et fine lunaris laboris acsi minuta casus equa essent ante et post medium eclipsis. In quarta autem sunt quantitates angulorum in principio et fine more acsi minuta more utrobique essent eadem et Luna in longitudine media.

25. Flexus tenebrarum sive in Solis sive in Lune defectu patenter assignare.

Propter evidentiam ponemus circulum umbre super centrum E, et sit medietas duorum diametrorum ED EC EG ET. Et propter quantitates angulorum assignandas ponimus circulum exteriorem PNQK, et linea circuli signorum PEQ, et P respiciens occidens, Q oriens, F meridiem, N septentrionem. Si ergo contingat Lunam in aliquo notorum temporum eclipsis erit in circulo signorum, verbi gratia in principio eclipsis ut super punctum D tunc quidem ea ingrediente in umbram, flexus tenebrarum in ea respicit gradum orientem

1216 in[2] – detectionis[1]] detectionis principio *N* sit] fit P_7 **1219** sit] sit in solari *M* latitudo Lune] Lune latitudo *P* **1220** visus locus] locus visus *N* duorum] duarum *N* **1221** huius] huiusmodi *M* **1223** medio] *corr. ex* fine P_7 eclipsis] eclipsis angulus *N* **1224** hos autem] autem hos *K corr. ex* hos ante *M* reperies] invenies *N* **1226** solaris – lunaris] lunaris sive solaris *M* cum] *om. N* **1227** noti[2]] noti sunt *N* eadem] ea P_7K **1229** etiam extendi] extendi etiam P_7 extendi – Luna] in Luna extendi *M* **1230** in[2]] *om.* P_7K acsi minuta] *corr. ex* ac finitam P_7 **1231** eadem essent] essent eadem *PN* **1232** et] et in *M* laboris] eclipsis *N* **1233** equa essent] essent equa *M* equa] eque *P corr. ex* que P_7 *corr. ex* a qua *K* equalia *N* **1234** et] et in *M* **1236** in[1]] *om. M* in[2]] *om. MN* **1237** super centrum] *corr. ex* super umbre *P* **1238** duorum] duarum *N* EC] ET *N* quantitates] *corr. ex* quantitatem P_7 **1239** ponimus] ponemus *N* PNQK] PXQK P_7 *corr. ex* PNK *M* **1240** occidens] occidens et *N* F] *corr. in* B P_7 N] *corr. in* X P_7 **1241** notorum – eclipsis] nodorum ipsum eclipsim *M* erit] esse *M* esse *corr. ex* †...† *N* (erit *Ba unclear abbreviation* E_1) **1243** in] *om. N* tenebrarum] *om. N* **1244** gradum] gradus *P corr. ex* gradus *N* the same ways, the angles at the end of the eclipse and at the beginning of the uncovering will be known with it supposed that G is the place of uncovering and D the moon's place at the end of the eclipse. Certainly, at the middle of the eclipse, it is clear that the angle that is sought is right, namely when the moon's center is upon line EZK.

In turn, for solar eclipses let the moon's apparent latitude be ZE at the beginning, point D the moon's apparent place, and ED the half of the two diameters. It is clear, therefore, that we seek angle DEB. And the investigation of this is not dissimilar to the earlier one whether D is the moon's apparent place at the beginning of the eclipse or at the end. Certainly, at the middle of the eclipse what is sought is right.

Moreover, in order to have these angles at hand, you will need the table that is entitled 'reflexion' or 'inclination of the darkness in each eclipse', And it is entered through the number of the digits of the eclipse whether solar or lunar. For when the digits of the eclipse are known, the latitude is also known, and through it the angles are known. And there are 4 columns in the same table. One containing the number of digits of the eclipse as far as it actually is able to be extended in the moon [i.e. in a lunar eclipse]. And indeed, in the second the numbers that are placed opposite contain the quantities of the angles at the beginning and at the end of a solar eclipse as if the minutes of immersion were the same before and after and the moon were at mean distance. And in the third are indeed the quantities of the angles at the beginning and the end of a lunar eclipse [*lit*., labor] as if the minutes of immersion were equal before and after the middle of the eclipse. Moreover, in the fourth are the quantities of the angles at the beginning and end of the delay as if the minutes of delay were the same on both sides and the moon at mean distance.

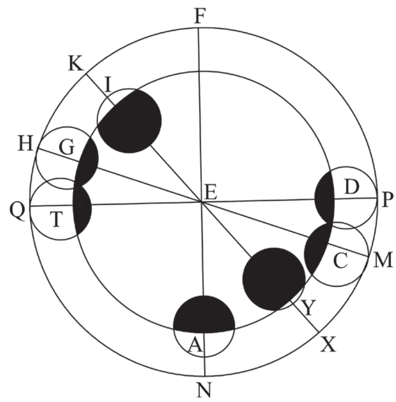
25. To clearly designate the directions of the darkness whether in an eclipse of the sun or the moon.

For the sake of clarity, we will suppose the shadow's circle upon center E, and let the half of the two diameters be ED, EC, EG, and ET. And in order to designate the quantities of the angles, we posit an exterior circle PNQK, line PEQ of the ecliptic, and P facing the setting point, Q the rising point,[83] F south, and N north. Therefore, if it happens that the moon at any of the known times of the eclipse will be[84] on the ecliptic, for example at the beginning of the eclipse as upon point D with indeed it entering the shadow then, the direction of the darkness in it faces the rising degree in the direction of

[83] Here and in much of this proposition 'oriens' and 'occidens' should not be taken to mean 'east' and 'west.'

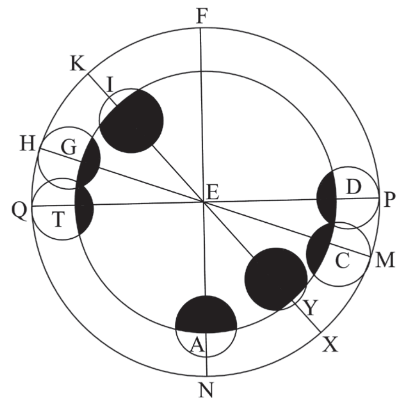
[84] The 'erit' is incorrect grammatically, but it appears to be original.

versus punctum Q, nam arcus transiens super duo centra est ipse orbis signorum. Et si in fine eclipsis Luna fuerit in orbe signorum ut super punctum T Luna quidem exeunte ab umbra, tunc flexus tenebrarum in ea respicit directum gradum occidentem versus punctum P cum linea super duo centra transiens sit ipse orbis signorum. Arcus autem orizontis inter gradum orientem vel gradum occidentem et circulum equinoctialem interceptus — quicumque gradus oriens vel occidens fuerit — notus est ex quarta propositione secundi libri. Si ergo locus orizontis ubi oritur Aries sive Libra notus fuerit atque ubi occidit, omnes orientes estivales et hiemales et omnes occidentes – hiemales dico et estivales – noti sunt. Nam estivales orientes vel occidentes sunt loca orizontis in quibus septentrionalia signa oriuntur vel occidunt. Hiemales orientes vel occidentes sunt loca orizontis in quibus septentrionalia signa oriuntur vel occidunt. Equalis vero oriens vel occidens dicitur ubi caput Arietis sive Libre oritur vel occidit.



Quod si Luna in aliquo temporum in orbe signorum non fuerit, invenienda est quantitas anguli in ipso tempore, ut verbi gratia in principio eclipsis sit super punctum C. Erit ergo angulus quesitus CEP cuius quantitatem assignare potest arcus PM. Tunc ergo flexus tenebrarum cum ingreditur umbram erit ad partem orientis in partem contrariam latitudinis a gradu oriente distans secundum quantitatem arcus HQ, qui equatur arcui invento PM. Est enim linea super duo centra transiens vergens in illam partem. Et si in fine eclipsis extra circulum signorum fuerit exempli gratia super punctum G, extremitas tenebrarum partem occidentis respicit et declinat ad partem contrariam latitudinis secundum quantitatem anguli assignati. Et si in principio more extra lineam signorum fuerit ut verbi gratia iuxta punctum Y, extremitas partis adhuc tegende ad

1248 quidem] quoque *M* **1249** in – respicit] respicit in ea *N* directum] *om. M* directem *N* gradum] *corr. ex* angulum *K* **1253** vel] et *PN corr. ex* et *K* (vel *BaE*$_1$) **1257** propositione] *corr. ex* proportione *P*$_7$ **1258** occidit] occiderit *P* acciderit *N* **1259** dico – estivales[1]] et estivales dico *N* **1261/1262** Hiemales – occidunt] *marg.* (*in another hand*) *P om. P*$_7$ (*om. Ba*) **1261** septentrionalia] meridionalia *MN* (septentriona *E*$_1$) **1262** vel[2]] sive *N* occidens] occasus *PN* **1263** sive] vel *N* **1265** anguli] *om. N* ipso] eo *PN* ut] *om. MN* **1266** C] T *N* quesitus] *om. N* CEP] TEP *N* **1267** Tunc] *s.l. K* **1268** oriente] orientis *N* **1270** in[1]] ad *M* **1272** declinat] declinatio *P*$_7$ **1274** Y] X *PMN* (Y *BaE*$_1$) adhuc] *corr. in* ultimo *N* tegende] redeunde *P* regende *corr. ex* regente *K* tangende *M* occultate *N* (tegende *Ba* tegente *E*$_1$)

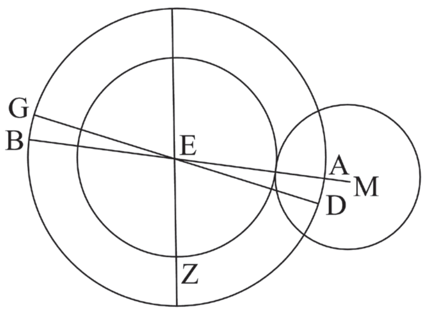


point Q, for the arc passing upon the two centers is the ecliptic itself. And if the moon is on the ecliptic at the end of the eclipse as upon point T with the moon indeed leaving the shadow, the direction of the darkness in it then faces the setting degree directed towards point P because the line passing upon the two centers is the ecliptic itself. Moreover, the arc of the horizon cut off between the rising degree or the setting degree and the equator – whatever degree is rising or setting – is known from the fourth proposition of the second book. Therefore, if the place of the horizon where Aries or Libra rises is known and where it sets, all the summer and winter rising points and all the setting points – I mean winter and summer – are known. For the summer rising or setting points are the places of the horizon where the northern signs rise or set. Winter rising or setting points are the places of the horizon where the northern[85] signs rise or set. And indeed, where the beginning of Aries or of Libra rises or sets is called an equal rising or setting point.

But if the moon in any of the times is not on the ecliptic, the quantity of the angle at that time must be found, as for example let it be upon C at the beginning of the eclipse. Therefore, the sought angle will be CEP, the quantity of which arc PM can designate. Therefore, at the time when it enters the shadow, the direction of the darkness will be in the direction of the east in the direction opposite the latitude, standing apart from the rising degree according to the quantity of arc HQ, which is equal to found arc PM. For the line passing upon the two centers tends in that direction. And if at the end of the eclipse, it will be beyond the ecliptic, for example upon point G, the extremity of the darkness faces the direction of the west and declines in the direction opposite the latitude according to the quantity of the designated angle. And if at the beginning of the delay, it is beyond the ecliptic [*lit.*, line of the signs], as for example near point Y, the extremity of the part still to be covered tends

[85] This should be 'meridionalia', but the mistake appears to have been have been made by the author or early in the transmission. The confusion caused by such a mistake may be the reason that this sentence was omitted in *P*, P_7, and *Ba*.

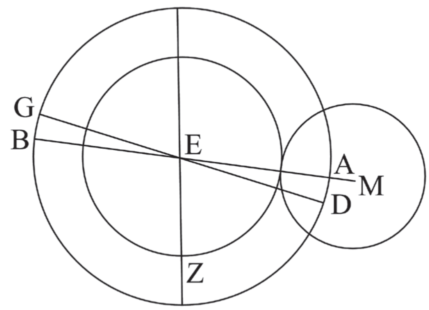
occidentis partem vergit et declinat a puncto tunc occidente secundum quantitatem anguli PEX, cui subtenditur arcus PX notus. Et si in fine more fuerit iuxta punctum I, principium detectionis parti orientis concurrit declinans quidem a puncto tunc oriente in partem sue latitudinis secundum quantitatem anguli QEK noti. Et si in medio eclipsis non fuerit tota eclipsimata, ut si fuerit super punctum A, tunc in contrariam partem latitudinis declinant umbre versus eam partem orizontis ad quam descendit circulus transiens super polos zodiaci et centrum Lune. Et sic quidem se habent flexus tenebrarum Lune.



Propter Solis tenebras ponimus circulum solaris corporis super centrum E et circulum exteriorem propter quantitates angulorum determinandas. Et A quidem sit pars occidentis, B orientis. Si ergo in aliquo temporum eclipsis Luna in signorum circulo fuerit, ut verbi gratia in principio super punctum M, tunc pars Solis que tegi incipit dirigitur ad punctum tunc occidens. Et si in fine eclipsis hoc contigerit, extremitas obscuri dirigitur ad punctum tunc oriens. Quod si Luna in aliquo temporum eclipsis latitudinem habuerit, ut verbi gratia in principio sit super punctum D, tunc secundum quantitatem anguli inventi in partem occidentis declinat obscuritas, et hoc in partem latitudinis Lune. Et si hoc in fine eclipsis fuerit, declinat in partem orientis secundum quantitatem assignati anguli, et hoc semper in eandem partem latitudinis Lune. Et si in medio eclipsis Sol non totus obscuratur, pars eius obscura eam partem orizontis respicit ad quam descendit circulus super centrum Solis et visum locum Lune et polos zodiaci transiens. Si vero Sol totus obscuratur in eclipsis medio, non habent partem tenebre eo quod non est circulus transiens super duo loca Solis

1275 occidentis partem] partem occidentis *N* occidentis] *corr. ex* occidentes P_7 **1277** iuxta] *marg.* (*perhaps other hand*) *P* I] *corr. ex* Q *K* L *M* orientis concurrit] *corr. ex* orientis occurrit *M* occurit orientis *N* **1278** partem] parte *M* **1279** eclipsimata] eclipsata P_7MN **1280** A] N *N* **1280/1281** umbre – eam] tenebre versus eandem *M* **1285** circulum] *corr. ex* centrum *N* **1286** determinandas] determinandas super idem centrum *M* **1287** sit pars] pars sit *M* occidentis] *corr. ex* orientis *N* **1288** B] B vero *M* aliquo] alio *K* **1289/1290** signorum circulo] circulo signorum *N* **1291** M] A *N* **1293** occidens] *corr. ex* occidentis *K* **1294** hoc] *om. N* contigerit] contingerit *K* obscuri] obscura *PMN* (obscurata *Ba* obscuri E_1) **1295** habuerit] habuit *PN* ut] non *P* gratia] *om. PK* **1299** assignati anguli] anguli assignati *M* **1300** obscura] obscurata *MN* **1302** eclipsis medio] eclipsi medio *K* eclipsi media *M* in the direction of the west and declines from the point then setting according to the quantity of angle PEX, which known arc PX subtends. And if it is near point I at the end of the delay, the beginning of the uncovering runs in the direction of the east, indeed declining from the point then rising in the direction of its latitude according to the quantity of known angle QEK. And if the whole is not eclipsed at the middle of the eclipse, as if it is upon point A, then the shadows decline in the direction opposite the latitude towards that part of the horizon to which the circle passing upon the zodiac's poles and the moon's center descends. And thus indeed are the directions of the moon's darkness disposed.

For the sun's darkness we place the circle of the solar body upon center E and an exterior circle for determining the quantities of the angles. And indeed let A be the direction of the setting point, B the of the rising point. Therefore, if at any of the times of eclipse, the moon is on the ecliptic, as for example upon point M at the beginning, then the part of the sun that begins to be hidden is directed towards the point then setting. And if this happens at the end of the eclipse, the extremity of the obscured part is directed towards the point then rising. But if the moon at any of the times of the eclipse has a latitude, as for example let it be upon point D at the beginning, then the obscurity declines in the direction of the west according to the quantity of the found angle, and this in the direction of the moon's latitude. And if this is at the end of the eclipse, it declines in the direction of the east according to the quantity of the assigned angle, and this always in the same direction as the moon's latitude. And if the whole sun is not obscured at the middle of the eclipse, its obscured part faces the part of the horizon to which the circle passing upon the sun's center, the moon's apparent place, and the zodiac's poles descends. But if the whole sun is obscured at the middle of the eclipse, the darkness has no direction because there is not a circle passing upon the two places of the sun and moon. For the



et Lune. Nam visus locus Lune tunc ipse locus Solis. Et inclinationes quidem tenebrarum sic se habent.

Explicit hic sextus liber et sexti glosa textus.

1304 tunc] tunc et *M* Solis] Solis est *N* **1305** habent] habent et cetera *N* **1306** Explicit – textus] explicit liber sextus P_7K explicit Almagesti minor finitus anno Christi 1434^{o} *M* Laus deo qui mihi favisti ceptis imponere finem. Laus et honor tibi sint astrorum eterne volutor. *N* moon's apparent place is then the very place of the sun. And thus indeed are the inclinations of the darkness disposed.

Here ends the sixth book and the gloss of the sixth text.

Part III

Commentary on the Text and Figures

Commentary on the Text

Book I

Preface. This passage does not have any closely corresponding passages in the *Almagest* or Albategni's *De scientia astrorum*. It summarizes some of the main conclusions of *Almagest* I.3–8, but its language bears more similarities to Martianus Capella's *De nuptiis Philologiae et Mercurii* and other early works than to Gerard's translation of the *Almagest*.[1] It is very possible that the preface was not an original part of the *Almagesti minor* or was taken by the author from another source. Besides the differences in wording, it is clearly arranged very differently than the beginnings of the other books. The other books begin with lists of principles, but here the text weaves together several principles into a few sentences. From the remaining books, one would expect to find some sort of a list, as is found in P_{16} (see Appendix). It is almost certain, however, that P_{16}'s scribe wrote that list with the normal preface in front of him. One might also expect the preface to include definitions of more of the major astronomical terms that are used in Book I, such as 'declinatio', 'circulus signorum', and 'equinoctialis.' That the preface is written in *P* in a different hand than the rest of the text could possibly be seen as an indication of its later addition to the work; however, the preface's presence in manuscripts from every part of the stemma shows that it must have been either original or a very early addition.

I.1. This corresponds to the first proof of *Almagest* I.9 (1515 ed., f. 5v, 1st and 2nd paragraphs). This is only the bare outline of a proof, but the propositions cited are ones required for Ptolemy's proof. Although it is not stated, it is

[1] Passages with similar wording in Martianus Capella include *De nuptiis philologiae et Mercurii*... VI § 590: 'Formam totius terrae non planam ... sed rotundam, *globosam* etiam ...'; and *De nuptiis* VIII, § 814 '... terram *in medio imoque defixam* aternis coeli raptibus circumcurrens circulari quadam ratione discriminat... sed suis fluctibus adhaerentes naturas *undiquesecus globoso* ambitu orbibusque diffundi' (Dick and Préaux, *Martianus Capella*, pp. 292 and 430, my emphases). Martianus also uses the word or phrase 'undiquesecus' several times, e.g. in VI, § 599 and 601, as well as 'machina' to refer to the universe and 'torqueo' for the action of the heavens, e.g. II § 201 (Dick and Préaux, *Martianus Capella*, pp. 76 and 296–97). Macrobius, in his *Commentarium in somnium Scipionis*, Liber I, Ch. 9, § 10 and Ch. 14, § 23, uses 'ἀπλαυής' to mean the sphere of the fixed stars or the outermost sphere (Eyssenhardt, *Macrobius*, pp. 523 and 544). This word is also found with the same meaning in glosses to Bede's *De natura rerum*, Pars I, Ch. 14 (Migne, *Patrologia Latina, Tomus XC: Venerabilis Bedae Tomus Primus*, pp. 218 and 222) among other places.

necessary for this proof that the figures inscribed in the circle are equilateral. Alternate proofs are found in *Ba* and *T*, and there is also an addition in W_1 (see Appendix).

I.2. This corresponds to a passage in *Almagest* I.9 (1515 ed., f. 5v, the 2nd full paragraph). The argument is the same as Ptolemy's, but it is less detailed. Alternate proofs are found in *Ba* and *T* (see Appendix).

I.3. This corresponds to a passage in *Almagest* 1.9 (1515 ed., f. 6r, the 1st paragraph). This is an outline of Ptolemy's proof. It is one of the few examples of a proof that is less complete than Ptolemy's proof, which even contains a generalized conclusion. Alternate proofs are found in *Ba* and *T* (see Appendix).

I.4. This corresponds to a section of *Almagest* I.9 (1515 ed., f. 6r, the 2nd paragraph). Again, the proof is barer than Ptolemy's. The dependence on Gerard's translation is suggested by the similarity between the phrases 'AD facta communi' here and 'facta AD communi' in the *Almagest*. Alternate proofs are found in *Ba* and *T* (see Appendix).

I.5. This corresponds to a passage of *Almagest* I.9 (1515 ed., f. 6r, the 3rd paragraph). The argument generally follows Ptolemy's. Ptolemy provides a generalized statement of what was proved, but the wording does not match that of the enunciation here. A possible connection to Gerard's translation is seen in phrases such as the *Almagesti minor*'s 'residui arcus de semicirculo' and the *Almagest*'s 'arcus residui semicirculi', both of which are used to refer to supplements. Alternate proofs are found in *Ba* and *T* (see Appendix).

I.6. The first paragraph corresponds to a passage of *Almagest* I.9 (1515 ed., f. 6v, the 1st paragraph). The argument follows that of the *Almagest*. The second to fourth paragraphs correspond to calculations and values given after the proofs in *Almagest* I.9 (1515 ed., ff. 5v–6r) that correspond to *Almagesti minor* I.1–5. While Ptolemy calculates the values of arcs after each proof, the *Almagesti minor* separates the discussion of values from the proofs and gives all of the values in this one passage. The fifth, sixth, and seventh paragraphs correspond to a passage in *Almagest* I.9 (1515 ed., f. 6v, the 1st full paragraph). The argument here is basically that of the *Almagest*, but the upper limit for the size of the chord of 1° is calculated further than in the *Almagest* – to $1^p\ 2'\ 50''\ 40'''$ instead of $1^p\ 2'\ 50''$. The argument here shows that the chord of 1° is greater than $1^p\ 2'\ 50''$ and less than $1^p\ 2'\ 50''\ 40'''$. By calculating the latter value more precisely, the author is able to avoid Ptolemy's seemingly contradictory statement that the chord of 1° is at one time greater and at one time less than 1^p $2'\ 50''$. However, Group 3 has the shorter, alternate passage that is closer to the argument of the *Almagest*. Group 3 also has an addition at the end of the proposition that describes *Almagest* I.11's table of arcs and chords (1515 ed., ff. 7r–8v). For Group 3's alternate passage and addition, as well as additions and alternate texts from *Ba* and *T*, see the Appendix.

I.7. This corresponds to the first proof of *Almagest* I.12 (1515 ed., f. 9v, the 1st full paragraph). While the proof is more threadbare than that in the *Almagest*, it is essentially the same argument, and the author's concern for generalization is clear from the long and awkward enunciation. This proof is the first of the six lemmata for the Menelaus Theorem, the two versions of which are I.13 and I.14. While some other commentaries explain the use of compound ratios, the author, like Ptolemy himself, does not address the issue and assumes that his readers will understand the implications of having ZD as a 'middle' between GD and EH.[2] Alternate proofs are found in *Ba* and *T* (see Appendix).

I.8. This corresponds to the second proof of *Almagest* I.12 (1515 ed., f. 9v, the 2nd full paragraph). Again, the proof is essentially that of the *Almagest* with some rearrangement of steps. Alternate proofs are found in *Ba* and *T* (see Appendix).

I.9. This corresponds to the third proof of *Almagest* I.12 (1515 ed., f. 9v, the 3rd full paragraph). This very short proof matches the argument of Ptolemy. Mistakes in the figures in several manuscripts may have led to textual errors in manuscripts including *K* and *M*. The word 'sine' is used here for the first time in this work although it is not defined until the end of I.16.

I.10. This corresponds to the fourth proof of *Almagest* I.12 (1515 ed., f. 9v, the 4th full paragraph). The argument follows that of Ptolemy. The author uses the concept of the denomination of a ratio in order to explain a step that Ptolemy does not explain. An alternate proof is found in *T* (see Appendix).

I.11. This corresponds to the fifth proof of *Almagest* I.12 (1515 ed., the paragraph going from f. 9v to f. 10r). The argument follows that of Ptolemy. An alternate proof is found in *T* (see Appendix).

I.12. This corresponds to the sixth proof of *Almagest* I.12 (1515 ed., f. 10r, the 1st complete paragraph). The proof is essentially that of Ptolemy. This proof and I.10, as well as the corresponding proofs in the *Almagest*, are not used and appear to be remnants of a pre-Ptolemaic use of the Menelaus Theorem.[3] The 'greater' in the enunciation appears to be a mistake as there is no reason why arc GB is necessarily greater than arc AB. Perhaps 'maior' is taken in the sense of 'antecedent.' Alternatively, the author may be relying too much upon the appearance of the figure. An alternate proof is found in *T* (see Appendix).

I.13. This is the disjunct Menelaus Theorem, which is in *Almagest* I.12 (1515 ed., f. 10r, the 2nd full paragraph). The proof outlines many of the steps but fol-

[2] For the Menelaus Theorem's tie to compound ratios in the Middle Ages, see Zepeda, *The Medieval Latin Transmission*.

[3] Sidoli, 'The Sector Theorem Attributed to Menelaus', p. 60.

lows Ptolemy's argument. Like Ptolemy, the author does not consider the cases in which HB and AD are parallel or meet on the other side of the sphere.[4] The references to propositions make it clear that the numbering of propositions was original. *T* has a long addition to the proof (see Appendix).

I.14. This is the second of the two proofs that together were referred to as the 'Menelaus Theorem', the 'sector figure', or the 'kata', which is also found spelled 'katha', 'catha', or 'alkata.' This is the 'conjoined kata', and the previous proposition is the 'disjunct kata.' While Ptolemy lays out the conclusion of this theorem at the end of *Almagest* I.12 (1515 ed., f. 10r, the 2nd full paragraph), he does not provide a proof; however, from the lemmata Ptolemy provides, it is clear that Ptolemy intended a proof similar to that given here. A diagram that matches this proof conceptually is found in at least some of the early manuscripts containing Gerard's translation of the *Almagest*, but the diagram letters used in this proof do not match those in that figure (e.g. Paris, BnF, lat. 14738, f. 15r) or the statement given by Ptolemy. This appears to have caused some confusion. Some copyists seem to have tried to make sense of this text with regard to the letters of Ptolemy's text and the diagram for the other part of the Menelaus Theorem. In manuscript *M*, the changes are not carried out far enough to make the argument work properly, while manuscript *N* has a similarly reworked figure but changes the text such that it gives the same mathematical argument as in the standard text. Again, the author does not give a proof that covers all the cases. Indeed, there are 13 different cases because the two lines EG and HA could be parallel to each other or meet on the opposite side of the diagram, as could the pair GZ and HI and the pair EZ and HT. While the proofs for many of these cases can be carried out with the two cited propositions, not all of them can. While some commentators proved more or all cases, other commentators provided universal proofs, i.e. ones that applied to any configuration of the figure.[5] *T*'s added treatment of the Menelaus Theorem is much more complete (see Appendix).

I.15. This corresponds to the first paragraph of *Almagest* I.12 (1515 ed., ff. 9r-v). The instruments are basically the same as those in the *Almagest* with the simple difference that the *Almagest*'s first instrument has its outer plate in a circular form while the *Almagesti minor*'s is square. This does not affect the function of the instrument, and probably arises from a misunderstanding of the rel-

[4] For examples of proofs for the other cases, see Lorch, *Thābit ibn Qurra. On the Sector-Figure*; and Zepeda, *The Medieval Latin Transmission*.

[5] An example of an *Almagest* commentary that does treat all possible cases is the 'Vatican Commentary', which is found in Vatican, BAV, Vat. lat. 3100 and Vatican, BAV, Vat. lat. 6795. For an examination of this work, its treatment of the Menelaus Theorem, and a partial transcription, see Zepeda, *The Medieval Latin Transmission*, pp. 222–51 and 573–636. Universal proofs are found in Thābit ibn Qurra's *De figura sectore*, versions of which have been edited and discussed in Björnbo, 'Thabits Werk über den Transveralensatz'; and Lorch, *Thābit ibn Qurra. On the Sector-Figure*.

ative clause 'cuius superficies sint quadrate', which Gerald Toomer understands as meaning that the ring 'has a rectangular cross-section.'[6] This proposition contains some of the first pieces of evidence that the author was relying upon Gerard's translation of the *Almagest*: 'per instrumenti artificium'/'per artificium instrumenti', 'lingule'/'linguulas', and 'laterem scilicet ligneum vel lapideum vel eneum quadratum'/'laterem lapideum aut ligneum quadratum', as well as the shared words 'tortuositate', 'piramidales', and 'grossitie.'[7] The two words 'tornatiles piramidales' are particularly telling. In the corresponding location of the *Almagest*, the A-Klasse of Gerard's translation has 'piramidales', and in the B-Klasse Gerard changed this to 'tornatiles.'[8] It thus appears that the *Almagesti minor*'s author used an *Almagest* manuscript that bore both Gerard's first choice and his subsequent correction. The non-Ptolemaic values of the declination probably come from Pseudo-Thebit's *De motu octave spere* (for the Indians' value),[9] Albategni's *De scientia astrorum*;[10] and a table of declinations that was part of the Toledan Tables (for Arzachel's value).[11]

I.16. This corresponds to *Almagest* I.13 (1515 ed., ff. 10r-v). This argument is set up as a metrical analysis rather than a calculation, as Ptolemy has. Unlike the prior proofs, which are more bare than Ptolemy's, this proof is more formal with a clearly distinguishable corollary, exemplification, construction, specification, argument, and conclusion. The author does not outline all of his steps here explicitly, but the main argument is similar to Ptolemy's. The method of dealing with compound ratios is different than Ptolemy's. From the outline of the proof, the steps pertaining to compound ratios can be seen:

(sin AZ : sin AB) comp. of (sin ZT : sin HT) & (sin EH : sin BE)	(I.14),
but sin ZT = sin BE,	
therefore, (sin AZ : sin AB) comp. of (sin BE : sin HT) & (sin EH : sin BE)	
or (sin AZ : sin AB) comp. of (sin EH : sin BE) & (sin BE : sin HT)	(commutative property).
But, (sin EH : sin HT) comp. of (sin EH : sin BE) & (sin BE : sin HT)	(def. or property of compound ratios)
therefore, sin AZ : sin AB :: sin EH : sin HT	(something like *Elements* I, c.n. 2).

[6] *Almagest*, 1515 ed., f. 9r; and Toomer, *Ptolemy's Almagest*, p. 61.

[7] *Almagest*, 1515 ed., f. 9r.

[8] Here my representative of the A-Klasse is Paris, BnF, lat. 14738, f. 14r, and my representative of the B-Klasse is Florence, BML, Plut. 89 sup. 45, f. 9r.

[9] Millás Vallicrosa, *Estudios sobre Azarquiel*, pp. 499–500.

[10] Albategni, *De scientia astrorum* Ch. 4 (1537 ed., f. 8r).

[11] Pedersen, *The Toledan Tables*, Table BA21, pp. 961–64. Pedersen explains that this value is not the one that Arzachel seems to have usually used and that this value seems to have been later inserted into some manuscripts of the Cb and Cc versions of the *Canons*.

Ptolemy merely says to subtract a known composing ratio from the known composed ratio in order to find the unknown composing ratio, but he does not specify what that means. The author here, on the other hand, outlines a process that more clearly relies on the insertion of a middle to create a compound ratio. The two main theories of compound ratios differed on whether this was the definition of compounding or only a property, but even those who defined compounding through denominations proved as one of the primary propositions that the insertion of middles leads to a statement of composition.[12] The last step in the argument relies upon it being clear that two things made up of the same things are equals. This is similar to the common notion that if equals are added to equals, the wholes are equals. The rule given in the corollary matches that in Albategni's *De scientia astrorum* Ch. 4 (1537 ed., f. 8v), but it is worded differently.

I.17. This corresponds to *Almagest* I.14 (1515 ed., ff. 11r-v). While this proposition uses the same figure and general line of argumentation as the *Almagest*, it is a metrical analysis, and it also deals with compound ratios differently. As in the previous chapter, Ptolemy subtracts a known composing ratio from the known composed ratio to find the remaining composing ratio. The author here does not speak of the subtraction of ratios, but moves from the statement of composition to a proportion. He seems to be calling upon a prior known fact that if (A : B) comp. of (C : D) & (E : F), then (A × D ÷ C) : B :: E : F. While the truth of this statement can be easily seen from algebraic manipulation of symbols, it would not have been obviously true to a medieval mathematician.[13] This rule and its derivation can be found in a set of notes by Campanus that are found in two *Almagest* manuscripts.[14] The rule for finding right ascensions that is found in the corollary matches that in *De scientia astrorum* Ch. 5 (1537 ed., f. 9r), but it is worded differently.

Book II

II.1. This corresponds to parts of *Almagest* II.2 and II.3 (1515 ed., f. 12r's 1st full paragraph and f. 12v's 1st full paragraph). The author chose to change Ptolemy's order of proofs (this is the third proof in *Almagest* II), possibly because of the three quantities that are of chief concern in the first four proofs (i.e. the pole's altitude, the arc on the horizon between rising points, and half the

[12] For example, Pseudo-Jordanus and Campanus' treatises on compound ratio both prove this property as their second proposition. Busard, 'Die Traktate De proportionibus', pp. 206 and 213–14. Also, see Zepeda, *The Medieval Latin Transmission*.

[13] For a recreation of a possible justification for this fact, see Zepeda, *The Medieval Latin Transmission*, pp. 179–80 n. 361.

[14] Vatican, BAV, Barb. lat. 336; and Paris, BnF, lat. 7256, 'Method 2C' in Zepeda, *The Medieval Latin Transmission*, pp. 157–58 and 409–10.

difference between the time of the shortest and equinoctial day), the pole's altitude is the most easily observed. Because this is the first of a set of proofs that use the same figure and that are laid out in a different order than Ptolemy's, the *Almagesti minor*'s author gives here the common part of these proofs that Ptolemy provides in his first proof (*Almagest* II.2) of the series. Again the lettering and general argument is similar to Ptolemy's, but this is a metrical analysis. Also, from the order of steps in the corollary, the process for finding the unknown term in the statement of composition seems to be similar to that of I.17. Although the author does not provide enough details to be certain of exactly how he proceeds, it is clear that the author does something equivalent to first concluding from a statement that a compound ratio is composed of others, that (1^{st} term × 4^{th} ÷ 3^{rd}) : 2^{nd} :: 5^{th} : 6^{th} and then using the rule of three to find the unknown 5^{th} term. Although the author states that he will find the length of the shortest day, he follows Ptolemy in stopping short of this goal and finding instead the difference between the shortest day and the equinoctial day.

II.2. This corresponds to *Almagest* II.2 (1515 ed., f. 12r). Because this is not the first of the series of four proofs as it is in the *Almagest*, the preliminary parts of the proof up to the pointing out of the sector figure are not given here and are merely assumed from the previous proof. The method here of dealing with compound ratios does not follow Ptolemy's and is similar to that above in I.16.

II.3. This corresponds to part of *Almagest* II.3 (1515 ed., the paragraph going from f. 12r to f. 12v). This has the same figure letters and basic argument that Ptolemy has; however, it is a metrical analysis and the compound ratio is treated as in I.17 above. The author does not explain how each of the known terms in the compound ratio are known. Arc ET is known by hypothesis, and then through II.2 EH can be found along with its complement HB.

II.4. This corresponds to a section of *Almagest* II.3 (1515 ed., f. 12v, the 2^{nd} full paragraph). It uses the same letters and general argument as Ptolemy does, except that it deals with ratios as in I.16 instead of subtracting ratios as Ptolemy appears to do.

II.5. This corresponds to a passage in *Almagest* II.3 (1515 ed., f. 12v, the 3^{rd} full paragraph). The argument follows that of the *Almagest* and uses the same figure.[15]

II.6. This corresponds to *Almagest* II.5 (1515 ed., f. 13r) and to the first part of *De scientia astrorum* Ch. 10 (ff. 14r-v). The rules in the corollary are taken from Albategni, who did not give proofs, and they only apply to the upright gnomon. That the enunciation refers to both types of gnomons while the rule does not apply to both may have been misleading to readers of the *Almagesti*

[15] The 1515 edition's figure has some points labeled differently, but the *Almagesti minor* matches the labels in Paris, BnF, lat. 14738, f. 19r.

minor, and this also suggests the possibility that the author did not understand the subject completely. The proofs of the two parts of the proposition are somewhat similar to Ptolemy's, but they were created by the author to not only find the sought quantities, but also to prove Albategni's rules. The author uses a figure that has the sines of the relevant arcs of altitude, unlike Ptolemy's figure, and he argues through similar triangles instead of through circumscribed right triangles as Ptolemy does. While most of the points common to the *Almagesti minor*'s figure and the *Almagest*'s have the same labels, each has several lines and points that are needed for their respective proofs and that are not in the other.[16] There are similarities to Albategni's text. Although Ptolemy only offers a sketch of the proof for the converse part of the proposition, the author here gives a detailed argument for it, and Albategni provides rules for this converse part of the proposition. The author of the *Almagesti minor* gives a proof that shows the validity of one of these rules, but he does not explicitly state the rule itself. Also, while Ptolemy does not mention the horizontal gnomon or the 'turned' shadow at all, Albategni treats them and gives rules for finding the 'umbra versa' (this term is used by Albategni) from the sun's altitude and vice versa, and the author of the *Almagesti minor* mentions them. However, he does not provide proofs concerning them.[17]

II.7. This corresponds to *Almagest* II.6 (1515 ed., f. 13v, the 1st paragraph) and to part of *De scientia astrorum* Ch. 6 (1537 ed., f. 9v). It is a much shorter discussion than in the *Almagest*, but the content does not stray from its source.

II.8. This corresponds to part of *Almagest* II.6 (1515 ed., f. 13v, the 1st paragraph) and the parts concerning the stars also corresponds loosely to a section of *De scientia astrorum* Ch. 6 (1537 ed., f. 10r). The contents of this proposition and the previous one are found intermingled in Ptolemy's text; the *Almagesti minor*'s author has separated the statements concerning the characteristics of the equator from those concerning other latitudes. While Ptolemy only describes the phenomena, the author gives explanations using a geometrical figure.

II.9. This corresponds loosely to parts of *Almagest* II.6 (1515 ed., ff. 13v–15v). While Ptolemy lists the properties for a number of different specific latitudes, the author proves this more general proposition.

[16] The figure in the 1515 edition has more differences than that in Paris, BnF, lat. 14738, f. 19v.

[17] Perhaps the reason that the author does not give the rules for the horizontal gnomon is that one of the rules is corrupt in Plato's translation (see Albategni, *De scientia astrorum* Ch. 10, 1537 ed., f. 14v; and *P*, f. 15r). In Nallino's translation, the passage makes mathematical sense (Nallino, *al-Battānī*, vol. I, p. 22).

II.10. This corresponds loosely to a section of *Almagest* II.6 (1515 ed., from f. 13v's last paragraph to f. 14r's 5th full paragraph) and to part of *De scientia astrorum* Ch. 6 (1537 ed., f. 10r). Ptolemy's second to sixth latitudes fall under the criterion of this proposition. While Ptolemy treats 39 different latitudes, Albategni and the author of the *Almagesti minor* treat special latitudes (e.g. the equator, the arctic circle, and the north pole) and the classes of latitudes that fall between these.

II.11. This corresponds to part of *Almagest* II.6 (1515 ed., from f. 14r's last paragraph through f. 15r's 13th full paragraph), and the second part of the proof also corresponds to part of *De scientia astrorum* Ch. 6 (1537 ed., f. 10v). While Ptolemy talks about the specific latitude 23° 51′ 20″, the author does not mention specific values. The enunciation only mentions the tropic, but this proposition treats the tropic and the class of latitudes beyond the tropic.

II.12. This corresponds loosely to a section of *Almagest* II.6 (1515 ed., from f. 15r's last full paragraph through the 5th full paragraph of f. 15v) and to *De scientia astrorum* Ch. 6 (1537 ed., f. 10v). Again, the author's treatment of the special latitude and then a general class is more in line with Albategni's approach than with Ptolemy's.

II.13. This corresponds to the end of *Almagest* II.6 (1515 ed., f. 15v, the 6th full paragraph) and to part of *De scientia astrorum* Ch. 6 (1537 ed., f. 10v).

II.14. This corresponds to part of *Almagest* II.7 (1515 ed., f. 15v, the chapter's 1st full paragraph). The figure and argument follow Ptolemy's.

II.15. This corresponds to passages in *Almagest* II.7 (1515 ed., from the last paragraph of f. 15v to f. 16r's 1st full paragraph, and from the last paragraph of f. 16r through the 1st paragraph of f. 16v). The proof begins similarly to the *Almagest*'s, although here our author explains at greater length how various arcs' oblique and right ascensions compare to each other. The author provides a proof using two figures that shows that the single figure used by Ptolemy matches the astronomical situation. Much of the argument centers on showing that the triangle HLE created for each of the two arcs of the ecliptic is indeed one triangle. The author has to argue that the conditions are met for there to be congruency of triangles through angle-side-side, which does not work universally.[18] The two-fold figure appears to have confused some scribes. *P*'s scribe thought that only the first figure was for this proof and that the second was for II.16. This appears to have led to the mislabeling of several points in these

[18] That the author states the qualifiers in a more general manner than the present situation specifies (e.g. angle HLE is right in both figures, but his qualification is that this angle is right or oblique) suggests that he is using a specific source here. The qualifications he states, however, do not match those of Menelaus' *Sphaerica*, I.13 (Vatican, BAV, Reg. lat. 1261, f. 226r).

figures. *M* used Ptolemy's figure, which contains both arcs of the ecliptic, but the lack of separate figures does not accord with the text.

II.16. This corresponds to a section of *Almagest* II.7 (1515 ed., f. 16r, from the 1st full paragraph to the last). The basic argument agrees with Ptolemy's; however, Ptolemy uses a slightly different figure that is not labeled in the same way as the preceding figure, while the *Almagesti minor*'s author reuses the first figure of II.15. Also, the author deals with the compound ratio as in I.17.

II.17. This corresponds to part of *Almagest* II.7 (1515 ed., f. 16v, the 1st full paragraph). The passage follows that in the *Almagest*, and some of the wording is very similar; e.g. the almost identical phrases 'quantam voluero' (*Almagesti minor* II.17) and 'quantum voluero' (1515 *Almagest*, f. 16v) are used for the same arc. The same figure is used.[19]

II.18. This corresponds to the second half of *Almagest* II.7 (1515 ed., from 16v's 2nd full paragraph to f. 17v). While the basic argument is the same, i.e. the same figure and the same sector figure are used, the ways of proceeding from the statements of composition are quite different. Ptolemy argues that by using the table of declinations, the ratio of the chord of double ET to the chord of EL can be determined for each 10° section of the ecliptic regardless of latitude. Because ET is known for each latitude, EL can then be found for a section of the ecliptic and a given latitude. Our author's use of I.17 allows him to quickly reach a proportionality and a simple rule for calculation. Note that here two ratios compose (in the active voice) another ratio, while before a ratio is always composed of others. The proof uses a premise similar to the common notion that if equals are subtracted from equals, equals remain; it is that if there are two statements of composition in which the composed ratio in one is the same as the composed ratio of the other, and in which a composing ratio of the one is the same as a composing ratio of the other, then the other composing ratios must be the same ratio. (In symbols, if ratio A is composed of ratio B and C while ratio A is also composed of ratio B and D, then ratios C and D are equal.)

II.19. This corresponds to *Almagest* II.9 (1515 ed., f. 19r). This generally accords with the methods of Ptolemy, but the author does not follow Ptolemy in giving a set of rules for the conversion of equal and unequal hours, nor does he mention the characteristics of locations based on their longitude. That both rules in the last paragraph have mistakes suggests that the errors are due to the author, not a subsequent scribe.

[19] *Almagest*, 1515 ed., f. 16v, has 'I' in place of 'K', but this change is not found in Paris, BnF, lat. 14738, f. 24v.

II.20. This corresponds to a short section in the middle of *Almagest* II.9 (1515 ed., f. 19r).

II.21. This corresponds loosely to a very short statement near the beginning of *Almagest* II.10 (1515 ed., f. 19v). This is the outline of a proof for a proportion that Ptolemy gives.

II.22. This corresponds to a section of *Almagest* II.10 (1515 ed., f. 19v, the 1st paragraph). The argument and figure follow those of Ptolemy. The use of Gerard's translation of the *Almagest* is seen in the specification – compare it to the *Almagest*'s 'Dico ergo quod angulus KHB equalis est angulo ZTE.'[20]

II.23. This corresponds to a proof in *Almagest* II.10 (1515 ed., f. 19v, the 2nd paragraph). The figure and argument follow that of Ptolemy. What looks to be the specification does not state the actual aim of the proof. The last step of the argument is not spelled out.

II.24. This corresponds to a section of *Almagest* II.10 (1515 ed., f. 19v, the 3rd paragraph). Much of the wording is taken almost word for word from Gerard's translation of the *Almagest*. Compare to the *Almagest*'s 'Et sit punctum ipsum A tropicum hiemale. Et describam supra polum A secundum spatium lateris quadrati medietatem circuli supra quam sint BED. Et quia orbis meridiei qui est ABGD est descriptus supra duos polos AEG et BED, erit arcus ED quarta circuli.'

II.25. This corresponds to part of *Almagest* II.10 (1515 ed., the paragraph going from f. 19v to f. 20r). This follows the *Almagest* very closely and much of the wording is taken directly from Gerard's translation. Both Ptolemy and the *Almagesti minor*'s author leave a step that is needed to establish that arc AZ is a quarter circle unstated. For the reason given, it is known that ED and AE are quarter circles, and then it can be argued that because AE is a quarter circle of the equator starting from an equinox, AZ will be a quarter of the ecliptic. The missing step caused some readers to change 'AZ' to 'AE', as in *K* and *M*. The author here does not stop at the angle found by Ptolemy, angle DAZ, but he breaks with Ptolemy's practice of finding the northeastern angle and continues to find the angle BAZ, which could be either the southeastern or northwestern angle.

II.26. This corresponds to the last part of *Almagest* II.10 (1515 ed., f. 20r, the 2 full paragraphs). This follows the text of the *Almagest* closely, and at times the wording is taken directly from Gerard's translation. The figure is mirrored from that in the *Almagest*, so the found angle is the northwestern angle instead of Ptolemy's usual northeastern angle.

[20] Paris, BnF, lat. 14738, f. 29v.

II.27. This corresponds to part of *Almagest* II.11 (1515 ed., f. 20v, the 1st paragraph). This is very close to the *Almagest*, and parts of it are taken directly from Gerard's translation.

II.28. This corresponds to a passage in *Almagest* II.11 (1515 ed., f. 20v, the 1st and 2nd full paragraph). This follows the *Almagest* very closely, and some of the wording is taken directly from Gerard's translation.

II.29. This corresponds to part of *Almagest* II.11 (1515 ed., f. 20v, the 2nd full paragraph). The argument is similar to that in the *Almagest*, and some of this proof is taken directly from Gerard's translation. Ptolemy uses one specific example, while the author of the *Almagesti minor* offers a general proof for the two cases when the latitude is less than and greater than the maximum declination. The argument for determining arcs DG and DB is slightly different than that in the *Almagest*, as Ptolemy does not use points K and T, the zeniths under the earth, and takes arc DZ as being known because the latitude is known. Also, the author of the *Almagesti minor* explains more clearly that the process involves addition or subtraction depending on the latitude.

II.30. This corresponds to a section of *Almagest* II.11 (1515 ed., the paragraph going from f. 20v to f. 21r). This follows the general argument of the *Almagest*, and some passages are taken directly from Gerard's translation; however, the way of dealing with the statement of composition is different. Ptolemy does not use the statement of composition that comes directly from the Menelaus Theorem, but instead uses one of its modes so that he can follows his *modus operandi* of subtracting ratios. The author of the *Almagesti minor* uses the statement of composition from the Menelaus theorem, but inverts the ratios so he can turn the statement of composition into a proportion.

II.31. This corresponds to part of *Almagest* II.12 (1515 ed., f. 21r, the 1st paragraph of the chapter). This follows the *Almagest* closely and some passages are taken directly from Gerard's translation.

II.32. This corresponds to a section of *Almagest* II.12 (1515 ed., from f. 21r's last paragraph to f. 21v's 1st full paragraph). This follows the *Almagest* closely and much of the text for the second case is taken directly from Gerard's translation.

II.33. This corresponds to proofs in *Almagest* II.12 (1515 ed., f. 21v, the 2nd and 3rd full paragraphs). The enunciation actually should only apply to the second case because in the first case the angles formed by the ecliptic and the circles of altitude do not exceed double angle DEZ by two right angles as the author states, but indeed are exceeded by angle DEZ by two right angles. That a mistake is made in both the first case and the enunciation show that the author misunderstood this and that the error is not just a scribal mistake. Our author presents the two cases in the reverse of Ptolemy's order, but besides the mistake in the case in which A is north, the proofs follow Ptolemy's and much of each is taken directly from Gerard's translation. The last paragraph seems to be the author's own creation.

II.34. This corresponds to proofs in *Almagest* II.12 (1515 ed., f. 21v, the 4th full paragraph). This follows the argumentation of the *Almagest*, and some text is taken directly from Gerard's translation. *N* has an addition of approximately 60 words that explains in more detail how arc AZ is known. This added text is only found in *N*, in the margins of *Pr* as a note, and as an addition on a separate leaf in *M*.

II.35. This corresponds to a section of *Almagest* II.12 (1515 ed., the paragraph going from f. 21v to f. 22r). The general line of argumentation, i.e. using the conjoined sector figure, matches that in the *Almagest* and some of the wording is clearly taken from Gerard's translation. The arrangement of this proof is unusual in that the rule is given after the proof, instead of immediately after the enunciation. This proof involves a new situation involving compound ratios. While there is not a common term in the composing ratios that allows for the sort of simplification done in I.16, matters can be simplified because the consequent of the composed ratio is equal to the consequent of one of the composing ratios. Using the modes of compound ratio would permit the author to rearrange the terms in a way that would allow him to use his method in I.16. If the author was Walter of Lille and if he was also the author of the treatise on compound ratios, as I have argued in the introduction, then he probably proceeded in that way. Another way that he could have reached this is the method that he usually applies in I.17 when there is an unknown in one of the composing ratios and the composing ratios cannot be dealt with as in I.16. Following this procedure, he would find that (sin BZ × sine TH ÷ sine TZ) : radius :: sine EH : radius. Since quantities that have the same ratio to the same third quantity are equal, the rule immediately follows and the additional process of finding a fourth proportional need not be done. The rule given here is essentially the same as the one given by Albategni in *De scientia astrorum* Ch. 39 (1537 ed., ff. 49v–50r) except that the order of operations is different and Albategni has an additional multiplication by a quarter circle and division by a quarter circle that cancel each other out. Albategni's rule thus involves six quantities and is more clearly derived from the sector figure although no such geometrical proof is provided in his work.

II.36. This corresponds to the last part of *Almagest* II.12 (1515 ed., f. 22r, the full paragraph). The basic proof in the first paragraph follows that of the *Almagest*, and some of the text is taken directly from Gerard's translation. As usual, the author of the *Almagesti minor* does not subtract ratios as Ptolemy does. Like the previous proposition, this proof is unusual in having the rule for calculation after the proof. The rule in the second paragraph is taken from *De* *scientia astrorum* Ch. 39 (1537 ed., f. 50v),[21] and some of the wording is taken directly from this source.

Book III

III.1. The first paragraph corresponds loosely to *Almagest* III.1 (1515 ed., ff. 26r–28v). While the topic is the length of the year, the two passages have different emphases. The *Almagest* and the *Almagesti minor* both explain that a year is the return of the sun to the same equinox or solstice and that the equinoxes should be observed for more accurate results; however, the bulk of *Almagest* III.1 consists of Ptolemy's evaluation of Hipparchus' observations that led him to ask whether years might be of varying lengths, and Ptolemy's discussion of the limits of instrumental precision that show that the year's length can be taken as constant. In the *Almagesti minor*, little is found on these topics. Instead, the author outlines the method of observing and calculating the time of an equinox, which Ptolemy does not do. In fact, the first paragraph of *Almagesti minor* III.1 corresponds much more closely to part of *De scientia astrorum* Ch. 27 (1537 ed., f. 26v), and some of the language matches Plato of Tivoli's translation of Albategni.

The second paragraph corresponds to a short passage in *Almagest* III.1 (1515 ed., f. 27v). With the theory of trepidation in mind, the author adds the caveat that it is only true that a longer interval between observations produces more accurate results if Ptolemy is correct that the year is of a constant length.

The third paragraph corresponds to a passage of *De scientia astrorum* Ch. 27 (1537 ed., ff. 26v–27v), and in a few instances, the wording matches this source. Also, while Ptolemy states that Hipparchus saw that the year was slightly less than 365 ¼ days,[22] here the author of the *Almagesti minor* follows Albategni's version of history, in which Hipparchus claimed that the year was 365 ¼ days although his observations should have shown him that this number was slightly too large.[23] While Albategni argues that his length of the year and Ptolemy's are close enough that the difference can be explained by Ptolemy's use of a fairly small length of time, and that there is no reason to think that the length of the year varies,[24] our author deviates from his source and uses the discrepancies between astronomers to argue that it is at least reasonable to believe the length of a year varies. This passage of the *Almagesti minor* appears

[21] Albategni, *De scientia astrorum*, 1537 ed. has an omission, but the complete rule is found in *P*, f. 47v.

[22] *Almagest*, 1515 ed., f. 26v.

[23] Albategni, *De scientia astrorum*, 1537 ed., f. 26v.

[24] Albategni, *De scientia astrorum* Ch. 52 (1537 ed., ff. 80v–81r).

to be the source of passages in Guillelmus Anglicus' *Astrologia* and of Grosseteste's *Compotus*.[25]

The fourth paragraph discusses Thebit's [i.e. Thābit ibn Qurra] theory of trepidation. The author gives the value for the mean year (rounded to seconds) that is given in *De anno solis*, which was commonly misattributed to Thebit, and although the text is not very clear, he iterates Pseudo-Thebit's claim that the anomalistic or solar year, rather than Ptolemy's tropical year, should be the measure of the true year.[26] The *Almagesti minor*'s author's source for Thebit's trepidation model seems to be the *De motu octave spere*, which was also misattributed to Thebit, but the *Almagesti minor*'s author offers few details of the trepidation model, which he perhaps did not understand well.[27] Thebit's value for the length of the year is found in some text accompanying the Toledan Tables in several manuscripts, and it can be extrapolated from the solar mean motion tables.[28] That the Toledan Tables are described as being of very recent composition may only mean that they were made recently compared to the works of the ancients. The use of the word 'novissime' and the attribution to Arzachel in connection with the Toledan Tables suggests the possibility that the author of the *Almagesti minor* read Raymond of Marseilles' *Liber cursuum planetarum*.[29]

III.2. This corresponds loosely to the last section of *Almagest* III.1 (1515 ed., f. 28v) and the last section of *De scientia astrorum* Ch. 27 (1537 ed., f. 27v). While Ptolemy (*Almagest* III.2) and al-Battānī only give tables for collected years, separate years, months, days, and hours, our author instructs the reader to also give values for fractions of hours, which suggests that the author had in mind the Toledan Tables.[30]

III.3. This corresponds to the first section of *Almagest* III.3 (1515 ed., from f. 29r to the 1st full paragraph of f. 29v). The argument follows Ptolemy's and some of the wording is taken directly from Gerard's translation. Ptolemy leaves the last steps of the first demonstration for the reader to complete, but the author here supplies the remaining steps. The scribes of *P*, *B*, and especially *K* have a variety of misspellings of the word 'epiciclus' in *Almagesti minor* III.3–7, but the scribes eventually learned to consistently spell it correctly.

[25] See Ch. 1 and 7 of the Introduction above; and Steele, *Opera hactenus inedita Rogeris Baconis*, pp. 213–16.

[26] Carmody, *The Astronomical Works of Thabit b. Qurra*, pp. 74–75.

[27] An edition of this is found in Millás Vallicrosa, *Estudios sobre Azarquiel*, pp. 496–509.

[28] Pedersen, *The Toledan Tables*, Tables QB11 and CA01, pp. 1577 and 1144–48.

[29] D'Alverny, Burnett, and Poulle, *Raymond de Marseille*, p. 200: '… novissime autem quendam Toletanum hac in doctrina perspicuum qui a quibusdam Azarchel vel Albateni nuncupatur super annos Arabum et super Toletum …'

[30] Pedersen, *The Toledan Tables*, p. 1148.

III.4. This corresponds to a short passage in *Almagest* III.3 (1515 ed., 29v, in the 1st full paragraph). The argument follows that of Ptolemy and some of the wording is taken from Gerard's translation. For the first part of the proposition, Ptolemy does not prove that angle AZB is smaller than angle GZD. The author here provides a short proof, and *M* and *N* have a further addition (also found in the margins of *Pr*). *P* has some problematic passages, and its scribe appears to have copied this proof rather carelessly.

III.5. This corresponds to a section of *Almagest* III.3 (1515 ed., from the bottom of f. 29v through f. 30r's 1st full paragraph). Much of the proof is taken directly from Gerard's translation. The author makes the apparent motions clearer by adding the circle PQX to represent the ecliptic, which is centered on the earth. *K* has a few careless errors in this passage.

III.6. This corresponds to a short passage and a proof of *Almagest* III.3 (1515 ed., from the bottom of f. 29v to the top of f. 30r and f. 30r's 2nd full paragraph). The argument follows that of Ptolemy and some wording is taken directly from Gerard's translation.

III.7. This corresponds to a section of *Almagest* III.3 (1515 ed., the paragraph going from f. 30r to f. 30v). The argument is from Ptolemy, and some wording is taken from Gerard's translation. In most of the witnesses, it is not clearly expressed that it is proved and not assumed that arcs KZ, AB, and EZ are equal.

III.8. This corresponds to part of *Almagest* III.3 (1515 ed., f. 30v, the two full paragraphs). The argument follows Ptolemy and much of this passage is taken directly from Gerard's translation.

III.9. This corresponds to a section of *Almagest* III.3 (1515 ed., the paragraph running from f. 30v to f. 31r). This follows Ptolemy's argument and some of this passage is taken directly from Gerard's translation. A small difference is that the author here includes the case where the stars are on the line of apsides.

III.10. This corresponds to the last part of *Almagest* III.3 and the first few sentences of III.4 (1515 ed., f. 31r, the 1st and 2nd full paragraphs). This follows Ptolemy's argument and some of the phrasing is similar to that of Gerard's translation. *K* has several errors that would have made it difficult for a reader to understand this proof.

III.11. This corresponds to sections of *Almagest* III.4 (1515 ed., from f. 31r's 1st full paragraph through f. 31v's full paragraph) and *De scientia astrorum* Ch. 28 (1537 ed., ff. 27v–29r). The value for the apogee's position attributed to Arzachel is not actually his. It could have been taken from the Toledan Tables or from canons on them.[31] That Arzachel's solar mean motion differed

[31] For this apogee position, see Pedersen, *The Toledan Tables*, Ca92, Cb141a, Table CA01, Table CE40, and Table DA01, pp. 256–59, 434–37, 1147–48, 1211, and 1222–23. This value

from Albategni's could have been derived from the different lengths of the year attributed to them above in III.1; however, the different values there are according to different definitions of a year and thus the values cannot be easily compared. Alternatively, the *Almagesti minor*'s author may have used the table of mean motion in the Toledan Tables as his source for Arzachel's value.[32] The Toledan Tables do not explicitly state the eccentricity, but our author could have easily seen from the greatest value in the table of the solar equation that the table was computed from Albategni's eccentricity or a value very close to it.[33] There are a couple of instances of phrasing that are taken from Gerard's translation. The general argument follows that of Ptolemy although Ptolemy's is a computation, while our author remains on the general level throughout the proof, only reporting parameters in the last paragraph. We see that here, as elsewhere, the author has a different approach to solving right triangles than Ptolemy does. While Ptolemy takes the side opposite the right angle as the diameter of a circle, the author takes this side as a radius. Albategni uses a similar procedure, although he uses point E as the center of the little circle.[34] This approach suggests that the author was more comfortable with working from sines to arcs and vice versa than with working from chords to arcs and vice versa. Some of the wording is clearly derived from Gerard's translation. *P*'s diagram has several errors that may have made the proof hard to follow for a reader. Note that here the author not only discusses the theory of trepidation, but proposes it as a cause of the differences in the location of the apogee as found by various astronomers. Later, in VI.10–11, he uses the apogee position that he attributes here to Arzachel, which suggests that he considers it to be better than Ptolemy's or Albategni's.

III.12. This corresponds to *Almagest* III.4 (1515 ed., the paragraph going from f. 31v to f. 32r). While Ptolemy calculates specific values, our author provides a general proof. Ptolemy calculates these values also for the epicyclic model (in the following paragraph of f. 32r), but our author omits this. The reason for this appears to be that some manuscripts of the *Almagest* omitted this paragraph, and the author of the *Almagesti minor* must have been using such an *Almagest* manuscript (see Ch. 1 of the Introduction). The *Epitome Almagesti*

was sometimes falsely attributed to Albategni, e.g. Raymond of Marseilles' *Liber cursuum,* written in 1141, Raymond's tables, (see d'Alverny, Burnett, and Poulle, *Raymond de Marseille*, pp. 194 and 340), and also among other tables in a couple of manuscripts (Pedersen, *The Toledan Tables*, Table DA01, pp. 1222–23).

[32] Pedersen, *The Toledan Tables*, Table CA01, pp. 1144–48. Interestingly, the values for the mean velocities in the Toledan Tables appears to be closely related to Albategni's values although the connection is not immediately clear because Albategni's include the motion of precession (Pedersen, *The Toledan Tables*, pp. 1140–41).

[33] Pedersen, *The Toledan Tables*, Table EA01, pp. 1245–49.

[34] Albategni, *De scientia astrorum*, 1537 ed., ff. 28v–29r.

also lacks a corresponding proposition for the epicyclic model, which is almost surely the result of Peurbach's use of the *Almagesti minor*.[35]

III.13. This corresponds to the first portion of *Almagest* III.5 (1515 ed., f. 32r's last paragraph and f. 32v's 1st paragraph) and loosely to part of *De scientia astrorum* Ch. 28 (1537 ed., f. 30v). Some of the wording is directly from Gerard's translation. Only one value remains from Ptolemy's example, but most of the argument follows Ptolemy's except that the author here solves for the angles and sides of triangles using circles that have a side of the triangle as the radius. This accords with Albategni's practice, not Ptolemy's. Besides this, however, there is no close connection to Albategni's corresponding proof.[36] The final steps of this proposition's last third are incorrect. The author finds TDL correctly, but he apparently does not realize that this is one of the sought angles, the angle of apparent motion, and that once this angle and the angle of the difference are known, the angle of mean motion should be known. He then proceeds as if lines DZ and TZ were parallel, saying that angle TDL is equal to angles DTK and ETZ, although it is not. At the end of the proof, he states that angle ADB has been found, but this had actually been found several steps earlier.

III.14. This corresponds to a passage in *Almagest* III.5 (1515 ed., f. 32v, the 2nd and 3rd paragraphs) and loosely to part of *De scientia astrorum* Ch. 28 (1537 ed., ff. 29r–30r). The argument follows that of Ptolemy but without the computation and with very little borrowed phrasing. Also, as usual, the author solves triangles by using sides as radii, instead of diameters.

III.15. This corresponds to part of *Almagest* III.5 (1515 ed., from f. 32v's last paragraph through f. 33r's 1st full paragraph) and loosely to a passage of *De scientia astrorum* Ch. 28 (1537 ed., ff. 30v–31r). The argument follows that of Ptolemy and a few passages show a dependence upon Gerard's translation.

III.16. The first three paragraphs of the proof correspond to the last portion of *Almagest* III.5 (1515 ed., f. 33r, the 2nd and 3rd full paragraphs) and loosely to part of *De scientia astrorum* Ch. 28 (1537 ed., f. 30r). The last paragraph loosely corresponds to 1515 *Almagest* III.6 (1515 ed., ff. 33r-v). The general argument is that of Ptolemy, but it does not employ his wording. The author's proofs are barer, and he makes larger steps without explaining implicit intermediary steps.

III.17. The first paragraph after the enunciation corresponds to *Almagest* III.8–9 (1515 ed., ff. 33v–34r) In this paragraph, the author outlines how

[35] Such a proposition would have been placed after III.15 in the *Epitome Almagesti*.

[36] Albategni's proofs that correspond loosely to *Almagesti minor* III.13–16 only show how to find the equation from the mean motion, not the other two parts of this proposition.

to find the place of the mean sun at an epoch and how to calculate the mean sun for any time at any longitude.

The following paragraphs telling how to calculate the position of the true sun at any time, correspond to parts of *De scientia astrorum* Ch. 28 (1537 ed., ff. 31r-v) and loosely to calculations in the *Almagest* III.5 (1515 ed., ff. 32r–33r). The rules depend upon whether the mean sun is less than 90°, exactly 90°, or more than 90° from the apogee. The rules for these first and third options can be derived from the metrical analyses of III.13 and III.15 respectively. These rules are clearly taken from Albategni with some wording taken directly from the source. While Albategni frames the rules in terms of the epicyclic model, presumably because he is providing a treatment of equations in general, not only the solar equation, the author here puts them in terms of the eccentric model. This modification of the rules from one model to the other is accomplished by our author by the mere substitution of the eccentricity where Albategni uses the epicycle's radius. Albategni has no rule for the case when the sun's mean motion is 90°, and thus it appears that the author formulated the one found here himself.

The author makes a small change in the third case. Albategni first subtracts 90° from the angle of mean motion and works through this angle, which he refers to either by 'residuum' and 'partes', and its complement, which he refers to as the 'perfectio partium.' On the other hand, the author here takes the mean motion's angle's supplement, which is referred to as the 'residuum' and which corresponds mathematically to Albategni's 'perfectio partium.' The complement of this 'residuum' is called the 'perfectio', and it corresponds mathematically to Albategni's 'residuum.' The difference in terminology could have caused some difficulty in comparing the two texts. In the manuscripts of Group 1, the supplement is not taken, but rather the excess of the angle of mean motion over 90°. It is closer to Albategni's text and argument. However, if this reading is chosen, the remainder of the paragraph is either incorrect mathematically or, at best, extremely confusing with unusual meanings of the words 'residuum' and 'perfectio.' A reader of *N* realized that there was a mistake and simply wrote 'Male stat' in the margin.

A set of three figures and accompanying marginal notes in most of the manuscripts of Group 2.A (*E* only has the figures) show an attempt to justify the rules in this proposition in a manner that does not harmonize with the geometrical proofs of III.13 and III.15. They are labeled very differently, and instead of using the smaller circles that are used in the earlier propositions (albeit understood, not drawn), these have extra lines and justify the rules through the use of similar triangles. Interestingly, the scribe of *T* realized that the three cases correspond to earlier propositions. He thought mistakenly that the second case was based on III.12, but that is for when the angle of true motion is 90°, not when the mean motion is 90°.

III.18. This corresponds to short passages in *Almagest* III.10 (1515 ed., f. 34v) and *De scientia astrorum* Ch. 29 (1537 ed., ff. 31v–32r). The author here gives a more accurate value for the daily mean motion than Ptolemy does in this chapter, but the value is found in *Almagest* III.2 (1515 ed., f. 29r).

III.19. This corresponds to a short section of *Almagest* III.10 (1515 ed., f. 34v). The author provides a geometric proof for statements that Ptolemy only asserts.

III.20. This proposition tells how to find the difference between the sun's mean motion and apparent motion in the ecliptic of any single day. While it is phrased in terms of a single day, the same method could be applied generally to other lengths of time.

III.21. This proposition corresponds to a short passage in the *Almagest* III.10 (1515 ed., f. 34v), but this is one of the rare times when the author contradicts Ptolemy. Ptolemy states that the greatest difference due to unequal ascensions in the declined sphere occurs in the times from solstice to solstice and that the greatest difference due to this will be the same as the difference between the longest and equinoctial day. *Epitome Almagesti* III.24 follows the *Almagesti minor* here, and even takes some of its wording directly.[37]

III.22. This corresponds to a short section in *Almagest* III.10 (1515 ed., f. 34v). Neither Ptolemy nor our author rigorously show where the difference begins or ends or exactly how large the greatest difference is. While Ptolemy says that the greatest difference is found over any pair of signs centered on an equinox or solstice point and that the greatest difference is about 4° 30′, our author writes that the arcs over which the greatest difference accrues start and end at the midpoints between the equinoxes and solstices and that the greatest difference is 5°. He could have calculated this easily from Ptolemy's tables of right ascensions. Albategni does not say where the differences start and finish, but he gives a more precise value for the maximum difference, 4° 27′.[38] Peurbach and Regiomontanus follow this proposition of the *Almagesti minor* closely, retaining some of its wording. They make the small change of not taking the exact midpoints of the signs in which the beginnings of addition and diminution occur; instead they take slightly different places, i.e. Taurus 16°, Leo 14°, Scorpio 16°, and Aquarius 14°.[39]

III.23. While the main part of this proposition has no corresponding passage in the *Almagest*, the last paragraph corresponds to sections of *Almagest* III.10 (1515 ed., f. 34v) and *De scientia astrorum* Ch. 29 (1537 ed., f. 31v).

[37] Venice, BNM, Fondo antico lat. Z. 328, ff. 28v–29r.

[38] Albategni, *De scientia astrorum* Ch. 29 (1537 ed., f. 32r).

[39] *Epitome Almagesti* III.27, Venice, BNM, Fondo antico lat. Z. 328, f. 29v.

III.24. This corresponds loosely to a small passage in *Almagest* III.10 (1515 ed., f. 34v). The *Epitome Almagesti* III.28 follows this proposition of the *Almagesti minor*, sometime taking words directly from it.[40]

III.25. This corresponds to the end of *Almagest* III.10 (1515 ed., ff. 34v–35r) and to part of *De scientia astrorum* Ch. 29 (1537 ed., ff. 32r-v). Ptolemy does not discuss what will occur if the radix of time is at the beginning of addition or of diminution, as our author does in the second paragraph. Peurbach and Regiomontanus have a similar discussion in *Epitome Almagesti* III.25 that uses some wording directly from the *Almagesti minor*.[41] *Da* has an addition (see the Appendix) following the end of Book III that discusses a table of the equation of time. The description of the table with its minimum at Aquarius 18° and additive value measured in degrees agrees well with Albategni's table, which was included among the Toledan Tables, however those have a maximum of 7° 54′ instead of 7° 52′.[42] *Da*'s addition summarizes some of *De scientia astrorum* Ch. 29 (1537 ed., f. 31v).

Book IV

Principles. Most of these definitions and postulates appear to be the author's original creation.

IV.1. This corresponds to the first section of *Almagest* IV.1 (1515 ed., f. 35v). This gives a geometrical representation of observations of the moon from the surface of the earth. While Ptolemy states some of the same results, he goes into less detail and does not explain with a figure.

IV.2. This corresponds to the second section of *Almagest* IV.1 (1515 ed., ff. 35v–36r). This has different language, but shares similar concepts with the *Almagest*.

IV.3. This corresponds to the first section of *Almagest* IV.2 (1515 ed., paragraph going from f. 36r to f. 36v). The proof in the first paragraph, i.e. the demonstration that when one finds an interval of time in which eclipses will always repeat with the same distance between them, this interval will be the time of a return of the moon's irregularity, appears to be the author's creation. The rest of the text generally follows the content of the *Almagest* with some matters (e.g. how the moon's complete motion completed during the interval is found from the sum of the sun's motion and the number of months) explained more simply than in the *Almagest*. The author includes two remarks that hint at the treatment in IV.5–6 of how the irregularities of the sun and moon require

[40] Venice, BNM, Fondo antico lat. Z. 328, ff. 29v–30r.

[41] Venice, BNM, Fondo antico lat. Z. 328, f. 29r.

[42] Nallino, *al-Battānī*, vol. II, pp. 61–64; and Pedersen, *The Toledan Tables*, Table BB11, pp. 968–75.

more selective criteria for selecting the eclipses used to determine the moon's return of diversities. *Da* has an addition to the text discussing how one finds the moon's mean motions of longitude and latitude [see the Appendix]. This corresponds to *Almagest* IV.3 (1515 ed., f. 37r). This addition is superfluous since the same topic is treated in *Almagesti minor* IV.7.

IV.4. This corresponds to a short passage of *Almagest* IV.2 (1515 ed., f. 36v, the 1st full paragraph). The content of this passage follows that of the *Almagest*.

IV.5. This and the next proposition correspond to intermingled passages in *Almagest* IV.2 (1515 ed., a section from the middle to bottom of f. 36v). In this proposition, some traces of the wording of Gerard's translation of the *Almagest* can be found. This proposition explains how in certain situations the criteria given in IV.3 for choosing eclipses for finding the return of the moon's diversities are insufficient (these criteria are that the intervals are equal and the moon has traveled an equal longitude in each); IV.6 provides additional criteria that must be met for the eclipses to be used to accurately find the moon's return of the irregularity. These propositions and the corresponding passage in the *Almagest* are very difficult to follow.[43] The first part of IV.5 gives an example of how the sun's anomaly can lead to inaccurate results in determining lunar periods. This is similar to Ptolemy's example, but is explained in more detail with the help of a geometrical figure. The second part of the proposition, in the last paragraph, deals with the requirement that the moon complete a full return of the irregularity. R_1, *Pr*, and L_1 contain marginal notes stating that in this last part of the proposition and in the subsequent proposition, the author misunderstood Ptolemy, and they refer to Geber for a correct understanding of what Ptolemy says.[44] However, Geber critiques Ptolemy's explanation of possible errors and necessary conditions, and he proposes his own set of simpler conditions instead.[45] Here the author of the *Almagesti minor* does not significantly misunderstand Ptolemy; he is only guilty here of omitting a case that Ptolemy describes (the case in which the two intervals both begin with the same irregularity or speed, but both end at another irregularity or speed) and of summarizing the last case.

IV.6. This corresponds to passages in the *Almagest* IV.2 (1515 ed., the paragraph going from f. 36v to f. 37r), parts of which correspond to the previous

[43] A good summary is found in Neugebauer, *A History of Ancient Mathematical Astronomy*, pp. 71–72 (accompanying figures on pp. 1223–25).

[44] 'Ab hoc loco usque in finem commenti sequentis propositionis scilicet sexte, actor iste non intellexit Ptolomeum ut patet per Gebrum.' R_1, f. 19r.

[45] Geber, *Liber super Almagesti*, 1534 ed., pp. 46–49. For a thorough discussion of Jābir's critique of Ptolemy, see José Bellver, 'Jābir b. Aflaḥ on the Four-Eclipse Method.' Interestingly, Peurbach and Regiomontanus chose to follow Geber, not Ptolemy, in *Epitome Almagesti* IV.4–5, Venice, BNM, Fondo antico lat. Z. 328, ff. 32r-v.

proposition. The first two paragraphs' content stays fairly close to the *Almagest* and traces of the wording of Gerard's translation remain. Most of the third paragraph appears to be the author's creation. As noted in the commentary on the previous proposition, some manuscripts include a note saying that it can be seen from Geber that the author misunderstood Ptolemy here, but the *Almagesti minor*'s author summarizes the *Almagest*'s corresponding passage without any major misunderstandings.

IV.7. This corresponds to *Almagest* IV.3 (1515 ed., ff. 37r-v). The content generally follows that of the *Almagest* but contains fewer values.

IV.8. This corresponds to part of *Almagest* IV.5 (1515 ed., f. 40r, the 1st and 2nd paragraphs). Our author does not report Ptolemy's discussion of the first and second anomalies that is at the beginning of this chapter in the *Almagest*, but instead he proceeds straight to this proof. Most of the proof follows Ptolemy's closely and some of the wording is clearly taken from Gerard's translation.

IV.9. The proof corresponds to the last section of *Almagest* IV.5 (1515 ed., f. 40v) and the last paragraph corresponds to a passage earlier in the chapter (1515 ed., f. 40r, the 1st paragraph). The proof follows the *Almagest*'s proof, and much of the text is taken word for word from Gerard's translation.

IV.10. This corresponds to a long passage in *Almagest* IV.6 (1515 ed., from f. 40v through the 1st paragraph of f. 42r). While most of the calculations have been replaced with more general discussion, this follows the *Almagest* fairly closely. A few passages are taken almost word for word from Gerard's translation. A difference appears in the first paragraph after the enunciation, in which the author provides a simpler lunar model than Ptolemy does by ignoring the motion of latitude for the meantime. In the remainder of the proposition, a few things are discussed in a different order and there are only vestiges of Ptolemy's calculations concerning the exact times and locations of the three observed eclipses; e.g. while our author reports the degrees in which the second and third eclipses occur, he does not do so for the first eclipse and thus the reader cannot calculate the distance between the first two eclipses from only what is given here. Also, while he reports many of Ptolemy's values for finding how much motion in the ecliptic is due to which sized arcs of the epicycle, the author omits enough that it would be difficult for a reader to follow him through the calculations. This, perhaps together with a mistaken value in Gerard's translation ('170' for '176') may have contributed to the many mistaken values found in our witnesses.

IV.11. This corresponds to a section of *Almagest* IV.6 (1515 ed., f. 42r, the 1st and 2nd full paragraphs). The argument is that of the *Almagest* except it serves as a metrical analysis instead of a computation. Here the author breaks from his normal practice and solves triangle DKN using a side as diameter, as Ptolemy does.

IV.12. This corresponds to a long passage in *Almagest* IV.6 (1515 ed., from the last paragraph of f. 42r through full paragraph of f. 43r). The general argument follows that of Ptolemy, and some passages are taken directly from Gerard's translation. The procedure here is almost identical to that given for the three ancient eclipses in IV.10 and only differs in the positions of the eclipses on the epicycle, so it is surprising that the author decided to give the proof in full. While Ptolemy had to go through the computation again since the starting values were different, the author here does not work through the computation, although he does give a few values along the way; therefore, the core of this proposition adds very little to what has already been explained by the author in IV.10.

IV.13. This corresponds to a passage in *Almagest* IV.6 (1515 ed., from f. 43r's last paragraph through f. 43v's full paragraph). This follows the argument of Ptolemy, but as a metrical analysis instead of a computation. While Albategni states in *De scientia astrorum* Ch. 28 that the radius of the moon's epicycle is $5^p\ 15'$, he restates it in connection with the simple equation and Ptolemy in Ch. 30.[46]

IV.14. The first paragraph of the proof corresponds to *Almagest* IV.7 (1515 ed., f. 43v). This follows the content of the *Almagest* closely, although there are a few less values given and the language is almost wholly changed. The last paragraph appears to be a paraphrase and explanation of a short passage of *De scientia astrorum* Ch. 30 (1537 ed., f. 35r) regarding the values of the mean motions of irregularity and of longitude. The author claims that Albategni and the Toledan Tables have a faster speed for the moon's mean motion of irregularity than Ptolemy does; however, Albategni clearly states that he retains Ptolemy's value, and the Toledan Tables agree with Ptolemy's value.[47] The mistake stems from our author's misreading of a sentence of *De scientia astrorum* that follows shortly after Albategni's statement that he accepts Ptolemy's value for the mean motion of irregularity: 'Eius autem portio nostri temporis portionis unius medietatem et quartam superaddebunt, quod ex ipsius itinere minuimus.'[48] This, however, refers not to a difference in the mean motion of irregularity, but to the accumulated difference in the motion of latitude found by Albategni, which our author reports in IV.16.[49]

[46] Albategni, *De scientia astrorum*, 1537 ed., ff. 31v and 33v.

[47] Albategni, *De scientia astrorum*, 1537 ed., ff. 35r: '… eiusque motus in differentia est motus, qui est in libro Ptolemaei prorsus …'; Pedersen, *The Toledan Tables*, Table CA21, pp. 1156–60.

[48] Albategni, *De scientia astrorum*, 1537 ed., f. 35v.

[49] Nallino, *al-Battānī*, vol. I, p. 255.

IV.15. This corresponds loosely to *Almagest* IV.8 (1515 ed., ff. 43v–44r). The subject matter matches that of Ptolemy, but the treatment here is much more general and shows no close connection to the wording of Ptolemy.

IV.16. This corresponds to *Almagest* IV.9 (1515 ed., the paragraph going from f. 44r to f. 44v). Our author does not summarize the first part of Ptolemy's chapter, in which he explains the problems with Hipparchus' method of finding the mean motion of latitude. The passage from the *Almagesti minor* matches the remainder of *Almagest* IV.9 rather closely in content, although many of the values are not reported. A few short passages are taken almost word for word from Gerard's translation. Since not all values are reported and much of the text is on a general level, a couple of errors in values entered into the text. The last paragraph paraphrases a statement of *De scientia astrorum* Ch. 30 (1537 ed., f. 35r).

IV.17. This corresponds to *Almagest* IV.9 (1515 ed., from f. 44v's first paragraph through the subsequent paragraph of f. 45r). This follows the argument of Ptolemy fairly closely. While some of it is taken word for word from Gerard's translation, it is more general and omits many of the values involved in the computation.

IV.18. This corresponds to a short passage in *Almagest* IV.9 (1515 ed., f. 45r, the 1st full paragraph). This agrees with the content of the *Almagest*, but with the difference that the author gives it in general terms while Ptolemy gives a computation.

IV.19. This corresponds loosely to a short passage *Almagest* IV.6 (1515 ed., f. 40r, 1st paragraph). In the course of describing his first lunar model, Ptolemy describes how the motion of the node is against the succession of signs and is the difference between the motions of latitude and longitude. Remember that in IV.10 the author of the *Almagesti minor* gave a simpler model that did not include the motion of the node. Although the text here describes Ptolemy's model accurately, at least one reader of the *Almagesti minor* thought that the text was mistaken, perhaps because it describes the motions differently than Ptolemy does.[50] The *Almagesti minor* does not have propositions that correspond to *Almagest* IV.10–11, which consists of tables and a lengthy discussion of why Hipparchus reached different results than Ptolemy did.

Book V

Principles. The definition of a star's place according to latitude is defined as the intersection of two circles that pass through the star's body, so the star's place according to latitude is merely the place of the star. Perhaps struggling

[50] R_1, f. 24r; *Pr*, f. 31r: 'Hoc est falsum et procedit ex malo intellectu Capitis Draconis…'

to find more general definitions, the author provides definitions of parallax in longitude and latitude that only apply when the moon is on the ecliptic. The definition of the moon's mean apogee summarizes the definition given in *Almagest* V.7 (1515 ed., f. 49r). The phrase 'equatio puncti', which is seldomly used by the author, is not used by Ptolemy nor by Albategni. Its source appears to be the Toledan Tables or their canons.[51]

V.1. This corresponds to the bulk of *Almagest* V.1 (1515 ed., f. 47r). The instrument and its use are essentially those described by Ptolemy. The minor changes include a slight change in the instrument – the apertures are placed on a rule instead of on a fifth ring, and in the last paragraph the author adds some justification for the method of observation and a short comment upon the effect of parallax. There is some shared vocabulary with Gerard's translation, but our author seems to have made a conscious effort to reword the description of the instrument and its use.

V.2. This corresponds to the first part of *Almagest* V.2 (1515 ed., ff. 47r-v). This explanation of the existence of a second irregularity follows the content of Ptolemy, but in greater detail and without a close dependency upon Gerard's wording.

V.3. This corresponds to the second half of *Almagest* V.2 (1515 ed., ff. 47v–48r). The content follows Ptolemy's with few differences and a few passages are taken almost directly from Gerard's translation. Our author clarifies Ptolemy's model by adding line EL from which the motions begin and by explaining the model in more general terms before referring to specific values for each motion and to the diagram.

V.4. This corresponds to *Almagest* V.3 (1515 ed., ff. 48r-v). This follows Ptolemy closely and most of the second paragraph is taken almost directly from Gerard's translation. The author passes quickly over the second example that Ptolemy uses, but he does include one value from it. He also provides the value for the apparent size of the epicycle's radius, which Ptolemy does not give until the following chapter.

V.5. This corresponds to *Almagest* V.4 (1515 ed., f. 48v). This follows the argument of *Almagest* although the wording is not taken directly from Gerard's translation. It is interesting that P_7 and *M* have an addition in the first sentence that makes the text closer to Gerard's translation. Perhaps the original was closer to Gerard's translation and the text in *P* and *K* reflects a further development, but since the text makes clear sense without this addition, I think it more likely that some scribes read and copied the *Almagesti minor* while

[51] Pedersen, *The Toledan Tables*, Ca95 and CcC01 and Table EA11, pp. 258, 727, and 1250–58.

reading the *Almagest* and made additions to it from the *Almagest*. Evidence of this is found in another variant in *M*, where 'ET' is replaced with 'ETB.' The latter agrees with the *Almagest*, but it is clearly not an original part of the *Almagesti minor*'s figure, because point B is not on line ET, as in the *Almagest*, but is on the epicycle.

V.6. This proof corresponds to the end of *Almagest* V.7 and the first sentence of V.8 (1515 ed., f. 51r) and to a section of *De scientia astrorum* Ch. 30 (1537 ed., f. 34r-v). While Ptolemy and Albategni work with a case in which the duplex elongation is more than 90°, the author of the *Almagesti minor* treats the case in which it is less than 90° and does not use wording from either source. Also, while his predecessors give their proofs, which are computations, in the middle of discussion of the table of complete lunar anomaly, the author of the *Almagesti minor* places it as a separate proof before any discussion of the moon's equation of portion. The last paragraph corresponds to the first passage of *Almagest* V.5 (1515 ed., ff. 48v–49r). The labels M and K for the points that the *Almagest* labels B and M respectively match Albategni's labeling, but the author takes nothing else from Albategni that is not in the *Almagest*.

V.7. This corresponds to *Almagest* V.5 (1515 ed., ff. 48v–50v). This passage is a metrical analysis and not a calculation as in the *Almagest*, but the argumentation follows that of the *Almagest* closely and there is some wording taken directly from Gerard's translation. In *Almagest* V.5–6 Ptolemy works from observations to a parameter and then from the parameter to the moon's position, but the values in his analysis and synthesis do not match perfectly. The author noticed these discrepancies and the inclusion of two different values for the sun's true place in V.5's second example, and he attempted to correct them. While he changes some other values to work with these changed values, he was not successful (and likely did not make an attempt) in harmonizing all the numbers.

V.8. This corresponds to part of *Almagest* V.6 (1515 ed., f. 50v) and to part of *De scientia astrorum* Ch. 30 (1537 ed., ff. 34v–35r). The argument follows that of the *Almagest*, but as a metrical analysis instead of a computation and without Gerard's language. Also, while Ptolemy gives the entire process by which the moon's position is found for a given time in *Almagest* V.6, our author follows Albategni in separating out the argument for finding the equation of portion. The remainder of the procedure for finding the moon's true position is shown in the following proposition.

V.9. The first paragraph corresponds to the last part of *Almagest* V.6 (1515 ed., 2nd half of chapter on f. 50v). The argument for finding the equation of anomaly is similar to that in the *Almagest*, but it is a metrical analysis instead of a computation. In terms of the geometrical diagram, the third, fourth, and fifth paragraphs repeat the procedure given in V.7 for finding the length of EB from the duplex longitude, whether it is smaller than, greater than, or equal to 90°. There are somewhat similar rules in *De scientia astrorum* Ch. 39 (1537 ed., f. 49r), but there are enough differences that it appears that our author did not base his rules upon these or only did so very loosely. The geometrical representations of these three cases are found in some manuscripts, including *B* and P_7. In the case where angle AEB is right, this merely requires the Pythagorean Theorem once, but in the other two cases, it must be used twice – first in the small triangle formed by dropping a perpendicular from the center of the eccentric to the line EB, and then in the larger triangle reaching to the epicycle's center, and additions and subtractions are needed. In the case in which the angle of the duplex longitude is obtuse, the little triangle (here BDG in B's third figure) does not contain the angle of the duplex longitude, but its supplement angle GBD, and the angle referred to as the 'complement' is angle BGD.

From the sixth to the eighth paragraphs, the author explains how to calculate the equation of portion in the three cases in which the duplex longitude is less than, greater than, or equal to 90° from the apogee. The geometrical justification for these directions can be found in the same figures from *B* that are used for paragraphs 3–5. In order to make the rules for the three cases similar to each other, the author makes the rule for the case where the duplex longitude is 90° more complex than it needs to be, as it is clear that the lines to the epicycle's center from the center of the eccentric and the point to which the mean apogee is directed are equal.

From the ninth paragraph to the twelfth, the author provide the rules for calculating the equation of anomaly. These rules for calculation have no corresponding passages in the *Almagest* although their geometrical bases could be derived from *Almagest* V.6 (1515 ed., f. 50v); however, they do correspond to rules for finding the distance from the earth to the moon given its duplex longitude and equated portion, which are in *De scientia astrorum* Ch. 39 (1537 ed., ff. 48v–49r). Although our author carries these a step further than Albategni and finds the equation of anomaly, many of the steps given are the same and some of the language follows Albategni's. Albategni also considers only the cases in which the equated portion is less than or greater than 90°, but our author also provides the rules for the remaining case in which it is 90°. In paragraph 11, the author consistently confuses the sine of the angle of the remainder and the sine of this angle's complement, and the rules for this case are thus incorrect. The rules are closely related to the geometrical arguments at the beginning of the proof, but a reader whose notes were copied in *B* and P_7 explained the geometrical basis for these rules very differently using several unnecessary lines and figures in the figures from *B*. Despite the mistakes and unneeded parts of the figures, the geometrical basis for the rules can be seen in these figures, which are found in several manuscripts.

The 13th paragraph corresponds loosely to *Almagest* V.7–8 (1515 ed., ff. 50v–51v), but the table described is not that of the *Almagest*. The description of this table matches one of the Toledan Tables much closer than the tables of Ptolemy or al-Battānī.[52] The order of the columns described in this column is as follows: 1) common numbers, 2) equation of portion, 3) proportional minutes, 4) excess of second diversity, 5) simple equation of anomaly, 6) latitude. In Ptolemy's table, the proportional minutes are placed after the simple equation of anomaly, and in al-Battānī's, the simple equation of anomaly is placed before the equation of portion. This Toledan Table does in fact have 180 rows, unlike Ptolemy's, which only has 45,[53] and the table and column headings in this table also match the text here better than Ptolemy's table does.[54] Also, Ptolemy goes into much greater depth about how the proportional minutes are found than the author of the *Almagesti minor* does.[55] Our author likely deviated from Ptolemy because the proportional minutes are values used for approximative calculations that are not fully built upon certain geometrical facts and because in V.6 he had already summarized much of the geometry that Ptolemy uses here.

The 14th paragraph corresponds to *Almagest* V.9 (1515 ed., f. 52r) and *De scientia astrorum* Ch. 36 (1537 ed., f. 47r). The order of the directions for calculation match Albategni's closer than Ptolemy's, but the wording does not closely follow either. The *Almagesti minor* in *Da* ends with paragraph 12, but it includes six additional notes, five attempting to explain how the values in each column of the table are found and one on similar tables for the planets [see the Appendix].[56]

V.10. This passage corresponds to *Almagest* V.10 (1515 ed., ff. 52r–53r). Most of this summarizes Ptolemy's arguments, and parts of the passage are copied directly from Gerard's translation. Although the mathematics and values in the first and third paragraphs follow Ptolemy's calculations, the author reaches a different conclusion regarding disregarding the equation of portion. The last paragraph concisely summarizes a passage in *De scientia astrorum* Ch. 42 (1537 ed., f. 60v), in which it is argued that disregarding the equation of portion could result in a perceptible error in the calculation of a true syzygy's time. He

[52] Pedersen, *The Toledan Tables*, Table EA11, pp. 1250–58; *Almagest*, 1515 ed., f. 51v; and Nallino, *al-Battānī*, vol. II, pp. 78–83. While al-Battānī's tables were likely not part of Plato's translation, the order of the columns can be gathered from *De scientia astrorum* Ch. 30 and Ch. 36 (1537 ed., ff. 33v–35v and 47r).

[53] Al-Battānī's table also had 180 rows.

[54] Some of the column headings are derived from Ptolemy and Albategni's text. We find 'equatio simplex' and 'longitudo propior' in Albategni, *De scientia astrorum* Ch. 36 (1537 ed., f. 47r), and 'diversitatas singularis' and 'superfluitas diversitas secunde super primam' in *Almagest* V.8 (1515 ed., f. 51v).

[55] *Almagest* V.7 (1515 ed., f. 51r). Albategni also details how the proportional minutes are found in *De scientia astrorum* Ch. 30 (1537 ed., f. 34r-v).

[56] *Da*, ff. 37v–38v.

does provide the value 40°, which is not mentioned by Albategni, so he appears to have recreated at least part of Albategni's calculation. Since this proposition shows that noticeable inaccuracy can result from arguing from eclipse observations if one does not take into account the moon's eccentric and the equation of portion, the author has thrown doubt upon the accuracy of some of the values derived from the first, simple lunar model. Perhaps not fully realizing the implications of this proposition or lacking the necessary mathematical skill, the author does not revisit Ptolemy's eclipses to determine whether any error was introduced by ignoring the equation of portion.

As here, the *Epitome Almagesti* V.12 notes that ignoring the equation of portion can lead to a noticeable error, but gives the maximum value of the error as 1/5 hour.[57] Although it is almost certain that the *Almagesti minor* provided the inspiration and the difference in the result may just reflect a difference in rounding, Peurbach or Regiomontanus performed the calculation themselves. Using Ptolemy or al-Battānī's tables, one finds that the difference in equations caused by ignoring the equation of portion is only about 6′ instead of the approximately 7′ 30″ that Albategni says it is. 6′ only causes a difference of about 12 minutes of time in the moon's travel.

V.11. This corresponds to *Almagest* V.12 (1515 ed., ff. 53r–54r) and *De scientia astrorum* Ch. 57 (1537 ed., ff. 89r–90r). The description of the instrument and its use match Albategni's closer than Ptolemy's. The most conspicuous differences are that Albategni and our author refer to a geometrical figure, which Ptolemy does not, and they divide the third rule into thirtieths and use a table of sines, while Ptolemy divides the upright rule into sixtieths for use with a table of chords. Some features of Ptolemy's instrument remain, such as the rules being 4 cubits long while Albategni's are 5 cubits. Also, Albategni makes this instrument primarily for measuring the sun's altitude and only notes briefly that it can be used for the moon, but our author puts his description of Albategni's version of the instrument into an otherwise close retelling of the passage from the *Almagest*. Although almost all of this proposition is derived conceptually from Ptolemy and Albategni, the wording is not taken directly from either. At least one passage, 'ut in cavatura alterius superduci possit sic ut linea FL media et linea HM in una sint plana superficie apparente', is found with only minor differences in a small work on this instrument titled 'Instrumentum ad inveniendum altitudinem Solis et stellarum', which is almost wholly taken from *De scientia astrorum* Ch. 57.[58] Perhaps this common passage was also found in some witnesses of *De scientia astrorum*.

[57] Venice, BNM, Fondo antico lat. Z. 328, ff. 43r-v.

[58] This work begins, 'Fac tres planas regulas de ligno vel ferro …' It is found in Vatican, BAV, Pal. lat. 1340, ff. 36v–37r, Vienna, ÖNB, 5303, ff. 261v–262v, and Vienna, ÖNB 5418, ff. 194r-v.

V.12. This proposition corresponds to a passage near the beginning of *Almagest* V.13 (1515 ed., f. 54r). The directions here are much more detailed than those in Ptolemy's calculation, and the author gives the rules for most of the different cases that can occur. The list of criteria for the observation, which are not clearly spelled out in *Almagest* V.13, may be derived from *De scientia astrorum* Ch. 39 (1537 ed., f. 48r) or from *Almagest* V.12 (f. 53v).

V.13. This corresponds to the first and last parts of *Almagest* V.13 (1515 ed., ff. 54r-v). This passage follows the *Almagest*'s argument closely although much is arranged as a metrical analysis, and parts are taken directly from Gerard's translation. The author is able to omit some of the steps of the calculation in the first paragraph because he has already given a treatment of how to find the parallax in the preceding proposition.

V.14. This corresponds to *Almagest* V.13 (1515 ed., ff. 54v–55r) and to part of *De scientia astrorum* Ch. 39 (1537 ed., f. 48v). The first paragraph after the enunciation follows the argument of Ptolemy closely although it is more of a metrical analysis than a calculation. Much of it is clearly taken directly from Gerard's translation. The second paragraph, which appears to be the original work of the author, shows how to find the distance of the moon from the earth at any point on the eccentric and epicycle. This provides a geometrical basis for rules of calculation given in V.9. It is similar to calculations of the moon's distance given by Ptolemy in *Almagest* V.17 (1515 ed., ff. 57r-v), but there Ptolemy only considers the particular situation in which the epicycle is assumed to be at the eccentric's apogee. The third paragraph explains how by adding and subtracting the size of the epicycle from the values for the eccentric's apogee and perigee, the four distances of the moon that are used in *Almagest* V.17 (1515 ed., f. 56v) are found. While Ptolemy outlines the addition that gives the first of these distances at the beginning of *Almagest* V.15 (f. 55v), he only lists the other distances in V.17. While Albategni lists the distances for the eccentric's apogee and perigee, the epicycle's radius, and the four 'termini' in terms of earth radii, he does not show how they are found.

V.15. This corresponds to *Almagest* V.14 (1515 ed., ff. 55r–55v). The argument here follows that of the *Almagest* fairly closely, but only a few similarities in wording remain.

V.16. This corresponds to the first half of *Almagest* V.15 (1515 ed., ff. 55v–56r), which is summarized in *De scientia astrorum* Ch. 30 (1537 ed., f. 38r). This follows the argument of the *Almagest* fairly closely although none of the text is taken directly from Gerard's translation.

V.17. This corresponds to the end of *Almagest* V.15 and V.16 (1515 ed., ff. 55v–56r), which is summarized in *De scientia astrorum* Ch. 30 (1537 ed., ff. 38r–39r), and the first two sentences correspond to a passage in *Almagest* V.14 (1515 ed., f. 55r). The argument follows that of the *Almagest* fairly closely, but discusses some matters in a different order. Some wording is taken directly from Gerard's translation.

V.18. The first paragraph seems to be the author's own explanation of how the volumes of spheres are found if their diameters are known. The second paragraph after the enunciation corresponds to part of *Almagest* V.16 (1515 ed., f. 56r), which is summarized in *De scientia astrorum* Ch. 30 (1537 ed., f. 38v). The author calculates value for the number of times that the earth's volume contains the moon's more accurately than Ptolemy and Albategni do. Much of the third paragraph is similar to a short section in *Almagest* V.17 (1515 ed., f. 56v). These passages discuss the same physical causes, but for different purposes. Ptolemy uses them to explain why the sun's varying distance from the earth, unlike the moon, has little effect upon the parallax; however, the author of the *Almagesti minor* uses them to show why Ptolemy thought that the sun's change in apparent diameter could be ignored while the moon's must be considered.

The remainder of the proposition is devoted to Albategni's findings concerning the apparent diameters of the sun, moon, and shadow, as well as the distance of the sun. Paragraphs 4–6 correspond to the passage on the apparent diameters in *De scientia astrorum* Ch. 30 (1537 ed., ff. 36r–37v). In addition to effectively replacing *Almagest* V.14 (or *Almagesti minor* V.15), this passage and its source collect in one place related material that Ptolemy gives in *Almagest* V.14 and VI.5. The rules for the calculation of apparent diameters from hourly motion most likely come from the canons to the Toledan Tables.

	Ptolemy	Albategni
Moon at epicyclic perigee	35′ 20″	35′ 20″
Moon at epicyclic apogee	31′ 20″	29′ 30″
Sun at perigee	31′ 20″	33′ 40″
Sun at apogee	31′ 20″	31′ 20″
Shadow's radius at moon's epicyclic apogee*	40′40″	38′ 20″
Shadow's radius at moon's epicyclic perigee*	46′	46′

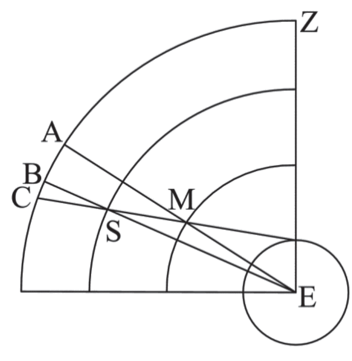
* With the sun at its apogee. Albategni states that the sun's varying distance makes the shadow's radius vary by 50″.

The last paragraph corresponds to a passage in *De scientia astrorum* Ch. 30 (1537 ed., ff. 39r-v) in which Albategni recalculates the distance of the sun according to his different values for the apparent diameters. The argument follows Albategni's. A small change is that the author of the *Almagesti minor* provides a different rational in calculating the length of the axis. Mention of the sun's epicycle, rather than its eccentricity, is obvious evidence of the reliance upon Albategni. A few sentences of this proposition are taken directly from Plato of Tivoli's translation of Albategni, but the author generally uses his own language when summarizing Albategni in this proposition.

V.19. The first two paragraphs correspond to the first paragraph of *Almagest* V.17 (1515 ed., ff. 56v–57r). The wording is changed, and Ptolemy's calculation is divided into a metrical analysis, found in the first paragraph, and a report of the resulting values, found in the second. The third paragraph provides a rule for calculation of the moon's parallax, which could be easily justified by the geometry of the first paragraph, that is taken from the rule given in *De scientia astrorum* Ch. 39 (1537 ed., f. 49v). Some words and phrases, including the use of 'chorda' to refer to sines, show the close dependence upon this source. More concerned with geometrical purity, the author of the *Almagesti minor* adds more steps to find the hypotenuse of a very thin triangle from the two others while Albategni is content to treat the long leg as identical to the hypotenuse. The fourth paragraph discusses some of the differences to this rule that would be needed to use it for the sun's parallax, and while it reports a conversion taken from *De scientia astrorum* Ch. 39 (1537 ed., f. 49v), it is mostly the author's own work. The remainder of the proposition treats the construction and use of the table of parallax found in *Almagest* V.18.[59] The fifth, seventh, and ninth paragraphs, which summarize the construction, correspond to a passage in *Almagest* V.17 (1515 ed., from f. 57r's full paragraph through f. 57v's 2nd full paragraph). The author of the *Almagesti minor* employs his own language. His explanations of how the values in columns 7, 8, and 9 in the table are found is simpler than Ptolemy's since he does not show here how the distances of the moon from the earth are calculated. He also uses different figures than those found in the *Almagest*. The sixth paragraph is the author's own explanation of how the tables are to be used if the moon is at one of the four terms. The eighth and tenth paragraphs contain the directions for using the table when the moon is in another place, and these paragraphs correspond to the first section of *Almagest* V.19 (1515 ed., f. 58v, the 1st paragraph) and to a section of *De scientia astrorum* Ch. 39 (1537 ed., ff. 50r-v). The passage gives essentially the same rules found in the sources, but it leaves out some of the explanations (e.g. whether to enter with the equated portion itself or with the difference between 360° and it) and adds some justification of the process. While most of this passage is in the author's own formulation, some wording is taken directly from Albategni (I discuss some peculiarities of Albategni's terminology in my commentary of the next proposition). The evaluation of this method in the last sentence appears to be the author's own judgement.

[59] This table was included with small differences among al-Battānī's tables (Nallino, *al-Battānī*, vol. II, pp. 93–94) and the Toledan Tables (Pedersen, *The Toledan Tables*, Table HD21, pp. 1407–08).

V.20. The first paragraph is the author's own explanation of the parallax of the moon to the sun on the circle of altitude. The figure that it uses is similar to one used by Ptolemy in *Almagest* V.13 and 17, but this one has circles for both the sun and moon and is labeled slightly differently. Albategni has a somewhat similar passage in *De scientia astrorum* Ch. 39 (1537 ed., f. 50v); however, here and in his rules for finding parallaxes from the table of *Almagest* V.18, Albategni uses different terminology and appears to have in mind a situation different from the one depicted in *Almagesti minor* V.20's figure. In the directions for finding the moon's parallax on the circle of altitude, Albategni refers more than once to the 'parallax of either the moon or the sun' ('diversitas aspectus utriusque Lunae, scilicet et Solis') where one would expect him to refer only to the moon's parallax. In no reasonable way can the found arc, which is BC in the *Almagesti minor* V.20's figure, be seen as the sun's parallax. Also oddly, while *Almagesti minor* V.20 instructs the reader to subtract the sun's parallax from the moon's in order to find the parallax of the moon to the sun (i.e. BC – CD = BD), Albategni directs his reader to subtract the sun's parallax from the parallax of the sun and moon in order to find the moon's parallax (which would appear nonsensical in the *Almagesti minor*'s figure: BD(?) – CD = BC).[60] These oddities, however, can be explained if Albategni has in mind a different situation in which the moon and sun are not along the same line directed to the earth's center, but are both in a line to the viewer's eye, i.e. during a solar eclipse (depicted below). In the first part of the passage, by the 'parallax of either the moon or the sun', he could mean arc AC, which marks the distance between the moon, which is the starting point of his calculations, and the point where the sun and moon both appear due to parallax, which would be unusual but not nonsensical. Also, some sense can be made of the odd subtraction if Albategni envisions this situation and if he refers to the arc between the true moon and the true sun (i.e. the ecliptic) when both appear at the same point as the 'parallax of the moon on the circle of altitude'; thus, in terms of the figure, the subtraction would be: AC – BC = AB. While this naming seems very unsuitable, Albategni may have chosen this system of naming because then both 'parallaxes' are measured from a point on the ecliptic. The author of the *Almagesti minor* did not follow this odd terminology, and the figure that he uses reflects a different situation than that which Albategni appears to have supposed.



[60] Albategni, *De scientia astrorum* Ch. 39 (1537 ed., f. 50v): 'Eam ex diversitate aspectus Solis et Lunae in altitudinis circulo quam in operis fine servasti minue, quodque remanserit, erit diversitas aspectus Lune in altitudinis circulo.'

The second paragraph corresponds to a passage in *De scientia astrorum* Ch. 39 (1537 ed., f. 50v) and only loosely to a single sentence in *Almagest* V.19 (1515 ed., f. 58v, the 1st paragraph). Ptolemy is able to obtain the values for solar parallax on the circle of altitude directly from his table in *Almagest* V.18 because he does not account for the varying distance of the sun from the earth. Albategni provides a rule for calculating the sun's parallax on the circle of altitude from the table in *Almagest* V.18 that attempts to account for the change in distance, as well as for the difference caused by his smaller value for the sun's greatest distance from the earth. The *Almagesti minor* here follows Albategni's directions but with only a few reminders of Albategni's wording. The author is right to emphasize that this rule is only approximative. The addition of 1/18 relies on the approximation that the change in the amount of the parallax is inversely proportional to the change of the distance of the sun from the earth. A source of greater inaccuracy, the degree of which the author appears to have not recognized, is the use of 13″ for the difference between the parallax at the sun's apogee and perigee. This is indeed the correct value for the maximum difference, which only occurs when the sun's elongation from the zenith is 90°. The use of this value in all cases skews the numbers dramatically when the elongation from the zenith is small and the sun is near its perigee. For example, when the sun is at its perigee (1070 earth radii) and its elongation from the zenith is 10°, its parallax according to this rule is about 12″ larger than it should be (≈45″ 43‴ instead of ≈33″ 28‴). Even with an elongation of 30°, with the sun at its perigee, the parallax calculated according to this rule is more than 6″ larger than it should be (≈1′ 42″ 43‴ instead of ≈1′ 36″ 23‴).

V.21. The rules for calculation in the third paragraph correspond to the rules given by Ptolemy in *Almagest* V.19 (1515 ed., f. 58v, the 2nd paragraph) and *De scientia astrorum* Ch. 39 (1537 ed., f. 50v). The use of 'chorda' several times to mean 'sine' suggests a closer tie to Albategni, but the wording is not taken directly from him. Neither Ptolemy nor Albategni include justification for their rules. The author thus appears to have developed the geometrical proofs of the first two paragraphs on his own. His concern with establishing these simple rules leads him to bring some approximation into his metrical analysis and to use the sector figure three times instead of only twice. He could have used the conjunct sector figure once in the second part of the proof, but the resulting rule for finding the parallax in longitude would have required that the parallax in latitude be found first and would have been more complex.

The tables of Theon discussed in the last paragraph were among al-Battānī's tables (Nallino, *al-Battānī*, vol. II, pp. 95–101 and 89).[61] The description of these tables corresponds to part of *De scientia astrorum* Ch. 39 (1537 ed.,

[61] For a thorough description of these tables and their use, see Rome, *Commentaires de Pappus et de Théon d'Alexandrie sur l'Almageste, Tome I,* pp. xlix–lv.

ff. 52r–53r). The author of the *Almagesti minor* uses his own wording, focuses on what the values of each column represent, and does not explain how the tables are used in as much detail as Albategni does. However, he does mention some specific values that are not in this passage of *De scientia astrorum*, and this suggests that he actually saw the tables. Although Plato's translation of *De scientia astrorum* lacked al-Battānī's tables discussed here, they, as well as many others, were part of the Toledan Tables.[62] Although the *Almagesti minor*'s author explains correctly how the table of correction is to be used, his explanation of what the values in the fourth and fifth columns represent appears to be incorrect or at best unclear. Indeed, the values in the fourth column do grow to 12′, but these 12′ are not the difference in the distance of the moon at the first and second term – that value would be approximately 9′ 40″ when the longest distance was 60′. Instead, the 60′ : 12′ ratio at issue here appears to come from an approximation of the ratio of the lunar parallax at the first term to the parallax added to this at the second term.[63] Similarly, the ratio of the maximum value in the fifth column, 32′, is the approximate addition of the lunar parallax of the eccentric's perigee over the parallax of the eccentric's apogee when the latter is held to be 60′.[64]

V.22. This proposition, which determines the angles and arcs that will be sought in V.23–25, corresponds to part of *Almagest* V.19 (1515 ed., f. 59r, the 1st whole paragraph). The argument of the proof in the first paragraph is similar to that of Ptolemy, but our author has little similarity in wording and uses a figure that is labeled slightly differently. The case of the proof in which the moon is to the south of the ecliptic (in the 2nd paragraph of this proposition) is not dealt with by Ptolemy, and it appears to be original to the *Almagesti minor*. Note that while Ptolemy follows this proof with a discussion of the flaws of Hipparchus' attempt to correct for the substitution of angles and arcs, our author has no such discussion.

V.23. This corresponds to *Almagest* V.19 (1515 ed., f. 59r's last full paragraph and the subsequent paragraph ending on f. 59v). While the content is similar to that of the *Almagest*, the wording is not taken directly from this source. While Ptolemy provides one construction for the parts of the figures of the rest of *Almagest* V.19 that remain the same, the *Almagesti minor*'s author gives V.23–

[62] Pedersen, *The Toledan Tables*, Tables HC and JC11, pp. 1380–1404 and 1437–40. The passage of Albategni on the tables and their use is also included in one set of canons (Pedersen, *The Toledan Tables*, Ca158–165, pp. 288–93).

[63] This can be easily confirmed by comparing the values of the third and fourth columns in the table of parallax in *Almagest* V.18 (1515 ed., f. 58r).

[64] The moon's apogee is 59^p in earth radii, and the perigee is 38^p 43′ in earth radii, so through simple trigonometry, their maximum parallaxes are found to be respectively approximately 58′ and 1° 29′.

25 their own constructions. Also, Ptolemy's mention of angles is confusing. He writes, 'The angle that is seen upon point D and point E is not different from the angle that is at B; therefore, the angles that will be from those lines described upon these points of the ecliptic will be right.'[65] While somewhat unclear, the meaning of this is that the angles that we seek for each of these locations of the moon, which are the angles of the ecliptic and the circles of altitude, as was shown in the previous proposition, are identical to the angle at B. The author of the *Almagesti minor* misunderstood this. Finding no angles at D and E because there is no triangle DTH or EMF as there was in the previous figure, he thought that Ptolemy must have been referring to the angle formed by the circle of altitude and the moon's declined circle.

V.24. This corresponds to a proof in *Almagest* V.19 (1515 ed., f. 59v, the 1st full paragraph). The argument is essentially that of Ptolemy; however, few traces of the wording remain, and the proof omits some steps that Ptolemy proves and explains steps that Ptolemy had implicitly used.

V.25. The geometric proof in the first paragraph corresponds to a proof in *Almagest* V.19 (1515 ed., f. 59v, the 2nd full paragraph), and some wording is taken directly from Gerard's translation. The author's argument is similar to Ptolemy's with some small differences and with more details. The second paragraph corresponds to Ptolemy's rule in *Almagest* V.19 (1515 ed., the paragraph going from f. 59v to f. 60r). Ptolemy provides a general rule and then an example in terms of the figure with values. The author of the *Almagesti minor* gives his rule in terms of the figure, but without values. He changes the rule by putting it in terms of sines instead of chords, but he makes many blatant mathematical mistakes. The third paragraph corresponds to a passage in *Almagest* V.19 (1515 ed., f. 59v, the 3rd full paragraph).

V.26. This is one of the cases where our author strays far from the order of the *Almagest*, as this proposition corresponds to a passage in *Almagest* VI.7 (1515 ed., f. 67r, the 1st and 2nd paragraphs). The text follows the general argument of Ptolemy, but does not use his wording. Neither Ptolemy nor the *Almagesti minor*'s author provide a geometrical proof to justify how one would determine the length of DG. A difference is that Ptolemy also gives a second scenario in which AB is the ecliptic and AG is the moon's declined circle, while our author does not.

V.27. This corresponds loosely to a very short passage in *Almagest* V.19 (1515 ed., the paragraph going from f. 58v to top of f. 59r), but the source is *De scien-*

[65] *Almagest*, 1515 ed., ff. 59r-v: 'Et erit angulus qui videtur super punctum D et punctum E non diversus ab angulo qui est apud B, ergo anguli qui erunt ex istis lineis descriptis super hec puncta orbis signorum erunt recti…'

tia astrorum Ch. 39 (1537 ed., f. 51r). Some of the wording is taken from Plato's translation of Albategni. Unlike Albategni and the author of the *Almagesti minor*, Ptolemy determines whether the parallax in longitude adds or subtracts based on whether the parallax in latitude is north or south and whether the angle taken from *Almagest* II.13 is acute or obtuse.

V.28. This corresponds loosely to a very short part of *Almagest* V.19 (1515 ed., f. 58v, 2nd paragraph); however, it corresponds much closer to part of *De scientia astrorum* Ch. 39 (1537 ed., f. 51r), and much of it is taken directly from Plato's translation.

Book VI

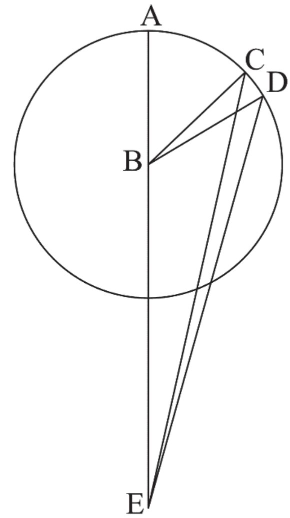
VI.1. The generalized rule in the first paragraph after the enunciation corresponds to Ptolemy's calculation found in *Almagest* VI.2 (1515 ed., ff. 60v–61r). The following four paragraphs correspond loosely to *Almagest* VI.3 and the first part of VI.4 (1515 ed., ff. 61r and 63r), in which Ptolemy explains how the tables of mean conjunction and opposition (1515 ed., ff. 61v–62v) are made and used. The texts show little similarity in wording, and there are changes in the content, such as the addition of rules for dealing with unequally sized years and months, the inclusion of more detailed instructions for using the table of months, and the omission of rules for using the tables of years. Al-Battānī had similar tables (Nallino, *al-Battānī*, vol. II, pp. 84–87), and he described their use in *De scientia astrorum* Ch. 42 (1537 ed., ff. 58r-v), but these instructions bear no close similarity to the ones in this proposition. The last paragraph describes tables found in the Toledan Tables (GA11–14), which are based on lunar years (of 354 and 355 days) and months (of 29 and 30 days) and which thus work in a different way than Ptolemy's or al-Battānī's tables.[66] While instructions for using these tables are found in the canons to the Toledan Tables,[67] the *Almagesti minor*'s author appears to have created the directions for constructing them that are found in this paragraph.

VI.2. This corresponds loosely to a short passage near the end of *Almagest* VI.4 (1515 ed., f. 63r). The method for finding the hourly motion according to 'true knowledge' appears to be the author's own work, and while the author says that the following approximative method is Ptolemy's, he is incorrect. To find the amount that the hourly motion differs from the mean hourly motion, Ptolemy's method is the following: 'We enter the table of the moon's anomaly [IV 10] with the anomaly at the moment in question, take the corresponding equation, and then determine the size of the increment in the equation [at

66 Pedersen, *The Toledan Tables*, pp. 1327–40.

67 Pedersen, *The Toledan Tables*, Ca126, Cb170, and Cc237, pp. 272–73, 448–53, and 686–89.

that point] corresponding to an increment of 1 degree in anomaly. We multiply this increment by the mean motion in anomaly in 1 hour, 0;32,40° …'[68] However, these instructions are expressed unclearly in Gerard's translation: 'Mittam numerum partium diversitatis Lune in hora quesita in tabula superfluitatis diversitatis Lune, et accipiam ex superfluitatibus que ei opponuntur additionis et diminutionis portionem diversitatis unius superfluitatum diversitatis, et multiplicabimus eam in motum diversitatis medium hore unius, qui est 32 minuta et 40 secunda.'[69] Failing to understand this difficult passage, the *Almagesti minor*'s author expresses a rule that relies on the 'near proportionality' of angles ABC : CBD :: AEC : CED. He supplies no geometrical argument for this. The method in the second paragraph is likewise the author's. The table described in the third paragraph is one of al-Battānī's, which was also included among the Toledan Tables.[70] The description of its construction comes neither from Albategni nor the canons to the Toledan Tables. That the author of the *Almagesti minor* states in the last paragraph that this method does not take into account the moon's second irregularity is evidence that he did not have al-Battānī's tables (except those included among the Toledan Tables), for Albategni does mention an adjustment, although unclearly, and has a small table to correct for the second irregularity.[71]



VI.3. The second paragraph, which is supposedly devoted to Ptolemy's approach to finding the time between the mean and true conjunctions and the place of the true conjunction, contains a method that differs from that of Ptolemy given in *Almagest* VI.4 (1515 ed., f. 63r). Ptolemy does not instruct to divide the distance by the moon's carrying beyond, and the source for this part of the

[68] Toomer, *Ptolemy's Almagest*, p. 282.

[69] *Almagest*, 1515 ed., f. 63r. In this printed version, the number 40 is corrected from 56.

[70] Nallino, *al-Battānī*, vol. II, p. 88; and Pedersen, *The Toledan Tables*, Table JA11, pp. 1410–12. The passage where Albategni refers to this table is in *De scientia astrorum* Ch. 42 (1537 ed., f. 59v).

[71] Albategni, *De scientia astrorum* Ch. 42 (1537 ed., f. 59v); and his small table found in Nallino, *al-Battānī*, vol. II, p. 88. While the text is corrupt in the printed version, it is also would have been unclear in *P*, f. 56r to any reader who did not have the table and its headings in front of him: 'Postquam motui Lune id quod inveneris ex secundis descriptis sub superfluo quod est inter Solem et Lunam via quam in ipso capitulo in ipsis tabulis docuimus superaddideris vel ex eo minueris.'

calculation may be a passage of *De scientia astrorum* Ch. 42 (1537 ed., f. 60v). The third paragraph, however, provides the method of Ptolemy, but the *Almagesti minor*'s author could be relying upon *De scientia astrorum* Ch. 42 (1537 ed., f. 59v), which relates this same method. The wording is close to neither Ptolemy's nor Albategni's. In the fourth paragraph, the author outlines Albategni's method from *De scientia astrorum* Ch. 42 (1537 ed., ff. 59r-v). As he did with Ptolemy's method, the author of the *Almagesti minor* supplies more exact steps before giving the approximative method of his source; while Albategni instructs his reader to find the equated portion by taking 7/24 of the distance between the sun and moon, the *Almagesti minor*'s author says to find the equation of point and then gives the option of approximating as Albategni does. The fifth and sixth paragraphs are likewise taken from *De scientia astrorum* Ch. 42 (1537 ed., f. 60v). Some of the wording is taken directly from the source, and the author adds some of his own explanations. In the final paragraph, the author attributes the desire to factor in the moon's second irregularity to an unknown group of scholars, but Albategni also instructs the reader to make an adjustment using one of his tables.[72] The *Almagesti minor*'s author did not know of this table of al-Battānī and referred instead to a similar table in the Toledan Tables, the use of which is discussed in some of the canons.[73] The language is not taken directly from any of these canons, and neither Albategni nor the canons explain that this adjustment is made to account for the second irregularity.

VI.4. This proposition corresponds to the bulk of *Almagest* VI.5 (1515 ed., ff. 63v–64v). The proposition does not take its wording directly from the source. Since the method for finding the moon's apparent diameter at perigee is the same as the one given in V.15 for finding it at apogee, the *Almagesti minor*'s author quickly summarizes the results of the first part of the chapter. Unlike Ptolemy, he treats lunar eclipses first, presumably because they are easier than solar eclipses since parallax is not involved. While many of the values from the *Almagest* are reported here, they are given a geometrical basis that is not provided in the *Almagest*. The small table or list of values is perhaps an addition to the text. Its Ptolemaic values correspond to a small table of values included in *Almagest* VI.3 in Gerard's translation.[74] In this proposition, there is confusion concerning Albategni's values for lunar eclipse limits. The manu-

72 Albategni, *De scientia astrorum* Ch. 42 (1537 ed., f. 59v); and his small table in Nallino, *al-Battānī*, vol. II, p. 88. See my commentary above on V.2.

73 Pedersen, *The Toledan Tables*, Table JA21, pp. 1413–14, and the canons Cb178, Cc 286 and Cc288, pp. 452–53 and 708–09.

74 In fact, this table is given twice in some *Almagest* manuscripts and the 1515 ed., ff. 61v and 62v.

scripts offer several different values for the maximum distance of a mean conjunction from the node: 14° 47′, 14° 43′, 13° 41′, and 9° 43′. Also, the value 14° 35′ is found in the little table, which may not be original. Albategni does not have a section of his text on how to find the limits of eclipses, but he does mention that they are found with his tables of conjunctions and oppositions.[75] In addition, a list of eclipse limits based upon a distance of 14° 47′ was included among his tables.[76] The source of these values is unclear. Plato of Tivoli does not appear to have translated al-Battānī's tables into Latin, and such values do not appear in the Toledan Tables. The values based upon 14° 47′ are found in one of the canons to the Toledan Tables, but there is no attribution there to Albategni.[77] Without a clear source, it thus seems most probably that the *Almagesti minor*'s author calculated the limits himself using Albategni's values for the diameters of the moon and shadow.

VI.5. These first two paragraphs correspond to sections within *Almagest* VI.5[78] (1515 ed., f. 64r, the 1st paragraph, and f. 64v, the 1st paragraph). The general argument follows Ptolemy's, but the author reports in parallel some values that come from Albategni or that result from calculations made with Albategni's values.

The source of the method in the last paragraph is unknown. Albategni does not explain how eclipse limits are found. Although there are some errors, the method in this proposition is to find the furthest distance that the moon can be from the node and still appear to touch the sun, then to locate the true conjunction that would have occurred shortly before this (or that would occur shortly after this), and from this to find the furthest place at which the mean conjunction could occur. This is an improvement to Ptolemy's method, in which the maximum difference between the true and mean conjunction is added not to the location of the true conjunction, but to the moon's true place at the time of the apparent conjunction. Thus Ptolemy's solar eclipse limits are not the furthest distances at which mean conjunctions can be from the node when there are eclipses. Geber noticed the same problem in the *Almagest* and made a similar correction.[79] Geber also noticed that if the parallax was away from the node, the limit turns out to be greater, but it appears that the author of the *Almagesti minor* did not realize this, as the parallax is only drawn

[75] Albategni, *De scientia astrorum* Ch. 43 (1537 ed., f. 61r).

[76] Nallino, *al-Battānī*, vol. II, p. 88; and these also occur in the Spanish translation found in Paris, Bibliothèque de l'Arsenal, 8322, f. 56v.

[77] Pedersen, *The Toledan Tables*, Ca190, pp. 306–07.

[78] In the 1515 edition of the *Almagest*, this chapter is incorrectly numbered as 'capitulum decimumquintum.'

[79] Geber, *Liber super Almagesti* V, 1534 ed., p. 72. A summary of this critique and correction of Ptolemy's eclipse limits is found in José Bellver, 'Jābir b. Aflaḥ on the Limits of Solar and Lunar Eclipses', pp. 3–27.

towards node B in the figure. That he does not fully correct Ptolemy as Geber does suggests that he came up with this partial correction on his own and that he had no knowledge of Geber's *Liber super Almagesti*.

It appears that the author of the *Almagesti minor* did not rely on a source for the Albategnian limits. Al-Battānī provided results only in his tables, which were not included in Plato's translation, and these values are also reported in the canons to the Toledan Tables.[80] However, even if the *Almagesti minor*'s author did have access to the tables or used that set of canons, the limits reached in this proposition do not agree with the values found in the tables. It seems most likely that the author of the *Almagesti minor* calculated these limits according to this method, which he seems to have derived himself, using Albategni's values for the sun's apparent diameter at perigee and the sun's greatest equation and Ptolemy's values for the maximum northern and southern parallaxes.[81] With these parameters and this method, one can successfully reach the eclipse limit to the south of the ecliptic, i.e. that using the maximum northern parallax. In terms of the figure, Albategni's value for AE is 34′ 30″, to which is added 8′, resulting in 42′ 30″ for AG. This is multiplied by 11.5, and BG is thus found to be 8° 8′ 45″. Because the parallax in longitude is 30′, this apparent conjunction is 2′ 30″ further from the node than the true conjunction,[82] so BF is 8° 6′ 15″. If 2° 34′, the maximum distance between true and mean conjunction, is added, the eclipse limit will be approximately 10° 40′, as is found in the text. For the southern parallax, the result cannot be explained as easily, but is perhaps as follows. First, 34′ 30″ is added to 58′ to reach 1° 32′ 30″. The author appears to have rounded this crudely, ignoring the seconds, which results in an error of 5′ 45″ in the eclipse limit. The rounded value 1° 32′ is multiplied by 11.5, which gives 17° 38′ for BG. A twelfth of the parallax in longitude 15′ is subtracted, so BF is approximately 17° 37′.[83] With 2° 35′ added as an approximation of the greatest distance between the mean and true conjunctions, the eclipse limit is thus approximately 20° 12′. *Epitome Almagesti* VI.7 attributes to Albategni the same solar eclipse limits that are found here.[84] It is possible that the table of eclipse limits was originally part of this work. It is found in the margins of manuscripts of Groups 2, 3, and 4: *K*, *D*, W_2, *M*, *W*, *T*, and W_1. On the other hand, its appearance in W_1 is almost

[80] Nallino, *al-Battānī*, vol. II, p. 88; Pedersen, *The Toledan Tables*, Ca189, pp. 306–07.

[81] Neugebauer, *A History of Ancient Mathematical Astronomy*, pp. 127–29 explains the deficiency of the values that Ptolemy uses.

[82] As Geber knew, by taking another configuration, the true conjunction indeed could be further from the node than the apparent conjunction, and this results in a slightly higher value for the eclipse limit; however, the author appears to have not realized that.

[83] The correct method would be to add a twelfth of 15′, but it appears more likely that the author would have made the same mistake here as in the case with northern parallax.

[84] Venice, BNM, Fondo antico lat. Z. 328, f. 53r.

surely due to contamination because E_1 does not have it. The fact that it is missing in so many manuscripts suggests that it is an addition. The Ptolemaic values are found in the small table of eclipse limits in the *Almagest* (1515 ed., ff. 61v and 62v).

VI.6. This corresponds to the first section of *Almagest* VI.6 (1515 ed., f. 64v, the 1st paragraph). While the argument is similar, Ptolemy does not provide a geometrical figure and justification. The wording is not taken directly from Gerard's translation.

VI.7. This corresponds only very loosely to a part of *Almagest* VI.7 (1515 ed., f. 67v, the last paragraph), but more closely to a passage in *De scientia astrorum* Ch. 43 (1537 ed., ff. 61r-v). This passage is more developed and explanatory than the corresponding passages of its sources, and the wording is the author's own. Our author probably moved this topic here because in the text corresponding to the following proof, Ptolemy only roughly approximates the apparent diameter of the moon when it is about 64° on either side of the epicycle's apogee.

VI.8. This corresponds to part of *Almagest* VI.6 (1515 ed., the paragraph going from f. 64v to f. 65r), and some of the wording is taken directly from Gerard's translation. The argument follows that of Ptolemy fairly closely although the *Almagesti minor*'s author refers to a geometrical diagram. While Albategni does not have a corresponding passage, our author calculates values according to his parameters, apparently using his tables of solar and lunar equation and hourly motion,[85] or perhaps working directly from the processes laid out above in V.9 and VI.2. Note that the author heeds his proof in V.10 and does factor in the equation of portion when calculating according to 'Albategni's work.'

VI.9. This corresponds to a passage in *Almagest* VI.9 (1515 ed., f. 65r, the 1st full paragraph). The argument follows that of Ptolemy, and there are a few close similarities in language. The author of the *Almagesti minor* includes some geometrical explanation while Ptolemy does not. Again, there is no corresponding passage in *De scientia astrorum*, but our author apparently performed the calculations himself probably using al-Battānī's tables of the equations of the sun and moon.[86]

85 Nallino, *al-Battānī*, vol. II, pp. 76, 78, and 80–81; however, the author of the *Almagesti minor* likely knew these tables through their inclusion among the Toledan Tables, see Pedersen, *The Toledan Tables*, Tables CA01, CA11, EA01 and EA11, pp. 1144–48, 1152–55, and 1245–58.

86 Nallino, *al-Battānī*, vol. II, pp. 78 and 80–81; or Pedersen, *The Toledan Tables*, Tables EA01 and EA11, pp. 1245–58.

VI.10. This corresponds to a section of *Almagest* VI.6 (1515 ed., the paragraph going from f. 65r to f. 65v), and some of it is taken directly from Gerard's translation. Again, there is no corresponding passage in *De scientia astrorum*, so the author of the *Almagesti minor* appears to have performed the calculations 'according to Albategni' himself. Note that our author does not find the places in the ecliptic according to Ptolemy's text; he uses the position for the sun's line of apsides that he attributed above to Arzachel in III.11. Our author quickly passes over the last part of the argument where Ptolemy discusses the latitudes that have large enough parallaxes for the found places of the ecliptic at the right times. Our author gives no indication of how he may have confirmed that the required parallax is found from the second clime northward, and perhaps, he repeated Ptolemy without redoing the laborious calculations for the shifted position of the ecliptic.

VI.11. This corresponds to part of *Almagest* VI.6 (1515 ed., f. 65r, the full paragraph). The general argument follows Ptolemy but with some added geometrical explanation, and there are a few sentences clearly derived directly from Gerard's translation. Once again, the author performs calculations 'secundum Albategni', although Albategni did not write about the repetition of eclipses. Note that our author is again using the location of the sun's line of apsides that he used in VI.10, and which was first reported in III.11. As in the last proposition, our author does not go through his process of determining what latitudes do or do not produce the required parallax according to Ptolemy and Albategni. The most likely option is that he checked al-Battānī's tables of parallax for different climes, which were also included among the Toledan Tables.[87]

VI.12. This corresponds to the last paragraph of *Almagest* VI.6 (1515 ed., f. 66r). The argument follows that of Ptolemy and some of the text is clearly derived directly from Gerard's translation. Perhaps because it is so similar to the operations of the preceding propositions, our author does not go through the process of how the approximately 30° of motion in latitude is found, while Ptolemy does go through the calculation. Unlike the preceding four propositions, the author does not perform the calculations 'secundum Albategni' here. The use of the terms 'obliqui' to refer to the antipodes and 'habitabilis' to refer to one of the two inhabitable zones of the earth suggest a possible connection to Cicero's *Somnium Scipionis*, which has these words used with similar meanings, although Gerard's translation of the *Almagest* also uses 'habitabilis.'[88]

VI.13. This passage has no directly corresponding passage in the *Almagest*; perhaps it could be seen as an expanded explanation of the passage in which

[87] Nallino, *al-Battānī*, vol. II, pp. 95–101; or Pedersen, *The Toledan Tables*, Tables HC, pp. 1380–1404.

[88] Zetzel, *Cicero, De Re Publica*, p. 89.

Ptolemy explains the theoretical background for the eclipse tables (*Almagest* VI.7, 1515 ed., ff. 67r-v). This passage does, however, correspond to a passage in *De scientia astrorum* Ch. 43 (1537 ed., ff. 61r-v).[89] While there is some similarity of language, and the general method is the same, our author provides a geometrical explanation and goes into much more detail than Albategni.

VI.14. These first two paragraphs correspond to passages in *Almagest* VI.7 (1515 ed., from f. 67r's last paragraph through f. 67v's full paragraph). The general outline of the proofs in the first two paragraphs match the computations of Ptolemy, but our author uses much more complex figures that are labeled differently, and he also treats eclipses with and without delay separately. The rules at the end of the second paragraph correspond to those given in *De scientia astrorum* Ch. 43 (1537 ed., ff. 61v–62r) although the language is different, and Albategni carries the rules further to reach times instead of distances. Also, Albategni sometimes uses the term 'morae minuta' to mean the difference between the radii of the shadow and the moon in these rules,[90] but the *Almagesti minor*'s author restricts its usage to the meaning expressed in the definition at the beginning of Book VI. The third paragraph on tables summarizes lunar eclipse tables such as those of Ptolemy, al-Battānī, or al-Zarqālī. It could be said to correspond very loosely to passages given in *Almagest* VI.9 or VI.10 (1515 ed., f. 67v, the last paragraph, or f. 69v) or to correspond more closely with a passage in *De scientia astrorum* Ch. 43 (1537 ed., ff. 63v–64r), but the discussion in the *Almagesti minor* is general enough that it was not necessarily written with one of these passages in mind.

The last two paragraphs correspond to Albategni's rules for finding the times of the various parts of eclipses more precisely in *De scientia astrorum* Ch. 43 (1537 ed., ff. 62r-v). Ptolemy does not show how to take the minutes of immersion and delay more accurately through treating the moon's path as declined, but Albategni and the *Almagesti minor*'s author do factor in the slant of the moon's transit during the eclipse. The *Almagesti minor* justifies Albategni's rules geometrically, and while Albategni's intent here is to find the times of the parts of the eclipse through the minutes of immersion and delay, our author focuses here only on the distances, i.e. the minutes of immersion and delay. The rules for finding the duration of the delay more accurately are corrupt in *De scientia astrorum*,[91] but our author either was able to reconstruct

[89] This printed edition mistakenly has the number 15 for 12 several times in this passage, implying that the digits are fifteenths of the moon's diameter and not twelfths; however, such mistakes are not found in *P*, f. 58r.

[90] See Nallino, *al-Battānī*, vol. I, pp. 97–98 and 275.

[91] Albategni, *De scientia astrorum*, 1537 ed., ff. 2r-v; Nallino, *al-Battānī*, vol. I, pp. 98–99; and *P*, ff. 58v–59r. Nallino, *al-Battānī*, vol. I, p. 275, provides an emended text that agrees conceptually with the *Almagesti minor*.

the mathematics correctly or had access to a non-corrupt or corrected copy of *De scientia astrorum*. These proofs share little wording in common with their source. The figure appears to be based off of a similar one that is used later in Albategni's chapter, which surely aided in the creation of our author's geometrical proof.[92]

VI.15. This corresponds only loosely to a short passage in the middle of *Almagest* VI.9 (1515 ed., f. 69v) and much more closely to passages in *De scientia astrorum* Ch. 43 (1537 ed., ff. 61v–62v and 64r). Albategni intertwines finding the various minutes of the eclipse and the times, but the author of the *Almagesti minor* separates Albategni's passage into two separate propositions, VI.14 and VI.15. The last paragraph has no corresponding passage in either source.

VI.16. This has no corresponding passages in Ptolemy, but it corresponds loosely to a passage in *De scientia astrorum* Ch. 44 (1537 ed., ff. 66v–67r). In the middle of the corresponding passage. Albategni gives rules that are somewhat similar to the methods given here; however, Albategni finds the true carrying beyond of the moon instead of the apparent motion of the moon, deals with time intervals of only 10 minutes or less, and includes the use of Theon's tables of parallax. Our author remains on a more general level than his source, and this proposition does not take its wording directly from Albategni. Also, while Albategni discusses the apparent carrying beyond in the midst of his directions for finding apparent conjunctions, the author of the *Almagesti minor* separates this into its own proof.

VI.17. Although this corresponds loosely to the first part of *Almagest* VI.10 (1515 ed., ff. 70r to top of 70v), in which Ptolemy gives a method for approximating apparent syzygies, the method found here has significant differences from Ptolemy's. It does, however, agree closely with Albategni's method found in *De scientia astrorum* Ch. 44 (1537 ed., ff. 66r–67v and 70r–71r). Our author adds the geometrical representations of time and space. A few phrases and terms, including 'diversitas prima/secunda/tercia' show a dependence upon Albategni, but most of the text is in the author's own words.

VI.18. There is no passage in the *Almagest* that corresponds closely to this passage. The source is *De scientia astrorum* Ch. 44 (1537 ed., ff. 67v–68r).[93] Our passage does not borrow Albategni's wording directly. The author does not go into as much detail as Albategni does about finding the apparent latitude of the moon and the apparent diameter of the sun. He makes sense of his source

[92] Albategni, *De scientia astrorum*, 1537 ed., f. 65v.

[93] As for the digits of lunar eclipses, the printed edition mistakenly has the number 15 instead of 12 several times in this passage and states that the digits are fifteenths of the sun's diameter instead of twelfths; however, such mistakes are not found in *P*, f. 64r.

although the *De scientia astrorum* has a few substantial errors here.[94] The geometrical representation in this proposition is the author's own creation.

VI.19. This corresponds to passages in *Almagest* VI.7 (1515 ed., f. 67r, the last paragraph) and *De scientia astrorum* Ch. 44 (1537 ed., f. 68r). While Ptolemy, unlike Albategni, does use a geometrical figure, the *Almagesti minor*'s author reuses the figure of VI.18 that represents the astronomical situation more clearly. He also provides more of a general proof that does not take wording directly from the *Almagest*. The rule at the end of the first paragraph accords with Ptolemy's method, but it is a paraphrase of Albategni's rule. In the last paragraph, the *Almagesti minor*'s author clearly had in mind primarily one of al-Battānī's tables, which was also included among the Toledan Tables;[95] however, the description of the table is general enough that he could have been simultaneously describing the table in *Almagest* VI.8 (1515 ed., f. 68v)[96] although with 31′ 20″ for the sun's diameter.

VI.20. This does not correspond closely to any passage in the *Almagest*, but it does correspond very closely to passages in *De scientia astrorum* Ch. 44 (1537 ed., ff. 68r and 71r). Our author gives a geometrical justification for the rules given by Albategni, and he takes some of the wording directly from his source. Interestingly, Albategni and the author of the *Almagesti minor* are not as exact here as they are in finding the precise minutes of immersion in lunar eclipses. The author could have easily applied the Pythagorean Theorem two more times as in VI.14 to find the lengths of KH and HT; however, calling upon V.26 (but with an incorrect or earlier numbering), he is content with the lengths KP and QT instead.

VI.21. The first two paragraphs correspond to a passage of *De scientia astrorum* Ch. 44 (1537 ed., ff. 68r–69r). Albategni's method is followed and some of the passage is taken directly from the source. A difference is that while Albategni outlines the process for finding the apparent carrying beyond of the moon each of the four times that it is used, the *Almagesti minor*'s author does not give any such instructions. The remaining two paragraphs correspond to the last section of *Almagest* VI.10 (1515 ed., ff. 70v–71r) and to part of *De scientia astrorum* Ch. 44 (1537 ed., ff. 71r-v). Unlike Ptolemy, the author remains on a general level instead of providing a calculation. He also makes it clear that the parallaxes in longitude are not always greater the nearer they are to the hori-

[94] Both Albategni, *De scientia astrorum*, 1537 ed., f. 67v and *P*, f. 63v have erroneous values for the combined radii of the sun and moon at apogee and perigee. Because Albategni does not state what the values represent, these errors make the passage difficult to comprehend.

[95] Nallino, *al-Battānī*, vol. II, p. 91; and Pedersen, *The Toledan Tables*, Table JE11, pp. 1472–74.

[96] The use of this table is explained in *Almagest* VI.10 (1515 ed., f. 70v).

zon, although he follows Ptolemy throughout the remainder of the proposition, disregarding the exceptional cases. The third paragraph is in the author's own words, but the fourth is almost entirely taken from Albategni.

VI.22. This corresponds to *Almagest* VI.7 (1515 ed., a short passage near the top of f. 68r and the last paragraph of the chapter going from 68r to 68v) and *De scientia astrorum* Ch. 43 (1537 ed., 62v–63r). The metrical analysis follows the argument of Ptolemy's calculation although the wording is not taken directly from the source. Also, while Ptolemy finds the obscured amounts for solar eclipses first, the *Almagesti minor*'s author begins with lunar eclipses. The second method for finding line GA is derived from Albategni's instructions. While Albategni devotes much time to discussing how to correct the minutes to take into account the varying apparent sizes of the moon and shadow during the eclipse, our author does not address this.

VI.23. This demonstration corresponds to a passages in *Almagest* VI.7 (1515 ed., f. 68r, the 1st paragraph) and *De scientia astrorum* Ch. 44 (1537 ed., ff. 69r–70r). The argument follows the general outline of the calculation in the *Almagest*, and a few words are taken directly from that source. Our author's passage is very condensed since the argument is almost identical to that of the preceding proposition. Unlike with lunar eclipses, the author does not calculate the areas of the circles, and indeed he does not even state that they are known. Albategni's set of directions is much more complex since he corrects for the apparent size of the sun and moon. The table discussed is either a table from *Almagest* VI.8 (1515 ed., f. 69v, discussed in a short passage of *Almagest* VI.9, near the bottom of f. 69v), or one of al-Battānī's (Nallino, *al-Battānī*, vol. II, 89, briefly described in *De scientia astrorum* Ch. 43 and 44, 1537 ed., ff. 64r and 71v), which is also found among the Toledan Tables.[97]

VI.24. This corresponds to the end of *Almagest* VI.11 (1515 ed., the paragraph going from f. 71v to f. 72r and f. 72r's 1st full paragraph) and to parts of *De scientia astrorum* Ch. 43 and Ch. 44 (1537 ed., ff. 63r-v and 70r). Our author follows the general method of Ptolemy, but does not take any wording directly from his source, he uses a different figure, and converts the calculation into a metrical analysis. This proposition has some similarities to Albategni's instructions: the author uses different latitudes for the different times of the eclipses, and as he often does, his method of solving right triangles involves making the hypotenuse a radius of a circle instead of a diameter. The table described corresponds to *Almagest* VI.12 (1515 ed., f. 72r), which was also included, with rounded values, among al-Battānī's tables and the Toledan Tables.[98] In the last

[97] Pedersen, *The Toledan Tables*, Table JC31a, pp. 1449–50.

[98] Nallino, *al-Battānī*, vol. II, p. 89; and Pedersen, *The Toledan Tables*, Table JC41, pp. 1453–55.

of these, it bore the heading 'Tabula reflexionis tenebrarum in utraque eclipsi', which suggests that the author had the Toledan Tables in mind. Albategni briefly explains the use of the table in *De scientia astrorum* Ch. 43 and 44 (1537 ed., ff. 64r-v and 71v), but he does not describe the process of finding the values to construct the table.

VI.25. This corresponds loosely to the first half of *Almagest* VI.11 (1515 ed., ff. 71r-v) and more closely to VI.13 (1515 ed., ff. 72r-v); but it is indeed closest to passages in *De scientia astrorum* Ch. 43 and 44 (1537 ed., ff. 63r–66r, 70r, and 71v). The content generally matches that of the *Almagest*, but the language is different and our author provides his own geometrical explanations using his own figures. Although the figure is different, it is similar to Albategni's eclipse figures in *De scientia astrorum* Ch. 43 and 44 (1537 ed., ff. 65v and 72r) in that south is at the top, north at the bottom, east to the left, and west to the right. Our author does not address a significant mistake in Ptolemy's and Albategni's treatments. Ptolemy and Albategni tell how to find the angle formed by the ecliptic and the great circle through the centers of the sun and moon or of the moon and shadow, and they instruct their readers when to take the quantity of this angle to the south or to the north from the rising or setting points; however, the value of the angle cannot simply be used as the value of an arc on the horizon since the angle at the center of the eclipse is not necessarily 90° from the horizon.[99] Our author follows Ptolemy in making this mistake. The author's definitions of winter and summer risings and settings do not match those given by Ptolemy or Albategni, who state that they are the places on the horizon where the summer and winter solstices rise, but they are similar to statements made by Albategni in *De scientia astrorum* Ch. 43.[100]

[99] See Neugebauer, *A History of Ancient Mathematical Astronomy*, p. 143.

[100] *Almagest*, 1515 ed., f. 71v; and Albategni, *De scientia astrorum* Ch. 7 and 43 (1537 ed., ff. 13r and 64v).

Commentary on the Figures

Book I

I.1. The figure is from *P*. *P* also has a first attempt at the figure that has a few errors and was marked 'falsa figura.' The figure is lettered slightly differently than the one from Gerard's translation of the *Almagest*. Point H here is labeled as point E in the *Almagest*. Point Z is point R in the 1515 printed edition of the *Almagest*, but the 1515 edition has 'r' consistently wherever the manuscripts of Gerard's translation have label 'z.' *K, Pr*, *T*, and *R* have two extra lines AC and CG, the sides of an inscribed triangle and hexagon respectively. *K*, *D*, *R*, and W_2 have an extra label E between D and Z. *B* does not have the whole circle, only the required semicircle. Some features of *K*'s figure are also found in *Pr*, *T*, *D*, *R*, and W_2. In *Pr*, E_1, W_1, *R*, and *Ba*, there are labels identifying which lines are the sides of which polygon. R_1 has added labels M and I along BH. W_2 has an added line AI, presumably the side of a triangle. *Ba* is labeled differently to match its alternate text (see Appendix). A second, added figure, which is also in the 1515 *Almagest* edition but not in Paris, BnF 14738, is in *Pr*, L_1, *Me*, and *Ba*. It consists merely of a right triangle inscribed in a circle.

I.2. The figure is taken from *P*. It is essentially identical to that in the *Almagest*. *N* contains two instances of this figure, one of which is unlabeled. In *Ba* the figure is labeled differently (see Appendix).

I.3. The figure is taken from *P*. It is essentially the same as that in the *Almagest*. *N* differs in having a complete circle.

I.4. I use *P*'s second figure. *P*'s first figure has line DZ not drawn as a perpendicular, so the figure is marked 'falsa.' The figure is essentially the same in almost all the manuscripts. R_1, *Pr*, *E*, E_1, W_1, W_2, and *Ba* were not drawn with DZ perpendicular to AG, but R_1, *Pr*, and W_1 were corrected. P_7 has line BZ instead of BG. *T* has the figure twice, and in both instances the labels D and G are reversed. In one instance, what should be line BG is omitted. In *D* the lines BG and BD were erased. *L* has an extra line from point B toward E. *Ba* is labeled very differently (see Appendix).

I.5. The figure is from *P*. The figure matches that in the *Almagest*, except that point H here is denoted E in Gerard's translation. *B* was missing line DH, but it was later corrected. *M* is missing lines DH and AB. *F*, L_1, *Me*, *Da*, *E*, *D*, and *W* are missing one or both of the lines DH and AB. *Ba* is labeled very differently and it had a few mistakes that were corrected (see Appendix).

I.6. The first is taken from the second attempt to draw it in *P*. *P*'s first figure has a curve (not an arc) from H to Z that does not pass through E. There is no point T. It appears that a later reader added the correct figure. *F*, R_1, *Pr*, L_1, *Me*, *E*, *T*, E_1, *L*, *D*, W_2, and *Ba* all have errors involving arc HET and confusion about points T and Z. For points H, T, and Z, *T* has the labels T, H, and C respectively. *Ba* is labeled differently (see Appendix). *Pr* and E_1 have an added figure to illustrate that sines and chords of double arcs have the same ratios.

The second figure is taken from *P*. *D* mislabels point A D. *Da* has two small added figures that have no clear connection to the text.

I.7. The figure is taken from *P*. Many of the manuscripts have the figured labeled with the name of the proof: 'alkata coniuncta' in *P*, *F*, R_1, *B*, P_7, *Da*, *E*, *K*, *D*, W_2, and *M*; 'alkatata coniuncta' in *W*; and 'kata coniuncta' in *N*, *Pr*, and *N*. *F* and R_1 include an extra line from A to Z, and *T* includes separated lines to illustrate the ratios below the figure. *Ba* has different labels for the points (see Appendix).

I.8. The figure is taken from *P*. The figure is labeled with the name of the proof: 'alkata disiuncta' in *P*, *B*, P_7, *Da*, *E*, E_1, *D*, and *M*; 'alkacata disiuncta' in *W*; and 'kata disiuncta' in *N*. R_1 has an added line from *B* to *G*. R_1 and *F* have an extra line that is perhaps there to correct this, and *Pr*, L_1, and *Me* have the figure drawn a second time as in *F* and R_1 but with a point labeled F. *Ba*'s figure is flipped, rotated, and labeled differently.

I.9. The figure is taken from *N*. There is ambiguity about which points of the figure are designated by labels D and Z in *P*, *N*, *F*, *Pr*, *Me*, L_1, P_7, *Da*, and *E*. *K*, *M*, *D*, W_2, and *W* have no label Z, and have AD perpendicular to DB. *B* and *T* have D at or near the lower extremity of the diameter. *Pr* has this figure twice, first with arc AB shorter than BG, and then with AB greater than BG. The first contains the extra lines AB and BG. E_1, W_1, and *Ba* have arc AB shorter than BG.

I.10. The figure is taken from *K*. *P* places this after I.11's figure, and DZ is drawn very faintly and not as a perpendicular. *F* and R_1 similarly place this after I.11's figure and have the sines drawn obliquely to the diameter (corrected in *F*). *M* and *W* have D very off-centered. *Da* lacks line DG and has an added figure consisting of triangle EDZ circumscribed. The figure is mirrored in E_1, W_1, *D*, and *Ba*. *D* has the labels A and G switched. *T*, which has an alternate proof, has an extra figure (see Appendix).

I.11. The figure is taken from *K*. In *P* the lines GH and BZ were not drawn as perpendiculars to the diameter and were then erased. Many of the other manuscripts also do not have one or both of the sines drawn perpendicularly to the diameter. W_1 started to draw the figure by I.10, but then drew it in the correct location. W_1 also mislabels points A and Z. Part of the figure has been cut off in *D*. W_2 has an incomplete figure that does not have the lines extended outside of the circle and that only labels point G.

I.12. The figure is taken from *P*. *N* and *F* add the sines of arcs AB and AG. *Ba* and W_1 do not label point A and have Z closer to the center of the circle than D is. *Ba* lacks label A.

I.13. The figure is taken from *K*. *P* and *B* are mirrored. *P* has a number of mistakes both in labeling (e.g. two points are labeled H and none is labeled K) and in the correct configuration of the lines and arcs (e.g. line TKL is not drawn, but there is a line ZH). R_1 and *F* have many of the same problems as *P*. *Pr* has point L at the apparent intersection of line AG and arc GZ, which suggests that the drawer did not understand the three-dimensionality of the figure. *Da* and *E* label the figure 'kata disiuncta.' *Da* lacks labels G, H, and T, and it draws many of the lines in the wrong places. *E* has this figure out of place near I.17. *T* includes two extra figures for its proofs of the cases that are not treated in the standard *Almagesti minor* (see Appendix). E_1 fails to make lines intersect correctly at L and K. For the latter, the drawer tried to correct the figure by adding a few angles into line HZ to make it pass through the intersection of DG and LT. W_2 does not make all the lines that should intersect at K actually do so. *Ba* has R for point K, and continues BH to point G.

I.14. The figure is taken from *B*. *P* has several missing labels and mislabeled points. It also has many lines drawn incorrectly. There is not even an arc GZI or point I, so it may have been difficult for a reader to even understand what the proof is attempting to show. *K* ran out of room to complete the figure and mistakenly puts O and T along line EZ at its apparent intersections with lines HI and HA. This shows that the illustrator did not properly understand the three-dimensionality of the figure. *M*, *N*, and the third figure of *Pr* have many points labeled differently (I, D, and O are respectively labeled D, K, and L). This relabeling matches the figure given in the *Almagest*. *M* lacks line GZD, and does not have the line that corresponds to HO passing through point A. R_1 and *F*'s figures have the same problems as *P*'s. *Pr* has the figure drawn three times. The first two instances are the standard figure and between them, a reader could understand the figure. The third figure, which is taken from the *Almagest*, has point H drawn very close to B, so it does not appear to be the sphere's center. *Me* forms points O (mislabeled K), K, and T, but they do not fall on a straight line. W_1 has a similar problem, and also part of the figure is cut off. *T* has two extra figures for its alternate text for this proposition, which are similar to ones in Thebit's *On the Sector Figure* (see Appendix).[1] *Da* and

[1] Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 59, 69, 129, 132, 145, and 147; and Knobloch, 'La Traduction Latine du Livre de Thābit ibn Qurra', pp. 565 and 573. The labels do not match those of the original Arabic or of the three Latin versions of this work.

E have the label 'kata coniuncta.' *Da* lacks labels B, H, G, and E, has extra labels F and D by points D and I respectively, and lacks or misdraws several of the lines and arcs. *E* has the figure drawn near I.16 and lacks line GEO. *E* also has the two added figures that are in *T* although they do not match the proof given in the text. *E* also has an extra figure that does not seem to have any connection to the text, but which is perhaps an attempt to draw the figure of one of Thebit's proofs.[2] There must have been some version of the proofs of Thebit in the manuscript from which *E* and *T* were copied. In *L*'s figure there is no line GZD, line ET does not pass through Z, line GO does not pass through E, and there is an extra line, EB. *D* is missing labels G and O, part is cut off, and lines and arcs do not clearly intersect at Z, as they should. W_2 follows *K* in the mistaken placement of O. *W* has the same mistakes as *M*.

I.15. The figures are taken from *K* with permission. These figures are almost surely later additions. Most manuscripts have no figures for this chapter, but *K*, W_2, *M*, and *W* have illustrations of the two instruments, *Da* has a picture of the first instrument, and *Pr* has two representations of a quadrant similar to the second instrument but by the text of III.1. *M* draws the two components of the first instrument separately. *K* and W_2 incorrectly have both pins of the second instrument on the uppermost edge instead of along the side.

I.16. The figure is taken from *P*. R_1, W_1, and *Ba* have astronomical labels. W_2 lacks points Z and T and arc ZHT. *T* draws separate lines outside of the figure to represent the quantities involved in the ratios.

I.17. The figure is taken from *P*. *B*, P_7, and *Da* include an extra line M drawn alongside the figure. *M* has the figure twice. *T* draws lines to represent each quantity in the compound ratio. In W_2 arc ZH is not extended all the way to the pole and no label Z is given. *Pr* includes an arc from the other pole to a point on EG. *Ba* and W_1 have astronomical labels.

Book II

II.1. The figure is taken from *P*. *B* and P_7 have mirrored figures, which fits the astronomy better. *M*, *Pr*, E_1, W_1, and *L* give astronomical labels. *Da* includes a separate line labeled M.

II.2. The figure is taken from *P*. Since it is identical to the figure for II.1, L_1, *Me*, E_1, *T*, *D*, and W_2 do not give a separate figure here. *Da* is mirrored and has a separate curved line labeled M. W_1 has astronomical labels. *Ba* reverses the labels B and D and has the figure mirrored.

[2] See my discussion of the figures of I.13. Also, see Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 65 and 131.

II.3. This proposition refers to the figure for II.2. A separate figure for II.3 is only given in *B*, *M*, L_1, *Me*, P_7, *Da*, W_1, *R*, and *W*. W_1 has astronomical labels.

II.4. This also uses the figure for II.2. Only W_1 has a figure given specifically for this proof.

II.5. The figure is taken from *P*. *B* has point H mislabeled as B. *N* is missing the labels B and D, and its S was perhaps mislabeled L and then corrected. *Da* has label N for Q. *Ba* lacks a figure for this proof. W_2 lacks three labels. *Pr* and W_1 have astronomical labels.

II.6. The figure is taken from *B*. I have made lines GZ and DZ meet, although the intersection is cut off in the margin. The figures are mirrored in *P*, *M*, *N*, *F*, L_1, *Me*, and *W*. *P* does not have line TD but instead has a line from T to another point on ED and has a line dropped perpendicularly upon FE. *P* also has a label K, but it is not clear which point it designates. *F* has the same problems, as did R_1 before the mistaken lines were erased. *M*, *N*, R_1, *Me*, *T*, and *W* have a perpendicular from D to EF. The figures in *K*, *M*, *Pr*, *E*, *T*, E_1, *W*, and W_2 all have problems with line CB: it is missing, it is drawn incorrectly and/or its label B does not clearly mark the correct point. *Pr* gives the figure twice, and its first figure mislabels points P L and C O respectively. *Me* and L_1 mislabel point H. *Da* mislabels point C Z, and then draws lines GZ as a chord of the circle and lacks label P. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.8–9. The figure is taken from *K*. In most manuscripts, one figure is given for II.8 and II.9 and it includes lines only used in the latter. There are two points labeled H, but an attentive reader would understand which was intended. *L*, *M*, and *W* have two different figures for II.8 and II.9. *P* has the second horizon, which is needed for II.9, drawn parallel to the first horizon instead of having them intersect at E. *P* also mislabels points N and P and omits labels F, H, Z, and V. *F* and R_1 have the same problems as *P*. In *M* and *W*, the first figure lacks points Y and R and lines PY and QR, so to follow II.8 a reader would need both figures. In *M* and *N*, the second figure has point V labeled N. *N* lacks label I, point Y, and line OX, and it has T instead of C. *N*, *Me*, and L_1 have line HF to the left of line CEG. *L*'s figures lack point V. W_1, which has the figure by II.14, has no label V and the labels F, H, and Z appear to mark points on the wrong horizon. *Me* and L_1 are lacking the second horizon needed for II.9. *Pr* labels several points differently. R_1 and E_1 have the line to the zenith drawn incorrectly, and W_1 omits both the zenith and the line drawn to it. *Pr* and E_1 label the zenith with Z. *Da* reverses labels O and P, switches labels B and Q, and labels R P. In *Ba* and *R*, the figure is mirrored. Astronomical labels are found in *Pr* and E_1.

II.14. The figure is taken from *P*. Only *N*, L_1, and *Da* draw LEM as a semicircle. In *Pr*, E_1, and *Ba*, the figure is mirrored. R_1 does not have label Z. W_2

does not have labels or mislabels A, B, and M. *Ba* labels K as C. Astronomical labels are found in *Pr*, W_1, and *Ba*.

II.15. The figures are taken from *K*. *P* gives figures that have arcs in the right configurations, but it does not label all points and several of the points are labeled incorrectly. *F* and R_1 have the same incorrect labels. *Pr* does the same for the first figure, but then gives a combined figure as in the *Almagest* and the normal second figure. There is a single figure for the proof, as in the *Almagest*, in *M*, *Me*, L_1, and *W*. *T*'s figure with arc HZ is mirrored, and it switches B and D. *E* only has the figure with arc HZ, but labels Z D. W_1 has the two figures and then the combined figure. In two of these, the label R is given in place of K. *Da* lacks label E in the first figure. In *Ba*, K is labeled R. There are astronomical labels in *Pr*, W_1, and *Ba*. In several manuscripts one of the figures is drawn by II.16, which may have been the effect and the cause of confusion.

II.16. This uses the first figure from I.15 taken from *K*. While several manuscripts put one of the figures for II.15 by the text of II.16, clever readers would understand that such a figure was needed for II.15 and that it is the figure usually placed first by II.15 that is reused for II.16. *Me* and L_1 mark the figures for II.15 as also belonging to II.16, but they also give an additional figure for II.16. This is taken from the *Almagest* (with M changed to Q) although its labeling does not match the text here in the *Almagesti minor*. *Da* has a figure with rectilinear triangles, but this belongs to a gloss.

II.17. The figure is taken from *P*. *N* switches labels H and Z. *Pr* has R for K. *Da* labels N H. *E* has no Z. E_1 has R instead of K and misplaces it and line RT (i.e. KT). W_1 has R for K and no D. *L* reverses B and D. In *Ba*, D is not labeled, H is labeled Q, and K is labeled R. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.18. The figure is taken from *P* with a small correction: where the figure has label L, *P* has S. The same mislabeling occurs in *F*. *L* switches labels B and D. W_1 lacks this figure. W_2 is missing labels A and B. Astronomical labels are found in *M*, R_1, L_1, *Pr*, *W*, and *Ba*.

II.22. The figure is taken from *P* with a small change. The figure is given twice in *P*, both times with the label O instead of E. The other main witnesses have this point labeled correctly, but the mislabeling is found in *F* and R_1. *N* is mirrored. E_1's figure is partly cut off in the margin and point G is labeled T. *D*'s point G has been cut off in the margin. W_1 and *Ba* lack labels A, D, E, and G. Astronomical labels are found in *N*, *Pr*, W_1, and *Ba*.

II.23. The figure is taken from *P*. *N* is mirrored. R_1 lacks label G. E_1's figure is partly cut off in the margin. *Ba* lacks label B. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.24. The figure is taken from *P*. The figures in *N*, W_1, and *Ba* are mirrored. Label D is cut off in the margin in *Pr* and E_1. *Pr*, E_1, W_1, and *Ba* have astronomical labels.

II.25. The figure is taken from *P*. W_1 lacks label Z. Astronomical labels are found in *Pr*, W_1, and *Ba*.

II.26. The figure is taken from *K*. *P* labels H M, as also do *F* and R_1. *M*, *N*, *W*, and *L* mirror the figure so that it is as in the *Almagest*. *E* does not have this figure. *T* lacks labels H and K and has an extra semicircle from B through E. E_1, W_1, and *Ba* have R for K. *Pr*, *L*, and W_1 have astronomical labels.

II.27. The figure is taken from *P*. R_1 switches labels H and L, and lacks E. *Da* lacks this figure. *E* lacks label G. In E_1 the part of the figure with label A has been cut off. W_1 is mirrored and lacks labels K and L. *R* has this figure twice. First it has it by the proper text, but with label A for K, and then it has a mirrored version of the standard figure by the text of II.18. *Ba* is mirrored and has R for K. *Pr* and W_1 have astronomical labels. *Da* lacks this figure.

II.28. The figure is taken from *K*. *P* has an extra label M near label G, as does R_1. *M*, *N*, R_1, L_1, *Me*, and *W* lack line ZD. *F* has M for G. *Da* lacks this figure. W_2 has H instead of G. In E_1 and *Ba*, the label G is cut off in the margin. *Ba* has C for E and has extra line BC. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.29. The figure is taken from *K*. In *P* and *F*, the label for B is L. *N* gives two versions of this figure. The first has the points on the circumference labeled differently and has arcs from E to each of these. The second figure is the standard figure with an extra arc, ET. R_1 labels B H. L_1 and *Me* have extra arcs ET and EK. L_1 has no label E and mislabels D B. *Pr* and *Ba* have R instead of K. *Da* lacks this figure. *E* lacks labels T and D. E_1 and W_2 lack labels T and K. *R* has an extra label H near D. *Ba* also lacks T and has O instead of G. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.30. The figure is taken from *K*. *P* has an extra label B next to label H and has a point F between Z and G, which perhaps represents the intersection of the meridian and equator. *F* and R_1 have the extra label F that is in *P*. The figure is mirrored in *Me* and L_1. *Da* lacks this figure. W_1 lacks G. W_2 lacks D and has the label E marking the point also labeled D. *W* lacks Z. *Ba* draws arc ZHT in such a way that it is not clear that it intersects the meridian at Z. Astronomical labels are found in *Pr*, E_1, W_1, *L*, and *Ba*.

II.31. The figure is taken from *P*. *Pr* has arc BD drawn incorrectly. *Me* labels E Z. *Da* lacks this figure. Point G is cut off in the margin of E_1. W_1 reverses D and E. *R* has arc AD end at D and the labels E and D apparently mark the same point. Astronomical labels are found in *Pr*, W_1, and *Ba*.

II.32. The figures are from *P*. The first figure in *M* and *W* has extra line HE. *N*'s first figure has A labeled D. *Me* and L_1 have the extra line HE in the first figure. L_1 is missing most of its labels and has the extra line HT in the first figure. *Me* and L_1's second figure lacks Z. *Da* lacks these figures. In *E*'s second figure, B is labeled T. E_1's first figure has D cut off in the margin, and the second figure's A is cut off in the margin. W_1's second figure is mirrored. E_1 and W_1 have R for K. *R* has Z at point E in the second figure. *L*'s second figure works, but it has A between B and G. *Ba* only has the first figure. It has some extra lines and labels, but these do not fit the second part of the proof. *Pr*'s figures for this and the following proposition are mixed, but there are labels saying which figure goes with which proof. Astronomical labels are found in *Pr*, E_1, W_1, and *Ba*.

II.33. The first figure is taken from *K* and the second from *P*. In *P*, *F*, and R_1, the first figure has arc BEK instead of GEK. *P*, *F*, and R_1 have the same extra third figure; it is for the second case but has arc DK instead of DE. In *B*, P_7, and *E*, the second figure has an extra arc BE, and K is on the extension of this, instead of GE. In *K*, *D*, *R*, and W_2, the second figure lacks K and T, which are not referred to in the text. *Pr*, E_1, W_1, and *Ba* have R instead of K. *Da* lacks these figures. W_1 and *Ba* have the first figure mirrored, and the second figure, which also is mirrored, appears near the text of II.36. *D* reverses labels T and H. *R*'s second figure lacks B. *W*'s first figure has arc BEK instead of AE and has Z on arc GE extended. *W*'s second figure is mirrored and has several mislabeled points. Astronomical labels are found in L_1 and E_1.

II.34. The figure is taken from *K*. In *P*, *F*, R_1, *Pr*, *Me*, L_1, and *L*, the figure is mirrored. *P*, *F*, and R_1 have a point F between B and Z that seems to be intended to serve as the intersection of the equator and the meridian. *Da* lacks this figure. W_1 has A labeled H. Astronomical labels are found in *Pr*, *Me*, L_1, E_1, W_1, *L*, and *Ba*.

II.35. The figure is taken from *B*. This figure is used for this proof and the following one. *P*, *F*, and R_1 lack D. *K* and *M* have the point at the end of semicircle ZHT labeled K, as also do *R*, *D*, W_2, and *W*. *M*, *N*, *Pr*, *Me*, L_1, *L*, and *W* have separate figures for II.36, so they do not include arc KLM, which is only needed in II.36. The figure is mirrored in *M*, L_1, and *W*. *Me* lacks G. *Da* lacks this figure. *E* does not have arc KEZ extended to A or G, and also lacks D. A and Z are cut off in the margin of E_1. W_1 and *Ba* have R for K. W_1 lacks E. *R* relabels K F. *Ba* has arc GEA drawn twice. Astronomical labels are found in *Pr* and E_1.

II.36. This shares a single figure with II.35 in most of the manuscripts, so see the description above for details concerning the shared figure. Separate figures for II.36 are found in *M*, *N*, *Pr*, *Me*, L_1, *L*, and *W*. *M* gives the figure for II.36

twice. The first lacks K. *N* has G at the end of arc ZHTL instead of AHEK. *Pr* has this figure three times. The first has the label D at M. The second has several errors. The third has astronomical labels, and has an extra label S at the end of arc ZHTL. *T* has the combined figure for II.35 and has it again by II.36, but without labels. *W* lacks labels K and A. *Me* lacks label T. L_1 lacks the labels K and L and has arc MLK mistakenly drawn to point B.

Book III

III.1. Only *Pr* has figures for this proposition. It has two labeled, geometrical representations of a quadrant with points marked on them for the equinoxes and solstices.

III.3. The figures are taken from *P*. *Da* lacks this figure. Parts of the figure in E_1 are cut off in the margins, but what remains matches the figures here. W_1 lacks labels H and K in the second figure. *R*'s first figure is flipped vertically. *Ba* has R for label K in the second figure. Astronomical labels are found in W_1 and *Ba*.

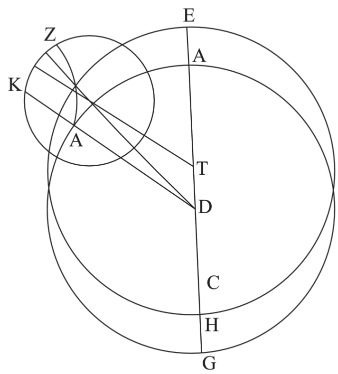
III.4. This proof reuses the figures from the previous proof, which I took from *P*. *P* and R_1 have the first of these figures drawn again for III.4, but mirrored and with X for Z. The first figure is also drawn again for this proposition in *B*, *Pr*, *T*, E_1, *Me* (mirrored), L_1 (mirrored and with a first unlabeled attempt), P_7, W_2 (mirrored and lacking label D), and *Ba*. The second figure is also redrawn in L_1 and *Ba*. A reader of *K* cleverly traced out some of the lines and labels from III.3's figures on the other side of the folio (these are thus mirrored).

III.5. The figure is taken from *K*. *P* labels points P and Q B and A respectively, as also does R_1. *M* has the figure twice, lacking labels E and Z in the second instance. *F* has the correct figure, but has a first attempt that lacks the outer circle (perhaps following the *Almagest*) and that consequently lacks points P, Q, and X. *Pr* has two instances of the figure, the second of which has an astronomical label. L_1 has L for point Q. Line EB is extended to the ecliptic in *Me* and L_1. *Da* lacks this figure. *E* has the label S for point Q and labels the corresponding point on the right side of the figure Q. E_1's point Q is cut off in the margin, but the scribe added a note saying where it should be. W_1 labels X Z. Before the standard figure, *R* has an unlabeled figure that is perhaps a misplaced attempt at III.11's figure. *L* gives the figure without circle PQX or those labels. W_2 is missing labels K and T. *Ba* has R for K and an astronomical label.

III.6. The figure is taken from *K*. *P* and *F* do not draw DHT tangent to the epicycle and lack label G. In *Pr*, *E*, *T*, E_1, W_1, and *Ba*, angle DAT is drawn very obtusely. *Pr*, E_1, W_1, and *Ba* have R instead of K. *Pr* has the figure drawn again by III.10 but with the epicycle and lines AT, AH, and DT drawn for two locations on the deferent and with B absent. *Me*'s figure is similar to this.

L_1 has an attempt at a figure as in *Me*, but it also lacks the tangent for one epicycle and misdraws it for the other. *Da* lacks the figure. *T* lacks label K. *D* has an extra point O on the deferent, which appears to mark where the apogee is. In W_2 line DH is not extended to T, and the label T appears to designate point K. *Ba* is mirrored, lacks label B, and has an extra line DK to show the direction of the apogee.

III.7. The figure is taken from *K*. *P*'s figure is faulty (see below), and the same mistaken figure is found in R_1 and *F*. *M* lacks label H. L_1 and *Ba* call Z R. *Da* lacks the figure. E_1 and W_1 have label R for K. *Ba* has label N for K and C for E. Astronomical labels are found in W_2 and *Ba*.



III.8. The figure is taken from *B*. *P* does not have line MN, but has lines BT and NT. It also draws line BZ incorrectly and mislabels point L H. *K* and *D* have the unnecessary line BT. *M* draws the figure twice, and contains the unnecessary line BM. In most of the figures, the smaller eccentric is drawn as large or almost as large as the concentric, but *N* draws it noticeably smaller. R_1 has the problems of *P*, except it lacks label A. *F* has all the problems of *P* and more, so it would have been very difficult for a reader to use it. After an unfinished first attempt, *Me* has the correct figure but with label D absent. L_1 and *Da* lack this figure. After a first rather incomplete first attempt, *E* has two further attempts. Both of these mistakenly put point T where E should be and thus have almost all of the lines drawn incorrectly. *T* has the figure twice. The first instance lacks line MN, but has additional lines BT and NT. The second instance is drawn correctly, except that it has line KZ instead of line KT. E_1 and W_1 have R for K, make Z and M the same point (which should not happen if eccentric LM is smaller than the concentric), and have additional line BT. *R* has Z and M coinciding, and has an extra line extending from the center of the epicycle, mislabeled D, to T. *L* has M and N coincide, and point G is cut off in the margin. W_2 does not have line DT passing through Z, and has extra line BT. It also puts M at the wrong point and thus has the incorrect

line NM. *W* has M marking the wrong point (although line MN is drawn geometrically correct) and has an extra line BM. *Ba* has an astronomical label, lacks labels A and L, and mislabels K R and Z M. Because M marks the wrong point, line MN is drawn incorrectly.

III.9. The figure is taken from *K*. *P* and *F* lack label G. The figure is mirrored in *M* and *W*. *Da* lacks the figure. *Ba* has an astronomical label.

III.10. The figure is taken from *B*. Points M and F, which are not used in the proof and which appear to represent the locations on the concentric where the epicycle's center is when the star is at apogee and perigee, are only found in *B* and P_7. The figure is mirrored in *P*, *N*, R_1, *F*, *L*, and *Ba*. *P* and *F* lack point G and have C for T. *K* and *M* lack T. R_1 has C for T. *Pr* has two instances of the figure. In the second of these, the epicycle is drawn near perigee G. *Me* has the figure with C for T. *Me* and L_1 draw the epicycle in a second location after it has passed the perigee. *Da* lacks the figure. *E* lacks G and has C for T. *T* has I for T. E_1 lacks label G. W_1 lacks B. *L* labels T C. W_2 and *W* lack T. *Ba* lacks B, has C for T, and has an astronomical label.

III.11. The figure is taken from *K* with one small correction – *K* mislabels R K. *P* is missing lines KVX and TOP, and has several mislabelings. It has A for Q, K for R, and Z for F. It also lacks a label for point A. *B* mislabels H B. *M* has the figure twice. R_1 and *F* have *P*'s problems. In addition, R_1 lacks labels X, S, and Z, and *F* has P mislabeled K. An attempt was made to correct the figure in R_1, but it still has several errors. *Pr* has a first attempt that has many errors, which the scribe marked with 'non valet', and then it has the figure drawn better twice. In these and in the figure in *Me*, C and T are reversed and there are no astronomical labels. *Pr*'s second correct figure is rotated counterclockwise 45°, as is the figure in E_1. The figure is missing in L_1 and *Da*. *E* lacks labels Q and L, and it has A for X and F for P. Part of *T*'s figure cannot be seen in my reproduction, but it appears to be correct. E_1 has R for K. W_1 lacks labels Q and L. *L* has Z for R and R for F. W_2 lacks lines KVX and POT and labels M and R. O and P appear to label the same point. *Ba* has R for K and has lines through points Q and L and through M and R. More astronomical labels are found in W_1.

III.12. The figure is taken from *K*. *P* draws the figure such that the angle at D is closer to being a right angle than the angle at D is. *M* and *W* have an extra circle drawn to represent the zodiac. *N* draws lines mirroring DB and EB on the right side of the figure. In R_1 and *F*, neither BD nor BE appears to be a right angle, although a reader added a small figure that is drawn better. *Da* does not have this figure. *T* appears to be missing label D. E_1 lacks label G. W_1 and *Ba* have the figure mirrored and have astronomical labels.

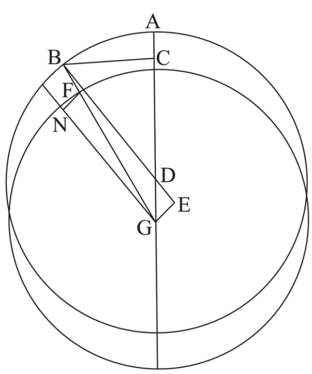
III.13. While the *Almagest* has a figure for the first third of this proposition and a second for the other two thirds, the author here combines these into one for the whole proposition. The figure is taken from *P* with one small change: *P* has X instead of Z. In *P*, *K*, *B*, R_1, *F*, *Pr*, *D*, and *R*, the angles at one or both K and L appear to be non-right. R_1 draws the figure as in *P*, but then redraws DK better. In *F* label Z appears to have been corrected from X. *Pr*, E_1, W_1, and *Ba* have R for K. *Da* has no figure. *E* lacks labels H and G. Part of the figure in *T* is not visible in my reproduction, but it appears to be correct. W_1 lacks label B. W_2 has C for T. Astronomical labels are found in W_1.

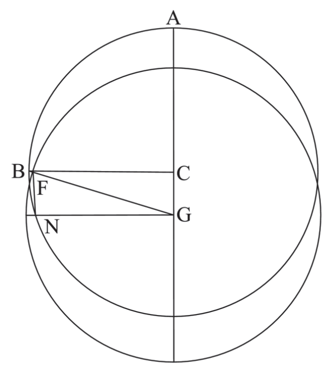
III.14. The figure is taken from *K*. *P* has an extra label Z that appears to mark K, and it lacks points G and L and line AL. *M* has the figure twice. The second instance lacks label G, as does the figure in *N*. R_1 has the same problems as *P*. *Pr* and *Me* have C for T. P_7 does not have line AL. *Da* lacks this figure. *E* lacks G and line AL. *T* has an additional diameter of the concentric, perhaps to represent the line of apsides. E_1 and W_1 have R for K. *D* lacks label G. *R* has D for Z. W_2 does not have L. *Ba* lacks K, H, and T.

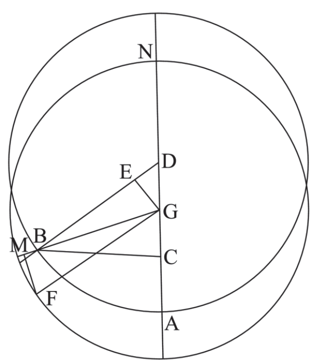
III.15. The figure is taken from *K*. The angle at L is drawn more as a right angle in most of the other manuscripts (exceptions are R_1 and *D*). *P* has C for T and places an additional point F on the eccentric to the right of Z. *M*, *N*, and *W* orient the figure so that the eccentric and ecliptic are placed as they are in the figure for III.13. The figure is also mirrored in *M* and *W*. R_1 and *F* have the same problems as *P*. *Pr*, which is mirrored, had a very acute angle L, but the scribe redrew line TL more accurately. In *Pr*, *Me*, and *R*, lines LD and TKZ do not meet at Z, but at B. L_1 does not have lines LDB and TKZ meet at all, and it lacks labels A and E. *Me* and L_1 have an extra line passing through D. *Da* does not have this figure. *E* labels T E and lacks A and E. E_1 and W_1 have mirrored figures. E_1 and *Ba* label R for K. W_1 has labels Z, K, and R by the point K. W_2, *W*, and *Ba* had lines LD and TK meet at B, but W_2 corrected the figure.

III.16. The figure, which combines two figures of the *Almagest* into one, is taken from *B* with a small change. *B* and P_7 have T for Z. *P* lacks line KH and label G, and it has X for Z. The label K appears to mark point T. AL is not drawn perpendicularly to line DH. *K* also puts K at T and has KH drawn as a chord. *M*'s figure is mirrored, as is *W*'s. *N* lacks labels G and Z. R_1 and *F* have the same problems as *P*. In R_1 label B was erased. *Pr* has no label L or line AL. It also has R for K. *Me* and L_1 have X for Z, C for T, and no label G. *Da* lacks this figure. *E* and *T* lack G and have T for Z. E_1 has AL drawn as a chord and has R for K. W_1 has point L beyond B, which is geometrically incorrect. It also has R for K, and lacks G. In *D*'s figure, K and T mark the same point and KH is drawn as a chord. *R* does not have L and T. *L* lacks E and G. W_2 has C for T and also has another label C near label H. In *Ba*, AL is drawn beyond the concentric. This figure also has R for K and lacks Z and G.

III.17. This proposition does refer to some points of III.15's figure, but it appears that the text originally had no figures drawn by this proposition. *L* gives this figure again here. *Da*, which did not give a figure by III.15, provides it here (with labels A and H missing). *F* has a crude figure that appears to have been added by a later reader containing a unique figure with additional sines drawn from the sun. A note gives the labels for astronomical quantities mentioned in the rules (e.g. 'arcus datus', 'eius sinus', etc.). It would not have been sufficient to provide a geometrical justification. *B*, P_7, *T*, and *E* have three additional figures (taken here from *B*), and a note reports, 'Iste tres figure huius operationis sunt preter principales et non de originali.' *Da* has the first two of these figures. *Da*'s first figure has M for N, has an extra label R on the extension of line GM, and lacks F. Its second figure has M for N. In *E* these figures are in the margin by III.15. The first two of its figures lack label A, and the second and third lack line MF.







III.19. The figure is taken from *P*. *M*, *N*, and L_1 mislabel C T. *Da* lacks the figure. In E_1 line CR is not extended to B and D. *R* appears to have C for R. W_2 lacks labels K, R, D, and G. *W* does not have D. *Ba* marks the lowermost point of the eccentric with T, and it also has astronomical labels.

Book IV

IV.1. The figure is taken from *M*. *P*, *K*, *D*, and W_2 do not have line DC passing through K. *P*, *B*, and *E* have the label G drawn as if it labeled a point on line EC. *N* has T for C and lacks line ET and label F. R_1, *F*, and *Pr* have the figure with the mistakes found in *P*, but R_1's scribe corrected its figure and *F* has a second, correct figure. *Me* and L_1 have T for C. *Da* lacks this figure. E_1 has R for K and has G marking a point on line EC. W_1 draws semicircles instead of whole circles. In *R*, K is not on DC, and point G is not on earth, but marks the intersection of DC and EB. *Ba* has only semicircles, has G on line EC, and does not put K on the moon's sphere.

IV.3. The figure is taken from *P*. There is no figure in *N*, *T*, *R*, and *W*. The figure is drawn vertically in *E* and W_1. Label A is cut off in the margin of *D*.

IV.5. The figure is taken from *K*. *P*'s figure mislabels P F, Q H, and T G. *B* lacks label K. *M* has the label C at the intersection of the eccentric and line EB. *N* lacks label K. *F* has a figure with *P*'s mistakes, but then has a better figure, which only lacks K. *Pr* and E_1 have R instead of K. L_1 mislabels both P and Q with L, and also lacks G. *Da* lacks the figure. L_1 has astronomical labels. W_1 lacks K. *R* is drawn with point Z very close to F, not as the center of the eccentric. *Ba* lacks labels H and K, and D has been cut off in the margin.

IV.8. The figure is taken from *K*. *P*, R_1, and *F*'s second figure have an extra line drawn from D between DT and DA. Its point Z is not on the eccentric. *M* has C for T. *N* only draws an arc of the eccentric, not the entire circle. It also extends DA to the eccentric and lacks line DZ. R_1, *F*, and L_1 extend line DA to the eccentric. In R_1, G is not quite on the deferent. *F* has the figure drawn twice. In the first figure, *K* has been cut off in the margin. *Pr* has the figure drawn three times. In the first, A is on the eccentric and there is no point K. In the second, only arc ZT of the eccentric is drawn. The third instance of the figure is placed by the text of IV.9. *Me* has a figure with many errors, but has a second correct figure. This second figure extends DA to the eccentric and lacks label K. *Da* lacks this figure. *E* has a mistaken, unlabeled figure, followed by a correct figure that extends DA to the eccentric and in which point K has been cut off in the margin. E_1 and W_1 have R for K. In *R* and W_2, the epicycle is drawn so large that it does not pass through the intersection that should be labeled Z, and another point is labeled Z. *L* extends DA to the eccentric. W_2 lacks labels E and K. *Ba*'s figure is mirrored and has F for A. It places point Z on the concentric circle instead of on the deferent and has line EZ instead of GZ and line HE instead of DZ.

IV.9. The figures are taken from *K*. *P*'s first figure has label G nearer to line DZ than ED, and it lacks label K. In *B* and P_7, the first figure is mirrored, and the second figure lacks D. *M*'s first figure has an added eccentric circle, and its second lacks D. *R_1*, *Me*, and *L_1* lack label K in the first figure. *F* has an extra label G marking the intersection of the deferent and line DZ in the first figure. *Pr* has the figures twice. The second instances of the figures, which are in the margins of IV.10 lack labels K and D in the first and second figures respectively. *P_7* has two instances of the second figure, both lacking label D. *Da* does not have either figure. *E* lacks label D in the second figure. In *E_1* and *W_1*, the first figure lacks label K, and the second figure has R for K. In *W_2* the first figure is mirrored, it lacks line GZ and point E has been cut off in the margin. In *W* the first figure has the eccentric passing through Z drawn, and the second figure lacks point D. *Ba*'s first figure lacks K, and its second figure, which is mirrored, has R for K.

IV.10. The figure is taken from *K*. In *P* lines DG and GA are drawn as one line and line DZ is drawn to a point on the epicycle's circumference to the right of A. Point *T* is labeled C and G is not labeled. There is also a line parallel to EH slightly above it. *M* has the figure three times, and *N* has it twice. *F* and *R_1* have most of the problems of *P*. *R_1* makes the additional mistake of putting H on line DZ. *Pr* has the figure three times. While the first two are drawn correctly, the third has line DG to the right of DA, which does not match the eclipses that are used for this proposition. With this configuration, line EH should be lower than EZ, but it is drawn above. This same figure is also found in *Me* and *L_1*. *Da* lacks this figure. *E* lacks line EG. *T* has the same problems with lines DG, GA, and DZ that *P* does. *E_1* has R for K. *W_1* lacks label A. *W* has the figure twice. Both lack line EG, and the second lacks label E. *Ba* has many problems. In its figure, which is mirrored, it switches L and B, has no label for point G, and labels E G. It also reverses H and T. *T* has a note on the eccentric model and there is a figure depicting it.

IV.11. The figure is taken from *P*. *Pr* and *E_1* have R for K, lack M and E, and also have extra line LN. *Da* does not have this figure. *W_1* is mirrored and has R for K. In *Ba* the figure, which is placed near the beginning of IV.10, is mirrored and has E for K.

IV.12. The figure is taken from *B*. *B* marks which eclipse is first, second, and third, but I have not replicated these. In almost all figures, it would not be clear which point M labels without consulting the text, and in several, such as *P* and *N*, M appears to mark a point on line EZ. *P* draws EH very crookedly. *M* has the figure twice. *N* has the figure drawn correctly, but in such a way that T is on line MK. *F* draws the figure twice. The first has an extra line drawn from H to a point between E and T. The second figure appears to have been added later. *Pr* has R for K and H for B. *L_1* lacks label M and line DG, and it has point H on line DL. The figure is not found in *Da*. *E* has B instead of H and misplaces it on line DB. It also lacks label T and line GT. *T* is missing labels G and T. *E_1* has R for K and has M labeling the intersection of

EZ and DG. *W_1* is mirrored and has R for K. It also has H upon point G, so there is no separate line EH. *R* has an unlabeled figure, which is followed by a labeled figure. This second figure has N instead of K, and it has lines ETN and NB instead of EB. *W_2* mislabels point G and draws what should be lines TG and EG from a point on line DB, instead of from what should be point G. *Ba*'s figure, which is placed by IV.10, has R for K and lacks M.

IV.13. The figure is taken from *P*. *B* has labels by A and B indicating which eclipses they are. *Pr* extends line SK to the opposite point of the epicycle, which is labeled F. *Da* lacks this figure. *E_1*, *W_1*, and *Ba* have R for K. In *Ba* the figure is drawn by the text of IV.11.

IV.17. The figure is taken from *K*. The figure is drawn mirrored from the way the motions are usually depicted (here the motion on the epicycle is counterclockwise and the epicycle is moved clockwise on the inclined circle). This configuration is also found in Gerard's translation of the *Almagest* in Paris, BnF, lat. 14738, f. 72r. *P* has H for B and has E for Z. *B* has the figure twice. *M*, *W_1*, *W*, and *Ba* have astronomical labels. *N* is mirrored, i.e. drawn in the normal configuration. *R_1* has E instead of Z. *F* appears to be missing D and Z (although they may be not visible due to the quality of my reproductions). *Me* reverses D and Z. *L_1* also reverses D and Z, and has B for H and K for B. *P_7* lacks label B. *Da* does not have this figure. *E* does not have a label for point B, and it has B for D. The figures in *W_1* and *Ba* are mirrored and have H for Z, Z for E, and E for H. *R* has H instead of B. *W_2* has H for A.

Book V

V.3. The two figures are taken from *K*. *P*'s second figure has the epicycle at the top of the figure drawn very off-center, and D has been cut off in the margin. In *M* the epicycle is drawn on center H in the first figure. *N* lacks the second figure. *R_1* does not have G in the first figure. *F* lacks label B in the first figure, and G is cut off in the second figure. In *Pr* the second figure is drawn twice, once by V.2 and once by V.3. The second figure lacks points B and G. *Me* has an incomplete first figure before the completed one. The second figure in *Me* and *L_1*, which is on an added leaf (f. 14r) by the text of III.1, has a small additional circle showing the movement of Z and notes two locations of Z upon it. *L_1* lacks G in the first figure. *Da* lacks these figures. *E* has an unlabeled first attempt at the second figure before the completed figure, which lacks B. In *T* the first figure has D for B and also lacks H and Z. *E_1* lacks H and G in the first figure, and in the second N labels the wrong point and H is on the wrong eccentric. *W_1* depicts the two locations of point Z in the second figure. *D*'s second figure has point D cut off in the margin. *R*'s first figure lacks label Z, and its second lacks B. *L*'s second figure does not have S and T along line BD, but on opposite sides of a diameter of the epicycle that is at an angle to BD.

W_2's second figure lacks N and has G for M, C for S, and D for G. *W* lacks G in the first figure, and has F for S in the second figure. *Ba*'s first figure is placed by V.1, and the second figure is placed by V.2. It also has C instead of S, and D is cut off in the margin. Astronomical labels showing directions are found in the first figures in *N*, *Pr*, *Me*, and *L_1*. In Gerard's translation of the *Almagest*, both figures are found with a few minor differences in labeling and with line EL lacking in the first figure.[3]

V.5. The figure, which here is taken from *K*, is for V.5–6. In *P*, EK is not drawn as a tangent to the epicycle, and line DI and its label appear to be one curved line. *B*, *P_7*, *E*, *T*, and *E_1* add the label R at the perigee of the epicycle upon center M. *M*, *W*, and *W_1* have a figure for V.5 that does not contain the epicycle and lines needed for V.6, and this figure also lacks H. *N* also lacks H and has the lines related to the epicycle on M flipped horizontally and vertically. Instead of line DI, *R_1* has a curve, and *F* has a line. *Me* has a curve, but the scribe corrected it. *Me* lacks H. *L_1* has this figure on an added leaf (f. 14v) by the text of III.2. It lacks H and line MK, and it has a curve for DI alongside the correct line. *Da* lacks this figure. *E* has B for H and no label at point B. It also has a perpendicular from K to ME. *E_1* lacks H and has L for I. *W_1* contains a second figure, which is essentially the standard figure mirrored, but with no label H, a second point B, L for I, and an added line from T to a point near E. *R* lacks label I. In *W_2* lines ME and KE do not meet properly at E. *Ba* has a figure with only what is required for V.5. This lacks H, has D and E in the wrong positions, and is mirrored.

V.6. This uses a figure shared with V.5, which is described above. This figure depicts another case than that in the *Almagest*, and it is simpler and has some differences in labeling (e.g. Ptolemy has M and B where the *Almagesti minor* has M and K respectively). Separate figures for V.6 are found only in *M*, *Pr*, *E_1*, *W_1*, *W*, and *Ba*. *M*'s figure is the standard figure for V.5–6, but with no point H and with L for I. *W* tries to replicate this figure; however, it draws the epicycle upon K, which it mislabels M, and it places label K upon a point on the epicycle's circumference. *Pr*'s figure has only the epicycle at M, not the one at G, and it has L for I. This figure is also mirrored from the standard V.5–6 figure. *E_1* redraws the figure it had for V.5 for this proposition. *W_1*'s figure only has the epicycle at M and has L for I, but is otherwise the standard V.5–6 figure. The epicycle appears to have been drawn incorrectly initially by the scribe. *Ba*'s figure is a mirrored from the standard V.5–6 figure, and has R for K, L for I, no H, and an extra point B. Labels B and T are apparently cut off in the margin.

[3] Paris, BnF, lat. 14738, ff. 76v–77r. In *Almagest*, 1515 ed., f. 47v–48r, the first figure contains the epicycle in several locations, as well as the circle upon which Z travels.

V.7. The first figure is taken from *K*. In *P*, R_1, and *F*, points H and M are drawn at the same location, and BL is not drawn as a perpendicular. *M* draws the figure four times. One is mirrored and another contains the normal figure and its mirror. This figure is drawn twice in *M*. *Me* reverses T and C. L_1 has label C for T and lacks a label for point T. *Da* lacks this figure. *E* has many misdrawn lines and incorrect labelings (including F for E). E_1 lacks A and G, has R for K and N for H, and draws points M and H at the same location. W_1 has many of the same problems as E_1 and has C for T and S for C, but it does have A and G. *D* has *P*'s problems and also has BC drawn as an extension of DB. *R* also draws BC as an extension of DB. W_2's figure is drawn correctly, but the labels T, H, and M are placed incorrectly. *W* draws H and M as the same point. *Ba* reverses the locations of points H and M on the epicycle. It also has R for K, and its labels T and G are cut off in the margin.

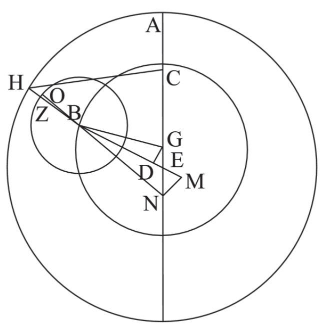
The second figure is also taken from *K*. In almost every figure, DB and BH are drawn as one line, although they should form an angle at B. *P*, R_1, and *F* have many problems: they draw angle AEB as an acute angle, draw KEZ as a curved line, draw lines DB, EB, and NB such that they do not intersect at the epicycle's center, and they add another line from E to the epicycle. *M* draws the figure three times. Of these one is mirrored and one contains both the standard depiction and its mirror image. All three figures have line NLH instead of line ELH, and in all three angle AEB is acute, which leads to angle K appearing acute instead of right or to K being placed between E and B. *N*, *Pr*, W_1, and *L* are the only manuscripts that do not draw DB and BH as a straight line (the bend is very slight). *Pr* has this figure twice. In the first of these and in *Me*, G is not labeled, and a circle representing the ecliptic or the moon's inclined circle is included. L_1 and *Da* lack this figure. *E* lacks T and has F for S. *T* has L for S and redraws and relabels much of the diagram. Although most of it is correct, the erased lines and labels make it harder to interpret. E_1 has R for K and lacks label T. W_1 has O for E and Z for K, and it lacks T. *D* draws angle ADE as an acute angle, and G is cut off in the margin. *R* draws angles AEB and DEK as acute angles. *W* has NLH instead of ELH and lacks E. It also makes angle AEB acute, and has lines from D to two points labeled K, one on each side of AG. *Ba* has R for K and lacks T. M, Z, and H are cut off in the margins.

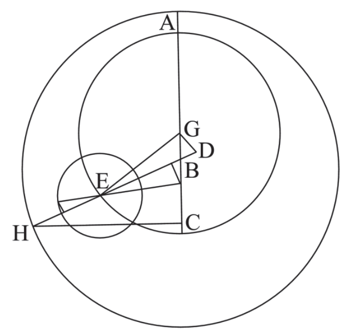
V.8. The figure is taken from *K*. In *P*, R_1, *F*, and *Me* the figure is drawn by V.9 and the figure for V.9 is drawn by V.8. In *P*'s figure, E appears to be the center of the eccentric, and point S is labeled H. *B* has H for N. *M* has a mirrored figure, in which angles AEB and DKE are drawn acute, a second point K is added between E and B, and line ES is drawn instead of NS. R_1 and *F* have *P*'s problems, and they also have line BZ as an extension of DB instead of EB.

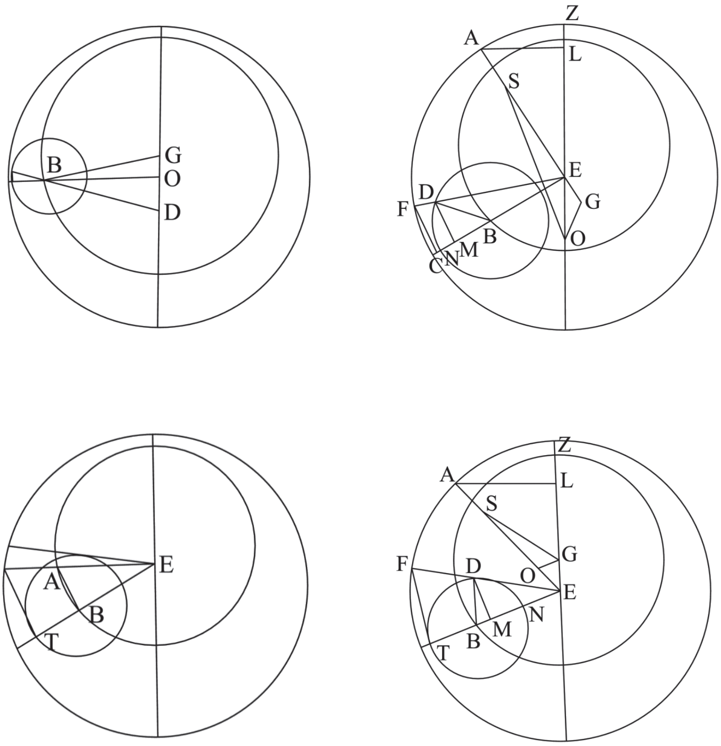
R_1 also has label A for N and an extra label A by D. *Da* lacks this figure. *E*, *T*, and W_1 draw BZ as an extension of DB and BM as an extension of EB. E_1 also places point D at a location off the diameter AG, and it has label H for M and M for N. *T* has an extra line AK. *R* draws BZ as an extension of DB and lacks label K. *L* has a combined figure for V.8–9 that contains the lines needed for both. E_1, W_1, and *Ba* have R for K. *W*'s figure is like M's, but it only has one point K on line EB. *Ba* lacks A, and most of the epicycle and all of the labels on it are cut off in the margin.

V.9. The figure is taken from *K*. *P*, R_1, and *F* have the figure once by V.9 and once by V.8. In both, the lines that should pass through B do not all do so. In *B*, P_7, *E*, *T*, E_1, and W_1, line BH is drawn as an extension of DB, but a note in *B* and P_7 explains that this is a coincidence. *B*, P_7, and *E* have six additional figures. *M*, *Da*, *L*, and *W* lack the figure. *N* depicts the moon's location H on the other side of line ZE. In R_1's second figure, line BZ is drawn as an extension of DB. *F* has label K for N in the first figure, and it lacks A in the second. *Pr* has four additional figures, which differ from those in *B*, P_7, and *E*. *Me*'s figure is placed by V.8. *Me* and L_1 lack label G. *Da* lacks this figure. *E* has a first, unfinished attempt at the figure. W_1 does not have label L or line HL. In *D* there is label K for N, and G is cut off in the margin. *R* has BLZ as an extension of DB. *Ba* draws line BH as an extension of DB, and it lacks L and line HL.

Below, the six added figures from *B*, P_7, and *E*, which are noted to be additional figures in these three manuscripts, are taken from *B*. The figures in *E* have some minor differences in labeling. *Pr* has two figures for the case in which the duplex longitude is less than 90°, followed by a figure for each of the cases in which it is greater than and equal to 90°. These four figures of *Pr* do not match the additional figures in other manuscripts, but they are similar in that they contain circles centered on the earth and additional lines representing the sines of various angles.







V.10. The figures are taken from *K*. In all manuscripts except *M*, *Me*, L_1, *E*, W_1, and *W*, the figures are reversed. They contain some lines that Ptolemy uses but that are not mentioned in this proposition. In the first figure in *P* and W_2, line ET is not drawn tangent to the epicycle. In the second figure of *P*, *F*, and *T*, line LN is not drawn. The second figure in *B* and P_7 lacks EL and has R instead of N. *N* combines both figures into one. It has label S instead of Z and lacks label G and lines MD, ZS, and ES. *Pr* has the first figure twice, and the second instance lacks label T. *Pr* gives the second figure three times. In the first instance of the second figure, L and N are reversed, and in the second instance, M and line DM are lacking. The third does not contain A, G, or the eccentric circle. In the second figure, *Me* and L_1 have I instead of N and lack the eccentric. L_1 also lacks A. *E*'s first figure lacks G. Its second figure has N for M, and lacks L, N, and G, as well as lines LN and EL. *T* lacks lines LN and EL in the second figure. W_1's second figure lacks S, has I for N, and has label N at the intersection of BD and the epicycle. *D*'s second figure lacks E, and has B cut off in the margins. *R*'s second figure lacks lines MD and DN, lacks label N, places E and M by point D, and has an extra label C on line DB. *L* combines both figures into one. W_2's second figure lacks line MD and places M near point D. In the second figure of *W* and *Ba*, line LM is drawn instead of EL. In *Ba*'s first figure, G is missing and A is cut off in the margin. *Ba* lacks S in the second figure, which is by V.9.

V.11. The figure is taken from *P*. I have supplied point M, which is cut off in the margin in *P*. *K*'s figure adds a label S between H and C. *B* draws the rules and the base separately. The part of the figure with the rules is mirrored and H is cut off in the margin. There is no label C. In *M*, *N*, *W*, W_1, and *Ba*, the figures are illustrations of the physical machine. *M*, *N*, and *W* add a feature mentioned by Ptolemy, plates that protrude from the upright rule that are used with the plumb, which is also depicted. Near point L, *M* has an extra label K, which is a point referred to in Albategni, but not in the *Almagesti minor*. *M* and *W* also show the divisions on HM. *N* has E for C and C for G, and also incorrectly labels the base's top instead of the side facing east. R_1, *F*, and *L* have E for M and have an extra label S between H and C. R_1 also puts G near L. *Pr* has the extra label S. *Me* and L_1 have E for M, and *Me* depicts the fins with the apertures. A and B are cut off in the margins of P_7 and *D*. W_1 and *Ba* share the same physical depiction, in which ABDG is the the top surface of the base and there is no label C. *R* depicts the apertures, adds short lines dividing HM into 30 parts, and has the extra label S. W_2 places the figure by V.13 and has the extra label S. *W* has the extra S, lacks line HC, and has the labels C and D at points D and G respectively.

V.13. The figure is taken from *P*. I have supplied label T, which is cut off in the margin. *M*, *E*, *L*, W_2, and *W* lack label B. *N*'s figure is mirrored. E_1 misplaces G upon line AZ. E_1, W_1, and *Ba* have R instead of K. *R* lacks label K, has an extra concentric circle, and mistakenly draws AB before AL was correctly drawn. The figure is placed near V.11 in W_2. In *Ba* an extra label G is placed along AZ, and T is cut off in the margin.

V.14. The figure is taken from *K*. *P*, R_1, and *F* draw BEH as an extension of line ZB instead of MB. *P* and *F* also label C E. *B*, P_7, and *E* draw BH as an extension of DB. *M* and *W* do not extend BK to Z. *M* has the figure a second time with the addition of epicycles drawn at A and G. *N* divides the figure into two. The first has only the points and lines required for the first paragraph, and point G is cut off in the margin. The second lacks points L and lines BL, EL, and BF, and it also has lines FC and EF. *Me* lacks G and T, and it has C for N. L_1 lacks G, C, and T, and it has C for N. *E* has three unlabeled attempts to draw this figure and two labeled ones. These two lack labels G and N and have S for Z and Q for K. They also locate L on the diameter of the eccentric, not on the epicycle, and thus line BL is drawn incorrectly. *T* mislabels F S, and draws EM as an extension of FE. E_1 has labels T and K by the wrong points, and it lacks M and lines DM and EM. W_1 and *Ba* have the epicycle drawn near the apogee, so the perpendiculars, which are labeled DN and ZM, are drawn differently. They also have T and R (for K) labeling the wrong points. Also, BL is drawn as an extension of DB. *L* has line EK instead of EL. *R* lacks labels A and B, places label T away from any intersections, and draws BF in line with ZB. In W_2's figure, F and H are cut off in the margin, B is mislabeled T, K is mislabeled B, and there is no line BF. *W* draws HB and BK in one line. *Ba* lacks B, and G is cut off in the margin.

V.16. The figure, which is shared with V.17, is taken from *P*. I have supplied label A, which is cut off in the margin. In *P* the label C on line QF is perhaps E. *K*, *T*, E_1, and W_2 draw a concentric circle inside of HTE. *M* and E_1 have label R instead of K. R_1 and *F* have an extra label E to the right of point E. R_1, *F*, *Me*, and L_1 have C labeled E. E_1 is mirrored. In *D* and *W*, point A is cut off in the margin. W_2 and *W* lack label L. *Ba* has a unique figure for V.16 that only has the parts of the figure required for this proposition. In *Ba*'s second figure (for V.17), label S is cut off in the margin. Astronomical labels are found in *P*, *B*, *M*, R_1, *F*, *R*, *L*, and *W*.

V.17. The figure was discussed above. *Pr* has an additional figure that accompanies a gloss (L_1 has the same figure by V.18) and has the figure for VI.5 here.

V.19. The three figures are taken from *P*. I have made a small correction in the first – G is labeled S in this manuscript. *B*, *M*, P_7, *E*, *T*, E_1, and *W* lack label B. The figure is mirrored in *N*. R_1, *F*, and *Pr*'s first instances of the figure have S instead of G. *Pr* has the figure drawn twice. R_1 has H instead of L. *E* has label L but lacks line AL. E_1 has R for K, and it has additional labels marking the end of the incomplete circles depicting the earth, moon's orb, and the orb of the fixed stars. *R* draws lines KH and AT in such a way that their intersection D does not fall upon the moon's sphere.

The second figure, which is different than the figure in the *Almagest*, has labels P and C that are not mentioned in the text. *M* and *W* have an additional label Q marking the point to the left of P. *N* and *L* do not have the unnecessary labels P and C. *F* has B in place of H. *E* has A instead of P. L_1 has X and T for P and C respectively. In W_1 and *Ba*, the figure is flipped vertically, and there is an extra label F where GZ meets the circle's other side. This second figure is placed by V.18 in W_2. *Ba* lacks label P. *Pr* also includes the figure from the *Almagest*.

In many of the manuscripts, the third figure, which also is different than the one in the *Almagest*, has Z apparently at the center, although E should be the center. *M* and *W* include many other lines drawn similarly to ZB and ZD. *N* has an incompletely labeled figure before the standard one. L_1 and *Me* have the extra line DE. *Ba*'s figure is mirrored, and it lacks G. *Pr* also includes the figure from the *Almagest*.

V.20. The figure, which is not in the *Almagest*, is taken from *P*. In *N* the figure is mirrored. *N*, R_1, and *F* mislabel point C E. R_1 and *F* place the label G at a point on line MB. In *F* the intersection of MD and KE is not placed on the sun's sphere. *Pr* has the figure drawn twice. L_1 lacks label K and reverses labels C and T. *Me* has C for T and labels point C E and then corrects this to T. W_1 has R for K and lacks labels E and N. *D* has label D instead of M. *R* has O for B. The figure is incorrect and unfinished in *Ba*. It is marked 'mala.'

V.21. The figure is taken from *P*. *N*'s figure is flipped vertically. R_1 and *F* lack labels Z, G, and E. *Pr* has a first, incomplete attempt and then a more complete figure that only lacks label P. L_1, *Me*, and *E* lack label P, and L_1 also lacks D. W_1 has R instead of K. In *D*'s figure, Z is cut off in the margin. *Ba* lacks H and has R instead of K. Astronomical labels are found in W_1 and *Ba*.

V.22. The figure is taken from *B*. *P*, *K*, R_1, and *F* place N at the intersection of ZF and AD, they lack any label at C, and they have a label C instead of E. They also have a label E at the intersection of AE and ZH extended. *P*, R_1, and *F* also lack F. *M* has the figure twice. *N* reverses the place of A and G, and draws lines AD and AE from the point it labels A. *N* also extends MD and FH until they meet. R_1 and *F* have a label N where H should be. R_1 has labels H and T where B should be, and it lacks line HT. *F* has H instead of B. The figure is drawn four times in *Pr*. While none of these figures is labeled completely correctly, a reader could have understood the proposition with the use of multiple figures. L_1 lacks labels A, C, and B. *Me* lacks a label for point C. *E* flips the figure vertically, has O for C, and lacks line TH. *T*'s figure has many problems. G is placed upon line AD. The intersection of AD and ZE is labeled O. There are no labels C or N. ZD is extended to AE, and the intersection is labeled E. E_1 and the two instances of the figure in W_1 have R for K and lack N. E_1 also adds points Q and P at the endpoints of AE and AD, and Z is cut off in the margin. One of the figures in W_1 is mirrored. The other figure in W_1 and a second figure in *Ba* draw the arcs from Z as arcs, but they draw arc ZM instead of ZF. *D*'s figure reverses H and K, misplaces N, and cuts off Z in the margin. *R* and W_2 have numerous misdrawings and mislabelings. Among the most egregious of *R*'s mistakes are that it has a line ADZ and that it does not have a line from Z through D and H. Among W_2's errors are that it places H and T at the wrong points, and it lacks line HT. It also has labels E and K at points that should not be labeled, but lacks labels at points C, K, and N. *Ba*'s first figure is mirrored. It lacks labels E and N, and it has T for K. *Ba*'s second figure, which is placed near V.28, lacks label K and N, as well as line HT.

V.23. The figure is taken from *P*. *B* and P_7 have C for G. *N*'s figure is mirrored, and it has additional lines AD and AE. *T* lacks label G. E_1 has the figure a second time near V.25. *Ba*'s figure is place by V.22.

V.24. The figure is taken from *P*. It is mirrored in *N* and *Ba*. *Pr* has the figure in its normal place and also by V.22. *N* also includes a spherical representation of the same situation, but it is labeled differently and includes additional arcs needed for a more complete proof found in a gloss.

V.25. The figure, which is the mirror of the one in the *Almagest*, is taken from *P*. *N*'s figure is flipped vertically, includes extra lines AD and AE, and extends line DE to a point C, which is labeled 'polus zodiaci.' *Pr* has the figure twice. The second is mirrored. L_1 only has a few lines of the figure with no labels. E_1 and W_1 have label R for K and also have an extra label P on the extension of line ZD. E_1 also has label T for E. W_1's figure is mirrored. *L* reverses labels G and T, and it includes additional lines AD and AE. In *Ba* the figure is flipped vertically, and it has R for K. It also has an extra label A along line ZD, and T is cut off in the margin.

V.26. The figure is taken from *P*. The figure is not found in *M*, E_1, W_1, *W*, and *Ba*. The figure is mirrored in *N*.

Book VI

VI.4. The figure, which has no corresponding figure in the *Almagest*, is taken from *P*. *B*, R_1, *Pr*, *Me*, P_7, *E*, and W_2 lack label H. *M* marks out degrees along AZ. *N* lacks T. The figure is not labeled in L_1. Label E is cut off in *D*.

VI.5. The figure is taken from *B*, but I have added the label F. Almost none of the figures have label F in the correct place and many lack it. It is not found in *P*, *K*, *B*, R_1, *F*, L_1, *Me*, P_7, *D*, *R*, *L*, and W_2. It is placed between D and G in *M*, E_1, W_1, *W*, and *Ba*. It is correctly drawn in *N*, *E*, and *T*. It is below D in *Pr*. *P*, R_1, *F*, and *R* have label B and G in place of label A. *P* and *F* draw the circles of the sun and moon overlapping instead of just touching. *P*, *K*, R_1, *F*, L_1, *T*, *D*, and W_2 give astronomical labels marking the ecliptic and the declined circle. *K* has an extra label G by A. *M* has the figure twice, once mirrored. L_1 and *Me* lack D and B. *Me* also has an additional label L below point D. *E* lacks line DE. The figure is mirrored in W_1, *D*, *L*, *W*, and *Ba*. *D* has an extra label G by point A. *Ba* lacks label B.

VI.6. The figure, which is also used in VI.8–11, is taken from *P*, although *P* lacks label G. The figure is placed near VI.8 in *P*, R_1, *T*, E_1, *R*, and *L*, and it is by VI.7 in *F* and *Pr*. The figure is mirrored in *N*, and it has T instead of C. *F* lacks label G. *B* and P_7 repeat the figure near V.8. *Pr* repeats the figure without all of the labels by VI.10 and again by VI.11. W_1 and *R* have B in place of H. Label A is cut off in the margin of *D*. G is cut off in the margin in *Ba*. Astronomical labels are found in *M* and *W*.

VI.13. The figure, which is also used in VI.14, is taken from *K*. *P*'s figure lacks labels Z and I, and it has A instead of N. H is cut off in the margin. *P*, *F*, and *L* have the line TAE drawn incorrectly so that it does not pass through points T or E. *M*, *N*, and *W* give separate figures for VI.13 that do not include all the points that are only used in VI.14. *M* and *W*'s figures for I.13 include all of the lines but do not provide labels for any of the points below line KH. *N*'s figure for I.13 has only the lines and labels that are mentioned in the text of VI.13. R_1, *Me*, and *Pr* have N on the circumference of the larger circle. *F* has A instead of N and T instead of Z. L_1 lacks N. E_1 has H for N and lacks F. W_1 has R for K and Q for I. Labels M, H, and E are cut off in the margins of *D*. *R* lacks Z. W_2 lacks D. *Ba* has the figure drawn twice by VI.13. In both instances, labels G, T, K, and I are cut off in the margins. *E*, which ends in VI.8, lacks this figure and the following ones.

VI.14. While the first figure used in this proposition is the one used in VI.13, *M*, *W*, and *N* provide separate figures for VI.13 and the beginning of VI.14. In *M*, *W*, and *N*, the first figure of VI.14 reverses the labels for the transit with a delay so that the moon moves from right to left. Also, *M* and *W* have L instead of I. *N* leaves out line AT, has S instead of U, and places N on the circumference of the outer circle.

The second figure is taken from *B*. Label P is only found in *B*, P_7, and *T*. In *P* and *F*, the figure is drawn so that intersection G is on the circumference of the shadow's circle. *P*, R_1, *Pr*, L_1, and *Me* have C instead of M. *P*, R_1, and *F* have an incomplete second attempt at drawing this figure. *M* has this figure twice, and in both, as well as in *W*'s figure, label Z is omitted. *N*'s figure, which is very incomplete, only includes the lines and labels needed for finding the combined minutes of immersion and delay, and it lacks the parts of the figure needed for finding the minutes of delay more accurately. R_1, *F*, *Pr*, and *R* have label E instead of B, and R_1 and *Pr* have B along line AT. In L_1 line FB is not drawn to point B properly. L_1 and *Me* draw the line of the moon's transit parallel to the equator. E_1 lacks point I and incorrectly includes lines EG and TF instead of lines IG and TD. W_1 and *Ba*, the latter of which provides the figure twice, have TF instead of TD and MI instead of ME. Point T is cut off in the margins of *D*. *R* has label D instead of T.

VI.17. The figure is taken from *K*. *P* lacks D and has K in place of A. *B* has D for P. D is cut off in the margin of *F*. *T* lacks Q, and has B for F. E_1 has F for P. C and F are cut off in the margin of *D*. *R* has O instead of C. W_2 lacks F and C. *W* lacks D and has the two curved lines intersecting at A and C. A and D are cut off in the margin of *Ba*.

In most of the manuscripts, the same figure is used for the two cases in which the conjunctions are near the rising or near the setting, but in the second figure the proper motions of the moon and of the sun and the flow of time move from right to left. To avoid this, a separate mirrored figure for the second case is added in *K*, *N*, *T*, *D*, and W_2. In *K* and W_2, the second figure has the label Q for B and lacks P and Q. *T* lacks B. *D* lacks Q and P, and its points F and

C are cut off in the margin. W_2 also has I for N, Q for M, X for Z, and C for G.

VI.18. The figure, which is also used for VI.19, is taken from *P*, but with some small corrections – *P*, R_1, *F*, *Pr*, L_1, and *Me* lack label Z and have E instead of C. Label P, which is not mentioned in VI.18 or VI.19, is not included in *B*, *M*, *N*, *Pr*, or P_7. G is cut off in the margin of *B*. *M* and *W* place F at the intersection of circle XNZ and line KH, and they also lack M. *N*'s figure is flipped vertically. *Pr*, L_1, and *Me* have T or C for X. E_1 and *Ba* have R instead of K. W_1 lacks this figure. *D* lacks Z. *L* lacks C and places X along line ET. W_2 has C for G and has F for P. *Ba* has an extra label Q at the intersection of ET and arc NZ.

VI.20. The figure is taken from *K*. In *P*, R_1, *F*, *Pr*, and *Me*, the transit is drawn parallel to line AEG. *P*, R_1, *F*, *Pr*, L_1, *Me*, and *R* have F instead of P. A is cut off in the margin of *B*. *M* and *W* include a label B as in the previous figure. *N*'s figure is flipped vertically and is drawn with K closer to the ecliptic than T. It also has the label B at the uppermost point and C at the lowermost point, as well as points M and N as in the previous figure. In L_1 there is no label T, and point K is drawn on the inner circumference. Also, the line of the transit tilts the other way, and the perpendiculars are drawn very obliquely. *T* omits labels G and T. In E_1 and *Ba*, there is label R for K, N for H, H for P, and an extra label P at the intersection of line EH and the inner circle. E_1 also has F and K at the top and bottom of the figure. The figure is lacking in W_1. *D* has B for H.

VI.22. The figure is taken from *P*. *K*, E_1, and *D* lack label B. *M*, *R*, and *W* do not have label H. Both the astronomical labels are missing in *N*, W_1, and *Ba*, and 'luna' is missing in *R*. R_1, W_1, and *Ba* lack labels B and H. E_1 and *Ba* have R for K.

VI.23. The figure, which is essentially identical to the preceding one, is taken from *P*. The astronomical labels are lacking in *B*, *N*, P_7, W_1, and *Ba*. *N* lacks H and K, and it depicts a greater eclipse in which Z is above E and D is below T. R_1, *F*, *Pr*, L_1, and *Me* have T for Z. *F* and *R* lack H. E_1, W_1, and *Ba* have R for K.

VI.24. The figure is taken from *P*. Point N is corrected from A in *P*. *N*'s figure is mirrored, and it has T for C. It also lacks labels K, N, and G, as well as lines EG and NG. It also has an additional line from D, perhaps representing the moon's transit. R_1, *Pr*, L_1, *Me*, and W_2 have an additional point A at the top of the figure. *F* and *Pr* have A instead of N. E_1, W_1, and *Ba* have R for K.

VI.25. The first figure is taken from *K*, but I have made some corrections because *K* has C for I, Z for T, and an extra label K on line EF. *K* has an unlabeled first attempt at this figure. *P* has Z for T, has B for A, and it lacks Y. The lunar circle at A is not centered on the north-south line in *P* and R_1. *B* and P_7 lack F and N. *M* has L for I. *N*'s figure is very incomplete. Its outer circle is the combined radii, not a larger circle, and it also lacks many labels and most of the circles representing the moon at various situations. R_1, *F*, and *R* have Z for T and S for F. R_1, *F*, W_1, and W_2 have an extra label K on line FE. R_1, *F*, and *R* lack labels A, N, X, Y, C, M, and D. *F* and *R* also lack P. The figure is unlabeled in *Pr* and *D*. *T* has E for I and B for H. E_1 has R for K, and labels H and Q are cut off in the margin. In E_1 and W_2, there is an eighth circle representing the moon. W_1 has Z for K, L for G, and Y for I. W_1, *L*, and *Ba* lack all seven circles representing the moon. *L* has several labels and lines in different places, and it also has extra lines. W_2 has an unlabeled figure, and then in its labeled figure, it has L for K. *W* has B for I. The figure is entirely lacking in L_1 and *Me*. *Ba* has many mislabelings: G for Y, E for C, O for E, L for G, Z for K, and Y for I, and it has an extra label R on line FE.

The second figure is taken from *K*. The figure is unlabeled and incomplete in *P*, R_1, and *F*. *M* and *W* lack Z. Instead of this figure, *N* has the incomplete figure from VI.24. E_1 lacks B. W_1 is mirrored, and it lacks Z and line EZ. W_2 has Z on the outer circle and a label N on the other endpoint of line ZE. *Ba* is mirrored, and it has O for E and lacks Z. The second figure is lacking in *B*, *Pr*, L_1, *Me*, P_7, *R*, and *L*. *D*, which does not have the end of the proposition, also lacks the second figure.

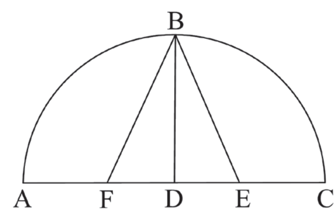
APPENDIX: ALTERNATE TEXTS & ADDITIONS

Alternate Preface

P_{16} (5r): Formam celi spericam esse. Motum celi circularem circa terram undique volvi. Terram in medio celi imoque defixam, que etsi omnium cadentium tam gravitate corporis quam quantitate ponderis sit maxima ideoque immobilis; tamen eius crassitudo comparatione infinitatis aplani et respectu distantie suorum luminum insensibilis est et centri vicem obtinet. †Quia occurrat† ratio comprobavit. Duos in celestibus principales motus et sibi invicem contrarios, quorum alter ab oriente in occidentem semper uniformiter contentione per paralellos et sibi invicem et equinoctiali qui omnium esse spatiosissimus equidistantes totum mundi corpus movet †et† circumvolutio circa polos celestis spere consistit indefesse; alter e contrario Solem, Lunam, et erraticas circa alios polos circumducit.

Alternate Proofs of I.1

Ba (221r):[1] Sit semifigura ABC. Dyameter sit AC divisa in duo equalia [CD] et AD. Erigatur perpendiculariter ad circumferentiam protracta. Sumatur medius punctus DC et sit E, et ab illo ad B recta ducatur. Inde sic CDB est rectus, quadratum EB est equale quadratis DB et DE. Ergo EB est maior ED. Addatur et [subtrahe] ex AD lineam que [faciat] ED equalem EB, et sit DF. Et ab F ducatur recta ad B. Inde sic DC dividitur in duo equalia et ei addatur DF quedam in longum, ergo per secundum Euclidis quod fit ex ductu CDF in DF additum quadratum DE est equale quadrato EF. [Sed] quadratum FE equale quadratum EB. Item quod fit ex ductu CDF in DF cum quadrato DE addito est equale quadrato EB. Sed quadratum EB est equale quadratis DB DE. Ergo illud quod fit ex ductu CDF in DF cum quadrato DE addito est equale quadratis DB et DE. Ergo ablato



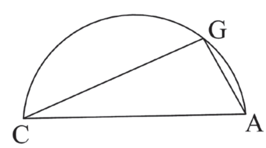
15 CD] MD *Ba* **21** subtrahe] *I add this to make mathematical sense of the passage.* **22** faciat] facti ac *Ba* **25** Sed] si *Ba*

[1] Because it is obvious that *Ba* was copied very carelessly from its exemplar, I do not present the text as it appears in *Ba*; instead, I put many of its clear errors in the apparatus and put the text as it appears to have stood in *Ba*'s exemplar in brackets.

quadrato inde DE est illud quod fit ex ductu CDF in DF equale quadrato DB, ergo [quadrato] DA. Ergo patet sic quod ista linea[2] sit secta secundum proportionem addentem medium ad duo extrema. Ergo per sextum et quintum Euclidis[3] que est proportio CDF ad AD eadem est CD ad FD. Sed CD est latus exagoni per [quarti] penultimam,[4] ergo DF est latus decagoni illius circuli; et hoc habemus per nonam terdecimi Euclidis. Inde sic dyameter notum, ergo eius medietas est nota, [ergo] DB est notum. Sed illud quod fit ex ductu CDF in [DF] equale quadrato DB, ergo illud est notum. [Sed] CD est notum, ergo DF est notum; vel CD est notum, ergo DE.[5] Sed quadratum [BE] est equale quadratis DB et DE, et BD est notum et DE est notum. Ergo BE est notum, ergo EF est notum. Et ED est notum, ergo DF est notum. Ergo latus decagoni est notum per penultimam primi Euclidis.

Et quadratum BF est equale quadratis BD et [DF], sed BD est notum et similiter DF. Ergo BF est notum, ergo latus pentagoni est notum. Quod illud sit latus pentagoni sic constat. Quadratum BF est equale quadrato BD et DF, et unum istorum est semidyameter alicuius circuli, reliquum est latus decagoni illius circuli. Ergo per decimam terdecimi Euclidis est tertium latus est [latus] pentagoni.

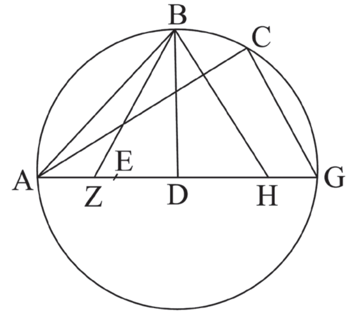
Latus quadrati similiter notum erit. Notus est dyameter cuius quadratum est equale quadratis laterum duorum quadrati illius circuli. Et illa latera sunt equalia et illa pariter accepta sunt nota. Ergo quadratum utriusque est notum cum sint equalia.



Latus trianguli est notum etiam quod sic videtur. Sic [AG] linea erit latus exagoni quod probatur esse notum. Et quod sit triangulus ex dyametro AC et AG et GC, et quadratum AC est notum et equale quadratis CG et GA et quadratum GA est notum. Ergo GC est notum. [Equaliter] quecumque corda cum aliqua predictarum cordarum et dyameter recto facit, scilicet complet semiperiferia, notum erit. Notum, quia dyameter est notus et quelibet predictarum est nota, ergo tertium latus est notum, cum dyameter quadre sit equale quadre illorum duorum laterum quia unum est notum. Et sic perfecta, consistit propositum.

29 DF] DF est *Ba* **30** quadrato] quadratis *Ba* **33** quarti] quartum *Ba* **35** ergo] g *Ba* **36** DF[1]] DB *Ba* Sed] si *Ba* **37** BE] B *Ba* **41** DF] EF *Ba* **45** latus[2]] latera *Ba* **49** duorum] duorum quadratorum *Ba* **53** AG] CG *Ba* **56** Equaliter] equalitatem *Ba*

[2] *i.e.* CF [3] The relevant proposition is *Elements* VI.17. [4] *i.e. Elements IV.15.* [5] In this sentence we see our scribe putting in two readings where he is unsure which is correct. The latter is the correct reading mathematically.



T (67ra): Verbi gratia, lineetur ABG semicirculus super AG diametrum divisum ad punctum D in duo equalia. Et ab eodem erecta perpendiculari ad punctum B, divisaque DG semidiametro in duo equalia ad punctum H. H et B linea recta adiectis.[6] Quia ergo BH longior est AD et minus longa AH per primum Geometrie, ex AH abscidatur HZ equalis scilicet BH. Igitur B et Z linea recta adiectis dico quod DZ latus est decagonis et BZ pentagoni et DB exagoni. Et protracta AC linea latere trigoni per quartum Geometrie,[7] dico quod CG latus est exagoni, et AB latus tetragoni. Que omnia nota esse sic astrue. AG est nota ex ypothesi, ergo DG eius medietas est nota, ergo DH est nota, et DB est nota quia equatur DG, ergo BH est notum, quia quantum potest[8] DB et DH tantum potest BH per ducarnom.[9] Ergo HZ nota ut BH; et DH nota, ergo DZ nota. Sed DZ latus est decagoni ut probatur per sextam secundi libri et ducarnom et nonam 13^{i}.

Item DG est notum, ergo CG latus exagoni ei equale per quartum Geometrie est notum. Sed item AG notum et CG notum, ergo AC latus trigoni notum. Item AG notum, ergo AB notum per ducarnom utrinque. Item DB est notum et DZ notum, ergo BZ notum, latus scilicet pentagoni per 13^{um} Geometrie. Ergo tam AB tetragoni latus quam BZ latus pentagoni quam AC latus trigoni quam DZ latus decagoni quam CG latus exagoni notus est si nota sit AG circuli ABG diameter. Intellectis illis lateribus poligoniorum circulo †circumscriptorum.†

Ex hoc etiam manifestum est quod dicitur in corollario, scilicet quod in semicirculo inscripto aliquo predictorum laterum noscitur corda superflui

84 decagoni] *corr. ex* exagoni *T*

[6] The author of these alternate proofs uses 'adiectis' as a second person verb. [7] Here and throughout the following proofs, the scribe writes his references to Euclid's *Elements* in a confusing manner. He writes a capital 'G' for 'Geometrie', but it appears identical to the way he writes '6'. He usually refers only to books of the *Elements*, not propositions, but he occasionally does give specific references. Thus, it would have been difficult for the reader to realize whether he was supposed to read '4 6' or '4 G', and the possible meanings could be 'quartum propositum sexti', 'quartum propositum Geometrie', 'quarti libri sextum propositum', or 'quartum librum Geometrie'. I have expanded references according to what the mathematics calls for. [8] 'Potest' here means 'squared'. A similar use of the verb is found in *Almagest* I.9: 'Et similiter quoniam latus pentagoni potest supra latus hexagoni cum latere decagoni ..'. (1515 ed., f. 5v). [9] This refers to the Pythogorean Theorem (*Elements* I.47 or I.46 depending upon the version). The spelling here (the word is spelled out in full in I.5) differs from the spellings that Kunitzsch, '"The Peacock's Tail"', pp. 208-9, gives: 'dulcaron' or 'dulcarnen/-on/-an/-un'.

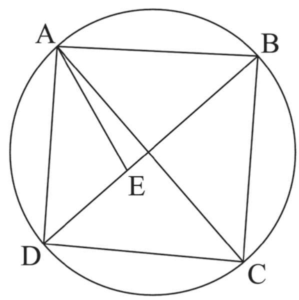
arcus. Cave tamen ne cuiuslibet intellegas quoniam hoc factum est. Non enim BC in semicirculo prefigurato per corollarium cognoscitur.

Addition to I.1

W_1 (35v): Sic patet. Ex 6 enim secundi Euclidis apperet id quod sit ex ductu AZ[10] in ZD cum quadrato ED[11] equale est quadrato ZE, ergo etiam quadrato BE, ergo etiam quadratis BD et DE. Ergo dempto communi quadrato scilicet linee DE, quod fit ex ductu AZ in ZD equale est quadrato BD sive DA. Ergo linea AZ divisa est super D secundum proportionem habentem medium et duo extrema. Ergo ex nona 13 Euclidis ZD est latus decagoni. Constat autem quod latus AD latus est hexagoni.[12] Latus enim quadrati inscripti circulo duplat potentialiter dimidium diametri. Ergo ex 10 eiusdem ZB latus est pentagoni, sed latera BD et DE[13] nota. Ergo quadrata nota.[14]

Alternate Proofs of I.2

Ba (221r): Sit circulus ABCD. Latera quadranguli scilicet rectanguli[15] sint AB primum, sit BC secundum, CD tertium, DA quartum. Dyametri sunt AC et BD. Item sit CD maius BC. Si essent latera equalia, facilis esset probatio. Ergo CAD sit maior CAB quia cadit in maiorem arcum. Resecetur ad equalitatem illius per lineam [AE]. Hoc facto sic procede. Primo BAE angulus est equalis DAC angulo, et EBA angulus est equalis ACD angulo. Ergo tertius angulus ACD trianguli est equalis tertio angulo BEA. Trianguli ergo illi trianguli sunt similes, ergo que est proportio CD ad CA eadem est BE ad BA. Sic DC primum, CA secundum, BE tertium, BA quartum; ergo quod fit ex ductu CD



108 CAB] *corr. ex* AB *Ba* **109** ad] *iter. Ba* AE] AC *Ba* **111** DAC] *corr. ex* ADC *Ba* **112** angulo] et CBA *add. et del. Ba*

[10] Throughout this addition, the point labeled G in the figure is referred to as A. [11] Although *W_1* has the standard figure and refers to point H in the standard text of this proposition, this addition always refers to point H as E, as it is in Gerard's translation of the *Almagest*. [12] Perhaps the previous two sentences were copied in the wrong order. If so, the mathematical reasoning would be better. [13] This should be 'DZ'. [14] The scribe appears to have copied the two last sentences in the wrong order. This addition is perhaps derived from Gerard's translation of the *Almagest* and interlinear or marginal notes accompanying it. Such notes could easily be read in the incorrect order. [15] The author of this proof falsely believes that the angles must be right. This is perhaps an instance of the particular figure drawn misleading the mathematician. A note in the margin in another hand points out the error here.

in BA est equale ei quod fit ex ductu CA secundi in BE tertium. Item sumatur alius triangulus. EAD angulus est equalis angulo CAB, et EDA est equalis ACB angulo quia cadunt in equos arcus. Ergo tertius est equalis tertio scilicet CBA angulus AED angulo. Ergo ABC et AED trianguli sunt similes, ergo latera sunt proportionalia. Ergo que est proportio BC primum ad CA secundum eadem est ED tertium ad DA quartum. Ergo quod fit ex CB primum in AD quartum est equale ei quod fit ex CA secundum in EB[16] tertium. Sed prius habuimus quod illud fit ex CD in AB. Ergo si bene meminerimus prime secundi libri Euclidis, illud quod fit ex ductu AC in DB est equale illis pariter acceptis que fiunt ex ductu AC [in] BE et CA in ED. Sic ergo illud quod fit ex ductu AC in BD est equale illis pariter acceptis que fiunt ex CD in AB et BC in CA, quod proposuimus probandum.

Si autem dicitur quod BAC angulus sit equalis CAD angulo, facilis erit probatio. Puncto †usque† sectionis appellato E, et prius sumptis istis triangulis ABC et ADC ad probandum propositum. Deinde istis triangulis sumptis ACD et ABC ad probandum propositum. Deinde istis triangulis sumptis ACD ABC erit facilime ut prius. Patet propositum.[17]

T (67ra): Esto enim quadrilaterum cuius duo diameteri AG et BD. Aut angulus ABD angulo GBD equatur aut si sit maior primo. Et ex eo ABD abscidatur EBA angulus equus angulo GBD per primum Geometrie. Sunt igitur duo trianguli EBA et GBD quorum duo anguli unius scilicet EBA et EAB duobus alterius angulis scilicet GBD et GDB equantur ex dispositione et tertio Geometrie. Ergo tertius equabitur tertio per primum Geometrie, ergo trianguli sunt similes per sextum Geometrie. Ergo latera eorum prout respiciunt equos angulos sunt proportionalia ex eodem Geometrie. Ergo est proportio eadem AB ad EA que BD ad DG ex eodem Geometrie. Sit igitur AB primum, EA secundum, BD tertium, GD quartum. Que est proportio primi ad secundum eadem est tertii ad quartum; ergo quod fit ex ductu primi in quartum equum est ei quod fit ex ductu secundi in tertium per sextum Geometrie. Ergo rectangulum quod continetur sub AB et DG equatur rectangulo quod sub EA et BD continetur. Item duo anguli EBA et DBG equantur ex premissis; ergo sumpto communiter angulo EBD utrimque angulus ABD angulo EBG fiet equalis per primum Geometrie. Sunt igitur duo trianguli ABD et EBG quorum unius duo anguli scilicet ABD et ADB duobus angulis alterius scilicet EBG et EGB sunt

116 ex ductu] *iter. Ba* **117** triangulus] †nunc sit ABC trianguli† *add. et del. Ba* angulus] *corr. ex* triangulus *Ba*

[16] The author of this proof makes an error here and writes 'EB' where he should have 'ED'. This mistake causes him to lose the remainder of the argument and to reach an incorrect conclusion [17] The proof of this last case is not made clear since the scribe wrote all triangles with point C when he was supposed to have point E for every other one.

equales ex premissis et tertio Geometrie. Ergo tertius tertio adequatur. Ergo mediis omissis latera eorum sunt proportionalia per sextum Geometrie. Ergo que est proportio BG ad EG eadem est BD ad AD. Que est proportio BG primi ad EG secundum eadem est BD tertii ad AD quartum. Ergo quod fit ex ductu BG primi in AD quartum equatur ei quod fit EG secundi et BD tertium per sextum Geometrie. Ergo quod fit ex ductu BD in EG equatur ei quod fit ex ductu BG in AD. Sed item id quod fit ex ductu eiusdem BD in EA ei quod fit ex ductu AB in DG. Ex premissis ergo coniunctim quod fit ex ductu BD in EA et in EG equatur rectangulis que sub AB et DG et eis que sub AD et BG continentur pariter acceptis. Sed quod fit ex ductu BD in EA et in EG equum est ei quod fit ex ductu BD in AG per secundum [Geometrie]. Ergo rectangulum quod sub BD et AG continetur eis que sub AB et DG et sub AD et BG continentur pariter acceptis adequatur. Ergo rectangulum quod sub duabus diametris continetur eis que sub oppositis lateribus continentur equabitur, quod erat propositum.

Alternate Proofs of I.3

Ba (221v):[18] Sit ABCD semicirculus. Sint DC et DB corda maior et corda minor, et sint note per ypothesim. Sit BC corda arcus maioris ad minoris excessus, quod probabo esse notam. Ducatur primus una linea a termino minoris scilicet C ad A; a B alia ad A. Hoc facto habemus quadrilaterum inscriptum. Inde sic quadratum AD valet quadratum DB et quadratum AB quia opponitur recto. Et quadratum AD est notum et notum est quadratum [DB, ergo] notum est quadratum AB. Eadem est probatio quod AC est notum. Et CA est notum. Ergo illud quod fit ex AC in BD est notum, et illud quod fit ex AB in [CD] est notum. Ergo quod fit ex AD in BC est notum. Sed AD est notum, ergo BC est notum.

T (67ra): Sit enim AB et AG nota in semicirculo ABGD. Dico ergo quod corda BG nota. Quod sic probatur. Angulus ABD est rectus per 30^{am} [tertii Euclidis].[19] Eadem ratione angulus AGD rectus. Quadratum AD equatur quadratis AB et BD per ducarnom, et quadratum AD notum per primam huius. Ergo quadratum AB et BD nota, et quadratum AB notum quia AB nota. Ergo quadratum BD notum, ergo BD notum. Eadem ratione GD notum. Item AG et BD sunt note. Ergo rectangulum quod sub eis continetur notum. Ergo rec-

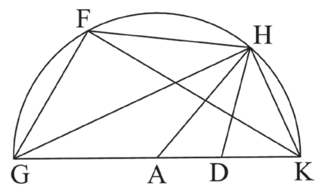
152 eadem est] *sup. lin. T* **155** per – Geometrie] *sup. lin. T* **160** Geometrie] secundi (*other hand*) *T* **171** DB ergo] D BG *Ba* **173** CD] C illud *Ba*

[18] *Ba*'s scribe made a mistake and placed this after the enunciation of I.4. [19] This refers to *Elements* III.31 in the Greek and modern versions, but the numbering here matches that in medieval versions, such as those of Robert of Ketton and Campanus.

tangula AD et BG et AB et GD nota per proximam huius. Sed rectangulum AB in GD notum quia utraque nota per ypothesim et premissa. Ergo rectangulum AD in BG notum, et AD nota per primam huius. Ergo BG nota, et BG est corda differentie AB et AG notis ex ypothesi, ergo notis in semicirculo et cetera. Et hoc est propositum.

Alternative proofs of I.4

Ba (221r-v):[20] Sit semicirculus GFHK. FK sit corda nota, KH medietas totalis et eius corda, FH corda secunde medietatis. Ab F ad G ducitur linea recta; ad G ab H. Item sumatur in GK dyametro equalis GF que sit GA. †Quoniam† [sic] huiusmodi trianguli FGH et huius GHA FG et GH latera sunt



equalia GH et GA, GH est commune, et anguli equilateribus contenti sunt equales, ergo per quartam primi Euclidis basim basi est equalis FH et HA. Sed FH equale KH, ergo HA est equale KH. Ergo anguli super basem sunt equales que [sunt] K A trianguli HAK per quintam primi Euclidis. Inde ab AHK angulo ducitur perpendicularis ad GK. Illa cadet inter A et K recto quia si non, probatur quod acutus est maior recto. Illa perpendicularis sit HD. Inde HAD trianguli etiam duo anguli sunt equales duobus angulis DHK trianguli et latus commune est [interiacens]. Ergo recta latera sunt equalia et recti anguli sunt equales; ergo AD equalis DK. Inde GK opponitur GFK angulo recto, ergo eius quadratum est equale quadratis in GF et FK pariter sumptis. Et GK est notum, ergo illa equalia pariter sumpta sunt nota. Sed FK per ypothesim est nota, ergo GF est notum. Ergo et GA ei equale est notum. Igitur AK est notum, ergo utraque eius medietas, scilicet tam AD quam DK, est nota. Item ab angulo orthogono GHK ducitur perpendiculariter scilicet HD ad suam basem; ergo per sextum Euclidis quod fit ex GD in DK est equale quadrato DH. Sed illud quod fit ex GD in [DK] est notum; ergo HD notum. Item quadratum HA est equale quadrato AD et DH quorum utrumque est notum. Ergo quadratum AH est notum; ergo KH est notum cum sit equale ei, ad quod tendimus.

T (67ra-b): Verbi gratia, in semicirculo ABGD ex ypothesi est BD nota, cuius arcus medius punctus G. Dico ergo BG et DG utraque est nota. Quod sic astrues. Redacta AD diametro ad quantitatem AB minoris per [primi] ter-

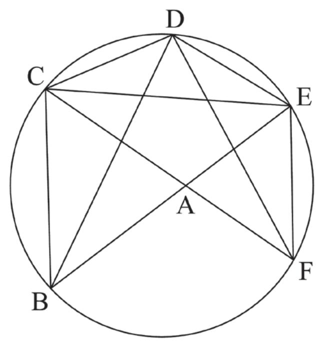
190 et eius] *del. Ba* **194** sic] fit *Ba* **203** interiacens] interiacentes *Ba* **211** DK] DH *Ba* ergo HD] *corr. ex* ergo †...† *Ba* **217** primi] primum et *T*

[20] *Ba* has this after the enunciation of I.3.

tium Geometrie et AG linea cum utraque [coniuncta], latera AB et AG lateribus AE et AG equabuntur, et anguli sub illis contenti equantur per tertium Geometrie. Ergo EG basi BG equalis per primum Geometrie. Quare EG et BG et GD invicem equabuntur. Item BD nota ex ypothesi. Ergo AB nota per ducarnom; ergo AE sibi equalis nota. Et AD nota per primam huius. Ergo ED nota; ergo utraque eius medietas scilicet EZ et ZD nota. Erigatur ergo ZG a puncto Z perpendiculariter. Sic angulus AGD rectus est per Geometrie tertium, et ab eodem descendit ZG linea perpendiculariter cadens ad basim AD in AGD triangulo. Ergo rectangulum quod sub AD et ZD continetur equum est rectangulo quod ex GD procreatur per sextum Geometrie. Sed rectangulum quod sub AD et †ZD† continetur notum quia utrumque eorum notum per premissa. Ergo rectangulum GD notam; ergo GD notum erit. Ergo et BG sibi equalis nota. Hoc autem erat propositum.

Alternate proofs of I.5

Ba (221v): Exemplum, sit CAF dyameter, CDEF semiperiferia. A sit centrum, FE una cordarum nota, ED alia, DF subtensa totali arcui DEF, quam probabimus esse notam. A D ad C ducatur recta, et sit DC. Ab E ad oppositum punctum circumferentis per A ducatur recta scilicet ad B. Et a D ad B ducatur recta primum; a C ad [B] est linea recta ducatur; et a C ad E recta ducatur. Inde CAF est notum et FE notum per ypothesim, ergo EC est notum. BAE dyameter est notum et DE per ypothesim est notum, ergo BD est notum. Habemus igitur quod huiusmodi paralellogrami BCDE dyametri se secantes sunt noti, ergo illa pariter sumpta que fiunt ex oppositis lateribus sunt noti, scilicet BC in DE et BE in DC. Sed BC est notum quia [DF] est notum, et DE est notum. Ergo quod fit ex BC in DE est notum; ergo quod fit ex BE in CD est notum. [Sed BE est notum, ergo CD est notum.][21] Sed CAF est notum, ergo DF est notum. Quadratum enim CF quod est notum est equale quadre simul sumptis quadratis DC et DF, quorum unum est notum scilicet CD. Et habemus ergo propositum.



218 coniuncta] coniuncata *T* **239** B] D *Ba* **241** FE] *corr. ex* E *Ba* **247** DF] DE *Ba*

[21] The mathematical argument calls for text similar to this. The similarity of the endings would have made this an easy passage to omit.

T (67rb): Ex ypothesi AB et BG nota. Dico quod AG nota. Perfecto enim semicirculo ABGD cuius diametri AD et BH, adiectis et rectis lineis BD et GH et GD et DH. Sic probatur. Angulus ABD rectus est per tertium Geometrie. Ergo AD potest[22] †equare† AB et BD per ducarnom. Et tam AD diameter nota per primam huius quam AB ex ypothesi; ergo BD nota. Eisdem argumentis GH nota, sed et eisdem DH nota. Est igitur quadrangulum BGDH cuius due diagonales scilicet BD et GH note sunt ex premissis, ergo et rectangulum quod sub eisdem BD et GH continetur est notum. Sed rectangulum BD et GH equum est rectangulis BG in DH et GD in BH per secundum huius libri. Ergo illa duo rectangula simul sumpta sunt nota, et rectangulum BG in DH notum quia earum utraque nota ex premissis et ypothesi. Ergo rectangulum GD in BH est notum. Et BH notum; ergo GD notum. Et AD diameter nota. Ergo AG nota per ducarnom, et [est] media. Hoc autem erat propositum ostendere.

Alternate Proofs and Additions in I.6

Group 3 [*The following alternate text for the passage in the sixth paragraph from 'Sed ad hunc numerum ...' to '... fuerit postponitur'. I take this and the addition from KM.*]: Unde corda AG gradum unum puncta 2 secundas 50 minime complebit, que quidem summa AD 47 puncta et secundas 8 fere sesquitertia est, que ergo nunc maior nunc minor unius gradus corda alio respectu consistit. Optimum visum est huiusmodi cordam partis unius punctorum 2 secundorum 50 merito reputari.

[*The following is an addition to the proof.*]

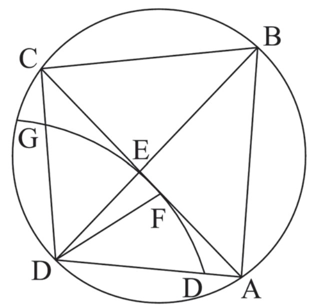
Quia tamen earum numerus et quantitas facilius ex oculo in subiecta figura deprehenditur, et scitu valde necessaria est in tabulis per ordinem disponantur ita ut unaqueque linea 4 contineat, quia hucusque satis congrua est extensio. In prima itaque tabula partes arcuum et earum numerus subdimidii gradus augmento deorsum describuntur. In secunda vero partes cordarum non sine punctis et secundis ad prescriptos arcus pertinentium sub certo numero deponuntur. In tertia quidem partes tricesime ipsius differentie que inter quaslibet duas occurrit cordas collocantur, numero vero punctorum que ad unum minutum attinent sub certa veritate et ad oculum deprehenso ab uno usque ad xxx singulos singulorum que inter duas consistunt cordas particulas. Ob hoc et

269 secundas] secunda *M* **270** complebit] complebis *M* **271** que] quia *M* nunc[1]] *s.l. K* unius] unius eiusdem *M* alio] *corr. ex* eiusdem *K* **272** Optimum] optime *M* huiusmodi] huius *M* **273** secundorum] secundarum *K* **275** tamen] cum *M* et quantitas] *iter. et del. M* **279** augmento] agmento *K* **280** deponuntur] *corr. in* disponuntur *K* disponuntur *M* **282** minutum] minimum *K* **284** singulos] singulas *M*

[22]As earlier, *T* uses 'potest' to indicate a line's square.

rursus oportuna videtur talis dispositio esse ut dum per hanc si quid erroris de numero vel quantitate cordarum tabula ipsa contineat agnoscatur, et dictorum ratione verisimiliter corrigatur, ut videlicet cordam arcus duplicati et certam cordam habentis prius cognoveris, aut saltem differentia qua certi arcus certas cordas habentes differunt precognita, vel si quemlibet arcum qui ad perfectionem semicirculi deest per arcum certum et certe corde presciveris. Et ad hunc quidem modum tabule ordinentur.

Ba (121v): Date corde sint AB CB. Angulus ABC subtendatur basim AC. ABC angulo diviso per equalia per lineam BD et tam ab A quam a C recta ducatur scilicet ad D, scilicet AD et [CD]. Necessario essent equales cum sint corde equalium arcuum. Sint enim anguli ABD DBC equales. Sit E punctus sectionis DB et AC. Constat quod [AE] est maior EC. Ergo ducta perpendiculari a D ad AC cadet super [AE], et dividet basim per equalia sic quod in †T†. Ponitur pedes circini in D quia †oppositum† in recto angulo, et secundum quantitatem DE fiat portio circuli vel circulus.



Hoc facto constat quod illa periferia non tanget [EZ] quia tam CD quam ED est maior [DZ]. Protrahitur ergo †DZ† ad istam periferiam, et sit DF. Hoc facto habebimus duos sectores et 2 triangulos: DEF DEG sectores, †DCE† [DEZ] triangulos. Inde fit DFE sector maior est triangulus DEZ; ergo maior est proportio [DEF] sectoris ad DEG sectorem quam [ZDE] trianguli ad DEG secundum [quintum] Euclidis. Sed DEC triangulus est maior DEG sectore; ergo maior est proportio DEZ ad DEG quam ad DEC. Igitur a primo[23] maior est proportio DEF ad [DGE quam ad DEC]. Sed que est sectoris ad sectorem eadem est [FDE] anguli ad EDG angulum; ergo maior est anguli ad angulum quam trianguli and triangulum. Sed que est triangulus ad triangulum est [EZ] ad EC per primum sexti Euclidis. Ergo maior est proportio FDE angulus ad EDG angulum quam ZE ad EC. Ergo coniunctim maior est proportio ZDC angulus ad EDC angulum quam FC linee ad EC.[24] Ergo maior est proportio

286 et] *corr. ex* est *M* **288** certas] *om. M* **296** CD] ED *Ba* **300** AE] AC *Ba* **301** AE] AC *Ba* **304** portio] *corr. ex* proportio *Ba* **305** EZ] et *Ba* **306** DZ] D et *Ba* **308** DEZ[1]] DE et *Ba* **309** DEF] DCF *Ba* ZDE] ZDC *Ba* **310** quintum] secundum *Ba* **311/312** DEZ – proportio] *iter. Ba* **312** DGE – DEC] DGC quam D ad EC *Ba* **313** FDE] FDC *Ba* **314** EZ] E *Ba*

[23] The phrase 'a primo' seems to mean 'a fortiore' here. [24] The transformation here is not simply taking the ratios coniunctim.

dupli FDC ad EDC angulum quam dupli FC ad EC. Sed duplum FDC est ADC angulum duplum, et FC duplum FC est AC, †utrique† cuiuslibet sani capitis. Igitur maior est proportio anguli ADC ad angulum EDC quam AC ad EC. Ergo disiunctim, igitur maior est proportio ADE ad EDC angulum quam AE ad EC lineam. Sed que est AE ad EC eadem est AB cum BC per ultimam quinti[25] Euclidis. Sed que est ADE anguli ad EDC angulum est arcus AB ad arcum BC per ultimam sexti eiusdem. Ergo [minor] est proportio AB ad BC quam arcus ad arcum, quod proponimus.

T (67r): Posito enim ABGD circulo in quo AB linea minor quam BG existat, dico quod minor est proportio BG ad AB rectam lineam quam arcus BG ad arcum AB. Subtensa enim AG linea et angulo ABG diviso per duo equalia periferia BD usque ad circumferentiam protractam, adiectis et AD et GD per rectas lineas. Probatur quod ED non est perpendicularis ad AG ex triangulorum superpositione periferia. A puncto igitur D ad AG ducatur perpendicularis DC que necessario inter E et G probatur incidere per triangulorum similitudinem et superpositionem. Et ipsius angulus ergo ECD rectus, quare ED longius CD per primum Geometrie. Sed angulus AED maior est ECD per primum Geometrie, ergo idem est obtusus. Linea AD maior est linea ED et linea ED maior quam linea CD. Ergo si describatur circulus ED, secabit AD et non continget CD. Protrahatur TEH portio cuius D centrum, et DC in †continuum† protracta contingat H. Inde sic aliqua est proportio sectoris EHD ad sectorem TED, et aliqua ad triangulum AED. Igitur maior est proportio sectoris EHD ad sectorem TED quam eiusdem EHD ad triangulum AED quoniam AED totum ad TED sectorem per quintum Geometrie. Et item maior est proportio EHD sectoris ad AED triangulum quam ECD trianguli partis EHD ad eundem AED triangulum ex eodem [Geometrie]. Sed que est sectoris EHD ad sectorem TED eadem est anguli EDH ad angulum EDT; ergo maior est anguli EDH ad angulum EDT quam trianguli ECD ad triangulum AED per sextum Geometrie utrinque. Ergo maior anguli EDH ad angulum EDT quam basis EC ad basim AE per sextum Geometrie. Ergo coniunctim maior anguli ADH ad angulum EDA quam basis AC ad EA. Ergo maior anguli ADG dupli ad EDA quam AG basis ad EA. Ergo disiunctim maior EDG anguli ad EDA quam EG ad EA, ergo quam BG corde ad AB per sextum Geometrie. Ergo maior BG arcus ad AB arcum quam BG †ad,†[26] et cetera.

[*Text continues as normal with 'Nunc quorsum ...' There is an addition at the end of the proposition.*]

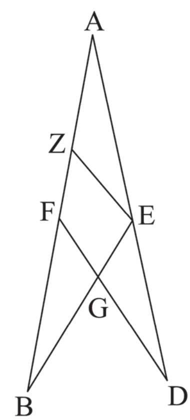
324 minor] maior *Ba* **345** AED] *corr. ex* ETD *T* **346** Geometrie] *s.l. T* EDT] *corr. ex* EDH *T* **348** EDA] *corr. ex* EDH *T*

[25] This should refer to *Elements* VI.3. [26] The omitted last quantity in this improportionality is AB.

(67va): Secundum igitur †predictorum† tenorem et ad maiorem dicendorum evidentiam fuit necessarium ut de arcubus et eorum cordis componerentur tabule ut quantolibet arcu cognito statim eiusdem arcus corde cognitio sequeretur. In prima ergo tabularum linea ponendus est arcus dimidii gradus sive 30 minutorum quod idem est, in cuius arcus directo ponenda est eius corda in tabula cordarum, per se tamen distincta sit. In secundo vero ordine, quoniam non refert sive ordo seu linea completur, sub tabula predicti arcus ponendus est duplus arcus scilicet unius gradus vel 60 minutorum quod idem est, in cuius arcus iterum directo ponenda est eiusdem corda. Sed quoniam inter primum arcum et sequentem multi arcus possunt inveniri quia fere 30, de quibus inter predictos nulla est mentio, ideo et quoniam si continue ponerentur, nimia esset †confusitudo† et prolixitas, ea propter consideravit Tholomeus quod ad huiusmodi arcus cordas inveniret et unius corde ad aliam in tabula primam differentiam et superfluitatem. Verbi gratia primus arcus nullius gradus est 30 minutorum cuius corda nullius gradus et 31 minutorum et 25 secundorum est. Secundo loco in tabula de arcubus sub predicto arcu scribitur arcus unius gradus cuius corda est unius gradus 2 minutorum 50 secundorum. Posset igitur contingere quod arcus inter duos arcus predictos nec esset primo dictus nec secundus ut si esset 31 minutorum; tunc non haberet cordam primam nec secundam quia corda eius maior est quam prima et minor quam secunda. Et ideo ad huiusmodi cordam reperiendam constituta est una tabula.

Alternate Proofs of I.7

Ba (222r): Sint AB AD linee descendentes ab angulo et ab AB ad AD reflectatur quedam linea usque ad E. Et a D ad AB alia reflectatur usque ad F, et secent se in G. Ab E in Z producitur equidistans GF usque ad AB. Quoniam ADF et [AEZ] trianguli sunt similes per secundum [sexti] Euclidis, igitur que est proportio DA ad EA est DF ad EZ. Sed illa producitur ex proportione DF ad GF et GF medie inter illas ad EZ. Sed proportio GF ad EZ est proportio GB ad EB quia BZE et BFG trianguli sunt similes. Ergo que est proportio GB ad FG eadem est BE ad EZ. Ergo permutatim que est proportio BG ad BE est GF ad EZ. Ergo proportio DA ad EA surgit ex proportionibus DF ad FG et GB ad BE, quod proponimus.

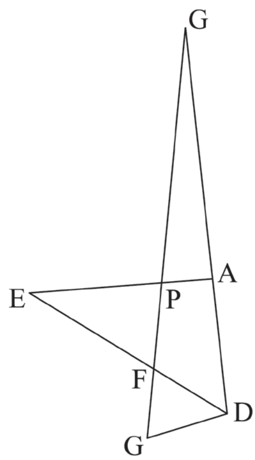


366 ad[2]] *s.l. T* **378** Et[1]] *s.l. Ba* **380** AEZ] AEZ et *corr. ex* AE et *Ba* **381** sexti] secundi *Ba* **383** GF[2]] *corr. ex* †...† *Ba* **384** GB] *corr. ex* †GD† *Ba* **388** DF] *corr. ex* †AF† *Ba* FG] *corr. ex* †BG† *Ba*

T (68ra): Exempli causa descendentibus ab angulo A AG et AB, reflexa que [a G] in AB ad punctum D. Item reflexa que [a B] in AG ad punctum E, a quo E protracta EH equidistanter GD. Dico quod proportio GA ad EA producitur ex proportione linee GD ad lineam ZD et proportione linee BZ ad lineam BE. Est enim triangulus AGD cuius latera secat EH equidistans basi GD. Ergo secat eadem proportionaliter per sextum Geometrie. Ergo proportio GA ad EA tanquam proportio GD ad EH, inter quas ZD linea statuatur media, quoniam proportio GD ad EH constat ex proportione GZ[27] ad ZD et ZD ad EH per epistolam Hameti de proportionalitate[28] et per librum Walteri Flandrensis de proportionibus.[29] Sed proportio ZD ad EH est tanquam proportio BZ ad BE per sextum Geometrie propter triangulum [BEH].[30] Ergo proportio GA ad EA est tanquam proportio GZ ad ZD et BZ ad BE. Hoc autem erat propositum.

Alternate Proofs of I.8

Ba (222r): Sint DC DE linee ab angulo descendentes. A C reflexitur in A, sit CA. Ab E sit EF, et secet CA in P. Protrahitur equidistans CA a D donec concurrat [EF] pertracta versus F, et concursus sit in G. Inde sic istius trianguli EGD PA secat latera proportionaliter [equidistans] basi GD. Patet per [secundam][31] sexti Euclidis. Ergo que est proportio EA ad AD eadem est EP ad PG. Sit PF medium inter PG et EP. Ergo EP ad PG fit ex proportione EP ad PF et PF ad PG. Ergo proportio EA ad DA ex eisdem constituitur. Sed proportio FP ad PG est proportio CF ad CD quod postea probabimus. Ergo habemus propositum. Sic autem probamus quod †misimus† ad probandum.



391 a G] AG *T* a B] AB *T* **398** Walteri Flandrensis] *corr. ex* Walterum Flandrensem *T* **400**] BEA *T* **401** et] *iter. et corr. T* **406** EF] CEF *Ba* **408** equidistans] equidistant *Ba* **409** per secundam] secunde per primam *Ba* **412** ad PF] *iter. et del. Ba* **414** FP] *corr. ex* F *Ba* **416** probandum] *corr. ex* divisimus *Ba*

[27] This should be 'GD'. The mistake here results in the author of this alternate proof reaching the incorrect conclusion. [28] Edited in Schrader, *The Epistola de proportione et proportionalitate of Ametus Filius Iosephi*. [29] This work is most likely Pseudo-Jordanus's *De proportionibus*, which immediately precedes the *Almagesti minor* in *T*. [30] That *T*'s scribe made multiple mistakes in this proof that appear to be scribal errors leads me to think that perhaps the scribe was not the author of this alternate proof. [31] It appears that the exemplar has a correction here, and the scribe, not understanding, added 'secunde' in the wrong place.

CFP GFD trianguli sunt similes. Ergo que est proportio CF ad FD ea est PF ad FG. Ergo coniunctim que est proportio CF ad CD eadem est PF ad PG, quod querimus.

T (68ra): Exempli gratia protractis ab angulo A GA et BA et conversim in se reflexis ad puncta D et E. Protractaque ab angulo A linea scilicet AH equidistanter EB donec concurrat GDH ad punctum H. Dico quod proportio GE ad EA producitur ex proportione GZ ad ZD et proportione BD ad BA lineam. Triangulus igitur est AGH cuius latera secat EZ lineam proportionaliter. Per secundum sexti ergo proportio GE ad EA est tanquam proportio GZ ad ZH, inter que medium statuatur ZD cuius proportio est ad DH tanquam BD ad DA per sextum Geometrie. Sunt enim trianguli AHD et BDZ similes. Ergo coniunctim proportio ZD ad ZH tamquam BD ad BA. Ergo proportio GE ad EA tamquam proportio GZ ad ZD et BD ad BA. Hoc autem erat propositum ostendere. Hec demonstratio vocatur alkata disiuncta ad differentiam disiuncte que sequetur.

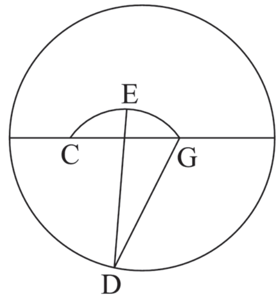
Alternate Proofs of I.9

Ba (222r): Sint arcus [ABG]. AG sit corda continuans terminorum illorum, BD [semidyameter] illius circuli. A G ad [H] ad BD protrahatur perpendiculariter. Ab A ad [Z] alia perpendicularis ad BD et sic [AZ]. E simul sectio AG et BD. Inde simul [GHE AEZ] sunt similes trianguli. Ergo que est proportio [AZ] ad EA eadem est GH ad EG; ergo permutatim. Sed que est GH ad [AZ] ea est dupla arcus unius ad duplum arcus alterius. Duplorum enim et subduplorum eadem est proportio. Sic constat de proposito. Perpendiculares enim subduple sunt corde arcuum duplorum.

T (68rb): Verbi gratia in circulo ABG AB et BG arcus continuantur, a quorum communi termino B diameter BD procedit, cui perpendicularis GH linea medietas corde arcus duplicantis arcum GB. Itemque sit AZ perpendicularis super eamdem diametrum, et sit sinus arcus AB. Quare fient trianguli GEH et AEZ similes. Ergo proportio AZ ad HG est tanquam proportio AE ad EG per quartum sexti. Ergo que est proportio dupli AZ ad duplum HG eadem est AE ad EG. Ergo BD sic secat AG in puncto E ut predictum est. Est AG linea duos eorum terminos non continuos continuans. Quare propositum habetur.

433 ABG] ABHG *Ba* **434** semidyameter] †sit† dyameter *Ba* H ad] AD *Ba* **435** Z] G *Ba* AZ] A et *Ba* **436** GHE AEZ] GH EA EC *Ba* **437** AZ[1]] A et *Ba* AZ[2]] A et *Ba* **447** sic secat] *iter. et del. T* Est[3]] est A *T*

Alternate Proof of I.10



T (68rb): Exempli causa in circulo ABG sit DZ perpendicularis ad AG cordam arcus AG noti. Quoniam ergo duo [latera] trianguli AZD et GZD duobus AZ et ZG invicem adequatur per tertium Geometrie et ZD latus commune continentque illa latera rectos angulos et est ypothenusa utrinque nota, oportet utrumque illorum triangulorum et angulis et lineis notum esse per primum Geometrie et tertium et per primam huius. Item proportio GE ad EA per premissam et ypothesim est nota. Ergo proportio coniuncta GA ad EA addita unitate denominatori proportionis disiuncte fiet nota. Ergo AE nota, ergo EZ et ZD et DE linee note respectu diametri circuli magni. Quia DA magni circuli semidiameter notus et ZA medietas AG note nota et EA nota, quare EZ nota, et ZD per bicornum[32] et primum huius. Constituta ergo DZ [et ZE] nota erit proportio DZ ad ZE, et angulus EZD rectus. Ergo ED notum, et triangulus EDZ notus lineis et angulis. Multiplica igitur ZD per ZE et totius extrahe radicem et habebis ED per ducarnom.[33] Itemque describatur circulus ad quantitatem DE; excedet DZ. Quare †cum† ad parvum ciclum protracta erit EZ, sinus illius arcus que nota est, ergo eius dupla corda scilicet arcus minoris circuli nota. Quare arcus ille totalis [minoris] circuli notus per propositionem de corda huius. Quare medietas eius nota ex eodem. Quare angulus eiusdem arcus notus scilicet angulus EDZ. Sed eadem ratione angulus ADZ notus. Quare angulus BDA notus. Sed et totalis GDA notus ex ypothesi. Ergo GDZ notus per primum Geometrie, et ZDE notus. Ergo GDE notus. Ergo GDB notus; ergo arcus GB notus per sextum Geometrie. Eadem ratione arcus AB notus, et hoc erat propositum.

Ex hinc quoque manifestum est quod proposito quocumque triangulo ortogonio si proportio cuiuscumque lateris eius ad quodcumque eiusdem latus nota fuerit, ipsum quoque triangulum et lineis et angulis notum esse necesse est, et ypothenusa de 60 constituta idest semidiametro.

458 triangulorum] triangulorum utrumque *T* **463** semidiameter] *corr. ex* diameter *T* **465** et ZE] †de† ZO *T* **471** minoris] mioris *T*

[32] *i.e.* the Pythagorean Theorem. [33] ZD and ZE should not be multiplied. Their squares should be added together.

Alternate Proof of I.11

T (68va): Exempli causa in circulo ABG GH sinus est arcus GA quia dupla corda ad GH dupli arcus ad [AB] corda erit. Cui GH equidistat BZ sinus arcus BA inclusi lineis concurrentibus, quarum altera GBE preter centrum transiens arcum GA secat, altera scilicet HAE secundum diametrum extracta concurrentibus in E. Fient trianguli GEH totalis et BEZ partialis similes per primum et sextum Geometrie. Ergo que est proportio GH ad BZ ea est GE ad BE. Sed que est GH ad BZ ea est corde dupli arcus GA ad cordam dupli arcus BA. Inferas ergo propositum.

Alternate Proofs of I.12

T (68va): Verbi gratia in ABG circulo propositus arcus AG cuius pars BG nota. Ergo BG corda nota per primum huius. Ergo ZB medietas corde arcus GB noti nota erit. Item DB semidiameter nota; ergo totus triangulus DZB ortogonius notus est et lineis et angulis per corollarium penultimi. Item proportio GE ad BE nota per proximam et ypothesum. Quare per penultimum tertii Euclidis EA nota. Sed ex his sequitur quod ZE nota. Item DA nota quia AH nota, ergo DH nota et AD nota. Item proportio GB ad BE nota. Sed que est GB ad BE ea est AH ad AE. Et AH est nota. Ergo AE nota. Et AD nota; ergo ED nota. Est igitur triangulus EZD ortogonius cuius duorum laterum idest ZE et DE proportio nota. Ergo ideo triangulus notus est lineis et angulis per antepenultimum, ergo angulus ZDE notus. Sed angulus ZDB notus ex premissis. Ergo angulus ADB notus; ergo arcus AB notus, quod erat propositum.

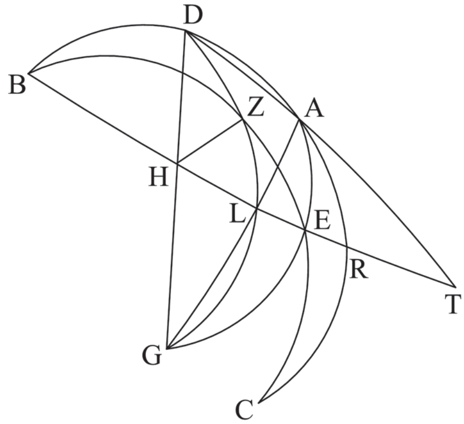
Addition to Proof of I.13[34]

T (68va) *The text matches the standard version through almost the whole proof (up to '... communis sectio linea') but then deviates and adds proofs of other cases*: ... communis sectio linea recta TKL scilicet. Qua protracta sic argumentare. A puncto A descendunt recte linee AT et AG, a quorum terminis due alie reflectuntur in easdem ad puncta D et L. Ergo proportio GL ad LA producitur ex GK ad KD et TD ad TA per kata disiunctam. Sed proportio GL ad LA que corde dupli arcus GE ad cordam dupli EA idest sinus GE ad sinum EA per nonum huius, et que est GK ad KD sinus GZ arcus ad sinum ZD ex eodem. Ergo proportio sinus GE ad sinum EA producitur ex proportione sinus GZ

484 AB] †OH† *T* **488** GH – BZ] *corr. ex* BZ ad GH *T* **489** GH – BZ] *corr. ex* BZ ad GH *T* **492** AG] *corr. ex* AB *T*

[34] Richard Lorch edited *T*'s texts of I.13 and I.14 (Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 376-381), but I give my own transcriptions here because I read a few words differently than Lorch does and so that one can have all of *T*'s alternate texts in one place.

ad sinum ZD et TD ad TA per quintum Geometrie. Sed que proportio TD ad TA ea est sinus BD ad sinum BA per undecimum huius et conversam proportionem. Ergo proportio sinus GE ad sinum EA producitur ex proportione sinus GZ ad sinum ZD et proportione sinus BD ad sinum BA. Sed eadem est proportio sinuum et cordarum duplorum arcus. Et sic habes propositum.

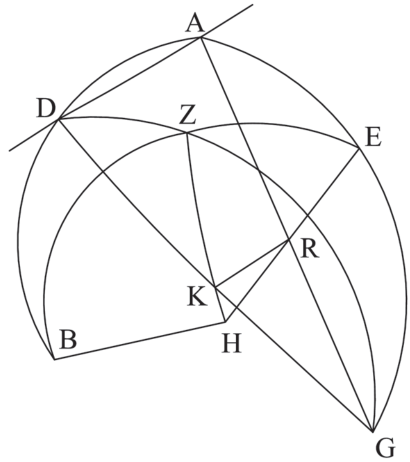


Poterit autem contingere AD et HB ex parte arcus AB non cuncurrant, sed ex parte arcus AG ut †subiecta†[35] monstrat et secunda dispositio. Dico item sicut prius quod proportio corde duplantis arcus GE ad cordam ipsius arcus EA componitur ex gemina proportione ex ea videlicet quam habet corda arcus ad GZ dupli ad cordam arcus ipsum ZD duplantis et ex ea que est corde arcus qui est duplus ad DB ad cordam arcus ad ipsum BA duplantis. Quod probatur protractis BA et BE arcubus dum in opposito puncto B sese item intersecent per Theodosium spere ad punctum scilicet C. Sunt igitur duorum magnorum orbium arcus DG et DC a puncto D [descendentes] a quorum reliquis terminis scilicet C et G duo arcus in eosdem reflectuntur ad puncta Z et A et ex parte arcus AC concurrunt AD et HB linee ad punctum T. Ergo per predictam dispositionem proportio sinus [GZ] ad sinum DZ producitur ex proportione sinus GE ad sinum EA et sinus CA ad sinum CD. Ex quo sic proportio sinus [GZ] primi ad sinum DZ secundi producitur ex proportione sinus GE tertii ad sinum EA quarti et proportione sinus CA quinti ad sinum CD sexti. Ergo ex libro proportionum Walteri proportio GE tertii sinus ad sinum EA quarti producitur ex proportione sinus [GZ] primi ad sinum DZ secundi et proportione sinus CD sexti ad sinum CA quinti. Sed sinus CD idem est quod BD, et sinus CA est idem quod sinus BA. Ergo proportio sinus GE ad sinum EA producitur ex proportione sinus GZ ad sinum DZ et sinus BD ad sinum BA. Et sic habes iterum propositum.

535 descendentes] descendentibus *T* **538** GZ] G *corr. ex* †G..† *T* **539** CA] *corr. ex* †CR† *T* **540** GZ] G *corr. ex* †G..† *T* **541** CA] *corr. ex* †CD† *T* **543** GZ] G *corr. ex* †G..† *T* **544** CA] *corr. ex* †CR† *T* **546** GZ] *corr. ex* G *T*

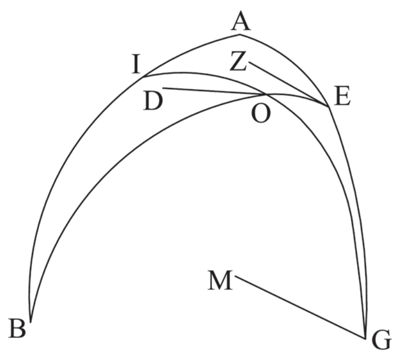
[35] Lorch reads 'subita' (Lorch, *Thābit ibn Qurra. On the Sector-Figure*, p. 377). [36]Instead of 'super lineam H ergo', Lorch reads 'super lineam HG' with the 'HG' corrected into 'HB' (*Thābit ibn Qurra. On the Sector-Figure*, pp. 378-9).

Amplius potest esse quod linee AD et HB sint paralelle ut tertia monstrat dispositio. Triangulus ADG secat superficiem BZE arcus ergo secundum rectam lineam per undecimum Geometrie. Fit ergo illa linea KR. Item linea AD equidistat linee HB, et triangulus ADG est super H. Ergo[36] potest constitui equidistans ADG; sit illa superficies S. Inde sic superficies S equidistat superficiei ADG, et EZB arcus secat illarum utramque, triangulum secundum KR lineam et superficiem S secundum HB lineam. Ergo KR et HB sunt equidistantes per undecimum Geometrie. Ergo KR et AD equidistant ex eodem. Ergo que est proportio GR ad AR ea est GK ad KD per sextum Geometrie. Ergo que est proportio sinus GE ad sinum EA eadem est sinus GZ ad sinum ZD. Sed proportio sinus BD ad sinum BA est equalitas quia sic est in quolibet arcu semicirculo minori. Ergo proportio sinus GE ad sinum EA producitur ex proportione sinus GZ ad sinum ZD et proportione sinus BD ad sinum BA quia omnis proportio [producitur] ex se et equalitate.



Addition to Proof of I.14

T (68rb) *Again, the standard text is followed for most of the proof (up to 'Hac igitur linea'), but then the text deviates from the standard version and gives proofs of other cases*: ... Hac igitur linea idest TDO protracta sic argue propositum. Proportio GO ad EO producitur ex proportione GD ad ZD et proportione TZ ad TE per kata coniuncta in rectis lineis. Sed proportio GO ad EO eadem est que sinus GA ad sinum EA. Similiter GD ad ZD eadem que sinus GI ad sinum [ZI] per undecimum huius. Ex eodem proportio TZ ad TE eadem est que sinus BZ ad sinum BE et per quintum Geometrie. Ergo proportio sinus GA ad sinum EA eadem componitur ex proportione sinus GI ad sinum ZI et sinus BZ ad sinum BE. Hoc autem erat

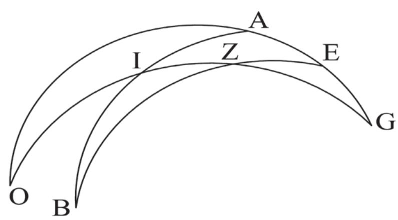


556 super] super lineam *T* **565** eadem] *marg.* (*other hand*) *T* **579** ZI] *om.* *T*
583 eadem] est *add. et del. T*

propositum. Quoniam idem est in arcuum duplorum cordis et in proportionibus arcuum sinibus demonstrare. Eadem enim hinc inde est proportio.

Sed quoniam modi sunt alii quamplurimi, ideo generaliter probetur sic. Dispositis GA et BA et reflexis GI et BE ut prius, sic equidistantibus GM et DO et ZE a punctis G et E et O perpendiculariter ad superficiem arcus AB protractis, deinde argumentare quoniam proportio GM ad ZE constat ex proportione GM ad DO et DO ad ZE. Sed que est GM ad ZE ea est sinus GA ad sinum EA. Similiter que est GM ad DO eadem est sinus GI ad sinum OI, et que est DO ad ZE ea est sinus BO ad sinum BE. Ergo que est sinus GA ad sinum EA ea est sinus GI ad sinum OI et sinus BO ad sinum BE, quod erat propositum ostendere.

Quod autem prima sit vera sic habeto. Propositis duobus arcubus semicirculo minoribus, unius quorum superficies super alterius superficiem perpendiculariter cadat, in eisque duobus punctis perpendiculariter signatis a quibus equidistantes protrahantur, que est proportio unius earum ad reliquam eadem est sinus totalis arcus ad sinum partialis, quod arcubus ad rectos angulos sese secantibus ex se argues. In aliis vero ex undecimo et sexto Geometrie per trigonos similes.[37]



Si vero kata disiunctam probare volueris, protrahe arcus GA et GI donec iterum concurrant ad punctum O. Proposito igitur sinus GA ad sinum EA constat ex proportione sinus GI ad sinum ZI et proportione sinus BZ ad sinum BE ex premissis. Item kata est ex arcu BE et OE et BA et OZ. Ergo proportio sinus BE ad sinum EA constat ex proportione sinus OZ ad sinum ZI et sinus BZ ad sinum BE per premissa. Sed sinus BE est sinus GE et sinus OZ est sinus GZ. Ergo proportio sinus GE ad sinum EA constat ex proportione sinus GZ ad ZI sinum et sinus BZ ad sinum BE. Et sic habes kata disiunctam in curvis.[38]

610 ex[1]] *s.l. T*

[37] Lorch writes that this is a sketch of a lemma given by Thabit (Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 360 and 384), and it probably is an attempt to recreate that proof, but there are mistakes and much of this passage is not intelligible. Here the plane of one arc is said to be perpendicular to the plane of the other, which is true only in a special case of Thabit's lemma (Lorch, *Thābit ibn Qurra. On the Sector-Figure*, p. 63). [38] This proof has mistakes, as Lorch explains (Lorch, *Thābit ibn Qurra. On the Sector-Figure*, pp. 360 and 384).

Addition after Book III

Da (23v-24r): Explicit liber tertius. Sequuntur quedam additiones quas ego Magister Anthonius hic inseravi...

Aditio. Tabula equationis dierum cum noctibus suis sic componitur. Quere arcum a Sole pertransitum secundum verum motum ultra unam revolutionem, eo existente in una aliquo certo gradu sicut in primo puncto Capricorni. Quere arcum quem in die ista motu proprio pertransivit, et illius arcus assentiones nota quas scribe in directe illius gradus. Et hoc vocatur equatio diei illius. Et similiter facies de aliis gradibus. Vel sic facilius: primo quere equationem unius gradus. Deinde quere equationem gradus sequentis, et illorum vide differentiam quam adde secunde equatione. Et habebis tertiam. Vel minue si tabula in ista parte non processerit. Et sic poteris facere et formare 5 vel sex lineas et item postea invenire aliam differentiam sicut prius. Hec est autem quorumdam scientia peritorum de hac tabula et eius formatione. Albategni vero dicit quod dies media sive equalis dicitur tempus integre revolutionis firmamenti cum addito illius arcus quem Sol secundum suum motum medium †interim† pertranssivit. Et quia motus Solis medius semper est equalis, ideo dies hoc modo consimiliter dicuntur equales. Dies diversa dicuntur integra revolutio sive tempus integre revolutionis cum tanta parte quanta Sol interim pertransit vero motu. Et quia motus verus est inequalis, ideo dies isti secundum inequalitatem continue variantur. Isti vero dies diversi et mediocres quandoque sunt equales, et hoc quando scilicet unus gradus ecliptise pertranssitus a Sole equale accidit ad meridianum cum 1 gradu equalis eis correspondente. Et hoc querit primo circa medium Aquarii et circa medium Leonis, et ideo in directo 18 gradus Aquarii minima equatio dierum in tabulis invenitur, quoniam quia[39] dies diversus quandoque maior est medio, quandoque minor. Et ratio omnium predictorum in predictorum in antedictis capitulis huius tertii libri manifestatur.

Ad inducendum igitur diem medium, qui est dies astronomicus secundum quem sunt omnium motuum tabule constitute, in diem diversum qui est dies secundum veritatem composita fuit tabula per hunc modum. Quere elevationem medii motus Solis in illo gradu in quo fuerit per circulum directum, et eum serva. Deinde quere verum motum Solis in principio illius diei ad quem volueris hoc inquirere et in fine eiusdem. Et minori maiori deposito, residui quere elevationem, quam conferes cum elevatione ex motu medio reservato. Deme minorem de maiore, et residuum erit equatio illius gradus ad suum diem, que equatio in horas et minuta horarum redacta minui debet de die medocri

622 assentiones] *i.e.* ascensiones **626** secunde] corr. ex prime *Da* **637** ecliptise] *i.e.* ecliptice **638** cum] peragrandu *add. et del. Da*

[39] One of these two conjunctions is a mistake.

si elevatio medii motus fuerit maior quam elevatio vera, si vero minor adde. Et sic ad quamlibet gradum sodiaci poteris invenire dierum equationes que nichil aliud est quam differentia inter quantitatem diei mediocris ad quantitatem diei diverse sunt. Differunt quelibet pars parva sicut in sigulis diebus; ascendet tamen ex multis revolutionibus usque ad 7 gradus et 52 minuta. Et incipit autem hec tabula secundum rei veritatem a 18 gradu Aquarii, et formatur secundam[40] regulam predictam accipiendo semper gradum sequentem cum precedente sicut fit in ascencionibus signorum. Et nota quod tempus proveniens ex hac equatione semper debet addi, cuius rei causa est quia equatio dierum in tabulis constituta sumit initium a 19 gradu Aquarii versus principium additis[41] ad diem mediocrem ut ex eo fiat differens. Si fuerit posita e converso, fiat e converso et cetera.

Addition after IV.3

Da 25v: Additio. Ad inveniendum medium motum Lune in una die, numerum dierum equalium lunationis qui est 29 dies 12 hore 44 minuta 3 secunda 16 tertia per motum Solis in una die multiplica, reducendo totum ad idem genus et numero inde producto adde totum circulum idest 360 gradus in eandem speciem fractionis. Et proveniet medius motus Lune ad unam mense lunarem, quem divide per numerum dierum mensis lunaris, et exibit motus unius diei. Sed quia in mense lunari sunt multe fractiones, ideo ad hac divisionem artificialiter faciendum reduc totum mensem lunarem ad unum genus et divide ipsum per quantitatem unius diei ad idem genus reducti. Et habebis in numero quotiens quanta pars est unus dies de toto mense. Per hunc ergo numerum quotiens divide medium motum Lune ad istum mensem, et habebis medium motum competentem uni diei, per quem inveniri potest medium motum ad quamlibet aliam differentiam temporis ut in horis et minutis et cetera. Quantitas autem mensis lunaris ex consideratione eclipsium est inventus.

Modus autem per quem inveni medium argumentum est iste: multiplica totum circulum in 269, hoc est per numerum revolutionis diversitatis superius argumenti in quibus fit reductio ad similem coniunctionem. Et redic[42] per numerum dierum qui continetur 251 mensibus lunaribus procedendo in hac divisione sicut prius dicebatur in motu medio Lune. Et habebis in numero quotiens medium argumentum Lune in una die, per quem procedes ad motum in aliis temporum differentiis inquirendi et cetera. Item 20051[43] est numerus mensium in quibus coniunctiones ad statum similem reducuntur.

653 sodiaci] *i.e.* zodiaci

[40] This is probably a mistake for ‘secundum’. [41] This should probably be ‘additionis’.
[42] The meaning is to divide. [43] This is an error for ‘251’.

Additions to V.9

Da (37v-38v): ⟨1⟩ Additio. Ad componendum tabulam equotionis centri Lune. Sic fac: primo scias centrum duplex Lune medium. Et si predictum centrum minus quarta fuerit, tunc scias sinum eius rectum et sinum illius quod ei deficit ad perfectionem quarte. Et utrumque per quantitatem distantie duorum centrorum multiplica, et per 60 partire. Et quod ex utroque pervenerit serva. Deinde semidiametrum ecentrici in se multiplica, et adde ad illud quod provenit ex divisionis[44] sinus perfectionis, et aggregatum serva quia ipsum est linea inter centrum epicicli existens in illo situ. Deinde super hanc lineam sic inventam adde quod provenerit ex sinu arcus dati prime quadrata.[45] Et duorum quadratorum simul iunctorum radicem queras et servas. Per quam divides illud quod ex sinu arcus provenerit in 60 multiplicatum, et habebis in numero quotiens sinum equationis quesite.[46]

Si vero arcus longitudinis duplicatus fuerit quarta presice, ex semidiametri ecentrici in se multiplicato distantiam duorum centorum in se multiplicatam deme, et residui radicem serva, que erit linea inter centrum orbis signorum et centrum epicicli, quem in se multiplicato.[47] Et numero inde producto adde distantiam duorum centorum in se ductam et totius aggregati radicem elice. Deinde multiplica distantiam duorum centrorum per 60 gradus, et productum divide per radicem; et habebis in numero quotiente sinum equationis quesite, cuius accipias circuli portionem, et patebit equatio.

Si vero arcus longitudinis duplicatus erit plus quarta et minus semicirculo, tunc sinum eius rectum et eius quod ei deficit ad completionem medietatis circuli quere, quorum primus nominabitur sinus arcus datis, alter vero sinus perfectionis. Et utrumque in distantiam duorum centrorum multiplica, et per 60 gradus partire. Et uterque numeri quotiens prime serva. Deinde ex semidiametro ecentrici in se multiplicato minue sinum arcus dati in se multiplicatum, et ex radice residui subtrahe quod provenerit ex sinu perfectionis in se ducto. Et residuum serva. Nam ipsum est linea a centro orbis signorum usque ad centrum epicicli, que notetur quia ipsum est linea EB. A qua remove id quod provenerit ex sinu perfectionis, et residui quadrati[48] addas cum quadrato eius quod ex sinu dati arcus. Proveniat totius aggregati radicem.[49] Radicem elice. Deinde id quod ex sinu dati arcus proveniat in 60 multiplicatum per inventam radicem partire. Et habebis sinum equationis quesite, cuius invenies circuli portionem et patebit quesitum.

688 equotionis] *i.e.* equationis **700** presice] *i.e.* prescise

[44] This should be 'divisione'. [45] This should be 'quadratum'. [46] There are numerous mathematical errors in this paragraph. [47] This is probably a mistake for 'multiplica'. [48] This should probably be 'quadratum'. [49] This should be 'radix'.

⟨2⟩ Ad componendum minuta proportionalia. Tabula vero minutorum proportionalium componitur isto modo. Consideretur quantitas linee que est inter centrum terre et centrum epicicli eo existente in auge differentis. Et scire potest quod non est quantitas ecentricis. Nota iterum quod est quantitas semidiametri epicicli, qui est 5 gradus et 15 minuta. Deinde consideretur quantitas eiusdem linee centro epicicli existente in opposito augis per regulas iam predictas, et minuatur a quantitate linee que est ad augem. Et residuum dividatur in 60 partes, que sunt minuta proportionabilia. Deinde consideretur quantitas eiusdem linee centro epicicli existente in quacumque alia parte ecentrici, et consideretur in quod[50] de dictis minutis linea que est ad augem excedat illas que sunt ad alia loca. Et compatebunt minuta proportionalia secundum quemlibet datum suum. Hoc autem facias accipiendo tantam partem de 60 quantus est ille partialis excessus de totali excessu, quod scire poteris considerando talem excessum pro primo partialem excessum pro secundo 60 partitio. Exemplum procedendo per unam 4 proportionalium quantum et cetera.

⟨3⟩ Ad componendum equationes argumenti. Tabulam equationum argumenti vero compones isto modo. Si partes equate fuerint quarta presice, lineam EB in se multiplicatam semidiametro epicicli qui est 5 gradus et 15 minutorum in se multiplicate superadde; et collecti radicem elice, et serva. Post hoc 5 partes et 15 minuta in 60 multiplica, et numerum inde productum per servatam radicem divide sive partire. Et quod ex divisione provenerit erit sinus equationis quesite.

Si vero arcus portionis †vel† argumenti equati fuerit minor quarta, accepta[51] sinum arcus dati, quem minue de quarta; et residui etiam sinum queras, qui dicitur sinus perfectionis. Et utrumque diversum, sive quodlibet per se, per semidiametrum epicicli multiplica scilicet 5 gradus et 15 minuta; et productum divide per 60, et numerum quotientem serva. Deinde numerum quotiens qui ex sinu perfectionis evenerit adde quantitate linee EB, accipiendo lineam EB per distantiam centri epicicli in defferente a centro orbis signorum prout doctrina superius ostendebat. In tabulis autem quibus utimur, accipitur centrum epicicli sui situ in auge defferentis, et totum collectum ex numero quotiens ipsius sinus perfectionis et ex linea EB quadra, idest in se multiplica. Et super huius quadratum adde quadratum numeri qui provenit ex divisione sinus dati arcus. Et totius collecti radicem quere et serva. Post hoc numerum qui ex divisione sinus dati arcus provenerit per 60 multiplica, et quod ex hac multiplicatione provenerit per servatam radicem partire. Et quod provenerit erit sinus equationis quesite, cuius habita portione patebit equatio.

724 differentis] *i.e.* deferentis **738** presice] *i.e.* prescise **755** divisione] demisione divisione *Da*

[50] The text appears to be corrupt here. [51] This should probably be ‘accipe’.

Quod si arcus dati portionis argumenti fuerit plus quarta, subtrahe inde quartam scilicet 90 gradus, et residui quere sinum, qui vocatur sinus dati arcus. Item ipsum residuum subtrahe de 90, et eius quod remanserit quere sinum, qui vocatur sinus perfectionis. Et utrumque multiplica per semidiametrum epicicli, et productum in numero per 60 gradus partire, divisim utrumque numerum per se notas vel servas. Et quod ex divisione sinus perfectionis provenerit a quantitate linee EB subtrahe, et residuum quadra idest in se multiplica. Et super huius quadratum illius numeri quotiens qui provenerat ex divisione sinus dati arcus adde. Totius aggregati radicem servas. Post hoc numerum qui ex sinu habitu arcus provenerit per 60 multiplica, et quod fuerit ex hac multiplicatione productum divide per radicem prius servatam. Et exibit sinus equationis quesite. Et si fuerit portio maior semicirculo, per residuum operare.

⟨4⟩ Ad componendum tabula diversitatis diametri. Tabula diversitatis diametri componitur sic. Querantur equationes argumenti centro epicicli existente in opposito augis et differentia inter eas in tabula diversitatis dyametri conscribantur. Verbi gratia centro epicicli existente in auge et portione equata existente unius gradus, inveniatur equatio argumenti. Iterum centro epicicli existente in opposito augis et portione equata existente unius gradus inveniatur equatio argumenti. Et subtracta minore a maiore equatione residuum erit equatio diversitatis diametri correspondentis unius gradui. Et similiter fiat de duobus gradibus et consimiliter de omnibus quousque tota tabula compleatur.

Possunt etiam equationes centri formare per additionem, licet non est ita presice. Adde qualibet gradus 6 minuta et cetera. Vel accipiatur tota equatio que est 13 gradus et 30 minuta, et multiplicetur per numerum graduum quorum equationem queris. Et productum dividatur per 90, et patebit equatio illius gradus correspondi. Et scire debes quod iste equationes crescunt usque ad 4 integra signa et ex inde decrescunt usque perveniatur ad finem. Similiter possunt fieri equationes argumenti per additionem ut in mediis motibus, et cressit usque ad 3 signa et exinde decressit usque ad finem. In primo autem gradu pone 4 minuta et 50 secunda, et per hunc numerum formare possunt alii subsequentes.

⟨5⟩ Ad componendum tabulam latitudinis Lune. Tabula latitudinis Lune componitur sicud tabula declinationis Solis ab equatare scilicet multiplica sinum arcus dati in sinum maxime latitudinis Lune que est 5 gradus, et productum divide per 60, et exibit sinus latitudinis quesite et cetera.

⟨6⟩ De tribus superioribus planetis. Tabula trium superiorum veri motus componitur sicud equationes Lune et ceterum. Et si es theoricus, poteris etiam componere sic tabulam Veneris et Mercurii. Compositio autem tabularum stationis prime est talis. Multiplica sinum stationis prime que est in auge per quantitatem linee egredientis a centro terre per centrum epicicli in auge.

781 presice] *i.e.* prescise **791** equatare] *i.e.* equatore

Et productum dividatur per quantitatem linee egredientis a centro terre per centrum epicicli eo existenti alibi ubicumque volueris. Et exibit sinus stationis prime ad illum statum. Quantitas autem linearum predictarum invenitur per regulam de equatione centri superius assignata. Statio vero prima in auge differentis in tabulis est nota et cum instrumentis materialibus invenitur.

Sequitur 'Artificium et cetera' et est de textu.

802 differentis] *i.e.* deferentis **803** est] *s.l. Da*

Glossary of Select Words and Phrases

In the following glossary of terms, the second principle part and the gender are only provided when the word is unusual or is not commonly found in dictionaries. I note the first occurrence of each meaning of a term. When searching for an entry, keep in mind that the words in phrases are often found in different orders in the text although I only provide one entry and that some entries are grouped together (e.g. 'applicatio media' and 'applicatio vera' are found under 'applicatio').

altitudo, altitude or height above the horizon (used often in the phrase **altitudo poli**) [II.1]; height of a physical object [I.15].

angulus differentie, the angle of the difference of the true and apparent motions (used without further qualification only in solar theory) [III.5].

angulus motus apparentis, the angle of the sun's apparent motion [III.13], synonymous with *angulus motus diversi*.

angulus motus diversi, the angle of the sun's irregular motion [III.13], synonymous with *angulus motus apparentis*.

angulus motus medii, the angle of the sun's mean motion [III.13].

angulus latitudinis, the angle facing parallax in latitude in the triangle bound by the parallax on the circle of altitude, parallax in latitude, and parallax in longitude [V.21].

angulus longitudinis, the angle facing the parallax in longitude in the triangle mentioned in the preceding entry [V.21].

annus, year [II.7]; more precisely, the solstitial year [III.1]; the sidereal year is mentioned once [III.1]; **annus solaris**, solar year [III.17]; **annus Solis equalis**, mean solar year [III.2]; tables are made for **anni collecti**, collected years, or for **anni disgregati** or **anni expansi**, expanded or separated years [III.2].

antemeridianus, before the meridian [II.33].

applanes, **applani**, outermost sphere of the heavens [I preface]. This word (often spelled 'aplanes'), which is derived from Greek, is found with this meaning in Macrobius and other earlier authors.

applicatio, syzygy [V.10]; **applicatio media** mean syzygy [V.10]; **applicatio vera,** true syzygy [V.10].

arcus diei, the arc of a day, i.e. the arc of the equator that measures the length of the day [II.1], synonymous with *arcus diurnus*; **arcus diei minimi/maximi**, the arc of the shortest/longest day [II.1].

arcus differentie (**duorum motuum**), the arc between the true and apparent places on the concentric [III.5].

arcus diurnus, diurnal arc [II.7]; see *arcus diei*.

arcus motus apparentis, the arc of apparent motion [III.5].

arcus noctis, the arc of a night, i.e. the arc of the equator that measures the length of the night [II.19]; also, **arcus nocturnus** [II.7].

argumentum equatum, the moon's elongation from the true apogee on its epicycle (used only once) [V principles]; see *portio*.

argumentum Solis, the sun's distance from apogee on the eccentric according to mean motion [III.17].

ascendo, to ascend, rise [II.1, III.21].

ascendens, the ascendant [II.1, II.19], synonymous with *oriens*, *pars ascendens*, *pars oriens*, and *punctum orientis*.

ascenscio, ascension, i.e. the arc of the equator with which an arc of the ecliptic rises [I.17, II.14]; synonymous with *ascensus*, *elevatio*, and *ortus*.

ascensus, **-us**, *m.*, ascension [I.17]; see *ascensio*.

augis oppositum, perigee [III.6]; see *longitudo propior*.

australis, southern [II.1].

auster, **austri**, *m.*, south [II.15].

aux, **augis**, *f.*, apogee [III.6]; see *longitudo longior*.

axis, axis of a cone (often in the phrase **axis umbre**, the cone of the earth's shadow) [V.17]; a part of an instrument [V.1].

caput, a person's head [I.15]; the beginning of a zodiacal sign [II.11]; the ascending lunar node [IV.16], also referred to once as **caput draconis** [IV.19].

Cauda, descending lunar node [IV.16].

celum, heavens [I preface].

cenit, *indecl.*, zenith [II.8]; most often **cenit capitum** [II principles]; the synonyms *summitas capitum/capitis* and *polus orizontis* (see *polus*) are used infrequently.

circulus altitudinis, circle of altitude, i.e. the great circle passing through the zenith and a given point [II.6]; also *orbis altitudinis*.

circulus brevis, epicycle (used rarely) [V.7]; see *epiciclus*.

circulus concentricus, concentric circle [III.3]; the substantive **concentricus** is used often to denote a concentric circle [III.3].

circulus declinans, the moon's inclined circle [IV principles]; also *orbis declinans*.

circulus declivis, in some instances this could possibly mean any inclined circle, but it is clear from context that this often refers specifically to the ecliptic [I.16] or to the moon's inclined circle [IV.18]; **declivis** can be found alone to mean ecliptic [I.16].

circulus ecentricus, eccentric circle [III.7]; see *ecentricus*.

circulus egressus, the heading of a column of an eclipse table is **circuli egressi** (perhaps referring to an eccentric circle or the movement along this circle) [V.19].

circulus longitudinis, the great circle passing through a heavenly body and the poles of the ecliptic [V.22]. (This circle determines the star's longitude on the ecliptic, and although defined for any heavenly body, this is only used for the moon.)

circulus medii diei, meridian [II.31]; see *meridianus*.

circulus meridianus, meridian [I.17]; see *meridianus*.

circulus parvus, epicycle [III.6], see *epiciclus*; also used to denote other small circles [III.1, V.4].

circulus signorum, ecliptic [I.17]; lesser used synonyms are *orbis signorum* and *zodiacus*.

clepsedra aquarum, water clock [V.15].

clima, **-tis**, *n.*, clime [II.1].

coequatio partis Lune, the portion of the moon's apparent motion's deviation from mean motion that is due to its epicycle (used only once) [V.9]; see *simplex equatio*.

consideratio, observation [I.15].

coniunctio, conjunction of sun and moon [IV.3]; the word is used non-technically to refer to a combination [VI.12]; **media coniunctio**, mean conjunction [V principles]; **vera coniunctio**, true conjunction [VI.2], and **visa coniunctio**, apparent conjunction [VI principles].

continuitas signorum, the order of the zodiacal signs, used only the phrase 'secundum continuitatem signorum', i.e. from west to east [IV.10], synonymous with *successio signorum*.

corniculatus, horned, used with moon to denote crescent moon [V.6]; see *exesus*.

cursus, **-us**, *m.*, course, motion [III.16]; intersection [V.21].

cursus apparens, apparent motion [III.25].

cursus diversitatis in epiciclo, the moon's motion on its epicycle [IV.14]; the same meaning is conveyed by **equalis cursus diversitatis** [IV.12] and **medius cursus diversitatis (in epiciclo)** [IV.14]; see *motus diversitatis* and *portio*.

cursus diversus, irregular motion of sun or moon [III.17].

cursus equatus, equated motion, i.e. sun or moon's true or apparent place when derived from calculation [III.17].

cursus latitudinis, the moon's motion on its inclined circle [VI.12]; **cursus verus in latitudine** [IV.16] or **cursus verus latitudinis** [VI.8], the moon's true course in latitude; **medius cursus latitudinis**, the moon's mean course of latitude [IV.16]; see *motus latitudinis*.

cursus medius, mean motion (used in both solar and lunar theory) [III.16].

cursus medius longitudinis, the moon's mean course of longitude [IV.12]; also, **cursus medius in longitudine** [IV.12]; see *motus longitudinis*.

cursus verus, true motion [IV.16].

declinatio, declination, inclination [V.23]; used often, but not exclusively, to refer to the distance between the ecliptic and equator measured by an arc of a great circle passing through the equator's poles [I.15, I.16]; in lunar theory this word is used both to refer to the 'turning aside' of the epicycle's diameter indicating the true apogee [V.7], see *reflexio*, and also to the moon's distance from the ecliptic [V.12]. See *maxima declinatio* and *maxima declinatio ad septentrionem*, which have other meanings.

declivis: see *circulus declivis*.

defectus, **-us**, *m.*, eclipse (used only twice) [IV principles]; see *eclipsis*.

deferens, this participle is used only once, denoting that the epicycle is carrying the planet [III.4]. Note that it is not used in the standard meaning of 'deferent', i.e. it does not refer to the deferent circle on which the epicycle is carried.

definita minuta detectionis, the precise minutes of the uncovering of an eclipse, i.e. the minutes of immersion after the middle of the eclipse [VI.20]; see *minuta casus*.

dies, **diei**, sometimes *m.*, sometimes *f.*, the time that the sun is above the horizon [II.1]; the combined period of a day and a night [III.1].

dies differens, diverse days [III.18].

dies mediocris, average days [III.18].

digitus (**eclipsis**), digit of eclipse, i.e. a twelfth of the diameter of the sun or moon [IV.16].

distantia centrorum, eccentricity [III.7].

diversitas, general meaning of difference or irregularity, used in many contexts [I.15, III.1]; sometimes used to mean *diversitas aspectus* [V.19] and *medius motus diversitatis* (see *motus diversitatis*) [IV.10].

diversitas aspectus, parallax [II.36]; sometimes just *diversitas* [V.19]; **diversitas aspectus** (**Lune/Solis**) **in circulo altitudinis** [V principles] or **diversitas in circulo altitudinis** [V.19], parallax on the circle of altitude; **diversitas aspectus Lune ad Solem in circulo altitudinis** [V principles] or **diversitas aspectus Lune ad Solem** [V.20], parallax of the moon to the sun on the circle of altitude; **diversitas aspectus** (**Lune**) **in longitudine** [V principles] or **diversitas aspectus longitudinis** [VI.21], parallax in longitude; and **diversitas aspectus** (**Lune**) **in latitudine** or **diversitas aspectus latitudinis** [V principles, VI.10], parallax in latitude.

diversitas simplex, the moon's simple irregularity, i.e. that due to its epicycle (used only once) [IV.9]; see *prima diversitas*.

ecentricus, eccentric circle [III.3]; synonymous with *orbis ecentricus* and *circulus ecentricus*.

eclipsimo, **-are**, to eclipse (always used in passive voice) [VI.18].

eclipsis, **-is**, *f.*, eclipse [II.36]; **solaris eclipsis**, solar eclipse [II.36]; **lunaris eclipsis**, lunar eclipse [IV principle]; **particularis eclipsis**, partial eclipse (used only once) [VI.13]; **universalis eclipsis**, total eclipse (used only once) [VI.13]; other words to denote eclipses that are used rarely are *defectus* and *lunaris labor*.

eclipso, **-are**, to eclipse (always used in passive voice) [IV.17].

eclipticus, capable of having an eclipse; used here only in the phrases *termini ecliptici* and **coniunctio ecliptica** [VI.10].

elevatio, height or elevation above horizon [II.9]; ascension, the arc of the equator that rises with a given arc of the ecliptic [I.17], synonymous with *ascensio*.

emisperium, hemisphere [II.7].

epiciclus, epicycle [III.3]; **orbis epicicli** [III.8]; *circulus brevis* and *circulus parvus* are used rarely.

equabilis, uniform (to describe motion) [III principles].

equalis, equal [I.1]; mean [III.1]; smooth, even [I.15]; uniform [III.3].

equalis occidens, the point of the horizon where the equinoxes set [VI.25].

equalis oriens, the point of the horizon where the equinoxes rise [VI.25].

equaliter, equally [II.7]; uniformly [III.3].

equatio, the difference of mean and true motions [III.17]; any correction, as in the equation of time [III.25] or the improvement of the value for the moon's mean motion of diversity [IV.14].

equatio argumenti, the arc between the lunar epicycle's true and mean apogees (used once) [V principles]; see *equatio portionis*.

equatio diversitatis, the heading of a column in the tables for the equation of the moon [V.9].

equatio medie diversitatis, the arc between the lunar epicycle's true and mean apogees [V principles]; see *equatio portionis*.

equatio portionis, the arc between the lunar epicycle's true and mean apogees [V principles]; synonymous with *equatio medie diversitatis*, *equatio argumenti*, and *equatio puncti*.

equatio puncti, the arc between the lunar epicycle's true and mean apogees [V principles]; see *equatio portionis*.

equatio singularis, the portion of the moon's apparent motion's deviation from mean motion that is due to its epicycle (used only once) [V.9]; see *simplex equatio*.

equator diei, equator (used infrequently) [II.27]; see *equinoctialis*.

equinoctialis, *adj*., equatorial, as in **circulus equinoctialis** [I.16] or **linea equinoctialis** [I.17]; equinoctial, as in **punctum equinoctiale** [II.14] or **punctus equinoctialis** [II.15] or **equinoctialis dies** [II.1].

equinoctialis, **-is**, *m*., equator [I preface]; synonyms are *circulus equinoctialis* and *linea equinoctialis* (see previous entry) and rarely *equator diei* and *rectus circulus*.

equinoctium, equinox [I.16]; often **punctum equinoctii** [II.14]; **autumpnale equinoctium**, autumnal equinox [III.1], synonymous with *punctum autumpnale* and *punctum equalitatis autumpnalis*; **vernale equinoctium**, vernal equinox [III.11], synonymous with *punctum vernale*.

equo, **-are**, to equal (used passively to mean 'is equal to') [I.2]; to correct [IV.3].

erratica, one of the five planets (used only once) [I preface].

exesus, eaten up, hollowed out, used with moon to denote crescent moon [V.6]; synonymous with *corniculatus*.

facies, **faciei**, *f*., face of the moon (used only once) [IV principles].

finis detectionis, the time at which an eclipse ends [VI principles]; see *finis eclipsis*.

finis eclipsis, the time at which an eclipse ends [IV.2]; synonymous with *finis detectionis*.

finis more, the time at which totality ends in an eclipse [VI principles]; see *principium detectionis*.

fixa lumina, fixed stars [I preface].

flexus tenebrarum, the direction of darkness in an eclipse [VI principles].

gibbosus, gibbous, as in the lunar phase [V.6]; see *protumidus*.

gnomo, **gnomonis**, *m*., gnomon [II.6]; **gnomo erectus**, upright gnomon [II.6]; **gnomo iacens**, horizontal gnomon [II.6].

gradus, **-us**, *m*., degree [I.6], see *pars*; rarely used to denote the 120th of the diameter [I.6], see *pars*; a rung in a column of a table [VI], synonymous with *scala*.

hora, hour [II.12]; **hora equalis**, an equal hour, i.e. a 24th of a day [II.19], synonymous with **hora recta**, a right hour [I.17]; **inequalis hora**, an inequal hour [II.19] and **hora temporalis**, temporal or seasonal days [II.20], i.e. a twelfth of the time between the sun's rising and setting or vice versa; thus, these latter can be either **hora diurna** [II.19] or **hora nocturna** [II.19].

impletio media, mean opposition (used only twice) [V principles]; see *oppositio*.

inclinatio tenebrarum, inclination of the darkness in an eclipse [VI.24], synonymous with *flexus tenebrarum*.

initium eclipsis, the time at which an eclipse begins [VI.21]; see *principium eclipsis*.

instrumentum, instrument [I.15].

kata, *f. indecl.*, the plane or spherical sector figure, i.e. the Menelaus Theorem, or one of the figures used in it (always used in other of the following phrases) [I.13]; **kata coniuncta**, the conjunct sector figure [I.14]; **kata disiuncta**, the disjunct sector figure [I.13].

latitudo, width [I.15]; latitude (from either equator or ecliptic) [II principles]; **latitudo regionis**, the latitude of a location on earth [II principles]; the author uses **latitudo Lune** [IV principles], but because of parallax, he also distinguishes between **vera latitudo** (**Lune**), i.e. the moon's true latitude [V.22], and **visa** (**Lune**) **latitudo**, i.e. the moon's apparent latitude [V.22].

linea medii diei, meridian [II.31]; the same is denoted by **linea medii celi** [II.33]; see *meridianus*.

lingula, a part of an instrument that projects out [I.15].

livellus, (perhaps **livellum**), level (apparently different than a plumb line) [I.15].

locus medius, place according to mean motion [IV.16].

locus secundum cursum medium, place according to mean course [III.17].

locus (**stelle**) **secundum latitudinem**, (a star's) place according to latitude (used primarily with moon) [V principles]; also **locus latitudinis** [V.1].

locus (**stelle**) **secundum longitudinem**, (a star's) place according to longitude (used primarily with moon) [V principles]; also **locus longitudinis** [V.1].

locus verus, place according to true motion [III.17].

longitudo, longitude, either along the equator or ecliptic [II principles]; sometimes it is used to refer to apogee or perigee [III.4]; but it is also used in a non-technical sense to refer to length [I.15] or distance [II.5]; **longitudo regionis**, the longitude of a location on earth [II principles].

longitudo duplex, double the mean distance of sun and moon [V.3].

longitudo longior, apogee [III.3]; synonymous with *aux*; on the moon's epicycle, there are two types of apogee: first, the **longitudo longior equalis** [V principles] or **longitudo longior equata** [V.18], i.e. the point on the moon's epicycle from which its mean motion is reckoned; secondly, the **longitudo longior (epicicli) vera** [V principles] or **longitudo vera epicicli** [V.7], i.e. the point on the moon's epicycle that is furthest from earth.

longitudo media, mean distance; usually used in the context of a eccentric circle to mean the points 90° from the apogee according to apparent motion [III.19]; the author also uses the term in context of an epicycle to mean the line tangent to the epicycle [VI.2]. Both meanings can be united under one definition because the distances from the earth to these points are mean proportionals between the distances from the earth to the apogee and perigee, as can be seen from Euclid's *Elements* III.31 and III.36.

longitudo propior, perigee [III.3]; **longitudo propinquior** (used only once) [V.4]; synonymous with *augis oppositum*; the moon's epicycle has both the **longitudo propior equalis** [V.7] or **longitudo propior media** [V.14] and also the **longitudo propior vera** [V.14].

luminis orba, *adj.*, bereft of light, used with the moon to denote new moon [V.6].

Luna, moon [I preface].

lunaris, lunar [IV principles].

lunaris labor, lunar eclipse (used only once) [VI.24]; see *eclipsis*.

lunatio, lunation [IV.14]; **equalis lunatio**, mean lunation [IV principles].

maxima declinatio, the greatest distance between two inclined circles; this usually refers to declination of ecliptic from equator [I.15] but also to the declination of the moon's inclined circle to the ecliptic [VI.4]; it is used in lunar theory as shorthand for *maxima declinatio ad septentrionem* [IV.17].

maxima declinatio ad/in septentrionem, used to mark the northernmost point on the moon's inclined circle [IV.17]; also, **maxima declinatio circuli ad septentrionem** [IV.17]; **maxima declinatio septentrionalis** [IV.17]; **maxima declinatio circuli declinantis versus septentrionem** [V.3]; **maxima declinatio ab orbe signorum versus septemtrionem** [V.11]; **maxima declinatio septemtrionalis** [V.13]; *maxima declinatio* is used to mean this point [IV.18].

media diversitas equata, the moon's elongation on epicycle from true apogee (used once) [V principles]; see *portio*.

media distantia Solis et Lune, the distance between the sun and moon according to their mean motions [IV.7]. Synonymous with *simplex longitudo*.

media eclipsis, the middle of an eclipse [IV.2]; also **medium eclipsis** [IV.15].

media nox, midnight [III.17].

medium celi, middle heaven [II.19]; also **punctum celum medians** [II.32] or rarely **medium celum** [II.1]; the author sometimes distinguishes further by using the terms **medium celi super terram** [II.19], **medium sub terra celum** [II.1]; the author sometimes refers to the degree at the middle heaven by the phrases **pars medii celi** [II.19], **gradus medii celi** [II.30], or **pars medians celum sub terra** [II.19].

medius motus portionis, synonymous with *motus diversitatis* (used only once) [VI.2].

mensis, month [III.2]; also **mensis lunaris** [V.2]; the author sometimes specifies a mean month by **mensis** (**lunaris**) **equalis** [VI.6] or **mensis medius** [VI.11].

meridianus, meridian [I.15], synonymous with *circulus medii diei*, *linea medii diei*, *meridies*, and many of the constructions with the *adj. meridianus*.

meridianus, *adj.*, meridian, used in **linea meridiana** [I.15], *circulus meridianus*, and **arcus meridianus** [II.22]; noon, as in **umbra meridiana** [II.7]; south, southern [II.7], synonymous with *australis* and *meridionalis*.

meridies, **meridiei**, *m.*, meridian [I.15], also **circulus meridiei** [II.1], **orbis meridiei** [II.31], and **linea meridiei** [II.32]; noon [I.15]; south [II.7].

meridionalis, southern, south (used only once) [II.19], synonymous with *meridianus*.

minuta affinitatis, column in the tables for lunar and solar eclipses [VI.14].

minuta casus, the minutes of immersion in an eclipse [VI principles]. See also *minuta more* and *definita minuta detectionis*.

minuta more, the minutes of delay in an eclipse [VI principles]; although the author sometimes discusses the **minuta totius more**, he more often refers to the **minuta dimidii more** or the **minuta more dimidie** [VI.14]. Note that the author refers to the combined minutes of immersion and of half of the delay by **minuta more et casus simul**, **minuta casus et dimidii more** (**simul**), or **minuta casus et more** [VI.14]. The author also sometimes distinguishes between the **minuta more ante medium eclipsis** and the **minuta more post medium eclipsis** [VI.14]; likewise, he refers to **minuta casus et more ante eclipsim** (**mediam**) and the **minuta casus et more post eclipsim** (**mediam**) [VI.14, VI.15].

minuta proportionalia, proportional minutes found in tables of the equation of the moon [V.9].

minutum, **-ti**, *n*., a sixtieth of any type of unit, as in the following: a sixtieth of a degree [I.15]; a sixtieth of a day [III.1]; a sixtieth of an hour [III.2]; a sixtieth of a part of the diameter (i.e. 1/60 of its 120th) [I.6], see *punctum*.

mora, the delay or duration of totality of an eclipse [V.17]; duration of time [II.15].

morula, a brief delay or duration of totality that may occur in a solar eclipse (used only once) [VI.18].

motus apparens, apparent motion [III.3].

motus Capitis, motion of the dragon's head [VI.3]; **medius motus Capitis** (**Draconis**), [IV.19]; synonymous with *motus nodi*.

motus diversitatis, moon's motion on its epicycle [V.9], although first used to refer to its irregular motion either on the epicycle or its eccentric [IV principles]; also, **medius motus diversitatis** [IV.3], which is sometimes shortened to *diversitas* [IV.10], and **motus prime diversitatis** (used only once) [V.3]; see *cursus diversitatis in epiciclo* and *portio*.

motus diversus, irregular motion [III.1]; also **diversus motus apparens** (rarely used) [III.3].

motus medius, mean motion [III principles].

motus latitudinis, the moon's motion on its declined circle [IV principles]; also **medius motus latitudinis** [IV.7]; more specifically, the author uses **verus motus latitudinis** [V.13] and **motus latitudinis equatus** [V.12]. See *cursus latitudinis*.

motus longitudinis, the moon's motion along the ecliptic [IV principles]; also, **motus in longitudine** [VI.3]; **motus in longum** (used for the moon and once for the sun) [IV.3]; **medius motus longitudinis** [IV.3] or **medius motus** (**Lune**) **in longitudine** [IV.7], mean motion in longitude, synonymous with *cursus medius longitudinis*.

motus nodi, synonymous with *motus Capitis* [IV.19].

motus primus, the first motion, i.e. daily motion of fixed stars upon the world's poles [II.7].

mundana machina, the universal machine (used once) [I preface].

mundus, world, universe [III.3].

nodus, node, i.e. where the moon's declined circle meets the ecliptic [IV principles]; **nodus Capitis**, the ascending node, and **nodus Caude**, the descending node [IV.16]. The shortened versions **Caput** and **Cauda** are used [IV principles].

nota declinationis diametri epicicli, the point to which the epicycle's mean apogee is directed (only used once) [V.14].

numeri communes, common numbers, i.e. numbers in a column that represent more than one astronomical object used for the entrance into the table [V.9].

obliquus, oblique, both as in more than 90° in **angulus obliquus** [V.25] or tilted as in *spera obliqua* [II principles]; **obliqui** is used once to refer to the people of the southern hemisphere [VI.12].

observatio, observation, used rarely both for astronomical observation [III.1] and once for a mental observation [I.6]. The more normal word for astronomical observation is *consideratio*.

occasus, **-us**, *m*., setting [II.7].

occidens, **-tis**, *m*., west [I preface]; the setting point [II.19].

occidentalis, western [II.28].

occidentes estivales, places of horizon where northern signs set [VI.25].

occidentes hiemales, places of horizon where southern signs set [VI.25].

occido, to set [II.7].

octava spera, the eighth sphere, i.e. the sphere of the fixed stars; only mentioned in the phrases **motus octave spere ante et retro** and **motus octave spere mobili** [III.1].

oppositio, opposition of sun and moon [IV.3], synonymous with *preventio*; **media oppositio**, opposition according to the sun and moon's mean motions [V principles], synonymous with *impletio media* and *media preventio* (see *preventio*); **oppositio vera**, true opposition [V.10, VI principles].

orbicularis, circular [I preface]; note that it does not appear to mean spherical.

orbis, circle [I.13]; note that **orbis** is not used to denote a sphere.

orbis altitudinis, circle of altitude [II.31]; see *circulus altitudinis*.

orbis declinans, the moon's inclined sphere [VI.10]; see *circulus declinans*.

orbis ecentricus, eccentric circle [III.4]; see *ecentricus*.

orbis signorum, ecliptic [II.23]; see *circulus signorum*.

ordo signorum, the order of the zodiacal signs, i.e. from west to east [IV.19].

oriens, **-tis**, *m*., east [I preface]; the rising point or the ascendant [II.1, II.19], see *ascendens*.

orientalis, eastern [II.1].

orientes estivales, places of horizon where northern signs rise [VI.25].

orientes hiemales, places of horizon where southern signs rise [VI.25].

orior, **oriri**, to rise [I.17].

orizon, **-tis**, *m*., (*nom./acc. plur.* **orizonta**), horizon [I.15]; **orizon declivis**, the declined horizon [II principles]; **orizon rectus**, right horizon [II.15].

ortus, **-us**, *m*., the point on the horizon where a certain object rises [II.1, II.2]; an ascension or act of rising [I.17], see *ascensio*.

pars, a part [I.7]; a 120th of the diameter [I.6]; a degree [I.6], see *gradus*.

pars ascendens, ascending degree [II.19]; see *ascendens*.

pars oriens, ascending degree [II.19]; see *ascendens*.

particularis eclipsis: see *eclipsis*.

permeatio, traverse, movement [III.5].

pinna, literally a feather or fin, but here a small protruding part of an instrument [I.15]; also, **pinnula** [I.15].

planeta, **-e**, *m*., planet (apparently including the sun and moon) [III principles].

plenus, full, used with moon to denote full moon [V.6].

polus, pole [I preface]; **polus** (**circuli**) **equinoctialis** [I.16], **polus equinoctia-**

lis [II.15], **polus equatoris diei** [II.32], **polus mundi** [V.1], and **polus primi motus** [II.7], the poles of the universe; **australis polus** [II.1] and **polus meridianus** [II.18] (not to be confused with **polus meridiani**, the pole of the meridian [II.29]), south pole; **polus septentrionalis**, north pole [II.5]; **polus zodiaci** [II.12] or **polus circuli signorum** [IV principles], the poles of the ecliptic; **polus orizontis**, the zenith [II principles], see *cenit*.

portio, a part or portion (non-technical) [I.6]; the sun's mean motion measured from apogee [III.17]; the moon's motion on its epicycle [V.8], synonymous with *cursus diversitatis in epiciclo* and *medius motus diversitatis*; **portio equata**, the moon's elongation from true apogee on its epicycle [V principles], synonymous with *argumentum equatum* and *media diversitas equata*; **simplex portio**, the moon's motion on its epicycle disregarding the equation of portion [V.10].

postmeridianus, after the meridian [II.33].

preventio, an opposition (of sun and moon) [V principles]; **media preventio**, mean opposition [V principles]; see *oppositio*.

prima diversitas, the first diversity of the moon, i.e. that due to its epicycle [IV.9]; synonymous with *diversitas simplex* and *singularis diversitas*.

principium additionis, where the equation of time begins to grow [III.24].

principium detectionis, the time at which totality ends [VI principles]; see *finis more*.

principium diminutionis, where the equation of time begins to decrease [III.24].

principium eclipsis, the time at which an eclipse starts [IV.2], lesser used synonyms are *initium eclipsis* and *principium obscurationis*.

principium more, the time at which the totality begins in an eclipse [VI principles].

principium obscurationis, the time at which an eclipse begins [VI principles]; see *principium eclipsis*.

proselidis, **-is**, *f.*, column of a table [VI.23]; see *tabula*.

protumidus, swollen, gibbous, as in the lunar phase [V.6]; see *gibbosus*.

punctum, geometrical point [I.13]; a sixtieth of a part (used in the measure of chords), i.e. a sixtieth of a 120th of the diameter [I.6], see *minutum*.

punctum autumpnale, autumnal equinox [II.15]; see *equinoctium*.

punctum equalitatis autumpnalis, the autumnal equinox (used only once) [III.17]; see *equinoctium*.

punctus medius, the middle point of a line or arc [I.1]; one of the two points on an eccentric circle at the intersection of its circumference and the line passing through the center of the world at right angles to the lines of apsides (used only twice) [III.5].

punctum orientis, the point of rising, i.e. the ascendant [II.34]; see *ascendens*.

punctum vernale, spring equinox [II.15]; see *equinoctium*.

quadrans, **-tis**, *m.*, a quarter circle [I.16]; an instrument [III.1].

quantitas (**prime**) **diversitatis**, the radius of the moon's epicycle [IV.10]; note that **quantitas secunde diversitatis** is used also, but it does not refer to a specific line and can be understood by the normal meanings of the words in the phrase [V.4].

radius, ray (of light) [II.6]; note that it is not used for the radius of a circle.

radix, square root [II.6]; shorthand for *radix temporis* [III.17].

radix temporis, radix or root of time, epoch [III.17]; usually merely *radix* [III.17].

rectus circulus, equator (used once) [I.17]; see *equinoctialis*.

reflexio, the 'turning aside' of the diameter of the moon's epicycle caused by difference between the true and mean apogees [V.7]; the same is denoted by *declinatio*.

reflexio tenebrarum, the heading of the table of inclinations of the darkness in eclipses (used only once) [VI.24].

regula, rule, part of an instrument [V.1]; a set of directions [I.16].

reversio, return [III.1]; **reversio diversitatis**, the return of an irregularity, i.e. the completion of a cycle of the moon's first irregularity [IV.3].

revolutio, revolution, i.e. a movement through 360 degrees [II.1]; **revolutio diversitatis**, return of an irregularity or perhaps revolutions on the moon's epicycle [IV.3]; **revolutio latitudinis**, revolutions of latitude [IV.3]; **revolutio longitudinis**, revolutions of longitude [IV.3].

scala, entry or rung in a column of a table [V.9]; see *gradus*.

sectionum figure, the figures of divisions (this appears to refer to the astrologically significant positions, i.e. opposition, conjunction, trine, quadrate, sextile) [V.2].

secunda diversitas, the moon's irregularity due to its eccentric circle [V.2].

semiplenus, half full, used with moon to denote half moon [V.6].

septentrio, **-nis**, *m*., north [II.7].

septentrionalis, northern [II.5].

signum, sign of the zodiac [II.2].

simplex equatio (**Lune**), the portion of the moon's apparent motion's deviation from mean motion that is due to its epicycle [IV.13]; lesser used synonyms are *coequatio partis Lune* and *equatio singularis*.

simplex longitudo, simple longitude, i.e. the mean distance between sun and moon (used only once) [IV.7].

simplex portio: see *portio*.

singularis diversitas, the moon's irregularity due to its epicycle (used rarely) [V.2]; see *prima diversitas*.

Sol, sun [I preface].

solaris, solar [II.36].

solstitialis, solstitial [III.1]; most commonly in **punctum solstitiale** [III.1]; see *solstitium*, *tropicum*, and *tropicus*.

solstitium, solstice [I.15]; **hiemale solstitium**, winter [I.16]; **estivum solstitium** [III.11] or **solstitium estivale** [I.16], summer solstice; see *solstitialis*, *tropicum*, and *tropicus*.

spera declivis, the declined sphere, i.e. anywhere where the zenith is not on the equator [II principles]; synonymous with *spera obliqua*.

spera obliqua, the oblique sphere [II principles]; synonymous with *spera declivis*.

spera recta, the right sphere [I.17].

stella, star [II.7]; it is sometimes used in contexts that exclude the fixed stars [III.3].

stella fixa, fixed star [III.1].

successio signorum, the succession of signs, used only in the phrase 'secundum successionem signorum', i.e. from west to east [II.19], synonymous with *continuitas signorum*.

summitas capitum/capitis, the zenith [I.15, II.6]; see *cenit*.

superfluitates longitudinis propioris, a heading of a column in the table of lunar equation (used once) [V.9].

superlatio, the moon's carrying beyond, i.e. the excess of its motion over the sun's motion [VI principles]; also, **superlatio Lune** [VI principles]; **media superlatio Lune**, the moon's mean carrying beyond [VI principles]; **visa superlatio** (**Lune**), the moon's apparent carrying beyond [VI principles]; **vera superlatio** (**Lune**), the moon's true carrying beyond [VI.3], and once **Lune vera superatio** [VI.20].

tabula, a table or more precisely a column of a table [III.1]; a synonym used rarely is *proselidis*.

tabulo, **-are**, to tabulate or to make a table of something [V.19].

tempus, time [I.15]; time-degrees (used rarely) [III.22].

tenebre, **-arum**, *plur.*, the darkness appearing on the sun or moon during an eclipse [IV.4].

terminus, endpoint [I.7]; limit [VI principles]; one of the four chief distances of the moon from the earth [V.19]; **terminus primus**, with the moon at the epicycle's apogee while the epicycle is at the eccentric's apogee; **terminus secundus**, with the moon at the epicycle's perigee while the epicycle is at the eccentric's apogee; **terminus tertius**, with the moon at the epicycle's apogee while the epicycle is at the eccentric's perigee; **terminus quartus**, with the moon at the epicycle's perigee while the epicycle is at the eccentric's perigee [V.19].

termini ecliptici (**solares/lunares**), *plur.*, the limits of the moon's distance from the nodes at which an eclipse can occur [VI principles]; also **lunares termini** and **solares termini** [VI.6].

terra, earth [I preface].

tropicum, tropic [I.15]; **tropicum hiemale** [II.24]; **tropicum estivum** [II.24]; **tropicum Cancri** [III.12]; see *solstitialis*, *solstitium*, and *tropicus*.

tropicus, tropical, used only in the following: **punctum tropicum**, tropic point [II.15], **punctum tropicum estivum**, summer tropic point [III.21], and **tropicum punctum hiemale**, winter tropic point [III.21]; see *solstitialis*, *solstitium*, and *tropicum*.

umbra, shadow [I.15]; **umbra iacens**, shadow cast on a horizontal surface [I.6]; **umbra versa**, shadow cast on a vertical surface [I.6].

universalis eclipsis: see *eclipsis*.

verificatio, the act of correction, i.e. taking irregularities into account in the calculations of a celestial object's motion [V.7].

verifico, **-are**, to correct, i.e. to calculate the sun or moon's place according to its true motion [VI.3].

verus locus, true place [III.17]; note that this term is used for both calculated and observed positions.

verus locus Lune in celo, the moon's true place in the heaven [IV principles].

verus locus Lune in circulo signorum, the moon's true place in the ecliptic, determined by the circle passing through the moon's true place and the ecliptic's poles [IV principles].

visus locus, apparent place [III.7].

visus motus, apparent motion [III.3].

zodiacus, zodiac, ecliptic [I.17].

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The *Almagesti minor* is one of the most important works of medieval astronomy. Probably written in northern France circa 1200, it is a Latin summary of the first six books of Ptolemy's astronomical masterpiece, the *Almagest*. Also known to modern scholars as the "*Almagestum parvum*", the *Almagesti minor* provides a clear example of how a medieval scholar understood Ptolemy's authoritative writing on cosmology, spherical astronomy, solar theory, lunar theory, and eclipses. The author incorporated the findings of astronomers of the Islamic world, such as al-Battānī, into the framework of Ptolemaic astronomy, and he altered the format and style of Ptolemy's astronomy in order to make it accord with his own ideals of a mathematical science, which were primarily derived from Euclid's *Elements*. The *Almagesti minor* had a profound effect upon astronomical writing throughout the 13^{th}-15^{th} centuries, including the work of Georg Peurbach and Johannes Regiomontanus. In this first volume of the *Ptolemaeus Arabus et Latinus* text series, Henry Zepeda offers not only a critical edition of this little-studied text, but also a translation into English, analysis of both the text and its geometrical figures, and a thorough study of the work's origins, sources, and long-lasting influence.

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