Ptolemaeus
Arabus et Latinus

# PTOLEMAIC TRADITION AND ISLAMIC INNOVATION: THE ASTRONOMICAL TABLES OF KŪSHYĀR IBN LABBĀN

Benno van Dalen

BREPOLS

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# The Astronomical Tables of Kūshyār ibn Labbān

# Ptolemaeus Arabus et Latinus

## Texts

Volume 2



Ptolemaeus Arabus et Latinus (PAL) is a project of the Bavarian Academy of Sciences and Humanities and the University of Würzburg. As part of the Academies' Programme, PAL is jointly funded by the Federal Republic of Germany and the Free State of Bavaria.

PAL is sponsored by the Union Académique Internationale (UAI).

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Cover design by Hilde Verhelst, T'Hi, Lier, Belgium.

Cover illustrations:
Woodcut (detail) from *La geografia di Ptolemeo Alessandrino*, Venice, 1548. Copy from Munich, Bayerische Staatsbibliothek. Aries (detail) from al-Ṣūfī's *Book of Constellations* (Arabic), MS Paris, Bibliothèque nationale de France, arabe 5036, fol. 109r.



D/2021/0095/72

ISBN 978-2-503-59341-8

Printed on acid-free paper.

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# Preface

Astronomers in the Islamic world produced more than 200 handbooks with tables for calculating all mathematical quantities needed by the practising astronomer or astrologer. These quantities included arcs on the celestial sphere such as the solar declination, the length of the day and the rising times of fixed stars; the positions in ecliptic longitude and latitude of the Sun, Moon and planets; the time, duration and magnitude of solar and lunar eclipses; the times of visibility of the Moon and the planets; the retrograde arcs of the planets, and a range of specifically astrological functions. These astronomical handbooks were called *zīj*es in Arabic and Persian and they were among the most important categories of Islamic scientific literature, representing some of the most significant developments in theoretical and applied mathematics as well as in theoretical and observational astronomy. In the early stages of the history of Islamic astronomy, *zīj*es were modelled after Indian astronomical works under the name *Sindhind* (from Sanskrit *siddhānta*). However, from the early ninth century onwards, Greek astronomy, as it had come down to the Islamic world especially in the works of the second-century Alexandrian scholar Claudius Ptolemy, soon became predominant.

The two major works by Ptolemy that influenced the compilation of *zīj*es in the Islamic world were the *Almagest* and the *Handy Tables*. The *Almagest* explained every aspect of Ptolemy's geocentric geometrical planetary models in detail, including the observations from which the parameters of the models could be determined, the construction of the instruments that were used for such observations, and the geometrical proofs for the calculations of the planetary positions on the basis of the models. The *Almagest* remained the main reference work for any Islamic astronomer until, in the eastern part of the Islamic world, it was gradually replaced by its most influential revision, written by the polymath Naṣīr al-Dīn al-Ṭūsī in the middle of the thirteenth century. On the other hand, as its name already indicates, the *Handy Tables* were more suited for practical work, providing only a more convenient set of tables than that found in the *Almagest* and instructions for their use. The *Handy Tables*, which are not extant as such in Arabic or Latin, became the model for most *zīj*es written in Arabic and Persian, but astronomers in the Islamic world would continue to refer to the *Almagest* for the underlying details of the models and for certain types of tables.

Islamic astronomy was highly productive in the early part of the Abbasid dynasty (ninth to eleventh centuries). From this period around 60 handbooks with tables are known, but only a very small number of these are extant. *Zīj*es

modelled after the *Sindhind* continued to be written occasionally throughout this period, but by the year AD 900 *zīj*es based on Ptolemy's *Almagest* and *Handy Tables* had fully matured and constituted a standard that would undergo no fundamental changes for many centuries to come. Around the year 1000, three astronomers in different parts of the Islamic world brought the art of the compilation of *zīj*es up to a level that comes close to modern standards of scientific methodology. Abū l-Wafāʾ in the Abbasid capital Baghdad, Ibn Yūnus in the capital of the Fatimid dynasty in Cairo, and al-Bīrūnī in Iran and later in Afghanistan under the Ghaznavids excelled in evaluating the details of earlier observations that they used, in deriving more accurate parameter values from their own observations, in providing extensive methods and complete proofs for all their calculations, and in computing tables with a much greater accuracy than their predecessors.

It was in this same period that the main character of this book, Kūshyār ibn Labbān, from the province of Gilan in north-western Iran, made a career for himself as the author of mathematical and astronomical works tailored to a less specialised audience. Kūshyār wrote one of the earliest extant treatises on decimal arithmetic as well as a very popular introductory work on astrology and a treatise on the astrolabe, a small metal instrument that allows a two-dimensional representation of the situation of the heavens at any desired time. Kūshyār also wrote two *zīj*es: one in the Indian tradition, of which only six pages have survived in a composite manuscript in Mumbai, and another in the Ptolemaic tradition, which he called *al-Zīj al-Jāmiʿ* (*The Comprehensive Zīj*). In the writing of this Ptolemaic *zīj* he was strongly influenced by *al-Zīj al-Ṣābiʾ* (the *Sabian Zīj*, *c.* 900) composed by al-Battānī, an astronomer from Raqqa in present-day Syria who was famous for his observations. Unlike most other Arabic and Persian *zīj*es, Kūshyār included in the four books of the *Jāmiʿ Zīj* not only tables with instructions for their use but also, though less thoroughly than his three great contemporaries, explanations of the underlying planetary models and proofs for the calculations involved. As will be shown in this book, Kūshyār clearly adhered to the Ptolemaic standards set in the earlier Arabic *zīj*es, but also introduced various types of innovations in the layout of the tables and the computation of certain functions. The popularity of the *Jāmiʿ Zīj* is shown by the fact that some 15 manuscripts containing the entire work, or at least several of its four books, have survived until this day, while several of Kūshyār's innovations were adopted in later Islamic astronomical handbooks.

Of the more than one hundred Arabic and Persian astronomical handbooks that still exist in manuscript libraries all around the world, fewer than a dozen have been satisfactorily edited, translated or studied in detail. As the experience of researchers over the last 200 years has shown, almost every surviving *zīj* contains either information about earlier works that may be lost, unexpected

technical details, innovations not known from other sources, tables based on otherwise unknown parameter values, or a combination of these features. Mathematical analyses of tables are needed in order to uncover also the characteristics that are not mentioned in the accompanying text or the headings, or are given incorrectly. By combining all these types of data, *zīj*es provide invaluable information about the development of astronomy in the Islamic world, of which a fuller picture can only be obtained by investigating many more of the surviving works.

Since the category of *zīj*es constitutes one of the most important representatives of the Ptolemaic heritage in Islamic astronomy, it finds a natural place within Ptolemaeus Arabus et Latinus (PAL), the project in which I have been involved since its inception in 2013. Under the auspices of this project I plan to complete my *New Survey of Islamic Astronomical Handbooks with Tables* in the coming years. Furthermore, a database of astronomical tables with tools for their edition and analysis is currently being developed as an international collaborative effort (see below). PAL's publication plan also includes a critical edition of (part of) a *zīj*. Full editions and studies of al-Bīrūnī's monumental *Masʿūdic Canon* (*c.* 1030) and al-Khāzinī's *Sanjarī Zīj* (*c.* 1120) are among the main desiderata of historians of Islamic astronomy, but will unfortunately be impossible within the time constraints of our project. For these and other reasons we decided to carry out a critical edition and analysis of the tables from Kūshyār's *Jāmiʿ Zīj*. Although Kūshyār's scholarship does not reach the same heights as some of his famous colleagues, a study of this kind would not only offer a window into the daily practices of a much larger category of Islamic astronomers and astrologers, but also seemed more practicable because it would be able to build on existing recent work.

Mohammad Bagheri, himself hailing from the Iranian province of Gilan, has investigated Kūshyār's astronomical works extensively since the 1990s. For his PhD dissertation at Utrecht University he prepared a critical edition of Books I and IV of the *Jāmiʿ Zīj* with an English translation and commentary. His colleague Hanif Ghalandari at the Institute for History of Science in Tehran is currently working on an edition and translation of Book III. For a full treatment of Kūshyār's Ptolemaic *zīj* the only remaining desideratum thus was an edition of the tables in Book II with their textual elements, to be supplemented by translations of the titles of the tables, explanatory texts and any relevant marginal notes, and an analysis of the mathematical characteristics and the origin of those tables that were not simply copied from Ptolemy or al-Battānī. With Mohammad Bagheri's encouragement, I decided to embark on this task in early 2014.

However, what seemed to be a rather straightforward enterprise at the time soon became much more substantial. First of all, I decided early on to introduce a new methodology for critically editing mathematical tables. Rather than

simply providing individual apparatus entries for every single deviating tabular value, I wanted to make clearly visible the ranges of variants in the available manuscripts that have a common cause. This includes, among others, entire columns that were copied in the wrong place, series of digits or values that were copied one or more rows too high or too low because of the omission or repetition of certain values, and the systematic small differences found in most of the mean motion tables because some of the subtables were computed anew to be included in a later version of Kūshyār's *zīj*. For such groups of errors I introduced special markers in the edition of the tables and a special notation in the apparatuses. This method not only provides more insight into the type of errors that were made in the process of copying, but also drastically reduces the total number of entries in the apparatuses. For textual elements that occur in basically the same form in a series of tables (especially the month names and the headings of the planetary mean motions and equations) I established 'general editions' that bring together the variants in all these occurrences, in some cases involving more than one hundred witnesses.

Besides the types of errors just mentioned, the manuscripts show further differences that can only be explained by assuming that there existed three or four somewhat different versions of the *Jāmiʿ Zīj*. It will be shown that all of these probably originated with Kūshyār himself and point to a long process of compilation of the *zīj* during which various tables were recomputed and replaced. Sorting out these different versions and establishing which of the variant tables can be reliably attributed to Kūshyār became an important secondary goal of my work on the tables. Then, in late 2018, when the edition was basically finished, I gained access to the only Judaeo-Arabic copy of the *Jāmiʿ Zīj* that includes Book II (the black-and-white photocopies that had been available before that time were not good enough to be used for the edition). Better late than never, I learned the Hebrew alphabet, and, realising that the Judaeo-Arabic manuscript not only represents an interesting early stage of the development of the *Jāmiʿ Zīj* but also provides very reliable copies of all the tables, I decided to include it in the edition. The result of all these efforts now lies before the reader.

*

This book was written as part of my work for the project Ptolemaeus Arabus et Latinus at the Bayerische Akademie der Wissenschaften and the Julius-Maximilians-Universität Würzburg. This project is financed by the Akademienprogramm of the Union der deutschen Akademien der Wissenschaften for a period of 25 years starting from 2013.

Mohammad Bagheri (Tehran and Rasht, Iran; formerly at the Encyclopaedia Islamica Foundation in Tehran) has stimulated my research on Kūshyār since we first met at the International Congress of History of Science in Zaragoza in 1993. He also actively supported my work on the edition of Book II of

the *Jāmiʿ Zīj* and contributed preliminary editions of the table of contents, the section on finding the original equations, and the textual elements of all the tables. In the final stage he checked the typeset version of several tables and texts and answered numerous questions on details.

The *Dustūr al-munajjimīn*, an Ismāʿīlī *zīj* written in the early twelfth century, possibly in Alamut, adopted many text passages and tables from Kūshyār's *Jāmiʿ Zīj*. I investigated the tables in this work for the project 'Der Dustūr al-munağğimīn als Quelle für die Geschichte der Ismāʿīliyya und ihre astronomisch-astrologischen Vorstellungen. Analyse, Edition und Übersetzung', funded by the Deutsche Forschungsgemeinschaft (DFG) from 2009 to 2015 and led by Eva Orthmann, then at Bonn University. I am grateful to her and to Petra Schmidl for allowing me to include some of the as yet unpublished results of that investigation in this book.

Special thanks are due to David Sulzberger of Ahuan Islamic Art for providing me with the Judaeo-Arabic copy of the *Jāmiʿ Zīj* which includes the entirety of the tables of Book II in a very reliable copy. I am also grateful to Laura Light of *Les Enluminures* and to Anthony J. Turner for helping me contact the current owner of the manuscript.

For nearly two decades, Rafał Ziołkowski programmed the Windows version *ZijManager* of my earlier DOS programs TA and MM for editing and analysing astronomical tables from the Ptolemaic tradition. Funding and technical difficulties have until now prevented us from publishing a final version of the program, but I have made a constant and highly profitable use of it when entering, analysing and recomputing Kūshyār's tables. The most convenient features of *ZijManager* have profoundly influenced the development of DISHAS ('Digital Information System for the History of Astral Sciences'), the online database of astronomical and astrological tables resulting from the project TAMAS ('Tables Analysis Method for the history of Astral Sciences'), a cooperative endeavour carried out under the projects *ALFA* ('Shaping a European Scientific Scene: Alfonsine Astronomy' at the Paris Observatory, PI Matthieu Husson), *HAMSI* ('History of Astronomical and Mathematical Sciences in India' at the University of Canterbury in Christchurch, New Zealand, PI Clemency Montelle) and PAL.

All members of the Ptolemaeus Arabus et Latinus project between 2014 and 2021 contributed to this work in one way or another, by discussions during team meetings, by providing answers to individual questions, and more generally by creating a stimulating work environment. I am very grateful to Claudia Dorl-Klingenschmid for carefully checking the edition of Kūshyār's tables against the raw data files, sorting out the bibliographical references and index entries, and correcting various types of mistakes in the text of Parts I and IV. I am also grateful to our research assistants Bastian Gröppmaier, Paul Hullmeine, Nadine Löhr and Emanuele Rovati for painstakingly controlling

some of the most complicated parts of the critical edition. Hayim Malkhasy helped me find my way in the Judaeo-Arabic manuscript of the *Jāmiʿ Zīj* before I gained access to the manuscript itself and thus to better reproductions. He also assisted with numerous other daily tasks, especially those involving administrative matters and languages that even for our project may be considered exotic.

The late Professor Paul Kunitzsch was very helpful in solving problems related to Kūshyār's star tables and in reading some difficult Judaeo-Arabic passages. Unfortunately, he passed away when the preparation of this book was in its final stages. Mohammad Mozaffari was always prepared to provide me with detailed answers to questions concerning observations and the interpretation of their results and other related issues on which he has published so extensively over the last ten years. Pouyan Rezvani assisted me in my attempts to decipher several unclear inscriptions and seals on the title pages of the eight manuscripts of the *Jāmiʿ Zīj* that I used, as well as the colophons of the Judaeo-Arabic manuscript. Furthermore, he meticulously checked the text edition in its final stages and suggested numerous small corrections. Mazen Amawi made problems of Arabic grammar in corrupt passages of the *Jāmiʿ Zīj* look trivial in comparison with the explanations in traditional text books. I am also extremely grateful to Jan Hogendijk, David Juste, Dag Hasse, Matthieu Husson, Rich Kremer and Sonja Brentjes for their very useful comments on sections of preliminary versions of this book; to David King, Petra Schmidl, Tzvi Langermann and Glen Van Brummelen for their helpful replies to specific questions on details, and to Stefan Müller for his advice on, and help with, numerous computer issues. Of course, I remain fully responsible for any errors and deficiencies that this book still contains. Flora Vafea made numerous trips to the Dār al-kutub and spent many hours negotiating with the authorities at the library in order to provide me with excellent colour photographs of the three Cairo manuscripts. I would further like to thank Gesine Yildiz for providing me with photographs of primary and secondary sources kept at the Institut für Geschichte der Arabisch-Islamischen Wissenschaften in Frankfurt am Main.

More in the background, my supervisors over the course of many years, especially Henk Bos and Jan Hogendijk (Utrecht), David King (Frankfurt) and Michio Yano (Kyoto), all played a vital role in shaping my research interests and skills that lie at the basis of the research in this book.

*

The editions of tables and paratexts in Parts II and III of this book were typeset using XeLaTeX and the package ArabXeTeX, programmed by François Charette on the basis of Klaus Lagally's ArabTeX. The Arabic was typeset in the font Scheherazade of SIL Language Technology. I am very grateful to Michael Maudsley for his careful correction of the English of Parts I and IV and to Bart Janssens and his colleagues at Brepols and Cultura for a very pleasant collaboration and the production of a high-quality book.

# Technical notes

## Sexagesimal numbers

Nearly all numbers in Kūshyār's *Jāmiʿ Zīj*, and in Arabic and Persian astronomical works in general, are given with an integer part in decimals and the fractional part in sexagesimals. I use the standard notation for these numbers which has been established in numerous publications on the history of ancient and medieval astronomy, namely with a semicolon representing the sexagesimal point and commas separating the fractional digits. For example, al-Battānī's (and hence Kūshyār's) length of the tropical year is 365;14,26 days, which stands for $365 + {}^{14}/_{60} + {}^{26}/_{3600} \approx 365.240556$ days. Zodiacal signs and other units of 30 degrees are indicated by a superscript 's', as in $2^{s}24;54,36°$.

## Transliteration of Arabic and Persian

I use the rules for transliteration of Arabic as used in Brill's *The Encyclopaedia of Islam. THREE* and published on their website. For Persian, an extension of the system for Arabic is used except for personal names that appear in different forms on the title pages of publications (e.g., Monzavi or Mozaffari) and place names that are commonly known and written differently.

## Dates

All years in the Islamic calendar are explicitly indicated to be 'Hijra'. If known, and relevant, I will also explicitly indicate whether Hijra dates are given in the civil or in the astronomical variant of the calendar. Similarly, dates in the Syrian and Persian calendars are indicated to be relative to the Seleucid era (also called the Era of Alexander) or the Yazdigird epoch. Years without an indication of an era or calendar are always Julian or Gregorian; unless otherwise mentioned, they are Julian before October 1582 and Gregorian after that month. A year given in the form 1047/8 indicates a Syrian, Arabic or Persian year that falls in these two Julian years. Whenever a date includes a Syrian, Arabic, Persian or Julian/Gregorian month name, the indication of the type of calendar may be omitted since it can be inferred from the month. For an overview of the months in the Syrian, Arabic and Persian calendars (as well as the names of the zodiacal signs), see the general edition of month names on pp. 265–69.

## 'Islamic'

I use the terms 'Islamic science', 'Islamic scholars', etc., in the wide sense of science practised, or scholars working, in the Islamicate world, i.e., without any direct religious connotation, and independent of any linguistic identity and social background.

# Sigla

Manuscripts containing Book II of Kūshyār's *Jāmiʿ Zīj*

**B** Berlin, Staatsbibliothek Preußischer Kulturbesitz, Or. quart. 101/1

**C** Cairo, Dār al-kutub, *mīqāt* 400

**$C_1$** Cairo, Dār al-kutub, *mīqāt* 188/2

**$C_2$** Cairo, Dār al-kutub, *mīqāt* 691

**F** Istanbul, Süleymaniye Kütüphanesi, Fatih 3418/1

**H** Ahuan Islamic Art, MS 40 (Judaeo-Arabic)

**L** Leiden, Universiteitsbibliotheek, Or. 8

**Y** Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3

Other manuscripts and works

**D** Paris, Bibliothèque nationale de France, arabe 5968: *Dustūr al-munajjimīn* (*c.* 1110)

**E** Escorial, RBMSL, árabe 908: al-Battānī, *Ṣābiʾ Zīj* (7th c. Hijra)

**N** Nallino, *Al-Battānī sive Albatenii*, vol. II: edition of the tables from the *Ṣābiʾ Zīj*

**A** Ptolemy, *Almagest*

**T** Ptolemy, *Tetrabiblos*

**r** recomputed or reconstructed tabular value

# Part I

# Introduction

## I.1. Historical background

Astronomers in the lands that were conquered by Muslim troops in the seventh and eighth centuries inherited several astronomical systems. These included, for example, a pre-Islamic Arabic system of timekeeping by means of the so-called *anwāʾ*, the risings and settings of a set of 28 asterisms. Furthermore, in Sasanian Iran various astronomical systems for determining the positions of the Sun, Moon and planets were available which, like Indian systems, were based on the concept of great conjunctions of the heavenly bodies. In the mid-eighth century scholars in the central part of the Islamic world started to occupy themselves intensively with astronomy. The earliest sources with which they became thoroughly familiar were works from India in the tradition of the seventh-century mathematician and astronomer Brahmagupta, which were brought to the Abbasid court in Baghdad around the year AD 775 and were translated into Arabic with the aid of members of an embassy from Sind (the region around Karachi in present-day Pakistan). They became known under the name 'Sindhind', reflecting the meaning of the Sanskrit word *siddhānta*, an astronomical treatise, and at the same time expressing their origin from Sind and from Hind (i.e., India).

Towards the end of the eighth century, the two most influential astronomical works by the second-century Greek scholar Ptolemy, the *Almagest* and the *Handy Tables*, were translated into Arabic. Whereas Indian astronomical treatises were entirely textual and written in verse, Ptolemy's works included large sets of numerical tables that could be used by a practising astronomer or astrologer to perform in a straightforward manner all necessary calculations of arcs on the celestial sphere and the positions of the Sun, Moon and planets, and hence to cast horoscopes. The *Almagest* (in Greek: *Mathèmatikè syntaxis*, in Arabic *al-Majisṭī* from the Greek *megistos*, 'the greatest') explained in detail Ptolemy's geocentric geometrical models for the motions of the planets, the way in which their underlying parameters could be determined from observations, and proofs for the calculations needed to determine planetary positions on the basis of these models. By means of the extensive tables a user could not only find planetary positions but also calculate the time and magnitude of solar and lunar eclipses and the beginning and end of the phases of retrograde motion and the visibility of the planets using only a very few basic operations such as addition, subtraction and an occasional multiplication. For complicated functions of multiple variables Ptolemy used a way of tabulation that is known among modern historians of science as 'Ptolemaic interpolation'. This makes use of the fact that such functions usually change much faster as a function of one of the variables, whereas the influence of the other variables can be sufficiently well represented by approximations. Ptolemy's *Handy Tables* (in Greek *Prokheiroi kanones*, in Arabic simply known as *al-Qānūn*, the 'canon' and often associated with Ptolemy's fourth-century commentator Theon of Alexandria)

omitted the explanations of the models and the proofs and thus represented a further development towards the practical use of the tables, especially by astrologers. While the late eighth-century *Almagest* translation was soon superseded by new translations, the translation of the *Handy Tables* must have been heavily used by the early Islamic astronomers but became obsolete once all relevant tables had been copied into the earliest Arabic astronomical handbooks. For these reasons neither of the early translations has survived.

The Abbasid caliph al-Maʾmūn, who reigned from 813 till his death in 833, enthusiastically promoted scholarly activities by supporting translations of Greek works found in the conquered lands or obtained through contacts with the Byzantine empire. The *Almagest* was even translated twice during al-Maʾmūn's reign, first by a certain al-Ḥasan ibn Quraysh in what was later referred to as the 'old Arabic' or 'old Maʾmūnic translation'. Some traces of this translation can be recovered from later works, but overall it was considered unsatisfactory; therefore al-Maʾmūn commissioned another translation from al-Ḥajjāj ibn Yūsuf ibn Maṭar, who is also known for his translation of Euclid's *Elements*, with the assistance of Sirjis ibn Hiliyyā al-Rūmī. This translation is still extant in manuscripts preserved in Leiden (complete) and in London (only the first half).

It was soon realised that, nearly seven centuries after its composition, the *Almagest* and the *Handy Tables* no longer reproduced the celestial phenomena accurately in all cases (as a matter of fact, some of Ptolemy's parameters had been erroneous in his own time). For this reason, in the late 820s, al-Maʾmūn also founded the earliest Islamic observatories in the outskirts of Baghdad and on a hill outside Damascus. A number of well-known scholars carried out systematic observational programmes at these two sites, which provided improvements in the parameters especially for the Sun and the Moon, but also for the five planets visible to the naked eye. The activities at the two observatories ended with al-Maʾmūn's untimely death in the war with Byzantium, but several scholars who had been involved in the observations finished astronomical handbooks with tables modelled after Ptolemy's *Handy Tables*.

These handbooks became known as *zīj*es (pronounce: *zeech*, Arabic plural *azyāj* or *zījāt*). This word has long been understood as signifying the warps of a loom because of their similarity with a tabular grid, but it has recently been argued that it is a translation of the Sanskrit word *tantra* that was used to mean an astronomical treatise in the Indian tradition (i.e., without tables) before the word *siddhānta* became commonly used. A *zīj* typically consists of a large set of tables for all astronomical functions needed by the practising astronomer or astrologer, together with a set of instructions for the use of these tables, which were sometimes assembled at the beginning of the work and sometimes interspersed with the tables. Already in the 1950s the grand old man of the history of Islamic mathematical astronomy, Edward S. Kennedy (1912–2009),

identified more than 100 Arabic and Persian *zīj*es from the late eighth to the fifteenth century, around half of which have survived in manuscripts up to our time. Later work by David A. King, Julio Samsó, Bernard R. Goldstein and others extended this number to over 200 astronomical handbooks, also including *zīj*es in Hebrew. To date, however, very few of the surviving *zīj*es have been properly edited, translated into a western language or studied in detail.

The early Abbasid period was particularly fruitful in terms of the production of *zīj*es. Titles of some 60 works from between 800 and 1050 are known, but only a very small number of these still exist in manuscripts, and several only in a modified form from centuries later. The earliest extant *zīj* is the *Mumtaḥan Zīj* (*Verified Astronomical Handbook*) by Yaḥyā ibn Abī Manṣūr, one of the leading astronomers who made observations under the caliph al-Maʾmūn. Yaḥyā was of Persian descent but grew up in Baghdad, where his father had a position at the Abbasid court, and converted to Islam in 818. Besides planetary tables that are closely modelled on Ptolemy's *Handy Tables*, the *Mumtaḥan Zīj* contains eclipse materials that have been found to be of Indian/Persian origin, which was typical for this early stage of Islamic astronomy. In both surviving thirteenth-century manuscripts of this work we find later additions of several kinds, ranging from trigonometrical tables to the mean motion tables for Mars of the famous tenth-century observer Ibn al-Aʿlam. The name '*mumtaḥan*' ('verified') continued to be used both for the observations carried out under al-Maʾmūn and for several *zīj*es that resulted from them.[1]

The best-known *zīj* of Indian type was the *Sindhind Zīj* by Muḥammad ibn Mūsā al-Khwārizmī, one of the scholars in the service of the caliph al-Maʾmūn and his successors during the first half of the ninth century. Al-Khwārizmī was arguably one of the most influential scholars of his time, especially in the western Islamic world and in the Latin Middle Ages. Two of his mathematical treatises were translated into Latin and were influential in Europe. His treatise on algebra (in Arabic: *al-jabr wa-l-muqābala*) gave its name to the discipline and his treatise on arithmetic led to the coining of the word 'algorithm', derived from his personal name. Al-Khwārizmī is also associated with a reworking of Ptolemy's *Geography*, including coordinates for thousands of localities in Eurasia and Africa, which formed the basis of numerous later geographical tables in the Islamic world. Al-Khwārizmī's *zīj* was not rooted entirely in the *Sindhind* tradition, but was a mixture of Indian, Persian and Greek influences. In the eastern Islamic world it was soon superseded by the *Mumtaḥan Zīj* and others, but it was transmitted to Muslim Spain and reworked for the coordinates of Cordoba by Maslama al-Majrīṭī around the year 1000. As a result, sev-

[1] General references for this section are listed on pp. 11–12. References to manuscripts of the *zīj*es discussed here, and to the most important secondary literature on these works, are provided in the Quick reference guide for *zīj*es on pp. 521–27.

eral of his disciples and some later Andalusian scholars continued to write *zīj*es in the Indian tradition. The *Sindhind Zīj* has survived in the Latin translation by Adelard of Bath. Together with the *zīj* of al-Battānī (for which see below), al-Khwārizmī's *zīj* was highly influential in European astronomy in the form of the *Toledan Tables*.

Among the astronomers who took part in the Mumtaḥan observations was the brilliant young scholar Ḥabash al-Ḥāsib, who hailed from Marw in present-day Turkmenistan, served at least four Abbasid caliphs between 825 and 870, and is said to have died as a centenarian. As a young scholar he wrote two *zīj*es, one in the Persian and one in the Indian tradition, which are now both lost. After the death of al-Maʾmūn he made further observations and eventually finished his important Ptolemaic *zīj* around AD 870, during the period when Samarra in northern Iraq was the temporary seat of the Abbasid government. This *zīj* was later referred to as the *Arabic Zīj* because it tabulated its mean motions for the Arabic calendar, or as the *Damascene Zīj* because it was based on observations made in Damascus. Again the planetary tables follow Ptolemy's *Handy Tables* closely, but in other parts the Indian influence can be clearly recognised. Ḥabash displayed his technical skills in particular in a recursive process for solving a problem equivalent to the Kepler equation for finding the lunar parallax. Like the *Mumtaḥan Zīj*, Ḥabash's *zīj* is extant in two manuscripts; one of these is closer to the original work, whereas the other includes more later influences.

In the second half of the ninth century, a new translation of Ptolemy's *Almagest* was also prepared in Baghdad by Isḥāq ibn Ḥunayn al-ʿIbādī, of Arab-Christian descent and the son of another physician and important translator of Greek works into Arabic, Ḥunayn ibn Isḥāq. The original translation by Isḥāq was still available to Ibn al-Ṣalāḥ in the twelfth century but is now thought to be lost. The most widely disseminated version of the *Almagest* became the revision of Isḥāq's translation made by Thābit ibn Qurra (*c.* 830–901), a member of the Sabian religious sect whose family came from Harran (now just across the Turkish border from Syria). Thābit translated scientific works both from the Syriac and from the Greek and also contributed original treatises in mathematical fields such as geometry and number theory. Isḥāq's translation of the *Almagest* revised by Thābit is extant in around ten manuscripts, mostly copied in the western Islamic world and in several cases containing only half of what appears to have been a two-volume set. Recently, Dirk Grupe identified another version of the Arabic *Almagest*, which was associated exclusively with Thābit ibn Qurra by the thirteenth-century polymath Naṣīr al-Dīn al-Ṭūsī, director of the famous observatory in Maragha in north-western Iran. This version is partially extant in a manuscript in Jaipur and was translated into Latin by a certain ʿAbd al-Masīḥ of Winchester (four books of this translation survive in Dresden).

Roughly at the same time as Ḥabash al-Ḥāsib finished his Ptolemaic *zīj* and Thābit ibn Qurra revised the *Almagest* translation by Isḥāq ibn Ḥunayn, the Sabian Muḥammad ibn Jābir al-Battānī, a private astronomer in Raqqa in north-western Syria and the son of a well-known instrument maker from Harran, started the extensive observations for which he would become famous. He recorded these in his *Sabian Zīj* around the year 900. Al-Battānī's *zīj* is the earliest extant work that can be considered to be entirely in Ptolemy's tradition. Al-Battānī confirmed the highly accurate value 23;35° of the obliquity of the ecliptic that was also observed under al-Maʾmūn but was not yet used in Yaḥyā ibn Abī Manṣūr's *Mumtaḥan Zīj*. He calculated accurate new values for the solar eccentricity and the longitude of the solar apogee from observations of the lengths of the seasons. He took most of the planetary equations, the planetary latitudes and stations and the tables for parallax directly from Ptolemy, and included around half of the fixed stars from Ptolemy's star catalogue in the *Almagest* in his own star table with an increase in the ecliptic longitudes of 11;10°.

From bio-bibliographical works and quotations from later astronomers, we know the titles of many further *zīj*es written in the ninth and tenth centuries or the names of their authors. In some cases the results of observations by these astronomers were quoted, in others technical details of the methods they described in their *zīj*es were presented. Ibn al-Aʿlam, who was in the service of the Buyid emir ʿAḍud al-Dawla (r. 949–983), was another astronomer famous for his observations and is said to have provided the best tables for the positions of Mars from all *zīj*es available at the time. In fact, his mean motions of Mars were included in the extant late recensions of Yaḥyā ibn Abī Manṣūr's *Mumtaḥan Zīj*, and several other tables and parameters have survived in other works, but no complete copy of his *ʿAḍudī Zīj* exists. Most of the other *zīj*es written in this period are also lost, most probably because they were surpassed by later ones that were more authoritative, more complete, more up-to-date or simply more user-friendly.

Between the years 970 and 1030 three of the most magnificent *zīj*es in the history of Islamic astronomy were written at places very far from each other. In Baghdad, Abū l-Wafāʾ al-Būzjānī, who lived from 940 to 997/8, wrote several works on mathematics as well as a *magnum opus* called *al-Majisṭī* that aimed to update Ptolemy's *Almagest* with the latest developments in Islamic mathematical astronomy, especially in trigonometry and spherical astronomy. The sole surviving manuscript of this work in Paris presents dozens of methods for calculating all kinds of arcs on the celestial sphere, supplemented with numerical examples that are consistently carried out to four sexagesimal places. Unfortunately, all tables are omitted from this manuscript, but some of them have been identified by the present author in the late thirteenth-century *zīj* by Ibn Maḥfūẓ al-Baghdādī; furthermore, in this book it will be shown that some

more tables of Abū l-Wafāʾ were included in the *zīj* of al-Bīrūnī (see below). The chapters on the planetary models and the corresponding tables are missing from the Paris manuscript and would constitute a major find for the history of Islamic astronomy if they were to be located in hitherto unidentified manuscripts. Occasional mentions of Abū l-Wafāʾs planetary parameters in later sources do not allow us to reliably restore the mathematical characteristics of his models.

In Cairo, the astronomer Ibn Yūnus was in the service of the Fatimid caliph al-Ḥākim. Shortly before he died in AD 1009, Ibn Yūnus finished his major work, the *Ḥākimī Zīj*. Only two quarters of this work are extant in manuscripts in Leiden and Oxford (with a total of nearly 700 pages), but they give a clear picture of the author's impressive scholarship. Particularly important is Ibn Yūnus's extensive discussion of observations and tables of his Islamic predecessors. He starts with the observations made under al-Maʾmūn, but is also the major source of information for astronomers such as the Bānū Āmājūr (*c.* AD 900), a father and son with their freed slave who made observations in Baghdad and Shiraz over the course of many years. Although Ibn Yūnus copied part of the planetary equations from the *Handy Tables*, he also computed several tables anew on the basis of the results of his own observations. The accuracy of the tables in the *Ḥākimī Zīj* is already impressive, but Ibn Yūnus furthermore compiled separate tables of the sine and the solar declination with values to four sexagesimal places on intervals of single minutes. Together with the already somewhat outdated *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr, Ibn Yūnus was the main source for the work of many astronomers in Egypt and Yemen over the course of the following three centuries.

A third important astronomer from around the year 1000 was al-Bīrūnī, who was born in the Khwarazm region, just south of the Aral Sea, in AD 973. Due to the political turmoil of the time he worked for various patrons in different places and was forced to spend the second half of his life in the service of the Ghaznavid rulers Maḥmūd and Masʿūd in present-day Afghanistan. This allowed him to travel through large parts of India (including present-day Pakistan) and to become familiar with its culture and science. Al-Bīrūnī was a polymath who wrote important works on topics ranging from mineralogy, pharmacology and meteorology, through chronology and geography, to mathematics, astronomy and astrology. In his younger years he corresponded with Abū l-Wafāʾ in order to make simultaneous observations of a lunar eclipse in AD 997, which allowed the difference in geographical longitude between Khwarazm and Baghdad to be determined. It thus seems possible that he received a copy of *al-Majisṭī* directly from the author; he repeatedly quoted it in his own work and, as we will see in this book, he copied several tables from it into his *zīj*. In the 990s, al-Bīrūni also visited Rayy, near modern-day Tehran, where he met the senior scholar al-Khujandī and must have admired the

first large-scale Islamic masonry instrument—the sextant that was named after the Buyid ruler Fakhr al-Dawla (d. 997).

Al-Bīrūnī finished his impressive *zīj* very late in his life, when he was nearly 60 years old. *Al-Qānūn al-Masʿūdī* (*The Masʿūdic Canon*), as it was called, surpasses Ptolemy's *Almagest* in detail and thoroughness. Al-Bīrūnī presents long lists of the results of observations of his Greek and Islamic predecessors and assesses them carefully in order to determine his own planetary parameters on the basis of the most reliable historical observations and his own new ones. He describes the proofs for the computation of his tables in detail, and adds many additional types of tables that were not found in the earliest Islamic *zīj*es. However, like his earlier contemporaries Abū l-Wafāʾ and Ibn Yūnus, he stayed firmly within the Ptolemaic tradition: he appears to have computed many of the planetary equations anew, but on the basis of Ptolemy's original parameters. *Al-Qānūn al-Masʿūdī* is extant in approximately 15 complete copies—more than any other *zīj* up to the second half of the thirteenth century.

It was against the background sketched above that Kūshyār ibn Labbān started his career as a mathematician and astronomer. His *nisba* al-Gīlānī makes clear that he, or his family, hailed from the province of Gīlān on the south-western coast of the Caspian Sea. Born most probably in the 960s, the earliest traces of Kūshyār's astronomical activities can be dated to the 990s. He himself mentions that he made an observation of a Saturn-Mars conjunction in AD 993, and al-Bīrūnī relates that Kūshyār was present in Rayy when he visited al-Khujandī. At that time Kūshyār must have been somewhat more experienced than the young aspiring scholar al-Bīrūnī, but probably had not yet acquired the title 'master' (*kiyā*) by which he would later be referred. We may assume that Kūshyār spent most of the rest of his life in the central northern part of Iran. He calls Rayy, together with Jurjān and Ṭabaristān (both on the south-eastern coast of the Caspian Sea), 'our regions'. He also sets up the mean motion tables in his *zīj* for the meridian of Jurjān (100 km north-east of modern-day Gorgan) with the convenient longitude of 90°, although this does not necessarily indicate that he lived and worked in this city. The latest date associated with Kūshyār's life is the year 1025, when he finished an autograph copy of his *zīj*. A reference by his student al-Nasawī indicates that he had died by the year 1047/8.

Although Kūshyār's mother tongue may have been Gilaki, the Persian dialect of the region Gilan, and though he was undoubtedly fluent in Persian, he wrote his scientific works in Arabic. He authored one of the earliest surviving treatises on decimal arithmetic and also a highly popular introduction to astrology that is extant in more than 50 manuscripts found in libraries all over the world. Furthermore, he wrote a treatise on the astrolabe that likewise survives in many copies as well as two astronomical handbooks with tables. In the

introduction of his astrological work, Kūshyār refers to these as the *Bāligh Zīj* (*Extensive Zīj*) and the *Jāmiʿ Zīj* (*Comprehensive Zīj*). Mohammad Bagheri, who has made extensive studies of all aspects of Kūshyār's life and work over the last 25 years, was able to study and copy the fragment of the *Bāligh Zīj* found in a manuscript in Mumbai. It became clear that this work was written in the Indian *Sindhind* tradition, since it deals with world years and great conjunctions. It was thus one of the last *Sindhind zīj*es written in the eastern part of the Islamic world.

On the other hand, the *Jāmiʿ Zīj*, which consists of four books or treatises (in Arabic: *maqālāt*), is written entirely in the Ptolemaic tradition. Unlike most other extant works, the *Jāmiʿ Zīj* provides not just a practical part with tables for all operations needed by the practising astronomer or astrologer, accompanied by instructions for their use, but also a theoretical part. Book I (the instructions) and Book II (the tables) make up the practical part (*al-juzʾ al-ʿamalī*), and Book III (on models) and Book IV (on proofs) the theoretical part (*al-juzʾ al-ʿilmī*). The fact that Book III and its final unnumbered chapters on sizes and distances and on technical terminology were of independent interest is clear from a number of manuscripts that include only these parts of the work. Kūshyār's explanations of the models and the proofs for his calculations are certainly not as thorough as those made by his three great contemporaries; however, the number of surviving copies of his *zīj* suggests that it provided an accessible alternative to those highly academic works. Significant parts of the *zīj* are extant in as many as 15 manuscripts—more than any other *zīj* before the mid-thirteenth century except for al-Bīrūnī's *al-Qānūn al-Masʿūdī*.

Explicit references, both in the text and in the tables, show that the *Jāmiʿ Zīj* was heavily influenced by al-Battānī's *zīj* for Raqqa. However, as we will see in this book, although Kūshyār stayed firmly within the Ptolemaic tradition, he was definitely no mere copyist: he introduced changes in the layout of the tables for the planetary equations that became common in the following centuries. Similarly, his systematic use of 'displacements', a technique that simplifies the calculation of planetary positions by removing the need to decide whether the planetary equations should be added or subtracted depending on other variables of the planetary models, was commonly applied in later *zīj*es. As Glen Van Brummelen showed in 1998, Kūshyār also attempted a technical innovation with a new type of interpolation for calculating the lunar and planetary equations of anomaly as a substitute for Ptolemy's type of interpolation introduced in the *Almagest*. Although we will see in this book that this method was based on a commonly applied principle for computing new tables from existing similar ones, its accuracy was insufficient especially for the planets Mars and Venus and so it was adopted by very few later authors of *zīj*es. As we will see, Kūshyār was also the earliest Islamic astronomer to compute tables for finding the time of true conjunctions and oppositions and for the lunar distance from the Earth, which are both used in the calculation of eclipses.

## Bibliography

For more information on the Islamic astronomers introduced in the overview above, see in particular the relevant articles in *The Encyclopaedia of Islam. New Edition* (*EI*² in the general bibliography at the end of this book), the *Dictionary of Scientific Biography* (*DSB*), the *Biographical Encyclopedia of Astronomers* (*BEA*), and *The Encyclopaedia of Islam. THREE* (*EI*³). For full details of Kūshyār's life and works, see Section I.2 of this book, and for earlier descriptions and analyses of his astronomical tables, see the references in the commentary in Part IV.

The tradition of Ptolemy's *Almagest* in Arabic as well as an important part of the Latin tradition have been sorted out on the basis of bio-bibliographical sources and manuscripts in several major publications by Paul Kunitzsch that are listed below. This includes the traces that can be found, in various later works, of *Almagest* versions that are now lost, in particular the Syriac translation and the 'old Arabic translation', which were still available in the twelfth century. For the study of the *Almagest* the edition of the Greek text by Heiberg, the German translation by Manitius, the more recent English translation by Gerald Toomer and the technical studies by Olaf Pedersen and Otto Neugebauer (all included below) are likewise indispensable. Editions of the Arabic translations by al-Ḥajjāj and Isḥāq/Thābit and the Latin translation by Gerard of Cremona are currently being prepared by my colleagues Pouyan Rezvani and Colette Dufossé under the auspices of the Ptolemaeus Arabus et Latinus project.

On *zīj*es (Islamic astronomical handbooks with tables) in general, see: Kennedy, *A Survey of Islamic Astronomical Tables*; numerous studies by Kennedy and his colleagues and students in King and Kennedy, *Studies in the Islamic Exact Sciences*; King and Samsó, 'Astronomical Handbooks and Tables'; the *EI*² article 'Zīdj̲'; Mercier, 'From Tantra to Zīj'. For details of the *zīj*es mentioned above and some others frequently referred to in this book, see the Quick reference guide on pp. 521–27. For observational activities by Islamic astronomers, see Sayılı, *The Observatory in Islam* and the recent work by Mohammad Mozaffari.

On Ptolemy's *Almagest* in Greek, see: Heiberg, *Syntaxis mathematica*; Manitius, *Des Claudius Ptolemäus Handbuch*; Toomer, *Ptolemy's Almagest*; Rome, *Commentaires de Pappus et de Théon*; Pedersen, *A Survey of the Almagest*; Neugebauer, *HAMA*; Britton, *Models and Precision*; Van Brummelen, *Mathematical Tables*.

On Ptolemy's *Handy Tables*, see: Heiberg, *Opera astronomica minora*, pp. 157–85; van der Waerden, 'Die Handlichen Tafeln'; Stahlman, *The Astronomical Tables*; Neugebauer, *HAMA*; Tihon, *Le "Petit Commentaire" de Théon*; Mogenet and Tihon, *Le «Grand Commentaire» de Théon I;* Tihon, *Le «Grand Commentaire» de Théon II/III*; Tihon, *Le «Grand Commentaire» de Théon IV*; Tihon, *Les* Tables Faciles *1a*; Mercier, *Ptolemy's Handy Tables 1b*; Lempire, *Le commentaire astronomique*.

On the Arabic tradition of Ptolemy's *Almagest*, see: Kunitzsch, *Der Almagest*; Kunitzsch, *Ibn aṣ-Ṣalāḥ*; Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog*; Grupe, 'The "Thābit Version"'; Grupe, *The Latin Reception of Arabic Astronomy*; Thomann, 'The Oldest Translation'.

On Indian and Sasanian-Persian astronomy and their influence on Islamic astronomy, see: Kennedy, 'The Sasanian Astronomical Handbook'; Kennedy and van der

Waerden, 'The World-Year'; Kennedy et al., *The Book of the Reasons*; several articles by David Pingree reprinted in Pingree and Steele, *Pathways into the Study*; the *EI²* article 'Sindhind'. See further the references for al-Khwārizmī's *Sindhind Zīj* in the Quick reference guide on p. 521.

Others: Articles 'Anwā'' in *EI²* and *EI³* and Kunitzsch, *Untersuchungen zur Sternnomenklatur* (for pre-Islamic Arabic astronomical lore); Kennedy, 'Parallax Theory', esp. pp. 51–52 (on Ḥabash's iterative algorithm for solving the Kepler equation).

## I.1.1. Editing Kūshyār's *Jāmiʿ Zīj*

Books I and IV of Kūshyār's *Jāmiʿ Zīj* were edited and translated by Mohammad Bagheri in his doctoral dissertation at Utrecht University (2006) and were later published in Frankfurt am Main (2009). An edition of Book III by Hanif Ghalandari of Tehran University is currently underway. A thorough treatment of Book II was thus the only remaining desideratum towards a full study of the *Jāmiʿ Zīj*. The primary purpose of the present book is therefore to provide a critical edition of the tables with their accompanying texts (the paratexts) on the basis of the eight extant manuscripts that include all or most of the tables from Book II, as well as a historical and technical commentary. Of the eight manuscripts, only one contains no other part of the *Jāmiʿ Zīj* (and omits several tables as well). Four manuscripts include only the instructions from Book I besides the tables. Three further manuscripts include all four books, whereas the only Judaeo-Arabic manuscript omits Book III. Of the seven manuscripts that include Book I besides Book II, three have significant gaps in the instructions.

Like Mohammad Bagheri in his edition of Books I and IV, I have attempted to reproduce the tables in the *Jāmiʿ Zīj* in what was most probably their original form. During my work on the edition it became increasingly clear that this was a more difficult undertaking than it appeared at first sight. The complete manuscript of the *zīj* from the collection of the Fatih mosque in the Süleymaniye Library in Istanbul was the most plausible candidate for this original version, because it is the earliest surviving witness of the tables, generally has a relatively small number of scribal errors, and it lacks several modifications and improvements that Kūshyār most likely made himself during his later work on the *zīj*.

The other seven manuscripts contain not just what may be considered scribal variations of the tables in the Fatih manuscript, but also independent versions of several of the tables. This includes an extensive sine table with values also for minutes of arc rather than only for integer degrees; tables for the oblique ascension and the equation of daylight for a slightly different geographical latitude; a star table with coordinates for 20 additional stars; and, finally, numerous inconspicuous but consistent differences in the mean motion tables, which have mathematical but no astronomical significance and make clear that

all these tables were newly computed at some point. Although the various characteristics of the tables allow us to divide the eight manuscripts into three or four groups, a detailed study of the group that most likely represents the latest developments in Kūshyār's work on the *Jāmiʿ Zīj* shows that their copyists also had one or more manuscripts from the earlier groups at hand, and every now and then adopted elements from them. Thus, the secondary goal of my edition became to sort out these different mathematical characteristics and to outline the relations between the three or four groups of manuscripts as accurately as possible.

In accordance with the goal mentioned above, in my edition of the tables I have generally accepted the values from the Fatih manuscript whenever these did not contain obvious scribal or other types of errors and whenever the other manuscripts did not contain obviously better readings. In these decisions I also bore in mind the mathematical correctness of the tabular values, that is, whether the values in the manuscripts were close to recomputed values or whether the errors that they contain fitted in the overall error pattern of the table. (More details of my editorial criteria are presented in the introduction to the edition in Part II of this book.)

Once digital versions of the tables in all extant manuscripts are available, it would be an easy matter to let a computer program produce both a critical edition on the basis of selection criteria for the values to be accepted in each case, and a full apparatus with the variants found in all other witnesses. However, rather than giving all deviating values separately, I have grouped together the ones that can be seen to have a common cause. This includes cases in which an entire column of values or digits was miscopied in the wrong place, and especially so-called 'slides'. While copying a column of values, scribes frequently skipped certain values or repeated them; as a result, a whole range of values would be copied some rows too high or too low before the mistake was discovered, often only towards the end of the column. In these cases I have inserted only a single entry in the apparatus of the table, indicating where the slides took place and how many rows (or, in some cases, columns) were involved. The places where these slides occurred are marked in the tables by means of special symbols, and additional variants in the same values are indicated in the apparatus by a plus sign to remind the reader that they are also subject to a variant involving multiple tabular values. Similarly, I have used 'general variants', given at the beginning of the apparatus of each table, to summarise the systematic differences between the versions of many subtables of mean motion tables in the *Jāmiʿ Zīj*. In nearly every case, as many as a third of the values in these subtables are one unit smaller or larger in one group of manuscripts than in the others. These general variants are indicated in the apparatus by an asterisk, and the values that they relate to are marked with a small asterisk in the table.

For repetitive textual elements I have followed a similar approach. Thus, in Part III of this book, I first provide 'general editions' of elements such as the

Persian, Syrian and Arabic month names and the titles and column headers of types of tables that appear a number of times in the *Jāmiʿ Zīj*, namely the mean motions and the planetary equations. As a result, all variants in the 116 witnesses for the titles and column headers of the mean motion tables and in the even larger number of witnesses for the Persian month names are assembled in a single place in the edition. In the text edition I have included explanatory texts to the tables (sometimes written between the columns of the tables, sometimes in the margins) which appear to have become part of the manuscript tradition of Kūshyār's *zīj*, or which are of particular technical or historical interest. On the other hand, I have generally omitted marginal notes in later hands.

In order to make the tables more accessible and to give the reader an impression of the types of techniques that Kūshyār used for computing his tables, of the innovations that he applied, of the parameter values that he incorporated, as well as of the earlier sources that he used and the later works that he influenced, I provide an extensive commentary for each table in Part IV of this book. The reader is referred to the available literature for general descriptions of the planetary models and other theoretical ideas underlying Ptolemaic astronomy, but aspects that are typical of Kūshyār and the way in which he worked will be discussed here in detail. Besides the innovations already mentioned above, we will also obtain a clearer picture of the working practices of an Islamic astronomer around the year 1000. We will see that Kūshyār is likely to have been engaged in an observational programme of some significance and may have adjusted some of his tables, especially those for Mars, on the basis of the results. We will also see how he performed calculations up to an accuracy of six sexagesimal fractional places (corresponding to eleven decimal places) in order to convert al-Battānī's mean motion parameters for the Syrian (i.e., Julian) calendar to his own for the Persian calendar. Rather than simply copying the tables for the planetary equations, latitudes, and stations from Ptolemy or al-Battānī, we will find multiple indications that Kūshyār smoothened the tables of his predecessors by adjusting the tabular values in such a way that the second-order tabular differences became almost constant. The commentary will thus clearly show how, in computing his astronomical tables, Kūshyār on the one hand stayed firmly within the Ptolemaic tradition as handed down in the *Almagest* and *Handy Tables*, but on the other hand made improvements of several kinds, both in the tabular values themselves and in the layout of the tables, which were adopted by later astronomers and to some extent became the standard for later *zīj*es.

## I.2. Kūshyār ibn Labbān: life and works

Abū l-Ḥasan Kūshyār ibn Labbān ibn Bāshahrī al-Jīlī, sometimes given the title *kiyā* (Persian for 'king', 'lord' or 'master'), hailed from the province Gilan on the south-western edge of the Caspian Sea.[2]

More or less definite dates in Kūshyār's lifetime can be deduced from a number of sources. First, Kūshyār is not included in the *Fihrist*, the bio-bibliographical work prepared by Ibn al-Nadīm around AD 987 and supplemented until AD 995. This may indicate that Kūshyār became well-known only after that time.[3] Two of the five manuscripts of the *Jāmiʿ Zīj* at the Dār al-kutub in Cairo include at the end of Book I a quotation from an autograph by Kūshyār in which he states that he observed and recomputed a Saturn-Mars conjunction on Thursday, 6 July 993.[4] In his book on spherical trigonometry *Maqālīd ʿilm al-hayʾa* (*Keys of the Science of Astronomy*), al-Bīrūnī (973–*c.* 1050) relates how he met al-Khujandī (d. *c.* 1000) and Kūshyār in Rayy near modern-day Tehran,

[2] Most of what we know about Kūshyār has been summarised in the introductory chapters of the editions of two of his most important works, namely Yano, *Kūšyār Ibn Labbān's Introduction*, and Bagheri, *az-Zīj al-Jāmiʿ*. Additional information, especially from Iranian sources, can be found in Bagheri, 'Mabḥath-i taqwīm', pp. 21–24. See further (in chronological order of appearance): Steinschneider, *Die hebraeischen Uebersetzungen*, pp. 565–66 (§ 352); Suter, *Die Mathematiker und Astronomen*, pp. 83–84 (no. 192); Suter, 'Nachträge und Berichtigungen', p. 168; Ghorbani, *Riyāḍīnānān-i Irānī*, Chapter 14, pp. 169–94; the *DSB* article by Saidan (outdated on Kūshyār's astronomical works); Ullmann, *Die Natur- und Geheimwissenschaften*, pp. 334–35; Sezgin, *GAS*, vol. V, pp. 343–45 and 404; vol. VI, pp. 246–49 and 294; vol. VII, pp. 182–83, and vol. XIII, pp. 277–79; Matvievskaya and Rozenfeld, *Matematiki i astronomy*, vol. II, pp. 216–17 (no. 192); the *EI²* article 'Kūshiyār b. Labān' by Jaouiche; Ghorbani, *Zindagīnāmah-yi rīyāḍīdānān*, 2nd ed., pp. 414–20 (no. 134); the *ENWC* article by Michio Yano; the *EIr* article 'Gušyār Gilānī' by Pingree; Rosenfeld and İhsanoğlu, *Mathematicians, Astronomers, and other Scholars*, pp. 118–19 (no. 308); and the *BEA* article by Bagheri. Abdullazade, *Kushyar Dzhili* (in Russian) discusses Kūshyār's life and his treatment of various mathematical and astronomical topics. On Kūshyār's works and their extant manuscripts, see also: Brockelmann, *Geschichte der arabischen Litteratur*, vol. I, pp. 222–23 (2nd ed. pp. 252–53, English transl. *History of the Arabic Written Tradition*, p. 220) and Brockelmann, *Geschichte der arabischen Litteratur. Supplementbände*, vol. II, pp. 397–98; Krause, 'Stambuler Handschriften', pp. 472–73 (no. 192); Storey, *Persian Literature*, pp. 42–43 (no. 77); King, *A Survey of the Scientific Manuscripts*, pp. 45–46 (B70) and 232 (Plate XII); and Monzavi, *Fihristvara*, pp. 2852–53 and 2957.

[3] Note, for comparison, that Ibn al-Nadīm includes Abū l-Wafāʾ (940–997/8) and Abū Sahl Wayjan al-Qūhī (fl. 969–989), but not al-Khujandī (fl. 994, d. *c.* 1000), al-Sijzī (fl. 969/70, d. *c.* 1020) and Abū l-Wafāʾ's student Abū Naṣr Ibn ʿIrāq (fl. from *c.* 1000 onwards, probably died between 1027 and 1036). See Flügel, *Kitâb al-Fihrist* and Dodge, *The Fihrist*.

[4] This quotation appears in Cairo, Dār al-kutub, *mīqāt* 691, fol. 24v (see Plate 2b) and in Cairo, Dār al-kutub, *mīqāt* 400, fol. 41r–v (fol. 42r–v according to Bagheri's count). It was first noticed and translated into English in Bagheri, 'Kūshyār ibn Labbān's Glossary', p. 145. This report will be further discussed in Section IV.8.3.

most probably in the last decade of the tenth century.[5] Section I.8.10 of the *Jāmiʿ Zīj* presents the days of the heliacal risings of the lunar mansions in the Syrian year 1320 Alexander, which corresponds to AD 1009. The Leiden manuscript of the *zīj* indicates the Syrian year 1321 Alexander (AD 1010) next to the Lent table (see Plate 3). In the month Bahman of 393 Yazdigird (January 1025) Kūshyār wrote in Suhruj (near Bastam in north-eastern Iran) the original from which the Alexandria manuscript of Books III and IV of his *Jāmiʿ Zīj* was copied,[6] and in 416 Yazdigird (1047/48) his student Abū l-Ḥasan ʿAlī ibn Aḥmad al-Nasawī (Rayy, 11th c.) refers to him in terms that make it clear that he was no longer alive.

Some other sources and data give less definite indications. The seventeenth-century *Taʾrīkh-i Māzandarān* reports an astrological prediction of the death of the Ziyarid ruler Wushmgīr that Kūshyār seems to have made in Jurjān in Muḥarram 357 (December 967/January 968).[7] Bagheri notes that this date is too early with regard to dates that can be reliably associated with Kūshyār, and surmises that he was instead in the service of Wushmgīr's son Qābūs, who reigned over Ṭabaristān and Jurjān during two periods between 978 and 1012.

The star table in the *Jamiʿ Zīj* displays coordinates for the beginning of the year 301 Yazdigird (AD 932/3), whereas a set of apogee longitudes in some of the manuscripts of the *zīj* are for 331 Yazdigird (AD 962/3). However, none of these dates need to be very strongly tied to the date of compilation of the work (the epochs of planetary and stellar positions in *zīj*es may generally at least be assumed to precede the date of compilation by at most the number of years of the cycle used for the mean motions). The star coordinates in Kūshyār's astrological handbook *al-Madkhal fī ṣināʿat aḥkām al-nujūm* (see below) are for the beginning of the year 361 Yazdigird (AD 992/3), which, as we can see from the other dated information related to Kūshyār, almost certainly fell within his period of activity. The longitude 24° Gemini of the solar apogee underlying the table for the equation of time in the *Jāmiʿ Zīj* was reached in AD 997 according to Kūshyār's (and al-Battānī's) parameters.[8]

[5] See Debarnot, *Al-Bīrūnī. Kitāb Maqālīd*, pp. 100/1 and 102/3; see also ibid., pp. 142/3 and 144/5, and Berggren, 'Spherical Trigonometry', pp. 16–17. Kūshyār himself mentions Rayy together with Jurjān and Ṭabaristān as 'our regions' (*diyārnā*) in Chapter I.1.2 of the *Jāmiʿ Zīj*; see Bagheri, *az-Zīj al-Jāmiʿ*, p. 9 (translation) and Arabic p. 10. In Chapter I.8.8 of the *zīj*, he reproduces a calculation of the *qibla* at Rayy; see ibid., p. 107 (translation), p. 113 (commentary) and Arabic p. 70.

[6] See Bagheri, 'Mabḥath-i taqwīm', p. 23.

[7] Bagheri, *az-Zīj al-Jāmiʿ*, p. xiv. The historical city Jurjān was situated at the location of present-day Gonbad-e Gabus, *c.* 100 km north-east of the modern-day Gorgan, which was called Astārābād in medieval times.

[8] van Dalen, *Ancient and Mediaeval Astronomical Tables*, p. 140.

Taqizadeh was the first to draw attention to an example for year transfers in Section I.7.5 of the *Jāmiʿ Zīj*, which places the beginning of the transfers in the year 332 Yazdigird (AD 963/4) and then finds the transfer in 389 Yazdigird (AD 1020/1), i.e., the 58th year reckoned from this beginning.[9] Ghorbani followed Taqizadeh in interpreting 332 Yazdigird as the approximate time at which Kūshyār wrote the *Jāmiʿ Zīj*.[10] However, Bagheri's suggestion that 332 Yazdigird might in fact refer to Kūshyār's year of birth, and that 389 Yazdigird might be the time at which he wrote this particular section of the *Jāmiʿ Zīj* at the age of 57, is plausible and agrees with the other known dates about his life and work.[11]

Kūshyār left a relatively small number of works, all written in Arabic, but some of them were important and highly popular. The *Kitāb fī Uṣūl ḥisāb al-Hind* (*Book on the Principles of Hindu Reckoning*) is one of the oldest surviving works in Arabic on arithmetic by means of Hindu numerals.[12] The *Kitāb al-Asṭurlāb* (*Book on the Astrolabe*) is extant in a dozen Arabic manuscripts and in one manuscript of a medieval Persian translation.[13] Kūshyār's highly popular handbook of astrology, known as *al-Madkhal fī ṣināʿat aḥkām al-nujūm* (*Introduction to the Art of Astrology*) or *Mujmal al-uṣūl fī aḥkām al-nujūm* (*Summary of the Principles of Astrology*), is extant in dozens of manuscripts scattered in

[9] Taqizadeh, *Gāhshumārī*, p. 227. The section concerned is found in Bagheri, *az-Zīj al-Jāmiʿ*, pp. 97 (translation), 101–02 (commentary) and Arabic p. 63. It appears in manuscripts **F** (fol. 30v), **H** (fol. 19v), **C** (fol. 26v), **Y** (fols 252v–253r), **B** (p. 27), and Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1 (fol. 22r). $\mathbf{C_2}$ lacks the chapter concerned and **L** omits the example from its Chapter 71. All sigla are introduced in Section I.4.

[10] Ghorbani, *Riyāḍīnānān-i Irānī*, p. 169 (with a typo 322 for 332 but the correct corresponding Hijra year); Ghorbani, *Zindagīnāmah-yi rīyāḍīdānān*, 2nd ed., p. 414.

[11] Bagheri, *az-Zīj al-Jāmiʿ*, p. xiii.

[12] This work is extant in Arabic in Istanbul, Süleymaniye Kütüphanesi, Ayasofya 4857/7, fols 267v–282v, and also in a Hebrew translation with commentary by Shālôm ben Joseph ʿAnābī. The Arabic text was edited in Saidan, 'Risālatān fī l-ḥisāb al-ʿarabī' and translated into English in Levey and Petruck, *Kūshyār ibn Labbān*. *Principles of Hindu Reckoning*. Of the earlier Arabic treatises on arithmetic, the one by al-Khwārizmī (*c.* 825) is only extant in Latin, whereas the one by al-Uqlīdīsī (*c.* 950) was identified and later on edited and translated into English by Saidan; see his 'The Earliest Extant Arabic Arithmetic', *al-Fuṣūl fī l-ḥisāb*, and *The Arithmetic of al-Uqlīdisī*.

[13] The Arabic text was edited and translated into Japanese by Taro Mimura in his bachelor thesis at Kyoto Sangyo University and was published in Bagheri and Mimura, *Risālah-yi Usṭurlāb*, pp. 1–53 (Japanese) and pp. 111–69 (Arabic). The medieval Persian translation was edited in Bagheri, 'Tarjima-yi fārisī-i kuhan', which was reprinted in Bagheri and Mimura, *Risālah-yi Usṭurlāb* together with a facsimile of the unique Tashkent manuscript.

libraries all over the world and was translated into Persian, Turkish and Chinese. The Arabic text was edited and translated into English by Michio Yano.[14]

In the first paragraph of his *al-Madkhal fī ṣināʿat aḥkām al-nujūm*, as well as in Section III.20 of this work, Kūshyār mentions that he had written two astronomical handbooks with tables, namely the *Jāmiʿ Zīj* and the *Bāligh Zīj*.[15] This is confirmed by Shams al-munajjim al-Wābkanawī (Shirwan in north-western Iran, *c.* 1300), who mentions a large number of earlier *zīj*es in the introduction of his *Muḥaqqaq Zīj*.[16] Since the early nineteenth century the *Jāmiʿ Zīj* has been known to be extant in several western libraries, most importantly in Leiden and Berlin, but the status of the *Bāligh Zīj* remained unclear until the 1990s. Kennedy suggested that some obvious differences between the manuscripts of the *Jāmiʿ Zīj* (e.g., the different numbering of the chapters in Books I and IV, different sine tables, and the use of slightly different geographical latitudes) might be due to the fact that one set of manuscripts in fact contains the *Bāligh Zīj*.[17] However, when Mohammad Bagheri consulted the manuscript Mumbai, K. R. Cama Oriental Institute, R I.86 (previously Mulla Firuz Library, Astr. 86), which, according to the 1873 catalogue by Rehatsek,[18] contains several works by Kūshyār, on fols 148v–151r he found a section entitled *Fī istiʿmāl adwār al-kawākib ʿalā madhhab al-hind min Zīj al-Bāligh li-Kūshyār* (*On the Use of Planetary Cycles according to the Method of the Indians, from the Bāligh Zīj by Kūshyār*) followed by four pages of corre-

[14] Yano, *Kūšyār Ibn Labbān's Introduction*, which also includes an edition of the Chinese translation prepared on the orders of Hong Wu, the first emperor of the Ming dynasty, in AD 1383. Chapter III.21 on *tasyīr*s ('prorogations') was also edited and translated into English in Yano and Viladrich, 'Tasyīr Computation' with elucidations of the computations taking into account descriptions by al-Battānī and al-Bīrūnī. Although the *Madkhal* mimics Ptolemy's *Tetrabiblos* in its division into four books and also follows it partially in terms of its contents, the largest part is a treatment of typical Islamic astrological topics such as world periods and Saturn-Jupiter conjunctions, for which Kūshyār most likely made use of the work of his Islamic precursors such as Abū Maʿshar and Māshāʾallāh. The consistent use of the astrological houses is another feature not yet found in Greek astrology. All in all, Kūshyār's *Madkhal* contains very few original contributions, but is a clear and systematic representation of all relevant topics in Islamic astrology.

[15] See Yano, *Kūšyār Ibn Labbān's Introduction*, pp. 6/7: *wa-qad taqaddama la-nā fī dhālika kitābān sammaynāhumā al-Zīj al-Jāmiʿ wa-l-Bāligh*, and pp. 216/217: *wa-qad bayyannā kayfiyyat istikhrājihi* (referring to the ascendant of a year transfer) *fī l-Zījayn al-Jāmiʿ wa-l-Bāligh*. Some modern authors have read '*al-Zīj al-Jāmiʿ wa-l-Bāligh*' as the name of a single *zīj*.

[16] Istanbul, Süleymaniye Kütüphanesi, Ayasofya 2694, fol. 3r, line 11. Also the bio-bibliographers al-Bayhaqī (1106–1174) and Ḥājjī Khalīfa (1609–1657, quoting *al-Madkhal fī ṣināʿat aḥkām al-nujūm*) mention Kūshyār as the author of two different *zīj*es; see Shafīʿ, *Tatimma*, pp. 83–84 (no. 43); Meyerhof, 'ʿAlī al-Bayhaqī's Tatimmat', pp. 157–58 (no. 43); Flügel, *Lexicon bibliographicum*, vol. III, pp. 563–64; and Yaltkaya and Bilge, *Kashf al-ẓunūn*, col. 968.

[17] Kennedy, 'A Survey of Islamic Astronomical Tables', p. 125, items 7 and 9.

[18] Rehatsek, *Catalogue Raisonné*, pp. 43–44.

sponding tables.[19] The text gives a brief description of the cycle of world days starting with a great conjunction of all planets, apogees and nodes at the end of Pisces and then, for each of the heavenly bodies, explains the calculation of the periods of return, of daily mean motions, and of mean longitudes at several points in time. It also explains how to calculate the number of conjunctions between two heavenly bodies during one world cycle and the amount of time and number of degrees between two consecutive conjunctions. The first two pages of tables give for all planets, apogees and nodes, as well as for Regulus, the number of rotations during a world cycle, the daily mean motion, and the mean longitudes at the Era of the Flood (18 February 3102 BC) and the Yazdigird epoch. The last two pages of tables display a list of mean Saturn-Jupiter conjunctions divided according to triplicity into four groups of 12 or 13 conjunctions each, giving both the time (to seconds of an hour) and the ecliptic degree (to seconds of a degree) at which the conjunctions take place. This table starts with the conjunction of AD 571 (watery triplicity) and ends with that of AD 1544 (airy triplicity). The text explicitly mentions the common Indian sidereal year of $365^{d}6;12,9^{h}$. All further numerical data in the text as well as in the tables are also in full agreement with those given by al-Bīrūnī in his *India* for al-Fazārī's and Yaʿqūb ibn Ṭāriq's translation of the *Sindhind*. It thus seems clear that Kūshyār's *Bāligh Zīj* was one of the so-called *Sindhind zīj*es that were written in parallel to the Ptolemaic *zīj*es by a number of Islamic scholars during the ninth, tenth and eleventh centuries.[20]

[19] See Bagheri, *Three Treatises*, pp. 42–43 for a facsimile of the two pages of text and Bagheri, 'A Chapter from Kūshyār's Lost Zīj' for an edition and explanation of the text. See further the entry on the *Bāligh Zīj* in my forthcoming updated survey of Islamic astronomical handbooks, which will also describe the four pages of tables in more detail based on a handwritten copy made by Mohammad Bagheri.

[20] cf. pp. 3, 6–7, 11–12 and 521.

## I.3. The *Jāmiʿ Zīj*

Kūshyār ibn Labbān's *al-Zīj al-Jamiʿ* ('*Comprehensive Zīj*') is a typical Islamic astronomical handbook consisting of explanatory text and a large set of tables.[21] Like all of Kūshyār's works, it was written in Arabic, but a Persian translation of the instructions for using the tables was prepared in AD 1090/1 by Muḥammad ibn ʿUmar ibn Abī Ṭālib Tabrīzī at the request of a certain Jaʿfar ibn Ayyāz.[22] Unlike most other Islamic *zīj*es, the textual part of the *Jāmiʿ Zīj* consists not only of instructions (in Book I) for using the tables (in Book II) and carrying out some further basic computations that practising astronomers or astrologers need, but also includes extensive explanations of the cosmological models underlying the tables (in Book III) and geometrical proofs for the calculations (in Book IV). Kūshyār based himself on the planetary parameters of the *Ṣābiʾ Zīj* by the famous observer al-Battānī (Raqqa in Syria, *c.* 900), but introduced several types of more or less successful innovations. In comparison with the thorough scholarly approach found in the *zīj*es of his older contemporaries Abū l-Wafāʾ (Baghdad, 940–997/8) and Ibn Yūnus (Cairo, d. 1009), as well as with the encyclopaedic character of *al-Qānūn al-Masʿūdī* by his later contemporary al-Bīrūnī (Ghazna and several other places, 973–*c.* 1050), Kūshyār's *Jāmiʿ Zīj* is less rigorously formulated and clearly intended for a wider audience with a less solid knowledge of the basics of mathematics and astronomy. The popularity of the work is demonstrated not only by the existence of the Persian translation, but also by the more than a dozen extant Arabic manuscripts of the work, a similar number to those of al-Bīrūnī's *al-Qānūn*, but far more than the single surviving incomplete manuscript of Abū l-Wafāʾ's *al-Majisṭī* and the two partial manuscripts of Ibn Yūnus's *Ḥākimī Zīj*.

We have already seen that the example for year transfers in Section I.7.5 of the *Jāmiʿ Zīj* suggests that Kūshyār worked on this part of the *zīj* in AD 1020/1. Furthermore, the Alexandria manuscript is stated to have been copied from an autograph by Kūshyār finished in the month Bahman of the Yazdigird year 393 (January 1025). We may thus assume that Kūshyār was actively working on the *zīj* between 1020 and 1025. On the other hand, he stated that he observed and recomputed the Saturn-Mars conjunction of AD 993, and in the Leiden manuscript of the *zīj* the Lent table marks the Seleucid year 1321 Alexander, corresponding to AD 1009/10. Al-Bayhaqī acknowledges that Kūshyār compiled tables for the correction of the true position of Mars, which are found in five

[21] For a description of a *zīj* and an overview of the most important early Islamic *zīj*es, see pp. 3–11 and 521–27.

[22] See Bagheri, 'The Persian Version'. For further references, see the description of the unique Leiden manuscript of this translation on pp. 32–33.

of the eight extant manuscripts that contain the tables.[23] It thus seems plausible that Kūshyār was already working on the *Jāmiʿ Zīj* several decades before he completed the autograph version of AD 1025.

In fact, my critical edition of the tables from the *Jāmiʿ Zīj* will show that they were distributed in three or even four somewhat different versions. This phenomenon has received relatively little attention in the modern literature, but does not appear to have been uncommon among medieval scientific works in both Arabic and Latin. Ibn al-Nadīm notes that several astronomers, in particular al-Khwārizmī (Baghdad, *c.* 825), Yaḥyā ibn Abī Manṣūr (Baghdad, *c.* 830) and al-Battānī (Raqqa, *c.* 900) wrote their *zīj* in two versions (*nuskha*).[24] The present author noticed that several sections attributed to al-Battānī in the Leipzig manuscript of the *Mumtaḥan Zīj* by Yaḥyā ibn Abī Manṣūr are not included in the *Ṣābiʾ Zīj* as published by Nallino and might hence stem from a different version of his *zīj*.[25] José Bellver details the modifications that Jābir ibn Aflaḥ (Seville, early 12th c.) made in the course of several revisions of his *al-Kitāb fī l-Hayʾa* (more generally known as the *Iṣlāḥ al-Majisṭī*).[26] A good example of astronomical work in progress can also be found in the fifth book of Levi ben Gerson's major work *The War of the Lords* (in Hebrew). Although Levi indicated 1328 as the year of completion of this book, he continued to add the results of observations made until 1340 (included in only a small number of the surviving manuscripts) and, on the basis of these results, adjusted his planetary models and added alternative versions of tables already present.[27] Finally, Henry Bate testifies that he continued to update his *Tables of Mechelen* after preparing a first version before 1280.[28]

In the case of Kūshyār's *Jāmiʿ Zīj* the obvious differences between some of the surviving manuscripts (in Books I and IV, most strikingly, the different arrangement of sections and chapters, in Book II the occurrence of a standard sine table with values for 1, 2, 3, ..., 90° besides a much more extensive one and the use of geographical latitude 36° as opposed to 35;30°) made Kennedy and

[23] See Shafīʿ, *Tatimma*, p. 84; Meyerhof, ʻʿAlī al-Bayhaqī's Tatimmat', p. 158; the edition of Table 30b, and Section IV.8.2 of the commentary.

[24] Flügel, *Kitâb al-Fihrist*, vol. I, pp. 274, 275 and 279; Dodge, *The Fihrist*, vol. II, pp. 652, 653 and 661, and Suter, 'Das Mathematiker-Verzeichniss', pp. 29 and 35. For al-Battānī, Ibn al-Nadīm states explicitly: 'he had ⟨the following⟩ books: the *zīj* in two copies, the first and the second, and the second is better than the first' (*wa lahu min al-kutub: kitāb al-zīj wa-huwa nuskhatān ūlā wa-thāniya wa-l-thāniya ajwad min al-ūlā*, my translation).

[25] See Nallino, *al-Battānī sive Albatenii* (Arabic text in vol. III, Latin translation in vol. I) and van Dalen, 'A Second Manuscript', pp. 16, 18, 20, 23, 28 and 30–33.

[26] Bellver, 'The Arabic Versions', pp. 188–198.

[27] Goldstein, *The Astronomical Tables*, e.g., pp. 12 and 30 (item 28) and Goldstein, 'A New Set', e.g., p. 386 (sentences 17–21). I am grateful to Paul Hullmeine for drawing my attention to this example.

[28] Steel et al., *The Astrological Autobiography*, p. 46.

others hypothesise that some of these manuscripts are in fact of the *Bāligh Zīj* rather than the *Jāmiʿ*. This possibility has now been ruled out by Bagheri's and my own inspection of the six pages from this work extant in Mumbai (see pp. 18–19). In Part I of this book, I will analyse in detail the differences between the tables in the eight surviving manuscripts of the *Jāmiʿ Zīj* that include Book II and will claim that many of them are most likely due to reworkings by Kūshyār himself. In his edition of Books I and IV, Mohammad Bagheri also concluded the existence of at least two versions of the *Jāmiʿ Zīj* on the basis of the different subdivisions of these books and several differences in the mathematical proofs.[29]

Because of the ease of access to the manuscripts in European libraries, the *Jāmiʿ Zīj* received attention from modern scholars from the first half of the nineteenth century onwards.[30] Ideler reproduced and translated into German the sections on the numbers of days between the well-known eras from Chapter I.1.1 and on the history of the Persian calendar from Chapter I.1.2.[31] In his monumental five-volume work on the history of geography, Lelewel edited and transliterated the geographical table from the *Jāmiʿ Zīj* (here Table 54) and used it for his analysis of the development of Islamic geographical knowledge.[32] Wiedemann translated the introduction of the *zīj* as it is included in the catalogue entry for the Leiden manuscript into German.[33] The Arabic text of Chapter III.32 of the *Jāmiʿ Zīj*, also copied separately under the title *Fī Maqādīr al-abʿād wa-l-ajrām* 'On the Magnitudes of the Distances and ⟨sizes of celestial⟩ Bodies', was published by the Osmania Oriental Publications Bureau in Hyderabad in a volume with other contemporaneous treatises.[34]

After Kennedy had described the contents of the *Jāmiʿ Zīj* in his 1956 survey of Islamic *zīj*es,[35] specific sections and tables from the *zīj* were studied and published by various scholars. Saliba edited and analysed Kūshyār's Lent table (Table 7 in the present book) and Berggren studied his method for calculating the direction of Mecca in Chapter I.8.8 of the *zīj* (corresponding to Chapter I.78

[29] Bagheri, *az-Zīj al-Jāmiʿ*, pp. xxxviii–xxxix.

[30] The following survey of secondary literature on the *Jāmiʿ Zīj* is partially based on the overview in Bagheri, *az-Zīj al-Jāmiʿ*, pp. xviii–xix.

[31] Ideler, *Handbuch der mathematischen und technischen Chronologie*, vol. II, pp. 547–48 and 623–31 (with commentary on pp. 631–33).

[32] Lelewel, *Géographie du moyen âge*, Tome 1, Appendix III, pp. 178–85 (Kūshyār is introduced but, due to the incorrect date of AD 1060 attached to him, placed in an inappropriate historical context on p. xlviii of the 'Prolégomènes').

[33] Wiedemann, 'Einleitungen (Zweite Mitteilung)', p. 132.

[34] *Rasāʾiluʾl-mutafarriqa*, *risāla* 11 (19 pp.). This treatise was edited, translated into English and commented upon in Bagheri et al., 'Kūshyār ibn Labbān Gīlānī's Treatise'.

[35] Kennedy, 'A Survey of Islamic Astronomical Tables', pp. 125 and 156–57.

in the Leiden manuscript and Chapter I.82 in the Berlin manuscript).[36] In the 1987 Festschrift for Professor Kennedy, Berggren translated, summarised, and commented upon the entire Section IV.3 of the *Jāmiʿ Zīj*, which deals with spherical trigonometry.[37] In 1988 Kennedy published an analysis and recomputation of the table for the equation of time (here Table 15), which was supplemented by the present author some years later.[38] In various case studies in my doctoral dissertation I also considered the parameters and accuracy of several of the spherical-astronomical tables from the *Jāmiʿ Zīj* when comparing them with tables in other works, especially the *zīj* of Ibn Maḥfūẓ al-Baghdādī.[39] In his unpublished master thesis, Toshiaki Kashino, a student of Michio Yano at Kyoto Sangyo University, edited the sections on finding the longitude of the Sun, the Moon and the five planets from all four books of the *Jāmiʿ Zīj*.[40] Under the auspices of of a DAAD research project carried out at the Institute for History of Science in Frankfurt am Main, Glen Van Brummelen analysed Kūshyār's tables for the planetary equations and identified the use of a peculiar type of Ptolemaic interpolation that has not been found in any other *zīj*es.[41]

Around 1990 Mohammad Bagheri, like Kūshyār a native of the Iranian province of Gilan, started his extensive investigations of all of Kūshyār's works and the collection of copies of manuscripts of these works. His activities culminated in the publication of a full edition with English translation and extensive commentary of Books I and IV of the *Jāmiʿ Zīj*.[42] Bagheri also provided a description of the Persian translation of Book I by Muḥammad ibn ʿUmar ibn ʿAlī Ṭālib Tabrīzī (AD 1090/1), which is extant in the manuscript Leiden, Universiteitsbibliotheek, Or. 523/1.[43] An edition of Book III is currently being

[36] Saliba, 'Easter Computation', pp. 197–98 and 209, and Berggren, 'The Origins of al-Bīrūnī's Method', pp. 7–8.

[37] Berggren, 'Spherical Trigonometry'.

[38] Kennedy, 'Two Medieval Approaches', pp. 2–4, and van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 134–41. In reconstructing the computation of the equation of time, I also noted properties of the table of the right ascension in the Fatih manuscript (ibid., p. 137; here Table 45) and of the solar equation (ibid., pp. 137–38; here Table 16).

[39] van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 172–73, 175–76, 178 and 187. More information will be given in the commentary to the tables concerned.

[40] Kashino, *Planetary Theory*. This includes a critical edition and English translation of the explanatory texts and marginal notes to Table 12 and of Section II.56 on finding the original planetary equations. Furthermore, Kashino transcribed Tables 12–36 (without 15) from the Fatih manuscript.

[41] Van Brummelen, 'Mathematical Methods'.

[42] Bagheri, *az-Zīj al-Jāmiʿ*, based on the doctoral dissertation Bagheri, *Books I and IV* defended at the University of Utrecht in 2006. Parts of this work were also published separately in Bagheri, 'Kūshyār ibn Labbān's Glossary'; Bagheri, 'Kūshyār ibn Labbān's Account'; and Bagheri, 'Mabḥath-i taqwīm'.

[43] Bagheri, 'The Persian Version'.

prepared by Hanif Ghalandari (Institute for History of Science, Tehran University) in collaboration with Mohammad Bagheri.[44]

This book intends to fill the one remaining gap in the treatment of Kūshyār's *Jāmiʿ Zīj* by providing a critical edition with translation and commentary of the 55 tables in Book II, together with the table of contents preceding the tables and the section on finding the original planetary equations from Kūshyār's displaced equations that follows the tables in three of the manuscripts. In order to make the tables accessible to as wide an audience as possible, I present the critical edition in Part II of this book with tabular values transliterated in Arabic-European numerals and with translated titles and column headers.[45] A critical edition of all Arabic textual elements in and around the tables (including titles and headers, instructions written between columns or in the margins, marginal notes that have become part of the manuscript tradition, etc.) can be found in Part III. A commentary with explanations of aspects of the tables that have not yet been treated in the literature is included in Part IV. The book is completed by the bibliography, a glossary of technical terminology, and indexes of subjects, parameters, historical dates, historical persons, historical works, manuscripts, modern persons, and projects and institutions.

[44] See already Ghalandari, 'Maqāla-yi siwum'.

[45] Note that in the sources most of the numbers in the tables are in the Arabic alphabetical (*abjad*) notation. Numbers that are given in Hindu numerals include only the arguments of the subtables for collected years in three chronological tables and in some of the manuscripts tangent values and their differences when they are larger than 60; see also pp. 76–77.

## I.4. The manuscripts

I have used the following eight manuscripts for the critical edition of the tables in Book II of Kūshyār's *Jāmiʿ Zīj* and their textual elements:

**B** = Berlin, Staatsbibliothek Preußischer Kulturbesitz, Or. quart. 101/1

*References*: Ahlwardt, *Verzeichniss der arabischen Handschriften*, vol. V, pp. 203–06 (no. 5751, with a complete table of contents) and p. 149 (no. 5663). https://ismi.mpiwg-berlin.mpg.de/codex/36938.

*Contents*: Book I (without chapters 9–69) and Book II with numerous additional tables; pp. 2–221 (of a total of 436 pp.), with Book II on pp. 35–159 and p. 220. All through Book II we find a series of sloppily written Persian marginal notes in a user's hand. See Plates 9 and 12.

*Origin*: Estimated by Ahlwardt to date from *c.* AD 1300.

This manuscript has a çarkuşe binding with battal ebru (marble) decoration and has original endbands in yellow, red and blue. The cover is heavily damaged by insects. The folios are consistently bound in quinions.[46]

The part of the manuscript containing the *Jāmiʿ Zīj* is written in a clear *naskh*. The first folio (pp. 1–2, with the text of the *zīj* starting on p. 2) was damaged and restored with new margins of much later paper and some further small pieces of paper glued onto p. 1 in order to cover a number of holes. Page 1 contains two texts by the same hand that copied the *Jāmiʿ Zīj*. The first of these, written vertically in the upper part of the page, cut off at its left side and generally barely legible, has as its heading 'On finding the true position of (the comet) Kaid' (*Fī maʿrifa taqwīm al-Kayd*). The second text, also partially illegible due to the damage to the page, explains how to find the longitude of a fixed star for any given year from the table for the year 442 Yazdigird (1073/4) at the end of 'this book' in 'this *zīj*' by adding or subtracting the precessional motion to seconds taken from a 'table of the travel of the planets (illegible) the apogees' (*jadwal masīr al-kawākib … l-awjāt*). The last few lines of this text discuss cases where the Sun is in the ascendant and one of the five planets in one of the houses and thus appears to be of an astrological nature. Apparently, the scribe took these instructional texts on rather different topics from a different *zīj*, unknown to us. Page 1 further contains an ex libris stamp 'Ex. Biblioth. Regia Berolinensi' at the bottom left and the shelfmark written out in full at the top. The text on pages 2 to 9 shows mostly interlinear and partially marginal notes in Latin by a user (translating, especially, all names

[46] I am grateful to Nadine Löhr for her inspection of the manuscript and her codicological description.

and numbers). Since these notes are particularly dense in Chapters 1 and 2, they may be by Christian Ludwig Ideler, who translated these chapters into German (see p. 22 and footnote 31).

The last pages in the Berlin manuscript that belong to the actual *Jāmiʿ Zīj* are pages 158–159 (star table) and page 220 (Chapter 56 on the original equations), but the texts and tables on a variety of topics on pages 160–219 were copied together with the *zīj* by the same hand (these additional tables and texts are here listed in Section I.6.3). Page 221 contains a table displaying geographical regions for each of the zodiacal signs written by a different hand in 832 Hijra (1428/9). Pages 222–435 contain a Persian treatise on astrology in five *maqāla*s by al-Ḥasan ibn ʿAlī, known as Abū Naṣr al-munajjim al-Qummī.[47] Sezgin dates al-Qummī to the tenth century and calls this particular treatise 'a Persian translation of the base text' of the author's Arabic *al-Madkhal ilā ʿilm aḥkām al-nujūm*. At the end of the copy, which was finished in Jumādā l-ākhira 806 Hijra (December 1403), it is referred to as *Kitāb-i Madkhal dar ʿilm-i nujūm*. Since al-Qummī's treatise was copied by a somewhat sloppy *naskh* hand different from the first part of the manuscript, this date is not valid for the *Jāmiʿ Zīj*.

The Berlin manuscript has four different page or folio numberings which provide us with information relevant to our purposes. Two of these numberings (including the modern one in pencil, as also found in numerous other Arabic manuscripts of the Staatsbibliothek) are page numbers that run in parallel over the entire manuscript and hence postdate the joining of the *Jāmiʿ Zīj* with al-Qummī's *Madkhal*. On the pages that appear upside down (namely pp. 205–206 and 216) the modern page numbers appear at the top of the manuscript pages (i.e., upside down with respect to the tables), the older ones above the tables (i.e., upside down with respect to the bound volume). For pp. 205–206 this may indicate that the leaf was incorrectly bound before the modern numbering was applied; but since p. 215 appears correctly, it is possible that p. 216 was simply copied upside down.

The other two numberings in the manuscript are a folio numbering in Hindu-Arabic numerals and one in Arabic-European numerals which, in spite of a rather large number of irregularities, generally run parallel up to p. 214. These folio numbers are written in the top right corner of the verso pages, but cannot be recognised on pp. 1–14, i.e., before the huge gap in the text of Book I. They then run from fol. 34 to 51 (pp. 16–48), from fol. 139 to 147 (pp. 50–66)

[47] cf. Ullmann, *Die Natur- und Geheimwissenschaften*, pp. 332–33; Sezgin, *GAS*, vol. VII, pp. 174–75, and King, *A Survey of the Scientific Manuscripts*, p. 44 (B60). The Persian translation of al-Qummī's treatise was edited from this sole surviving copy and commented upon in Zanjānī, *Tarjama-yi al-Madkhal*.

and from fol. 52 to 136 (pp. 68–214). We may conclude that the *Jāmiʿ Zīj* as found in the Berlin manuscript was originally part of a larger manuscript (with folios numbered in Hindu-Arabic numerals as well as Arabic-European equivalents) containing at least 27 folios preceding the *Jāmiʿ Zīj*. This manuscript contained further additional tables after the current page 218 or 220, which were later placed in between the original fols 51 and 52. This is also confirmed by the clear difference in the traces of use (especially the presence of stains due to the turning over of the pages or humidity) between the current pages 48 and 67 on the one hand and the intervening nine leaves on the other. Note that three small slips of paper (pp. 17–18, 169–170, 175–176) were inserted in the manuscript at an early point: they show all four types of page or folio numbering and were in fact written by the main hand. The first of these slips contains a section omitted from the main text on p. 16; the other two slips indicate the planetary phases of Saturn, Jupiter and Mercury and thus most probably belong to Ibn al-Aʿlam's tables for the planetary equations found on the preceding pages.

**C** = Cairo, Dār al-kutub, *mīqāt* 400

*References*: King, *Fihris al-makhṭūṭāt*, vol. I, p. 62 (for all the Cairo manuscripts of the *Jāmiʿ Zīj*, see also *idem*, vol. II, pp. 104–06 (no. 2/1/7), with partial tables of contents and incipits); King, *A Survey of the Scientific Manuscripts*, pp. 45–46 (B70) and Plate XII on p. 232 (displaying Tables 13 and 14 on fols 50v–51r of this manuscript).

*Contents*: Books I and II; 85 fols (fols 1–3 bound between fols 36 and 37), with Book II on fols 42r–85r. See Plates 5 and 10.

*Origin*: Estimated by King to date from *c.* 650 Hijra (*c.* AD 1250).

Written in an elegant *naskh*; the text is generally dotted, but, as we will see, in the tables most of the dots on *nūn* are omitted (and occasionally substituted by a different hand). The chapter numbers in Book I are written with *abjad* numerals (باب آ, باب بَ, etc.). Of the original title page on fol. 1r only a small piece is extant, partially overglued with other pieces of paper. The only legible words in the title are *al-zīj al-maʿrūf | bi-zīj l...* The title page further contains several notes of possession without dates (for the names, see King's catalogue entry), a drawing of a plant, and a vague, illegible imprint of a blue seal. This is apparently the same as the seal of the Egyptian Khedival Library (the earlier name of the Dār al-kutub) on fol. 85v,[48] which further contains a partial calen-

[48] https://chesterbeatty.ie/islamic-seals-database/, seal no. 437. The seals database of the Chester Beatty Library was consulted before its redevelopment in 2019; the database was not

dar conversion table in a user's hand. The bottoms of the pages in the second half of the manuscript were damaged by woodworm or other causes and were partially cut off, occasionally making the bottom part of the tables illegible. The binding is partially loose, so that many pages are only kept together by attached strips of paper.

There are occasional additions and marginal notes to the text, and numerous calculations by several users (presumably students practising their skills) in the margins of the tables. The dates provided with these calculations are between 1000 and 1110 Hijra (AD 1591–1699), between 1072 and 1083 Yazdigird (AD 1702–1714) and, in one case, on an inlaid leaf with a recomputed solar mean motion table (fol. 49), for 1252 and 1255 Yazdigird (AD 1882–1886). To the headings of the planetary mean motion tables a user added corrections that must be applied to use the tables in Yemen. Under the table for the solar equation, more precise values for this function were written that were taken from the *zīj* of Ulugh Beg (Samarqand, *c.* AD 1440). To the geographical table a user added the coordinates for Ḥamā (halfway between Damascus and Aleppo) from al-Battānī's *Ṣābiʾ Zīj* (longitude 69;30° and latitude 35;20°).

**$C_1$** = Cairo, Dār al-kutub, *mīqāt* 188/2

*References*: King, *Fihris al-makhṭūṭāt*, vol. I, p. 53; King, *A Survey of the Scientific Manuscripts*, pp. 45–46 (B70).

*Contents*: Most of Book II (all tables except the chronological and scattered other ones, but with the section on finding the original equations); fols 11r–49v (of a total of 49 fols). See Plates 8 and 15.

*Origin*: Estimated by King to date from *c.* 1200 Hijra (*c.* AD 1800).

Written in a clear *naskh*. The folios are unnumbered (except for fol. 49r) and I have adopted the numbering implied by King. The title page (fol. 1r) mentions only the title of the treatise from which a number of tables are included on fols 1v–10v, namely the *Kitāb Nuzhat al-abṣār* by Muḥammad al-Wafāʾī, a student of Ibn Abī l-Fatḥ al-Ṣūfī (Cairo, *c.* AD 1450). This part of the manuscript, copied by an earlier hand, contains various tables for timekeeping for the latitude 27° of Asyut in central Egypt.[49] The title page further contains two undated owner's notes, one of which is repeated on fol. 10v, and the other is by the Ottoman general (*sarʿaskar*) and later Egyptian viceroy Ibrāhīm ibn

yet back online when this book went to print. The numbers given for the seals are the ones that were valid at the time.

[49] cf. King, *In Synchrony with the Heavens I*, p. 316 (with further mentions on pp. 100, 111 and 155).

Muḥammad ʿAlī (1789–1848), who owned hundreds of manuscripts now kept in the Dār al-kutub (including $\mathbf{C_2}$ below). Furthermore, the title page contains a *fāʾida* on the extraction of square roots, a note on spherical astronomy, and the same seal of the Egyptian Khedival Library also found in **C** (a vaguer impression of this seal can be recognised on fol. 49v). In this manuscript, the tables from the *Jāmiʿ Zīj* do not show any traces of use. A marginal note in the main hand on fol. 14v, introduced by *fī l-zīj al-thānī*, was apparently taken from a second copy of the *Jāmiʿ Zīj* that the scribe had at hand (see explanatory text B1 to Table 10 on p. 276).

$\mathbf{C_2}$ = Cairo, Dār al-kutub, *mīqāt* 691

*References*: King, *Fihris al-makhṭūṭāt*, vol. I, p. 120; King, *A Survey of the Scientific Manuscripts*, pp. 45–46 (B70).

*Contents*: Parts of Books I and II; 46 fols, with Book II on fols 2v and 25r–46v. See Plates 2a, 2b and 11.

*Origin*: Estimated by King to date from *c.* 700 Hijra (*c.* AD 1300).

Written in an elegant *naskh*, fully dotted. The manuscript was recently foliated by the staff of the Dār al-kutub. Both Books I and II are incomplete and the leaves of the manuscript are not in order. The correct order can be restored as follows: *Book I*: fols 4–11 (table of contents, Chapters I.1–12), fols 12–22 (Chapters I.53–78), fol. 3 (Chapters I.81–83) and fols 23–24 (Chapter I.83–end). *Book II*: fol. 2 (title page and table of contents, see Plate 2a), fols 43–44 (Tables 21–24), fols 37–42 (Tables 24–27), fols 25–36 (Tables 27–36), and fols 45–46 (Tables 36–37). Thus, from Book II the complete tables for the planetary mean motions and equations are present, as well as the tables for the mean motion of the lunar node and for the lunar latitude. The second title page (fol. 2r) gives the title *al-Maqāla al-thāniya fī l-jadāwil* in the main hand besides librarians' notes and a stamp of the Egyptian Khedival Library (repeated on fol. 46v (see Plate 11), different from the seal in **C** and $\mathbf{C_1}$). The first title page (fol. 1r) includes an owner's note by General Ibrāhīm (see above under $\mathbf{C_1}$), a library label, the inaccurate description 'splendid treatise on timekeeping' (*risāla jalīla fī l-mīqāt*) of the contents of the manuscript, which is repeated, and a circular seal with two-word inscription which is likewise repeated on fol. 46v. Folios 4–19 are rather heavily water-damaged. The text of Book I has occasional marginal corrections in the main hand. A user recomputed the intervals between the eras in the margins of Chapter I.1 (fols 5v–6r). Book I ends on fol. 24r with the important observation report of a Saturn-Mars conjunction in AD 993 based on a note in Kūshyār's own hand (cf. p. 15, footnote 4, Section IV.8.3, and Plate 2b). The tables do not show any trace of use, but there are large red ink smudges on fols 41v–42r.

**F** = Istanbul, Süleymaniye Kütüphanesi, Fatih 3418/1

*References*: *Defter-i Fatih*, p. 196; Krause, 'Stambuler Handschriften', p. 472 (no. 192); http://ktp.isam.org.tr/ktpgenel/recordlistb.php?Bolum=Fatih &Demirbas=003418.

*Contents*: Books I to IV (with part of the table of contents of Book I and Chapters I.1.1 to I.2.2 missing); fols 1v–175v (of a total of 227 fols), with Book II covering fols 37r–90r. See Plates 1 and 6.

*Origin*: Copied in 545 Hijra (AD 1150/1) in the 'house (*mskn*) of Ḥarīth' in Samarqand.

Written in a generally clear *naskh*, not fully dotted; in the colophons, marginal notes and certain other places the writing becomes hasty and undotted. Numerous marginal notes in the main hand, mostly labelled '*ḥāshiya*', sometimes '*nuskha*', in Book I; occasional marginal corrections in Books III and IV. Colophons at the end of Books I (fol. 36v), II (fol. 90r), III (fol. 131r) and IV (fol. 175v) of the *Jāmiʿ Zīj* state that the copying of these books was completed on 5 Ramaḍān, 21 Dhū l-qaʿda, 9 Ramaḍān and 18 Ramaḍān respectively of the year 545 (December 1150–March 1151). Thus it appears that the tables were copied only after the three books of text. This might also explain why only the colophon of Book II mentions the location where the copy was made. The name of the owner and copyist (*ṣāḥibuhu kātibuhu*) appears most clearly and completely on fol. 177r (under the title of the second treatise in this manuscript) as Maḥmūd ibn Aḥmad ibn al-Ḥusayn al-Muʿallimī al-Samarqandī. The first two folios of the manuscript were heavily damaged by woodworm. The title page (fol. 1r, see Plate 1) underwent heavy repair, but still shows what may have been the original title and author's name: *Kitāb al-Zīj al-Jāmiʿ | wa-huwa arbaʿ maqālāt, al-ūlā minhā wa-hiya hadhihi fī ḥisāb al-abwāb | min taʾlīf al-kiyā Abī l-Ḥasan Kūshyār ibn …* (remainder cut off). Another restored part of the title page, apparently partially glued over the previous part, contains in large writing three repetitions of the phrase *yā kabīkaj* ('Oh Asiatic crowfoot', an invocation to protect the book from damage by woodworm, etc.) and an owner's statement. The title page also carries the seal of the Ottoman sultan Mahmud I (r. 1730–1754)[50] and two others that I have not been able to identify. In the same hand as the *Jāmiʿ Zīj* preceding it, fols 177r–226v contain an incomplete copy of Kūshyār's *al-Madkhal fī ṣināʿat aḥkām al-nujūm* that breaks off in the middle of Chapter III.21 (just before the end of § 9 in Yano's edition). Fol. 227r–v contains in a different *naskh* hand a worked example in Persian of spherical-astronomical/astrological calculations for a solar

[50] https://chesterbeatty.ie/islamic-seals-database/, seal no. 124.

longitude of $10^s 28;10°$ (but without indication of a date). Fol. 227v carries the pointed oval seal of the Ottoman sultan Bayezid II (r. 1481–1512).[51]

**H** = Ahuan Islamic Art, MS 40 (Judaeo-Arabic)

*References*: Langermann, 'Arabic Writings', p. 151. Vinograd et al., *Catalogue of Rare and Antique Hebrew Books*, p. 48 (no. 68). https://www.nli.org.il/he/manuscripts/NNL_ALEPH000182675/NLI. http://www.textmanuscripts.com/medieval/astronomy-hebrew-60473.

*Contents*: Books I, II and IV (with Chapters I.24–56 missing); fols 1v–82r and 85v–118v (of a total of 128 fols), with Book II on fols 24r–82r. Book I consists of 85 *bāb*s and Book IV of 66 *bāb*s without the larger subdivisions into *fuṣūl* found in most other manuscripts of the *Jāmiʿ Zīj*. Book II is complete, but the tabular differences were omitted from the tables of lunar and planetary equations. See Plates 14 and 16.

*Origin/provenance*: Copied at Sanʿā, Yemen in 1499 (colophon on f. 119r), based on a copy of an autograph. Modern binding with Hebrew imprint. Ex libris stamp of Yaḥyā al-Qāfiḥ (in Arabic and in Hebrew script) dated '13…' (illegible). The manuscript was auctioned by the Jewish Treasures Auction House in Zürich and by Les Enluminures before it became part of the collection of Ahuan Islamic Art.

Besides the *Jāmiʿ Zīj*, this Judaeo-Arabic manuscript contains part of an unidentified treatise on algebra (fols 120v–125v) and some further small fragments (with a drawing of a scale on fol. 83v). The chapter numbers in Books I and IV were for the most part omitted; only for the first chapters of Book I were they written, in Arabic *abjad* notation in red. For further chapters of Book I they were supplemented in the margin in red *abjad* numerals in Arabic and later on in Hebrew script. The manuscript is heavily damaged towards the end, especially at the top of the pages, which was repaired with strips of modern paper. Several pages in other parts of the manuscript are worn and partially very difficult to read. Twentieth-century post-its with calculations in Hebrew script are attached to the pages containing the mean motion tables for the Sun and the Moon.

This manuscript was previously MS 15 in the collection of Rabbi Yosef Qāfiḥ, who spent his youth in Yemen, became the leading scholar of the Yemenite community in Jerusalem, and died there in 1999. The National Library of Israel has a black-and-white microfilm of the manuscript; photocopies that were made from this film were barely legible and certainly could not

[51] https://chesterbeatty.ie/islamic-seals-database/, seal no. 2.

be used for the edition. After Qāfiḥ's death the manuscript was auctioned and changed owner several times before it became part of a collection of instruments and manuscripts of Ahuan Islamic Art. I am very grateful to David Sulzberger for making the manuscript available to me and for allowing me to scan it. All variants found in this manuscript, to which I refer by the siglum **H** for 'Hebrew (characters)', were added to my edition of tables and text in the final stage of the work on this book. It turned out that the copies of the tables in this manuscript are close to **F** and in general have the fewest errors of all eight witnesses. Since two notes (fol. 82v (see Plate 16) and fol. 119r) mention that it was made from a copy of an autograph by Kūshyār, it is of particular importance for judging whether the changes found in the tables can be attributed to Kūshyār himself.[52]

**L** = Leiden, Universiteitsbibliotheek, Or. 8

*References*: De Jong and de Goeje, *Catalogus Codicum Orientalium*, pp. 84–86 (no. 1054, with a transcription of the introduction of Book I and a rough table of contents); Voorhoeve, *Handlist of Arabic Manuscripts*, p. 405 (basic information listed under the work title); Witkam, *Inventory of the Oriental Manuscripts*, vol. I, p. 18.

*Contents*: Books I to IV; fols 1r–124r (of a total of 144 fols, fol. 40r only contains an empty frame), with Book II covering fols 21r–82r and numerous additional tables after Book IV on fols 124v–144r. See Plates 3 and 4.

*Origin*: Copied in Muḥarram 634 (September/October 1236).

Written in a clear, occasionally elegant *naskh*, frequently undotted. The diagrams in Books III and IV are neatly drawn, but the tables in Book II are drawn in an unprofessional manner. The numerous grammatical errors in the titles of the tables suggest a scribe of non-Arab origin. From fol. 55 onwards all leaves show water damage, mostly at the bottom. The title page (fol. 1r) includes the title of the work (*Kitāb al-Zīj al-Jāmiʿ*) and the name of the author (*allafahu Kūshyār ibn Labbān ibn Bāshahrī al-Jīlī raḥmat Allāh ʿalayhi*) in the same hand as the main text. Furthermore, statements of change of ownership include (from top to bottom) the dates and places 920 ⟨Hijra⟩ (1514/5) in Bāb al-sulṭān (Aleppo?), Dhū l-ḥijja 892 (November/December 1487) in Constanti-

[52] Langermann, 'Arabic Writings', p. 151 mentions three further extant Judaeo-Arabic copies of the *Jāmiʿ Zīj*. These contain Book III as well as the treatise on bodies and distances and the introductory chapter with a glossary of technical terms that were sometimes appended to the 30 chapters of Book III but also circulated separately from the *zīj*. The four manuscripts together thus preserve exactly the entire *Jāmiʿ Zīj* in Hebrew characters with some lacunae.

nople, Rajab 664 (April/May 1266) in Irbil, and Rabīʿ al-awwal 702 (October/November 1302). The date of completion of the revision of this copy is included in the colophon at the end of Book IV on fol. 124r. Fols 124v–144r contain numerous tables and some text passages in the same hand as the *Jāmiʿ Zīj*. With some exceptions (namely, copies of the tables for the right ascension and the equation of daylight for 36° as found in **F** and of the geographical table) these do not belong to the *Jāmiʿ Zīj* itself, but cover topics related to those dealt with in the *zīj*. A complete list of these additional tables, which also include several from the lost *Fākhir Zīj* by Kūshyār's student al-Nasawī, can be found below in Section I.6.2.

**Y** = Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3

*References*: *Yeni Cami Kütüphanesinde*, p. 41; Krause, 'Stambuler Handschriften', p. 472 (no. 192); http://ktp.isam.org.tr/ktpgenel/recordlistb.php?Bolum=Yeni Cami&Demirbas=000784.

*Contents*: Books I to IV (Chapters IV.1–9 and IV.34–43 missing); fols 230r–362r (of a total of 376 fols), with Book II on fols 257r–311r followed by some additional tables. See Plates 7 and 13.

*Origin*: Estimated by Krause to date from the twelfth century AD.

Written in an elegant *naskh*, generally dotted. This important manuscript contains three other works:

1) Fols 1v–69r: the *Kitāb al-Qaḍāʾ ʿalā l-mawālīd* (*Book of Judgments on Nativities*, also referred to as *Pentateuch*) by Dorotheus of Sidon (Dhūruthiyūs al-Miṣrī, 1st c. AD), translated into Arabic by ʿUmar ibn Farrukhān al-Ṭabarī (Baghdad, *c.* AD 800). The only other known copy of this work is Berlin, SBPK, Or. oct. 2663. See Sezgin, *GAS*, vol. VII, pp. 32–38 and the edition of the Arabic text with English translation in Pingree, *Dorothei Sidonii Carmen astrologicum*.
2) Fols 69v–229v: one of the only two extant copies of the Ptolemaic *zīj* by Ḥabash al-Ḥasib (Damascus/Samarra, *c.* 870) referred to as the *Damascene Zīj* (since it was based on observations made at the observatory founded by the caliph al-Maʾmūn in Damascus) or the *Arabic Zīj* (since it uses the Arabic calendar for its planetary mean motion tables). The other copy of this *zīj*, which differs significantly from this one, is MS Berlin, SBPK, Wetzstein I 90 (Ahlwardt no. 5750). See p. 522 and Debarnot, 'The *Zīj* of Ḥabash al-Ḥāsib'.

4) Fols 362v–376v: a copy of Kūshyār's *Kitāb al-Asṭurlāb*.

All four works were copied in the same clear *naskh*, generally dotted. None of the four colophons give a date of copying and none of the four treatises

were annotated in any way (with the exception of two marginal notes to Book III of the *Jāmiʿ Zīj*). Many of the pages were worm-eaten and have been repaired with slips of paper. The heavily damaged and restored title page (fol. 1r) calls the volume *Kitāb al-Durar al-muthammana* (*Book of the Precious Pearls*, followed by an illegible line) and sums up the contents as 'the *zījes* of Ḥabash and Kūshyār, astrology and many other useful topics' (... *wa-fīhi aḥkām wa-ashyā kathīra wa-ghayr dhalika min al-fawāʾid al-ḥasana wa-l-farāʾid al-mustaḥsanat*). It includes a user note with the date 919 Hijra (AD 1513/4) and a seal of the *waqf* of the Ottoman sultan Ahmed III Khān Ghāzī (r. 1703–1730) accompanied by the date 1137 Hijra (AD 1724/5). The same seal is found on fol. 376v.

Bagheri listed these other manuscripts of the *Jāmiʿ Zīj* that he used for his edition and translation of Books I and IV:[53]

- Alexandria, al-Maktaba al-baladiyya, 4285 *jīm* [Books III and IV, 73 fols, copied in 566 Hijra (AD 1170/1) from an autograph dated Bahman 393 Yazdigird (January 1025)].
- Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1 [Book I, fols 1v–26r (of a total of 30 fols), copied in 1169 Hijra (AD 1755/6)].
- Cairo, Dār al-kutub, *riyāḍa* Talʿat 102/3 [Book IV, fols 49v–88v (of a total of 126 fols), copied in 1128 Hijra (AD 1715/6)].
- Istanbul, Süleymaniye Kütüphanesi, Bağdatlı Vehbi 893 [Book IV, 76 fols, copied from an autograph in 427 Hijra (AD 1035/6)].
- Moscow, Rossiyskaya Gosudarstvennaya Biblioteka (Russian State Library), Fond 179, MS No. 154/1 [Book III (30 chapters plus the glossary and the chapter on sizes and distances, interrupted shortly after the beginning of Chapter III.17 by Chapters I.1–19 of Kūshyār's *al-Madkhal fī ṣināʿat aḥkām al-nujūm*) and Book IV (in 8 *fuṣūl*), fols 36v–46v and 54r–111r (of a total of 217 fols), copied in 525 Hijra (AD 1130/1)].
- Birmingham, Cadbury Research Library, Mingana 1496 [Gottschalk et al., *Catalogue of the Mingana Collection*, vol. IV, p. 356; identified as part of the *Jāmiʿ Zīj* by Sezgin (collection of microfilms of the Institute for the History of Arabic-Islamic Science in Frankfurt am Main). Book III, Chapters 9, 10, 22, 30 and 31, fols 1r-14r (of a total of 48 ff.), copied in the nineteenth century and using a modern notation for fractions in the other parts of the manuscript].

[53] Bagheri, *az-Zīj al-Jāmiʿ*, p. xxxvii.

Further manuscripts known to Mohammad Bagheri or me include only separate copies of Chapters III.31 and III.32 or other small fragments. The Persian translation of Book I of the *Jāmiʿ Zīj* is extant in:

- Leiden, Universiteitsbibliotheek, Or. 523/1 [de Jong and de Goeje, *Catalogus Codicum Orientalium*, pp. 87–88 (no. 1056); Witkam, *Inventory of the Oriental Manuscripts*, vol. I, p. 230. Book I; fols 1v-26v (of a total of 73 fols),[54] copied in 689 Hijra (AD 1290/1)]. The Persian translation of the *Jāmiʿ Zīj* was briefly described in Bagheri, 'The Persian Version', while Section I.1 on chronology was edited in Bagheri, 'Mabḥath-i taqwīm', pp. 30–43. A number of tables have been appended to the text of Book I and at the end of the manuscript, but none of these stem from Book II of the *Jāmiʿ Zīj*. They include:

  1) A table of *tasyīr*s (prorogations) and *intihāʾ*s (terminal points) for world-years, with columns for the Era of the Flood and the year 796 Yazdigird (1426/7), but only the first row of values filled in (fols 26v–27r).
  2) Numbers of days of Jalālī years and months, in groups of 28 collected years, 1 to 28 extended years, and the twelve Persian months of 30 days (fol. 71r). Like all known tables of this type, it is in fact calculated for the Julian calendar, with the insertion of a leap day every fourth year.[55]
  3) *Notae* of the months in the Byzantine, Arabic and Persian calendars (fol. 71v). The *notae* are given for Seleucid years 1100, 1200, ..., 1900 and for Hijra and Yazdigird years 100, 200, ..., 900, with additive values for tens of years, single years and months.
  4) Table for finding the Christian Lent (fol. 71v, written sideways under the *notae* table). This small table gives the date of the beginning of Lent as a function of the position in the 19-year lunar cycle (vertically) and the weekday of an unspecified date (horizontally). All except two Lent dates in the columns for the years 7, 10 and 18 in the cycle are indicated in red and are hence to be taken in Adhār (March).

[54] Both de Jong & de Goeje and Witkam state that the manuscript is in complete disorder and provide the correct order of the folios. However, Lameer, 'Dū zīj-i fārsī', which describes in particular the second treatise in the manuscript, found the leaves of the actual codex to be in the correct order, which suggests that in the meantime they had been rebound. The scans kindly provided by the library also show the manuscript in the correct order. I follow the correct folio numbering given in pencil in the bottom margin of the recto pages.

[55] cf. van Dalen, 'The Malikī Calendar'.

5) Hours of the distance of the Sun (fol. 72r). This table gives the time in hours, minutes and seconds needed by the Sun to traverse distances of 1, 2, 3, …, 60 minutes of arc (vertically) as a function of the daily true solar motion (57, 58, …, 61′, horizontally).

6) Oblique ascension for latitude 38°, with values to seconds and tabular differences (fols 72v–73r).

7) Table for finding the day of the Christian Lent, different from Kūshyār's Table 7 (fol. 73v). Although the format of the table is the same as Kūshyār's, this is a copy of al-Bīrūnī's *Chronicon*; cf. Saliba, 'Easter Computation', pp. 188–189 and 200.

8) Second half of an oblique ascension table for latitude 36;30° with values to minutes and tabular differences (fol. 74r).

9) Tables for nativities during the day and during the night, both giving years, months, days and hours in subtables for pairs of partner (*mushārik*) planets, as well as for the two lunar nodes (fols 74v–75r).

10) Table for finding the lunar mansions as a function of the lunar longitude (fol. 75v).

## I.5. The tables

Book II of the *Jāmiʿ Zīj* consists of a table of contents, around 50 folios of tables, and a chapter on finding the original planetary equations from Kūshyār's displaced ones. For the sake of consistency I will use the numbering of the tables from 1 to 55 found in the table of contents of **F**, which is also used in Bagheri, *az-Zīj al-Jāmiʿ*. The table of contents in all manuscripts (edited in Part III, pp. 259–63) names each table as well as the chapter on finding the original equations (in **F** numbered 56, in **CC**$_2$ 44) a *nawʿ*, literally 'type' or 'kind'. Only in the incomplete manuscript **C**$_2$ do the tables themselves have a consistent numbering in *abjad*, which corresponds to the table of contents in this manuscript (cf. Plate 2a). In **C** also, *abjad* numbers are found above several of the tables (others appear to have been cut off in the process of restoration), but these do not correspond exactly to the table of contents in **C** but rather to the one in **C**$_2$ (on this, see the remarks in the caption to Table A below and the section on differences in the table numbering on p. 59).

The practical disadvantage of using the numbering from **F** is that the lunar and planetary equations are thus indicated by a single number, making it inconvenient to distinguish the first and second equation, interpolation minutes, and variation at nearest and furthest distance. Nevertheless, I preferred to use the simplest way to indicate variations of a table, namely by appending small letters to their number, for designating variants of tables that are so different from the main table that they need to be edited separately. The most obvious examples of this are the two versions of the sine table (Tables 8 and 8a), the variations in the mean motions and equations of Mars (Tables 28–30 vs. 28a–30b), the spherical-astronomical tables based on different geographical latitudes (36° vs. 35;30°, Tables 46 and 48 vs. 46a and 48a) and the two different star tables (Tables 55 and 55a). Table A below displays the titles, the numbers in the respective tables of contents and the locations in the manuscripts of all tables belonging to Kūshyār's main set.

Table A: Comparative table of the contents of Book II of Kūshyār's *Jamiʿ Zīj*

The first two columns display the table number in the edition (which follows manuscript **F**) and the tabulated function. Then for each of the eight manuscripts the number of the table in the table of contents (missing only from **C**$_1$) and the first folio or page number of the table are given (if the table occupies non-consecutive page or folio numbers due to disorder or a re-organisation of the manuscript, the entire range of folio or page numbers is displayed). An asterisk indicates that a tabular frame is present but does not contain any tabular values (although in several cases the title and column headers).

*Remarks*: In **C** the red table numbers in the table of contents appear to have been inserted particularly carelessly after the black table headings were copied. Thus number

17 appears twice, numbers for the tangent of the solar declination and the prorogations were omitted, and then numbers 40–42 were skipped to compensate for these three mistakes. Without these, the numbering would have been identical to that in **C**$_2$ (and hence with **YB** up to Table 16). The correct table numbers are indicated in *abjad* at the top of the pages, but many of these were cut off, probably when the manuscript was restored. Fol. 49 in **C** is an inlaid leaf with a recomputation of the solar mean motion table in an elegant hand. Fol. 85v is filled with a chronological table with lines drawn without a ruler in a rather sloppy hand.

In **C**$_2$ the *abjad* numbers of the tables are also correctly indicated at the top of the pages on which they begin. Note, however, that the folios containing the tables are not in order, so that the corresponding page numbers show several jumps. In **C**$_1$ the table of contents is not included and no numbers are written above the tables either, so that no numbering for the tables can be known for certain. However, since *abjad* 44 is written above the chapter on the original equations and the three Cairo manuscripts are generally very close, we may assume that **C**$_1$ followed the same table numbering as **CC**$_2$.

The table of contents in **Y** is given in tabular form. The actual order of the chronological tables in this manuscript is different from the order in the table of contents.

The differences in the numbering of the tables in the eight manuscripts are mostly due to the following irregularities:

- the taking together under a single number of the mean motion tables for the Moon and each of the five planets in **CC**$_2$;
- the taking together under a single number of the mean motion tables *plus* the equations for the Moon and each of the five planets in **L**;
- the assignment of a single number to all tables for planetary latitudes and stations in **L**; and
- the insertion in the table of contents of a universal table for the equation of daylight in **F** and **L** (although the tabular frame has been left empty in **F** and the table is not included at all in **L**).

The fact that the numberings of the tables in **C** and **C**$_2$ deviate from each other in part of the table of contents is due to the following three irregularities in the generally rather sloppy copy **C**:

- the repetition of no. 17 for the lunar nodes in **C** (causing a difference of 1 with respect to **C**$_2$ for all of the planetary tables);
- the omission of a number for the tangent of the first declination in **C** (raising the difference with respect to **C**$_2$ up to 2);
- the omission of a number for the table of prorogations in **C** (raising the difference with respect to **C**$_2$ up to 3, which is restored by skipping from 39 for the geographical table straight to 43 for the star table);

Overall, especially in **CL** it seems quite clear that the numbers were inserted in red in a highly arbitrary way after the titles of the tables had been written in black. In general, since the numbering of the tables is not included with the tables themselves except in the Cairo manuscripts, it is probable that it does not stem originally from Kūshyār but was later added by copyists.

| | *Section or table in the edition* | **F** (fols) | **H** (fols) | **C** (fols) | **$C_1$** (fols) | **$C_2$** (fols) | **Y** (fols) | **L** (fols) | **B** (pp.) |
|---|---|---|---|---|---|---|---|---|---|
| | Introduction and table of contents | 37v | 24r | 42v | – | 2v | 257r | 21r | 35 |
| | **Chronological tables** | | | | | | | | |
| 1 | Days of Syrian years and months | 1 38r | 1 25v | 1 43r | – | 1 – | 1 257v | 1 21v | 1 36 |
| 2 | Days of Arabic years and months | 2 38v | 3 27r | 3 44r | – | 3 – | 2 258v | 3 22v | 2 37 |
| 3 | Days of Persian years and months | 3 39r | 5 27v | 5 45r | – | 5 – | 3 259v | 5 23v | 3 38 |
| 4 | *Notae* of Syrian years and months | 4 39v | 2 26r | 2 43v | – | 2 – | 4 258r | 2 22r | 4 39 |
| 5 | *Notae* of Arabic years and months | 5 40r | 4 26v | 4 44v | – | 4 – | 5 259r | 4 23r | 5 40 |
| 6 | *Notae* of Persian years and months | 6 40v | 6 28r | 6 45v | – | 6 – | 6 260r | – | 6 41 |
| 7 | *Notae* of the Christian lent | 7 41r | – | – | – | – | – 260v | 6 24r | – |
| | **Trigonometric tables** | | | | | | | | |
| | *Sine* | | | | | | | | |
| 8 | - for every degree of arc | 8 41v | 7 28v | 7 46r | 13r | 7 – | 7 261r | – | 7 42 |
| 8a | - for minutes of arc | – | | – | – | – | – | 7 24v | – 49–63 |
| 9 | Versed sine | 9 42r | 8 29r | 8 46v | 13v | 8 – | 8 261v | – 33v | 8 43 |
| | *First tangent* | | | | | | | | |
| 10 | - with arguments up to 45° | 10 43r | – | – | – | – | – | 8? 32r | – |
| 10 | - with arguments up to 60° | – | – | – | – | – | 9 262v | 8? 34v | 9 45 |
| 10 | - with arguments up to 90° | – | 9 30r | 9 47v | 14v | 9 – | – | – | – |

| Section or table in the edition | | **F** (fols) | | **H** (fols) | | **C** (fols) | | **C$_1$** (fols) | **C$_2$** (fols) | | **Y** (fols) | | **L** (fols) | | **B** (pp.) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | Second tangent | 11 | 43v | 10 | 30v | 10 | 48r | 15r | 10 | – | 10 | 263r | 9? | 35r | 10 | 46 |
| | - only the table for feet | | | | | | | | | | | | 9? | 32r | | |
| **Solar and lunar tables** | | | | | | | | | | | | | | | | |
| 12 | Preliminaries of the mean motions | 12 | 44r | 11 | 31r | 11 | 48v | 15v | 11 | – | 11 | 263v | 16 | 35v | 11 | 47 |
| 13 | Solar mean motion | 13 | 44v | 12 | 31v | 12 | 50r | 16r | 12 | – | 12 | 264r | 18 | 36r | 12 | 48/67 |
| 14 | Apogee motion | 14 | 45v | 13 | 32v | 13 | 50v | 16v | 13 | – | 13 | 265r | 17 | 37r | 13 | 68 |
| | *Equation of time* | | | | | | | | | | | | | | | |
| 15 | - on intervals of 6° with interpolation coefficients | 15 | 46r | – | | – | | – | – | | 14 | 265r | – | | – | |
| 15a | - on intervals of 1° | – | | 14 | 32v | 14 | 50v | 16v | 14 | – | – | | 19 | 37v | 14 | 69 |
| 15b | - expressed in lunar mean motion | – | | | | – | | – | – | | | 268r | | 39v | | 75 |
| 16 | Solar equation | 16 | 46v | 15 | 33v | 15 | 51v | 17v | 15 | – | 15 | 265v | 20 | 38r | 15 | 70 |
| 17 | Lunar mean motion | 17 | 48v | 16 | 35v | 16 | 53r | 19r | 16 | – | 16 | 266v | 21 | 39r | 16 | 74 |
| 18 | Lunar mean anomaly | 18 | 49v | 17 | 36v | 16 | 53v | 19v | 16 | – | 17 | 268v | 21 | 40v | 17 | 76 |
| 19 | Double elongation | 19 | 50v | 18 | 37v | 16 | 54r | 20r | 16 | – | 18 | 269v | 21 | 41v | 18 | 78 |
| 20 | Lunar equations | 20 | 51v | 19 | 38v | 17 | 54v | 20v | 17 | – | 19 | 270v | 21 | 42v | 19 | 80 |
| 21 | Mean motion of the lunar node | 21 | 54v | 20 | 41v | 17 | 57r | 23r | 18 | 43r | 20 | 273v | 22 | 45r | 20 | 85 |
| **Planetary tables** | | | | | | | | | | | | | | | | |
| 22 | Mean motion of Saturn | 22 | 55v | 21 | 42v | 18 | 57v | 23v | 19 | 43v | 21 | 274v | 23 | 45v | 21 | 86 |
| 23 | Mean anomaly of Saturn | 23 | 56v | 22 | 43v | 18 | 58r | 24r | 19 | 44r | 22 | 275v | 23 | 46v | 22 | 88 |
| 24 | Equations of Saturn | 24 | 57v | 23 | 44v | 19 | 58v | 24v | 20 | 44v/37r | 23 | 276v | 23 | 47v | 23 | 90 |

| *Section or table in the edition* | | **F** (fols) | | **H** (fols) | | **C** (fols) | | **$C_1$** (fols) | **$C_2$** (fols) | | **Y** (fols) | | **L** (fols) | | **B** (pp.) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | Mean motions of Jupiter | 25 | 60v | 24 | 48v | 20 | 61v | 27v | 21 | 39v | 24 | 279v | 24 | 50v | 24 | 96 |
| 26 | Mean anomaly of Jupiter | 26 | 61v | 25 | 49v | 20 | 62r | 28r | 21 | 40r | 25 | 280v | 24 | 51v | 25 | 98 |
| 27 | Equations of Jupiter | 27 | 62v | 26 | 50v | 21 | 62v | 28v | 22 | 40v/25r | 26 | 281v | 24 | 52v | 26 | 100 |
| 28 | Mean motions of Mars | 28 | 65v | 27 | 54v | 22 | 65v | 31v | 23 | 25v | 27 | 284v | 25 | 55v | 27 | 106 |
| 29 | Mean anomaly of Mars | 29 | 66v | 28 | 55v | 22 | 66r | 32r | 23 | 26r | 28 | 285v | 25 | 56v | 28 | 108 |
| 30 | Equations of Mars | 30 | 67v | 29 | 56v | 23 | 66v | 32v | 24 | 26v | 29 | 286v | 25 | 57v | 29 | 110 |
| 30 | - standard variations | | 69v | | 58v | – | | – | – | | | 288v | | 59v | | 114 |
| 30a | - adjusted variations in the Cairo manuscripts | – | | – | | | 68v | 34v | | 28v | – | | – | | – | |
| 30b | - correction of the true position of Mars | – | | – | | | 69v | 35v | | 29v | – | | | 60v | | 116 |
| 31 | Mean motions of Venus | 31 | 70v | 30 | 60v | 24 | 70v | 36v | 25 | 30v | 30 | 289v | 26 | 61v | 30 | 118 |
| 32 | Mean anomaly of Venus | 32 | 71v | 31 | 61v | 24 | 71r | 37r | 25 | 31r | 31 | 290v | 26 | 62v | 31 | 120 |
| 33 | Equations of Venus | 33 | 72v | 32 | 62v | 25 | 71v | 37v | 26 | 31v | 32 | 291v | 26 | 63v | 32 | 122 |
| 34 | Mean motions of Mercury | 34 | 75v | 33 | 66v | 26 | 74v | 40v | 27 | 34v | 33 | 294v | – | 66v | 33 | 128 |
| 35 | Mean anomaly of Mercury | 35 | 76v | 34 | 67v | 26 | 75r | 41r | 27 | 35r | 34 | 295v | – | 67v | 34 | 130 |
| 36 | Equations of Mercury | 36 | 77v | 35 | 68v | 27 | 75v | 41v | 28 | 35v/45r | 35 | 296v | – | 68v | 35 | 132 |
| | *Lunar latitude* | | | | | | | | | | | | | | | |
| 37 | - with values to minutes | 37 | 80v | 36 | 72v | 28 | 78v | 44v | 29 | 46v | – | | – | | – | |
| 37a | - with values to seconds and tabular differences | – | | – | | – | | – | – | | 36 | 299v | 27 | 71v | 36 | 138 |

| | Section or table in the edition | **F** (fols) | **H** (fols) | **C** (fols) | $\mathbf{C_1}$ (fols) | $\mathbf{C_2}$ (fols) | **Y** (fols) | **L** (fols) | **B** (pp.) |
|---|---|---|---|---|---|---|---|---|---|
| | *Planetary latitudes and first stations* | | | | | | | | |
| 38 | - Saturn | 38 81r | 37 73r | – | – | – | 37 300r | 28 72r | 37 139 |
| 39 | - Jupiter | 39 81v | 38 73v | – | – | – | 38 300v | 28 72v | 38 140 |
| 40 | - Mars | 40 82r | 39 74r | – | – | – | 39 301r | 28 73r | 39 141 |
| 41 | - Venus | 41 82v | 40 74v | – | – | – | 40 301v | 28 73v | 40 142 |
| 42 | - Mercury | 42 83r | 41 75r | – | – | – | 41 302r | 28 74r | 41 143 |
| 38–42a | - all latitudes in one table | – | – | 29 79r | 45r | 30 – | – | – | – |
| 38–42b | - all stations in one table | – | – | 30 79v | 45v | 31 – | – | – | – |
| **Spherical-astronomical tables** | | | | | | | | | |
| | *First and second declination* | | | | | | | | |
| 43 | - together in one table | 43 83v | 42 75v | – | – | – | 42 302v | – 74v | 42 144 |
| 43a | - first declination<br>in a separate table | – | – | 31 80r | 11r | 32 – | – | 10 32v | – |
| 43b | - second declination<br>in a separate table | | – | 32 81r | 12r | 34 – | – | 11 33r* | – |
| 44 | Tangent of first declination | 44 84r | 43 77r | – 80v | 11v | 33 – | 43 304r | 12 76r | 43 147 |
| | *Right ascension* | | | | | | | | |
| 45 | - less accurate | 45 84v | 44 77v | 33 81v | 12v | 35 – | – | – | – |
| 45a | - more accurate | – | | – | – | – | 44 304v | 13 76v | 44 148 |
| | *Oblique ascension* | | | | | | | | |
| 46 | - for latitude 36° | 46 85r | 46 78v | – | – | – | – | 14 – | – |
| 46a | - for latitude 35;30° | – | – | – | – | – | 46 306r | – 78r | 46 151 |
| 46b | - with equation of daylight,<br>for latitude 35;30° | – | – | 34 82r | – | 36 – | – | – | – |

| | Section or table in the edition | **F** (fols) | **H** (fols) | **C** (fols) | **$C_1$** (fols) | **$C_2$** (fols) | **Y** (fols) | **L** (fols) | **B** (pp.) |
|---|---|---|---|---|---|---|---|---|---|
| | *Equation of daylight* | | | | | | | | |
| 47 | - maximum equation of daylight | 47 85v* | – | – | – | – | – | 13 – | – |
| 48 | - for latitude 36° | 48 85v | 45 78r | – | – | – | – | 14 – | – |
| 48a | - for latitude 35;30° | – | – | – | – | – | 45 305v | – 77v | 45 150 |
| | - with oblique ascension, for latitude 35;30° | – | – | 34 82r | – | 36 – | – | – | – |
| **Other tables** | | | | | | | | | |
| 49 | Hourly solar and lunar motion and the diameters of the Sun, Moon and shadow of the Earth | 49 86r | 47 79r | 35 83r | 46r | 37 – | 47 307r | 79r | 47 153 |
| 49bis | Conjunction and Opposition | – | – | – | – | – | 311v | 29 – | 211–214, 206–205, 217 |
| 50 | Lunar distance from the Earth | 50 86v | 48 79v | 36 83v | 46v | 38 – | 48 307v | 30 79v | 48 154 |
| 51 | Solar parallax | 51 87r | 49 80r | 37 84r | 47r | 39 – | 49 307r | 31 80r | 49 155 |
| 52 | Equation for the magnitude of eclipses | 52 87r | 50 80r | 38 84r | 47r | 40 – | 50 307r | 32 80r | 50 155 |
| 53 | Prorogations (*tasyīrs*) | 53 87r | 52 81r | – 84r | 47r | 41 – | 52 308v | 34 81r* | 52 157 |
| 54 | Geographical table | 54 87v | 51 80v | 39 84v | 47v | 42 – | 51 308r | 33 80v* | 51 156 |
| | - same table at the end of manuscript **L** | – | – | – | – | – | – | 132r | – |
| 55 | Star table | 55 88r | 53 81v | 43 85r | 48r | 43 – | 53 310v | 35 81v* | 53 158 |
| 56 | Section: Finding the original equations from the displaced ones | 56 89r | – | 44 – | 48v | 44 – | – | – | – 220 |

## I.6. Additional tables

Manuscripts **YLB** contain additional tables copied by the main hand of these manuscripts at the end of Book II (**YB**) or at the end of the whole *zīj* (**L**). Since several of these tables are of considerable interest, either on their own or in relation to the *Jāmiʿ Zīj*, I here present full lists of these tables accompanied by some remarks. It can be seen that both **Y** and **L** include spherical astronomical tables for latitudes between 29 and 33°, suggesting that these manuscripts or some of their ancestors were prepared for use in southern Iran (especially Isfahan and Shiraz). All three manuscripts add tables of types not originally provided by Kūshyār, especially tables of feasts and fasts in various calendars and a range of astrological functions, including the equalisation of the houses and year transfers.

### I.6.1. Additional tables in the Yeni Cami manuscript

The Yeni Cami manuscript adds to the basic set of tables of the *Jāmiʿ Zīj*, among others, two tables that are also found as additional tables in the Berlin manuscript and may have been among the additions made by Kūshyār himself in the version of the *zīj* extant in manuscripts **YLB**. Furthermore it contains a table of year transfers based on al-Battānī's year-length and a set of spherical-astronomical tables for latitudes 30°, 32;23° (Isfahan) and 33°.

- [fols 309r–310r] Table of middle and small prorogations for days 1, 2, 3, …, 30 for each of the months 1 to 12, plus an additional single-digit column for the *epagomenae* in which the arguments and digits of these five values are written above each other and filled up with a zero. The table is identical to that in **B**, pp. 63–65, except that the latter omits the column for the *epagomenae*, and it is explicitly referred to in the text of Book I in **L**. It is therefore possible that this extended version of Table 53 was inserted by Kūshyār himself in the recension of the *Jāmiʿ Zīj* that survives in manuscripts **YLB** (cf. Section I.7). This is discussed in more detail in the commentary on Table 53 in Section IV.12.
- [fols 311v–313v] Table of Conjunction and Opposition (see Plate 13): the 'part' and the 'hours of the distance' as a function of degrees of the elongation for each lunar velocity from 11;50 to 14;50°/day, with a column for 15;0°/day left empty. The values and headers of this table are identical to those in **B**, pp. 211–214, 206–205, 217, and the table is included in the table of contents in **L**. Therefore there is reason to believe that this was another table added by Kūshyār in the version of the *Jāmiʿ Zīj* that is extant in manuscripts **YLB** (cf. Section I.7). The table is therefore edited as Table $49^{\text{bis}}$ and is discussed further and analysed in the commentary on that table in Section IV.11.
- [fol. 314] Table of oblique ascensions for latitude 33°.

- [fol. 315] Table of oblique ascensions for Isfahan, latitude 32;23° (the title gives sine and cosine values of this latitude to four sexagesimal places).
- [fol. 316] Table of equatorial degrees corresponding to seasonal hours (‘parts of the hours’ *ajzāʾ ⟨al-⟩sāʿāt*) for latitude 32;23°.
- [fol. 317r] Table of the solar altitude at noon for latitude 32° (the table is indeed for latitude 32;0°, not for 32;23°).
- [fol. 317v] Table of the number of equal hours of daylight for latitude 32;23°.
- [fol. 318r] Table of the equation of daylight for latitude 30° (the heading appears to have 30;5°, but the table was clearly computed for 30;0°).
- [fol. 318v] Table of the ascendants of year transfers, which tabulates the ‘ascensions of the year’ (*maṭāliʿ al-sinīn*) for arguments 1, 2, 3, ..., 120 (the tabular values are plain multiples modulo 360° of the excess of revolution 86;36° corresponding exactly to al-Battānī’s and Kūshyār’s year length of 365;14,26 days).

### I.6.2. Additional tables (and some texts) in the Leiden manuscript

In the Leiden manuscript additional tables were copied in the main hand after the colophon of Book IV on fol. 124r. These include the less accurate right ascension and the equation of daylight for latitude 36° that are found in manuscripts **FH**. Among the additional tables are also alternatives for tables that were left blank in Book II, namely a geographical table basically identical to the one in the other manuscripts of the *Jāmiʿ Zīj* and a smaller star table with additional types of data for AD 981. Of interest also are a list of daily mean motions and the tables of mean motions and equations for Mars from al-Nasawī’s lost *Fākhir Zīj*.

- [fol. 124v] Table of the oblique ascension for Bardsir, latitude 32;5° (this table is in fact based on a latitude very close to the mentioned value, unlike the equation of daylight for 32;0° in **Y**, fol. 318r, to which it is hence not related).
- [fol. 125r] Astrological tables: lots for nativities, terms according to the Egyptians, lords of triplicities and decans, and the *darījān* (cf. fol. 144r).[56]

[56] For the astrological concept of *darījān*, see Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 130/131 (Arabic text and English translation) and 420 (Arabic-Latin glossary).

- [fols 125v–126r] Tables of feasts and fasts in the Christian, Islamic and Persian calendars (these tables display some differences from the ones in **B**, pp. 164, 194–195).
- [fol. 126v, right] Table giving for each of the 28 lunar mansions their longitude (*wasaṭ al-ṭūl*) and 'its half' (*niṣf dhalika*), their latitude (*wasaṭ al-ʿarḍ*) and northern or southern direction (*jihāt*), and their magnitude (*ʿadad*), with instructions in the same hand.
- [fol. 126v, left] Table of the shadow (i.e., cotangent, here entitled *jadwal al-aqdām*) for a gnomon length of 7 feet (already found on fols 32r and 35r; cf. Table 11). To this table the main hand has added the same explanatory text on prayer times found with the table on fol. 32r (see explanatory text B3 on p. 279).
- [fol. 127r] Table of the sexagesimal places resulting from division and multiplication, and two small tables for the *qisma* of Mercury and the five planets (said to 'be calculated from the velocity' *tuʿmilu min al-buht*, both containing only zeroes and ones, with a line of instructions).[57]
- [fols 127v–130r] Table for the equalisation of the houses (*taswiyat al-buyūt*) as a function of the longitude of the ascendant, displaying for every degree of the ecliptic the longitudes of the second to sixth houses (with an indication of the terms next to the arguments).
- [fol. 130v] Table of oblique ascensions for latitude 29;30° (presumably for Shiraz, cf. fol. 142v).
- [fols 131 and 135–137] Sections on lunar eclipses and solar eclipses followed by tables for their calculation according to the view of al-Bīrūnī (the manuscript once calls him Abū l-Rayḥān al-Rūmī). These include a table of lunar latitude (maximum 5;0°) on fol. 137r and a possibly unrelated table of equal hours for the latitude 29° of Shiraz.
- [fol. 132r] The geographical table from the *Jāmiʿ Zīj* (here edited as Table 54; note that the frame intended for this table on fol. 80v has only a title and column headers).
- [fol. 132v] Table of longitudes, latitudes, declinations, degrees of transit and temperaments of 30 fixed stars for the Seleucid year 1293

[57] For the astrological concept of *qisma*, see Burnett et al., *Al-Qabīṣī (Alcabitius)*, pp. 124–129 (Arabic text and English translation) and 450 (Arabic-Latin glossary), and Elwell-Sutton, *The Horoscope of Asadullāh Mīrzā*, pp. 84–86.

(AD 981/2) (the longitudes are 13;0° degrees larger than those in the *Almagest*).

- [fol. 133, sideways] Table of parallax in longitude and latitude for the geographical latitude 30;22° of the third climate.
- [fol. 134r] Table of ordinary right ascensions with values to minutes (this is Kūshyār's less accurate table found in **FHCC$_1$**, here edited as Table 45, with only minor differences).
- [fol. 134v] Table of normed right ascensions with inaccurate values to seconds.[58]
- [fol. 136r] Table of the lunar latitude (with maximum 5;0°, possibly belonging to the eclipse materials on fols 131 and 135).
- [fols 138r–142r] Tables from the *Fākhir Zīj* (of Kūshyār's student al-Nasawī): mean motion, mean anomaly, first equation, second equation and variations (*ikhtilāf*) with interpolation coefficients for Mars; and a list of daily mean motions of all the planets.
- [fol. 142v, right] Table of the maximum solar altitude for the latitude 29;30° of Shiraz, with instructions in the main hand.
- [fol. 142v, left] Tables of the equation of time expressed in time (with interpolation minutes) and in lunar mean longitude, with instructions in the main hand. The first table is close, but not identical to Kūshyār's Table 15. The second table is identical to Table 15b (in **L** found on fol. 39v). See further the commentary in Section IV.6.
- [fol. 143r] Table of the equation of daylight for latitude 36;0°, with instructions in the left margin (the values in the first quadrant are practically identical to those in Table 48 as further found only in **FH**, but the Leiden copy has symmetrical values for every degree up to 360).
- [fol. 143v] Table of the equatorial degrees corresponding to a seasonal hour for latitude 36;0° (as a function of the solar longitude, with values for every degree up to 360).
- [fol. 144r] Astrological tables: lots, triplicities, decans and *darījān* (cf. fol. 125r).

[58] This table was computed by means of linear interpolation within intervals of 3°, most likely for obliquity 23;35°, but with numerous irregularities. The same table is found in the Leipzig manuscript of the *Mumtaḥan Zīj* and in the Istanbul copy of the *zīj* of Ḥabash al-Ḥāsib; see van Dalen, 'A Second Manuscript', p. 21 and Debarnot, 'The *Zīj* of Ḥabash al-Ḥasib', p. 47.

## I.6.3. Additional tables (and some texts) in the Berlin manuscript

**B** adds a highly interesting set of tables after the standard set belonging to the *Jāmiʿ Zīj* but before the section on the original equations that concludes Book II in three of the eight manuscripts. Pages 160–163, possibly together with pages 167–179, appear to contain a coherent set of texts and tables investigating the accuracy of the *zīj*es of Yaḥyā ibn Abī Manṣūr, Ḥabash al-Ḥāsib, al-Battānī and Ibn al-Aʿlam, the major Islamic astronomers and observers up to the year AD 970. The accompanying planetary tables make it possible to compute many of the planetary positions according to each of the four authors. Some of the material can be dated to the early twelfth century (i.e., the time when al-Khāzinī wrote his impressive *Sanjarī Zīj* in Marw in present-day Turkmenistan), but this makes it curious that Ibn Yūnus and especially al-Bīrūnī were not included in the comparison. The star table on pp. 182–184 can also be dated to this period and may hence belong to the same work, probably together with the intervening table for the normed right ascension. Another interesting set of tables in **B** extends the planetary tables from the *Jāmiʿ Zīj* in such a way that they can be used at al-Battānī's locality Raqqa. Finally, **B** has two additional tables in common with **Y** that may have been added by Kūshyār to the version of his *zīj* represented by manuscripts **YLB**.

- [pp. 49–63 (originally after p. 218 or 220)] Extensive sine table with values for minutes of arc (here edited as Table 8a). This table is also found in **L** (see Plate 4) and is attributed to Kūshyār in the *Dustūr al-munajjimīn*, an early twelfth-century Ismāʿīlī *zīj* that also incorporates other tables and texts from the *Jāmiʿ Zīj*.[59]
- [pp. 63–65 (immediately following the sine table)] Table of middle and small prorogations for days 1, 2, 3, ..., 30 for each of the months 1 to 12. This extends Kūshyār's Table 53 to 360 days. The table is identical to the table in **Y**, fols 309r–310r, which, however, also adds a single-digit column for the *epagomenae*. Since this table is referred to in the text of Book I in **L**, it is possible that Kūshyār himself added this table in the recension of his *zīj* extant in manuscripts **YLB**. This is discussed further in the commentary on Table 53 in Section IV.12.
- [p. 160] Planetary mean positions at the beginning of the year 483 Yazdigird (16 February 1114) according to Yaḥyā ibn Abī Manṣūr (Baghdad), Ḥabash al-Ḥāsib (Baghdad), al-Battānī (Raqqa) and Ibn al-Aʿlam (Baghdad).

[59] For more information on this work, see the Quick reference guide on pp. 525–26.

- [p. 161] The same for noon of the Yazdigird epoch (16 June 632).
- [pp. 162–163] Text belonging to the previous two tables (it seems possible that the planetary equation tables on pp. 167–179 also belong to these materials).
- [p. 164] Table of feasts and fasts in the Islamic calendar.
- [p. 165] Table of western (*shuhūr al-maghrib*, i.e., January, February, etc.), Soghdian, Roman (i.e., Latin), Coptic and Jewish month names and of the dates of Jewish Passover (فسح *sic*!) in the 19-year cycle of the Jewish calendar.
- [p. 166] Table of the Christian Lent (with a double-argument table as a function of the day of the week and the Golden Number and a second table with as arguments years 1, 2, 3, ..., 95, i.e., very different from Kūshyār's Table 7).
- [p. 167] Table of the solar equation according to Ibn al-Aʿlam (a double-entry table with a maximum value of 2;0,10°). This table is indeed the same as the one attributed to Ibn al-Aʿlam in the early fourteenth-century *Ashrafī Zīj* by Sayf-i munajjim-i Yazdī al-Kamālī, which collects the planetary mean motions and equations from a series of earlier *zīj*es (Paris, Bibliothèque nationale de France, suppl. persan 1488, fol. 236v).
- [pp. 168–172 and 174–176] Tables of the equation of centre for Saturn, Jupiter and Mercury and the equation of anomaly for Mercury according to Ibn al-Aʿlam (pp. 169–170 and 175–176 are inserted slips of paper with instructions for applying these equations). The tables for Mercury are indeed the same as the ones attributed to Ibn al-Aʿlam in the *Ashrafī Zīj* (Paris, Bibliothèque nationale de France, suppl. persan 1488, fol. 237r–v; the tables for Saturn and Jupiter are not included in this work).
- [p. 173] Table of the solar equation according to al-Battānī and Ḥabash al-Ḥāsib (with maximum 1;59,10°; the table is close to al-Battānī's table except for some ranges of the argument, but is clearly different from Ḥabash, who has a maximum equation of 1;59,0°).
- [p. 177] Table of the solar mean motion with collected / extended years 478, 479, 480, ..., 508 Yazdigird (AD 1109–1139), values to sexagesimal thirds for years and months and values to sexagesimal fourths for days and hours (the table appears to be based on the value for the daily mean motion of Yaḥyā and Ḥabash).
- [pp. 178–179] 'Table of the equation of the Sun / solar mean motion', in fact displaying the true solar longitude as a function of the mean

solar longitude (based on parameters that can be associated with Yaḥyā ibn Abī Manṣūr).[60]

- [pp. 180–181] Table of normed right ascensions.
- [pp. 182–184] Table of 75 fixed stars (arranged entirely by increasing longitude, with the longitudes from Ptolemy's *Almagest* adjusted by +14;29°, i.e., for the beginning of the twelfth century AD). Cf. the comments below to the star table on pp. 188–189, which has identical longitudes for the stars that the tables have in common.
- [p. 185] 'First table for the equation of transits' (tabulating the sine of the degree of transit for 1, 2, 3, …, 180°).
- [pp. 186–187] Table for calculating the ascendant by night for 34 fixed stars (five numbered columns contain for each star the intermediate and final results of these calculations, which are explained by instructions in the headers of the columns). See King, *In Synchrony with the Heavens I*, pp. 141–42 and 887–96 (with reproductions).
- [pp. 188–189] Table of 25 fixed stars with longitudes and latitudes as well as declination, right ascension, zenith distance (*al-inḥirāf ʿan samt al-raʾs*), altitude at culmination, half arc of daylight, and four further quantities (the longitudes are identical to those in the table on pp. 182–184). This table was studied in the unpublished preprint Girke, 'Die Sterntafel des al-Balḫī' and is described in King, *In Synchrony with the Heavens I*, pp. 76–78 and 887–96 (with reproductions). A gloss states that the table was originally compiled by a certain al-Balkhī in the Seleucid year 1310 (AD 998/9), but then adjusted for the Seleucid year 1424 (AD 1112/3) by adding 1;29° to the longitudes. Thus the original table displayed the same coordinates as the star table among the additional tables in **L** (fol. 132v).
- [pp. 190–193] Table of the eastern and western ascendants for every degree of altitude up to the respective maxima, for Sirius (maximum altitude 45°), Capella (77°), Vega (86°?) and Arcturus (85°). The gloss to the previous table makes it clear that the two tables belong together

[60] In van Dalen, 'A Table for the True Solar Longitude' (a somewhat extended version of van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 76–86), I showed that this table belongs to the tradition of Yaḥyā ibn Abī Manṣūr. The underlying solar equation was computed by the 'method of declinations' of Persian-Sasanian or early Islamic origin for a maximum equation of 1;59,56°. Thus it is basically the same as the solar equation attributed to Yaḥyā in the *Ashrafī Zīj*; cf. Kennedy, 'The Solar Equation'. The table in **B** is further based on the Mumtaḥan apogee longitude of 82;40°, and values for non-multiples of 5° were calculated by 'distributed linear interpolation' (see pp. 528–29).

and that the present one is not complete. See King, *In Synchrony with the Heavens I*, pp. 77–78 and 887–96 (with reproductions).

- [pp. 194–197] Table of Persian, Christian and Jewish feasts and fasts.
- [pp. 198–204, 216] Sections on finding the *namūdhār* of the 'falling of the sperm' according to Hermes and Vettius Valens, partially attributed to al-Kindī, accompanied by two tables for the duration of the stay (*makth*) of the fetus in the mother's womb 'and its equation' and two more for 'the equation for the Moon above / below the horizon'. See King, 'A Hellenistic Astrological Table', esp. pp. 689, 691, 700 n. 85 and Appendix A on pp. 702–03.
- [pp. 211–214, 205–206, 217] Table of Conjunction and Opposition: the 'part' and the 'hours of the distance' as a function of degrees of the elongation for each lunar velocity from 11;50 to 14;50°/day, with instructions in the same hand in the margin of p. 211.[61] The values and headers in this table are identical to those in **Y**, fols 311v–313v and the table is referred to in Book I in manuscript **L**. Since it is therefore plausible that this is an original addition by Kūshyār in the version of his *zīj* extant in manuscripts **YLB**, it is edited here as Table 49[bis] and analysed in the commentary on that table in Section IV.11.
- [pp. 207–210] Tables of mean positions for collected years 1, 21, 41, ..., 581 Yazdigird and of the corrections to these positions (*fī mā bayn al-ṭūlayn*) needed for geographical longitudes 61, 62, 63, ..., 90°. Both types of tables are given for the solar and lunar mean longitude, the lunar anomaly, the double elongation, the longitude of the lunar node and the mean longitude of Saturn (on pp. 207–209); only the second type is given for all other mean motions (on p. 210). The tables give displaced positions in full agreement with Kūshyār's, but they are for al-Battānī's longitude 73;15° for Raqqa; they thus provide exactly the subtables that one needs to replace Kūshyār's tables for a longitude of 90° by the same type of tables for Raqqa.[62] Note that the subtables for collected years of the mean motion tables for Jupiter, Mars, Venus and Mercury in the main part of manuscript **B** were corrected to the val-

[61] Apparently the folios in this part of manuscript **B** were not in order at a very early stage. The set of tables shows too little internal coherence for the order to be reliably restored in its entirety, but at least it can be seen that pp. 198–204, 216*, 215 and pp. 211–214, 206*, 205*, 217–218 must have appeared in that order before the manuscript received its very first foliation (pages indicated with an asterisk are found upside down in the bound manuscript).

[62] The long marginal note in a user's hand on p. 207 shows an awareness that the table is for Raqqa, although this is not explicitly indicated in the table itself.

ues for Raqqa that would have been contained in the subtables missing here; for a full discussion see Section IV.5.3.

- [p. 215] Circular table of the azimuth of the ascension for the seven climates taken from al-Battānī's *Ṣābiʾ Zīj*.[63] This table is discussed and reproduced in King, *In Synchrony with the Heavens I*, p. 112.
- [p. 217] Table of the astrological concept of twelfths with an example in the same hand.
- [p. 218] Table of planetary latitudes with columns 'northern' and 'southern' for each of the superior planets, columns *al-mayl* 'deviation' and *al-inḥirāf* 'slant' for Venus and Mercury, plus a shared column of interpolation minutes (these tables are clearly different from any based on Ptolemy's *Almagest*; the underlying parameters appear to be Ptolemaic, but all functions except for the slant of the inferior planets were computed by a different formula).
- [p. 219] Table of 65 fixed stars with longitudes 12°32′ larger than those in Ptolemy's *Almagest* (a marginal note in a barely legible, different hand states that these are for 331 Yazdigird).

[63] Escorial, RBMSL, árabe 908, fol. 196v; Nallino, *al-Battānī sive Albatenii*, vol. II, p. 92.

## I.7. The different versions of the *Jāmiʿ Zīj*

The comparative list of the tables in Book II of the *Jāmiʿ Zīj* (Table A on pp. 37–43), a closer look at the layout and contents of the tables, and numerous variants in the critical edition will make clear that the eight Arabic manuscripts that contain the entirety (**FHCYLB**) or at least part (**$C_1C_2$**) of the tables belong to three or even four somewhat different versions of the *Jāmiʿ Zīj*. Like Mohammad Bagheri,[64] I will argue that manuscript **F** is probably the closest to the original version of the *Jāmiʿ Zīj* and its tables because 1) it is the oldest extant manuscript that contains the tables; 2) its tabular values generally have very few scribal errors, and 3) its set of tables is coherent and shows the best agreement with the instructions in Book I. In this section we will furthermore see that many of the differences in the tables in the other versions of the *zīj* are probably modifications of the tables in **F**. **H** is very close to **F** but already includes some of the changes that are also found in **$CC_1C_2$**, which are likewise associated with Yemen. The three Cairo manuscripts are obviously closely related, not only in terms of changes in layout and contents of some of the original tables, but also with regard to certain new elements not found in any of the other five manuscripts. Besides several further, more conspicuous common features, the most notable characteristic of **YLB** is the large number of newly computed subtables of the mean motion tables, making these more accurate than the ones in the other manuscripts.

In order to be able to give a more detailed specification of the characteristics of each of the three or four groups of manuscripts, I will first present a complete overview of the differences between the eight manuscripts.

### I.7.1. Differences between the tables arranged by type

#### Inclusion, omission and order of tables

- *Chronology* (Tables 1–7): **FB** first give the tables for the numbers of days in the three main calendars, and only then the three tables of *notae*; **HCYL** place the table of numbers of days and the table of *notae* together for each of the three calendars.[65] **L** omits the table of *notae* for Persian years.[66] Only **FYL** include the table of the Christian Lent

[64] Bagheri, *az-Zīj al-Jāmiʿ*, pp. xxxviii-xxxix.

[65] Judging from its table of contents, the latter arrangement was originally also found in **$C_2$**, and hence most likely in **$C_1$** as well. In **Y** the table of contents follows the order of **FB**, but the arrangement of the tables is as in **HCL**.

[66] The calculation of these *notae* is very easy and is explained in full in the last part of Chapter I.1.5; see Bagheri, *az-Zīj al-Jāmiʿ*, pp. 11–12 (translation) and Arabic p. 12. Note that the instructions as edited by Bagheri from the manuscripts **Y** (fol. 234v) and Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1 (fol. 5v), and as also found in the Leiden manuscript of the Persian translation (fol. 6r), are given for the old version of the Persian calendar (with the

(see Plate 3 for the table in **L**; this table is missing from the table of contents in **Y**).

- *Sine* (Tables 8 and 8a): The standard sine table with values for every degree is contained in all manuscripts except **L**, which instead includes the much more extensive table covering 15 pages with values for 1, 2, 3, ..., 15, 18, 21, ..., 60 minutes to be added to the sines of every integer degree from 0 to 90 indicated at the top of the columns (for the first page of this table, see Plate 4). This same table is found in **B**, in addition to the regular sine table and followed by three pages of tables for middle and small prorogations that are different from Table 53 but are also found among the additional tables in **Y**. As explained in the description of **B** in Section I.4, the nine leaves with these two tables were at an early stage (i.e., before the copy of the *Jāmiʿ Zīj* was bound together with the Persian translation of the introduction to astrology by al-Qummī) moved from the additional tables following Book II of the *Jāmiʿ Zīj* to their current position, thereby separating the two halves of the solar mean motion table (already before this move, the leaf with the current pp. 61 and 62 was inserted with front and back reversed). The insertion of these nine folios after page 48, rather than immediately after the sine table on page 42, may have been due to the fact that the first 48 pages of the manuscript constituted a whole number of quires; and the moving of the last two pages of the table of prorogations together with the sine table may be explained by the fact that the prorogations would otherwise have been split, since the first page of the prorogation table appears on the obverse of the last page of the sine table. Note that the extensive sine table is also included in the *Dustūr al-munajjimīn*, where it is explicitly attributed to Kūshyār; this copy has also been used in the edition of this table on pp. 97–111. The table of prorogations is further discussed in the commentary to Table 53 in Section IV.12.

- *Lunar mean motion* (Table 17): Only in **YLB** does the table for the lunar mean motion incorporate a small table (here edited as Table 15b on p. 123) for the equation of time expressed in lunar mean motion for every six degrees of solar mean longitude (but without interpolation coefficients).

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*epagomenae* following the eighth month Abān), whereas the instructions in Chapter I.5 in **C** (fol. 6v), $\mathbf{C}_2$ (fol. 9r) and **B** (p. 11) and Chapter I.4 in **L** (fol. 4v) are for the later version of the calendar (the entire chronological section is missing from Book I in **F**). The table of Persian *notae* includes values for both versions of the Persian calendar in all five manuscripts that contain the table.

- *Maximum equation of daylight* (Table 47): **F** includes an empty frame for a table for the maximum equation of daylight as a function of geographical latitudes from 16 to 45°. In **L** an entry for this table is included in the table of contents, but the table itself was omitted.
- *Correction of the true position of Mars* (Table 30b): The three Cairo manuscripts together with **LB** include the table for the 'correction of the true position of Mars' (*iṣlāh taqwīm al-mirrīkh*) that al-Bayhaqī attributes to Kūshyār (see Plate 10, the references given in footnote 23, and the analysis in Section IV.8.2). The instructions in manuscript **C** refer explicitly to this correction in a passage that is not included in Bagheri's edition and translation but is here edited on p. 431.
- *Conjunctions and Oppositions*: This table appears as an additional table in **YB** (see Plate 13) and is included in the table of contents of **L**. Its use is described in a paragraph appended to the relevant chapter in Book I in manuscripts **Y** and **L**. It is thus probably an original addition by Kūshyār and has been edited in Part II as Table 49$^{bis}$. For further information, see the commentary on this table in Section IV.11.

Arrangement of the tables and subtables

- *Days of Persian years and months* (Table 3): **FH** write the numbers of days for the old version of the Persian calendar (in which the five *epagomenae* were inserted after the eighth month Ābān instead of at the end of the year) in the main column for the months, and those for the later version (used from around Kūshyār's time onwards) next to it. **CYLB** write the numbers of days for the later version of the Persian calendar in the main column for the months; **CB** write the numbers of days for the old version next to this column, **YL** omit them entirely.
- *Tangent* (Table 10): The table of the first tangent in **F** has arguments up to 45°, the one in **YLB** up to 60°, and the one in **HCC$_1$** up to 90° (see Plate 5).[67]
- *Preliminaries of the mean motions* (Table 12): Only **F** includes a subtable with al-Battānī's planetary positions for the Seleucid year 931 and the Hijra epoch and arranges the four subtables as in the edition on p. 116 (see Plate 6). In all other manuscripts we find the derived posi-

[67] The tangent values for the common parts of the table generally agree. **L** in addition provides, in the same part of Book II, another tangent table with arguments up to 45° and basically identical values (i.e., the table from **F**), as well as another cotangent table for gnomon length 7 feet (but without the one for 12 fingers). Another copy of the latter table, with numerous scribal errors and the same explanatory note on prayer times (cf. note B3 in the text edition on p. 279), is included among the additional tables of **L**.

tions for Raqqa at the Yazdigird epoch at the top right, the motions in 20 Syrian years at the top left and the calculated daily mean motions at the bottom right, whereas the bottom left quadrant contains explanatory texts (cf. Plate 7). The numbers of full rotations for the motions in 20 years are indicated in **FHCC$_1$** (with a separate column heading *adwār* 'rotations' only in **HCC$_1$**), but not in **YLB**.[68] In all manuscripts, Table 12 comes with a number of explanatory texts apparently belonging to, or at least associated with, the original *Jāmiʿ Zīj*, but in **F** there is a particularly large number of such texts written rather sloppily in the main hand between the subtables, partially vertically or upside down (cf. Plate 6). In **HCC$_1$YLB** the apogee longitudes at the Yazdigird epoch are written between the two subtables on the right side of the page, whereas **F** writes these vertically in the left margin and in addition writes al-Battānī's apogee longitudes upside down between the upper two subtables.

- *Apogee motion* (Table 14): Only **F** gives three sets of apogee longitudes next to the table. These are for the Yazdigird epoch, for al-Battānī's epoch (the Seleucid year 1191, AD 879/80) and for the year 331 Yazdigird.
- *Mean motions*: In **FHYLB** alternative values for the old version of the Persian calendar are written below those for the later version in the subtables for months of all mean motion tables; in **FYLB** the values for the old version are in red (see Plate 9), in **H** those for the later version. **CC$_1$C$_2$** squeeze all mean motion tables onto a single page by placing the longitude corrections under the subtable for months and omitting the values for the old variant of the Persian calendar (see Plate 8). In **LB**, Table 21 for the motion of the lunar nodes was squeezed onto a single page without changing its arrangement. **YLB** add a mean motion value for 600 single years in all mean motion tables by squeezing three values into the last two lines of the subtable (see Plate 9).
- *Equation of time* (Table 15): In **FY** the table for the equation of time has values for every six degrees of solar mean longitude, accompanied by interpolation coefficients; in **HCC$_1$LB** the equation of time is given for every single degree and it can be verified that the values for non-multiples of six degrees are in full agreement with the interpolation coefficients given in **FY**.

[68] Note that these numbers of rotations were needed for calculating the daily mean motions from those in 20 Syrian years (cf. explanatory text D to Table 12, edited on p. 284, which was included both in **F** and in **L**).

- *Planetary equations*: **H** omits the tabular differences from the lunar and planetary equations (but not those from the sine, versed sine and tangent, the solar equation and the first and second declinations).
- *Planetary latitudes and stations* (Table 38–42): for each of the five planets, **FHYLB** have a combined table for the latitudes *and* the first station (with separate table numbers except in **L**), whereas **CC₁** have one table displaying the planetary latitudes of all five planets with a shared column of interpolation coefficients and one table displaying the planetary stations of all five planets, both with their own table number.
- *Spherical astronomy, declinations* (Table 43): **FHYB** have a combined table for the first and second declination (with tabular differences), whereas **CC₁** have two separate tables each with their own table number and the tangent of the declination placed in between the two.[69] **L** includes a copy of the separate declination tables before its planetary tables (with the values for the second declination omitted) and a copy of the combined tables together with the other spherical astronomical tables; its table of contents lists only the separate tables.

Tables computed anew

- *Mean motion tables*: Whereas most of the other differences can be easily recognised from the tables themselves or their headings, one type of changes in the mean motion tables can only be identified by a close inspection of the tabular values. Thus it can be seen that a large number of subtables of the mean motion tables, not only those for collected and extended years but also less significant ones such as those for hours, were computed anew in manuscripts **YLB** (see Section IV.5.2 of the commentary for a complete overview and analysis). In most cases the changes amount to single units in the least significant digits of the tables (seconds for the solar and lunar mean longitude, minutes for all other mean motions), but they often concern at least a dozen values in multiple subtables and are obviously systematic. The fact that they cannot be the result of scribal errors (such as the confusion of consecutive *abjad* numbers 3 and 4 or 6 and 7) is also very clear from several differences involving multiple digits, such as that in the lunar mean motion in longitude in 51 hours (28;0,0 vs 27;59,59°). In nearly all cases the subtables in **YLB** can be seen to be more accurate than those in **FHCC₁C₂** for the epoch positions and daily mean motions laid out by Kūshyār in Table 12. Only for Mars are the differences larger than sin-

[69] **C₂** does not actually contain these tables, but its table of contents suggests the same structure.

gle units. Since the variations for Mars in the three Cairo manuscripts are also different from those in the other witnesses and since all manuscripts except **F**, **H** and **L** include tables for correcting the true position of Mars, the Mars tables are discussed and analysed in more detail in Section IV.8. Note that **YLB** also extend the subtable for single years of each mean motion table from 500 to 600 years. **B** constitutes a particularly interesting source, since its values were collated with a manuscript with mean motion tables from the family **FHCC$_1$C$_2$** but with the subtables for collected years laid out for al-Battānī's locality Raqqa. In some cases tabular values in **B** can be seen on the scans to have been corrected to those related to **FHCC$_1$C$_2$**, while in other cases the deviating digits were written next to the subtables in **B** (see Plate 9). Details of these collations are discussed in Section IV.5.3.

- *Planetary equations of Mars, variation* (Table 30a): The tables for the variation in the equation of anomaly of Mars in **CC$_1$C$_2$** are different from those in all other manuscripts. It is as yet unclear whether these differences are related to the correction of the true position, since the latter is included not only in **CC$_1$C$_2$** but in **LB** as well (cf. the commentary in Section IV.8.2).
- *Lunar latitude* (Table 37): The table for the lunar latitude in **FHCC$_1$C$_2$** has values to minutes only and no tabular differences (cf. Plate 11), whereas the one in **YLB** has values to seconds with tabular differences.
- *Spherical astronomy, ascensions* (Tables 45–48): **FHCC$_1$** contain an inaccurate table of right ascensions in which one third of the tabular values have an error of one or two minutes, whereas **YLB** have a more accurately computed table with only eight errors of one minute among the 90 values in the first quadrant.[70] **FH** include separate tables for the oblique ascension and the equation of daylight for latitude 36°, which may be for Raqqa or the fourth climate in general. **YLB** have separate tables for the same functions but for latitude 35;30° (see Plate 12), presumably for Rayy near Tehran, where Kūshyār is known to have stayed and met al-Khujandī and al-Bīrūnī (see p. 9).[71] **C** has (and, judging from the table of contents of **C$_2$**, the other two Cairo manuscripts probably

[70] **L** also includes the less accurate right ascension table among its additional tables.

[71] Note that in **L** the tables for the oblique ascension and the equation of daylight are combined in the table of contents under no. 14, but are in fact two separate tables. It may thus be assumed that the omitted no. 15 should have been assigned to the table for the equation of daylight. Note that the table of contents in **L** also gives the latitude underlying these two tables as 36° instead of the actual 35;30°. This is one of several indications that a manuscript close to **F** was also used by the scribe of **L**, others being the inclusion in the table of contents of the absent table for the maximum equation of daylight and the inclusion among its addi-

also had) a combined table for oblique ascensions and the equation of daylight with the same values for latitude 35;30° as found in **YLB**.[72]

- *Prorogations* (Table 53): A more extensive version of this table is present as an additional table in **YB** and is mentioned in the explanatory text in Book I of **L**.
- *Fixed stars* (Table 55): In the table of fixed stars for the year 301 Yazdigird at the end of Book II, **FHYB** include a selection of 48 stars. Not all of these are included in the selection of 60 stars that we find in the table in **C**$\mathbf{C_1}$ for the same epoch (see Plate 15). In several cases the coordinates in **C**$\mathbf{C_1}$ conform better to Ptolemy's *Almagest*. Kunitzsch has discussed the differences in the star names between the two versions of the table and their origin (cf. Section IV.14 of the commentary). In **L** the star table was left empty, while a star table among the additional tables in this manuscript displays declinations and degrees of transit along with longitudes, latitudes and temperaments and is for a different epoch (although also during Kūshyār's lifetime).

Numbering of the tables in the tables of contents

As can be seen from Table A, the numbering of the tables in the tables of contents differs considerably between the three groups of manuscripts **F**, **C**$\mathbf{C_2}$ and **HYLB** (for the table of contents in $\mathbf{C_2}$, see Plate 2a; note that the table of contents does not survive in $\mathbf{C_1}$), and to some extent even within these groups. As a result, the last table of Book II, that of the fixed stars, is numbered 55 in **F**, 43 in **C**$\mathbf{C_2}$, 53 in **HYB** and 35 in **L**.[73] Besides being due to the changes in order and the restructuring of certain tables as specified above, the differences also arise because of the irregularities in the numbering in several manuscripts, as mentioned in the caption to Table A on pp. 37–38. Since the numbering is most likely due to later scribes, it will not be considered further as a means to distinguish groups of manuscripts.

Further small differences

The manuscript copies of Kūshyār's tables can further be grouped on the basis of less notable differences, such as those in terminology and in tabular values.

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tional tables of the less accurate right ascension table and the table for the equation of daylight for 36° from **F** (see above).

[72] As will be shown in Section IV.10 of the commentary, the oblique ascension for latitude 35;30° was computed on the basis of the accompanying table for the equation of daylight for that latitude. However it was not computed from the accurate right ascension table for obliquity 23;35° that precedes the oblique ascensions in every manuscript, but from a less accurate table for the Ptolemaic obliquity value 23;51°.

[73] In **F** the section on the original equations is numbered 56, in **C**$\mathbf{C_2}$ 44. In **YLB** it is not included in the table of contents.

For example, in the tables for the planetary equations the tabular differences or interpolation coefficients are called *tafāḍul* in **FCC$_1$C$_2$** and *ḥiṣṣat al-daraja* in **YLB**. In the names of the arguments of these tables, **F** precedes 'centrum' (*al-markaz*) or 'anomaly' (*al-khāṣṣa*) by 'degrees' (*darajāt*), **YLB** by 'parts' (*ajzāʾ*) and **CC$_1$C$_2$** by neither. Furthermore, in **F** the locations of the mean, nearest and furthest distance in the equant or epicycle are indicated with their numbers of degrees in the headers of the columns for the zodiacal signs concerned; in all other manuscripts they are indicated between or next to the tabular values. Finally, throughout Book II of the *Jāmiʿ Zīj* we find occasional variants in tabular values that are shared, especially, by manuscripts **CC$_1$C$_2$** or **YLB**. All deviations of the types described here can be found in the editions of tables and texts in Parts II and III.

## I.7.2. Differences between the tables grouped by manuscripts

Having given a complete overview of the differences between the tables in the eight manuscripts in the previous section, I will now arrange the most significant differences according to groups of manuscripts. It is already clear that **CC$_1$C$_2$** and **YLB** constitute groups within which the manuscripts share various characteristics not found in others. **F** is different from both of these groups and, as I will argue in detail on p. 70, most likely constitutes the most archaic version of the *Jāmiʿ Zīj*. It is difficult to decide whether **H**, which on the one hand is very close to **F** but on the other already includes some characteristics found in either **CC$_1$C$_2$** or **YLB**, should be considered a separate group. As we will see, the tables in **H** do not have any characteristics that are not also found in one of the other groups except for the omission of tabular differences from the tables of lunar and planetary equations, which might have been done by any scribe. In order to obtain the full picture, I will not only put together the characteristics of the three or four groups that we have already identified, but also those that are shared between groups, and will create a separate mixed category for characteristics that do not strictly follow the boundaries of the groups identified. Characteristics in the list below that are not explicitly associated with one or more manuscripts are found in all manuscripts of the group concerned; if I mention manuscripts explicitly, this usually means that one or two of the manuscripts in the group do not have the characteristic(s) concerned (partially also because these manuscripts do not contain the particular table, especially in the cases of **C$_1$** and **C$_2$**). An asterisk after an item indicates that the changes could have been made by a copyist, as opposed to a skilled astronomer or Kūshyār himself. Even less specific differences, such as those in the numbering of the tables in the tables of contents and in individual tabular values, have been disregarded. The relevant differences are also indicated in Table B, which is designed to allow an overview of the occurrence of certain properties in multiple groups at a glance.

Table B: Characteristics of the tables in the four groups of manuscripts.

| *characteristic* | **F** | **H** | **$CC_1C_2$** | **YLB** |
|---|---|---|---|---|
| type of sine table | every degree | every degree | every degree | **Y**: every degree, **L**: every minute, **B**: both tables |
| arguments of the tangent table | up to 45° | up to 90° | up to 90° | up to 60° |
| preliminaries of the mean motions: | | | | |
| – different arrangement, with al-Battānī's epoch positions | × | – | – | – |
| – rotations of motions in 20 years | × | × | × | – |
| – headers *adwār* 'rotations' | – | × | × | – |
| – mean motions in 20 years rounded to fifths | – | × | × | – |
| arguments of the equation of time | every 6° | every degree | every degree | **Y**: every 6°, **LB**: every degree |
| mean motion tables: | | | | |
| – squeezed onto a single page | – | – | × | **YB**: lunar nodes |
| – values for the months in the early Persian calendar | × | × | – | × |
| – values for 600 single years | – | – | – | × |
| – more accurate computation of numerous subtables | – | – | – | × |
| – daily mean motions for Mars different from al-Battānī | × | × | × | – |
| – equation of time expressed in lunar mean motion | – | – | – | × |
| omission of tabular differences from all planetary equations | – | × | – | – |
| correction of true position of Mars | – | – | × | × (but not in **Y**) |
| different variation of Mars | – | – | × | – |
| adjustment of 16 values in the equation of centre for Mercury | × | – | – | – |
| lunar latitude | values to minutes | values to minutes | values to minutes | values to seconds, with differences |
| planetary latitudes and stations | 5 tables | 5 tables | 2 tables | 5 tables |
| first and second declinations | combined, with tabular differences | combined, with tabular differences | separate | combined, with tabular differences (**L** has both tables) |
| right ascensions | inaccurate | inaccurate | inaccurate | accurate table |
| oblique ascension and equation of daylight | 36°, separate | 36°, separate | 35;30°, combined | 35;30°, separate tables |
| maximum equation of daylight | unfilled table | – | – | – (but in toc of **L**) |
| conjunctions and oppositions | – | – | – | × |
| extensive table of prorogations | – | – | – | × |
| table of fixed stars for the year 301 Yazdigird | 48 stars | 48 stars | 60 stars | 48 stars (unfilled in **L**) |

Characteristics found only in manuscript **F**

- Table 10: The tangent table has arguments up to 45°. (A copy of this table is included as a second tangent table in manuscript **L**.)
- Table 12: **F** arranges the table of preliminaries for the mean motions differently and is the only manuscript to include a subtable for al-Battānī's mean positions at the Seleucid and Hijra epochs. It also contains a much larger set of explanatory texts in between the subtables and in the margins.
- Table 36: An adjustment by one or two minutes of 16 tabular values around the maximum of the first equation of Mercury, but not of the corresponding tabular differences.
- Table 47: A table for the maximum equation of daylight as a function of geographical latitudes from 16 to 45° (left unfilled, mentioned by **L** in its table of contents but not included).

Characteristics found only in manuscript **H**

- *Planetary equations*: The tabular differences are omitted from the tables of lunar and planetary equations.*

Characteristics shared by manuscripts **FH** but not found in any others

- Tables 46 and 48: The oblique ascension and the equation of daylight are tabulated for a latitude of 36° (for Raqqa or the fourth climate in general).

Characteristics shared by manuscripts **CC$_1$C$_2$** but not found in any others

Note that most of these characteristics, indicated by asterisks, are of the kind that did not require the intervention of an astronomer who knew the exact details of the tables (like Kūshyār himself), to make the changes.

- Table 30: The variations of Mars at nearest and furthest distance have significantly different values from the other five manuscripts.
- Tables 38–42a: **CC$_1$** have one table for the latitudes of the five planets (with a shared column of interpolation coefficients) and one for their stations.*
- Table 43: **CC$_1$** have separate tables for the first and for the second declination, with the tangent of the declination placed in between. Judging from the table of contents, **C$_2$** will have had the same arrangement. A copy of these separate tables is also found in **L** (inserted in a different part of Book II from where the combined table is found).*

- Tables 46a and 48a: **C** has (and **$C_1C_2$** most likely had) a combined table for the oblique ascension and the equation of daylight, in which each value of the equation of daylight appears for four symmetric arguments ($x$, $180 - x$, $180° + x$ and $360° - x$ for every $x \in [0,90]$).*
- Table 55a: The star table in **$CC_1$** lists 60 stars on a single page, omitting some of the 48 stars from the table in **FHYB** but including a set of stars not found in those manuscripts.

Characteristics shared by manuscripts **$CC_1C_2$** but also found in **F** and/or **H**

- Table 9: The tangent table in **$HCC_1$** has arguments up to 90° (for the table in **C**, see Plate 5).
- Table 12: **$FHCC_1$** include a column of full rotations in the subtable for the mean motions in 20 years; in **$HCC_1$** this column has a header: *adwār* 'rotations'.
- Table 12: In **$HCC_1$** the mean motions in 20 Syrian years are rounded to sexagesimal fifths.*
- Tables 28 and 29: In **$FHCC_1C_2$** the daily mean motions underlying the mean longitude and mean anomaly of Mars (but not the positions at the Yazdigird epoch) are essentially different from those of al-Battānī that were used in **YLB**.
- Table 37: **$FHCC_1C_2$** have a table for the lunar latitude to minutes only and without tabular differences (note, however, that the table in **FH** has arguments 1–90°, whereas the table in **$CC_1C_2$** has arguments 1–180° with symmetrical values for arguments beyond 90°).*
- Table 45: **$FHCC_1$** have an inaccurate table for right ascensions with thirty errors of $\pm 1'$ and two errors of $\pm 2'$ in the first quadrant. A copy of this table is also found among the additional tables in **L**.

Characteristics shared by manuscripts **YLB** but not found in any others

- Table 9: The tangent table has arguments up to 60°.
- Mean motion tables:
  - A value for 600 years has been added to the subtable for single years in all tables.*
  - Many of the subtables were computed anew in a more accurate way and thus show systematic differences with respect to the other witnesses.
  - Table 17: The table for the lunar mean longitude incorporates a subtable for the equation of time expressed in lunar mean motion.

- Table 37: The table for the lunar latitude has values to seconds and tabular differences, whereas all other manuscripts have a table with values to minutes without differences.
- Table 45a: The table of right ascensions is much more accurate than the one in the other manuscripts and has only eight errors of ±1′ in the first quadrant.
- Tables 49$^{bis}$: A table of conjunctions and oppositions is included for the calculation of the position and time of true syzygies.
- Table 53: A version of the table of prorogations with values for every day of a year is provided in addition to the small table found in all manuscripts of the *Jāmiʿ Zīj*.

Characteristics shared by manuscripts **YLB** but also found in **FH**

- Tables 38–42: The latitudes and first station of each planet are given in a single table (rather than giving one table for all planetary latitudes and one table for all stations as in manuscripts **CC$_1$**).*
- Table 43: The first and second declinations are combined in a single table with tabular differences.*
- Tables 46 and 48: The oblique ascension and the equation of daylight (for latitude 36° in **FH** and for latitude 35;30° in **YLB**) are given in two separate tables (rather than being combined as in **CC$_1$**).*
- Table 55: The star table lists 48 stars on two pages (in **L** it has been left empty), which are not all included in the table of 60 stars found in manuscripts **CC$_1$**.

Characteristics shared by manuscripts **YLB** but also found in **CC$_1$C$_2$**

- Tables 46a and 48a: The oblique ascension and the equation of daylight are tabulated for a latitude of 35;30° (Rayy?) instead of for 36° as in **FH**.

Mixed characteristics

I will refer by ‘mixed characteristics’ to the properties of the eight manuscripts that cross the boundaries of the three or four groups that have been outlined above.

- Tables 8 and 8b: **L** includes the extensive sine table with values for minutes of arc, whereas **FHCC$_1$Y** only give a sine table with values for every degree. **B** contains both tables.
- Table 15: **FY** give the equation of time for every six degrees with interpolation coefficients, **HCC$_1$LB** tabulate the equation of time for every

degree of the ecliptic on the basis of the interpolation coefficients found in **FY**.

- Table 30b: **CC$_1$C$_2$LB** include a table for correcting the true position of Mars, which is not contained in **FHY**. Only **C** includes instructions for the use of this table (**L** omits these instructions, whereas the chapter concerned is not contained in manuscripts **C$_1$**, **C$_2$** and **B**).

### I.7.3. The origin of the different versions of the tables in the *Jāmiʿ Zīj*

On the basis of the above overviews of the differences between the witnesses, we can conclude that there is a clear division of the eight manuscripts that include the tables of the *Jāmiʿ Zīj* into three or four groups. Whereas the tables in manuscripts within each of the groups **FH**, **CC$_1$C$_2$** and **YLB** obviously have more characteristics in common than manuscripts from two of these groups, each pair of groups also shares properties not found in the third group. Although **H** is very close to **F**, it follows all other manuscripts with regard to the layout and contents of the table of preliminaries for the mean motions, and it joins **CC$_1$C$_2$** in tabulating the first tangent for arguments 1–90° (as compared to 1–45° in **F** and 1–60° in **YLB**). This is why I have not made a definite decision on whether to make **F** and **H** two separate 'groups' or a single one.

In this section I will gather information that allows us to judge which of the variant tables found in the eight manuscripts of the *Jāmiʿ Zīj* may be assumed to be the original ones, and which variant tables can be assumed to derive from Kūshyār himself. In this way I also hope to be able to establish which of the groups of manuscripts may be considered as representing the earliest version of the *Jāmiʿ Zīj*. I will take it for granted that the tables from the basic set of 55 tables included in all complete manuscripts of Book II (here edited in Part II), including at least one version of all tables of which different versions are found in the three or four groups, stem from Kūshyār. This is a reasonable assumption since, in spite of the presence of some 'mixed characteristics' (see above), the tables in all manuscripts form a coherent and consistent set that is in agreement with the extant tables of contents of Book II and the instructions in Book I. In fact, practically every type of table included in the basic set is also referred to in Book I, but no others.

For each of the tables of which different versions are contained in the eight manuscripts, I will now discuss whether we may assume that one of the versions was earlier than the other(s), and which versions can be reliably associated with Kūshyār.

- Tables 8 and 8a: The sine table with values for every degree appears in all manuscripts of Book II of the *Jāmiʿ Zīj* except **C$_2$** (which preserves only the planetary tables) and **L** (which includes only the alter-

native sine table). A copy of this table in the Escorial manuscript of the ninth-century *Mumtaḥan Zīj* is attributed to Kūshyār. But since this manuscript contains a thirteenth-century recension of the original work, this may at most be taken to mean that the table was included in certain manuscripts of the *Jāmiʿ Zīj*. The sine table with values for minutes, contained in **L** and inserted as an additional table in **B**, is explicitly attributed to Kūshyār in the *Dustūr al-munajjimīn*, which also incorporates many other text passages and tables from the *Jāmiʿ Zīj* with generally correct attributions. Instructions for the use of this specific table are included in the main text of Book I in manuscript **L**. We have no concrete evidence to suggest which of the two sine tables was earlier, but it seems plausible that Kūshyār would have first computed a small table and only later on expanded it to a larger set of arguments; and similarly, for some other types of tables, that he may first have computed a less accurate version, which he later replaced by a more accurate one. In both situations the new computation required Kūshyār to carry out more extensive calculations but resulted in a table that was far more convenient for the user.

- Table 10: The tangent table with arguments up to 45° in manuscripts **F** and **L**, which is also found in the *Dustūr al-munajjimīn* with an explicit attribution, can be firmly associated with Kūshyār. The explanatory text in Book I in manuscripts **FYL** states explicitly that the tangent was tabulated only up to 45°, although the table in **YL** in fact has arguments up to 60°. As well as in $\mathbf{CC_1}$, the table with arguments up to 90° is included in manuscript **H**, which is based on an early copy of an autograph. The values up to 45° are in full agreement in all seven witnesses of the *Jāmiʿ Zīj* and in the *Dustūr* except for obvious scribal errors. It is thus plausible that all three versions of the tangent table stem from Kūshyār himself.
- Table 12: The table of preliminaries for the mean motions in **F** has a different format from the other six witnesses. It includes al-Battānī's epoch positions for the Hijra epoch and the Seleucid year 931 (AD 619/20) in a slightly different arrangement and has a much larger number of marginal notes. The Byzantine epoch positions and some of the additional instructions were needed to derive Kūshyār's mean motion parameters from al-Battānī's *Ṣābiʾ Zīj*, but were no longer necessary once the parameters had been calculated. It thus seems plausible that the extensive table of preliminaries in **F** was the original one and the copy in the other six witnesses a later simplification. Since this simplification appears in all three groups, including an early copy of an autograph (**H**), it is likely to have originated with Kūshyār himself.

- Table 15b: As shown in Section IV.6, this subtable for the equation of time expressed in lunar mean longitude, included as part of the table for lunar mean longitude in manuscripts **YLB**, does not fit in very well with the separate table for the equation of time (expressed in minutes and seconds of time) that is found in every manuscript of the *Jāmiʿ Zīj*. Therefore there is no particular reason to assume that this table stems from Kūshyār himself.
- *Mean motion tables*: As discussed in detail in Section IV.5.2, in many of the subtables we find numerous small but systematic differences between **FHC$C_1$$C_2$** on the one hand and **YLB** on the other. Except for the mean motions of Mars (cf. the next item), these differences have no astronomical significance, only a theoretical, mathematical one. The computation of both sets of tables required knowledge of Kūshyār's epoch values and daily mean motions listed in Table 12 in all manuscripts, but not of al-Battānī's data from which these parameters were derived (as included only in **F**). The tables in **YLB** show a clearly better agreement with an accurate modern recomputation than the tables in **FHC$C_1$$C_2$**. Since it seems unlikely that someone would compute basically correct tables anew in a less accurate way, it is plausible that the tables in **FHC$C_1$$C_2$** were the original ones and hence stem from Kūshyār. This hypothesis is supported by the fact that the original version of Table 12 is contained in **F**. It seems unlikely that an astronomer other than the original author would have made the effort to calculate all the mean motion tables anew in order to make the minor theoretical improvements that we find in the tables in **YLB**. I therefore assume that these adjustments also stem from Kūshyār himself (but cf. the next item).
- Tables 28/29: As discussed in detail in Section IV.8, the mean motions in longitude and anomaly of Mars in **FHC$C_1$$C_2$** were computed from daily parameters clearly different from al-Battānī's and leading to a difference of 1° in Kūshyār's time. On the other hand, the recomputed tables in **YLB** are based on al-Battānī's daily mean motions for Mars and are hence in full agreement with the way in which all other mean motion tables in **YLB** were computed. One might expect that Kūshyār first calculated his tables for Mars on the basis of al-Battānī's parameters and only later adjusted them on the basis of new observations or insights, thus challenging my assumption above that the mean motion tables in **FHC$C_1$$C_2$** were earlier. However, the interplay of the various differences in the Mars tables (mean motions, correction of the true position, adjusted variation tables) and their relation to Kūshyār's own observational report of a Saturn-Mars conjunction is so complicated

that further research is necessary before any definite conclusions can be drawn.

- Table 30b: The table for the correction of the true position of Mars as found in **$CC_1C_2LB$** is attributed to Kūshyār by al-Bayhaqī (see pp. 20–21) and, in an adjusted version of it, in the anonymous late thirteenth-century *Sulṭānī Zīj* (see footnote 118 on p. 430). Since the Mars tables in **FH** do not include the correction, these manuscripts may be assumed to represent an earlier version. Why the copyist of **Y** did not include the correction is unclear, and the question of whether the alternative values for the variation in the equation of anomaly found in the three Cairo manuscripts also stem from Kūshyār needs to be investigated further (cf. the commentary in Section IV.8).
- Table 37: As shown in Section IV.9, the values to minutes in the lunar latitude table in **$FHCC_1C_2$** were most likely rounded from the ones to seconds in **YLB**. This implies that the table to seconds must have been available when the table to minutes was copied in **$FHCC_1C_2$**, and that we cannot conclude that the more accurate table in **YLB**, which also provides tabular differences, was a later development.
- Table 45: The table for the right ascension in **$FHCC_1$** (with 30 errors in every quadrant caused by the use of linear interpolation on intervals of six degrees) is significantly less accurate than the table in **YLB** (with eight errors of one minute). The latter table was derived from al-Battānī's table for the normed right ascension and is explicitly attributed to Kūshyār in the *Dustūr al-munajjimīn*. We cannot decide which of the two versions of this table was earlier. One might have expected Kūshyār to start by copying al-Battānī's table rather than by computing one of his own; or he may have realised later on that the table found in **$FHCC_1$** was less accurate than al-Battānī's and replaced it.
- Tables 46 and 48: The tables for the equation of daylight and the oblique ascension in **FH** are for a geographical latitude of 36° (for Raqqa or the fourth climate in general), and those in **CYLB** for latitude 35;30° (possibly intended for Rayy). The table for the equation of daylight for 36° was computed by means of the same type of linear interpolation as the right ascension table in **$FHCC_1$**. The oblique ascension table for 36° was computed by subtracting the equation of daylight for this latitude from the right ascension table included in **$FHCC_1$**. The table for the equation of daylight for 35;30° may have been computed as a homothetic version of a table for a different latitude (but not from Kūshyār's own table for 36°). The oblique ascension table for 35;30° is consistent with the equation of daylight but was further computed from a right ascension table for the Ptolemaic obliquity

23;51° (*sic*!) rather than from the accompanying, more accurate right ascension table derived from al-Battānī. This situation does not allow us to draw firm conclusions about the originality or sources of these tables. It seems likely that the tables for 35;30° were included in some versions of the *Jāmiʿ Zīj* for use in Rayy. Since their methods of computation and underlying obliquity are very different from what we have found in other tables of Kūshyār, they may well have been taken from a different source (possibly al-Khujandī?). Because the tables are explicitly mentioned with their latitudes in the text of Book I in manuscripts **FCYL** (although in **L** with the incorrect latitude 36°), it is reasonable to assume that both versions were intrinsic parts of the *Jāmiʿ Zīj* and thus were provided by Kūshyār himself.

- Table $49^{bis}$: The table of conjunctions and oppositions is found as an additional table in manuscripts **YB**, and instructions for its use are included in the main text of Book I in manuscripts **YL**. It is thus clear that this table was not part of the version of the *Jāmiʿ Zīj* represented by the groups **FH** and $\mathbf{CC_1C_2}$. The table has the peculiar range of vertical arguments 1, 2, 3, ..., 15, 18, 21, ..., 60 which we have seen in Kūshyār's sine table with values for minutes (Table 8a), but which is not found in any of the other *zīj*es that I have inspected. Furthermore, this table was copied in the *Dustūr al-munajjimīn* and is attributed there to Kūshyār. We may thus conclude that this table was most likely computed by Kūshyār himself.
- Table 53: Whereas all the witnesses contain the same small table of progations, manuscripts **YB** also include among their additional tables an extended version of this table with values for every single day of the year. This version is mentioned in the explanatory text of Book I in **L** as well as in a quotation from 'another copy' of the *Jāmiʿ Zīj* in **F** (see Section IV.12), which suggests that it was an early addition to the *zīj*. Thus it seems probable that this table also stems from Kūshyār himself.
- Table 55: The two star tables (in **FHYB** on the one hand and in $\mathbf{CC_1}$ on the other) give the same coordinates for the same epoch and the same other types of data (including the indications *q* or *qāṭiʿ* (lit. 'cutter') for malefic stars). Furthermore, they share several deviations from the *Almagest* and/or al-Battānī's star catalogue. Thus it is probable that both tables stem from Kūshyār. Since both tables include stars that are not found in the other one, we cannot venture any suggestions about which one was earlier.

Certain other variants in the tables, such as the addition of mean motion values for 600 single years, the expansion of the table for the equation of time to

every degree of the ecliptic and the rearrangement of the declination tables with the addition of tabular differences, could have been made by a later astronomer on the basis of information found in the original tables. But since these modifications appear in the coherent sets of tables together with the ones above that can be attributed to Kūshyār himself, it is probable that these changes are also due to him. However, the situation is different for the numerous changes in the layout of the tables found in the three Cairo manuscripts, which may very well have been carried out by a scribe receiving instructions from an astronomer unfamiliar with the underlying details of the tables.

To summarise, we had already established that the manuscripts of Book II of Kūshyār's *Jāmiʿ Zīj* contain a highly coherent and consistent basic set of tables that is in full agreement with the tables of contents found in six of the eight surviving manuscripts that contain Book II, and with the instructions in Book I. The tables in **FH** are generally correct with only small numbers of scribal errors. **F** was copied in AD 1150/1 and is thus the oldest extant manuscript including Book II. **H**, although relatively late, was copied from a copy of an autograph that included comments on the mean motion tables dating from the late eleventh century. Accordingly, both **F** and **H** represent an early stage of the manuscript tradition of the *Jāmiʿ Zīj*.

In addition, I have now argued that the version of the table of preliminaries for the mean motions included only in **F** was very probably the original one and that the set of mean motion tables found in **FHCC$_1$C$_2$** is likely to have been earlier than the more accurate set in **YLB**. It is also plausible that the table for the correction of the true position of Mars, which is not included in **FH**, was a later addition to the original tables. For the other tables with multiple versions in the eight manuscripts we have not been able to make definite decisions about which versions were earlier, but in several cases the versions found in **FH** and **CC$_1$C$_2$** have a smaller range of arguments or less accurate tabular values than those in **YLB**, making it plausible that the latter were expanded or newly computed versions of the former. Thus we may conclude that **F** most likely contains the earliest known version of Kūshyār's *Jāmiʿ Zīj*, that **H** is very close to **F** although it already includes some changes found in the other manuscripts, and that **CC$_1$C$_2$** and **YLB** are later versions in which several tables from the main set were replaced by alternative ones.

In most cases I have been able to show that the variant versions of tables from the basic set of 55 tables found in the eight surviving manuscripts of Book II are likely to stem from Kūshyār. Thus we may conclude with reasonable confidence that the three versions of the sets of tables in Book II of the *Jāmiʿ Zīj* that are represented by the groups **FH**, **CC$_1$C$_2$** and **YLB** can all be associated with Kūshyār himself (with the exception of the elementary changes in the layout of the tables in **CC$_1$C$_2$**), and that only the majority of the addi-

tional tables found at the end of manuscripts **YLB** were supplemented by later astronomers or copyists. The small differences and contaminations that we find in manuscripts **YLB** and to a lesser extent in **F** can be attributed to the use of multiple manuscripts by the respective scribes. In particular, marginal notes in **F**, $\mathbf{C_1}$ and **B** explicitly refer to variants in other manuscripts that were consulted by the copyist; the table of contents and the additional tables in **L** contain some elements that belong to the group **FH**, and **B** shows clear traces of collation with, on the one hand, a manuscript close to **L** and, on the other, a manuscript related to the group $\mathbf{FHCC_1C_2}$ (but with mean positions for Raqqa instead of the meridian of 90°, which might stem from the earliest stages of Kūshyār's work on the *zīj*).

The most difficult differences to explain are the ones listed under the category 'Mixed characteristics' in Section I.7.2 (see pp. 64–65). In all three cases (omission of the sine table with values for minutes of arc, tabulation of the equation of time on intervals of 6° with interpolation coefficients, omission of the correction of the true position of Mars), **Y** follows **FH** rather than **LB**. On the basis of the currently available information it cannot be decided whether Kūshyār also distributed an intermediate version of his *zīj* somewhere between **FH** and **LB**, or whether this mix-up was the result of the occasional use of a manuscript from the group **FH** by the scribe of **Y**.

# Part II

# Tables

Part II of this book provides a critical edition of the 55 tables included in Book II of Kūshyār ibn Labbān's *Jāmiʿ Zīj* on the basis of the seven available Arabic manuscripts in Arabic characters and the single Judaeo-Arabic manuscript. In order to make the edition accessible to readers who do not have Arabic, the numbers in the tables have been transliterated into Arabic-European numerals and the titles and headers as well as occasional explanatory texts for which the edited tables leave sufficient space have been translated into English. Some standard text elements appearing in many of the tables, especially subcolumn headers such as 'parts', 'degrees', 'minutes', 'fourth sexagesimal position (before the sexagesimal point)', etc. are not translated in full in the edited tables but have been rendered by symbols such as $^{p}$, $^{\circ}$, $'$ and $^{(4)}$. A full list of such symbols can be found on p. 257. Numbers of the signs from 0 to 11 (given in alphanumerical notation in the manuscripts) are printed in bold at the top of the columns in the edition. Note that these signs are often not identical to zodiacal signs, as in the case when they refer to mean and true ecliptic longitudes: they may signify groups of 30 degrees of several variables, including the mean and true centrum and anomaly. Tabular values in red are printed in italics in the edition. The Arabic text of all titles, headers, explanatory texts and the most important marginal notes (especially those that seem to have become part of the manuscript tradition) are critically edited in Part III, together with the tables of contents of Book II of the *Jāmiʿ Zīj* and the section on finding the original planetary equations that was inserted at the end of the book in some of the surviving manuscripts. Consequently, variants in the textual elements of the tables are generally *not* indicated in the apparatuses to the edition of the tables (for example, as variants of the translations), but can be found in Part III.

The vast majority of numbers in Islamic astronomical tables are written in the alphabetical *abjad* notation, in which the numbers 1 to 9 are represented by the letters *alif* ا, *bāʾ* ب, *jīm* ح, …, *ṭāʾ* ط, the numbers 10 to 90 by the letters *yāʾ* ے , *kāf* ک, *lām* ل, …, *ṣād* ص, and the numbers 100 to 1000 by the letters *qāf* ق, *rāʾ* ر, *shīn* ش, …, *ghayn* غ.[1] All other numbers up to 1999 can then

[1] For the *abjad* notation of numbers and the scribal errors to which its use may lead, see Irani, *Arabic Numeral Forms*; Destombes, 'Les chiffres coufiques'; King, *The Ciphers of the Monks*, Appendix C, especially pp. 295–97; Kunitzsch, *Der Sternkatalog*, vol. I, pp. 19–21; van Dalen, *Ancient and Mediaeval Astronomical Tables*, p. 7 and the table on p. 8; and soon also van Dalen, 'The Geographical Table'. Slightly different *abjad* numbers were used in the Maghrib and al-Andalus; for their origin, see Thomann, *Scientific and Archaic Arabic Numerals*.

Whenever I give *abjad* numbers in their original Arabic form (occasionally in the apparatuses of the tables in Part II and repeatedly in the edition of the textual elements in Part III), I use the form of the numbers that is most commonly found in the manuscripts, namely ɀ for 0, ح (rather than ج) for 3, ے (rather than ي) for 10, and ک (rather than ك) for 20. I omit the single dots on *bāʾ* (ب 2) and *zāʾ* (ز 7) and the two dots under *yāʾ* (ي 10) in compounds because they are hardly ever written in the manuscripts and are generally not needed to unambiguously identify the numbers.

be written as combinations of these base numbers, for example 1962 = *ghayn-ẓāʾ-sīn-bāʾ* غظسب. For larger numbers, various solutions are found in the manuscripts which have not yet been systematically inventoried (one of these is the use of *ghayn-ghayn* غغ for 2000). However, in astronomical manuscripts *abjad* numerals are only rarely used for numbers above 360, the number of degrees in a circle. For larger numbers, especially the numbers of days between calendar epochs given in chronological sections and the numbers of single or collected years in chronological tables, generally Hindu-Arabic numerals are used. Thus in the eight manuscripts of Book II of Kūshyār's *Jāmiʿ Zīj* we find Hindu numerals in particular in the arguments of the subtables for collected years in the tables displaying the numbers of days corresponding to years and months in the three main calendars (Tables 1–3), in which these arguments range respectively from 924 to 1736 Syrian years, 30 to 900 Arabic years and 28 to 840 Persian years. However, Hindu numerals are *not* used for the arguments of the table for *notae* in the Arabic calendar (ranging from 1 to 210) or for the arguments of the subtables for collected years of the mean motion tables (ranging from 1 to 581 Yazdigird).

The only other table in the *Jāmiʿ Zīj* in which Hindu numerals are used in a regular way is the one for the tangent for radius of the base circle 60 (Table 10, cf. Plate 5). In **F** this table has arguments from 1 to 45°, which implies that the tangent never exceeds 60. In manuscripts **YLB** the tangent table has arguments up to 60°; here **Y** writes the integer part of the tangents for arcs 45° and larger consistently in *abjad* numerals, **B** does this in Hindu numerals, and **L** writes only the integer parts of the tangent of 45° (i.e., 60) as a Hindu numeral. In **HCC$_1$** the tangent table has arguments up to 90°, so that both the tangent and the tabular differences run into the thousands (no tangent table is contained in manuscript **C$_2$**). In **H** the integer part of the tangent is displayed in Hindu numerals from argument 61 onwards (where it happens to exceed 100 for the first time), and in **CC$_1$** from argument 46 onwards. In all three sources the integer part of the tabular differences is displayed in Hindu numerals as soon as it exceeds 60, i.e., from argument 83° onwards. Note that in the shadow tables for gnomon lengths 7 and 12 (i.e., the cotangents in Table 11) the integer part of the tabular value is at most 687 and is written in *abjad* numerals in all sources.

A highly uncommon use of Hindu numerals can be found in manuscript **B** in the subtable for months of the tables for the lunar mean anomaly (Table 18) and the mean anomaly of Saturn (Table 23). Here the zodiacal signs, degrees and minutes of the four values for the old version of the Persian calendar are written with Hindu numerals in red, whereas the values for the main version (in black) are in the usual *abjad* notation (see Figure 1).

In my edition, the variants of arguments and tabular values found in the eight manuscripts are given as footnotes to the tables. The apparatus entries give

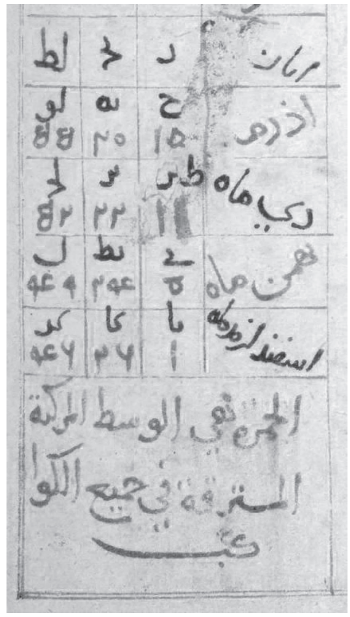

Figure 1: Extract from the table for lunar mean anomaly in manuscript **B** with tabular values in Hindu numerals, © Staatsbibliothek zu Berlin – Preußischer Kulturbesitz, Orientabteilung, Ms. or. quart. 101, p. 76.

the sigla of the witnesses containing a variant, followed by the deviating digit(s) found in these witnesses. These digits are indicated by the same abbreviations used in the subcolumn headers of the tables (see p. 257). For example, the entry ‘[1] **CC$_1$B** 47‴’ in the apparatus to the first page of the solar mean motion (Table 13) indicates that the witnesses **C**, **C$_1$** and **B** have the value $11^s$29;45,47 for 1 extended year instead of the mathematically correct $11^s$29;45,46. The apparatus entry ‘[24] **C$_1$** $11^s$ 37″’ indicates that **C$_1$** has the value $11^s$12;37,45 (with one common scribal error and one less common error) for 200 single years instead of $10^s$12;34,45, as in all other witnesses. The following abbreviations are used in the apparatuses:

| | |
|---|---|
| add. | ‘adds’ or ‘inserts’ |
| om. | ‘omits’ |
| ill. | illegible |
| dam. | illegible due to any type of damage to the page (including stains, wormholes, parts of the paper that were eroded or cut off, etc.) |

The variants are always given in the order **FHCC$_1$C$_2$YLB**, thus grouping the related witnesses **CC$_1$C$_2$** and **YLB** and reflecting that the three Cairo manuscripts are somewhat closer to **F** and **H** than **YLB** (cf. Section I.7). The following criteria are used to decide on a particular reading in cases where the sources do not agree:[2]

1) *Scribal errors*: Most of the variants in numerical values in manuscripts of Arabic or Persian astronomical works are the result of scribal mistakes. Due to the similarity of several letters in the Arabic alphabet and the fact that some of these letters are only distinguished by the presence or absence of one, two or three dots, numerous confusions between numbers are plausible, which in most cases can be easily recognised in the variants.[3] When deciding on the preferred reading for a particular tabular value, the possibility that some of the variants in the manuscripts are due to scribal confusions is always tacitly considered. For example, if in a particular tabular value a majority of the witnesses has a digit 7 ز, which is also plausible on mathematical grounds (cf. below), a digit 50 ن

[2] For a similar systematic approach to the edition of mathematical tables in Arabic manuscripts, see van Dalen and Pedersen, ‘Re-Editing the Tables’, pp. 407–10.

[3] For overviews of possible scribal errors, see the references given in footnote 1 on p. 75.

in the same position in a minority of the sources is highly likely to be a scribal error for 7. Errors resulting from more complicated types of scribal mistakes such as the miscopying of a range of tabular values into incorrect rows of the same column, into adjacent columns, or even into a different table, are more difficult to recognise individually. Whenever the cause of such complex scribal errors can be detected, they will be indicated as a single entry in the apparatus and further comparisons of the tabular values will be made on the basis of a restoration of the wrongly copied values to their correct positions. (Further details of such errors and the way in which they are included in the apparatuses will be discussed below.)

2) *Manuscript evidence*: The primary purpose of my edition is to reconstruct the original tables from Kūshyār's *Jāmiʿ Zīj*, of which, as we have seen in Section I.7.3, the Fatih manuscript and the Judaeo-Arabic copy can be considered to be the most accurate representations. For this reason preference is generally given to values from **FH**. Since for the vast majority of the tables the three Cairo manuscripts have basically the same values as **FH**, evidence from $\mathbf{CC_1C_2}$ is taken to support **FH**. This implies that whenever **F** or **H** has a value that is mathematically incorrect (cf. below), and the corresponding value in the Cairo manuscripts is mathematically plausible, I will accept the reading from $\mathbf{CC_1C_2}$, also in cases where the value from **F** or **H** is *not* an obvious scribal mistake for the one in the Cairo manuscripts. On the other hand, I will prefer a mathematically correct reading from **F** or **H** even if a majority of the other manuscripts have a different, mathematically less plausible reading. As we have seen, **YLB** have numerous systematic deviations from the base version of the *Jāmiʿ Zīj* as represented by the Fatih, Judaeo-Arabic and Cairo manuscripts, especially in the mean motion tables. In several cases these deviations are so numerous that I had to decide to edit the (sub)tables from **YLB** separately. In many other cases the deviations are indicated by so-called 'general variants' covering a range of interrelated variants in a single entry in the apparatus. For the deviations in the mean motion tables both the general variant in the apparatus and the affected tabular values are marked by small asterisks. Although in many tables **YLB** generally follow $\mathbf{FHCC_1C_2}$, the possibility of systematic deviations should always be kept in mind when judging whether in a particular case **YLB** can be expected to support the readings from **FH** (and $\mathbf{CC_1C_2}$).

3) *Mathematical correctness*: While the correctness of prose text primarily needs to be judged by grammatical and semantic criteria, for numbers in tables (as well as numbers in texts that are the result of computations) mathematical criteria provide a powerful additional tool. Many

tables in the *Jāmiʿ Zīj*, including all planetary equations, have an internal mathematical check in the form of columns of tabular differences. These tabular differences are generally in full agreement with the values for the tabulated function and can therefore be used to correct scribal errors in the main values. In some cases it can be seen that the tabular differences were checked against the table by a scribe or collator of the manuscript.[4]

For many tables a modern recomputation can help to judge which variants are historically the most likely. Usually the algorithm by which a medieval table was computed can be approximated quite well by the modern equivalent of the tabulated function: that is, the table is quite accurate by modern standards. Examples in the *Jāmiʿ Zīj* are the sine table, which has only a single error of one unit in 90 values, and the mean motion tables, which were accurately computed from al-Battānī's parameters as given by Kūshyār in Table 12 (but see pp. 57–58 and the commentary in Section IV.5.2 for a discussion of the variations in these tables between manuscripts **FHC$C_1$$C_2$** on the one hand and **YLB** on the other). In such tables, we expect every tabular value to be very close to the exact modern value, and witnesses further removed to be subject to an error. In other tables in the *Jāmiʿ Zīj*, such as the solar declination, around half of the tabular values show an error with respect to a modern recomputation, but all these errors are at most three units in the least significant digit (here, seconds). In such tables, we expect all tabular values to have at most relatively small errors, and witnesses with significantly larger errors are likely to be incorrect.

In further tables of the *Jāmiʿ Zīj*, such as the solar equation, most of the tabular values are in error, and in some parts of the table the errors are as large as 17 units in the least significant digit (here again, seconds). However, the errors tend to occur in clusters, i.e., groups in which they have the same sign and in which their size changes more or less continuously (producing small humps in a graphical representation of the errors).[5] In such cases we expect the tabular values to fit within

[4] The tables in the *Jāmiʿ Zīj* show several cases where some of the manuscripts deviate both in a tabular value and consistently in the corresponding tabular differences. See, for example, the sine of 23° with the corresponding differences for 23 and 24° in Table 8, the versed sines of 13, 67 and 113° in Table 9, the tangent of 37° in Table 10, the lunar first equation for $5^s28°$ in Table 20, the first equation of Jupiter for $8^s25°$ in Table 27, etc. All these cases are explicitly mentioned in the apparatuses to the tables.

[5] Such patterns may often be assumed to stem from the use of linear or second order interpolation on intervals of, for example, 5, 6, 10 or 15°. See, for example, Van Brummelen, *Mathematical Tables*, pp. 13–14 (on distributed linear interpolation), 24–26, 40–45 and numerous examples throughout; van Dalen, *Ancient and Medieval Astronomical Tables*, pp. 11–12, 76–

> these particular error patterns.[6] For example, if the errors before and after an uncertain value are respectively +2, +5, +8 and +12, +11 and +8 units, we may expect a value with an error of approximately +11 units for the uncertain one.
>
> Note that it is possible that, by using these mathematical criteria, in some cases a value is accepted for the edition that was never used by Kūshyār himself: namely, if the original scribal or computational error already occurred in his notes or autograph copy of the *zīj*. But this is a risk that the mathematician-editor is willing to take.

Whereas in general all relevant variants are mentioned in the apparatuses, I have not included erroneous digits that were corrected by the main hands of the eight manuscripts. In cases in which a reading was not entirely clear, due to a correction, an ink blob, damage to the page or uncertain shapes of letters, I have only inserted an entry in the apparatus if it seemed impossible that the intended number was equal to the accepted reading.

Although apparatuses based on footnotes make the tables somewhat more cumbersome to read, especially if there is a very large number of variants, I have opted for this system because it allows me to combine certain types of variants that affect multiple arguments or tabular values in a single apparatus entry. Typical examples of these are what the late Fritz S. Pedersen dubbed a 'slide' and what other scholars (including myself) have referred to as 'shifts'.[7] These occur when the scribe, while copying a column of digits or values, either repeated or skipped one or more digits or values.[8] Especially in the case of very slowly changing tabular values (for example, the tables of interpolation minutes for the lunar and planetary equations) such mistakes were often noted only when the scribe came near the end of the column. If he had mistakenly skipped some values, resulting in an 'upward slide', he would either continue with the correct values, thus repeating some he had already copied, or fill up the empty spaces with zeroes or other nonsensical values. If he had mistakenly repeated some values, resulting in a downward slide, he would continue

78, 187–91 and some further examples. See also Van Brummelen, 'The Numerical Structure'; van Dalen¸ 'On Ptolemy's Table', esp. pp. 119–21 and 128–33, and van Dalen, 'Islamic and Chinese Astronomy', pp. 351–53. Cf. also Dorce, 'The *Tāj al-azyāj*'.

[6] As a matter of fact, this general formulation also pertains to the two previously mentioned cases of generally correct tables with occasional errors and tables with mostly small errors and occasional larger ones.

[7] See, for example, Pedersen, *The Toledan Tables*, Part I, pp. 30–32, and van Dalen, 'The *Zīj-i Nāṣirī*', pp. 851–852. To avoid confusion with the 'shift' involved in displaced tables for the planetary equations, I have since adopted Pedersen's use of the term 'slide'.

[8] As we will see later on, in some cases copyists proceeded along rows or even by blocks of tabular values. To avoid complicating the following explanations unnecessarily, I have formulated them for the more common case of copying by row.

with the correct ones, thus possibly causing a jump in the tabular values that the tabulated function would not normally display. Other scribal mistakes that affect multiple tabular values are the copying of an entire erroneous column of digits or values, possibly even from a different table.[9] Rather than overloading the apparatuses with individual, incoherent variants for each of the tabular values involved in such mistakes, in my edition they are represented by a single footnote mark and apparatus entry. The footnote mark is placed at the first value involved, accompanied by one of the symbols ↓, → or ↓→, in order to indicate whether the range of erroneous values is part of a column, a row, or is two-dimensional (i.e., a block). The last value of the range is marked respectively with ⊥, ⊣ or ┘. The apparatus entry specifies which arguments or values are involved and the type of the error (for examples, see below). Note that not all values that are part of a slide are necessarily erroneous. In certain parts of tables for slowly changing functions, such as the interpolation minutes and in some cases the tabular differences of the lunar and planetary equations, three or four consecutive values may be identical. This implies that an upward or downward slide of one row affects only every third or fourth value in the column. In the apparatuses, such slides will nevertheless be indicated as a single variant whenever they can be distinctly recognised as a slide and cause errors in at least three different values; if only two values are affected, the errors will be entered separately.[10]

Slides and other variants involving a range of arguments or tabular values are indicated in the apparatuses as follows:

⟨sigla⟩ ⟨range of arguments or tabular values⟩: ⟨error specification⟩

[9] Note, for example, that **C** copies into the first twelve rows and the entire last column of tabular differences on the second page of the table of the versed sine (Table 9) the corresponding values from the first page. Furthermore, in the second column of tabular differences of the tangent (Table 10), **Y** introduces two consecutive slides in the minutes, and instead of the correct seconds it copies the minutes of the second tangent for gnomon length 7 from Table 11 with numerous additional scribal errors. The columns of tabular differences for signs 3 and 4 in the table for the first equation of Venus (Table 33, p. 188) in **L** in fact contain the columns of differences for signs 2 and 3. Finally, in **C** the column of tabular differences for sign 11 in this same table (p. 189) contains the corresponding differences from the table of the second equation, which are of a different order of magnitude.

[10] An example of a slide affecting only a small part of a range of tabular values can be seen in the fifth column of the table for the first equation of Mars (Table 30 on p. 172): the tabular differences in the Fatih manuscript were apparently slid upwards by two rows for arguments $4^s7$–21° (or possibly even for arguments $4^s4$–22°), but only six values out of these 15 (or even 19) are now erroneous. Several types of slides, including a vertical one, can be seen in the table for the variation at nearest distance of Venus (Table 33 on p. 192). One of the very few examples of a simultaneous slide of tabular values and their differences can be seen in the columns for sign 9 of the lunar second equation (Table 20 on p. 137).

Here the range of arguments or tabular values can be specified as follows:

| | |
|---|---|
| arguments 10–20: | the variant affects only a column of arguments |
| $4^s$ 10–20°: | the variant affects the tabular values for arguments $4^s$ 10–20° |
| $4^s$ 10–20° (minutes): | the variant affects only the minutes of the tabular values for arguments $4^s$ 10–20° |
| 2–$3^s$ 10–12°: | the variant affects the tabular values for arguments 10–12° in the columns for signs 2 and 3 |

In a mean motion table the subtable involved is specified as follows:

| | |
|---|---|
| arguments 10–20 hours: | the variant affects the arguments of the subtable for hours |
| collected years 301–381: | the variant affects the values for 301, 321, …, 381 Yazdigird in the subtable for collected years |
| 10–20 hours: | the variant affects the tabular values for 10–20 hours in the subtable for hours |
| longitudes 80–90°: | the variant affects the values for longitudes 80–90° in the subtable for longitude differences |

Note that the footnote marks, arrows and end symbols also make clear which arguments or tabular values are affected by any composite variant.

The slides are specified as follows in the apparatus entries (*n* is a positive integer number):

| | |
|---|---|
| slide[+*n*] | upward slide of *n* rows |
| slide[−*n*] | downward slide of *n* rows |
| slide[+*n* cols] | forward horizontal slide of *n* columns (to the *left* in the original Arabic table, to the *right* in the edited table) |
| slide[−*n* cols] | backward horizontal slide of *n* columns (to the *right* in the original Arabic table, to the *left* in the edited table) |

For example, the variant '**L** longitudes 90–98°: slide[+2]' in the second half of the solar mean motion table (Table 13 on p. 108) indicates that in the Leiden manuscript the values for longitudes 92–100° appear in the rows for arguments 90–98° (two rows too high), most probably because the scribe skipped the values for 90 and 91° (which in this case can be easily explained because the values for 89° and 91° are the same).

Besides slides and miscopies of entire columns, the apparatuses also include so-called 'general variants' that affect multiple arguments or tabular values in a less coherent way. Again the purpose of such variants is to show structural differences between the witnesses rather than simply to give any variant individually. The 'general variants' are expressed in words at the beginning of each apparatus and may, for instance, be of the following forms:

'**F** frequently omits the dots on *nūn* in the tabular differences.'

'**B** writes the names of the zodiacal signs instead of *abjad* numbers.'

'* In **YLB** the values for 7, 13, 20, 26, 33, 39, 46, 52, and 59 hours are one minute smaller.'

In the third example the variants are fully specified and the values in each source can be reconstructed from the apparatus entry; the affected tabular val-

ues are marked by a small asterisk. In the first example the values involved are all tabular differences with an occurrence of *nūn* (in digits from 50 to 59) in one of their sexagesimal positions, with unspecified exceptions. In neither case are the variants included in the apparatus as individual variants, nor are the exceptions. However, the omission of the dots on *nūn* will be included in the apparatus whenever the value concerned also has another variant, if another witness likewise omits the dot, or if the omission of the dot confirms an error in other witnesses.[11] Note that the general variant in the first example above (cf. Table 8 on p. 96) implies both that tabular differences in **F** that do carry the dot on *nūn* are not specifically indicated and that the omission of dots on *nūn* in the sine values (as opposed to the tabular differences) would be indicated.

In many cases one or more of the arguments or tabular values affected by a slide or general variant may have further individual variants. To indicate this, the sigla in the individual variants are marked with a plus sign. A plus sign *preceding* a siglum indicates that the tabular value concerned is also affected by a previously mentioned general variant or slide in a *different* witness, while a plus sign *following* a siglum indicates that it is also affected by a general variant or slide in the *same* witness.[12] For example (from the second half of the table of the solar mean motion, Table 13 on p. 118):

* In **YLB** the values for 16, 22, 34–35, 40–42, ..., and 58 hours are one second larger.
[21] **L** longitudes 90–98°: slide[+2]
[23] +**C** 57′ **Y**+ 55′ ... [value for 22 hours, also affected by the general variant in **YLB**]
[28] +**B** 29″. [value for longitude 97°, also affected by the slide in **L**]

Several of the eight witnesses show copying characteristics that may lead to specific kinds of errors. In particular, **C** has two peculiarities that lead to several types of general variants and numerous apparatus entries. Most strikingly the scribe of this manuscript omitted the vast majority of dots on *nūn* from arguments and values in nearly all tables, causing the digits 51, 52, ..., 59 to be read as 11, 12, ..., 19.[13] In some cases it can be recognised that the dots were supplemented, apparently by a different hand, and often clearly thicker than

[11] As an example of this latter case, see the tangent for argument 37° in Table 10 on p. 114. The variant 18″ in **HC** is here taken to confirm the variant 58″ found in all other sources except **L**$_1$. Although the tabular differences in these same sources confirm the variant 58″, I have nevertheless preferred the mathematically correct 45;12,48 since the table otherwise contains only errors of one second, and occasionally two.

[12] The plus sign is not added if the general variant affecting the same tabular value is of an unspecific type, such as '**C** omits most dots on *nūn*.'

[13] Whereas composite *abjad* numbers such as نا (51) can be easily confused with يا (11, often written without the dots), the shape of an isolated or final *nūn* ن (50) is unambiguous and clearly different from *yāʾ* ے (10). However, ن (50) is often confused with *zāʾ* ز (7) and, especially if it is written without dot, with *lām* ل (30).

the dots of the main hand.[14] However, it is my impression that this occurs especially in parts of tables where the dotless *nūn* can easily be recognised as representing 50 rather than 10, for example because the tabular values increase very slowly. Thus it seems possible that the corrections were made on the basis of an inspection of the tabular values themselves rather than on a collation with a better copy of the *Jāmiʿ Zīj*. For tables in which the dots on *nūn* are generally correctly supplied, the occasional omissions of the dots will appear in the apparatus as variants of the form '**C** 1*x*' for a sexagesimal digit '5*x*' in the edition (where *x* is a number from 1 to 9). For all other tables a general variant of the form '**C** mostly omits the dots on *nūn*' is presumed, but is not explicitly given in order not to overburden the apparatuses. Individual deviations from this rule are not registered, except in the situations already mentioned above.

In the tables in the three Cairo manuscripts and in the Judaeo-Arabic manuscript the tabular values are separated by horizontal lines after every third row (in the other four manuscripts these horizontal lines are drawn after every second row). As a second peculiarity of manuscript **C**, a large number of 'block slides' show that the scribe copied the tabular values in blocks of three rows, proceeding horizontally. A clear example of this can be seen in Table 4 (variant 5 in the apparatus), where the scribe, while copying the three rows for 7–9 Syrian years, skipped the column for Kānūn I, copied the values from the columns for Kānūn II to Ayyār one column too far to the right (to the left in the edition), and filled up the three rows in the column for Ayyār with nonsensical values 0, 1 and 2. Note that no other rows besides those for vertical arguments 7–9 Syrian years were affected by this error. In the table of the second equation for Mercury (Table 36 on p. 203) all the values in two consecutive blocks (for arguments $6^s\,18$–$23°$) were slid upwards by three rows, the height of one block.

Further evidence for the blockwise copying in **C** can be seen in the occurrence of numerous transpositions involving two or three consecutive values within a block. Transpositions of sexagesimal digits between two consecutive values (e.g., 2;31 / 2;35 copied as 2;35 / 2;31) or of the units of a single value (e.g., 85;16 copied as 15;86) are occasionally found in most witnesses, but the

[14] For example, in many of the tables of 'minutes of proportions' (interpolation coefficients) for the planetary equations, the dots are corrected on ranges of digits between 50 and 59. Furthermore, in the table of the mean anomaly for Mercury very thick dots were added on the values for 19 and 20 extended years. In the table of the mean motion for Saturn the dots on all arguments, originally in red, were supplemented in black. Although these arguments do not include *nūn*s, possible ambiguity also arises due to the omission of the dots on *fāʾ* ف (80) and *qāf* ق (100), *sīn* س (60) and ش *shīn* (300), and *tāʾ* ت (400) and *thāʾ* ث (500). In addition, v-shaped symbols were written above *rāʾ* ر (200) to distinguish it from *zāʾ* ز (7). Of course, the arguments of nearly every mathematical table are easy to restore because of the highly regular pattern that they follow, but when looking up a single tabular value the lack of dots can easily lead to the choice of a wrong value.

former type as well as transpositions involving three consecutive values (e.g., 6–7–6 copied as 7–6–7) are found much more often in **C**. This seems to indicate that the scribe read three values from a column at once and then wrote all three values down, occasionally in an incorrect order, without checking against the original.[15] In the apparatuses the copying of digits *x–y–x* as *y–x–y* is indicated as a 'block transposition'; for all other types of transpositions the variants are given individually.

In many tables in **C** traces of correction can be observed, also of long ranges of tabular values and in both the main hand and in others. Corrections of copying mistakes are often carried out by writing the correct letter over the erroneous one if it has a similar shape, e.g., in order to change 14 يد to 15 يه. In other cases a diacritical dot is added to indicate the correct letter, for example, a dot over 6 و corrects it to 7 ز (leaving the small loop of the *wāw* unchanged), a dot under 8 ح corrects it to 3 ج, and 51 نا, 52 نب, etc. are corrected to 11 يا, 12 يب, etc. and even 28 to 18 (in the mean anomaly of Mars in 1 extended year) by adding two dots under the *nūn* or *kāf*. Two forms of *kāf* appear to be used arbitrarily in the composite forms for numbers from 21 to 29: the standard form as in كا with the upper rightward stroke omitted, and the form that is more commonly used for the number 20 in astronomical tables as in كا.

Being written in Hebrew characters, scribal errors in manuscript **H** tend to be of a different kind than those in the other manuscripts. Very common scribal errors in numbers written in Arabic *abjad* notation can be distinguished confidently in Hebrew. For example, the common confusion of 1*x* ي (or ى) and 5*x* ن is very unlikely between the Hebrew equivalents י and נ, and the slightly less common confusion between 3 ج (or ح) and 4 د is similarly unlikely between Hebrew ג and ד. This means, for example, that in the erroneous tangent value 45;12,58 for argument 37° in Table 10 (p. 114), the variant 18″ in **C** may be due to the fact that the scribe of this manuscript omitted most of the dots on *nūn*, but the same variant in **H** makes it very likely that this error also occurred in other manuscripts in Arabic script. On the other hand, the Hebrew script introduces new possibilities for scribal errors such as the confusion of 5 ה and 8 ח and that of 2 ב and 20 כ. **H** writes *abjad* zero and Hindu numerals as in Arabic script, with the only exception that for Hindu zero it uses the *abjad* symbol ح . *Abjad* numbers above 400 are written in the standard way for Hebrew, namely as the sum of multiples of 400 plus a remainder: that is, 400 = ת, 500 = תק, 600 = תר, 700 = תש, 800 = תת, and 1000 = תתר. This avoids letters that are written with a dot (cf. the easy confusion of Arabic 400 ت and 500 ث).

[15] Examples of this can be seen for arguments $11^s$ 21–23° in the first equation of Saturn (Table 24 on p. 147) and for arguments $6^s$ 21–23° in the second equation of Jupiter (Table 27 on p. 161). A similar peculiar mistake is also the triple 3;59 / 3;59 / 4;0 for arguments $8^s$ 9–11° in the second equation of Mercury (Table 36 on p. 203), which was copied as 3;0 / 3;59 / 4;59.

The other manuscripts have only occasional copying peculiarities. **C**$_1$ generally writes the Hindu numeral 5 (٥) instead of *abjad* 5 (ه) in the column headers of tables that have (zodiacal) signs as their horizontal argument, including the right ascension, the equation of time and the planetary first and second equations and variations (but not the interpolation functions). The final 2 ب in composite *abjad* numbers (but not an isolated 2) is written with a small downward stroke at the end (e.g., لب). **LB** write the argument 200 ر in the subtables for extended years in practically all mean motion tables as ٢ ر, apparently to avoid confusion with 7 ز. In the tables for the planetary interpolation coefficients, which have a double entry, **FL** write the second set of arguments (to the left side of the table) upside down. **L** also does this in the tables for the latitudes and stations of the planets.

The eight manuscripts of the *Jāmiʿ Zīj* show different ways of dealing with the repeated occurrences of digits or entire tabular values. **F** writes out every single digit in each table. **H** generally does the same with the exception of columns that contain (almost) exclusively leading zeroes or, in the columns for the planetary stations in Tables 38–42, exclusively the same number of zodiacal signs; in such columns the zero or number of signs is only written for the first of the three values in each block. **C** for the most part writes out every single digit in each table, but from the table for the equation of daylight (Table 48a) onwards it writes the most significant digit of the tabular values or the arguments only if they differ from the preceding value or argument.[16]

**C**$_1$ in principle writes every single sexagesimal digit in all tables, but it combines pairs or triples of consecutive identical digits into one vertically expanded shape (which may also cross the horizontal lines between every three rows of tabular values) whenever the digits contain at least one element that is easily vertically expandable (see Figure 2). Such elements include units 1 ا, 3 ج, 8 ح and 9 ط and the multiples of ten 20 ك and 30 ل, but not 1 and 20 when they appear on their own. The third Cairo manuscript, **C**$_2$, of which only the planetary mean motions and equations and the lunar latitude have survived, writes out every single digit.

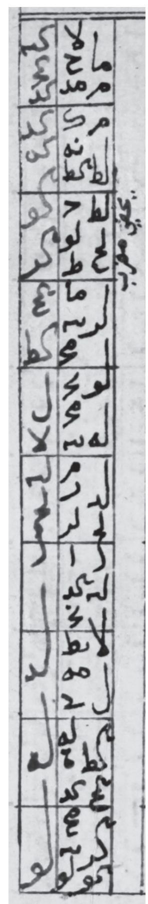

Figure 2: Fragment of the table for the second equation of Mercury in manuscript **C**$_1$ with several examples of vertically expanded elements of composite digits. Cairo, Dār al-kutub, *mīqāt* 188, fol. 42v.

[16] In the column for the hourly motion of the Sun in Table 49 and in the last column of prorogations in Table 53, both of which have identical leading digits almost throughout, **C** writes these leading digits only for the first of the three values in each block, as in **H**.

Of all eight witnesses of Kūshyār's tables, **Y** is the one that most frequently omits repetitive digits. Of long stretches of repeated digits (including entire columns of identical digits as they may appear in tables of slowly changing functions) the scribe often only writes the first and last occurrence (always writing a digit in the top and bottom rows of every column); in other cases he writes only every second or fourth digit (i.e., one in every cell or every two cells separated by horizontal lines). If a digit is repeated only two or three times, the scribe may also write only the first occurrence. It is in particular in this last case where the omission of any digit would lead to the following digits being read incorrectly as the digit preceding the one omitted. In a number of cases this is indicated explicitly in the apparatus.[17] **L** generally writes all digits but in some tables follows similar strategies as **Y** to avoid repetition. The scribe of **L** shows another scribal pecularity not found in any of the other witnesses, namely that in columns for arguments or tabular values consisting of only a single digit he often writes these in the corners of the cells, often alternating cells with a top-right / bottom-left arrangement with cells with a top-left / bottom-right arrangement (see Figure 3). Finally, **B** in most tables writes every single digit, but occasionally follows a strategy similar to **Y** for repeated digits: namely, the scribe either writes only the first and last occurrence of a range of identical digits or, especially for long series of zeroes, writes only every second occurrence (i.e., one in each cell separated by horizontal lines).

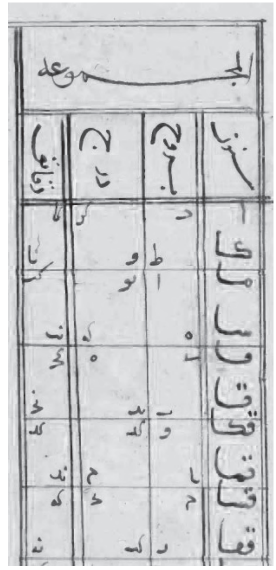

Figure 3: Alternate arrangement of digits in the corners of the cells in manuscript **L** (here in the table of the mean anomaly of Mars). Leiden, Universiteitsbibliotheek, Or. 8, fol. 56v.

In the edition of the tables all repetitive digits are included, and errors in individual manuscripts that lead to misreadings of further digits are indicated in the apparatuses.

[17] An example of a case where the omission of a single digit in **Y** will lead to many following ones being read incorrectly is the first equation of Venus (Table 33 on p. 189), in which the omission of the interpolation coefficient 2 for argument $6^s\,6^\circ$ causes the next eight values to be read as 2 instead of 1. In the table for the solar equation (Table 16 on p. 124) the column of equation values for $1^s$ in **Y** is entirely unusable: the scribe not only wrote the degrees (which are all equal to zero) just in the first and last rows, but also the minutes (which range from 58 to 17). In the column for $10^s$ in the table of the variation at furthest distance for Venus, the omission of some repeated 'one's causes thirteen consecutive values to be read as 0;0 instead of 1;0. As explained above, the scribe of **C** only omitted repetitive digits from some tables towards the end of Book II. However, he did this in such a sloppy way that both the equation of daylight (Table 48a) and the columns for the diameters of the Moon and the shadow of the Earth in Table 49 contain multiple cases in which values would be misread.

# Sigla

Manuscripts containing Book II of Kūshyār's *Jāmiʿ Zīj*

**B** Berlin, Staatsbibliothek Preußischer Kulturbesitz, Or. quart. 101/1

**C** Cairo, Dār al-kutub, *mīqāt* 400

**$C_1$** Cairo, Dār al-kutub, *mīqāt* 188/2

**$C_2$** Cairo, Dār al-kutub, *mīqāt* 691

**F** Istanbul, Süleymaniye Kütüphanesi, Fatih 3418/1

**H** Ahuan Islamic Art, MS 40 (Judaeo-Arabic)

**L** Leiden, Universiteitsbibliotheek, Or. 8

**Y** Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3

Other manuscripts and works

**D** Paris, Bibliothèque nationale de France, arabe 5968: *Dustūr al-munajjimīn* (*c.* 1110)

**E** Escorial, RBMSL, árabe 908: al-Battānī, *Ṣābiʾ Zīj* (7th c. Hijra)

**N** Nallino, *Al-Battānī sive Albatenii*, vol. II

**A** Ptolemy, *Almagest*

**T** Ptolemy, *Tetrabiblos*

**r** recomputed or reconstructed tabular value

General variants for all tables

- In nearly all tables, **C** generally omits the dots on *nūn* from the arguments and the tabular values, so that numbers 51, 52, ..., 59 read as 11, 12, ..., 19. The same occurs occasionally in other manuscripts. Such omissions are only indicated in the apparatuses for tables in which most of the dots were correctly inserted, or whenever other manuscripts also omit the dots.
- Manuscripts **$CC_1C_2$** squeeze all mean motion tables onto a single page. They omit the variants for the early version of the Persian Yazdigird calendar from the subtable for months and place the subtable for longitude corrections in two small columns with 15 entries each under the subtable for months. Manuscripts **LB** compress the table for the motion of the lunar node to a single page but do not further change its format.
- The Judaeo-Arabic manuscript **H** omits the columns for tabular differences from all lunar and planetary equations (but not from the solar equation).

**Table 1: Days of Syrian years and months**
*Sources*: **F** fol. 38r, **H** fol. 25v, **C** fol. 43r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ - , **Y** fol. 257v, **L** fol. 21v, **B** p. 36.

Days of the Syrian Years, to be taken with the completed years and months

| Days of collected years | |
|---|---|
| years[1] | (4) (3) (2) (1) |
| 924 | 1,33,44,51 |
| 952 | 1,36,35,18 |
| 980 | 1,39,25,45 |
| 1008 | 1,42,16,12[7] |
| 1036 | 1,45, 6,39 |
| 1064[8] | 1,47,57, 6[9] |
| 1092 | 1,50,47,33 |
| 1120 | 1,53,38, 0[11] |
| 1148 | 1,56,28,27[12] |
| 1176 | 1,59,18,54[13] |
| 1204 | 2, 2, 9,21 |
| 1232 | 2, 4,59,48 |
| 1260 | 2, 7,50,15 |
| 1288 | 2,10,40,42 |
| 1316 | 2,13,31, 9 |
| 1344 | 2,16,21,36 |
| 1372 | 2,19,12, 3 |
| 1400 | 2,22, 2,30 |
| 1428 | 2,24,52,57 |
| 1456 | 2,27,43,24 |
| 1484 | 2,30,33,51 |
| 1512 | 2,33,24,18[14] |
| 1540 | 2,36,14,45 |
| 1568[16] | 2,39, 5,12 |
| 1596 | 2,41,55,39 |
| 1624 | 2,44,46, 6[18] |
| 1652[19] | 2,47,36,33[20] |
| 1680[21] | 2,50,27, 0[22] |
| 1708 | 2,53,17,27[23] |
| 1736 | 2,56, 7,54 |

| Days of extended years | | |
|---|---|---|
| years[2] | | (3) (2) (1) |
| 1 | | 0, 6, 5 |
| 2 | | 0,12,10 |
| 3 | ly[6] | 0,18,16 |
| 4 | | 0,24,21 |
| 5 | | 0,30,26 |
| 6 | | 0,36,31 |
| 7 | ly | 0,42,37[10] |
| 8 | | 0,48,42 |
| 9 | | 0,54,47 |
| 10 | | 1, 0,52 |
| 11 | ly | 1, 6,58 |
| 12 | | 1,13, 3 |
| 13 | | 1,19, 8 |
| 14 | | 1,25,13 |
| 15 | ly | 1,31,19 |
| 16 | | 1,37,24 |
| 17 | | 1,43,29 |
| 18 | | 1,49,34 |
| 19 | ly | 1,55,40 |
| 20 | | 2, 1,45 |
| 21 | | 2, 7,50 |
| 22 | | 2,13,55[15] |
| 23 | ly | 2,20, 1 |
| 24 | | 2,26, 6 |
| 25 | | 2,32,11[17] |
| 26 | | 2,38,16 |
| 27 | ly | 2,44,22 |
| 28 | | 2,50,27 |

| Days of the months | | |
|---|---|---|
| months[3] | (2) (1) | [4] ⟨in a⟩ leap ⟨year⟩[5] |
| Tishrīn I<br>31 | 0,31 | |
| Tishrīn II<br>30 | 1, 1 | |
| Kānūn I<br>31 | 1,32 | |
| Kānūn II<br>31 | 2, 3 | |
| Shubāṭ<br>28 | 2,31 | *32* |
| Ādhār<br>31 | 3, 2 | *3* |
| Nīsān<br>30 | 3,32 | *33* |
| Ayyār<br>31 | 4, 3 | *4* |
| Ḥazīrān<br>30 | 4,33 | *34* |
| Tammūz<br>31 | 5, 4 | *5* |
| Āb<br>31 | 5,35 | *36* |
| Aylūl<br>30 | 6, 5 | *6* |

This era is earlier than the Arabic ⟨one⟩ by 1,34,38,20 days, and it is earlier than the Persian ⟨one⟩ by 1,35,38,44 days.

[1] Arguments written with Hindu numerals, with leading zeroes for the first three. [2] **F** omits the indications *kāf* ('ly') for leap years; **H** incorrectly places them next to the years 2, 6, 8, 11, 14, 17, 20, 23, and 26; **C** slides them one row upwards to the years 2, 6, ..., 26. [3] **FHCL** omit the lengths of the months. [4] **YLB** omit the values for a leap year (cf. explanatory text A on p. 270). [5] +**C** om. label [6] **Y** om. [7] **Y** $39^{(2)}$ [8] **F** 1004 [9] **C** $17^{(2)}$ [10] **H** $32^{(1)}$ [11] **C** $13^{(3)}$ [12] **FC** $16^{(3)}$ [13] **C** $19^{(3)}$ [14] **C** $38^{(1)}$ [15] **C** $15^{(1)}$ [16] **Y** 5568 [17] **F** $34^{(2)}$ [18] **C** thirds and fourths dam. [19] **C** 1252 [20] **C** seconds, thirds and fourths dam. [21] **C** 1380 [22] **C** $36^{(2)}$ (correction in different hand) [23] **F** $16^{(2)}$ **C** $13^{(3)}$.

**Table 2: Days of Arabic years and months**

*Sources*: **F** fol. 38v, **H** fol. 27r, **C** fol. 44r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 258v, **L** fol. 22v, **B** p. 37.

Days of the Arabic Years, to be taken with the completed years and months

| Days of collected years | | Days of extended years | | | Days of the months | |
|---|---|---|---|---|---|---|
| years[1] | (4) (3) (2) (1) | years[2] | | (3) (2) (1) | months[3] | (2) (1) |
| 30 | 0, 2,57,11 | 1 | | 0, 5,54 | Muḥarram 30 | 0,30 |
| 60 | 0, 5,54,22 | 2[4] | ly | 0,11,49 | Ṣafar 29 | 0,59 |
| 90 | 0, 8,51,33 | 3 | | 0,17,43 | Rabīʿ I 30 | 1,29 |
| 120 | 0,11,48,44[5] | 4 | | 0,23,37 | Rabīʿ II 29 | 1,58 |
| 150 | 0,14,45,55 | 5 | ly | 0,29,32 | Jumādā I 30 | 2,28 |
| 180 | 0,17,43, 6 | 6 | | 0,35,26 | Jumādā II 29 | 2,57 |
| 210 | 0,20,40,17 | 7 | ly[6] | 0,41,21 | Rajab 30 | 3,27 |
| 240 | 0,23,37,28 | 8 | | 0,47,15[7] | Shaʿbān 29 | 3,56 |
| 270[8] | 0,26,34,39 | 9 | | 0,53, 9 | Ramaḍān 30 | 4,26 |
| 300 | 0,29,31,50 | 10 | ly | 0,59, 4 | Shawwāl 29 | 4,55 |
| 330[9] | 0,32,29, 1 | 11 | | 1, 4,58[10] | Dhū l-qaʿda 30 | 5,25 |
| 360 | 0,35,26,12 | 12 | | 1,10,52 | Dhū l-ḥijja 29 | 5,54 |
| 390 | 0,38,23,23 | 13 | ly | 1,16,47[11] | | |
| 420 | 0,41,20,34[12] | 14 | | 1,22,41 | This era is earlier than the Persian ⟨one⟩ by 1,0,24 days. | |
| 450 | 0,44,17,45 | 15 | ly[13] | 1,28,36 | | |
| 480 | 0,47,14,56 | 16 | | 1,34,30 | | |
| 510 | 0,50,12, 7 | 17 | | 1,40,24 | | |
| 540 | 0,53, 9,18 | 18 | ly | 1,46,19 | | |
| 570 | 0,56, 6,29 | 19 | | 1,52,13[14] | | |
| 600 | 0,59, 3,40 | 20 | | 1,58, 7[15] | | |
| 630 | 1, 2, 0,51 | 21 | ly | 2, 4, 2 | | |
| 660 | 1, 4,58, 2 | 22 | | 2, 9,56 | | |
| 690 | 1, 7,55,13 | 23 | | 2,15,50 | | |
| 720 | 1,10,52,24 | 24 | ly[16] | 2,21,45 | | |
| 750 | 1,13,49,35 | 25 | | 2,27,39[17] | | |
| 780 | 1,16,46,46 | 26 | ly | 2,33,34[18] | | |
| 810 | 1,19,43,57 | 27 | | 2,39,28 | | |
| 840 | 1,22,41, 8 | 28 | | 2,45,22 | | |
| 870 | 1,25,38,19 | 29 | ly | 2,51,17[19] | | |
| 900 | 1,28,35,30 | 30 | | 2,57,11 | | |

[1] Arguments written with Hindu numerals, with leading zeroes for the first three (except in **Y**). [2] **F** omits the indications *kāf* ('ly') for leap years; **Y** indicates the leap years as for the Syrian calendar (3, 7, 11, ..., 27). [3] **FHCL** omit the lengths of the months. [4] **C** writes the argument '2' and *kāf* ('ly') before the '1' in the first row. [5] **H** $1^{(3)}$ [6] **B** om. *kāf* ('ly') [7] **C** $55^{(1)}$ [8] **B** 280 (corrected to 270 in black) [9] **B** 310 (corrected to 330 in black) [10] **C** $18^{(1)}$ [11] **H** $44^{(1)}$ [12] **Y** $35^{(1)}$ [13] **H** appears to have erased the *kāf* and replaced it by a black *kāf* in the row for 16 years; **B** om. [14] **C** $12^{(2)}$ [15] **C** $18^{(2)}$ [16] **B** om. *kāf* ('ly') [17] **H** $24^{(2)}$ [18] **H** $33^{(1)}$ [19] **H** $14^{(1)}$.

**Table 3: Days of Persian years and months**
*Sources*: **F** fol. 39r, **H** fol. 27v, **C** fol. 45r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 259v, **L** fol. 23v, **B** p. 38.

| Days of the Persian Years, to be taken with the completed years and months | | | | | | |
|---|---|---|---|---|---|---|
| Days of collected years | | Days of extended years | | Days of the months | | |
| years[1] | (4) (3) (2) (1) | years | (3) (2) (1) | months | (2) (1) | [2] marginal gloss |
| 28[3] | 0, 2,50,20 | 1 | 0, 6, 5 | Farwardīn | 0,30 | |
| 56 | 0, 5,40,40 | 2 | 0,12,10 | | | |
| 84 | 0, 8,31, 0 | 3 | 0,18,15 | Urdībihisht | 1, 0 | |
| 112 | 0,11,21,20 | 4 | 0,24,20 | | | |
| 140 | 0,14,11,40[4] | 5 | 0,30,25 | Khurdād | 1,30 | |
| 168 | 0,17, 2, 0 | 6 | 0,36,30 | | | |
| 196 | 0,19,52,20 | 7 | 0,42,35 | Tīr | 2, 0 | |
| 224 | 0,22,42,40 | 8 | 0,48,40 | | | |
| 252 | 0,25,33, 0 | 9 | 0,54,45[5] | Murdād | 2,30 | |
| 280 | 0,28,23,20 | 10 | 1, 0,50 | | | |
| 308 | 0,31,13,40 | 11 | 1, 6,55[6] | Shahrīwar | 3, 0 | |
| 336[7] | 0,34, 4, 0 | 12 | 1,13, 0 | | | |
| 364 | 0,36,54,20 | 13 | 1,19, 5 | Mihr | 3,30 | |
| 392 | 0,39,44,40 | 14 | 1,25,10 | | | |
| 420 | 0,42,35, 0 | 15 | 1,31,15 | Ābān | 4, 5 | 0 |
| 448 | 0,45,25,20 | 16 | 1,37,20 | | | |
| 476 | 0,48,15,40 | 17 | 1,43,25 | Ādhar | 4,35 | 30 |
| 504 | 0,51, 6, 0 | 18 | 1,49,30 | | | |
| 532 | 0,53,56,20 | 19 | 1,55,35 | Day | 5, 5 | 0 |
| 560 | 0,56,46,40 | 20 | 2, 1,40 | | | |
| 588 | 0,59,37, 0 | 21 | 2, 7,45 | Bahman | 5,35 | 30 |
| 616 | 1, 2,27,20[8] | 22 | 2,13,50 | | | |
| 644[9] | 1, 5,17,40 | 23 | 2,19,55 | Isfandārmudh | 6, 5 | 5 |
| 672 | 1, 8, 8, 0 | 24 | 2,26, 0[10] | | | |
| 700 | 1,10,58,20[11] | 25 | 2,32, 5 | | | |
| 728 | 1,13,48,40 | 26 | 2,38,10 | ⟨The values in⟩ the marginal gloss are for the epagomenal days in Isfandārmudh māh | | |
| 756 | 1,16,39, 0 | 27 | 2,44,15[12] | | | |
| 784 | 1,19,29,20 | 28 | 2,50,20 | | | |
| 812[13] | 1,22,19,40 | | | | | |
| 840 | 1,25,10, 0 | | | | | |

[1] Arguments written with Hindu numerals, with leading zeroes for the first three (except in **Y**).
[2] **YL** omit the values for the case that the *epagomenae* are located in Ābān (i.e., the older version of the Yazdigird calendar) and place the values for the new version in the table itself; **CB** exchange the values for the old and new versions. [3] **Y** *abjad* 3 instead of Hindu 2 [4] **Y** $11^{(3)}$ [5] **F** $14^{(2)}$ [6] **C** $15^{(1)}$ [7] **C** 326 [8] **F** $24^{(2)}$ [9] **Y** 642 [10] **F** $31^{(1)}$ [11] **C** $18^{(2)}$ [12] **C** $55^{(1)}$ [13] **Y** *abjad* 3 instead of Hindu 2.

**Table 4: *Notae* of Syrian years and months**
*Sources*: **F** fol. 39v, **H** fol. 26r, **C** fol. 43v, **$C_1$** -, **$C_2$** -, **Y** fol. 258r, **L** fol. 22r, **B** p. 39.

| | *Nota* of the Syrian Months, to be taken with the incomplete year | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Syrian months | | | | | | | | | | | |
| [1] | | 31 | 30[2] | 31 | 31 | 28 | 31 | 30 | 31 | 30 | 31 | 31 | 30 |
| [3] | Syrian years | Tishrīn I | Tishrīn II | Kānūn I | Kānūn II | Shubāṭ | Ādhār | Nīsān | Ayyār | Ḥazīrān | Tammūz | Āb | Aylūl |
| | 1 | 2 | 5 | 0 | 3 | 6 | 6 | 2 | 4 | 0 | 2 | 5 | 1 |
| | 2 | 3 | 6 | 1 | 4 | 0 | 0 | 3 | 5 | 1 | 3 | 6 | 2 |
| ly | 3 | 4 | 0 | 2 | 5 | 1 | 2 | 5 | 0 | 3 | 5 | 1 | 4 |
| | 4 | 6 | 2 | 4[4] | 0 | 3 | 3 | 6 | 1 | 4 | 6 | 2 | 5 |
| | 5 | 0 | 3 | 5 | 1 | 4 | 4 | 0 | 2 | 5 | 0 | 3 | 6 |
| | 6 | 1 | 4 | 6 | 2 | 5 | 5 | 1 | 3 | 6 | 1 | 4 | 0 |
| ly | 7 | 2 | 5 | 0[5]↕→ | 3 | 6 | 0 | 3 | 5[6] | 1 | 3 | 6 | 2 |
| | 8 | 4 | 0 | 2 | 5 | 1 | 1 | 4 | 6[7] | 2 | 4 | 0 | 3 |
| | 9 | 5 | 1 | 3 | 6 | 2 | 2 | 5⌟ | 0[8] | 3 | 5 | 1 | 4 |
| | 10 | 6 | 2 | 4 | 0 | 3 | 3 | 6 | 1 | 4 | 6 | 2 | 5 |
| ly | 11 | 0 | 3 | 5 | 1 | 4 | 5 | 1 | 3 | 6 | 1 | 4 | 0 |
| | 12 | 2 | 5 | 0 | 3 | 6 | 6 | 2 | 4 | 0 | 2 | 5 | 1 |
| | 13 | 3 | 6 | 1 | 4 | 0 | 0 | 3 | 5 | 1 | 3 | 6 | 2 |
| | 14 | 4 | 0 | 2 | 5 | 1 | 1 | 4 | 6 | 2 | 4 | 0 | 3 |
| ly | 15 | 5 | 1 | 3 | 6 | 2 | 3 | 6 | 1 | 4 | 6 | 2 | 5 |
| | 16 | 0 | 3 | 5 | 1 | 4[9] | 4 | 0 | 2 | 5 | 0 | 3 | 6 |
| | 17 | 1 | 4 | 6 | 2 | 5 | 5 | 1 | 3 | 6 | 1 | 4 | 0 |
| | 18 | 2 | 5 | 0 | 3 | 6 | 6 | 2 | 4 | 0 | 2 | 5 | 1 |
| ly | 19 | 3 | 6 | 1 | 4 | 0 | 1 | 4 | 6 | 2 | 4 | 0 | 3 |
| | 20 | 5 | 1 | 3 | 6 | 2 | 2 | 5 | 0 | 3 | 5 | 1 | 4 |
| | 21 | 6 | 2 | 4 | 0 | 3 | 3 | 6 | 1 | 4 | 6 | 2 | 5 |
| | 22 | 0 | 3 | 5[10] | 1 | 4 | 4 | 0 | 2 | 5 | 0 | 3[11]↓ | 6 |
| ly | 23 | 1 | 4 | 6 | 2 | 5 | 6 | 2 | 4[12] | 0 | 2 | 5 | 1 |
| | 24 | 3 | 6 | 1 | 4 | 0 | 0 | 3 | 5 | 1 | 3 | 6 | 2 |
| | 25[13]↓ | 4 | 0 | 2 | 5[14] | 1 | 1 | 4 | 6 | 2 | 4 | 0⊥ | 3 |
| | 26 | 5 | 1 | 3 | 6 | 2 | 2 | 5 | 0 | 3 | 5 | 1 | 4 |
| ly | 27 | 6 | 2 | 4 | 0 | 3 | 4 | 0 | 2 | 5[15] | 0[16] | 3 | 6[17] |
| | 28⊥ | 1 | 4 | 6 | 2 | 5 | 5 | 1 | 3 | 6 | 1 | 4 | 0 |

[1] **L** omits the lengths of the months. [2] **Y** 31 [3] **CY** omit the indications *kāf* ('ly') for leap years. [4] **Y** 3 [5] **C** years 7–9, months Kānūn I–Nīsān: slide[−1 col.] [6] **C** 0 (end of slide) [7] **C** 1 (end of slide) [8] **C** 2 (end of slide) [9] **Y** 3 [10] **C** 0 [11] **Y** years 22–25, month Āb: slide[+1] [12] **L** 3 [13] **C** arguments 25–28: slide[−1] (argument '28' is written in an otherwise empty twenty-ninth row of the table) [14] **F** 0 [15] **C** dam. [16] **C** dam. [17] **FH** 5.

**Table 5:** ***Notae* of Arabic years and months**
*Sources*: **F** fol. 40r, **H** fol. 26v, **C** fol. 44v, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 259r, **L** fol. 23r, **B** p. 40.

*Nota* of the Arabic Years,
to be taken with the incomplete year and the incomplete month

| [1] | years | days | years | days | years | days | years | days | years | days | years | days | years | days | months | days[2] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 5 | 31 | 3 | 61 | 1 | 91 | 6 | 121 | 4 | 151 | 2 | 181 | 0 | Muḥarram | 0[3] |
| ly | 2 | 2 | 32 | 0 | 62 | 5 | 92 | 3 | 122 | 1 | 152 | 6 | 182 | 4 | | |
| | 3 | 0 | 33 | 5 | 63 | 3 | 93 | 1 | 123 | 6 | 153 | 4[4] | 183 | 2 | Ṣafar | 2 |
| | 4 | 4 | 34 | 2 | 64 | 0 | 94 | 5 | 124 | 3 | 154 | 1 | 184 | 6 | | |
| ly | 5 | 1 | 35 | 6 | 65 | 4 | 95 | 2 | 125 | 0[5] | 155 | 5 | 185 | 3 | Rabīʿ I | 3 |
| | 6 | 6 | 36 | 4 | 66 | 2[6] | 96 | 0 | 126 | 5[7] | 156 | 3 | 186 | 1 | | |
| ly | 7 | 3 | 37 | 1[8] | 67 | 6 | 97 | 4 | 127 | 2 | 157 | 0 | 187 | 5 | Rabīʿ II | 5 |
| | 8 | 1 | 38 | 6 | 68 | 4 | 98 | 2 | 128 | 0 | 158 | 5 | 188 | 3 | | |
| | 9 | 5 | 39 | 3[9] | 69 | 1 | 99 | 6 | 129 | 4[10] | 159 | 2 | 189 | 0 | Jumādā I | 6 |
| ly | 10 | 2 | 40 | 0 | 70 | 5 | 100 | 3[11] | 130 | 1 | 160 | 6 | 190 | 4[12] | | |
| | 11 | 0 | 41 | 5 | 71 | 3 | 101 | 1 | 131 | 6 | 161 | 4[13] | 191 | 2 | Jumādā II | 1 |
| | 12 | 4 | 42 | 2 | 72 | 0 | 102 | 5 | 132 | 3[14] | 162 | 1 | 192 | 6 | | |
| ly | 13 | 1 | 43 | 6 | 73 | 4 | 103 | 2 | 133 | 0 | 163 | 5[15] | 193 | 3 | Rajab | 2 |
| | 14 | 6 | 44 | 4 | 74 | 2 | 104 | 0 | 134 | 5 | 164 | 3 | 194 | 1 | | |
| ly | 15 | 3 | 45 | 1 | 75 | 6 | 105 | 4 | 135 | 2 | 165 | 0 | 195 | 5 | Shaʿbān | 4 |
| | 16 | 1 | 46 | 6 | 76 | 4 | 106 | 2 | 136 | 0 | 166 | 5 | 196 | 3 | | |
| | 17 | 5 | 47 | 3 | 77 | 1 | 107 | 6 | 137 | 4 | 167 | 2 | 197 | 0 | Ramaḍān | 5 |
| ly | 18 | 2 | 48 | 0 | 78 | 5 | 108 | 3 | 138 | 1 | 168 | 6 | 198 | 4 | | |
| | 19 | 0 | 49 | 5 | 79 | 3 | 109 | 1 | 139 | 6 | 169 | 4 | 199 | 2 | Shawwāl | 0 |
| | 20 | 4 | 50 | 2 | 80 | 0 | 110 | 5 | 140 | 3 | 170 | 1 | 200 | 6 | | |
| ly | 21 | 1 | 51 | 6 | 81 | 4 | 111 | 2 | 141 | 0 | 171 | 5 | 201 | 3 | Dhū l-qaʿda | 1 |
| | 22 | 6 | 52 | 4 | 82 | 2 | 112 | 0 | 142 | 5 | 172 | 3 | 202 | 1 | | |
| | 23 | 3 | 53 | 1 | 83 | 6 | 113 | 4 | 143 | 2 | 173 | 0 | 203 | 5 | Dhū l-ḥijja | 3 |
| ly | 24 | 0 | 54 | 5 | 84 | 3 | 114 | 1 | 144 | 6 | 174 | 4 | 204 | 2 | | |
| | 25 | 5 | 55 | 3 | 85 | 1 | 115 | 6 | 145 | 4[16] | 175 | 2 | 205 | 0 | | |
| ly | 26 | 2[17] | 56 | 0 | 86 | 5 | 116 | 3 | 146 | 1 | 176 | 6 | 206 | 4 | | |
| | 27 | 0 | 57 | 5 | 87 | 3 | 117 | 1 | 147 | 6 | 177 | 4 | 207 | 2 | | |
| | 28 | 4 | 58 | 2 | 88 | 0 | 118 | 5 | 148 | 3[18] | 178 | 1 | 208 | 6 | | |
| ly | 29 | 1 | 59 | 6 | 89 | 4 | 119 | 2 | 149 | 0 | 179 | 5 | 209 | 3 | | |
| | 30 | 6 | 60 | 4 | 90 | 2 | 120 | 0 | 150 | 5 | 180 | 3 | 210 | 1 | | |

[1] The indications *kāf* ('ly') for leap years are only included in **HL**. [2] **FB** om. [3] **C** 5 [4] **F** 3 [5] **C** 5 [6] **F** 0 [7] **C** 6 [8] **F** 2 [9] **F** 4 [10] **F** 3 **H** 7 [11] **H** 2 (?) [12] **F** 3 [13] **C** 3 [14] **F** 4 [15] **C** 0 [16] **F** 3 [17] **C** 1 [18] **C** dam.

**Table 6: *Notae* of Persian years and months**
*Sources*: **F** fol. 40v, **H** fol. 28r, **C** fol. 45v, **$C_1$** -, **$C_2$** -, **Y** fol. 260r, **L** -, **B** p. 41.

| *Nota* of the Persian Months, to be taken with the incomplete year | | | | | | | |
|---|---|---|---|---|---|---|---|
| | the seven years | | | | | | |
| months | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Farwardīn | 3 | 4 | 5 | 6 | 0 | 1 | 2 |
| Urdībihisht | 5 | 6 | 0 | 1 | 2 | 3 | 4 |
| Khurdād | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Tīr | 2 | 3 | 4 | 5 | 6 | 0 | 1 |
| Murdād | 4 | 5 | 6 | 0[1] | 1 | 2 | 3 |
| Shahrīwar | 6 | 0 | 1 | 2 | 3 | 4 | 5 |
| Mihr | 1 | 2 | 3 | 4 | 5 | 6 | 0 |
| Ābān | 3 | 4 | 5 | 6 | 0 | 1 | 2 |
| Ādhar | 3<br>*5*[2]↓ | 4<br>*6* | 5<br>*0* | 6<br>*1* | 0<br>*2* | 1<br>*3* | 2<br>*4* |
| Day | 5<br>*0* | 6<br>*1* | 0<br>*2* | 1[3]<br>*3* | 2[4]<br>*4* | 3<br>*5* | 4<br>*6* |
| Bahman | 0<br>*2* | 1<br>*3* | 2<br>*4* | 3<br>*5* | 4<br>*6* | 5<br>*0* | 6<br>*1* |
| Isfandārmudh | 2<br>*4*⊥ | 3<br>*5* | 4<br>*6* | 5<br>*0*[5] | 6<br>*1* | 0<br>*2* | 1<br>*3*[6] |

⟨The values in⟩ red are for the epagomenal days in Isfandārmudh[7] māh.

**C** gives the *notae* for Sunday as '7' instead of '0'. **C** exchanges the black and red (italic in the edition) variants for the last four months, in agreement with the adjusted comment under the table (cf. note 7). [1] **F** 5 [2] **Y** Ādhar–Isfandārmudh: red (here: italic) values for year 1 made equal to the ones for year 2 [3] **C** 2 [4] **C** 3 [5] **H** ill. [6] **H** ill. [7] **C** Ābān.

**Table 7: *Notae* of the Christian Lent**

*Sources*: **F** fol. 41r, **H** -, **C** -, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 260v, **L** fol. 24r (see Plate 3), **B** -.

| years (vertically) | *Nota* of the Christian Lent, black from Shubāṭ, red from Ādhar — years (horizontally): 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 25[1] | 11 | *4* | 18 | 11 | 25 | 18 | 4 | 25 | 11 | *4*[2] | 18 | 11 | 25 | 18 | 4 | 25[3] | 11 | *4* |
| 2 | 24 | 10 | *3* | 17 | 10 | 24 | 17 | 3 | 24 | 10 | *3*[4] | 24 | 10 | *3* | 17 | 3 | 24 | 17 | *3* |
| 3 | 23 | 9 | *1* | 16 | 9[5] | *1* | 16 | 2 | 23 | 9 | *1* | 23 | 9 | *1* | 16 | 9 | 23 | 16 | *1* |
| 4 | 21 | 14 | 28 | 21 | 7 | 28 | 14 | 7 | 21 | 14 | *7* | 21 | 7 | 28 | 14 | 7 | 28 | 14 | *7* |
| 5 | 20 | 13 | 27 | 20 | 6 | 27 | 13 | 6 | 27 | 13 | *6* | 20 | 13 | 27 | 20 | 6 | 27 | 13 | *6* |
| 6 | 19 | 12 | *5* | 19 | 5 | 26 | 19 | 5 | 26 | 12 | *5* | 19 | 12 | 26 | 19 | 5 | 26 | 12 | *5* |
| 7 | 25 | 11 | *3* | 18 | 11 | 25 | 18 | 4 | 25[6] | 11 | *3* | 18 | 11 | 25 | 18 | 4 | 25 | 11 | *3* |
| 8 | 23 | 9 | *2* | 16 | 9 | *2* | 16 | 2 | 23 | 9 | *2* | 23 | 9 | *2* | 16 | 9 | 23 | 16 | *2* |
| 9 | 22 | 8 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *8* |
| 10 | 21 | 14 | 28 | 21 | 7 | 28 | 14 | 7 | 21 | 14 | *7* | 21 | 7 | 28 | 14 | 7 | 28 | 14 | *7* |
| 11 | 20 | 13 | 27 | 20 | 6 | 27 | 13 | 6 | 27 | 13 | *5* | 20 | 13 | 27 | 20 | 6 | 27 | 13 | *5* |
| 12 | 25 | 11 | *4* | 18 | 11 | 25 | 18 | 4 | 25 | 11 | *4*[7] | 18 | 11 | 25 | 18 | 4 | 25 | 11[8] | *4* |
| 13 | 24 | 10 | *3* | 17 | 10 | 24 | 17 | 3 | 24[9] | 10 | *3* | 24 | 10 | *3* | 17 | 3 | 24 | 17 | *3* |
| 14 | 23 | 9 | *1* | 16 | 9 | *1* | 16 | 2 | 23 | 9 | *1* | 23 | 9 | *1* | 16 | 9 | 23 | 16 | *1* |
| 15 | 22 | 8 | 29 | 22 | 8 | 29 | 15 | 8 | 22 | 15 | 29[10] | 22 | 8 | 29 | 15 | 8 | 22 | 15 | *7* |
| 16 | 20 | 13 | 27 | 20 | 6[11] | 27 | 13 | 6 | 27 | 13 | *6* | 20[12] | 13 | 27 | 20 | 6 | 27 | 13 | *6* |
| 17 | 19 | 12 | *5* | 19 | 5 | 26 | 19 | 5 | 26 | 12 | *5* | 19 | 12 | 26 | 19 | 5 | 26 | 12 | *5* |
| 18 | 25 | 11 | *4* | 18 | 11 | 25 | 18 | 4 | 25 | 11 | *4* | 18 | 11 | 25 | 18 | 4 | 25 | 11 | *4* |
| 19 | 24 | 10 | *2* | 17 | 10 | 24 | 17 | 3 | 24 | 10 | *2* | 24 | 10 | *2* | 17 | 3 | 24 | 17 | *2* |
| 20 | 22 | 8 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *8*[13] |
| 21 | 21 | 14 | 28 | 21 | 7 | 28 | 14 | 7 | 21 | 14 | *7* | 21 | 7 | 28 | 14 | 7[14] | 28 | 14 | *7* |
| 22 | 20 | 13 | 27 | 20 | 6 | 27 | 13 | 6 | 27 | 13 | *6* | 20 | 13 | 27 | 20 | 6 | 27 | 13 | *6* |
| 23 | 19 | 12 | *4* | 19 | 5 | 26 | 19 | 5 | 26 | 12 | *4*[15] | 19 | 12 | 26 | 19 | 5 | 26 | 12 | *4* |
| 24 | 24 | 10 | *3* | 17 | 10 | 24 | 17 | 3 | 24 | 10 | *3* | 24[16] | 10[17] | *3* | 17 | 3 | 24 | 17 | *3* |
| 25 | 23 | 9 | *1* | 16[18] | 9 | *1* | 16 | 2[19] | 23 | 9 | *1* | 23 | 9 | *1* | 16 | 9 | 23 | 16[20] | *1* |
| 26 | 22 | 8 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *1* | 22 | 8 | *1* | 15 | 8 | 22 | 15 | *8* |
| 27 | 21 | 14 | 28 | 21 | 7 | 28 | 14 | 7 | 21 | 14 | *6* | 21 | 7 | 28 | 14 | 7 | 28 | 14 | *6* |
| 28 | 19 | 12 | *5*[21] | 19 | 5[22] | 26 | 19 | 5 | 26 | 12 | *5* | 19 | 12 | 26 | 19 | 5 | 26 | 12 | *5* |

Values in red in the manuscripts are here set in italics. Errors in the computation of the table are underlined: the values 1 in rows 14 and 25 should be corrected to 2, and the values 9 in column 10 to 16 (cf. Section IV.1, pp. 350–52). [1] **Y** 26 [2] **F** 1 [3] **L** 22 [4] **F** 1 [5] **F** 19 [6] **Y** 29 [7] **Y** 7 [8] **L** 12 [9] **F** 25 [10] **Y** om. [11] **Y** 5 [12] **F** 27 [13] **F** 3 [14] **Y** 4 [15] **Y** 7 [16] **F** 17 [17] **F** 3 [18] **F** 19 [19] **F** 19 [20] **Y** 56 [21] **F** 6 [22] **F** 6.

## Table 8: Sine

*Sources*: **F** fol. 41v, **H** fol. 28v, **C** fol. 46r, $\mathbf{C}_1$ fol. 13r, $\mathbf{C}_2$ -, **Y** fol. 261r, **L** -, **B** p. 42, **r** = recomputation. **L** includes only the more extensive sine table here edited on pp. 97–111, **B** includes both types of sine tables.

Table of the Sine

| arc | sine | difference | arc | sine | difference | arc | sine | difference |
|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ |
| 1 | 1; 2,50 | 1; 2,50 | 31 | 30;54, 8 | 0;54, 8 | 61 | 52;28,38 | 0;30,57 |
| 2 | 2; 5,38 | 1; 2,48 | 32 | 31;47,43 | 0;53,35 | 62 | 52;58,37 | 0;29,59 |
| 3 | 3; 8,25 | 1; 2,47 | 33 | 32;40,42 | 0;52,59 | 63 | 53;27,37[1] | 0;29, 0 |
| 4 | 4;11, 7 | 1; 2,42 | 34 | 33;33, 6 | 0;52,24[2] | 64 | 53;55,40 | 0;28, 3 |
| 5 | 5;13,46 | 1; 2,39 | 35 | 34;24,53[3] | 0;51,47 | 65 | 54;22,42 | 0;27, 2 |
| 6 | 6;16,18 | 1; 2,32 | 36 | 35;16, 2 | 0;51, 9 | 66 | 54;48,46 | 0;26, 4 |
| 7 | 7;18,44 | 1; 2,26 | 37 | 36; 6,32 | 0;50,30 | 67 | 55;13,49[4] | 0;25, 3 |
| 8 | 8;21, 1 | 1; 2,17 | 38 | 36;56,23 | 0;49,51 | 68 | 55;37,52 | 0;24, 3[5] |
| 9 | 9;23,10 | 1; 2, 9 | 39 | 37;45,33[6] | 0;49,10 | 69 | 56; 0,53 | 0;23, 1 |
| 10 | 10;25, 8[7] | 1; 1,58 | 40 | 38;34, 2 | 0;48,29 | 70 | 56;22,54 | 0;22, 1 |
| 11 | 11;26,55 | 1; 1,47[8] | 41 | 39;21,49 | 0;47,47 | 71 | 56;43,52 | 0;20,58 |
| 12 | 12;28,29 | 1; 1,34 | 42 | 40; 8,52[9] | 0;47, 3 | 72 | 57; 3,48 | 0;19,56 |
| 13 | 13;29,49 | 1; 1,20 | 43 | 40;55,12[10] | 0;46,20[11] | 73 | 57;22,42 | 0;18,54 |
| 14 | 14;30,55 | 1; 1, 6 | 44 | 41;40,46 | 0;45,34 | 74 | 57;40,33 | 0;17,51 |
| 15 | 15;31,45 | 1; 0,50 | 45 | 42;25,35 | 0;44,49 | 75 | 57;57,20 | 0;16,47 |
| 16 | 16;32,18 | 1; 0,33 | 46 | 43; 9,37 | 0;44, 2[12] | 76 | 58;13, 4 | 0;15,44[13] |
| 17 | 17;32,32 | 1; 0,14 | 47 | 43;52,52 | 0;43,15 | 77 | 58;27,44 | 0;14,40 |
| 18 | 18;32,28 | 0;59,56 | 48 | 44;35,19 | 0;42,27 | 78 | 58;41,20 | 0;13,36 |
| 19 | 19;32, 3 | 0;59,35 | 49 | 45;16,57[14] | 0;41,38 | 79 | 58;53,51 | 0;12,31 |
| 20 | 20;31,16 | 0;59,13 | 50 | 45;57,46[15] | 0;40,49 | 80 | 59; 5,18 | 0;11,27 |
| 21 | 21;30, 7[16] | 0;58,51 | 51 | 46;37,44 | 0;39,58 | 81 | 59;15,41 | 0;10,23 |
| 22 | 22;28,35 | 0;58,28 | 52 | 47;16,50[17] | 0;39, 6 | 82 | 59;24,58 | 0; 9,17 |
| 23 | 23;26,39[18] | 0;58, 4[19] | 53 | 47;55, 5 | 0;38,15 | 83 | 59;33,10 | 0; 8,12 |
| 24 | 24;24,15[20] | 0;57,36[21] | 54 | 48;32,28 | 0;37,23 | 84 | 59;40,17 | 0; 7, 7 |
| 25 | 25;21,26 | 0;57,11 | 55 | 49; 8,57 | 0;36,29 | 85 | 59;46,18 | 0; 6, 1 |
| 26 | 26;18, 8 | 0;56,42 | 56 | 49;44,32 | 0;35,35[22] | 86 | 59;51,14[23] | 0; 4,56[24] |
| 27 | 27;14,22 | 0;56,14 | 57 | 50;19,13[25] | 0;34,41 | 87 | 59;55, 4 | 0; 3,50 |
| 28 | 28;10, 6 | 0;55,44 | 58 | 50;52,58 | 0;33,45 | 88 | 59;57,48 | 0; 2,44 |
| 29 | 29; 5,19 | 0;55,13 | 59 | 51;25,48 | 0;32,50[26] | 89 | 59;59,27[27] | 0; 1,39 |
| 30 | 30; 0, 0 | 0;54,41[28] | 60 | 51;57,41 | 0;31,53 | 90 | 60; 0, 0 | 0; 0,33 |

**F** frequently omits the dots on *nūn* in the tabular differences. [1] $\mathbf{C}_1$ 34″ [2] **F** 51′ [3] $\mathbf{C}_1$ 27′ [4] $\mathbf{C}_1$ 33′ [5] **F** 26′ [6] $\mathbf{C}_1$ 47′ [7] $\mathbf{C}_1$ 24′ [8] $\mathbf{C}_1$ 44″ [9] $\mathbf{C}_1$ 58′ [10] **B** 41° [11] **F** 20 corrected to 22″ (?) in black [12] **B** 7″ [13] **F** 45″ [14] **H** 17″ [15] **F** 56′ [16] **B** 30″ [17] $\mathbf{C}_1$ 46° [18] **YBr** 38″ [19] **YBr** 3″ (in correspondence with the variant in the sine value for 23°) [20] $\mathbf{C}_1$ 27′ [21] **YBr** 37″ (in correspondence with the variant in the sine value for 23°) [22] **F** 36′ [23] **F** 54″ [24] $\mathbf{CC}_1$ 17″ [25] $\mathbf{C}_1$ 18″ [26] **F** 34′ [27] $\mathbf{C}_1$ 22″ [28] $\mathbf{C}_1$ 56′.

**Table 8a: Sine for fractions of a degree (part 1)**

*Sources*: **L** fols 24v–31v (see Plate 4 for the first page), **B** pp. 49–63 (with part 14 before part 13 due to an incorrectly bound folio), **D** = *Dustūr al-munajjimīn*, MS Paris, BnF, arabe 5968, fols 29r–36r. **FHCC$_1$Y** include only the sine table with values for every integer degree from 1 to 90, here edited on p. 96, while **B** contains both sine tables (cf. p. 54 in the introduction).

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0°** | | **1°** | | **2°** | | **3°** | | **4°** | | **5°** | |
| 0; 0, 0 | | 1; 2,50 | | 2; 5,38 | | 3; 8,25 | | 4;11, 7[1] | | 5;13,46 | |
| 1 | 1, 3 | 1 | 1, 3 | 1 | 1, 3 | 1 | 1, 3 | 1 | 1, 3 | 1 | 1, 3 |
| 2 | 2, 6 | 2 | 2, 6 | 2 | 2, 6 | 2 | 2, 5 | 2 | 2, 5 | 2 | 2, 5 |
| 3 | 3, 8 | 3 | 3, 8 | 3 | 3, 8 | 3 | 3, 8 | 3 | 3, 8 | 3 | 3, 8 |
| 4 | 4,11 | 4 | 4,11 | 4 | 4,11 | 4 | 4,11 | 4 | 4,11 | 4 | 4,10 |
| 5 | 5,14 | 5 | 5,14 | 5 | 5,14 | 5 | 5,14 | 5 | 5,13 | 5 | 5,13 |
| 6 | 6,17 | 6 | 6,17 | 6 | 6,17 | 6 | 6,16 | 6 | 6,16 | 6 | 6,15 |
| 7 | 7,20 | 7 | 7,20 | 7 | 7,20 | 7 | 7,19 | 7 | 7,19 | 7 | 7,18 |
| 8 | 8,22 | 8 | 8,22 | 8 | 8,22 | 8 | 8,22 | 8 | 8,21 | 8 | 8,20 |
| 9 | 9,25 | 9 | 9,25 | 9 | 9,25 | 9 | 9,25 | 9 | 9,24 | 9 | 9,23 |
| 10 | 10,28 | 10 | 10,28 | 10 | 10,28 | 10 | 10,27 | 10 | 10,27 | 10 | 10,25 |
| 11 | 11,31 | 11 | 11,31 | 11 | 11,31 | 11 | 11,30 | 11 | 11,29 | 11 | 11,28 |
| 12 | 12,34 | 12 | 12,34 | 12 | 12,34 | 12 | 12,33 | 12 | 12,32 | 12 | 12,30 |
| 13 | 13,36 | 13 | 13,36 | 13 | 13,36 | 13 | 13,36 | 13 | 13,35 | 13 | 13,33 |
| 14 | 14,39 | 14 | 14,39 | 14 | 14,39 | 14 | 14,38 | 14 | 14,37 | 14 | 14,35 |
| 15 | 15,42 | 15 | 15,42 | 15 | 15,42 | 15 | 15,41 | 15 | 15,40 | 15 | 15,38 |
| 18 | 18,51[2] | 18 | 18,50 | 18 | 18,50 | 18 | 18,49 | 18 | 18,48 | 18 | 18,46 |
| 21 | 21,59 | 21 | 21,59 | 21 | 21,59 | 21 | 21,57 | 21 | 21,56 | 21 | 21,54 |
| 24 | 25, 8 | 24 | 25, 7 | 24 | 25, 7 | 24 | 25, 5 | 24 | 25, 4 | 24 | 25, 1[3]↓ |
| 27 | 28,16 | 27 | 28,16 | 27 | 28,16 | 27 | 28,13 | 27 | 28,12 | 27 | 28, 9 |
| 30 | 31,25 | 30 | 31,24 | 30 | 31,24 | 30 | 31,21 | 30 | 31,20 | 30 | 31,17 |
| 33 | 34,33 | 33 | 34,32 | 33 | 34,32 | 33 | 34,29 | 33 | 34,28 | 33 | 34,25 |
| 36 | 37,42 | 36 | 37,41 | 36 | 37,40 | 36 | 37,37 | 36 | 37,36 | 36 | 37,32⊥ |
| 39 | 40,50 | 39 | 40,49 | 39 | 40,49 | 39 | 40,46 | 39 | 40,44 | 39 | 40,40 |
| 42 | 43,59 | 42 | 43,58 | 42 | 43,57 | 42 | 43,54 | 42 | 43,52 | 42 | 43,47 |
| 45 | 47, 7 | 45 | 47, 6 | 45 | 47, 5 | 45 | 47, 2 | 45 | 47, 0 | 45 | 46,55 |
| 48 | 50,16 | 48 | 50,14 | 48 | 50,13 | 48 | 50,10 | 48 | 50, 8 | 48 | 50, 2 |
| 51 | 53,24 | 51 | 53,23 | 51 | 53,22 | 51 | 53,18 | 51 | 53,16 | 51 | 53,10 |
| 54 | 56,33 | 54 | 56,31 | 54 | 56,30[4] | 54 | 56,26 | 54 | 56,23 | 54 | 56,17 |
| 57 | 59,41 | 57 | 59,40 | 57 | 59,39 | 57 | 59,34[5] | 57 | 59,31 | 57 | 59,25 |
| 60 | 62,50 | 60 | 62,48 | 60 | 62,47 | 60 | 62,42 | 60 | 62,39 | 60 | 62,32 |

[1] **D** 8″ [2] **D** 50″ [3] **B** 5°24–36′ (minutes): 1 less (mistakenly copied from the argument column) [4] **D** 32″ [5] **D** 37″.

**Table 8a: Sine for fractions of a degree (part 2)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **6°** | | **7°** | | **8°** | | **9°** | | **10°** | | **11°** | |
| 6;16,18 | | 7;18,44 | | 8;21, 1 | | 9;23,10 | | 10;25, 8 | | 11;26,55 | |
| 1 | 1, 2 | 1 | 1, 2 | 1 | 1, 2 | 1 | 1, 2 | 1 | 1, 2 | 1 | 1, 2 |
| 2 | 2, 5 | 2 | 2, 5 | 2 | 2, 4 | 2 | 2, 4 | 2 | 2, 4 | 2 | 2, 3 |
| 3 | 3, 7 | 3 | 3, 7 | 3 | 3, 7 | 3 | 3, 6 | 3 | 3, 6 | 3 | 3, 5 |
| 4 | 4,10 | 4 | 4, 9 | 4 | 4, 9 | 4 | 4, 8 | 4 | 4, 7 | 4 | 4, 7 |
| 5 | 5,12 | 5 | 5,12 | 5 | 5,11 | 5 | 5,10 | 5 | 5, 9 | 5 | 5, 8 |
| 6 | 6,15 | 6 | 6,14 | 6 | 6,13 | 6 | 6,12 | 6 | 6,11 | 6 | 6,10 |
| 7 | 7,17 | 7 | 7,16 | 7 | 7,15 | 7 | 7,14 | 7 | 7,13 | 7 | 7,12 |
| 8 | 8,20 | 8 | 8,19 | 8 | 8,18 | 8 | 8,16 | 8 | 8,15 | 8 | 8,13 |
| 9 | 9,22 | 9 | 9,21 | 9 | 9,20 | 9 | 9,18 | 9 | 9,17 | 9 | 9,15 |
| 10 | 10,25 | 10 | 10,23 | 10 | 10,22 | 10 | 10,20 | 10 | 10,19 | 10 | 10,17 |
| 11 | 11,27 | 11 | 11,26 | 11 | 11,24 | 11 | 11,22 | 11 | 11,21 | 11 | 11,18 |
| 12 | 12,30 | 12 | 12,28 | 12 | 12,26 | 12 | 12,24 | 12 | 12,22 | 12 | 12,20 |
| 13 | 13,32 | 13 | 13,30 | 13 | 13,29 | 13 | 13,26 | 13 | 13,24 | 13 | 13,22 |
| 14 | 14,35 | 14 | 14,33 | 14 | 14,31 | 14 | 14,28 | 14 | 14,26 | 14 | 14,23 |
| 15 | 15,37 | 15 | 15,35 | 15 | 15,33 | 15 | 15,30 | 15 | 15,28 | 15 | 15,25 |
| 18 | 18,44 | 18 | 18,42 | 18 | 18,40 | 18 | 18,36 | 18 | 18,33 | 18 | 18,30 |
| 21 | 21,52 | 21 | 21,49 | 21 | 21,46 | 21 | 21,42 | 21 | 21,39 | 21 | 21,34[1] |
| 24 | 24,59 | 24 | 24,56 | 24 | 24,53 | 24 | 24,48 | 24 | 24,44 | 24 | 24,39 |
| 27 | 28, 7 | 27 | 28, 3 | 27 | 27,59 | 27 | 27,54 | 27 | 27,50 | 27 | 27,43 |
| 30 | 31,14 | 30 | 31,10 | 30 | 31, 6 | 30 | 31, 0 | 30 | 30,55 | 30 | 30,48 |
| 33 | 34,21 | 33 | 34,17 | 33 | 34,12 | 33 | 34, 6 | 33 | 34, 0 | 33 | 33,53 |
| 36 | 37,28 | 36 | 37,24 | 36 | 37,19 | 36 | 37,12 | 36 | 37, 5 | 36 | 36,58 |
| 39 | 40,36 | 39 | 40,30 | 39 | 40,25 | 39 | 40,18 | 39 | 40,11 | 39 | 40, 2 |
| 42 | 43,43 | 42 | 43,37 | 42 | 43,32 | 42 | 43,24 | 42 | 43,16 | 42 | 43, 7 |
| 45 | 46,50 | 45 | 46,44 | 45 | 46,38 | 45 | 46,30 | 45 | 46,21 | 45 | 46,12 |
| 48 | 49,57 | 48 | 49,51 | 48 | 49,44 | 48 | 49,36 | 48 | 49,26 | 48 | 49,16 |
| 51 | 53, 4[2] | 51 | 52,57 | 51 | 52,50 | 51 | 52,41 | 51 | 52,31 | 51 | 52,21 |
| 54 | 56,12 | 54 | 56, 4 | 54 | 55,57 | 54 | 55,47 | 54 | 55,37 | 54 | 55,25 |
| 57 | 59,19 | 57 | 59,10 | 57 | 59, 3 | 57 | 58,52 | 57 | 58,42 | 57 | 58,30 |
| 60 | 62,26 | 60 | 62,17 | 60 | 62, 9 | 60 | 61,58 | 60 | 61,47 | 60 | 61,34 |

[1] **D** 29′ [2] **L** 52′.

**Table 8a: Sine for fractions of a degree (part 3)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **12°** | | **13°** | | **14°** | | **15°** | | **16°** | | **17°** | |
| 12;28,29 | | 13;29,49 | | 14;30,55 | | 15;31,45 | | 16;32,18 | | 17;32,32[1] | |
| 1 | 1, 1 | 1 | 1, 1 | 1 | 1, 1 | 1 | 1, 1 | 1 | 1, 0 | 1 | 1, 0 |
| 2 | 2, 3 | 2 | 2, 2 | 2 | 2, 2 | 2 | 2, 1 | 2 | 2, 1 | 2 | 2, 0 |
| 3 | 3, 4 | 3 | 3, 4 | 3 | 3, 3 | 3 | 3, 2 | 3 | 3, 1 | 3 | 3, 0 |
| 4 | 4, 6 | 4 | 4, 5 | 4 | 4, 4 | 4 | 4, 3 | 4 | 4, 1 | 4 | 4, 0 |
| 5 | 5, 7 | 5 | 5, 6 | 5 | 5, 5 | 5 | 5, 3 | 5 | 5, 2 | 5 | 5, 0 |
| 6 | 6, 8 | 6 | 6, 7 | 6 | 6, 6 | 6 | 6, 4 | 6 | 6, 2 | 6 | 6, 0 |
| 7 | 7,10 | 7 | 7, 8 | 7 | 7, 7 | 7 | 7, 5 | 7 | 7, 2 | 7 | 7, 0 |
| 8 | 8,11 | 8 | 8,10 | 8 | 8, 7 | 8 | 8, 5 | 8 | 8, 3 | 8 | 8, 1 |
| 9 | 9,13 | 9 | 9,11 | 9 | 9, 8 | 9 | 9, 6 | 9 | 9, 3 | 9 | 9, 1 |
| 10 | 10,14 | 10 | 10,12 | 10 | 10, 9 | 10 | 10, 7 | 10 | 10, 3 | 10 | 10, 1 |
| 11 | 11,15 | 11 | 11,13 | 11 | 11,10 | 11 | 11, 7 | 11 | 11, 4 | 11 | 11, 1 |
| 12 | 12,17 | 12 | 12,14 | 12 | 12,11 | 12 | 12, 8 | 12 | 12, 4 | 12 | 12, 1 |
| 13 | 13,18 | 13 | 13,16 | 13 | 13,12 | 13 | 13, 9 | 13 | 13, 4 | 13 | 13, 1 |
| 14 | 14,20 | 14 | 14,17 | 14 | 14,13 | 14 | 14, 9 | 14 | 14, 5 | 14 | 14, 1 |
| 15 | 15,21 | 15 | 15,18 | 15 | 15,14 | 15 | 15,10 | 15 | 15, 5 | 15 | 15, 1 |
| 18 | 18,25 | 18 | 18,21 | 18 | 18,17 | 18 | 18,12 | 18 | 18, 6 | 18 | 18, 1 |
| 21 | 21,29 | 21 | 21,25 | 21 | 21,19 | 21 | 21,13 | 21 | 21, 7 | 21 | 21, 1 |
| 24 | 24,34 | 24 | 24,28 | 24 | 24,22 | 24 | 24,15 | 24 | 24, 7 | 24 | 24, 0 |
| 27 | 27,38 | 27 | 27,32 | 27 | 27,24 | 27 | 27,16 | 27 | 27, 8 | 27 | 27, 0 |
| 30 | 30,42 | 30 | 30,35 | 30 | 30,27 | 30 | 30,18 | 30 | 30, 9 | 30 | 30, 0 |
| 33 | 33,46 | 33 | 33,38 | 33 | 33,29 | 33 | 33,20 | 33 | 33,10 | 33 | 33, 0 |
| 36 | 36,50 | 36 | 36,41 | 36 | 36,32 | 36 | 36,21 | 36 | 36,10 | 36 | 36, 0 |
| 39 | 39,54 | 39 | 39,45 | 39 | 39,34 | 39 | 39,23 | 39 | 39,11 | 39 | 38,59 |
| 42 | 42,58 | 42 | 42,48 | 42 | 42,37 | 42 | 42,24 | 42 | 42,11 | 42 | 41,59 |
| 45 | 46, 2[2]↓ | 45 | 45,51 | 45 | 45,39 | 45 | 45,26 | 45 | 45,12 | 45 | 44,59 |
| 48 | 49, 6 | 48 | 48,54 | 48 | 48,41 | 48 | 48,27 | 48 | 48,12 | 48 | 47,58 |
| 51 | 52, 9⊥ | 51 | 51,57 | 51 | 51,43 | 51 | 51,29 | 51 | 51,13 | 51 | 50,58 |
| 54 | 55,13 | 54 | 55, 0 | 54 | 54,46 | 54 | 54,30 | 54 | 54,13 | 54 | 53,57[3] |
| 57 | 58,16 | 57 | 58, 3 | 57 | 57,48 | 57 | 57,32 | 57 | 57,14 | 57 | 56,57 |
| 60 | 61,20 | 60 | 61, 6 | 60 | 60,50 | 60 | 60,33 | 60 | 60,14 | 60 | 59,56 |

[1] **D** 13″ [2] **L** 12°45–51′ (minutes): 1 less (mistakenly copied from the argument column)
[3] **B** 54″.

**Table 8a: Sine for fractions of a degree (part 4)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **18°** | | **19°** | | **20°** | | **21°** | | **22°** | | **23°** | |
| 18;32,28 | | 19;32, 3 | | 20;31,16 | | 21;30, 7 | | 22;28,35 | | 23;26,38 | |
| 1 | 1, 0 | 1 | 0,59 | 1 | 0,59 | 1 | 0,59 | 1 | 0,58 | 1 | 0,58 |
| 2 | 1,59 | 2 | 1,59 | 2 | 1,58 | 2 | 1,57 | 2 | 1,56 | 2 | 1,56 |
| 3 | 2,59 | 3 | 2,58 | 3 | 2,57 | 3 | 2,56 | 3 | 2,55 | 3 | 2,53 |
| 4 | 3,59 | 4 | 3,57 | 4 | 3,56 | 4 | 3,55 | 4 | 3,53 | 4 | 3,51 |
| 5 | 4,58 | 5 | 4,57 | 5 | 4,55 | 5 | 4,53 | 5 | 4,51 | 5 | 4,49 |
| 6 | 5,58 | 6 | 5,56 | 6 | 5,54 | 6 | 5,52 | 6 | 5,49 | 6 | 5,47 |
| 7 | 6,58 | 7 | 6,55 | 7 | 6,53 | 7 | 6,51 | 7 | 6,47 | 7 | 6,45 |
| 8 | 7,57 | 8 | 7,55 | 8 | 7,52 | 8 | 7,49 | 8 | 7,46 | 8 | 7,42 |
| 9 | 8,57 | 9 | 8,54 | 9 | 8,51 | 9 | 8,48 | 9 | 8,44 | 9 | 8,40 |
| 10 | 9,57 | 10 | 9,53 | 10 | 9,50 | 10 | 9,47 | 10 | 9,42 | 10 | 9,38 |
| 11 | 10,56 | 11 | 10,53 | 11 | 10,49 | 11 | 10,45 | 11 | 10,40 | 11 | 10,36 |
| 12 | 11,56 | 12 | 11,52 | 12 | 11,48 | 12 | 11,44 | 12 | 11,38 | 12 | 11,34 |
| 13 | 12,56 | 13 | 12,51 | 13 | 12,47 | 13 | 12,43 | 13 | 12,37 | 13 | 12,31 |
| 14 | 13,55 | 14 | 13,51 | 14 | 13,46 | 14 | 13,41 | 14 | 13,35 | 14 | 13,29 |
| 15 | 14,55 | 15 | 14,50 | 15 | 14,45 | 15 | 14,40 | 15 | 14,33 | 15 | 14,27 |
| 18 | 17,54 | 18 | 17,48 | 18 | 17,42 | 18 | 17,35 | 18 | 17,27 | 18 | 17,20 |
| 21 | 20,53 | 21 | 20,46 | 21 | 20,39 | 21 | 20,31 | 21 | 20,22 | 21 | 20,13 |
| 24 | 23,52 | 24 | 23,43 | 24 | 23,35 | 24 | 23,26 | 24 | 23,16 | 24 | 23, 6 |
| 27 | 26,51 | 27 | 26,41 | 27 | 26,32 | 27 | 26,22 | 27 | 26,11 | 27 | 25,59 |
| 30 | 29,50 | 30 | 29,39 | 30 | 29,29 | 30 | 29,17 | 30 | 29, 5 | 30 | 28,52 |
| 33 | 32,49 | 33 | 32,37 | 33 | 32,25 | 33 | 32,12 | 33 | 31,59 | 33 | 31,45 |
| 36 | 35,47 | 36 | 35,34 | 36 | 35,22 | 36 | 35, 7 | 36 | 34,53 | 36 | 34,37 |
| 39[1] | 38,46 | 39 | 38,32 | 39 | 38,18 | 39 | 38, 3 | 39 | 37,47 | 39 | 37,30 |
| 42 | 41,44 | 42 | 41,29 | 42 | 41,15 | 42 | 40,58 | 42 | 40,41 | 42 | 40,22 |
| 45 | 44,43 | 45 | 44,27 | 45 | 44,11 | 45 | 43,53 | 45 | 43,35 | 45 | 43,15 |
| 48 | 47,41 | 48 | 47,24 | 48 | 47, 7 | 48 | 46,48 | 48 | 46,29 | 48 | 46, 7 |
| 51 | 50,40 | 51 | 50,21 | 51 | 50, 3 | 51 | 49,43 | 51 | 49,22 | 51 | 49, 0 |
| 54 | 53,38 | 54 | 53,19 | 54 | 52,59 | 54 | 52,38 | 54 | 52,16 | 54 | 51,52 |
| 57 | 56,37 | 57 | 56,16 | 57 | 55,55 | 57 | 55,33 | 57 | 55, 9 | 57 | 54,45 |
| 60 | 59,35 | 60 | 59,13[2] | 60 | 58,51 | 60 | 58,28 | 60 | 58, 3 | 60 | 57,37 |

[1] **D** 49 [2] **L** 19′.

**Table 8a: Sine for fractions of a degree (part 5)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **24°** | | **25°** | | **26°** | | **27°** | | **28°** | | **29°** | |
| 24;24,15 | | 25;21,26 | | 26;18, 8 | | 27;14,22 | | 28;10, 6 | | 29; 5,19 | |
| 1 | 0,57[1] | 1 | 0,57 | 1 | 0,56 | 1 | 0,56 | 1 | 0,55 | 1 | 0,55 |
| 2 | 1,55 | 2 | 1,54 | 2 | 1,53 | 2 | 1,52 | 2 | 1,51 | 2 | 1,50 |
| 3 | 2,52 | 3 | 2,51 | 3 | 2,49 | 3 | 2,48 | 3 | 2,46 | 3 | 2,45 |
| 4 | 3,49 | 4 | 3,47 | 4 | 3,46 | 4 | 3,44 | 4 | 3,42 | 4 | 3,39 |
| 5 | 4,47 | 5 | 4,44 | 5 | 4,42 | 5 | 4,40 | 5 | 4,37 | 5 | 4,34 |
| 6 | 5,44 | 6 | 5,41 | 6 | 5,38 | 6 | 5,36 | 6 | 5,32 | 6 | 5,29 |
| 7 | 6,41 | 7 | 6,38 | 7 | 6,35 | 7 | 6,32 | 7 | 6,28 | 7 | 6,24 |
| 8 | 7,39 | 8 | 7,35 | 8 | 7,31 | 8 | 7,27 | 8 | 7,23 | 8 | 7,19 |
| 9 | 8,36 | 9 | 8,32 | 9 | 8,28 | 9 | 8,23 | 9 | 8,19 | 9 | 8,14 |
| 10 | 9,33 | 10 | 9,29 | 10 | 9,24 | 10 | 9,19 | 10 | 9,14 | 10 | 9, 9 |
| 11 | 10,31 | 11 | 10,26 | 11 | 10,20 | 11 | 10,15 | 11 | 10, 9 | 11 | 10, 4 |
| 12 | 11,28 | 12 | 11,22 | 12 | 11,17 | 12 | 11,11 | 12 | 11, 5 | 12 | 10,58 |
| 13 | 12,25 | 13 | 12,19 | 13 | 12,13 | 13 | 12, 7 | 13 | 12, 0 | 13 | 11,53 |
| 14 | 13,23 | 14 | 13,16 | 14 | 13,10 | 14 | 13, 3 | 14 | 12,56 | 14 | 12,48 |
| 15 | 14,20 | 15 | 14,13 | 15 | 14, 6 | 15 | 13,59 | 15 | 13,51 | 15 | 13,43 |
| 18 | 17,12 | 18 | 17, 3 | 18 | 16,55 | 18 | 16,46 | 18 | 16,37 | 18 | 16,27 |
| 21 | 20, 4 | 21 | 19,53 | 21 | 19,44 | 21 | 19,34 | 21 | 19,23 | 21 | 19,11 |
| 24 | 22,55 | 24 | 22,44 | 24 | 22,33 | 24 | 22,21 | 24 | 22, 8 | 24 | 21,56 |
| 27 | 25,47 | 27 | 25,34 | 27 | 25,22 | 27 | 25, 9 | 27 | 24,54 | 27 | 24,40 |
| 30 | 28,39 | 30 | 28,24 | 30 | 28,11 | 30 | 27,56 | 30 | 27,40 | 30 | 27,24 |
| 33 | 31,30 | 33 | 31,14 | 33 | 30,59 | 33 | 30,43 | 33 | 30,26 | 33 | 30, 8 |
| 36 | 34,22 | 36 | 34, 4 | 36 | 33,48 | 36 | 33,30 | 36 | 33,11 | 36 | 32,52 |
| 39 | 37,13 | 39 | 36,54 | 39 | 36,36 | 39 | 36,17 | 39 | 35,57 | 39 | 35,36 |
| 42 | 40, 5 | 42 | 39,44 | 42 | 39,25 | 42 | 39, 4[2] | 42 | 38,42[3] | 42 | 38,20 |
| 45 | 42,56 | 45 | 42,34 | 45 | 42,13 | 45 | 41,51 | 45 | 41,28 | 45 | 41, 4[4] |
| 48 | 45,47 | 48 | 45,24 | 48 | 45, 1 | 48 | 44,38 | 48 | 44,13 | 48 | 43,47 |
| 51 | 48,38 | 51 | 48,13 | 51 | 47,49 | 51 | 47,24 | 51 | 46,58 | 51 | 46,31 |
| 54 | 51,29 | 54 | 51, 3 | 54 | 50,38 | 54 | 50,11 | 54 | 49,43 | 54 | 49,14 |
| 57 | 54,20 | 57 | 53,52 | 57 | 53,26 | 57 | 52,57[5] | 57 | 52,28 | 57 | 51,58 |
| 60 | 57,11 | 60 | 56,42 | 60 | 56,14 | 60 | 55,44 | 60 | 55,13 | 60 | 54,41 |

[1] **D** 56″ [2] **D** 7″ [3] **D** 44″ [4] **D** 7″ [5] **D** 56″.

**Table 8a: Sine for fractions of a degree (part 6)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **30°** | | **31°** | | **32°** | | **33°** | | **34°** | | **35°** | |
| 30; 0, 0 | | 30;54, 8 | | 31;47,43 | | 32;40,42 | | 33;33, 6 | | 34;24,53[1] | |
| 1 | 0,54 | 1 | 0,54 | 1 | 0,53 | 1 | 0,53 | 1 | 0,52 | 1 | 0,51 |
| 2 | 1,49 | 2 | 1,48 | 2 | 1,46 | 2 | 1,45 | 2 | 1,44 | 2 | 1,43 |
| 3 | 2,43 | 3 | 2,41 | 3 | 2,40 | 3 | 2,38 | 3 | 2,36 | 3 | 2,34 |
| 4 | 3,37 | 4 | 3,35 | 4 | 3,33 | 4 | 3,30 | 4 | 3,28 | 4 | 3,25 |
| 5 | 4,32 | 5 | 4,29 | 5 | 4,26 | 5 | 4,23 | 5 | 4,20 | 5 | 4,17 |
| 6 | 5,26 | 6 | 5,23 | 6 | 5,19 | 6 | 5,16 | 6 | 5,12 | 6 | 5, 8 |
| 7 | 6,20 | 7 | 6,17 | 7 | 6,12 | 7 | 6, 8 | 7 | 6, 4[2] | 7 | 5,59 |
| 8 | 7,15 | 8 | 7,10 | 8 | 7, 6 | 8 | 7, 1 | 8 | 6,56 | 8 | 6,51 |
| 9 | 8, 9 | 9 | 8, 7[3] | 9 | 7,59 | 9 | 7,53 | 9 | 7,48 | 9 | 7,42 |
| 10 | 9, 3 | 10 | 8,58 | 10 | 8,52 | 10 | 8,46 | 10 | 8,40 | 10 | 8,33 |
| 11 | 9,58 | 11 | 9,52 | 11 | 9,45 | 11 | 9,39 | 11 | 9,32 | 11 | 9,25 |
| 12 | 10,52 | 12 | 10,46 | 12 | 10,38 | 12 | 10,31 | 12 | 10,24 | 12 | 10,16 |
| 13 | 11,46 | 13 | 11,39 | 13 | 11,32 | 13 | 11,24 | 13 | 11,16 | 13 | 11, 7 |
| 14 | 12,41 | 14 | 12,33 | 14 | 12,25 | 14 | 12,16 | 14 | 12, 8 | 14 | 11,59[4] |
| 15 | 13,35 | 15 | 13,27 | 15 | 13,18 | 15 | 13, 9 | 15 | 13, 0 | 15 | 12,50 |
| 18 | 16,18 | 18 | 16, 8 | 18 | 15,57 | 18 | 15,46 | 18 | 15,36 | 18 | 15,24 |
| 21 | 19, 0 | 21 | 18,49 | 21 | 18,36 | 21 | 18,24 | 21 | 18,11 | 21 | 17,58 |
| 24 | 21,43 | 24 | 21,30 | 24 | 21,16 | 24 | 21, 1 | 24 | 20,47 | 24 | 20,31 |
| 27 | 24,25 | 27 | 24,11 | 27 | 23,55 | 27 | 23,39 | 27 | 23,22 | 27 | 23, 5 |
| 30 | 27, 8 | 30 | 26,52 | 30 | 26,34 | 30 | 26,16 | 30 | 25,58 | 30 | 25,39 |
| 33 | 29,50 | 33 | 29,32 | 33 | 29,13 | 33 | 28,53 | 33 | 28,33 | 33 | 28,12 |
| 36 | 32,32 | 36 | 32,13 | 36 | 31,51 | 36 | 31,30 | 36 | 31, 8 | 36 | 30,45 |
| 39 | 35,15 | 39 | 34,53 | 39 | 34,30 | 39 | 34, 7 | 39 | 33,43 | 39 | 33,19 |
| 42 | 37,57 | 42 | 37,34 | 42 | 37, 8 | 42 | 36,44 | 42 | 36,18 | 42 | 35,52 |
| 45 | 40,39 | 45 | 40,14 | 45 | 39,47 | 45 | 39,21 | 45 | 38,53 | 45 | 38,25 |
| 48 | 43,21 | 48 | 42,54 | 48 | 42,25 | 48 | 41,58[5] | 48 | 41,28 | 48 | 40,58 |
| 51 | 46, 3 | 51 | 45,34 | 51 | 45, 4 | 51 | 44,34[6]↓ | 51 | 44, 3[7] | 51 | 43,31 |
| 54 | 48,44 | 54 | 48,15 | 54 | 47,42 | 54 | 47,11 | 54 | 46,37 | 54 | 46, 3 |
| 57 | 51,26 | 57 | 50,55 | 57 | 50,21[8] | 57 | 49,47 | 57 | 49,12 | 57 | 48,36 |
| 60 | 54, 8 | 60 | 53,35 | 60 | 52,59 | 60 | 52,24⊥ | 60 | 51,47 | 60 | 51, 9 |

[1] **D** 22′ [2] **D** 3″ [3] **B** 4″ [4] **B** 49″ [5] **D** 18″ [6] **D** 33°51–60′ (minutes): 1 less (miscopied from the column for 35°?) [7] **D** 8″ [8] **D** 31″.

**Table 8a: Sine for fractions of a degree (part 7)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **36°** | | **37°** | | **38°** | | **39°** | | **40°** | | **41°** | |
| 35;16, 2 | | 36; 6,32 | | 36;56,23 | | 37;45,33 | | 38;34, 2 | | 39;21,49 | |
| 1 | 0,51 | 1 | 0,50 | 1 | 0,49 | 1 | 0,49 | 1 | 0,48 | 1 | 0,47 |
| 2 | 1,41 | 2 | 1,40 | 2 | 1,39 | 2 | 1,37 | 2 | 1,36 | 2 | 1,35 |
| 3 | 2,32 | 3 | 2,30 | 3 | 2,28 | 3 | 2,26 | 3 | 2,24 | 3 | 2,22 |
| 4 | 3,23 | 4 | 3,21 | 4 | 3,18 | 4 | 3,15 | 4 | 3,12 | 4 | 3, 9 |
| 5 | 4,14 | 5 | 4,11 | 5 | 4, 7 | 5 | 4, 4 | 5 | 4, 0 | 5 | 3,57 |
| 6 | 5, 4 | 6 | 5, 1 | 6 | 4,56 | 6 | 4,52 | 6 | 4,48 | 6 | 4,44 |
| 7 | 5,55 | 7 | 5,51 | 7 | 5,46 | 7 | 5,41 | 7 | 5,36 | 7 | 5,31 |
| 8 | 6,46 | 8 | 6,41 | 8 | 6,35 | 8 | 6,30 | 8 | 6,25 | 8 | 6,19 |
| 9 | 7,37 | 9 | 7,31 | 9 | 7,25 | 9 | 7,19 | 9 | 7,13 | 9 | 7, 6 |
| 10 | 8,27 | 10 | 8,21 | 10 | 8,14 | 10 | 8, 7 | 10 | 8, 1 | 10 | 7,53 |
| 11 | 9,18 | 11 | 9,11 | 11 | 9, 3 | 11 | 8,56 | 11 | 8,49 | 11 | 8,41 |
| 12 | 10, 9 | 12 | 10, 2 | 12 | 9,53 | 12 | 9,45 | 12 | 9,37 | 12 | 9,28 |
| 13 | 11, 0 | 13 | 10,52 | 13 | 10,42 | 13 | 10,34 | 13 | 10,25 | 13 | 10,15 |
| 14 | 11,50 | 14 | 11,42 | 14 | 11,32 | 14 | 11,22 | 14 | 11,13 | 14 | 11, 3 |
| 15 | 12,41 | 15 | 12,32 | 15 | 12,21 | 15 | 12,11 | 15 | 12, 1 | 15 | 11,50 |
| 18 | 15,13 | 18 | 15, 2 | 18 | 14,49 | 18 | 14,37 | 18 | 14,25 | 18 | 14,11 |
| 21 | 17,45[1] | 21 | 17,31 | 21 | 17,17 | 21 | 17, 3 | 21 | 16,48 | 21 | 16,33 |
| 24 | 20,16 | 24 | 20, 1 | 24 | 19,44 | 24 | 19,28 | 24 | 19,12 | 24 | 18,54 |
| 27 | 22,48 | 27 | 22,30 | 27 | 22,12 | 27 | 21,54 | 27 | 21,35 | 27 | 21,16 |
| 30 | 25,20 | 30 | 25, 0 | 30 | 24,40 | 30 | 24,20 | 30 | 23,59 | 30 | 23,37 |
| 33 | 27,51 | 33 | 27,29 | 33 | 27, 7 | 33 | 26,45 | 33 | 26,22 | 33 | 25,58 |
| 36 | 30,22 | 36 | 29,59 | 36 | 29,34 | 36 | 29,10 | 36 | 28,45 | 36 | 28,19 |
| 39 | 32,54[2] | 39 | 32,28 | 39 | 32, 2 | 39 | 31,36 | 39 | 31, 8 | 39 | 30,39 |
| 42 | 35,25 | 42 | 34,58 | 42 | 34,29 | 42 | 34, 1 | 42 | 33,31 | 42 | 33, 0 |
| 45 | 37,56 | 45 | 37,27 | 45 | 36,56 | 45 | 36,26 | 45 | 35,54 | 45 | 35,21 |
| 48 | 40,27 | 48 | 39,56 | 48 | 39,23 | 48 | 38,51 | 48 | 38,17 | 48 | 37,41 |
| 51 | 42,58 | 51 | 42,25 | 51 | 41,50 | 51 | 41,15 | 51 | 40,39 | 51 | 40, 2 |
| 54 | 45,28 | 54 | 44,53 | 54 | 44,16 | 54 | 43,40 | 54 | 43, 2 | 54 | 42,22 |
| 57 | 47,59 | 57 | 47,22 | 57 | 46,43 | 57 | 46, 4[3] | 57 | 45,24 | 57 | 44,43 |
| 60 | 50,30 | 60 | 49,51 | 60 | 49,10 | 60 | 48,29 | 60 | 47,47 | 60 | 47, 3 |

[1] **D** 44″ [2] **B** 14″ [3] **D** 45′.

**Table 8a: Sine for fractions of a degree (part 8)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **42°** | | **43°** | | **44°** | | **45°** | | **46°** | | **47°** | |
| 40; 8,52 | | 40;55,12 | | 41;40,46 | | 42;25,35 | | 43; 9,37 | | 43;52,52[1] | |
| 1 | 0,47 | 1 | 0,46 | 1 | 0,45 | 1 | 0,44 | 1 | 0,44 | 1 | 0,43 |
| 2 | 1,33 | 2 | 1,32 | 2 | 1,30 | 2 | 1,29 | 2 | 1,27 | 2 | 1,26 |
| 3 | 2,20 | 3 | 2,18 | 3 | 2,15 | 3 | 2,13 | 3 | 2,11 | 3 | 2, 8 |
| 4 | 3, 6 | 4 | 3, 3 | 4 | 3, 1 | 4 | 2,57 | 4 | 2,54 | 4 | 2,51 |
| 5 | 3,53 | 5 | 3,49 | 5 | 3,46 | 5 | 3,42 | 5 | 3,38 | 5 | 3,34 |
| 6 | 4,40 | 6 | 4,35 | 6 | 4,31 | 6 | 4,26 | 6 | 4,22 | 6 | 4,17 |
| 7 | 5,26 | 7 | 5,21 | 7 | 5,16 | 7 | 5,10 | 7 | 5, 5 | 7 | 5, 0 |
| 8 | 6,13 | 8 | 6, 7 | 8 | 6, 1 | 8 | 5,55 | 8 | 5,49 | 8 | 5,42 |
| 9 | 6,59 | 9 | 6,53 | 9 | 6,46 | 9 | 6,39 | 9 | 6,32 | 9 | 6,25 |
| 10 | 7,46 | 10 | 7,39 | 10 | 7,31 | 10 | 7,23 | 10 | 7,16 | 10 | 7, 8 |
| 11 | 8,33 | 11 | 8,25 | 11 | 8,16 | 11 | 8, 8 | 11 | 8, 0 | 11 | 7,51 |
| 12 | 9,19 | 12 | 9,10 | 12 | 9, 2 | 12 | 8,52 | 12 | 8,43 | 12 | 8,34 |
| 13 | 10, 6 | 13 | 9,56 | 13 | 9,47 | 13 | 9,36 | 13 | 9,27 | 13 | 9,16 |
| 14 | 10,52 | 14 | 10,42 | 14 | 10,32 | 14 | 10,21 | 14 | 10,10 | 14 | 9,59 |
| 15 | 11,39 | 15 | 11,28 | 15 | 11,17 | 15 | 11, 5 | 15 | 10,54 | 15 | 10,42 |
| 18 | 13,58 | 18 | 13,45 | 18 | 13,32 | 18 | 13,17[2] | 18 | 13, 4 | 18 | 12,50 |
| 21 | 16,17 | 21 | 16, 2 | 21 | 15,46 | 21 | 15,30 | 21 | 15,14 | 21 | 14,57 |
| 24 | 18,37 | 24 | 18,19 | 24 | 18, 1 | 24 | 17,42 | 24 | 17,24 | 24 | 17, 5 |
| 27 | 20,56 | 27 | 20,36 | 27 | 20,15 | 27 | 19,55[3] | 27 | 19,34 | 27 | 19,12 |
| 30 | 23,15 | 30 | 22,53 | 30 | 22,30 | 30 | 22, 7 | 30 | 21,44 | 30 | 21,20 |
| 33 | 25,34 | 33 | 25, 9 | 33 | 24,44[4] | 33 | 24,19 | 33 | 23,53 | 33 | 23,27 |
| 36 | 27,53 | 36 | 27,26 | 36 | 26,58 | 36 | 26,31 | 36 | 26, 3 | 36 | 25,34 |
| 39 | 30,11 | 39 | 29,42 | 39 | 29,13 | 39 | 28,42 | 39 | 28,12 | 39 | 27,41 |
| 42 | 32,30 | 42 | 31,59 | 42 | 31,27 | 42 | 30,54 | 42 | 30,22 | 42 | 29,48 |
| 45 | 34,49 | 45 | 34,15 | 45 | 33,41 | 45 | 33, 6 | 45 | 32,31 | 45 | 31,55 |
| 48 | 37, 7 | 48 | 36,31 | 48 | 35,55 | 48 | 35,17 | 48 | 34,40 | 48 | 34, 1 |
| 51 | 39,25 | 51 | 38,47[5] | 51 | 38, 8 | 51 | 37,28 | 51 | 36,49 | 51 | 36, 8 |
| 54 | 41,44 | 54 | 41, 2 | 54 | 40,22 | 54 | 39,40 | 54 | 38,57 | 54 | 38,14 |
| 57 | 44, 2 | 57 | 43,18 | 57 | 42,35 | 57 | 41,51 | 57 | 41, 6 | 57 | 40,21 |
| 60 | 46,20 | 60 | 45,34 | 60 | 44,49 | 60 | 44, 2 | 60 | 43,15 | 60 | 42,27 |

[1] **D** 53″ [2] **D** 12′57″ [3] **D** 59″ [4] **B** 43″ [5] **D** 46″.

**Table 8a: Sine for fractions of a degree (part 9)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **48°** | | **49°** | | **50°** | | **51°** | | **52°** | | **53°** | |
| 44;35,19 | | 45;16,57 | | 45;57,46 | | 46;37,44 | | 47;16,50 | | 47;55, 5 | |
| 1 | 0,42 | 1 | 0,41 | 1 | 0,40 | 1 | 0,39 | 1 | 0,39 | 1 | 0,38 |
| 2 | 1,24 | 2 | 1,22 | 2 | 1,21 | 2 | 1,19 | 2 | 1,17 | 2 | 1,15 |
| 3 | 2, 6 | 3 | 2, 3 | 3 | 2, 1 | 3 | 1,58 | 3 | 1,56 | 3 | 1,53 |
| 4 | 2,48 | 4 | 2,45 | 4 | 2,41 | 4 | 2,38 | 4 | 2,34 | 4 | 2,31 |
| 5 | 3,30[1] | 5 | 3,26 | 5 | 3,21 | 5 | 3,17 | 5 | 3,13 | 5 | 3, 9 |
| 6 | 4,12 | 6 | 4, 7 | 6 | 4, 2 | 6 | 3,56 | 6 | 3,52 | 6 | 3,47 |
| 7 | 4,54 | 7 | 4,48 | 7 | 4,42 | 7 | 4,36 | 7 | 4,30 | 7 | 4,24[2] |
| 8 | 5,35 | 8 | 5,29 | 8 | 5,22 | 8 | 5,15 | 8 | 5, 9 | 8 | 5, 2 |
| 9 | 6,17 | 9 | 6,10 | 9 | 6, 2 | 9 | 5,55 | 9 | 5,47 | 9 | 5,40 |
| 10 | 6,59 | 10 | 6,51 | 10 | 6,43 | 10 | 6,34 | 10 | 6,26 | 10 | 6,17 |
| 11 | 7,41 | 11 | 7,32 | 11 | 7,23 | 11 | 7,13 | 11 | 7, 5 | 11 | 6,55 |
| 12 | 8,23 | 12 | 8,14 | 12 | 8, 3 | 12 | 7,53 | 12 | 7,43 | 12 | 7,33 |
| 13 | 9, 5 | 13 | 8,55 | 13 | 8,43 | 13 | 8,32 | 13 | 8,22 | 13 | 8,11 |
| 14 | 9,47 | 14 | 9,36 | 14 | 9,24 | 14 | 9,12 | 14 | 9, 0 | 14 | 8,48 |
| 15 | 10,29 | 15 | 10,17 | 15 | 10, 4 | 15 | 9,51 | 15 | 9,39 | 15 | 9,26 |
| 18 | 12,34 | 18 | 12,20 | 18 | 12, 4 | 18 | 11,49 | 18 | 11,34 | 18 | 11,18 |
| 21 | 14,39 | 21 | 14,23 | 21 | 14, 4 | 21 | 13,47 | 21 | 13,29 | 21 | 13,11 |
| 24 | 16,45 | 24 | 16,25 | 24 | 16, 5 | 24 | 15,44 | 24 | 15,24[3] | 24 | 15, 3 |
| 27 | 18,50 | 27 | 18,28 | 27 | 18, 5 | 27 | 17,42 | 27 | 17,19 | 27 | 16,56 |
| 30 | 20,55 | 30 | 20,31 | 30 | 20, 5 | 30 | 19,40 | 30 | 19,14 | 30 | 18,48 |
| 33 | 23, 0 | 33 | 22,33 | 33 | 22, 5 | 33 | 21,37 | 33 | 21, 8 | 33 | 20,40 |
| 36 | 25, 4 | 36 | 24,35 | 36 | 24, 4 | 36 | 23,34 | 36 | 23, 3 | 36 | 22,32 |
| 39 | 27, 9 | 39 | 26,37 | 39 | 26, 4 | 39 | 25,30 | 39 | 24,57 | 39 | 24,23 |
| 42 | 29,13 | 42 | 28,39 | 42 | 28, 3 | 42 | 27,27 | 42 | 26,52 | 42 | 26,15 |
| 45 | 31,18 | 45 | 30,41 | 45 | 30, 3 | 45 | 29,24 | 45 | 28,46 | 45 | 28, 7 |
| 48 | 33,22 | 48 | 32,43 | 48 | 32, 2 | 48 | 31,20 | 48 | 30,40 | 48 | 29,58 |
| 51 | 35,26 | 51 | 34,44 | 51 | 34, 1 | 51 | 33,17 | 51 | 32,34 | 51 | 31,49[4] |
| 54 | 37,30 | 54 | 36,46 | 54 | 36, 0[5] | 54 | 35,13 | 54 | 34,27 | 54 | 33,41 |
| 57 | 39,34 | 57 | 38,47 | 57 | 37,59 | 57 | 37,10 | 57 | 36,21 | 57 | 35,32 |
| 60 | 41,38 | 60 | 40,49 | 60 | 39,58 | 60 | 39, 6 | 60 | 38,15 | 60 | 37,23 |

[1] **D** 32″ [2] **D** 25″ [3] **D** 22″ [4] **D** 43″ [5] **D** 35′.

**Table 8a: Sine for fractions of a degree (part 10)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **54°** | | **55°** | | **56°** | | **57°** | | **58°** | | **59°** | |
| 48;32,28 | | 49; 8,57 | | 49;44,32 | | 50;19,13[1] | | 50;52,58 | | 51;25,48[2] | |
| 1 | 0,37 | 1 | 0,36 | 1 | 0,35 | 1 | 0,34 | 1 | 0,33 | 1 | 0,32 |
| 2 | 1,14 | 2 | 1,12 | 2 | 1,10 | 2 | 1, 8 | 2 | 1, 6 | 2 | 1, 5 |
| 3 | 1,50 | 3 | 1,48 | 3 | 1,45 | 3 | 1,42 | 3 | 1,40 | 3 | 1,37 |
| 4 | 2,27 | 4 | 2,24 | 4 | 2,20 | 4 | 2,16[3] | 4 | 2,13 | 4 | 2, 9 |
| 5 | 3, 4 | 5 | 3, 0 | 5 | 2,55 | 5 | 2,50 | 5 | 2,46 | 5 | 2,41 |
| 6 | 3,41 | 6 | 3,36 | 6 | 3,30 | 6 | 3,24 | 6 | 3,19 | 6 | 3,14 |
| 7 | 4,18 | 7 | 4,12 | 7 | 4, 5 | 7 | 3,58 | 7 | 3,52[4] | 7 | 3,46 |
| 8 | 4,54[5] | 8 | 4,47 | 8 | 4,40 | 8 | 4,33 | 8 | 4,26 | 8 | 4,18 |
| 9 | 5,31 | 9 | 5,23 | 9 | 5,15 | 9 | 5, 7 | 9 | 4,59 | 9 | 4,50 |
| 10 | 6, 8 | 10 | 5,59 | 10 | 5,50 | 10 | 5,41 | 10 | 5,32 | 10 | 5,23 |
| 11 | 6,45 | 11 | 6,35 | 11 | 6,25 | 11 | 6,15 | 11 | 6, 5 | 11 | 5,55 |
| 12 | 7,22 | 12 | 7,11 | 12 | 7, 0 | 12 | 6,49 | 12 | 6,38 | 12 | 6,27 |
| 13 | 7,58 | 13 | 7,47 | 13 | 7,35 | 13 | 7,23[6] | 13 | 7,12 | 13 | 6,59 |
| 14 | 8,35 | 14 | 8,23 | 14 | 8,10 | 14 | 7,57 | 14 | 7,45 | 14 | 7,32 |
| 15 | 9,12 | 15 | 8,59 | 15 | 8,45 | 15 | 8,31 | 15 | 8,18 | 15 | 8, 4 |
| 18 | 11, 2 | 18 | 10,46 | 18 | 10,29 | 18 | 10,13 | 18 | 9,57 | 18 | 9,40 |
| 21 | 12,52 | 21 | 12,33 | 21 | 12,14 | 21 | 11,55 | 21 | 11,36 | 21 | 11,16 |
| 24 | 14,41 | 24 | 14,20 | 24 | 13,58 | 24 | 13,36 | 24 | 13,14 | 24 | 12,52 |
| 27 | 16,31 | 27 | 16, 7 | 27 | 15,43 | 27 | 15,18 | 27 | 14,53 | 27 | 14,28 |
| 30 | 18,21 | 30 | 17,54 | 30 | 17,27 | 30 | 17, 0 | 30 | 16,32 | 30 | 16, 4 |
| 33 | 20,10 | 33 | 19,40 | 33 | 19,11 | 33 | 18,41 | 33 | 18,10 | 33 | 17,39 |
| 36 | 21,59 | 36 | 21,27 | 36 | 20,55 | 36 | 20,22 | 36 | 19,48 | 36 | 19,14 |
| 39 | 23,49 | 39 | 23,13 | 39 | 22,38 | 39 | 22, 2 | 39 | 21,27 | 39 | 20,50 |
| 42 | 25,38 | 42 | 25, 0 | 42 | 24,22 | 42 | 23,43 | 42 | 23, 5 | 42 | 22,25 |
| 45 | 27,27 | 45 | 26,46 | 45 | 26, 6 | 45 | 25,24 | 45 | 24,43 | 45 | 24, 0 |
| 48 | 29,15 | 48 | 28,32 | 48 | 27,49 | 48 | 27, 4 | 48 | 26,20 | 48 | 25,35 |
| 51 | 31, 4 | 51 | 30,18 | 51 | 29,32 | 51 | 28,44 | 51 | 27,58 | 51 | 27, 9 |
| 54 | 32,52 | 54 | 32, 3 | 54 | 31,15[7] | 54 | 30,25 | 54 | 29,35 | 54 | 28,44 |
| 57 | 34,41 | 57 | 33,49 | 57 | 32,58 | 57 | 32, 5 | 57 | 31,13 | 57 | 30,18 |
| 60 | 36,29 | 60 | 35,35 | 60 | 34,41 | 60 | 33,45 | 60 | 32,50 | 60 | 31,53 |

[1] **D** 49′ [2] **D** 18″ [3] **D** 46″ [4] **D** 40″ [5] **D** 50″ [6] **B** 27″ [7] **D** 7″.

**Table 8a: Sine for fractions of a degree (part 11)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **60°** | | **61°** | | **62°** | | **63°** | | **64°** | | **65°** | |
| 51;57,41 | | 52;28,38 | | 52;58,37 | | 53;27,37 | | 53;55,40 | | 54;22,42 | |
| 1 | 0,31 | 1 | 0,30 | 1 | 0,29 | 1 | 0,28 | 1 | 0,27 | 1 | 0,26[1] |
| 2 | 1, 3 | 2 | 1, 1 | 2 | 0,59 | 2 | 0,57 | 2 | 0,55 | 2 | 0,53 |
| 3 | 1,34 | 3 | 1,31 | 3 | 1,28 | 3 | 1,25 | 3 | 1,22 | 3 | 1,19 |
| 4 | 2, 5 | 4 | 2, 1 | 4 | 1,57 | 4 | 1,54 | 4 | 1,50 | 4 | 1,46 |
| 5 | 2,37 | 5 | 2,32 | 5 | 2,27[2] | 5 | 2,22 | 5 | 2,17 | 5 | 2,12 |
| 6 | 3, 8 | 6 | 3, 2 | 6 | 2,56 | 6 | 2,50 | 6 | 2,44 | 6 | 2,39 |
| 7 | 3,39 | 7 | 3,32 | 7 | 3,25 | 7 | 3,19 | 7 | 3,12 | 7 | 3, 5 |
| 8 | 4,11 | 8 | 4, 3 | 8 | 3,55 | 8 | 3,47[3] | 8 | 3,39 | 8 | 3,32 |
| 9 | 4,42 | 9 | 4,33 | 9 | 4,24 | 9 | 4,16 | 9 | 4, 7[4] | 9 | 3,58 |
| 10 | 5,13 | 10 | 5, 3 | 10 | 4,53 | 10 | 4,44 | 10 | 4,34 | 10 | 4,25 |
| 11 | 5,45 | 11 | 5,34 | 11 | 5,23 | 11 | 5,12 | 11 | 5, 1 | 11 | 4,51 |
| 12 | 6,16 | 12 | 6, 4[5] | 12 | 5,52 | 12 | 5,41 | 12 | 5,29 | 12 | 5,18 |
| 13 | 6,47 | 13 | 6,34 | 13 | 6,21 | 13 | 6, 9 | 13 | 5,56 | 13 | 5,44 |
| 14 | 7,19 | 14 | 7, 5 | 14 | 6,51 | 14 | 6,38 | 14 | 6,24 | 14 | 6,11 |
| 15 | 7,50 | 15 | 7,35 | 15 | 7,20 | 15 | 7, 6 | 15 | 6,51 | 15 | 6,37 |
| 18 | 9,23 | 18 | 9, 5 | 18 | 8,47 | 18 | 8,31 | 18 | 8,12 | 18 | 7,56 |
| 21 | 10,56 | 21 | 10,36 | 21 | 10,15 | 21 | 9,55 | 21 | 9,34 | 21 | 9,14 |
| 24 | 12,30 | 24 | 12, 6 | 24 | 11,42 | 24 | 11,20 | 24 | 10,55 | 24 | 10,33 |
| 27 | 14, 3 | 27 | 13,37[6] | 27 | 13,10 | 27 | 12,44 | 27 | 12,17 | 27 | 11,51 |
| 30 | 15,36 | 30 | 15, 7[7] | 30 | 14,37 | 30 | 14, 9 | 30 | 13,38 | 30 | 13,10 |
| 33 | 17, 8 | 33 | 16,36 | 33 | 16, 4 | 33 | 15,33 | 33 | 14,59 | 33 | 14,28 |
| 36 | 18,41 | 36 | 18, 6 | 36 | 17,31 | 36 | 16,57 | 36 | 16,20 | 36 | 15,46 |
| 39[8] | 20,13 | 39 | 19,35 | 39 | 18,57 | 39 | 18,20 | 39 | 17,40 | 39 | 17, 3 |
| 42 | 21,46 | 42 | 21, 5 | 42 | 20,24 | 42 | 19,44 | 42 | 19, 1 | 42 | 18,21 |
| 45 | 23,18 | 45 | 22,34 | 45 | 21,51 | 45 | 21, 8 | 45 | 20,22 | 45 | 19,39 |
| 48 | 24,50 | 48 | 24, 3 | 48 | 23,17 | 48 | 22,31 | 48 | 21,42 | 48 | 20,56 |
| 51 | 26,22 | 51 | 25,32 | 51 | 24,43 | 51 | 23,54[9] | 51 | 23, 2 | 51 | 22,13 |
| 54 | 27,53 | 54 | 27, 1 | 54 | 26, 8 | 54 | 25,17[10] | 54 | 24,22 | 54 | 23,30 |
| 57 | 29,25 | 57 | 28,30 | 57 | 27,34 | 57 | 26,40 | 57 | 25,42 | 57 | 24,47 |
| 60 | 30,57 | 60 | 29,59 | 60 | 29, 0 | 60 | 28, 3 | 60 | 27, 2 | 60 | 26, 4[11] |

[1] **D** 25″ [2] **D** 26″ [3] **L** 46″ [4] **D** 4″ [5] **D** 3″ [6] **D** 36″ [7] **D** 6″ [8] **D** 49 [9] **D** 57″ [10] **L** 57″ [11] **D** 3″.

**Table 8a: Sine for fractions of a degree (part 12)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **66°** | | **67°** | | **68°** | | **69°** | | **70°** | | **71°** | |
| 54;48,46 | | 55;13,49 | | 55;37,52[1] | | 56; 0,53 | | 56;22,54 | | 56;43,52 | |
| 1 | 0,25 | 1 | 0,24 | 1 | 0,23 | 1 | 0,22 | 1 | 0,21[2] | 1 | 0,20 |
| 2 | 0,51 | 2 | 0,49 | 2 | 0,47 | 2 | 0,45 | 2 | 0,43 | 2 | 0,41 |
| 3 | 1,16 | 3 | 1,13 | 3 | 1,10 | 3 | 1, 7 | 3 | 1, 4 | 3 | 1, 1 |
| 4 | 1,42 | 4 | 1,38 | 4 | 1,34 | 4 | 1,30 | 4 | 1,25 | 4 | 1,21 |
| 5 | 2, 7 | 5 | 2, 2 | 5 | 1,57 | 5 | 1,52 | 5 | 1,47 | 5 | 1,42 |
| 6 | 2,32 | 6 | 2,26 | 6 | 2,20 | 6 | 2,14 | 6 | 2, 8 | 6 | 2, 2 |
| 7 | 2,58 | 7 | 2,51 | 7 | 2,44 | 7 | 2,37 | 7 | 2,29 | 7 | 2,22 |
| 8 | 3,23 | 8 | 3,15 | 8 | 3, 7 | 8 | 2,59 | 8 | 2,51 | 8 | 2,43 |
| 9 | 3,49 | 9 | 3,40[3] | 9 | 3,31 | 9 | 3,22 | 9 | 3,12 | 9 | 3, 3 |
| 10 | 4,14 | 10 | 4, 4[4] | 10 | 3,54 | 10 | 3,44 | 10 | 3,33 | 10 | 3,23 |
| 11 | 4,39 | 11 | 4,28 | 11 | 4,17 | 11 | 4, 6 | 11 | 3,55 | 11 | 3,44 |
| 12 | 5, 5 | 12 | 4,53 | 12 | 4,41 | 12 | 4,29 | 12 | 4,16 | 12 | 4, 4 |
| 13 | 5,30 | 13 | 5,17 | 13 | 5, 4 | 13 | 4,51 | 13 | 4,37 | 13 | 4,24 |
| 14 | 5,56 | 14 | 5,42 | 14 | 5,28[5] | 14 | 5,14 | 14 | 4,59 | 14 | 4,45 |
| 15 | 6,21 | 15 | 6, 6 | 15 | 5,51 | 15 | 5,36 | 15 | 5,20 | 15 | 5, 5 |
| 18 | 7,37 | 18 | 7,19 | 18 | 7, 0 | 18 | 6,42 | 18 | 6,23 | 18 | 6, 5 |
| 21 | 8,52 | 21 | 8,31 | 21 | 8,10 | 21 | 7,49 | 21 | 7,27 | 21 | 7, 5 |
| 24 | 10, 8 | 24 | 9,44 | 24 | 9,19 | 24 | 8,55 | 24 | 8,30 | 24 | 8, 6 |
| 27 | 11,23 | 27 | 10,56 | 27 | 10,29 | 27 | 10, 2 | 27 | 9,34 | 27 | 9, 6 |
| 30 | 12,39 | 30 | 12, 9 | 30 | 11,38 | 30 | 11, 8 | 30 | 10,37 | 30 | 10, 6 |
| 33 | 13,54 | 33 | 13,21 | 33 | 12,47 | 33 | 12,14 | 33 | 11,39 | 33 | 11, 5 |
| 36 | 15, 9 | 36 | 14,33 | 36 | 13,56 | 36 | 13,19 | 36 | 12,42 | 36 | 12, 5 |
| 39 | 16,23 | 39 | 15,44 | 39 | 15, 4 | 39 | 14,25 | 39 | 13,44 | 39 | 13, 4 |
| 42 | 17,38 | 42 | 16,56 | 42 | 16,13 | 42 | 15,30 | 42 | 14,47 | 42 | 14, 4 |
| 45 | 18,53 | 45 | 18, 8 | 45 | 17,22 | 45 | 16,36 | 45 | 15,49 | 45 | 15, 3 |
| 48 | 20, 7[6] | 48 | 19,19 | 48 | 18,30 | 48 | 17,41 | 48 | 16,51 | 48 | 16, 2 |
| 51 | 21,21 | 51 | 20,30 | 51 | 19,38 | 51 | 18,46 | 51 | 17,53 | 51 | 17, 0 |
| 54 | 22,35 | 54 | 21,41 | 54 | 20,45 | 54 | 19,51 | 54 | 18,54 | 54 | 17,59 |
| 57 | 23,49 | 57 | 22,52 | 57 | 21,53 | 57 | 20,56 | 57 | 19,56 | 57 | 18,57 |
| 60 | 25, 3 | 60 | 24, 3 | 60 | 23, 1 | 60 | 22, 1 | 60 | 20,58 | 60 | 19,56 |

[1] **L** om. **D** 40″ [2] **D** 24″ [3] **D** 47″ [4] **D** 7″ [5] **D** 23″ [6] **B** 50″.

**Table 8a: Sine for fractions of a degree (part 13)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63 (part 14 comes before part 13), **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **72°** | | **73°** | | **74°** | | **75°** | | **76°** | | **77°** | |
| 57; 3,48 | | 57;22,42 | | 57;40,33 | | 57;57,20 | | 58;13, 4 | | 58;27,44 | |
| 1 | 0,19 | 1 | 0,18 | 1 | 0,17 | 1 | 0,16 | 1 | 0,15 | 1 | 0,14 |
| 2 | 0,39 | 2 | 0,36 | 2 | 0,34 | 2 | 0,32 | 2 | 0,30 | 2 | 0,28 |
| 3 | 0,58 | 3 | 0,55 | 3 | 0,51 | 3 | 0,48 | 3 | 0,45 | 3 | 0,42 |
| 4 | 1,17 | 4 | 1,13 | 4 | 1, 9 | 4 | 1, 5 | 4 | 1, 0 | 4 | 0,56 |
| 5 | 1,37[1] | 5 | 1,31 | 5 | 1,26 | 5 | 1,21 | 5 | 1,15 | 5 | 1,10 |
| 6 | 1,56 | 6 | 1,49 | 6 | 1,43 | 6 | 1,37 | 6 | 1,30 | 6 | 1,24 |
| 7 | 2,15 | 7 | 2, 7 | 7 | 2, 0 | 7 | 1,53[2] | 7 | 1,45 | 7 | 1,38 |
| 8 | 2,35 | 8 | 2,26 | 8 | 2,17 | 8 | 2, 9 | 8 | 2, 1 | 8 | 1,52 |
| 9 | 2,54 | 9 | 2,44 | 9 | 2,34 | 9 | 2,25 | 9 | 2,16 | 9 | 2, 6 |
| 10 | 3,13 | 10 | 3, 2 | 10 | 2,51 | 10 | 2,41 | 10 | 2,31 | 10 | 2,20 |
| 11 | 3,33 | 11 | 3,20 | 11 | 3, 8 | 11 | 2,57 | 11 | 2,46 | 11 | 2,34[3] |
| 12 | 3,52 | 12 | 3,38 | 12 | 3,26 | 12 | 3,14 | 12 | 3, 1 | 12 | 2,48 |
| 13 | 4,11 | 13 | 3,57 | 13 | 3,43 | 13 | 3,30 | 13 | 3,16 | 13 | 3, 2 |
| 14 | 4,31 | 14 | 4,15 | 14 | 4, 0 | 14 | 3,46 | 14 | 3,31 | 14 | 3,16 |
| 15 | 4,50 | 15 | 4,33 | 15 | 4,17 | 15 | 4, 2 | 15 | 3,46 | 15 | 3,30 |
| 18 | 5,47[4] | 18 | 5,27 | 18 | 5, 8 | 18 | 4,50 | 18 | 4,30 | 18 | 4,11 |
| 21 | 6,44 | 21 | 6,21 | 21 | 5,59 | 21 | 5,37 | 21 | 5,15 | 21 | 4,52 |
| 24 | 7,41 | 24 | 7,15 | 24 | 6,49 | 24 | 6,25 | 24 | 5,59 | 24 | 5,34 |
| 27 | 8,38 | 27 | 8, 9 | 27 | 7,40 | 27 | 7,12 | 27 | 6,44 | 27 | 6,15 |
| 30 | 9,35 | 30 | 9, 3 | 30 | 8,31 | 30 | 8, 0 | 30 | 7,28 | 30 | 6,56 |
| 33 | 10,31 | 33 | 9,56 | 33 | 9,21 | 33 | 8,47[5] | 33 | 8,12 | 33 | 7,36 |
| 36 | 11,27 | 36 | 10,49 | 36 | 10,11 | 36 | 9,34 | 36 | 8,55[6] | 36 | 8,17 |
| 39 | 12,24 | 39 | 11,43 | 39 | 11, 1 | 39 | 10,20 | 39 | 9,39 | 39 | 8,57 |
| 42 | 13,20 | 42 | 12,36 | 42 | 11,51 | 42 | 11, 7 | 42 | 10,22 | 42 | 9,38 |
| 45 | 14,16 | 45 | 13,29 | 45 | 12,41 | 45 | 11,54 | 45 | 11, 6 | 45 | 10,18 |
| 48 | 15,12 | 48 | 14,21 | 48 | 13,30 | 48 | 12,40 | 48 | 11,49 | 48 | 10,58 |
| 51 | 16, 7 | 51 | 15,14 | 51 | 14,19 | 51 | 13,26 | 51 | 12,32 | 51 | 11,37 |
| 54 | 17, 3 | 54 | 16, 6[7] | 54 | 15, 9 | 54 | 14,12 | 54 | 13,14 | 54 | 12,17 |
| 57 | 17,58 | 57 | 16,59[8] | 57 | 15,58 | 57 | 14,58 | 57 | 13,57 | 57 | 12,56 |
| 60 | 18,54 | 60 | 17,51 | 60 | 16,47 | 60 | 15,44 | 60 | 14,40 | 60 | 13,36 |

[1] **D** 36″ [2] **D** 2′13″ [3] **B** 37″ [4] **D** 46″ [5] **D** 46″ [6] **D** 52″ [7] **L** 16″ [8] **D** 17′19″.

**Table 8a: Sine for fractions of a degree (part 14)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63 (part 14 comes before part 13), **D** fols 29r–36r.

| Sine | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **78°** | | **79°** | | **80°** | | **81°** | | **82°** | | **83°** | |
| 58;41,20 | | 58;53,51 | | 59; 5,18 | | 59;15,41 | | 59;24,58 | | 59;33,10 | |
| 1 | 0,13 | 1 | 0,12 | 1 | 0,11 | 1 | 0,10 | 1 | 0, 9 | 1 | 0, 8 |
| 2 | 0,26 | 2 | 0,24 | 2 | 0,22 | 2 | 0,19 | 2 | 0,17 | 2 | 0,15 |
| 3 | 0,39 | 3 | 0,36 | 3 | 0,32 | 3 | 0,29 | 3 | 0,26 | 3 | 0,23 |
| 4 | 0,52 | 4 | 0,47[1] | 4 | 0,43 | 4 | 0,39 | 4 | 0,34 | 4 | 0,30 |
| 5 | 1, 5 | 5 | 0,59 | 5 | 0,54 | 5 | 0,48 | 5 | 0,43 | 5 | 0,38 |
| 6 | 1,18 | 6 | 1,11 | 6 | 1, 5 | 6 | 0,58 | 6 | 0,52 | 6 | 0,45 |
| 7 | 1,31 | 7 | 1,23 | 7 | 1,16 | 7 | 1, 8 | 7 | 1, 0 | 7 | 0,53 |
| 8 | 1,43 | 8 | 1,35 | 8 | 1,26 | 8 | 1,17 | 8 | 1, 9 | 8 | 1, 0 |
| 9 | 1,56 | 9 | 1,47 | 9 | 1,37 | 9 | 1,27 | 9 | 1,17 | 9 | 1, 8 |
| 10 | 2, 9 | 10 | 1,59 | 10 | 1,48 | 10 | 1,37 | 10 | 1,26 | 10 | 1,15 |
| 11 | 2,22 | 11 | 2,11 | 11 | 1,59 | 11 | 1,46 | 11 | 1,35 | 11 | 1,23 |
| 12 | 2,35 | 12 | 2,22 | 12 | 2,10 | 12 | 1,56 | 12 | 1,43 | 12 | 1,30 |
| 13 | 2,48 | 13 | 2,34[2] | 13 | 2,20 | 13 | 2, 6 | 13 | 1,52 | 13 | 1,38 |
| 14 | 3, 1 | 14 | 2,46 | 14 | 2,31 | 14 | 2,15 | 14 | 2, 0 | 14 | 1,45 |
| 15 | 3,14 | 15 | 2,58 | 15 | 2,42 | 15 | 2,25 | 15 | 2, 9 | 15 | 1,53 |
| 18 | 3,52 | 18 | 3,33 | 18 | 3,14 | 18 | 2,53 | 18 | 2,34 | 18 | 2,15 |
| 21 | 4,30 | 21 | 4, 8 | 21 | 3,45 | 21 | 3,21 | 21 | 2,59 | 21 | 2,37 |
| 24 | 5, 8 | 24 | 4,42 | 24 | 4,17 | 24 | 3,50 | 24 | 3,24 | 24 | 2,58 |
| 27 | 5,46 | 27 | 5,17 | 27 | 4,48 | 27 | 4,18 | 27 | 3,49 | 27 | 3,20 |
| 30 | 6,24 | 30 | 5,52 | 30 | 5,20 | 30 | 4,46 | 30 | 4,14 | 30 | 3,42 |
| 33 | 7, 1 | 33 | 6,26 | 33 | 5,51 | 33 | 5,14 | 33 | 4,38 | 33 | 4, 3 |
| 36 | 7,38 | 36 | 7, 0 | 36 | 6,21 | 36 | 5,41 | 36 | 5, 2 | 36 | 4,24[3] |
| 39 | 8,16 | 39 | 7,34 | 39 | 6,52 | 39 | 6, 9 | 39 | 5,27 | 39 | 4,44[4] |
| 42 | 8,53 | 42 | 8, 8 | 42 | 7,22 | 42 | 6,36 | 42 | 5,51 | 42 | 5, 5 |
| 45 | 9,30 | 45 | 8,42 | 45 | 7,53 | 45 | 7, 4 | 45 | 6,15 | 45 | 5,26 |
| 48 | 10, 6 | 48 | 9,15 | 48 | 8,23 | 48 | 7,31 | 48 | 6,38 | 48 | 5,46 |
| 51 | 10,42 | 51 | 9,48 | 51 | 8,53 | 51 | 7,57 | 51 | 7, 2 | 51 | 6, 6 |
| 54 | 11,19 | 54 | 10,21 | 54 | 9,23 | 54 | 8,24 | 54 | 7,25 | 54 | 6,27 |
| 57 | 11,55 | 57 | 10,54 | 57 | 9,53[5] | 57 | 8,50 | 57 | 7,49 | 57 | 6,47[6] |
| 60 | 12,31 | 60 | 11,27 | 60 | 10,23 | 60 | 9,17 | 60 | 8,12 | 60 | 7, 7 |

[1] **D** 46″ [2] **D** 37″ [3] **D** 25″ [4] **D** 45″ [5] **B** 13″ [6] **B** 46″.

**Table 8a: Sine for fractions of a degree (part 15)**
*Sources*: **L** fols 24v–31v, **B** pp. 49–63, **D** fols 29r–36r.

| Sine | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **84°** | | **85°** | | **86°** | | **87°** | | **88°** | | **89°** | | **90°** |
| 59;40,17 | | 59;46,18 | | 59;51,14 | | 59;55, 4 | | 59;57,48 | | 59;59,27 | | 60; 0, 0 |
| 1 | 0, 6 | 1 | 0, 5 | 1 | 0, 4 | 1 | 0, 3 | 1 | 0, 2 | 1 | 0, 1 | |
| 2 | 0,13 | 2 | 0,11 | 2 | 0, 9 | 2 | 0, 6 | 2 | 0, 4 | 2 | 0, 2 | |
| 3 | 0,19 | 3 | 0,16 | 3 | 0,13 | 3 | 0, 9 | 3 | 0, 6[1] | 3 | 0, 3 | |
| 4 | 0,26 | 4 | 0,21 | 4 | 0,17 | 4 | 0,13 | 4 | 0, 8 | 4 | 0, 4 | |
| 5 | 0,32 | 5 | 0,27 | 5 | 0,21 | 5 | 0,16 | 5 | 0,10 | 5 | 0, 5 | |
| 6 | 0,38 | 6 | 0,32 | 6 | 0,26 | 6 | 0,19 | 6 | 0,12 | 6 | 0, 6 | |
| 7 | 0,45 | 7 | 0,37 | 7 | 0,30 | 7 | 0,22 | 7 | 0,14 | 7 | 0, 7 | |
| 8 | 0,51 | 8 | 0,43 | 8 | 0,34 | 8 | 0,25 | 8 | 0,17[2] | 8 | 0, 8 | |
| 9 | 0,58 | 9 | 0,48 | 9 | 0,38 | 9 | 0,28 | 9 | 0,19 | 9 | 0, 9 | |
| 10 | 1, 4 | 10 | 0,53 | 10 | 0,43 | 10 | 0,31 | 10 | 0,21 | 10 | 0,10 | |
| 11 | 1,10 | 11 | 0,59 | 11 | 0,47 | 11 | 0,34 | 11 | 0,23 | 11 | 0,11 | |
| 12 | 1,17 | 12 | 1, 4 | 12 | 0,51 | 12 | 0,38 | 12 | 0,25 | 12 | 0,12 | |
| 13 | 1,23 | 13 | 1, 9 | 13 | 0,55 | 13 | 0,41 | 13 | 0,27 | 13 | 0,13 | |
| 14 | 1,30 | 14 | 1,15 | 14 | 1, 0 | 14 | 0,44 | 14 | 0,29 | 14 | 0,14 | |
| 15 | 1,36 | 15 | 1,20 | 15 | 1, 4 | 15 | 0,47 | 15 | 0,31 | 15 | 0,15 | |
| 18 | 1,55 | 18 | 1,35 | 18 | 1,16 | 18 | 0,56 | 18 | 0,36 | 18 | 0,17 | |
| 21 | 2,13 | 21 | 1,50 | 21 | 1,28 | 21 | 1, 4 | 21 | 0,42 | 21 | 0,19 | |
| 24 | 2,32 | 24 | 2, 6 | 24 | 1,39 | 24 | 1,13 | 24 | 0,47 | 24 | 0,21 | |
| 27 | 2,50 | 27 | 2,21 | 27 | 1,51 | 27 | 1,21 | 27 | 0,53 | 27 | 0,23 | |
| 30 | 3, 9 | 30 | 2,36 | 30 | 2, 3 | 30 | 1,30 | 30 | 0,58 | 30 | 0,25 | |
| 33 | 3,27 | 33 | 2,50 | 33 | 2,14 | 33 | 1,38 | 33 | 1, 3 | 33 | 0,26 | |
| 36 | 3,44 | 36 | 3, 5 | 36 | 2,25 | 36 | 1,46 | 36 | 1, 7 | 36 | 0,27 | |
| 39 | 4, 2 | 39 | 3,19 | 39 | 2,37 | 39 | 1,53 | 39 | 1,12 | 39 | 0,29 | |
| 42 | 4,19 | 42 | 3,34 | 42 | 2,48 | 42 | 2, 1 | 42 | 1,16 | 42 | 0,30 | |
| 45 | 4,37 | 45 | 3,48 | 45 | 2,59 | 45 | 2, 9 | 45 | 1,21 | 45 | 0,31 | |
| 48 | 4,54 | 48 | 4, 2 | 48 | 3, 9 | 48 | 2,16 | 48 | 1,25 | 48 | 0,31 | |
| 51 | 5,11 | 51 | 4,15 | 51 | 3,19 | 51 | 2,23 | 51 | 1,28 | 51 | 0,32 | |
| 54 | 5,27 | 54 | 4,29 | 54 | 3,30 | 54 | 2,30 | 54 | 1,32 | 54 | 0,32 | |
| 57 | 5,44 | 57 | 4,42 | 57 | 3,40 | 57 | 2,37 | 57 | 1,35 | 57 | 0,33 | |
| 60 | 6, 1 | 60 | 4,56 | 60 | 3,50 | 60 | 2,44 | 60 | 1,39 | 60 | 0,33 | |

[1] **D** 5″ [2] **L** 16″.

**Table 9: Versed sine (first half)**

*Sources*: **F** fol. 42r–42v, **H** fol. 29r–29v, **C** fols 46v–47r, **$C_1$** fols 13v–14r, **$C_2$** -, **Y** fols 261v–262r, **L** fols 33v–34r, **B** pp. 43–44, **r** = recomputation.

| Table of the Versed Sine | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| arc | versed sine | difference | arc | versed sine | difference | arc | versed sine | difference |
| | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ |
| 1 | 0; 0,33 | 0; 0,33 | 31 | 8;34,12 | 0;31,53 | 61 | 30;54,41 | 0;54,41[1] |
| 2 | 0; 2,12 | 0; 1,39 | 32 | 9; 7, 2[2] | 0;32,50 | 62 | 31;49,54[3] | 0;55,13[4] |
| 3 | 0; 4,56[5] | 0; 2,44[6] | 33 | 9;40,47[7] | 0;33,45[8] | 63 | 32;45,38 | 0;55,44 |
| 4 | 0; 8,46 | 0; 3,50 | 34 | 10;15,28 | 0;34,41 | 64 | 33;41,52[9] | 0;56,14 |
| 5 | 0;13,42[10] | 0; 4,56 | 35 | 10;51, 3 | 0;35,35 | 65 | 34;38,34 | 0;56,42 |
| 6 | 0;19,43 | 0; 6, 1[11]↓ | 36 | 11;27,32 | 0;36,29 | 66 | 35;35,45 | 0;57,11 |
| 7 | 0;26,50 | 0; 7, 7 | 37 | 12; 4,55[12] | 0;37,23 | 67 | 36;33,21[13] | 0;57,36[14] |
| 8 | 0;35, 2 | 0; 8,12 | 38 | 12;43,10 | 0;38,15[15] | 68 | 37;31,25 | 0;58, 4[16] |
| 9 | 0;44,19 | 0; 9,17[17] | 39 | 13;22,16 | 0;39, 6 | 69 | 38;29,53 | 0;58,28 |
| 10 | 0;54,42 | 0;10,23 | 40 | 14; 2,14 | 0;39,58 | 70 | 39;28,44 | 0;58,51 |
| 11 | 1; 6, 9 | 0;11,27 | 41 | 14;43, 3 | 0;40,49 | 71 | 40;27,57 | 0;59,13 |
| 12 | 1;18,40 | 0;12,31 | 42 | 15;24,41 | 0;41,38 | 72 | 41;27,32 | 0;59,35 |
| 13 | 1;32,16[18] | 0;13,36[19] | 43 | 16; 7, 8 | 0;42,27 | 73 | 42;27,28 | 0;59,56 |
| 14 | 1;46,56 | 0;14,40[20] | 44 | 16;50,23 | 0;43,15 | 74 | 43;27,42 | 1; 0,14 |
| 15 | 2; 2,40 | 0;15,44[21] | 45 | 17;34,25 | 0;44, 2 | 75 | 44;28,15 | 1; 0,33 |
| 16 | 2;19,27 | 0;16,47 | 46 | 18;19,14[22] | 0;44,49 | 76 | 45;29, 5 | 1; 0,50 |
| 17 | 2;37,18 | 0;17,51 | 47 | 19; 4,48 | 0;45,34 | 77 | 46;30,11 | 1; 1, 6 |
| 18 | 2;56,12 | 0;18,54 | 48 | 19;51, 8 | 0;46,20 | 78 | 47;31,31 | 1; 1,20 |
| 19 | 3;16, 8 | 0;19,56 | 49 | 20;38,11 | 0;47, 3 | 79 | 48;33, 5 | 1; 1,34 |
| 20 | 3;37, 6 | 0;20,58 | 50 | 21;25,58 | 0;47,47 | 80 | 49;34,52 | 1; 1,47[23] |
| 21 | 3;59, 7 | 0;22, 1 | 51 | 22;14,27 | 0;48,29[24] | 81 | 50;36,50 | 1; 1,58[25] |
| 22 | 4;22, 8 | 0;23, 1 | 52 | 23; 3,37 | 0;49,10 | 82 | 51;38,59 | 1; 2, 9 |
| 23 | 4;46,11 | 0;24, 3 | 53 | 23;53,28 | 0;49,51 | 83 | 52;41,16 | 1; 2,17 |
| 24 | 5;11,14 | 0;25, 3 | 54 | 24;43,58 | 0;50,30 | 84 | 53;43,42 | 1; 2,26 |
| 25 | 5;37,18 | 0;26, 4 | 55 | 25;35, 7 | 0;51, 9 | 85 | 54;46,14 | 1; 2,32 |
| 26 | 6; 4,20 | 0;27, 2 | 56 | 26;26,54 | 0;51,47 | 86 | 55;48,53 | 1; 2,39 |
| 27 | 6;32,23 | 0;28, 3 | 57 | 27;19,18 | 0;52,24[26] | 87 | 56;51,35 | 1; 2,42[27] |
| 28 | 7; 1,23 | 0;29, 0 | 58 | 28;12,17 | 0;52,59 | 88 | 57;54,22[28] | 1; 2,47 |
| 29 | 7;31,22 | 0;29,59 | 59 | 29; 5,52 | 0;53,35 | 89 | 58;57,10 | 1; 2,48 |
| 30 | 8; 2,19 | 0;30,57⊥ | 60 | 30; 0, 0 | 0;54, 8 | 90 | 60; 0, 0 | 1; 2,50 |

[1] **F** 57′ [2] **L** 2′ [3] **$C_1$** 57″ [4] **F** 53″ [5] **F** 3′ [6] **YLB** 42″ [7] **LB** 17″ [8] **C** 53′ [9] **CLB** 12″ [10] **F** 40″ [11] **F** 6–30°: 5, 6, 7, ..., 29′ (i.e., minutes 1 less for arguments 6–20° and 29–30° and 2 less for arguments 21–28°) [12] **Y** 11° **L** 15″ [13] **C** 38′ **YLBr** 22″ [14] **YLBr** 37″ (in correspondence with the variant in the versed sine for 67°) [15] **F** 35″ **C** 55″ [16] **FYLBr** 3″ (in correspondence with the variant in the versed sine for 67°) [17] +**$C_1$** 57″ [18] **YLB** 15″ [19] +**YLB** 35″ (in correspondence with the variant in the versed sine for 13°) [20] +**YLB** 41″ (in correspondence with the variant in the versed sine for 13°) [21] +**$C_1$** 45″ [22] **C$C_1$** 59′ [23] **C** 40″ [24] **F** 39″ [25] **F** 2′18″ [26] **F** 34″ [27] **C** 2″ [28] **$C_1$** 24′.

**Table 9: Versed sine (second half)**
*Sources*: **F** fol. 42r–42v, **H** fol. 29r–29v, **C** fols 46v–47r, **C**$_1$ fols 13v–14r, **C**$_2$ -, **Y** fols 261v–262r, **L** fols 33v–34r, **B** pp. 43–44, **r** = recomputation.

Table of the Versed Sine

| arc | versed sine | difference | arc | versed sine | difference | arc | versed sine | difference |
|---|---|---|---|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ | | ° ′ ″ | ° ′ ″ |
| 91 | 61; 2,50 | 1; 2,50 | 121 | 90;54, 8 | 0;54, 8 | 151 | 112;28,38 | 0;30,57 |
| 92 | 62; 5,38 | 1; 2,48 | 122 | 91;47,43 | 0;53,35 | 152 | 112;58,37 | 0;29,59 |
| 93 | 63; 8,25 | 1; 2,47 | 123 | 92;40,42 | 0;52,59[1] | 153 | 113;27,37 | 0;29, 0 |
| 94 | 64;11, 7 | 1; 2,42 | 124 | 93;33, 6 | 0;52,24 | 154 | 113;55,40 | 0;28, 3[2] |
| 95 | 65;13,46 | 1; 2,39 | 125 | 94;24,53 | 0;51,47 | 155 | 114;22,42 | 0;27, 2 |
| 96 | 66;16,18 | 1; 2,32 | 126[3]↓ | 95;16, 2 | 0;51, 9 | 156 | 114;48,46 | 0;26, 4 |
| 97 | 67;18,44 | 1; 2,26 | 127 | 96; 6,32 | 0;50,30 | 157 | 115;13,49 | 0;25, 3 |
| 98 | 68;21, 1 | 1; 2,17 | 128 | 96;56,23 | 0;49,51 | 158 | 115;37,52 | 0;24, 3 |
| 99 | 69;23,10 | 1; 2, 9 | 129 | 97;45,33[4] | 0;49,10 | 159 | 116; 0,53 | 0;23, 1[5] |
| 100 | 70;25, 8 | 1; 1,58 | 130 | 98;34, 2 | 0;48,29 | 160 | 116;22,54 | 0;22, 1[6] |
| 101 | 71;26,55 | 1; 1,47 | 131 | 99;21,49 | 0;47,47 | 161 | 116;43,52 | 0;20,58 |
| 102 | 72;28,29 | 1; 1,34 | 132 | 100; 8,52 | 0;47, 3 | 162 | 117; 3,48 | 0;19,56 |
| 103 | 73;29,49 | 1; 1,20 | 133 | 100;55,12 | 0;46,20[7] | 163 | 117;22,42 | 0;18,54 |
| 104 | 74;30,55 | 1; 1, 6 | 134 | 101;40,46 | 0;45,34 | 164 | 117;40,33[8] | 0;17,51 |
| 105 | 75;31,45 | 1; 0,50[9] | 135 | 102;25,35 | 0;44,49[10] | 165 | 117;57,20[11] | 0;16,47 |
| 106 | 76;32,18 | 1; 0,33 | 136 | 103; 9,37 | 0;44, 2[12] | 166 | 118;13, 4 | 0;15,44 |
| 107 | 77;32,32 | 1; 0,14 | 137 | 103;52,52[13] | 0;43,15 | 167 | 118;27,44 | 0;14,40 |
| 108 | 78;32,28 | 0;59,56[14]↓ | 138 | 104;35,19 | 0;42,27 | 168 | 118;41,20 | 0;13,36 |
| 109 | 79;32, 3[15] | 0;59,35 | 139 | 105;16,57[16] | 0;41,38 | 169 | 118;53,51 | 0;12,31 |
| 110 | 80;31,16 | 0;59,13 | 140 | 105;57,46 | 0;40,49 | 170 | 119; 5,18 | 0;11,27 |
| 111 | 81;30, 7 | 0;58,51 | 141 | 106;37,44 | 0;39,58[17] | 171 | 119;15,41 | 0;10,23 |
| 112 | 82;28,35 | 0;58,28 | 142 | 107;16,50 | 0;39, 6 | 172 | 119;24,58 | 0; 9,17 |
| 113 | 83;26,39[18] | 0;58, 4[19] | 143 | 107;55, 5 | 0;38,15 | 173 | 119;33,10 | 0; 8,12 |
| 114 | 84;24,15 | 0;57,36[20] | 144 | 108;32,28 | 0;37,23 | 174 | 119;40,17[21] | 0; 7, 7 |
| 115 | 85;21,26 | 0;57,11 | 145 | 109; 8,57 | 0;36,29 | 175 | 119;46,18 | 0; 6, 1 |
| 116 | 86;18, 8 | 0;56,42 | 146 | 109;44,32 | 0;35,35 | 176 | 119;51,14 | 0; 4,56[22] |
| 117 | 87;14,22 | 0;56,14 | 147 | 110;19,13[23] | 0;34,41 | 177 | 119;55, 4[24] | 0; 3,50 |
| 118 | 88;10, 6 | 0;55,44 | 148⊥ | 110;52,58 | 0;33,45[25] | 178 | 119;57,48[26] | 0; 2,44 |
| 119 | 89; 5,19 | 0;55,13 | 149 | 111;25,48 | 0;32,50 | 179 | 119;59,27[27] | 0; 1,39 |
| 120 | 90; 0, 0 | 0;54,41⊥ | 150 | 111;57,41[28] | 0;31,53 | 180 | 120; 0, 0[29] | 0; 0,33 |

**C** miscopies onto the second page of this table all tabular values from the first twelve rows as well as the tabular differences from the entire last column of the first page of the table; variants in these repeated values have not been included. [1] **Y** 39″ [2] **+F** 4″ [3] **C** arguments 126–148°: slide[+1] [4] **+C**$_1$ 44′ [5] **+B** 22′ [6] **+B** 21′ [7] **C**$_1$ 34″ [8] **LB** 38″ [9] **F** 7″ [10] **C** 46′ **C**$_1$ 39″ [11] **L** 118° [12] **C** 19′ [13] **C** 104° [14] **F** 108–120°: 0′ [15] **B** 8″ [16] **CY** 17″ [17] **C** 38″ [18] **YLBr** 38″ [19] **+YLBr** 3″ (in correspondence with the variant in the versed sine for 113°) [20] **+YLBr** 37″ (in correspondence with the variant in the versed sine for 113°) [21] **C**$_1$ 16″ [22] **+F** 6′ **C**$_1$ 57″ **L** 16″ [23] **Y** 53″ [24] **YLB** 3″ [25] **Y** 44″ [26] **CC**$_1$ 17′ [27] **C** 11′14″ [28] **CC**$_1$ 17′ [29] **L** 119°.

**Table 10: First tangent**

*Sources*: **F** fol. 43r, **H** fol. 30r, **C** fol. 47v (see Plate 5), **$C_1$** fol. 14v, **$C_2$** -, **Y** fol. 262v, **$L_1$** = **L** fol. 34v, **$L_2$** = **L** fol. 32r, **B** p. 45. **$FL_2$** have arguments up to 45°, **$YL_1B$** up to 60°, **$HCC_1$** up to 90°.

The First Tangent, which is the reversed one for the calculation of the elementary quantities

| arc | tangent | diff. | arc | tangent | diff. | arc | tangent | diff. |
|---|---|---|---|---|---|---|---|---|
| | p ′ ″ | p ′ ″ | | p ′ ″ | p ′ ″ | | p ′ ″ | p ′ ″ |
| 1 | 1; 2,51 | 1; 2,51 | 31 | 36; 3, 6 | 1;24,38[1] | 61 | 108;14,34 | 4;19,12 |
| 2 | 2; 5,43[2] | 1; 2,52 | 32 | 37;29,33 | 1;26,27[3] | 62 | 112;50,37 | 4;36, 3 |
| 3 | 3; 8,40 | 1; 2,57 | 33 | 38;57,52 | 1;28,19 | 63 | 117;45,23 | 4;54,46[4] |
| 4 | 4;11,44[5] | 1; 3, 4 | 34 | 40;28,14 | 1;30,22[6] | 64 | 123; 1, 8[7] | 5;15,45 |
| 5 | 5;14,58 | 1; 3,14 | 35 | 42; 0,45 | 1;32,31 | 65 | 128;40,10 | 5;39, 2 |
| 6 | 6;18,22 | 1; 3,24 | 36 | 43;35,33 | 1;34,48[8] | 66 | 134;45,45 | 6; 5,35 |
| 7 | 7;22, 2 | 1; 3,40 | 37 | 45;12,48[9] | 1;37,15[10] | 67 | 141;21, 4 | 6;35,19 |
| 8 | 8;25,56[11] | 1; 3,54 | 38 | 46;52,38 | 1;39,50[12] | 68 | 148;30,20 | 7; 9,16[13] |
| 9 | 9;30,11 | 1; 4,15 | 39 | 48;35,13 | 1;42,35[14] | 69 | 156;18,22[15] | 7;48, 2 |
| 10 | 10;34,47 | 1; 4,36 | 40 | 50;20,45 | 1;45,32 | 70 | 164;50,59 | 8;32,37 |
| 11 | 11;39,45 | 1; 4,58 | 41 | 52; 9,27[16] | 1;48,42[17] | 71 | 174;15, 8 | 9;24, 9[18] |
| 12 | 12;45,12 | 1; 5,27[19] | 42 | 54; 1,27 | 1;52, 0 | 72 | 184;39,36[20] | 10;24,28 |
| 13 | 13;51, 7 | 1; 5,55[21] | 43 | 55;57, 4[22] | 1;55,37[23] | 73 | 196;15, 8 | 11;35,32 |
| 14 | 14;57,37 | 1; 6,30 | 44 | 57;56,29 | 1;59,25[24] | 74 | 209;14,39 | 12;59,31 |
| 15 | 16; 4,37[25] | 1; 7, 0 | 45 | 60; 0, 0 | 2; 3,31 | 75 | 223;55,22[26] | 14;40,43 |
| 16 | 17;12,17 | 1; 7,40 | 46 | 62; 7,54 | 2; 7,54 | 76 | 240;38,49 | 16;43,27[27] |
| 17 | 18;20,37[28] | 1; 8,20 | 47 | 64;20,30[29] | 2;11,36 | 77 | 259;53,27[30] | 19;14,38[31] |
| 18 | 19;29,43 | 1; 9, 6[32] | 48 | 66;38,12[33] | 2;28,42 | 78 | 282;16,39 | 22;23,12 |
| 19 | 20;39,35 | 1; 9,52 | 49 | 69; 1,19[34] | 2;23, 7 | 79 | 308;40,14 | 26;23,35 |
| 20 | 21;50,17 | 1;10,42 | 50 | 71;30,20 | 2;29, 1 | 80 | 340;16,34 | 31;36,20 |
| 21 | 23; 1,54 | 1;11,37 | 51 | 74; 5,39 | 2;35,19 | 81 | 378;49,26 | 38;32,52 |
| 22 | 24;14,30 | 1;12,36 | 52 | 76;47,47 | 2;42, 8 | 82 | 426;54,58[35] | 48; 5,32[36] |
| 23 | 25;28, 7 | 1;13,37 | 53 | 79;37,21 | 2;49,34[37] | 83 | 488;39,24[38] | 61;44,26 |
| 24 | 26;42,49 | 1;14,42 | 54 | 82;34,58 | 2;57,37 | 84 | 570;52, 0[39] | 82;12,36 |
| 25 | 27;58,43[40] | 1;15,54 | 55 | 85;41,19 | 3; 6,21 | 85 | 685;47,23[41] | 114;55,23 |
| 26 | 29;15,50 | 1;17, 7[42] | 56 | 88;57,12 | 3;15,53[43] | 86 | 858; 3,45[44] | 172;16,22[45] |
| 27 | 30;34,18[46] | 1;18,28[47] | 57 | 92;23,31 | 3;26,19[48] | 87 | 1144;52, 9 | 286;48,24[49] |
| 28 | 31;54, 9 | 1;19,51[50] | 58 | 96; 1,10 | 3;37,39[51] | 88 | 1718;14,18[52] | 573;22, 9[53] |
| 29 | 33;15,31 | 1;21,22 | 59 | 99;51,25 | 3;50,15[54] | 89 | 3437; 8,26 | 1718;54, 8[55] |
| 30 | 34;38,28 | 1;22,57 | 60 | 103;55,22 | 4; 3,57[56] | 90 | 216000;0,0[57] | 212562;51,34[58] |

For the systematic errors in **Y**, see the commentary (Section IV.3, p. 362). [1] **F** 23′33″ [2] **B** 53″ [3] **F** 17″ [4] **HC** 14′ [5] **F** 9′ [6] **F** 27″ [7] **$C_1$** $122^p$ [8] **F** 45″ [9] **$FC_1YL_2$** 58″ **HC** 18″ [10] **F** 55″ **$HCC_1L_2$** 25″ (in correspondence with the incorrect tangent value for 37°) [11] **$CL_2$** 16″ [12] **$FHCC_1L_2$** 40″ (in correspondence with the incorrect tangent value for 37°) [13] **C** $6^p$ [14] **F** 34″ [15] **$C_1$** 48′ [16] **$C_1$** 24″ [17] **F** 45″ [18] **$C_1$** 27′ [19] **$L_1$** 24″ [20] **C** $84^p$ [21] **FC** 15″ [22] **$C_1$** 5″ [23] **F** 57″ **B** 47″ [24] **$C_1$** 55′ [25] **C** 34″ [26] **C** 14′ [27] **$HCC_1$** 44′ [28] **$L_1$** 34″ [29] **Y** 9′ [30] **C** $258^p$33′ **$C_1$** 13′ [31] **$C_1$** 54′ [32] **Y** 40″ [33] **$C_1$** 8′ **B** 18′ [34] **$C_1$** 39″ [35] **H** 34′ [36] **$CC_1$** 37″ [37] **$C_1$** 37″ [38] **C** 40″ **$C_1$** $499^p$ [39] **$CC_1$** $580^p$ [40] **C** 13′ **$C_1$** 18′ [41] **$C_1$** $695^p$ [42] **$C_1$** 6″ **Y** 4″ [43] **$C_1$** 58″ [44] **C** $158^p$ **$C_1$** $859^p$ [45] **C** 24″ [46] **B** 38″ [47] **H** 28′ **Y** 27″ [48] **H** 25′ [49] **$C_1$** $296^p$ [50] **H** 59′ [51] **$C_1$** 53″ (?) [52] **$C_1$** $1716^p$ [53] **C** $5728^p$ **$C_1$** $1728^p$ [54] **$C_1$** 27″ (?) [55] **$C_1$** $5718^p$ [56] **HB** $3^p$ [57] **H** $3600^p$ [58] **H** nonsensical digits in a different hand **$CC_1$** 11′ **$C_1$** 24″.

**Table 11: Second tangent**

*Sources*: **F** fol. 43v, **H** fol. 30v, **C** fol. 48r, **$C_1$** fol. 15r, **$C_2$** -, **Y** fol. 263r, **$L_1$** = **L** fol. 35r, **$L_2$** = **L** fol. 32r (only the table for gnomon length 7 feet), **B** p. 46.

| The Second Tangent, which is the straight one for determining the noon shadows | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| altitude | tangent | | altitude | tangent | | altitude | tangent | |
| | fingers ′ | feet ′ | | fingers ′ | feet ′ | | fingers ′ | feet ′ |
| 1 | 687;26 | 401; 0[1] | 31 | 19;58 | 11;39 | 61 | 6;39 | 3;53 |
| 2 | 343;39[2] | 200;28[3] | 32 | 19;12 | 11;12 | 62 | 6;23 | 3;43 |
| 3 | 228;58 | 133;34 | 33 | 18;29 | 10;47 | 63 | 6; 7[4] | 3;34 |
| 4 | 171;37 | 100; 7 | 34 | 17;47 | 10;22[5] | 64 | 5;51 | 3;25 |
| 5 | 137; 9[6] | 80; 0 | 35 | 17; 8 | 10; 0 | 65 | 5;36 | 3;16 |
| 6 | 114;10 | 66;36 | 36 | 16;31 | 9;38 | 66 | 5;21 | 3; 7 |
| 7 | 97;47 | 57; 2 | 37 | 15;55 | 9;17 | 67 | 5; 6[7] | 2;59 |
| 8 | 85;23 | 49;48 | 38 | 15;22 | 8;58 | 68 | 4;51 | 2;50 |
| 9 | 75;46 | 44;12 | 39 | 14;49 | 8;39 | 69 | 4;36 | 2;41 |
| 10 | 68; 3 | 39;42 | 40 | 14;18 | 8;21 | 70 | 4;22 | 2;33 |
| 11 | 61;44 | 36; 1 | 41 | 13;48 | 8; 3 | 71 | 4; 8 | 2;25[8] |
| 12 | 56;27[9] | 32;56[10] | 42 | 13;20 | 7;47[11] | 72 | 3;54 | 2;17[12] |
| 13 | 51;58 | 30;19 | 43 | 12;52 | 7;30 | 73 | 3;40 | 2; 8 |
| 14 | 48; 8 | 28; 5 | 44 | 12;26 | 7;15 | 74 | 3;26 | 2; 0 |
| 15 | 44;47 | 26; 7 | 45 | 12; 0 | 7; 0 | 75 | 3;13 | 1;53 |
| 16 | 41;51 | 24;25 | 46 | 11;35 | 6;45 | 76 | 3; 0 | 1;45 |
| 17 | 39;15 | 22;54 | 47 | 11;11 | 6;31 | 77 | 2;46 | 1;37 |
| 18 | 36;56 | 21;33[13] | 48 | 10;48 | 6;18 | 78 | 2;33 | 1;29 |
| 19 | 34;51 | 20;20 | 49 | 10;26[14] | 6; 5 | 79 | 2;20 | 1;22 |
| 20 | 32;58 | 19;14 | 50 | 10; 4 | 5;52 | 80 | 2; 7 | 1;14 |
| 21 | 31;16 | 18;14 | 51 | 9;43 | 5;40 | 81 | 1;54 | 1; 6 |
| 22 | 29;42 | 17;20 | 52 | 9;23 | 5;28 | 82 | 1;41 | 0;59 |
| 23 | 28;16 | 16;29 | 53 | 9; 3 | 5;17 | 83 | 1;28 | 0;51 |
| 24 | 26;57 | 15;43 | 54 | 8;43 | 5; 5 | 84 | 1;16 | 0;44 |
| 25 | 25;44 | 15; 1[15] | 55 | 8;24 | 4;54 | 85 | 1; 3 | 0;37 |
| 26 | 24;36 | 14;21 | 56 | 8; 6 | 4;44[16] | 86 | 0;50 | 0;29 |
| 27 | 23;33 | 13;44 | 57 | 7;48 | 4;33 | 87 | 0;38 | 0;22 |
| 28 | 22;34 | 13;10 | 58 | 7;30[17] | 4;23 | 88 | 0;25 | 0;15 |
| 29 | 21;39 | 12;38 | 59 | 7;13 | 4;13 | 89 | 0;13 | 0; 8 |
| 30 | 20;47 | 12; 7 | 60 | 6;56 | 4; 3 | 90 | 0; 0 | 0; 0 |

[1] **C** $11^f$ **$L_2$B** $51^f$ [2] **F** $348^f$ [3] **C** $205^f$ [4] **$C_1$** 2′ [5] **F** 25′ [6] **F** 39′ [7] **F** 4′ [8] **F** 22′ [9] **$L_1$B** 24′ [10] **FC** 16′ [11] **$C_1$** 44′ [12] **Y** 57′ [13] **C** 38′ (with a dot added underneath in order to correct it to 33′) [14] **Y** 24′ [15] **F** $14^f$ [16] **$CC_1$** 43′ [17] **$L_1$** 4′.

**Table 12: Preliminaries of the mean motions**

*Sources*: **F** fol. 44r (see Plate 6), **H** fol. 31r, **C** fol. 48v, **C**$_1$ fol. 15v, **C**$_2$ -, **Y** fol. 263v (see Plate 7), **L** fol. 35v, **B** p. 47, **N** = Nallino, *al-Battānī sive Albatenii*, vol. II (for the *Ṣābi' Zīj*), **r** = reconstruction / recomputation.

Preliminaries of the Mean Motions

| | Byzantine epoch positions at Raqqa | Arabic epoch positions at Raqqa |
|---|---|---|
| planets | s ° ′ ″ | s ° ′ ″ |
| sun | $11^{s}$ 10;23,37 | $3^{s}$ 23;58, 4[1] |
| moon | $7^{s}$ 18;58, 5 | $3^{s}$ 29;43,46[3] |
| lunar anomaly | $10^{s}$ 12;14,10 | $3^{s}$ 16;30,40 |
| lunar nodes | $6^{s}$ 7;53,56 | $7^{s}$ 23;45,18[6] |
| Saturn | $2^{s}$ 27;15, 0 | $3^{s}$ 26;16, 0[8] |
| Jupiter | $8^{s}$ 20; 4, 0[10] | $11^{s}$ 2; 3, 0 |
| Mars | $3^{s}$ 27;58, 0[12] | $7^{s}$ 1;49, 0[13] |
| Venus anomaly | $7^{s}$ 21;11, 0 | $1^{s}$ 15; 7, 0[15] |
| Mercury anomaly | $8^{s}$ 23; 0, 0 | $2^{s}$ 13;23, 0[17] |

| | Planetary motions in 20 Syrian years according to what al-Battānī recorded in his *Zīj for Raqqa* |
|---|---|
| planets | r s ° ′ ″ ‴ iv v vi |
| sun | $20^{r}$ $0^{s}$ 0;11,10,14,35,31,30[2] |
| moon | $267^{r}$ $4^{s}$ 13;35,33,11, 4,59,45[4] |
| lunar anomaly | $265^{r}$ $1^{s}$ 9;41,59,15,13,58,15[5] |
| lunar nodes | $1^{r}$ $0^{s}$ 26;48,10,36,48, 0, 0[7] |
| Saturn | $0^{r}$ $8^{s}$ 4;42,44,20, 0, 0, 0[9] |
| Jupiter | $1^{r}$ $8^{s}$ 7;17,34,26, 0, 0, 0[11] |
| Mars | $10^{r}$ $7^{s}$ 18;22,10,49, 0, 0, 0[14] |
| Venus anomaly | $12^{r}$ $6^{s}$ 3;43, 2, 0, 0, 0, 0[16] |
| Mercury anomaly | $63^{r}$ $0^{s}$ 14;27,43,20, 0, 0, 0[18] |

| | Persian epoch positions for noon of the first day of the Yazdigird era, from al-Battānī's base parameters for the longitude of Raqqa, which is 73;15° |
|---|---|
| planets | s ° ′ ″ ‴ iv v vi |
| sun | $2^{s}$ 26;57,20,33,15, 3,54[19] |
| moon | $0^{s}$ 4;10,28,43,41, 7, 3[21] |
| lunar anomaly | $10^{s}$ 7; 4,26,53,17,57, 9 |
| lunar nodes | $2^{s}$ 5;41,57,29,53,26, 6[24] |
| Saturn | $7^{s}$ 27;41,43, 7,26,32,33[26] |
| Jupiter | $9^{s}$ 3;25,15, 6,27, 0,27[28] |
| Mars | $10^{s}$ 11;35,18,56,44,34, 3[30] |
| Venus anomaly | $3^{s}$ 29;59,54,13,16,50,15[32] |
| Mercury anomaly | $5^{s}$ 25;12, 3,56,47,16, 3[34] |

| | Planetary ⟨mean⟩ motions in one day |
|---|---|
| planets | ° ′ ″ ‴ iv v vi |
| sun | 0;59, 8,20,46,56,14[20] |
| moon | 13;10,35, 2, 7,14,13[22] |
| lunar anomaly | 13; 3,53,56,17,51,59[23] |
| lunar nodes | 0; 3,10,37,17,40,26[25] |
| Saturn | 0; 2, 0,35,50,48, 3[27] |
| Jupiter | 0; 4,59,16,54,54,57[29] |
| Mars | 0;31,26,40,15,11,13[31] |
| Venus anomaly | 0;36,59,29,27,42,45[33] |
| Mercury anomaly | 3; 6,24, 7,44,53,13[35] |

**HCC**$_1$ round the motions in 20 Syrian years to fifths; **YLB** omit the numbers of rotations. [1] **F** $24^{\circ}$ [2] **B** $0^{v}$ [3] **F** $48'$ [4] **F** $67^{r}$ **C** $66^{r}$ $13''$ **LB** $35''$ [5] **F** $65^{r}$ **B** $35^{vi}$ **N** $5^{iv}38^{v}$ $55^{vi}$ [6] **F** $4^{s}$ [7] **HC**$_1$ $5^{v}$ **YLB** $47^{iv}45^{v}30^{vi}$ **L** $56^{\circ}$ **N** om. [8] **N** $15'$ [9] **L** $0^{s}$ (due to slide) [10] **F** $7'$ [11] **C**$_1$ $30''$ [12] **F** $8^{s}$ [13] **N** $48'$ [14] **L** $13^{\circ}$ [15] **F** $16^{\circ}$ [16] **L** om. degrees, $18'$ [17] **F** $27'$ [18] **F** $26'$ **L** $18''$ [19] **CC**$_1$ $17'$ [20] **F** $54^{vi}$ [21] **C**$_1$ $5^{s}$ **B** $4^{vi}$ [22] **C**$_1$ $12'''$ **L** $17^{iv}$ [23] **C** $33''36'''$ **C**$_1$ $3^{\circ}$ **L** $16^{iv}$ [24] **Y** $17^{vi}$ [25] **L** $57^{iv}$ [26] **L** $13^{vi}$ **B** $4^{vi}$ [27] **C** $5''30^{iv}$ [28] **r** $28'$ **L** $31^{v}$ [29] **L** $5'$ [30] **C** $17'''32^{v}$ **C**$_1$ $57'''$ **L** $36'$ $16'''$ om. sixths [31] **C** $41^{v}$ **Y** $51^{v}$ **B** $0'$ [32] **Y** $53'''$ **L** om. sixths [33] **CC**$_1$ $34'$ **Y** $29''$ **B** $16'$ **r** $43^{v}15^{vi}$ [34] **FC** $16'''$ **L** $2'$ $4''$ $15'''$ om. fifths and sixths [35] **FC** $50'''$ **CC**$_1$ $7'$ **C**$_1$ $58^{v}$ **Y** $30'''$ corrected to $6'''$ (?).

**Table 13: Solar mean motion (first part)**
*Sources*: **F** fols 44v–45r, **H** fols 31v–32r, **C** fol. 50r, **C$_1$** fol. 16r (see Plate 8), **C$_2$** -, **Y** fol. 264r–264v, **L** fol. 36r–36v, **B** pp. 48 and 67.

Table of the Solar Mean Motion in Years and Months

| years | collected ⟨years⟩ s ° ′ ″ | years | extended ⟨years⟩ s ° ′ ″ | months | months s ° ′ ″ |
|---|---|---|---|---|---|
| 1 | 2$^s$ 24;54,36* | 1 | 11$^s$ 29;45,46[1] | Farwardīn | 0$^s$ 0; 0, 0 |
| 21 | 2$^s$ 20;10, 5* | 2 | 11$^s$ 29;31,33 | | |
| 41 | 2$^s$ 15;25,33* | 3 | 11$^s$ 29;17,19 | Urdībihisht | 0$^s$ 29;34,10 |
| 61 | 2$^s$ 10;41, 2[2] | 4 | 11$^s$ 29; 3, 6 | | |
| 81 | 2$^s$ 5;56,30 | 5 | 11$^s$ 28;48,52 | Khurdād | 1$^s$ 29; 8,21 |
| 101 | 2$^s$ 1;11,59 | 6 | 11$^s$ 28;34,39 | | |
| 121 | 1$^s$ 26;27,27 | 7 | 11$^s$ 28;20,25 | Tīr | 2$^s$ 28;42,31[3] |
| 141 | 1$^s$ 21;42,56* | 8 | 11$^s$ 28; 6,11[4] | | |
| 161 | 1$^s$ 16;58,24 | 9 | 11$^s$ 27;51,58 | Murdād | 3$^s$ 28;16,42 |
| 181 | 1$^s$ 12;13,53[5] | 10 | 11$^s$ 27;37,44 | | |
| 201[6] | 1$^s$ 7;29,21 | 11 | 11$^s$ 27;23,31[7] | Shahrīwar | 4$^s$ 27;50,52 |
| 221 | 1$^s$ 2;44,50[8] | 12 | 11$^s$ 27; 9,17[9] | | |
| 241 | 0$^s$ 28; 0,18[10] | 13 | 11$^s$ 26;55, 3[11] | Mihr | 5$^s$ 27;25, 2 |
| 261 | 0$^s$ 23;15,47* | 14 | 11$^s$ 26;40,50 | | |
| 281 | 0$^s$ 18;31,15 | 15 | 11$^s$ 26;26,36 | Ābān | 6$^s$ 26;59,13[12] |
| 301 | 0$^s$ 13;46,44* | 16 | 11$^s$ 26;12,23 | | |
| 321 | 0$^s$ 9; 2,12 | 17 | 11$^s$ 25;58, 9 | Ādhar | 7$^s$ 26;33,23[13] |
| 341 | 0$^s$ 4;17,41* | 18 | 11$^s$ 25;43,56[14] | | *8$^s$ 1;29, 5* |
| 361[15] | 11$^s$ 29;33, 9[16] | 19 | 11$^s$ 25;29,42 | Day | 8$^s$ 26; 7,33[17] |
| 381 | 11$^s$ 24;48,38[18] | 20 | 11$^s$ 25;15,29 | | *9$^s$ 1; 3,15* |
| 401 | 11$^s$ 20; 4, 6[19] | single years | | Bahman | 9$^s$ 25;41,44 |
| 421 | 11$^s$ 15;19,35* | | | | *10$^s$ 0;37,26*[20] |
| 441 | 11$^s$ 10;35, 3 | 40 | 11$^s$ 20;30,57[21] | Isfandārmudh | 10$^s$ 25;15,54[22] |
| 461 | 11$^s$ 5;50,32* | 60 | 11$^s$ 15;46,26 | | *11$^s$ 0;11,36*[23] |
| 481 | 11$^s$ 1; 6, 0 | 80 | 11$^s$ 11; 1,54 | The ⟨values in⟩ red are the mean motion⟨s⟩ set up with the *epagomenae* ⟨inserted after Ābān⟩ for all planets. | |
| 501 | 10$^s$ 26;21,29* | 100 | 11$^s$ 6;17,23 | | |
| 521 | 10$^s$ 21;36,57 | 200 | 10$^s$ 12;34,45[24] | | |
| 541 | 10$^s$ 16;52,26* | 300 | 9$^s$ 18;52, 8[25] | | |
| 561 | 10$^s$ 12; 7,54 | 400 | 8$^s$ 25; 9,30[26] | | |
| 581 | 10$^s$ 7;23,23* | 500 | 8$^s$ 1;26,53[27] | | |
| | | | [28] | | |

* In **YLB** the values for collected years 1, 21, 41, 141, 181, 221, 261, 301, 341, 381, 421, 461, 501, 541, and 581 are one second smaller. [1] **CC$_1$B** 47″ [2] **CB** 1″ [3] **C$_1$** 44′ [4] **C$_1$** 51″ [5] +**C$_1$** 33″ [6] **H** 101 [7] **C** 11″ [8] +**C** 7° [9] **C** 11′18″ [10] **F** 19″ [11] **C$_1$** 15″ **YL** 4″ [12] **C$_1$** 18″ [13] **Y** 24″ [14] **L** 40′ [15] **F** فشسا, **L** قسا corrected to شسا [16] **Y** 19″ **L** 10″ [17] **YL** 34″ [18] +**F** 25° [19] **F** 20′ [20] **Y** 0$^s$ 10° [21] **F** 56″ [22] **B** 55″ [23] **L** 0$^s$ (but given correctly in the margin) **B** 37″ [24] **C$_1$** 11$^s$ 37′ [25] **C$_1$** 12′ [26] **L** 15°25′2″ [27] **L** 16′ [28] **YLB** add a value 7$^s$ 7;44,15 (**L** 2°) for 600 years.

**Table 13: Solar mean motion (second part)**
*Sources*: **F** fols 44v–45r, **H** fols 31v–32r, **C** fol. 50r, **$C_1$** fol. 16r, **$C_2$** -, **Y** fol. 264r–264v, **L** fol. 36r–36v, **B** pp. 48 and 67.

Table of the Solar Mean Motion in Days, Hours, and Between Longitudes

| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
|---|---|---|---|---|---|---|---|---|
| | s ° ′ ″ | | ° ′ ″ | | ° ′ ″ | | | ′ ″ |
| 1 | $0^s$ 0; 0, 0 | 1 | 0; 2,28 | 31 | 1;16,23 | 71 | additive | 3, 7 |
| 2 | $0^s$ 0;59, 8 | 2 | 0; 4,56 | 32 | 1;18,51 | 72 | | 2,58[1] |
| 3 | $0^s$ 1;58,17 | 3 | 0; 7,24 | 33 | 1;21,19 | 73 | | 2,48 |
| 4 | $0^s$ 2;57,25 | 4 | 0; 9,51 | 34 | 1;23,46* | 74 | | 2,38 |
| 5 | $0^s$ 3;56,33 | 5 | 0;12,19 | 35 | 1;26,14* | 75 | | 2,28 |
| 6 | $0^s$ 4;55,42 | 6 | 0;14,47 | 36 | 1;28,42[2] | 76 | | 2,18 |
| 7 | $0^s$ 5;54,50 | 7 | 0;17,15 | 37 | 1;31,10 | 77 | | 2, 8 |
| 8 | $0^s$ 6;53,58 | 8 | 0;19,43 | 38 | 1;33,38 | 78 | | 1,58[3] |
| 9 | $0^s$ 7;53, 7 | 9 | 0;22,11 | 39 | 1;36, 6 | 79 | | 1,48 |
| 10 | $0^s$ 8;52,15 | 10 | 0;24,38[4] | 40 | 1;38,33* | 80 | | 1,39 |
| 11 | $0^s$ 9;51,23 | 11 | 0;27, 6 | 41 | 1;41, 1* | 81 | | 1,29 |
| 12 | $0^s$ 10;50,32 | 12 | 0;29,34[5] | 42 | 1;43,29* | 82 | | 1,19 |
| 13 | $0^s$ 11;49,40 | 13 | 0;32, 2[6] | 43 | 1;45,57 | 83 | | 1, 9[7] |
| 14 | $0^s$ 12;48,49 | 14 | 0;34,30 | 44 | 1;48,25 | 84 | | 0,59[8] |
| 15 | $0^s$ 13;47,57 | 15 | 0;36,58[9] | 45 | 1;50,53[10] | 85 | | 0,49 |
| 16 | $0^s$ 14;47, 5 | 16 | 0;39,25* | 46 | 1;53,20* | 86 | | 0,39[11] |
| 17 | $0^s$ 15;46,14 | 17 | 0;41,53 | 47 | 1;55,48* | 87 | | 0,30[12] |
| 18 | $0^s$ 16;45,22[13] | 18 | 0;44,21[14] | 48 | 1;58,16 | 88 | | 0,20 |
| 19 | $0^s$ 17;44,30 | 19 | 0;46,49 | 49[15]↓ | 2; 0,44[16] | 89 | | 0,10 |
| 20 | $0^s$ 18;43,39[17] | 20 | 0;49,17 | 50 | 2; 3,12 | 90 | | 0, 0[18]↓ |
| 21 | $0^s$ 19;42,47 | 21 | 0;51,45 | 51 | 2; 5,40[19] | 91 | subtractive | 0,10 |
| 22 | $0^s$ 20;41,55 | 22 | 0;54,12[20] | 52 | 2; 8, 7* | 92 | | 0,20 |
| 23 | $0^s$ 21;41, 4[21]↓ | 23 | 0;56,40 | 53 | 2;10,35* | 93 | | 0,30 |
| 24 | $0^s$ 22;40,12 | 24 | 0;59, 8 | 54 | 2;13, 3[22] | 94 | | 0,39 |
| 25 | $0^s$ 23;39,20 | 25 | 1; 1,36 | 55 | 2;15,31* | 95 | | 0,49 |
| 26 | $0^s$ 24;38,29 | 26 | 1; 4, 4 | 56 | 2;17,59 | 96 | | 0,59[23] |
| 27 | $0^s$ 25;37,37⊥ | 27 | 1; 6,32[24] | 57⊥ | 2;20,27 | 97 | | 1, 9[25] |
| 28 | $0^s$ 26;36,45 | 28 | 1; 8,59[26] | 58 | 2;22,54[27] | 98 | | 1,19⊥ |
| 29 | $0^s$ 27;35,54[28] | 29 | 1;11,27 | 59 | 2;25,22 | 99 | | 1,29[29]↓ |
| 30 | $0^s$ 28;35, 2 | 30 | 1;13,55[30] | 60 | 2;27,50 | 100 | | 1,39⊥ |

* In **YLB** the values for 16, 22, 34–35, 40–42, 46–47, 52–53, 55, and 58 hours are one second larger. [1] **C** 18″ **L** 53″ **r** 57″ [2] **$C_1$** 12″ [3] **C** 8″ [4] **C** 33″ [5] **Y** 35″ [6] **Y** 30″ [7] **F** 0′ [8] **C** 19″ [9] **C** 18″ [10] **Y** 13″ **L** 14′3″ [11] **C** 9″ [12] **F** 29″ [13] **L** 32″ [14] **L** 11″ [15] **Y** arguments 49–57 hours: slide[+1] [16] **F** 1° [17] **L** 34″ [18] **L** longitudes 90–98°: slide[+2] [19] **C** 0′ [20] **+C** 57′ **Y+** 55′ [21] **Y** 23–27 days (minutes): slide[+1] [22] **L** 10′ [23] **+C** 19″ [24] **L** 12″ [25] **+B** 29″ [26] **C** 19″ [27] **+H** 14″ **L+** 15″ [28] **L** 36′ [29] **L** longitudes 99–100°: om. (end of slide) [30] **L** 15″.

**Table 14: Apogee motion**

*Sources*: **F** fol. 45v, **H** fol. 32v, **C** fol. 50v, **C**$_1$ fol. 16v, **C**$_2$ -, **Y** fol. 265r, **L** fol. 37r, **B** p. 68, **r** = recomputation.

Motion of the Apogees

in years: units, tens, and hundreds

| units | ° ′ ″ |
|---|---|
| 1 | 0; 0,54 |
| 2 | 0; 1,48 |
| 3 | 0; 2,42 |
| 4 | 0; 3,36 |
| 5 | 0; 4,30 |
| 6 | 0; 5,24 |
| 7 | 0; 6,18 |
| 8 | 0; 7,12 |
| 9 | 0; 8, 6 |

| tens | ° ′ ″ |
|---|---|
| 10 | 0; 9, 0 |
| 20 | 0;18, 0 |
| 30 | 0;27, 0 |
| 40 | 0;36, 0 |
| 50 | 0;45, 0 |
| 60 | 0;54, 0 |
| 70 | 1; 3, 0 |
| 80 | 1;12, 0 |
| 90 | 1;21, 0 |

| hundreds | ° ′ ″ |
|---|---|
| 100 | 1;30, 0 |
| 200 | 3; 0, 0[4] |
| 300 | 4;30, 0 |
| 400 | 6; 0, 0 |
| 500 | 7;30, 0 |
| 600 | 9; 0, 0[5] |
| 700 | 10;30, 0 |
| 800 | 12; 0, 0 |

in months and days

| months | ′ ″ |
|---|---|
| 1 | 0, 5 |
| 2 | 0, 9 |
| 3 | 0,14 |
| 4 | 0,18 |
| 5 | 0,23 |
| 6 | 0,27 |
| 7 | 0,32 |
| 8 | 0,36 |
| 9 | 0,41 |
| 10 | 0,45 |
| 11 | 0,50 |
| 12 | 0,54 |

| days[1] | ′ ″ |
|---|---|
| 1[2] | 0, 0[3] |
| 2 | 0, 0 |
| 4 | 0, 1 |
| 6 | 0, 1 |
| 8 | 0, 1 |
| 10 | 0, 1 |
| 12 | 0, 2 |
| 14 | 0, 2 |
| 16 | 0, 2 |
| 18 | 0, 3 |
| 20 | 0, 3 |
| 22 | 0, 3 |
| 24 | 0, 4 |
| 26 | 0, 4 |
| 28 | 0, 4[6] |
| 30[7] | 0, 5[8] |

[9]

The positions of the apogees for the first day of the reign of Yazdigird ibn Shahriyār in Persian ⟨years (?)⟩ according to what al-Battānī stated in his *zīj*

| | | | |
|---|---|---|---|
| Sun | 2$^s$ 18;31 | Saturn | 8$^s$ 0;45 |
| Jupiter | 5$^s$ 10;45 | Mars | 4$^s$ 3;15 |
| Venus | 2$^s$ 18;31 | Mercury | 6$^s$ 17;45 |

The positions of the apogees for the beginning of the year 1191 of the Two-Horned, which is approximately the beginning of the year 249 Yazdigird, according to what al-Battānī wrote in his *zīj*

| | | | |
|---|---|---|---|
| Sun | 2$^s$ 22;14 | Saturn | 8$^s$ 4;28 |
| Jupiter | 5$^s$ 14;28 | Mars | 4$^s$ 6;58 |
| Venus | 2$^s$ 22;28[10] | Mercury | 6$^s$ 21;28 |

The positions of the apogees for the beginning of the year 331 Yazdigird

| | | | |
|---|---|---|---|
| Sun | 2$^s$ 23;28 | Saturn | 8$^s$ 5;42 |
| Jupiter | 5$^s$ 14;45[11] | Mars | 4$^s$ 8;12 |
| Venus | 2$^s$ 23;28 | Mercury | 6$^s$ 22;42 |

The values for 700 and 800 years are only part of the table in **F**. [1] **F** arguments given as 2, 4, 6, ..., 32; **C** arguments given as 16, 17, 18, ..., 30 (mistaken continuation of the arguments of the subtable for months, with arguments 13–15 still visible under the subheading 'days'). [2] **CC**$_1$ om. [3] **CC**$_1$ om. [4] **L** 31′ [5] **Y** 8° [6] **HY** 5″ [7] **H** written under the table [8] **H** om. **r** 4″ [9] The three lists of apogee positions are only found in **F** (but cf. explanatory texts A1 and A2 to Table 12). [10] Mistake for 2$^s$ 22;14. [11] Presumably a mistake for 5$^s$ 15;42.

**Table 15: Equation of time for every 6 degrees (with interpolation constants)**
*Sources*: **F** fol. 46r, **Y** fol. 265r. **HCC$_1$LB** contain a table for the equation of time with values for every degree (see Table 15a).

Equation of Time
This is for the apogee in 24° Gemini, to be taken with the solar mean motion and always to be subtracted from the assumed time

| solar mean motion | ⟨equation⟩ ′ ″ | share of degree ″ ‴ | solar mean motion | ⟨equation⟩ ′ ″ | share of degree ″ ‴ |
|---|---|---|---|---|---|
| $0^s$ 0 | 9,48 | 21,20 | $6^s$ 0 | 24, 8 | 19,20 |
| $0^s$ 6 | 11,52 | 20,40 | $6^s$ 6 | 26, 4[1] | 19,20 |
| $0^s$ 12 | 13,52 | 20, 0 | $6^s$ 12 | 27,44 | 16,40 |
| $0^s$ 18 | 15,52 | 20, 0 | $6^s$ 18 | 29,16 | 15,20 |
| $0^s$ 24 | 17,40 | 18, 0 | $6^s$ 24 | 30,24 | 11,20 |
| $1^s$ 0 | 19, 8 | 14,40 | $7^s$ 0 | 31,12 | 8, 0 |
| $1^s$ 6 | 20,16 | 11,20 | $7^s$ 6 | 31,32 | 3,20 |
| $1^s$ 12 | 21,12[2] | 9,20 | $7^s$ 12 | 31,24 | 1,20 |
| $1^s$ 18 | 21,48 | 6, 0 | $7^s$ 18 | 30,48[3] | 6, 0 |
| $1^s$ 24 | 21,56 | 1,20 | $7^s$ 24 | 29,40 | 11,20 |
| $2^s$ 0 | 21,44 | 2, 0 | $8^s$ 0 | 27,56 | 17,20 |
| $2^s$ 6 | 21, 8[4] | 6, 0 | $8^s$ 6 | 25,48 | 21,20 |
| $2^s$ 12 | 20,20[5] | 8, 0 | $8^s$ 12 | 23,16 | 25,20 |
| $2^s$ 18 | 19,16 | 10,40 | $8^s$ 18 | 20,32 | 27,20 |
| $2^s$ 24 | 17,56[6] | 13,20 | $8^s$ 24 | 17,24 | 31,20 |
| $3^s$ 0 | 16,44 | 12, 0 | $9^s$ 0 | 14,20 | 30,40 |
| $3^s$ 6 | 15,24 | 13, 0[7] | $9^s$ 6 | 11,12 | 31,20 |
| $3^s$ 12 | 14, 8 | 12,40 | $9^s$ 12 | 8,20 | 28,40 |
| $3^s$ 18 | 13,16 | 8,40 | $9^s$ 18 | 5,48 | 25,20 |
| $3^s$ 24 | 12,28 | 8, 0 | $9^s$ 24 | 3,40 | 21,20 |
| $4^s$ 0 | 12, 0[8] | 4,40 | $10^s$ 0 | 1,56 | 17,20 |
| $4^s$ 6 | 11,56 | 0,40 | $10^s$ 6 | 0,44 | 12, 0 |
| $4^s$ 12 | 12,12 | 2,40 | $10^s$ 12 | 0, 8 | 6, 0 |
| $4^s$ 18 | 12,52 | 6,40 | $10^s$ 18 | 0, 0 | 1,20 |
| $4^s$ 24 | 13,52 | 10, 0 | $10^s$ 24 | 0,20 | 3,20 |
| $5^s$ 0 | 15, 8 | 12,40 | $11^s$ 0 | 1, 8 | 8, 0[9] |
| $5^s$ 6 | 16,36 | 14,40 | $11^s$ 6 | 2,24 | 12,40 |
| $5^s$ 12 | 18,24 | 18, 0 | $11^s$ 12 | 3,52 | 14,40 |
| $5^s$ 18 | 20,12 | 18, 0 | $11^s$ 18 | 5,40 | 18, 0 |
| $5^s$ 24 | 22,12 | 20, 0 | $11^s$ 24 | 7,40 | 20, 0 |

[1] **Y** 3″ [2] **Y** 17″ [3] **Y** 32′ [4] **Y** 20′ [5] **Y** 19′ [6] **F** 16″ [7] **Y** 20‴ [8] **Y** 11′ [9] **F** 3″.

**Table 15a: Equation of time for every degree (first half)**
*Sources*: **H** fols 32v–33r, **C** fols 50v–51r, $\mathbf{C_1}$ fols 16v–17r (all three in three blocks of four signs), $\mathbf{C_2}$ -, **L** fol. 37v, **B** p. 69. Variants from the table in **FY** (see Table 15) are here included as well.

| Equation of Time | | | | | | |
|---|---|---|---|---|---|---|
| solar mean motion | **0**[1] | **1** | **2** | **3** | **4** | **5**[2] |
| | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ |
| 0[3]↓ | 9,48 | 19, 8 | 21,44 | 16,44 | 12, 0[4] | 15, 8[5] |
| 1 | 10, 9 | 19,19 | 21,38 | 16,31 | 11,59 | 15,23 |
| 2 | 10,29 | 19,31[6] | 21,32 | 16,17 | 11,59 | 15,37 |
| 3 | 10,50 | 19,42 | 21,26 | 16, 4 | 11,58 | 15,52 |
| 4 | 11,11 | 19,53 | 21,20 | 15,51 | 11,57 | 16, 7 |
| 5 | 11,31 | 20, 5 | 21,14 | 15,37 | 11,57 | 16,21 |
| 6 | 11,52 | 20,16 | 21, 8[7] | 15,24 | 11,56[8] | 16,36 |
| 7 | 12,12 | 20,25 | 21, 0 | 15,11 | 11,59 | 16,54 |
| 8 | 12,32 | 20,35 | 20,52[9] | 14,59 | 12, 1[10] | 17,12 |
| 9 | 12,52 | 20,44 | 20,44 | 14,46 | 12, 4 | 17,30 |
| 10 | 13,12 | 20,53 | 20,36 | 14,33 | 12, 7 | 17,48 |
| 11 | 13,32 | 21, 3 | 20,28[11] | 14,21 | 12, 9 | 18, 6 |
| 12 | 13,52 | 21,12[12] | 20,20[13] | 14, 8 | 12,12 | 18,24 |
| 13 | 14,12 | 21,18 | 20, 9[14]↓ | 13,59 | 12,19 | 18,42[15] |
| 14 | 14,32 | 21,24 | 19,59 | 13,51 | 12,25 | 19, 0 |
| 15 | 14,52 | 21,30 | 19,48 | 13,42 | 12,32 | 19,18 |
| 16 | 15,12 | 21,36 | 19,37 | 13,33 | 12,39 | 19,36[16] |
| 17 | 15,32 | 21,42[17]↓ | 19,27[18] | 13,25[19] | 12,45 | 19,54 |
| 18 | 15,52 | 21,48[20] | 19,16 | 13,16 | 12,52 | 20,12 |
| 19 | 16,10 | 21,49[21] | 19, 3⊥ | 13, 8 | 13, 2 | 20,32 |
| 20 | 16,28 | 21,51[22] | 18,49 | 13, 0 | 13,12 | 20,52 |
| 21 | 16,46 | 21,52 | 18,36 | 12,52[23] | 13,22 | 21,12 |
| 22 | 17, 4 | 21,53 | 18,23 | 12,44 | 13,32 | 21,32 |
| 23 | 17,22 | 21,55 | 18, 9 | 12,36 | 13,42 | 21,52 |
| 24 | 17,40 | 21,56 | 17,56[24] | 12,28 | 13,52 | 22,12 |
| 25 | 17,55 | 21,54 | 17,44 | 12,23 | 14, 5 | 22,31 |
| 26 | 18, 9 | 21,52 | 17,32 | 12,19 | 14,17 | 22,51 |
| 27 | 18,24 | 21,50[25] | 17,20 | 12,14[26] | 14,30 | 23,10 |
| 28⊥ | 18,39 | 21,48 | 17, 8 | 12, 9 | 14,43 | 23,29[27] |
| 29[28] | 18,53[29] | 21,46⊥ | 16,56 | 12, 5[30] | 14,55 | 23,49 |

**B** writes the names of the zodiacal signs instead of *abjad* numbers. [1] **C** $5^s$ [2] **C** $0^s$ [3] **C** arguments 0–28° (block for signs 4–7): slide[+1] [4] **Y** 11′ [5] **C** 3″ [6] **C** 0″ [7] **Y** 20′ [8] **C** 17″ $\mathbf{C_1}$ 57″ [9] $\mathbf{CC_1}$ 12″ [10] $\mathbf{C_1}$ 0″ [11] **L** 23″ [12] **Y** 17″ **L** 52″ [13] **Y** 19′ [14] **C** $2^s$ 13–19° (tens of minutes): ill. due to a strip of paper glued onto the page, **C**+ ?9′19″ [15] $\mathbf{CC_1}$ 45″ [16] **L** 16″ [17] **C** $1^s$ 17–29°: slide[−3] [18] +**L** 26″ [19] **C** units of seconds dam. [20] +**L** 18″ [21] +**L** 19″ [22] +**L** 11″ [23] **C** seconds dam. [24] **FC** 16″ [25] +**L** 55″ [26] **C** seconds dam. [27] **L** 59″ [28] **C** (block for signs 4–7): 30° (end of slide) [29] **L** 39″ [30] **C** seconds dam.

**Table 15a: Equation of time for every degree (second half)**

*Sources*: **H** fols 32v–33r, **C** fols 50v–51r, **$C_1$** fols 16v–17r (all three in three blocks of four signs), **$C_2$** -, **L** fol. 37v, **B** p. 69. Variants from the table in **FY** (see Table 15) are here included as well.

| Equation of Time | | | | | | |
|---|---|---|---|---|---|---|
| solar mean motion | **6** | **7** | **8** | **9** | **10** | **11** |
| | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ |
| 0 | 24, 8 | 31,12 | 27,56 | 14,20[1] | 1,56 | 1, 8 |
| 1 | 24,27[2] | 31,15 | 27,35[3] | 13,49 | 1,44 | 1,21 |
| 2 | 24,47 | 31,19 | 27,13 | 13,17 | 1,32 | 1,33 |
| 3 | 25, 6 | 31,22 | 26,52 | 12,46 | 1,20 | 1,46 |
| 4 | 25,25 | 31,25 | 26,31 | 12,15 | 1, 8 | 1,59 |
| 5 | 25,45[4] | 31,29 | 26, 9 | 11,43 | 0,56[5] | 2,11 |
| 6 | 26, 4[6] | 31,32 | 25,48[7] | 11,12 | 0,44 | 2,24 |
| 7 | 26,21[8] | 31,31 | 25,23 | 10,43 | 0,38 | 2,39 |
| 8 | 26,37[9] | 31,29 | 24,57[10] | 10,15 | 0,32 | 2,53[11] |
| 9 | 26,54 | 31,28[12] | 24,32 | 9,46 | 0,26 | 3, 8 |
| 10 | 27,11 | 31,27[13] | 24, 7 | 9,17 | 0,20 | 3,23 |
| 11 | 27,27 | 31,25 | 23,41 | 8,49[14] | 0,14[15] | 3,37 |
| 12 | 27,44 | 31,24 | 23,16 | 8,20 | 0, 8[16] | 3,52[17] |
| 13 | 27,59[18] | 31,18 | 22,49 | 7,55[19] | 0, 7 | 4,10 |
| 14 | 28,15 | 31,12 | 22,21 | 7,29 | 0, 5 | 4,28 |
| 15 | 28,30 | 31, 6 | 21,54[20] | 7, 4[21] | 0, 4 | 4,46 |
| 16 | 28,45 | 31, 0 | 21,27 | 6,39[22] | 0, 3 | 5, 4 |
| 17 | 29, 1 | 30,54 | 20,59 | 6,13 | 0, 1 | 5,22 |
| 18 | 29,16 | 30,48[23] | 20,32 | 5,48[24] | 0, 0 | 5,40 |
| 19 | 29,27[25] | 30,37 | 20, 1[26] | 5,27[27] | 0, 3 | 6, 0 |
| 20 | 29,39 | 30,25 | 19,29 | 5, 5 | 0, 7 | 6,20 |
| 21 | 29,50 | 30,14 | 18,58 | 4,44 | 0,10 | 6,40 |
| 22 | 30, 1 | 30, 3 | 18,27 | 4,23[28] | 0,13 | 7, 0 |
| 23 | 30,13 | 29,51 | 17,55 | 4, 1 | 0,17 | 7,20 |
| 24 | 30,24 | 29,40 | 17,24 | 3,40 | 0,20 | 7,40 |
| 25 | 30,32 | 29,23 | 16,53 | 3,23 | 0,28 | 8, 1 |
| 26 | 30,40 | 29, 5 | 16,23 | 3, 5 | 0,36 | 8,23 |
| 27 | 30,48 | 28,48 | 15,52 | 2,48 | 0,44 | 8,43 |
| 28 | 30,56 | 28,31 | 15,21 | 2,31 | 0,52 | 9, 4 |
| 29 | 31, 4 | 28,13 | 14,51 | 2,13 | 1, 0 | 9,26 |

**B** writes the names of the zodiacal signs instead of *abjad* numbers. [1] **F** 54′ [2] **$C_1$** 24″ [3] **$C_1$** 30″ [4] **L** 42″ **B** 44″ [5] **C** 17″ **$C_1$** 57″ [6] **YB** 3″ [7] **C** 49″ [8] **L** 27′ [9] **C** 27″ [10] **HC** 17″ [11] **C** 33″ [12] **L** 25″ [13] **L** 24″ [14] **LB** 48″ [15] **B** 15″ [16] **B** 10″ [17] **C** 17″ [18] **B** 29″ [19] **C** 16″ [20] **LB** 57″ [21] **C** 43″ [22] **C** 35″ [23] **Y** 32′ [24] **C** 28″ **LB** 43″ [25] **L** 24″ **B** 26″ [26] **$C_1$** 30″ [27] **B** 24″ [28] **LB** 28″.

**Table 15b: Equation of time expressed in lunar mean motion**

*Sources*: **Y** fol. 268r, **L** fol. 39v, **B** p. 75. This table is part of the table for the lunar mean motions. An identical copy of this table is found as an additional table in **L** fol. 142v.

| ⟨Equation of Time⟩ | | | |
|---|---|---|---|
| solar mean motion | the decrement ′ ″ | solar mean motion | the decrement ′ ″ |
| $0^s$ 6 | 6,35 | $6^s$ 6 | 14,16 |
| $0^s$ 12 | 7,41 | $6^s$ 12 | 15,22 |
| $0^s$ 18 | 8,47 | $6^s$ 18 | 16,28 |
| $0^s$ 24 | 9,53 | $6^s$ 24 | 17, 1 |
| $1^s$ 0 | 10,59 | $7^s$ 0 | 17,34 |
| $1^s$ 6 | 11,32 | $7^s$ 6 | 17,34 |
| $1^s$ 12 | 12, 5 | $7^s$ 12 | 17,34 |
| $1^s$ 18 | 12, 5 | $7^s$ 18 | 17, 1 |
| $1^s$ 24 | 12, 5 | $7^s$ 24 | 16,28 |
| $2^s$ 0 | 12, 5 | $8^s$ 0 | 15,22[1] |
| $2^s$ 6 | 12, 5 | $8^s$ 6 | 14,16 |
| $2^s$ 12 | 11,32 | $8^s$ 12 | 13,11 |
| $2^s$ 18 | 10,59 | $8^s$ 18 | 11,32 |
| $2^s$ 24 | 9,53[2] | $8^s$ 24 | 9,53 |
| $3^s$ 0 | 8,47 | $9^s$ 0 | 8,14 |
| $3^s$ 6 | 8,14[3] | $9^s$ 6 | 6, 2 |
| $3^s$ 12 | 7,41 | $9^s$ 12 | 4,56 |
| $3^s$ 18 | 7, 8 | $9^s$ 18 | 3,18[4] |
| $3^s$ 24 | 6,35 | $9^s$ 24 | 2,12[5] |
| $4^s$ 0 | 6,35 | $10^s$ 0 | 1, 6 |
| $4^s$ 6 | 6,35 | $10^s$ 6 | 0,33 |
| $4^s$ 12 | 6,35 | $10^s$ 12 | 0, 0 |
| $4^s$ 18 | 7, 8 | $10^s$ 18 | 0, 0 |
| $4^s$ 24 | 7,41 | $10^s$ 24 | 0, 0 |
| $5^s$ 0 | 8,14 | $11^s$ 0 | 0,33 |
| $5^s$ 6 | 8,47 | $11^s$ 6 | 1, 6 |
| $5^s$ 12 | 9,53 | $11^s$ 12 | 2,12 |
| $5^s$ 18 | 10,59 | $11^s$ 18 | 3,18 |
| $5^s$ 24 | 12, 5 | $11^s$ 24 | 4,24 |
| $6^s$ 0 | 13,11 | $0^s$ 0 | 5,29 |

[1] **Y** 27″ [2] **Y** 13″ [3] **L** 44″ [4] **Y** 58″ [5] **Y** 2″.

**Table 16: Solar equation (first quarter)**

*Sources*: **F** fols 46v–48r, **H** fols 33v–35r, **C** fols 51v–52v, $\mathbf{C_1}$ fols 17v–18v, $\mathbf{C_2}$ -, **Y** fols 265v–266r, **L** fol. 38r–38v, **B** pp. 70–73.

| Table of the Solar Equation | | | | | | |
|---|---|---|---|---|---|---|
| degrees of anomaly | **0** | differ-ences | **1** | differ-ences | **2** | differ-ences |
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 0 | 1;55,59 | 2, 0 | 0;58,41 | 1,44 | 0;16,39 | 1, 3 |
| 1 | 1;53,59 | 2, 0 | 0;56,57[1]↓ | 1,44 | 0;15,37 | 1, 2 |
| 2 | 1;51,58 | 2, 1 | 0;55,14[2] | 1,43 | 0;14,37 | 1, 0 |
| 3 | 1;49,58 | 2, 0 | 0;53,32 | 1,42 | 0;13,39 | 0,58 |
| 4 | 1;47,58 | 2, 0[3] | 0;51,52[4] | 1,40 | 0;12,43 | 0,56 |
| 5 | 1;45,59 | 1,59 | 0;50,13[5] | 1,39 | 0;11,50 | 0,53 |
| 6 | 1;43,59 | 2, 0 | 0;48,35[6] | 1,38 | 0;11, 0 | 0,50 |
| 7 | 1;42, 0 | 1,59 | 0;46,59[7] | 1,36 | 0;10,12 | 0,48 |
| 8 | 1;40, 1 | 1,59 | 0;45,24 | 1,35 | 0; 9,25 | 0,47 |
| 9 | 1;38, 2 | 1,59 | 0;43,51 | 1,33 | 0; 8,40 | 0,45 |
| 10 | 1;36, 3 | 1,59 | 0;42,19[8] | 1,32 | 0; 7,56[9] | 0,44[10] |
| 11 | 1;34, 5 | 1,58 | 0;40,48 | 1,31 | 0; 7,15 | 0,41 |
| 12 | 1;32, 8 | 1,57 | 0;39,17[11] | 1,31[12] | 0; 6,37 | 0,38 |
| 13 | 1;30,11 | 1,57[13] | 0;37,48 | 1,29 | 0; 6, 1[14]↓ | 0,36 |
| 14 | 1;28,14 | 1,57[15] | 0;36,20 | 1,28[16] | 0; 5,26 | 0,35 |
| 15 | 1;26,18 | 1,56 | 0;34,54[17] | 1,26 | 0; 4,53 | 0,33 |
| 16 | 1;24,22[18] | 1,56 | 0;33,30 | 1,24[19] | 0; 4,21 | 0,32 |
| 17 | 1;22,27[20] | 1,55[21] | 0;32, 7 | 1,23 | 0; 3,51 | 0,30 |
| 18 | 1;20,33 | 1,54 | 0;30,45 | 1,22 | 0; 3,25 | 0,26 |
| 19 | 1;18,40 | 1,53 | 0;29,24 | 1,21 | 0; 3, 1 | 0,24 |
| 20 | 1;16,47 | 1,53 | 0;28, 5 | 1,19 | 0; 2,39 | 0,22 |
| 21 | 1;14,54[22] | 1,53 | 0;26,47 | 1,18 | 0; 2,18⊥ | 0,21 |
| 22 | 1;13, 2 | 1,52 | 0;25,32 | 1,15 | 0; 1,59 | 0,19 |
| 23 | 1;11,11 | 1,51 | 0;24,19 | 1,13 | 0; 1,43 | 0,16 |
| 24 | 1; 9,22 | 1,49 | 0;23, 9 | 1,10[23] | 0; 1,30 | 0,13 |
| 25 | 1; 7,34 | 1,48 | 0;22, 1 | 1, 8 | 0; 1,19 | 0,11 |
| 26 | 1; 5,46 | 1,48 | 0;20,55[24] | 1, 6 | 0; 1,10 | 0, 9 |
| 27 | 1; 3,58[25] | 1,48[26] | 0;19,50 | 1, 5 | 0; 1, 2 | 0, 8 |
| 28 | 1; 2,11 | 1,47 | 0;18,46⊥ | 1, 4[27] | 0; 0,56 | 0, 6 |
| 29 | 1; 0,25 | 1,46[28] | 0;17,42 | 1, 4 | 0; 0,52 | 0, 4 |

[1] **C** 17″ **Y** 1ˢ 1–28°: om. minutes **Y**+ 37″ [2] +$\mathbf{C_1}$**L** 54″ [3] **C** 1′ [4] +**C** 12″ [5] +**L** 53″ [6] +**C**$\mathbf{C_1}$ 39″ [7] +**C** 19″ [8] +**FL** 59″ [9] **Y** 16″ [10] $\mathbf{C_1}$ 45″ [11] +**L** 57″ [12] **L** 30″ [13] **B** 56″ [14] **Y** 2ˢ 13–21° (minutes): slide[+1] [15] **B** 56″ [16] **L** units of seconds dam. [17] +**C** 14″ [18] **C** 27′ [19] **L** 27″ [20] **H** 26″ [21] **C** 15″ [22] **C** 14″ [23] **F** 9″ [24] +**L** 52″ [25] **C** 18″ [26] **L** 49″ [27] **F** 5″ [28] **F** 47″.

**Table 16: Solar equation (second quarter)**
*Sources*: **F** fols 46v–48r, **H** fols 33v–35r, **C** fols 51v–52v, $\mathbf{C_1}$ fols 17v–18v, $\mathbf{C_2}$ -, **Y** fols 265v–266r, **L** fol. 38r–38v, **B** pp. 70–73.

Table of the Solar Equation

| degrees of anomaly | mean distance 0° **3** | differences | **4** | differences | nearest distance 28° **5** | differences |
|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 0 | 0; 0,50 | incr. 0, 2 | 0;17, 4 | 1, 2[1] | 1; 2,26 | 1,55 |
| 1 | 0; 0,52 | 0, 2 | 0;18, 9 | 1, 5 | 1; 4,22 | 1,56 |
| 2 | 0; 0,57 | 0, 5 | 0;19,17 | 1, 8 | 1; 6,19 | 1,57 |
| 3 | 0; 1, 4 | 0, 7 | 0;20,29[2] | 1,12 | 1; 8,16 | 1,57 |
| 4 | 0; 1,12[3]↓ | 0, 8[4]↓ | 0;21,42 | 1,13 | 1;10,14 | 1,58 |
| 5 | 0; 1,22 | 0,10 | 0;22,57 | 1,15 | 1;12,12 | 1,58 |
| 6 | 0; 1,34 | 0,12 | 0;24,12 | 1,15 | 1;14,10 | 1,58 |
| 7 | 0; 1,47 | 0,13 | 0;25,28[5] | 1,16 | 1;16, 9 | 1,59 |
| 8 | 0; 2, 2 | 0,15 | 0;26,45[6] | 1,17 | 1;18, 8 | 1,59 |
| 9 | 0; 2,19 | 0,17 | 0;28, 5 | 1,20 | 1;20, 7 | 1,59 |
| 10 | 0; 2,39 | 0,20 | 0;29,28 | 1,23 | 1;22, 7 | 2, 0 |
| 11 | 0; 3, 2 | 0,23 | 0;30,53 | 1,25 | 1;24, 8 | 2, 1 |
| 12 | 0; 3,27 | 0,25 | 0;32,19 | 1,26 | 1;26,10 | 2, 2 |
| 13 | 0; 3,55 | 0,28 | 0;33,46 | 1,27 | 1;28,12 | 2, 2 |
| 14 | 0; 4,25 | 0,30[7] | 0;35,15 | 1,29 | 1;30,14 | 2, 2 |
| 15 | 0; 4,57 | 0,32 | 0;36,45 | 1,30 | 1;32,17 | 2, 3 |
| 16 | 0; 5,31 | 0,34 | 0;38,17 | 1,32 | 1;34,22 | 2, 5 |
| 17 | 0; 6, 6 | 0,35 | 0;39,50 | 1,33 | 1;36,29[8] | 2, 7 |
| 18 | 0; 6,43 | 0,37 | 0;41,24[9] | 1,34[10] | 1;38,36 | 2, 7 |
| 19 | 0; 7,23 | 0,40 | 0;43, 0 | 1,36 | 1;40,43 | 2, 7 |
| 20 | 0; 8, 6 | 0,43 | 0;44,37[11] | 1,37 | 1;42,51 | 2, 8 |
| 21 | 0; 8,52 | 0,46[12] | 0;46,17 | 1,40 | 1;44,59 | 2, 8 |
| 22 | 0; 9,40 | 0,48 | 0;47,59 | 1,42 | 1;47, 7 | 2, 8 |
| 23 | 0;10,31 | 0,51 | 0;49,42 | 1,43[13] | 1;49,15 | 2, 8[14] |
| 24 | 0;11,23 | 0,52 | 0;51,27 | 1,45 | 1;51,24 | 2, 9 |
| 25 | 0;12,16⊥ | 0,53⊥ | 0;53,13 | 1,46 | 1;53,33 | 2, 9 |
| 26 | 0;13,10 | 0,54 | 0;55, 0 | 1,47 | 1;55,42 | 2, 9 |
| 27 | 0;14, 5 | 0,55 | 0;56,48 | 1,48 | 1;57,51 | 2, 9 |
| 28 | 0;15, 2 | 0,57 | 0;58,38 | 1,50 | 2; 0, 0 | 2, 9 |
| 29 | 0;16, 2 | 1, 0[15] | 1; 0,31[16] | 1,53 | 2; 2, 9 | 2, 9 |

[1] **F** 4″ [2] **F** 19″ [3] **L** $3^s$ 4–25°: slide[+1] [4] **L** $3^s$ 4–25°: slide[+1] (i.e., equations and differences have slid together) [5] **L** 24′ [6] **C** 27′ [7] +**F** 32″ [8] **C** 37′ [9] $\mathbf{CC_1}$ 25″ [10] $\mathbf{C_1}$ 37″ [11] **B** 34″ [12] +**C** seconds dam. [13] **Y** 44″ [14] **FB** 9″ [15] **F** 0′ [16] **F** 0″.

**Table 16: Solar equation (third quarter)**

*Sources*: **F** fols 46v–48r, **H** fols 33v–35r, **C** fols 51v–52v, **$C_1$** fols 17v–18v, **$C_2$** -, **Y** fols 265v–266r, **L** fol. 38r–38v, **B** pp. 70–73.

Table of the Solar Equation

| degrees of anomaly | 6 | differences | 7 | differences | mean distance 26° 8 | differences |
|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 0 | 2; 4,18 | 2, 9 | 3; 5, 0[1] | 1,48 | 3;46,50[2] | 0,55 |
| 1 | 2; 6,27 | 2, 9 | 3; 6,47 | 1,47 | 3;47,44 | 0,54 |
| 2 | 2; 8,36 | 2, 9 | 3; 8,33[3] | 1,46 | 3;48,37[4] | 0,53 |
| 3 | 2;10,45 | 2, 9 | 3;10,18 | 1,45 | 3;49,29 | 0,52 |
| 4 | 2;12,53[5] | 2, 8 | 3;12, 1 | 1,43 | 3;50,20[6] | 0,51 |
| 5 | 2;15, 1 | 2, 8 | 3;13,43 | 1,42 | 3;51, 8 | 0,48 |
| 6 | 2;17, 9 | 2, 8 | 3;15,23 | 1,40 | 3;51,54[7] | 0,46 |
| 7 | 2;19,17 | 2, 8 | 3;17, 0 | 1,37[8] | 3;52,37 | 0,43 |
| 8 | 2;21,24 | 2, 7 | 3;18,36 | 1,36 | 3;53,17 | 0,40 |
| 9 | 2;23,31 | 2, 7 | 3;20,10 | 1,34 | 3;53,54 | 0,37 |
| 10 | 2;25,38[9] | 2, 7 | 3;21,43 | 1,33 | 3;54,29[10] | 0,35 |
| 11 | 2;27,43 | 2, 5 | 3;23,15 | 1,32 | 3;55, 3 | 0,34 |
| 12 | 2;29,46 | 2, 3 | 3;24,45 | 1,30 | 3;55,35[11] | 0,32 |
| 13 | 2;31,48[12] | 2, 2 | 3;26,14 | 1,29 | 3;56, 5 | 0,30 |
| 14 | 2;33,50 | 2, 2 | 3;27,41[13] | 1,27 | 3;56,33 | 0,28 |
| 15 | 2;35,52 | 2, 2 | 3;29, 7 | 1,26 | 3;56,58[14] | 0,25 |
| 16 | 2;37,53 | 2, 1 | 3;30,32 | 1,25 | 3;57,21 | 0,23 |
| 17 | 2;39,53 | 2, 0 | 3;31,55 | 1,23 | 3;57,41 | 0,20 |
| 18 | 2;41,52 | 1,59 | 3;33,15 | 1,20 | 3;57,58 | 0,17 |
| 19 | 2;43,51 | 1,59 | 3;34,32 | 1,17 | 3;58,13 | 0,15 |
| 20 | 2;45,50 | 1,59 | 3;35,48 | 1,16 | 3;58,26 | 0,13 |
| 21 | 2;47,48 | 1,58 | 3;37, 3 | 1,15 | 3;58,38 | 0,12 |
| 22 | 2;49,46 | 1,58 | 3;38,18 | 1,15[15] | 3;58,48[16] | 0,10 |
| 23 | 2;51,44 | 1,58 | 3;39,31 | 1,13 | 3;58,56 | 0, 8 |
| 24 | 2;53,41[17] | 1,57 | 3;40,43 | 1,12 | 3;59, 3 | 0, 7 |
| 25 | 2;55,38 | 1,57[18] | 3;41,51 | 1, 8 | 3;59, 8 | 0, 5 |
| 26 | 2;57,34[19] | 1,56 | 3;42,56 | 1, 5 | 3;59,10 decr. | 0, 2 |
| 27 | 2;59,29 | 1,55 | 3;43,58 | 1, 2 | 3;59, 8 | 0, 2 |
| 28 | 3; 1,22 | 1,53 | 3;44,58 | 1, 0 | 3;59, 4 | 0, 4[20] |
| 29 | 3; 3,12 | 1,50 | 3;45,55 | 0,57[21] | 3;58,58 | 0, 6 |

[1] **$C_1$** 7″ [2] **$CC_1$** 45′ [3] **C** 13″ [4] **$C_1$** 34″ [5] **C** 13″ [6] **C** 28″ [7] **C** 14″ **Y** 34″ [8] **C** 34″ [9] **C** 33″ **$C_1$** 18″ [10] **C** 55′ [11] **C** 36″ [12] **C** 43″ [13] **F** 26′ [14] **C** 18″ [15] **$FCC_1$** 14″ [16] **Y** 18′ [17] **C** seconds dam. [18] **Y** 56″ [19] **F** 56′ **C** 37″ **L** 17′ [20] **F** 3″ [21] **F** 1′56″.

**Table 16: Solar equation (fourth quarter)**
*Sources*: **F** fols 46v–48r, **H** fols 33v–35r, **C** fols 51v–52v, $\mathbf{C_1}$ fols 17v–18v, $\mathbf{C_2}$ -, **Y** fols 265v–266r, **L** fol. 38r–38v, **B** pp. 70–73.

Table of the Solar Equation

| degrees of anomaly | 9 | differ-ences | 10 | differ-ences | furthest distance 28° 11 | differ-ences |
|---|---|---|---|---|---|---|
| | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 0 | 3;58,50 | 0, 8 | 3;39, 5 | 1, 5 | 2;54,14 | 1,48 |
| 1 | 3;58,41 | 0, 9 | 3;37,59[1] | 1, 6 | 2;52,26 | 1,48 |
| 2 | 3;58,30 | 0,11 | 3;36,51[2] | 1, 8 | 2;50,38 | 1,48 |
| 3 | 3;58,17 | 0,13 | 3;35,41 | 1,10 | 2;48,49 | 1,49 |
| 4 | 3;58, 1 | 0,16 | 3;34,28 | 1,13 | 2;46,58[3] | 1,51 |
| 5 | 3;57,42 | 0,19 | 3;33,13[4] | 1,15 | 2;45, 6 | 1,52 |
| 6 | 3;57,21 | 0,21 | 3;31,55[5] | 1,18 | 2;43,13 | 1,53 |
| 7 | 3;56,59 | 0,22 | 3;30,36 | 1,19 | 2;41,20 | 1,53 |
| 8 | 3;56,35 | 0,24 | 3;29,15[6] | 1,21 | 2;39,27 | 1,53 |
| 9 | 3;56, 9 | 0,26 | 3;27,53[7] | 1,22 | 2;37,33 | 1,54 |
| 10 | 3;55,39 | 0,30 | 3;26,30 | 1,23 | 2;35,38 | 1,55 |
| 11 | 3;55, 7 | 0,32 | 3;25, 6 | 1,24 | 2;33,42 | 1,56 |
| 12 | 3;54,34 | 0,33 | 3;23,40 | 1,26 | 2;31,46 | 1,56 |
| 13 | 3;53,59 | 0,35 | 3;22,12 | 1,28 | 2;29,49 | 1,57 |
| 14 | 3;53,23 | 0,36 | 3;20,43 | 1,29 | 2;27,52[8] | 1,57 |
| 15 | 3;52,45 | 0,38 | 3;19,12 | 1,31 | 2;25,55 | 1,57 |
| 16 | 3;52, 4[9] | 0,41 | 3;17,41 | 1,31 | 2;23,57 | 1,58 |
| 17 | 3;51,20 | 0,44[10] | 3;16, 9 | 1,32 | 2;21,58 | 1,59 |
| 18 | 3;50,35 | 0,45 | 3;14,36 | 1,33 | 2;19,59 | 1,59 |
| 19 | 3;49,48 | 0,47 | 3;13, 1 | 1,35 | 2;18, 0 | 1,59 |
| 20 | 3;49, 0 | 0,48 | 3;11,25 | 1,36[11] | 2;16, 1 | 1,59 |
| 21 | 3;48,10 | 0,50 | 3; 9,47 | 1,38[12] | 2;14, 1 | 2, 0 |
| 22 | 3;47,17[13] | 0,53[14] | 3; 8, 8 | 1,39 | 2;12, 2 | 1,59[15] |
| 23 | 3;46,21 | 0,56[16] | 3; 6,28 | 1,40 | 2;10, 2 | 2, 0 |
| 24 | 3;45,23[17] | 0,58[18] | 3; 4,46 | 1,42 | 2; 8, 2 | 2, 0 |
| 25 | 3;44,23 | 1, 0 | 3; 3, 3 | 1,43 | 2; 6, 1 | 2, 1 |
| 26 | 3;43,21 | 1, 2 | 3; 1,19 | 1,44 | 2; 4, 1 | 2, 0 |
| 27 | 3;42,18 | 1, 3 | 2;59,35 | 1,44[19] | 2; 2, 1 | 2, 0 |
| 28 | 3;41,14[20] | 1, 4 | 2;57,49 | 1,46 | 2; 0, 0 | 2, 1 |
| 29 | 3;40,10[21] | 1, 4[22] | 2;56, 2 | 1,47[23] | 1;57,59[24] | 2, 1 |

[1] **FC** 19″ [2] **C** 11″ [3] **C** 18″ [4] **C** 33″ [5] **FC** 15″ [6] $\mathbf{CC_1}$ 14″ [7] **C** 13″ [8] $\mathbf{CC_1}$ 12″ [9] **Y** 6″ [10] **LB** 43″ [11] **C** 1″ [12] **C** 33″ [13] **C** 57″ [14] **C** 13″ [15] **F** 2′ [16] **C** 16″ [17] **C** minutes dam. [18] **C** 13″ [19] **Y** 45″ [20] **F** 34″ [21] **L** 1″ [22] **F** 5″ [23] **H** 46′ corrected to 47′ in black [24] **C** 19″.

## Table 17: Lunar mean motion (first part)

*Sources*: **F** fols 48v–49r, **H** fols 35v–36r, **C** fol. 53r, **$C_1$** fol. 19r, **$C_2$** -, **Y** fols 266v–268r, **L** fol. 39r–39v, **B** pp. 74–75. The additional subtable in **YLB** for the equation of time expressed in lunar mean motion is separately edited as Table 15b on p. 123.

Lunar Mean Motion in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ ″ | | s ° ′ ″ | | s ° ′ ″ |
| 1 | $11^s$ 25;33,42 | 1 | $4^s$ 9;23, 8[1] | Farwardīn | $0^s$ 0; 0, 0 |
| 21 | $2^s$ 3;16,20 | 2 | $8^s$ 18;46,16 | | |
| 41 | $4^s$ 10;58,58[2] | 3 | $0^s$ 28; 9,24 | Urdībihisht | $1^s$ 5;17,31 |
| 61 | $6^s$ 18;41,36 | 4 | $5^s$ 7;32,32 | | |
| 81 | $8^s$ 26;24,14 | 5 | $9^s$ 16;55,39[3] | Khurdād | $2^s$ 10;35, 2 |
| 101 | $11^s$ 4; 6,52[4] | 6 | $1^s$ 26;18,47[5] | | |
| 121 | $1^s$ 11;49,30 | 7 | $6^s$ 5;41,55 | Tīr | $3^s$ 15;52,33[6] |
| 141 | $3^s$ 19;32, 8 | 8 | $10^s$ 15; 5, 3 | | |
| 161 | $5^s$ 27;14,46[7] | 9 | $2^s$ 24;28,11 | Murdād | $4^s$ 21;10, 4 |
| 181 | $8^s$ 4;57,24[8] | 10 | $7^s$ 3;51,19[9] | | |
| 201 | $10^s$ 12;40, 2 | 11 | $11^s$ 13;14,27 | Shahrīwar | $5^s$ 26;27,35 |
| 221 | $0^s$ 20;22,40 | 12 | $3^s$ 22;37,35 | | |
| 241 | $2^s$ 28; 5,18 | 13 | $8^s$ 2; 0,43 | Mihr | $7^s$ 1;45, 6[10] |
| 261 | $5^s$ 5;47,56[11] | 14 | $0^s$ 11;23,51 | | |
| 281 | $7^s$ 13;30,34 | 15 | $4^s$ 20;46,58 | Ābān | $8^s$ 7; 2,37 |
| 301 | $9^s$ 21;13,12 | 16 | $9^s$ 0;10, 6 | | |
| 321 | $11^s$ 28;55,50[12] | 17 | $1^s$ 9;33,14[13] | Ādhar | $9^s$ 12;20, 9 |
| 341 | $2^s$ 6;38,28[14] | 18 | $5^s$ 18;56,22[15] | | *$11^s$ 18;13, 4* |
| 361 | $4^s$ 14;21, 6[16] | 19 | $9^s$ 28;19,30 | Day | $10^s$ 17;37,40 |
| 381 | $6^s$ 22; 3,44 | 20 | $2^s$ 7;42,38[17] | | *$0^s$ 23;30,35* |
| 401 | $8^s$ 29;46,22 | single years | | Bahman | $11^s$ 22;55,11 |
| 421 | $11^s$ 7;29, 0 | | | | *$1^s$ 28;48, 6* |
| 441 | $1^s$ 15;11,38 | 40 | $4^s$ 15;25,16 | Isfandārmudh | $0^s$ 28;12,42[18] |
| 461 | $3^s$ 22;54,16 | 60 | $6^s$ 23; 7,54[19] | | *$3^s$ 4; 5,37*[20] |
| 481 | $6^s$ 0;36,54[21] | 80 | $9^s$ 0;50,32 | | |
| 501 | $8^s$ 8;19,32 | 100 | $11^s$ 8;33,10 | | |
| 521 | $10^s$ 16; 2,10 | 200 | $10^s$ 17; 6,20 | | |
| 541 | $0^s$ 23;44,48 | 300 | $9^s$ 25;39,30 | | |
| 561 | $3^s$ 1;27,26[22] | 400 | $9^s$ 4;12,40 | | |
| 581 | $5^s$ 9;10, 4[23] | 500 | $8^s$ 12;45,50 | | |
| | | | [24] | | |

[1] **F** 28′ [2] **F** 8$^s$ **C** 18′18″ [3] **C** 15′ [4] **C** 12″ **B** 9$^s$ [5] **CC$_1$** 37″ **Y** 11′ [6] **C$_1$** 13″ [7] **C** 44″ **C$_1$** 47″
[8] **C** 17′ [9] **C** 11′ [10] **F** 6$^s$ [11] **C** 16″ [12] **F** 54′ **L** 15′ **LB** 48″ [13] **F** 34″ [14] **C$_1$** 26° **LB** 27″
[15] **C$_1$** 58° 16′ [16] **C** 54° [17] **C$_1$** 2° [18] **F** 2″ [19] **L** 50′ [20] **F** 57″ [21] **Y** 55″ [22] **H** 26′ [23] **YL** 3″
[24] **YLB** add a value 7$^s$ 21;19,0 for 600 years.

**Table 17: Lunar mean motion (second part)**

*Sources*: **F** fols 48v–49r, **H** fols 35v–36r, **C** fol. 53r, **$C_1$** fol. 19r, **$C_2$** -, **Y** fols 266v–268r, **L** fol. 39r–39v, **B** pp. 74–75.

| Lunar Mean Motion in Days, Hours, and Between Longitudes | | | | | | | |
|---|---|---|---|---|---|---|---|
| | days | | hours | | and their fractions | | in between longitudes |
| days | s ° ′ ″ | hours | ° ′ ″ | hours | ° ′ ″ | longitudes | ′ ″ |
| 1 | 0$^s$ 0; 0, 0 | 1 | 0;32,56 | 31 | 17; 1,10 | 71 | additive 41,43 |
| 2 | 0$^s$ 13;10,35 | 2 | 1; 5,53[1] | 32 | 17;34, 7 | 72 | 39,31 |
| 3 | 0$^s$ 26;21,10 | 3 | 1;38,49[2] | 33 | 18; 7, 3 | 73 | 37,20 |
| 4 | 1$^s$ 9;31,45 | 4 | 2;11,46 | 34 | 18;40, 0[3] | 74 | 35, 8 |
| 5 | 1$^s$ 22;42,20 | 5 | 2;44,42[4] | 35 | 19;12,56[5] | 75 | 32,56[6] |
| 6 | 2$^s$ 5;52,55[7] | 6 | 3;17,39 | 36 | 19;45,53[8] | 76 | 30,45[9] |
| 7 | 2$^s$ 19; 3,30[10] | 7 | 3;50,35 | 37 | 20;18,49 | 77 | 28,33 |
| 8 | 3$^s$ 2;14, 5 | 8 | 4;23,32 | 38 | 20;51,46[11] | 78 | 26,21 |
| 9 | 3$^s$ 15;24,40 | 9 | 4;56,28 | 39 | 21;24,42 | 79 | 24, 9[12] |
| 10 | 3$^s$ 28;35,15 | 10 | 5;29,25[13] | 40 | 21;57,39[14] | 80[15]↓ | 21,58[16] |
| 11 | 4$^s$ 11;45,50 | 11 | 6; 2,21 | 41 | 22;30,35 | 81 | 19,46 |
| 12 | 4$^s$ 24;56,25 | 12 | 6;35,17 | 42 | 23; 3,31 | 82⊥ | 17,34[17] |
| 13 | 5$^s$ 8; 7, 0[18] | 13 | 7; 8,14[19] | 43 | 23;36,28[20] | 83 | 15,22 |
| 14 | 5$^s$ 21;17,35 | 14 | 7;41,10 | 44 | 24; 9,24[21] | 84[22] | 13,11 |
| 15 | 6$^s$ 4;28,10 | 15 | 8;14, 7 | 45 | 24;42,21 | 85[23] | 10,59[24] |
| 16 | 6$^s$ 17;38,46[25] | 16 | 8;47, 3 | 46 | 25;15,17 | 86 | 8,47[26] |
| 17 | 7$^s$ 0;49,21 | 17 | 9;20, 0[27] | 47 | 25;48,14 | 87 | 6,35 |
| 18 | 7$^s$ 13;59,56 | 18 | 9;52,56[28] | 48 | 26;21,10 | 88 | 4,24[29] |
| 19 | 7$^s$ 27;10,31[30] | 19 | 10;25,53[31] | 49 | 26;54, 7 | 89 | 2,12[32] |
| 20 | 8$^s$ 10;21, 6 | 20 | 10;58,49 | 50 | 27;27, 3[33] | 90 | 0, 0 |
| 21 | 8$^s$ 23;31,41 | 21 | 11;31,46* | 51 | 28; 0, 0* | 91 | subtractive 2,12 |
| 22 | 9$^s$ 6;42,16[34] | 22 | 12; 4,42 | 52 | 28;32,56[35] | 92 | 4,24 |
| 23 | 9$^s$ 19;52,51[36] | 23 | 12;37,38* | 53 | 29; 5,52[37] | 93 | 6,35 |
| 24 | 10$^s$ 3; 3,26 | 24 | 13;10,35 | 54 | 29;38,49[38] | 94 | 8,47[39] |
| 25 | 10$^s$ 16;14, 1[40] | 25 | 13;43,31 | 55 | 30;11,45 | 95 | 10,59 |
| 26 | 10$^s$ 29;24,36 | 26 | 14;16,28[41] | 56 | 30;44,42[42] | 96 | 13,11 |
| 27 | 11$^s$ 12;35,11 | 27 | 14;49,24 | 57 | 31;17,38 | 97 | 15,22 |
| 28 | 11$^s$ 25;45,46 | 28 | 15;22,21 | 58 | 31;50,35 | 98 | 17,34[43] |
| 29 | 0$^s$ 8;56,21 | 29 | 15;55,17[44] | 59 | 32;23,31 | 99 | 19,46 |
| 30 | 0$^s$ 22; 6,56[45] | 30 | 16;28,14[46] | 60 | 32;56,28 | 100 | 21,58[47] |

* In **YLB** the values for 10, 21, 38, 40, and 51 hours are one second smaller, the value for 23 hours is one second larger. [1] **CY** 13″ **$C_1$** 58″ [2] **C** 58′ **$C_1$** 18′ [3] **C** 31″ [4] **L** 43″ [5] **C** 16″ [6] **C** 16″ [7] **CY** 15″ [8] **C** 13″ **$C_1$** 43″ [9] **F** 44″ [10] **C** 30′3″ [11] +**C** 11′ [12] **Y** 20″ [13] +**C** 28″ **$C_1$** 35″ [14] +**C** 17′ **$C_1$** 54′ [15] **C** arguments 80–82°: slide[+1] [16] **C** 18″ [17] **YLB** 33″ [18] **$C_1$** 5″ **B** 6′ [19] **C** degrees dam. [20] **$C_1$** 26′ [21] **C** 14″ [22] **C** 184 [23] **C** 141 (?) [24] **C** 19″ [25] **C** 45″ **$C_1$** 18° [26] **$C_1$** 44″ [27] **F** 11° [28] **L** 53′ **C** 12′16″ [29] **C** 25″ [30] **L** 30″ [31] **L** 26′ **C** 13″ [32] **Y** 2″ [33] **B** 26′ [34] **$C_1$** 15″ [35] **C** 16″ [36] **C** 59°12′11″ [37] **L** 9° [38] **C** 18′ [39] **H** 46″ [40] **C** 56° [41] **C** 56′ [42] **$C_1$** 47′ [43] **$C_1$** 37″ **YLB** 33″ [44] **L** 57′ [45] **C** 16″ [46] **F** 24″ [47] **C** 18″.

**Table 18: Lunar mean anomaly (first part)**
*Sources*: **F** fols 49v–50r, **H** fols 36v–37r, **C** fol. 53v, $\mathbf{C}_1$ fol. 19v, $\mathbf{C}_2$ -, **Y** fols 268v–269r, **L** fols 40v–41r, **B** pp. 76–77.

Lunar Mean Anomaly in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | $9^s$ 22;27* | 1 | $2^s$ 28;43 | Farwardīn | $0^s$ 0; 0 |
| 21 | $8^s$ 26;50 | 2 | $5^s$ 27;26 | | |
| 41 | $8^s$ 1;12* | 3 | $8^s$ 26; 9 | Urdībihisht | $1^s$ 1;57 |
| 61 | $7^s$ 5;35 | 4 | $11^s$ 24;52* | | |
| 81 | $6^s$ 9;57[1] | 5 | $2^s$ 23;36 | Khurdād | $2^s$ 3;54 |
| 101 | $5^s$ 14;20 | 6 | $5^s$ 22;19 | | |
| 121 | $4^s$ 18;42* | 7 | $8^s$ 21; 2 | Tīr | $3^s$ 5;51 |
| 141 | $3^s$ 23; 5[2] | 8 | $11^s$ 19;45 | | |
| 161 | $2^s$ 27;27* | 9 | $2^s$ 18;28 | Murdād | $4^s$ 7;48 |
| 181 | $2^s$ 1;50 | 10 | $5^s$ 17;11 | | |
| 201 | $1^s$ 6;12* | 11 | $8^s$ 15;54 | Shahrīwar | $5^s$ 9;45 |
| 221 | $0^s$ 10;35 | 12 | $11^s$ 14;37 | | |
| 241 | $11^s$ 14;57[3] | 13 | $2^s$ 13;20* | Mihr | $6^s$ 11;42 |
| 261 | $10^s$ 19;20 | 14 | $5^s$ 12; 3* | | |
| 281 | $9^s$ 23;42* | 15 | $8^s$ 10;47[4] | Ābān | $7^s$ 13;39 |
| 301 | $8^s$ 28; 5 | 16 | $11^s$ 9;30 | | |
| 321 | $8^s$ 2;27[5] | 17 | $2^s$ 8;13[6] | Ādhar | $8^s$ 15;36 |
| 341 | $7^s$ 6;50 | 18 | $5^s$ 6;56 | | *$10^s$ 20;55* |
| 361 | $6^s$ 11;12* | 19 | $8^s$ 5;39 | Day | $9^s$ 17;33[7] |
| 381 | $5^s$ 15;35 | 20 | $11^s$ 4;22 | | *$11^s$ 22;52*[8] |
| 401 | $4^s$ 19;57[9] | single years | | Bahman | $10^s$ 19;30 |
| 421 | $3^s$ 24;20 | | | | *$0^s$ 24;49* |
| 441 | $2^s$ 28;42* | 40 | $10^s$ 8;45 | Isfandārmudh | $11^s$ 21;27 |
| 461 | $2^s$ 3; 5 | 60 | $9^s$ 13; 7[10] | | *$1^s$ 26;46* |
| 481 | $1^s$ 7;27* | 80 | $8^s$ 17;30 | | |
| 501 | $0^s$ 11;50[11] | 100 | $7^s$ 21;52 | | |
| 521 | $11^s$ 16;12* | 200 | $3^s$ 13;45 | | |
| 541 | $10^s$ 20;35 | 300 | $11^s$ 5;37 | | |
| 561 | $9^s$ 24;57[12] | 400 | $6^s$ 27;30[13] | | |
| 581 | $8^s$ 29;20 | 500 | $2^s$ 19;22 | | |
| | | | [14] | | |

In **L** all four red (here: italicised) values for the early version of the Persian calendar are 1 minute larger; in **B** all digits of these values are written with Hindu numerals (sic!). * In **YL** (but not in **B**) the values for collected years 1, 41, 81, ..., 521 (but see note 12 for year 561) and the values for 4, 13 and 14 extended years are one minute larger. [1] +**C** 17′ [2] **B** 13° [3] +**C** 17′ [4] **B** 44′ [5] +**H** 26″ [6] $\mathbf{C}_1$ $7^s$ [7] **B** degrees repeated in column of signs [8] +**Y** 12′ [9] +**C** 17′ **Y**+ 53′ [10] **C** 50′ [11] **Y** $1^s$ [12] **Y** 18′ [13] $\mathbf{C}_1$ 24° [14] **YLB** add a value $10^s$ 11;15 (**L** 41°) for 600 years.

**Table 18: Lunar mean anomaly (second part)**

*Sources*: **F** fols 49v–50r, **H** fols 36v–37r, **C** fol. 53v, $\mathbf{C_1}$ fol. 19v, $\mathbf{C_2}$ -, **Y** fols 268v–269r, **L** fols 40v–41r, **B** pp. 76–77.

Lunar Mean Anomaly in Days, Hours, and Between Longitudes

| days | days<br>s ° ′ | hours | hours<br>° ′ | hours | and their fractions<br>° ′ | longitudes | in between longitudes<br>° ′ |
|---|---|---|---|---|---|---|---|
| 1 | 0ˢ 0; 0 | 1 | 0;33 | 31 | 16;53[1] | 71[2]↕ | additive 0;42[3] |
| 2 | 0ˢ 13; 4 | 2 | 1; 5 | 32 | 17;25 | 72 | 0;40[4] |
| 3 | 0ˢ 26; 8 | 3 | 1;38 | 33 | 17;58[5] | 73 | 0;37 |
| 4 | 1ˢ 9;12 | 4 | 2;11 | 34 | 18;31 | 74 | 0;35 |
| 5 | 1ˢ 22;16 | 5 | 2;43 | 35 | 19; 3[6] | 75 | 0;33 |
| 6 | 2ˢ 5;19[7] | 6 | 3;16 | 36 | 19;36 | 76 | 0;30 |
| 7 | 2ˢ 18;23 | 7 | 3;49 | 37 | 20; 9 | 77 | 0;28 |
| 8 | 3ˢ 1;27 | 8 | 4;21 | 38 | 20;41[8] | 78 | 0;26[9] |
| 9 | 3ˢ 14;31 | 9 | 4;54[10] | 39 | 21;14 | 79 | 0;24 |
| 10 | 3ˢ 27;35 | 10 | 5;27[11] | 40 | 21;47 | 80 | 0;22 |
| 11 | 4ˢ 10;39 | 11 | 5;59[12] | 41 | 22;19 | 81 | 0;20 |
| 12 | 4ˢ 23;43 | 12 | 6;32 | 42 | 22;52[13] | 82 | 0;17 |
| 13 | 5ˢ 6;47 | 13 | 7; 5 | 43 | 23;25[14] | 83 | 0;15 |
| 14 | 5ˢ 19;51 | 14 | 7;37 | 44 | 23;57[15] | 84 | 0;13 |
| 15 | 6ˢ 2;55 | 15 | 8;10 | 45 | 24;30 | 85 | 0;11 |
| 16 | 6ˢ 15;58 | 16 | 8;43 | 46 | 25; 2 | 86[16]↕ | 0; 9 |
| 17 | 6ˢ 29; 2 | 17 | 9;15 | 47 | 25;35 | 87 | 0; 7 |
| 18 | 7ˢ 12; 6 | 18 | 9;48 | 48 | 26; 8 | 88 | 0; 4 |
| 19 | 7ˢ 25;10 | 19 | 10;21 | 49 | 26;40 | 89 | 0; 2 |
| 20 | 8ˢ 8;14 | 20 | 10;53 | 50 | 27;13 | 90 | 0; 0[17] |
| 21 | 8ˢ 21;18 | 21 | 11;26 | 51 | 27;46 | 91 | subtractive 0; 2[18] |
| 22 | 9ˢ 4;22 | 22 | 11;59[19] | 52 | 28;18 | 92 | 0; 4 |
| 23 | 9ˢ 17;26[20] | 23 | 12;31 | 53 | 28;51[21] | 93 | 0; 7 |
| 24 | 10ˢ 0;30 | 24 | 13; 4 | 54 | 29;24 | 94 | 0; 9 |
| 25 | 10ˢ 13;34 | 25 | 13;37 | 55 | 29;56[22] | 95 | 0;11 |
| 26 | 10ˢ 26;37 | 26 | 14; 9 | 56 | 30;29 | 96⊥ | 0;13 |
| 27 | 11ˢ 9;41 | 27 | 14;42 | 57 | 31; 2 | 97[23] | 0;15 |
| 28 | 11ˢ 22;45 | 28 | 15;15 | 58 | 31;34 | 98⊥ | 0;17[24] |
| 29 | 0ˢ 5;49 | 29 | 15;47 | 59 | 32; 7 | 99[25]↕ | 0;20 |
| 30 | 0ˢ 18;53[26] | 30 | 16;20 | 60 | 32;40 | 100⊥ | 0;22 |

[1] **C** 13′ **Y** 56° [2] **L** arguments 71–98°: slide[+2] [3] **YLB** 41′ [4] **YLB** 39′ [5] **C** 18′ [6] **L** 4′ [7] **C** 29′ **YL** 20′ [8] **Y** 10° [9] **Y** 27′ [10] **C** 14′ [11] $\mathbf{C_1}$ 24′ [12] **C** 19′ [13] **C** 12′ [14] **YLB** 24′ [15] **FC** 17′ [16] +**C** arguments 86–96°: slide[+3] [17] **C** 5′ [18] **Y** 7′ [19] **C** 19′ [20] **Y** 27′ [21] **C** 11′ [22] **C** 16′ [23] **C** om. (end of slide) [24] **C** 57′ [25] **L** arguments 99–100°: om. (end of slide) [26] **F** 13′.

**Table 19: Double elongation (first part)**
*Sources*: **F** fols 50v–51r, **H** fols 37v–38r, **C** fol. 54r, **$C_1$** fol. 20r, **$C_2$** -, **Y** fols 269v–270r, **L** fols 41v–42r, **B** pp. 78–79.

Double Elongation in Years and Months

| years | collected ⟨years⟩ s ° ′ | years | extended ⟨years⟩ s ° ′ | months | months s ° ′ |
|---|---|---|---|---|---|
| 1 | 6$^s$ 13;19[1] | 1 | 8$^s$ 19;15 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 11$^s$ 8;13 | 2 | 5$^s$ 8;29 | | |
| 41 | 4$^s$ 3; 7[2] | 3 | 1$^s$ 27;44 | Urdībihisht | 0$^s$ 11;27[3] |
| 61 | 8$^s$ 28; 2* | 4 | 10$^s$ 16;59 | | |
| 81 | 1$^s$ 22;56[4] | 5 | 7$^s$ 6;14 | Khurdād | 0$^s$ 22;53 |
| 101 | 6$^s$ 17;50 | 6 | 3$^s$ 25;28 | | |
| 121 | 11$^s$ 12;44 | 7 | 0$^s$ 14;43 | Tīr | 1$^s$ 4;20 |
| 141 | 4$^s$ 7;39* | 8 | 9$^s$ 3;58[5] | | |
| 161 | 9$^s$ 2;33[6] | 9 | 5$^s$ 23;12[7] | Murdād | 1$^s$ 15;47[8] |
| 181 | 1$^s$ 27;27 | 10 | 2$^s$ 12;27 | | |
| 201 | 6$^s$ 22;22* | 11 | 11$^s$ 1;42 | Shahrīwar | 1$^s$ 27;13 |
| 221 | 11$^s$ 17;16[9] | 12 | 7$^s$ 20;56[10] | | |
| 241 | 4$^s$ 12;10 | 13 | 4$^s$ 10;11 | Mihr | 2$^s$ 8;40 |
| 261 | 9$^s$ 7; 5* | 14 | 0$^s$ 29;26 | | |
| 281 | 2$^s$ 1;59[11] | 15 | 9$^s$ 18;41 | Ābān | 2$^s$ 20; 7[12] |
| 301 | 6$^s$ 26;53[13] | 16 | 6$^s$ 7;55[14] | | |
| 321 | 11$^s$ 21;48* | 17 | 2$^s$ 27;10 | Ādhar | 3$^s$ 1;34 |
| 341 | 4$^s$ 16;42 | 18 | 11$^s$ 16;25 | | *7$^s$ 3;28* |
| 361 | 9$^s$ 11;36 | 19 | 8$^s$ 5;39[15] | Day | 3$^s$ 13; 0 |
| 381 | 2$^s$ 6;31[16] | 20 | 4$^s$ 24;54 | | *7$^s$ 14;54*[17] |
| 401 | 7$^s$ 1;25 | single years | | Bahman | 3$^s$ 24;27[18] |
| 421 | 11$^s$ 26;19 | | | | *7$^s$ 26;21* |
| 441 | 4$^s$ 21;14* | 40↓[19] | 9$^s$ 19;49[20] | Isfandārmudh | 4$^s$ 5;54 |
| 461[21] | 9$^s$ 16; 8* | 60 | 2$^s$ 14;43[22] | | *8$^s$ 7;48*[23] |
| 481 | 2$^s$ 11; 2 | 80 | 7$^s$ 9;37[24] | | |
| 501 | 7$^s$ 5;56 | 100 | 0$^s$ 4;32 | | |
| 521 | 0$^s$ 0;51[25] | 200 | 0$^s$ 9; 3 | | |
| 541 | 4$^s$ 25;45 | 300 | 0$^s$ 13;35 | | |
| 561 | 9$^s$ 20;39 | 400 | 0$^s$ 18; 6 | | |
| 581 | 2$^s$ 15;34* | 500⊥ | 0$^s$ 22;38[26] | | |
| | | | [27] | | |

* In **YLB** the values for collected years 1, 61, 81, 141, 201, 261, 321, 381, 441, 461, 521, and 581 are one minute smaller. [1] +**$C_1$** 59′ [2] **C** 4′ [3] **$C_1$** 24′ [4] +**C** 16′ **$C_1$** 24° [5] **F** 20° [6] **$C_1$** 13′ [7] **C** 0$^s$ 52′ [8] **$C_1$** 44′ [9] **H** 16° [10] **C** 16′ **YL** 57′ [11] **C** 19′ **LB** 11° [12] **$C_1$** 4′ **Y** 6′ [13] **C** 13′ [14] **L** 27° [15] **YL** 40′ [16] +**C** 4° [17] **LY** 55′ [18] **$C_1$** 24′ [19] **C** arguments 40–500 years: 23, 24, ..., 30 (continued from the extended years) [20] **Y** 43′ [21] **C** 401 (لا) [22] **Y** 49′ [23] **Y** 47′ [24] **$C_1$** 34′ [25] +**C** 11′ [26] **Y** 33′ [27] **YLB** add a value 0$^s$ 27;10 (**B** 9′) for 600 years.

**Table 19: Double elongation (second part)**

*Sources*: **F** fols 50v–51r, **H** fols 37v–38r, **C** fol. 54r, **$C_1$** fol. 20r, **$C_2$** -, **Y** fols 269v–270r, **L** fols 41v–42r, **B** pp. 78–79.

| Double Elongation in Days, Hours, and Between Longitudes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
| | s ° ′ | | ° ′ | | ° ′ | | | ° ′ |
| 1 | $0^s$ 0; 0 | 1 | 1; 1 | 31 | 31;30 | 71 | additive | 1;17 |
| 2 | $0^s$ 24;23 | 2 | 2; 2 | 32 | 32;31 | 72 | | 1;13 |
| 3 | $1^s$ 18;46 | 3 | 3; 3 | 33 | 33;32* | 73 | | 1; 9 |
| 4 | $2^s$ 13; 9 | 4 | 4; 4 | 34 | 34;33* | 74 | | 1; 5 |
| 5 | $3^s$ 7;32 | 5 | 5; 5 | 35 | 35;34[1]↓ | 75 | | 1; 1 |
| 6 | $4^s$ 1;54 | 6 | 6; 6 | 36 | 36;35[2] | 76 | | 0;57 |
| 7 | $4^s$ 26;17 | 7 | 7; 7 | 37 | 37;36* | 77 | | 0;53[3] |
| 8 | $5^s$ 20;40 | 8 | 8; 8 | 38 | 38;37*⊥ | 78 | | 0;49 |
| 9 | $6^s$ 15; 3 | 9 | 9; 9 | 39 | 39;38* | 79 | | 0;45 |
| 10 | $7^s$ 9;26 | 10 | 10;10 | 40[4] | 40;39* | 80 | | 0;41 |
| 11 | $8^s$ 3;49 | 11 | 11;11* | 41 | 41;40* | 81 | | 0;37[5] |
| 12 | $8^s$ 28;12 | 12 | 12;12* | 42 | 42;41* | 82 | | 0;33 |
| 13 | $9^s$ 22;35 | 13 | 13;13* | 43 | 43;42* | 83 | | 0;28 |
| 14 | $10^s$ 16;58 | 14 | 14;14* | 44 | 44;43* | 84 | | 0;24 |
| 15 | $11^s$ 11;20 | 15 | 15;15* | 45 | 45;44* | 85 | | 0;20 |
| 16 | $0^s$ 5;43 | 16 | 16;16* | 46 | 46;44 | 86 | | 0;16 |
| 17 | $1^s$ 0; 6 | 17 | 17;17* | 47 | 47;45 | 87 | | 0;12 |
| 18 | $1^s$ 24;29 | 18 | 18;17[6] | 48 | 48;46 | 88 | | 0; 8 |
| 19 | $2^s$ 18;52 | 19 | 19;18 | 49 | 49;47 | 89 | | 0; 4 |
| 20 | $3^s$ 13;15 | 20 | 20;19 | 50 | 50;48 | 90 | | 0; 0 |
| 21 | $4^s$ 7;38[7] | 21 | 21;20 | 51 | 51;49 | 91 | subtractive | 0; 4 |
| 22 | $5^s$ 2; 1 | 22 | 22;21 | 52 | 52;50 | 92 | | 0; 8 |
| 23 | $5^s$ 26;24 | 23 | 23;22 | 53 | 53;51 | 93 | | 0;12 |
| 24 | $6^s$ 20;46 | 24 | 24;23 | 54 | 54;52 | 94 | | 0;16 |
| 25 | $7^s$ 15; 9 | 25 | 25;24 | 55 | 55;53* | 95 | | 0;20 |
| 26 | $8^s$ 9;32 | 26 | 26;25 | 56 | 56;54* | 96 | | 0;24 |
| 27 | $9^s$ 3;55 | 27 | 27;26 | 57 | 57;55* | 97 | | 0;28 |
| 28 | $9^s$ 28;18 | 28 | 28;27 | 58 | 58;56* | 98 | | 0;33[8] |
| 29 | $10^s$ 22;41 | 29 | 29;28 | 59 | 59;57* | 99 | | 0;37 |
| 30 | $11^s$ 17; 4 | 30 | 30;29 | 60 | 60;58* | 100 | | 0;41 |

* In **YLB** the values for 11–17 (**B** 16–17), 33–45, and 55–60 hours are one minute smaller. **YLB** display all longitude corrections in minutes (i.e., as 77, 73, 69′, ...). [1] **+Y** 35–38 hours (degrees): slide[+1] [2] **Y**+ 36° (instead of 37°) [3] **B** 58′ [4] **Y** 31 [5] **C** 34′ **$C_1$** 36′ [6] **C** 18′ [7] **C** 18′ [8] **F** 38′.

**Table 20: Lunar first equation (first half)**
*Sources*: **F** fols 51v–52r, **H** fols 38v–39r, **C** fols 54v–55r, $\mathbf{C_1}$ fols 20v–21r, $\mathbf{C_2}$ -, **Y** fols 270v–271r, **L** fols 42v–43r, **B** pp. 80–81.

First Equation for the Moon

| degrees of the double elong. | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | **2** ° ′ | differences ′ | mean distance 24° **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 14; 0 | 9 | 18;23 | 9 | 22;36 | 8 | 26; 0 | 5 | 27; 4 | 1 | 23;22 | 13 |
| 1 | 14; 9 | 9 | 18;32 | 9 | 22;44 | 8 | 26; 5 | 5 | 27; 2 | 2 | 23; 8 | 14 |
| 2 | 14;17 | 8 | 18;40 | 8 | 22;51 | 7 | 26;10 | 5 | 27; 1 | 1 | 22;53 | 15 |
| 3 | 14;26 | 9 | 18;49 | 9 | 22;59 | 8 | 26;15 | 5 | 26;59 | 2[1] | 22;38 | 15 |
| 4 | 14;35 | 9 | 18;58 | 9 | 23; 7 | 8 | 26;19 | 4 | 26;56 | 3 | 22;22 | 16 |
| 5 | 14;44 | 9 | 19; 6 | 8 | 23;14[2] | 7 | 26;24 | 5 | 26;53 | 3 | 22; 5 | 17 |
| 6 | 14;53 | 9 | 19;15 | 9 | 23;22 | 8 | 26;28 | 4 | 26;50 | 3[3] | 21;48 | 17 |
| 7 | 15; 2 | 9 | 19;24 | 9 | 23;29 | 7 | 26;32 | 4 | 26;46 | 4 | 21;31 | 17 |
| 8 | 15;10 | 8 | 19;32 | 8 | 23;37 | 8 | 26;35 | 3 | 26;41 | 5 | 21;14 | 17 |
| 9 | 15;19 | 9 | 19;41 | 9 | 23;44 | 7 | 26;39 | 4 | 26;36 | 5[4] | 20;57 | 17 |
| 10 | 15;28 | 9 | 19;50 | 9 | 23;51 | 7 | 26;42 | 3 | 26;30 | 6 | 20;39 | 18 |
| 11 | 15;37 | 9 | 19;58 | 8 | 23;59 | 8 | 26;45 | 3 | 26;23 | 7 | 20;21 | 18 |
| 12 | 15;46 | 9 | 20; 7 | 9 | 24; 6 | 7 | 26;48 | 3 | 26;16 | 7 | 20; 3 | 18 |
| 13 | 15;55 | 9 | 20;15 | 8 | 24;13 | 7 | 26;51 | 3 | 26; 9 | 7 | 19;45 | 18 |
| 14 | 16; 3 | 8 | 20;24 | 9 | 24;20 | 7 | 26;53 | 2 | 26; 2 | 7 | 19;26 | 19 |
| 15 | 16;12 | 9 | 20;32 | 8 | 24;27 | 7 | 26;56 | 3 | 25;54 | 8 | 19; 8 | 18[5] |
| 16 | 16;21 | 9 | 20;41 | 9 | 24;34 | 7 | 26;58 | 2 | 25;46 | 8 | 18;49 | 19 |
| 17 | 16;30 | 9 | 20;49 | 8 | 24;41 | 7 | 27; 0 | 2 | 25;37 | 9 | 18;30 | 19 |
| 18 | 16;39 | 9 | 20;58 | 9 | 24;48 | 7 | 27; 2 | 2 | 25;29 | 8 | 18;11 | 19 |
| 19 | 16;48 | 9 | 21; 6 | 8 | 24;54 | 6 | 27; 3 | 1 | 25;20 | 9 | 17;51 | 20 |
| 20 | 16;56 | 8 | 21;15 | 9[6] | 25; 1 | 7 | 27; 5 | 2 | 25;11 | 9 | 17;32 | 19[7] |
| 21 | 17; 5 | 9 | 21;23 | 8[8] | 25; 7 | 6 | 27; 6 | 1 | 25; 2 | 9 | 17;12 | 20[9] |
| 22 | 17;14 | 9 | 21;31 | 8 | 25;14 | 7 | 27; 7 | 1 | 24;52 | 10 | 16;52 | 20 |
| 23 | 17;22 | 8 | 21;40 | 9[10] | 25;20 | 6 | 27; 8 | 1 | 24;43 | 9 | 16;31 | 21 |
| 24 | 17;31 | 9[11] | 21;48 | 8 | 25;27 | 7 | 27; 8 | 0 | 24;33 | 10 | 16;11 | 20 |
| 25 | 17;40 | 9 | 21;56 | 8 | 25;33 | 6 | 27; 8 | 0 | 24;22 | 11 | 15;50 | 21 |
| 26 | 17;48 | 8 | 22; 4 | 8 | 25;39 | 6 | 27; 7 | 1 | 24;11 | 11 | 15;28 | 22 |
| 27 | 17;57 | 9 | 22;12 | 8 | 25;45 | 6 | 27; 7 | 0 | 24; 0 | 11 | 15; 7 | 21 |
| 28 | 18; 6 | 9 | 22;20 | 8 | 25;50 | 5 | 27; 6 | 1 | 23;48 | 12 | 14;45[12] | 22[13] |
| 29 | 18;14 | 8 | 22;28 | 8 | 25;55 | 5 | 27; 5 | 1 | 23;35 | 13 | 14;23 | 22[14] |

**H** leaves the columns for tabular differences empty. [1] **L** 3′ [2] **C** 28° [3] **L** 4′ [4] **L** 6′ [5] **C** 19′ [6] **YL** 8′ [7] **F** 20′ [8] **YL** 9′ (in **L** corrected in black) [9] **F** 21′ [10] **L** 8′ (corrected in black) [11] **Y** om. (hence read as 8′) [12] **F** 55′ **Y** 46′ [13] **F** 12′ (in correspondence with the error in the equation for $5^s$ 28°) [14] **F** 32′ (in correspondence with the error in the equation for $5^s$ 28°).

**Table 20: Lunar first equation (second half)**
*Sources*: **F** fols 51v–52r, **H** fols 38v–39r, **C** fols 54v–55r, $\mathbf{C_1}$ fols 20v–21r, $\mathbf{C_2}$ -, **Y** fols 270v–271r, **L** fols 42v–43r, **B** pp. 80–81.

First Equation for the Moon

| degrees of the double elong. | nearest distance 0° **6** ° ′ | differences ′ | **7** ° ′ | differences ′ | mean distance 6° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 14; 0[1] | 23 | 4;38[2] | 14 | 0;56 | 2 | 2; 0 | 5 | 5;24 | 8 | 9;37 | 9 |
| 1 | 13;37 | 23 | 4;25[3] | 13 | 0;55 | 1 | 2; 5 | 5 | 5;32 | 8 | 9;46 | 9 |
| 2 | 13;15 | 22 | 4;12 | 13 | 0;54 | 1 | 2;10 | 5 | 5;40 | 8 | 9;54 | 8 |
| 3 | 12;53[4] | 22 | 4; 0 | 12 | 0;53 | 1 | 2;15 | 5 | 5;48 | 8 | 10; 3 | 9 |
| 4 | 12;32 | 21 | 3;49 | 11 | 0;53 | 0 | 2;21 | 6 | 5;56 | 8 | 10;12 | 9 |
| 5 | 12;10 | 22 | 3;38 | 11 | 0;52 | 1 | 2;27 | 6 | 6; 4 | 8 | 10;20 | 8 |
| 6 | 11;49 | 21 | 3;27 | 11 | 0;52 | 0 | 2;33[5] | 6 | 6;12 | 8 | 10;29 | 9 |
| 7 | 11;29 | 20 | 3;17 | 10 | 0;52 | 0 | 2;40 | 7 | 6;20 | 8 | 10;38 | 9 |
| 8 | 11; 8 | 21 | 3; 8 | 9 | 0;53 | 1 | 2;46 | 6 | 6;29 | 9 | 10;46 | 8 |
| 9 | 10;48 | 20 | 2;58 | 10 | 0;54 | 1 | 2;53 | 7 | 6;37 | 8 | 10;55[6] | 9 |
| 10 | 10;28 | 20 | 2;49 | 9 | 0;55 | 1 | 2;59 | 6 | 6;45 | 8 | 11; 4 | 9 |
| 11 | 10; 9 | 19 | 2;40 | 9 | 0;57 | 2 | 3; 6 | 7 | 6;54 | 9 | 11;12 | 8 |
| 12 | 9;49 | 20 | 2;31 | 9 | 0;58 | 1 | 3;12 | 6 | 7; 2 | 8 | 11;21 | 9 |
| 13 | 9;30 | 19 | 2;23 | 8 | 1; 0 | 2 | 3;19 | 7 | 7;11 | 9 | 11;30 | 9 |
| 14 | 9;11 | 19 | 2;14 | 9 | 1; 2 | 2 | 3;26 | 7 | 7;19 | 8 | 11;39 | 9 |
| 15 | 8;52 | 19 | 2; 6 | 8 | 1; 4 | 2 | 3;33 | 7 | 7;28 | 9 | 11;48 | 9 |
| 16 | 8;34 | 18[7] | 1;58 | 8 | 1; 7 | 3 | 3;40 | 7 | 7;36 | 8 | 11;57 | 9 |
| 17 | 8;15 | 19 | 1;51[8] | 7 | 1; 9 | 2 | 3;47 | 7 | 7;45 | 9 | 12; 5 | 8 |
| 18 | 7;57 | 18 | 1;44 | 7 | 1;12 | 3 | 3;54 | 7 | 7;53 | 8 | 12;14 | 9 |
| 19 | 7;39 | 18 | 1;37 | 7 | 1;15 | 3 | 4; 1 | 7 | 8; 2 | 9 | 12;23 | 9 |
| 20 | 7;21 | 18 | 1;30 | 7 | 1;18 | 3 | 4; 9 | 8 | 8;10 | 8 | 12;32 | 9 |
| 21 | 7; 3 | 18 | 1;24 | 6 | 1;21 | 3 | 4;16 | 7 | 8;19 | 9 | 12;41 | 9 |
| 22 | 6;46 | 17 | 1;19 | 5 | 1;25 | 4[9] | 4;23[10] | 7 | 8;28 | 9 | 12;50 | 9 |
| 23 | 6;29 | 17[11] | 1;14 | 5 | 1;28 | 3[12] | 4;31 | 8 | 8;36 | 8 | 12;58 | 8 |
| 24 | 6;12 | 17 | 1;10 | 4 | 1;32 | 4 | 4;38 | 7 | 8;45 | 9 | 13; 7 | 9 |
| 25 | 5;55 | 17 | 1; 7 | 3 | 1;36 | 4 | 4;46 | 8 | 8;54 | 9 | 13;16 | 9 |
| 26 | 5;38 | 17 | 1; 4 | 3 | 1;41 | 5 | 4;53[13] | 7 | 9; 2 | 8 | 13;25 | 9 |
| 27 | 5;22 | 16 | 1; 1 | 3 | 1;45 | 4 | 5; 1 | 8 | 9;11 | 9 | 13;34 | 9 |
| 28 | 5; 7 | 15 | 0;59 | 2 | 1;50 | 5 | 5; 9 | 8 | 9;20 | 9 | 13;43 | 9 |
| 29 | 4;52[14] | 15 | 0;58 | 1 | 1;55 | 5 | 5;16[15] | 7 | 9;28 | 8 | 13;51[16] | 8 |

**H** leaves the columns for tabular differences empty. [1] **F** 13° [2] **L** 28′ [3] **H** 22′ [4] **C** 33′ [5] $\mathbf{C_1}$ 38′ [6] **F** 56′ $\mathbf{C_1}$ 15′ [7] **F** 19′ [8] **C** 11′ [9] **Y** 3′ [10] $\mathbf{C_1}$ 28′ [11] **L** 57′ [12] **L** 4′ (corrected in black) [13] **F** 13′ **C** 33′ [14] **F** 5° [15] **H** 17′ (smudged, whole value repeated in red under the table) [16] $\mathbf{C_1}$ 11′.

**Table 20: Lunar second equation (first half)**

*Sources*: **F** fols 52v–53r, **H** fols 39v–40r, **C** fols 55v–56r, **$C_1$** fols 21v–22r, **$C_2$** -, **Y** fols 271v–272r, **L** fols 43v–44r, **B** pp. 82–83.

| Second Equation for the Moon | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| parts of the epicycle | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | **2** ° ′ | differences ′ | mean distance 5° **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ′ |
| 0 | 8; 0 | 5 | 5;41 | 4 | 3;51 | 3 | 3; 0 | 1[1] | 3;28 | 2 | 5;17[2] | 5 |
| 1 | 7;55 | 5 | 5;37 | 4 | 3;48 | 3 | 3; 0 | 0 | 3;31 | 3 | 5;22 | 5 |
| 2 | 7;50 | 5 | 5;32 | 5 | 3;46 | 2 | 3; 0 | 0 | 3;33 | 2 | 5;27 | 5 |
| 3 | 7;46 | 4 | 5;28 | 4 | 3;43 | 3 | 3; 0 | 0 | 3;36 | 3 | 5;32 | 5 |
| 4 | 7;41 | 5 | 5;24 | 4 | 3;40 | 3 | 2;59 | 1 | 3;39 | 3 | 5;37 | 5 |
| 5 | 7;36 | 5 | 5;19 | 5 | 3;38 | 2 | 2;59 | 0 | 3;41 | 2 | 5;42 | 5 |
| 6 | 7;31 | 5 | 5;15 | 4 | 3;35 | 3 | 2;59 | 0 | 3;44 | 3 | 5;47 | 5 |
| 7 | 7;26 | 5 | 5;11 | 4 | 3;33 | 2 | 2;59 | 0 | 3;47 | 3 | 5;52 | 5 |
| 8 | 7;22 | 4 | 5; 7 | 4 | 3;30 | 3 | 3; 0 | 1 | 3;50[3]↓ | 3 | 5;58 | 6 |
| 9 | 7;17 | 5 | 5; 3[4] | 4 | 3;28 | 2 | 3; 0 | 0 | 3;53 | 3 | 6; 3 | 5 |
| 10 | 7;12 | 5 | 4;59 | 4 | 3;26 | 2 | 3; 0 | 0[5] | 3;57⊥ | 4 | 6; 8 | 5[6]↓ |
| 11 | 7; 8 | 4 | 4;55 | 4[7] | 3;23[8] | 3[9] | 3; 1 | 1 | 4; 0 | 3 | 6;14 | 6 |
| 12 | 7; 3 | 5 | 4;51 | 4 | 3;21 | 2 | 3; 1 | 0[10] | 4; 3 | 3[11] | 6;19 | 5 |
| 13 | 6;58 | 5 | 4;47 | 4 | 3;19 | 2 | 3; 2 | 1 | 4; 7 | 4 | 6;25 | 6[12]↓ |
| 14 | 6;54 | 4 | 4;44 | 3 | 3;18 | 1 | 3; 3 | 1 | 4;10 | 3 | 6;30 | 5 |
| 15 | 6;49 | 5 | 4;40 | 4 | 3;16 | 2 | 3; 4 | 1 | 4;14 | 4 | 6;36 | 6 |
| 16 | 6;44 | 5 | 4;36 | 4[13] | 3;14 | 2[14] | 3; 5 | 1 | 4;18 | 4 | 6;41 | 5 |
| 17 | 6;40 | 4 | 4;33 | 3[15] | 3;13 | 1 | 3; 6 | 1[16] | 4;21 | 3[17] | 6;47 | 6⊥ |
| 18 | 6;35 | 5 | 4;29 | 4 | 3;11 | 2 | 3; 7 | 1 | 4;25 | 4 | 6;52 | 5 |
| 19 | 6;30 | 5 | 4;26[18] | 3 | 3;10 | 1 | 3; 8 | 1 | 4;29 | 4 | 6;58[19] | 6 |
| 20 | 6;26 | 4[20] | 4;22 | 4[21] | 3; 9 | 1 | 3;10 | 2 | 4;33 | 4 | 7; 3 | 5 |
| 21 | 6;21 | 5 | 4;19 | 3[22] | 3; 8 | 1 | 3;11 | 1 | 4;37 | 4 | 7; 9 | 6 |
| 22 | 6;16 | 5 | 4;16 | 3 | 3; 6 | 2 | 3;13 | 2 | 4;42 | 5 | 7;14 | 5 |
| 23 | 6;12 | 4[23] | 4;12 | 4[24] | 3; 5 | 1 | 3;14 | 1 | 4;46 | 4 | 7;20 | 6⊥ |
| 24 | 6; 7 | 5 | 4; 9 | 3 | 3; 4[25] | 1 | 3;16 | 2 | 4;50 | 4 | 7;25 | 5 |
| 25 | 6; 3 | 4 | 4; 6 | 3 | 3; 3 | 1 | 3;18 | 2 | 4;54 | 4 | 7;31 | 6 |
| 26 | 5;58 | 5 | 4; 3 | 3 | 3; 3 | 0 | 3;20 | 2 | 4;59 | 5 | 7;37 | 6 |
| 27 | 5;54[26] | 4 | 4; 0 | 3 | 3; 2 | 1 | 3;22 | 2 | 5; 3 | 4 | 7;43 | 6 |
| 28 | 5;50 | 4 | 3;57 | 3 | 3; 1 | 1 | 3;24 | 2 | 5; 8 | 5 | 7;48 | 5 |
| 29 | 5;45 | 5 | 3;54 | 3 | 3; 1 | 0 | 3;26 | 2 | 5;12 | 4 | 7;54 | 6 |

**H** leaves the columns for tabular differences empty. [1] **YL** 0′ [2] **$C_1$** 16′ [3] **F** 4$^{s}$ 8–10°: 4° [4] **F** 8′ **$C_1$** 4′ [5] **$C_1$** 1′ [6] **$C_1$** 5$^{s}$ 10–17°: slide[+1] [7] **$C_1$** 3′ [8] **C** 28′ [9] **$C_1$** 1′ [10] **$C_1$** 1′ [11] **$C_1$** 4′ [12] +**C** 5$^{s}$ 13–23°: slide[+1] [13] **$C_1$** 3′ [14] **$C_1$** 1′ [15] **$C_1$** 4′ [16] **$C_1$** 2′ [17] **$C_1$** 4′ [18] **C** 27′ [19] **F** 18′ [20] **F** 5′ [21] **Y** 3′ [22] **Y** 4′ [23] **F** 5′ [24] **$C_1$** 3′ [25] **$C_1$** 3′ [26] **F** 14′.

**Table 20: Lunar second equation (second half)**
*Sources*: **F** fols 52v–53r, **H** fols 39v–40r, **C** fols 55v–56r, $\mathbf{C_1}$ fols 21v–22r, $\mathbf{C_2}$ -, **Y** fols 271v–272r, **L** fols 43v–44r, **B** pp. 82–83.

Second Equation for the Moon

| parts of the epicycle | nearest distance 0° **6** ° ′ | differences ′ | **7** ° ′ | differences ′ | mean distance 25° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8; 0 | 6 | 10;43 | 5 | 12;32 | 3 | 13; 0 | 0 | 12; 9 | 3 | 10;19 | 4 |
| 1 | 8; 6 | 6 | 10;48 | 5 | 12;34 | 2 | 12;59 | 1 | 12; 6 | 3 | 10;15 | 4 |
| 2 | 8;12 | 6 | 10;52 | 4 | 12;36 | 2 | 12;59 | 0 | 12; 3 | 3 | 10;10 | 5 |
| 3 | 8;17 | 5 | 10;57 | 5 | 12;38 | 2 | 12;58 | 1 | 12; 0[1] | 3 | 10; 6 | 4 |
| 4 | 8;23 | 6 | 11; 1 | 4 | 12;40 | 2 | 12;57 | 1 | 11;57 | 3 | 10; 2 | 4 |
| 5 | 8;29 | 6 | 11; 6 | 5 | 12;42 | 2 | 12;57[2]↓ | 0[3] | 11;54 | 3 | 9;57 | 5 |
| 6 | 8;35[4] | 6 | 11;10 | 4 | 12;44 | 2 | 12;56 | 1[5]↓ | 11;51 | 3 | 9;53 | 4 |
| 7 | 8;40 | 5 | 11;14 | 4 | 12;46 | 2 | 12;55 | 1 | 11;48 | 3 | 9;48 | 5 |
| 8 | 8;46 | 6 | 11;18 | 4 | 12;47 | 1 | 12;54 | 1 | 11;44 | 4 | 9;44 | 4 |
| 9 | 8;51 | 5 | 11;23 | 5 | 12;49 | 2 | 12;52 | 2 | 11;41 | 3 | 9;39 | 5 |
| 10 | 8;57 | 6 | 11;27 | 4 | 12;50 | 1 | 12;51 | 1 | 11;38 | 3 | 9;34 | 5 |
| 11 | 9; 2 | 5 | 11;31 | 4 | 12;52 | 2 | 12;50 | 1 | 11;34 | 4 | 9;30 | 4 |
| 12 | 9; 8 | 6[6]↓ | 11;35 | 4 | 12;53 | 1 | 12;49 | 1 | 11;31 | 3 | 9;25 | 5[7] |
| 13 | 9;13 | 5 | 11;39 | 4 | 12;54 | 1 | 12;47 | 2 | 11;27 | 4 | 9;20 | 5 |
| 14 | 9;19 | 6 | 11;42 | 3 | 12;55 | 1 | 12;46 | 1 | 11;24 | 3[8] | 9;16[9] | 4 |
| 15 | 9;24 | 5 | 11;46 | 4[10] | 12;56 | 1 | 12;44 | 2 | 11;20 | 4 | 9;11 | 5 |
| 16 | 9;30[11] | 6 | 11;50 | 4 | 12;57 | 1 | 12;42 | 2 | 11;16 | 4 | 9; 6 | 5 |
| 17 | 9;35 | 5 | 11;53 | 3 | 12;58 | 1 | 12;41 | 1 | 11;13 | 3[12] | 9; 2 | 4[13] |
| 18 | 9;41 | 6⊥ | 11;57 | 4 | 12;59 | 1 | 12;39 | 2 | 11; 9 | 4 | 8;57 | 5 |
| 19 | 9;46 | 5 | 12; 0 | 3 | 12;59 | 0 | 12;37 | 2 | 11; 5 | 4 | 8;52 | 5 |
| 20 | 9;52 | 6 | 12; 3 | 3 | 13; 0 | 1 | 12;34 | 3 | 11; 1 | 4 | 8;48 | 4 |
| 21 | 9;57 | 5 | 12; 7 | 4 | 13; 0 | 0[14] | 12;32 | 2 | 10;57 | 4 | 8;43 | 5 |
| 22 | 10; 2 | 5 | 12;10 | 3[15] | 13; 0 | 0[16] | 12;30 | 2 | 10;53 | 4 | 8;38 | 5 |
| 23 | 10; 8 | 6 | 12;13 | 3 | 13; 1 | 1[17] | 12;27⊥ | 3⊥ | 10;49 | 4 | 8;34 | 4 |
| 24 | 10;13 | 5 | 12;16 | 3 | 13; 1 | 0 | 12;25[18]↓ | 2[19] | 10;45 | 4 | 8;29 | 5 |
| 25 | 10;18 | 5 | 12;19 | 3 | 13; 1 | 0[20] | 12;22 | 3[21] | 10;41 | 4 | 8;24 | 5 |
| 26 | 10;23 | 5 | 12;21 | 2 | 13; 1 | 0 | 12;20⊥ | 2[22] | 10;36 | 5 | 8;19 | 5 |
| 27 | 10;28 | 5 | 12;24 | 3 | 13; 0 | 1 | 12;17 | 3[23] | 10;32 | 4 | 8;14 | 5 |
| 28 | 10;33 | 5 | 12;27 | 3 | 13; 0 | 0 | 12;14 | 3 | 10;28 | 4 | 8;10 | 4 |
| 29 | 10;38 | 5 | 12;29 | 2 | 13; 0 | 0 | 12;12 | 2 | 10;23 | 5 | 8; 5 | 5 |

[1] **C** 5′ [2] **L** 9ˢ 5–23°: slide[+1] [3] **L** corrected to 1′ in black [4] $\mathbf{C_1}$ 36′ [5] **L** 9ˢ 6–23°: slide[+1] (in correspondence with the slide of the equations) [6] **C** 6ˢ 12–18°: slide[+1] [7] $\mathbf{C_1}$ minutes ill. [8] **Y** om. (hence read as 4′) [9] $\mathbf{C_1}$ 17′ [10] $\mathbf{C_1}$ 3′ [11] **Y** 32′ [12] **L** 4′ [13] **F** 5′ [14] **C** 1′ [15] **C** 4′ [16] **Y** 1′ [17] **Y** 0′ [18] **L** 9ˢ 24–26°: 23, 21, 19′ (adjustment at end of slide?) [19] **C** 1′ [20] **Y** 1′ [21] **C** 0′ **L** 2′ [22] **C** 0′ [23] **L** 2′.

**Table 20: Lunar variation of the nearest distance**
*Sources*: **F** fol. 53v, **H** fol. 40v, **C** fol. 56v, $\mathbf{C_1}$ fol. 22v, $\mathbf{C_2}$ -, **Y** fol. 272v, **L** fol. 44v, **B** p. 84.

| Variation of the Nearest Distance for the Moon | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the double elongation | **0** | **1** | **2** | **3** | **4** | **5** | parts of the double elongation |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | |
| 0 | Place of conjunction and opposition | 0; 8 | 0;33 | 1;12 | 1;52 | 2;27 | 30[1]↓↓ |
| 1 | | 0; 9 | 0;34 | 1;13 | 1;53 | 2;28 | 29 |
| 2 | | 0; 9 | 0;35 | 1;15 | 1;55 | 2;29 | 28 |
| 3 | | 0;10 | 0;36 | 1;16 | 1;56 | 2;29[2]↓ | 27 |
| 4 | | 0;11 | 0;38 | 1;17 | 1;57 | 2;30 | 26⊥ |
| 5 | | 0;11 | 0;39 | 1;19 | 1;59 | 2;31 | 25[3] |
| 6 | | 0;12 | 0;40 | 1;20 | 2; 0 | 2;32 | 24 |
| 7 | | 0;13 | 0;41 | 1;21 | 2; 1 | 2;33 | 23 |
| 8 | | 0;13 | 0;43 | 1;23 | 2; 3 | 2;33 | 22 |
| 9 | | 0;14 | 0;44 | 1;24 | 2; 4 | 2;34 | 21 |
| 10 | | 0;15 | 0;45 | 1;25 | 2; 5 | 2;35 | 20 |
| 11 | | 0;15 | 0;47 | 1;27 | 2; 7 | 2;35 | 19 |
| 12 | | 0;16 | 0;48 | 1;28 | 2; 8 | 2;36 | 18 |
| 13 | 0; 0 | 0;17 | 0;49 | 1;29 | 2; 9 | 2;36 | 17 |
| 14 | 0; 1[4] | 0;18 | 0;51 | 1;31 | 2;10 | 2;37 | 16 |
| 15 | 0; 1 | 0;18[5] | 0;52 | 1;32 | 2;11 | 2;37 | 15 |
| 16 | 0; 1 | 0;19[6] | 0;53 | 1;33 | 2;13 | 2;37 | 14 |
| 17 | 0; 2 | 0;20[7] | 0;55 | 1;35 | 2;14 | 2;38 | 13 |
| 18 | 0; 2 | 0;21 | 0;56 | 1;36[8] | 2;15 | 2;38 | 12 |
| 19 | 0; 2 | 0;22 | 0;57 | 1;37 | 2;16 | 2;38 | 11 |
| 20 | 0; 3 | 0;23 | 0;59 | 1;39 | 2;17 | 2;38 | 10 |
| 21 | 0; 3 | 0;24 | 1; 0 | 1;40 | 2;18 | 2;38⊥ | 9 |
| 22 | 0; 4 | 0;25 | 1; 1 | 1;41 | 2;19 | 2;39 | 8 |
| 23 | 0; 4 | 0;26 | 1; 3 | 1;43 | 2;20 | 2;39 | 7 |
| 24 | 0; 5 | 0;27 | 1; 4 | 1;44 | 2;21 | 2;39 | 6 |
| 25 | 0; 5 | 0;28 | 1; 5 | 1;45 | 2;22 | 2;39 | 5 |
| 26 | 0; 6 | 0;29 | 1; 7 | 1;47 | 2;23 | 2;39 | 4 |
| 27 | 0; 6 | 0;30 | 1; 8 | 1;48 | 2;24 | 2;40 | 3 |
| 28 | 0; 7 | 0;31 | 1; 9 | 1;49 | 2;25 | 2;40 | 2 |
| 29 | 0; 7 | 0;32 | 1;11 | 1;51 | 2;26 | 2;40 | 1⊥ |
| double elonga-tion | **11** | **10** | **9** | **8** | **7** | **6** | double elonga-tion |

[1] **C** arguments 30–1°: 0, 1, 2, ..., 29; **L** arguments 30–26°: slide[+1] [2] **F** $5^s$ 3–21°: slide[+1]
[3] **L** om. (end of slide) [4] $\mathbf{C_1}$ 0′ [5] **C** 19′ [6] **C** 20′ [7] **C** 22′ [8] **L** 35′.

**Table 20: Lunar interpolation minutes**

*Sources*: **F** fol. 54r, **H** fol. 41r, **C** fol. 56v, $\mathbf{C_1}$ fol. 22v, $\mathbf{C_2}$ -, **Y** fol. 273r, **L** fol. 44v, **B** p. 84.

Minutes of Proportions for the Moon

| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
|---|---|---|---|---|---|---|---|
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0[1]↓ | 0 | 32 | 54 | 60 | 50 | 28 | 30[2]↓ |
| 1 | 1 | 33 | 54 | 60 | 49 | 27 | 29 |
| 2 | 2 | 34 | 55 | 60 | 49 | 26 | 28 |
| 3 | 3 | 35 | 55 | 59 | 48 | 25 | 27 |
| 4 | 5 | 36 | 56 | 59 | 47 | 25 | 26 |
| 5 | 6 | 37 | 56 | 59 | 47[3] | 24 | 25[4]↓ |
| 6 | 7 | 38 | 57 | 59 | 46 | 23 | 24 |
| 7 | 8 | 39 | 57 | 59 | 45 | 22 | 23 |
| 8 | 9 | 40 | 58 | 58 | 45 | 21 | 22 |
| 9 | 10 | 40[5]↓ | 58 | 58 | 44 | 20 | 21 |
| 10 | 12 | 41 | 58 | 58 | 43 | 19 | 20⊥ |
| 11 | 13 | 42⊥ | 59 | 58 | 43 | 18 | 19 |
| 12 | 14 | 43 | 59 | 58 | 42 | 17 | 18 |
| 13 | 15 | 44 | 59 | 58 | 41 | 16 | 17 |
| 14 | 16 | 44 | 59 | 57 | 41 | 15 | 16 |
| 15 | 17 | 45 | 59 | 57 | 40 | 14 | 15 |
| 16 | 18 | 46 | 60 | 57[6] | 39 | 13 | 14 |
| 17 | 19 | 46 | 60 | 56 | 39 | 12 | 13 |
| 18 | 20 | 47 | 60 | 56 | 38 | 11 | 12 |
| 19 | 21 | 48 | 60 | 55 | 37 | 10 | 11 |
| 20 | 22 | 48 | 60 | 55 | 36[7] | 9[8] | 10 |
| 21 | 23 | 49 | 60 | 54 | 35 | 8 | 9 |
| 22 | 24 | 50 | 60 | 54 | 35 | 8 | 8 |
| 23 | 25 | 50 | 60 | 53 | 34 | 7 | 7 |
| 24 | 26 | 51 | 60 | 53 | 33 | 6 | 6 |
| 25 | 27 | 51 | 60 | 52 | 32 | 5 | 5 |
| 26 | 28 | 52 | 60 | 52 | 31 | 4 | 4 |
| 27 | 29 | 52 | 60 | 51 | 30 | 3 | 3 |
| 28 | 30 | 53 | 60 | 51 | 30 | 2 | 2 |
| 29⊥ | 31 | 53 | 60 | 50 | 29 | 1 | 1⊥ |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **C** arguments 0–29°: 30, 29, 28, …, 1; **B** arguments 0–29°: 1, 2, 3, …, 30 [2] **F** arguments 30, 29, …, 1: 29, 28, …, 0 [3] **F** 46′ [4] +**C** arguments 25, 24, …, 20: 23, 24, …28 [5] $\mathbf{C_1}$ $7^s$ 9–11°: slide[+1] [6] **F** 56′ [7] **C** 37′ [8] **L** 5′.

**Table 21: Mean motion of the lunar node (first part)**
*Sources*: **F** fols 54v–55r, **H** fols 41v–42r, **C** fol. 57r, $\mathbf{C_1}$ fol. 23r, $\mathbf{C_2}$ fol. 43r, **Y** fols 273v–274r, **L** fol. 45r, **B** p. 85.

| Mean Motion of the Lunar Node in Years and Months | | | | | |
|---|---|---|---|---|---|
| years | collected ⟨years⟩ s ° ′ | years | extended ⟨years⟩ s ° ′ | months | months s ° ′ |
| 1 | $2^s$ 5;42 | 1 | $0^s$ 19;20 | Farwardīn | $0^s$ 0; 0 |
| 21 | $3^s$ 2;15* | 2 | $1^s$ 8;39 | | |
| 41 | $3^s$ 28;47* | 3 | $1^s$ 27;59 | Urdībihisht | $0^s$ 1;35 |
| 61 | $4^s$ 25;19 | 4 | $2^s$ 17;18[1] | | |
| 81 | $5^s$ 21;52[2] | 5 | $3^s$ 6;38 | Khurdād | $0^s$ 3;11 |
| 101 | $6^s$ 18;24* | 6 | $3^s$ 25;58[3] | | |
| 121 | $7^s$ 14;56[4] | 7 | $4^s$ 15;17[5] | Tīr | $0^s$ 4;46 |
| 141 | $8^s$ 11;28[6] | 8 | $5^s$ 4;37 | | |
| 161 | $9^s$ 8; 1* | 9 | $5^s$ 23;57 | Murdād | $0^s$ 6;21 |
| 181 | $10^s$ 4;33[7] | 10 | $6^s$ 13;16 | | |
| 201 | $11^s$ 1; 5[8]↓ | 11 | $7^s$ 2;36 | Shahrīwar | $0^s$ 7;57 |
| 221 | $11^s$ 27;38* | 12 | $7^s$ 21;55 | | |
| 241 | $0^s$ 24;10* | 13 | $8^s$ 11;15 | Mihr | $0^s$ 9;32 |
| 261 | $1^s$ 20;42[9] | 14 | $9^s$ 0;34[10] | | |
| 281 | $2^s$ 17;14 | 15 | $9^s$ 19;54 | Ābān | $0^s$ 11; 7 |
| 301 | $3^s$ 13;47[11] | 16 | $10^s$ 9;14 | | |
| 321 | $4^s$ 10;19* | 17 | $10^s$ 28;33 | Ādhar | $0^s$ 12;42[12] |
| 341 | $5^s$ 6;51[13] | 18 | $11^s$ 17;53 | | *$0^s$ 12;58*[14] |
| 361 | $6^s$ 3;24* | 19 | $0^s$ 7;13 | Day | $0^s$ 14;18 |
| 381 | $6^s$ 29;56[15] | 20 | $0^s$ 26;32 | | *$0^s$ 14;34* |
| 401 | $7^s$ 26;28 | single years | | Bahman | $0^s$ 15;53 |
| 421 | $8^s$ 23; 0 | | | | *$0^s$ 16; 9*[16] |
| 441 | $9^s$ 19;33* | 40 | $1^s$ 23; 5 | Isfandārmudh | $0^s$ 17;28[17] |
| 461 | $10^s$ 16; 5 | 60 | $2^s$ 19;37 | | *$0^s$ 17;44* |
| 481 | $11^s$ 12;37 | 80 | $3^s$ 16; 9[18] | | |
| 501 | $0^s$ 9;10* | 100 | $4^s$ 12;41[19] | | |
| 521 | $1^s$ 5;42* | 200 | $8^s$ 25;23 | | |
| 541 | $2^s$ 2;14 | 300 | $1^s$ 8; 4 | | |
| 561 | $2^s$ 28;47[20] | 400 | $5^s$ 20;46 | | |
| 581 | $3^s$ 25;19*⊥ | 500 | $10^s$ 3;27 | | |
| | [21] | | [22] | | |

* In **YLB** the values for collected years 241, 301, 321, 361, 381, 441, 501, 521, 561, and 581 are 1 minute smaller; in **YL** also the values for collected years 21, 41, 81, 101, 161, 181, and 221 are 1 minute smaller. **L** omits the alternative values in red (here: in italics) from the subtable for months.
[1] $\mathbf{C_1}$ 16° [2] +**C** 12′ [3] **F** 18′ [4] **C** 16′ [5] $\mathbf{C_1C_2}$ 57′ [6] **L** 25′ [7] **L**+ om. (beginning of slide) [8] **L**+ 201–581 years: slide[−1] [9] +**C** 24°44′ [10] **YLB** 35′ [11] +**F** 46′ [12] **F** 2′ [13] +**C** 11′ [14] **F** 18′ [15] +**C** 16′ [16] **F** 41′ [17] **L** 47° [18] $\mathbf{CC_1C_2}$ 41′ **Y** 3′ [19] $\mathbf{CC_1C_2}$ 49′ [20] **Y**+ 18° [21] **L** add. $3^s$ 25;18 (value for year 581) under the table (end of slide) [22] **YLB** add a value $2^s$ 16;9 for 600 extended years.

**Table 21: Mean motion of the lunar node (second part)**
*Sources*: **F** fols 54v–55r, **H** fols 41v–42r, **C** fol. 57r, **C$_1$** fol. 23r, **C$_2$** fol. 43r, **Y** fols 273v–274r, **L** fol. 45r, **B** p. 85.

| Mean Motion of the Lunar Node in Days, Hours, and Between Longitudes | | | | | | | |
|---|---|---|---|---|---|---|---|
| | days | | hours | | and their fractions | | in between longitudes |
| days | s ° ′ | hours | ° ′ | hours | ° ′ | longitudes [1] | ′ ″ |
| 1 | 0ˢ 0; 0 | 1 | 0; 0 | 31 | 0; 4 | 71 | 0, 0 |
| 2 | 0ˢ 0; 3 | 2 | 0; 0 | 32 | 0; 4 | 72 | 0, 0 |
| 3 | 0ˢ 0; 6 | 3 | 0; 0 | 33 | 0; 4 | 73 | 0, 0 |
| 4 | 0ˢ 0;10 | 4 | 0; 1 | 34 | 0; 5 | 74 | 0, 0 |
| 5 | 0ˢ 0;13 | 5 | 0; 1 | 35 | 0; 5 | 75 | 0, 0 |
| 6 | 0ˢ 0;16 | 6 | 0; 1 | 36 | 0; 5 | 76 | 0, 0 |
| 7 | 0ˢ 0;19 | 7 | 0; 1 | 37 | 0; 5 | 77 | 0, 0 |
| 8 | 0ˢ 0;22 | 8 | 0; 1 | 38 | 0; 5 | 78 | 0, 0 |
| 9 | 0ˢ 0;25 | 9 | 0; 1 | 39 | 0; 5 | 79 | 0, 0 |
| 10 | 0ˢ 0;29 | 10 | 0; 1 | 40 | 0; 5 | 80 | 0, 0 |
| 11 | 0ˢ 0;32 | 11 | 0; 1 | 41 | 0; 5 | 81 | 0, 0 |
| 12 | 0ˢ 0;35 | 12 | 0; 2 | 42 | 0; 6 | 82 | 0, 0 |
| 13 | 0ˢ 0;38 | 13 | 0; 2 | 43 | 0; 6 | 83 | 0, 0 |
| 14 | 0ˢ 0;41 | 14 | 0; 2 | 44 | 0; 6 | 84 | 0, 0 |
| 15 | 0ˢ 0;44 | 15 | 0; 2 | 45 | 0; 6 | 85 | 0, 0 |
| 16 | 0ˢ 0;48 | 16 | 0; 2 | 46 | 0; 6 | 86 | 0, 0 |
| 17 | 0ˢ 0;51 | 17 | 0; 2 | 47 | 0; 6 | 87 | 0, 0 |
| 18 | 0ˢ 0;54 | 18 | 0; 2 | 48 | 0; 6 | 88 | 0, 0 |
| 19 | 0ˢ 0;57 | 19 | 0; 3 [2] | 49 | 0; 7 [3] | 89 | 0, 0 |
| 20 | 0ˢ 1; 0 | 20 | 0; 3 | 50 | 0; 7 | 90 | 0, 0 |
| 21 | 0ˢ 1; 4 | 21 | 0; 3 | 51 | 0; 7 | 91 | 0, 0 |
| 22 | 0ˢ 1; 7 | 22 | 0; 3 | 52 | 0; 7 | 92 | 0, 0 |
| 23 | 0ˢ 1;10 | 23 | 0; 3 | 53 | 0; 7 | 93 | 0, 0 |
| 24 | 0ˢ 1;13 | 24 | 0; 3 | 54 | 0; 7 | 94 | 0, 0 |
| 25 | 0ˢ 1;16 | 25 | 0; 3 | 55 | 0; 7 | 95 | 0, 0 |
| 26 | 0ˢ 1;19 | 26 | 0; 3 [4] | 56 | 0; 7 | 96 | 0, 0 |
| 27 | 0ˢ 1;23 | 27 | 0; 4 | 57 | 0; 8 | 97 | 0, 0 |
| 28 | 0ˢ 1;26 | 28 | 0; 4 | 58 | 0; 8 | 98 | 0, 0 |
| 29 | 0ˢ 1;29 | 29 | 0; 4 | 59 | 0; 8 | 99 | 0, 0 |
| 30 | 0ˢ 1;32 | 30 | 0; 4 | 60 | 0; 8 | 100 | 0, 0 |

[1] **Y** extends the arguments in the column for longitude differences to 102°, **L** omits all arguments in this column. [2] **F** 2′ [3] **YLB** 6′ **C$_2$** ill. [4] **CC$_1$C$_2$** 4′.

**Table 22: Mean motion of Saturn (first part)**
*Sources*: **F** fols 55v–56r, **H** fols 42v–43r, **C** fol. 57v, **$C_1$** fol. 23v, **$C_2$** fol. 43v, **Y** fols 274v–275r, **L** fols 45v–46r, **B** pp. 86–87.

| Table of the Mean Motion of Saturn in Years and Months | | | | | |
|---|---|---|---|---|---|
| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 7$^s$ 13;42[1] | 1 | 0$^s$ 12;14 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 3$^s$ 18;15* | 2 | 0$^s$ 24;27 | | |
| 41 | 11$^s$ 22;47 | 3 | 1$^s$ 6;41 | Urdībihisht | 0$^s$ 1; 0 |
| 61 | 7$^s$ 27;20 | 4 | 1$^s$ 18;55 | | |
| 81 | 4$^s$ 1;53[2] | 5 | 2$^s$ 1; 8[3] | Khurdād | 0$^s$ 2; 1 |
| 101 | 0$^s$ 6;26* | 6 | 2$^s$ 13;22 | | |
| 121 | 8$^s$ 10;58[4] | 7 | 2$^s$ 25;35 | Tīr | 0$^s$ 3; 1 |
| 141 | 4$^s$ 15;31[5] | 8 | 3$^s$ 7;49 | | |
| 161 | 0$^s$ 20; 4* | 9 | 3$^s$ 20; 3 | Murdād | 0$^s$ 4; 1 |
| 181 | 8$^s$ 24;36 | 10 | 4$^s$ 2;16 | | |
| 201 | 4$^s$ 29; 9 | 11 | 4$^s$ 14;30 | Shahrīwar | 0$^s$ 5; 1[6] |
| 221 | 1$^s$ 3;42[7] | 12 | 4$^s$ 26;43[8] | | |
| 241 | 9$^s$ 8;14 | 13 | 5$^s$ 8;57 | Mihr | 0$^s$ 6; 2 |
| 261 | 5$^s$ 12;47 | 14 | 5$^s$ 21;11 | | |
| 281 | 1$^s$ 17;20[9] | 15 | 6$^s$ 3;24[10] | Ābān | 0$^s$ 7; 2 |
| 301 | 9$^s$ 21;52 | 16 | 6$^s$ 15;38 | | |
| 321 | 5$^s$ 26;25[11] | 17 | 6$^s$ 27;51[12] | Ādhar | 0$^s$ 8; 2[13] |
| 341 | 2$^s$ 0;58* | 18 | 7$^s$ 10; 5 | | *0$^s$ 8;12* |
| 361 | 10$^s$ 5;30 | 19 | 7$^s$ 22;19 | Day | 0$^s$ 9; 3 |
| 381 | 6$^s$ 10; 3 | 20 | 8$^s$ 4;33[14] | | *0$^s$ 9;13* |
| 401 | 2$^s$ 14;36[15] | single years | | Bahman | 0$^s$ 10; 3 |
| 421 | 10$^s$ 19; 9* | | | | *0$^s$ 10;13* |
| 441 | 6$^s$ 23;41 | 40 | 4$^s$ 9; 5 | Isfandārmudh | 0$^s$ 11; 3[16] |
| 461 | 2$^s$ 28;14* | 60 | 0$^s$ 13;38 | | *0$^s$ 11;13* |
| 481 | 11$^s$ 2;47* | 80 | 8$^s$ 18;11 | | |
| 501 | 7$^s$ 7;19[17] | 100 | 4$^s$ 22;43 | | |
| 521 | 3$^s$ 11;52[18] | 200 | 9$^s$ 15;27 | | |
| 541 | 11$^s$ 16;25[19] | 300 | 2$^s$ 8;10 | | |
| 561 | 7$^s$ 20;57[20] | 400 | 7$^s$ 0;54[21] | | |
| 581 | 3$^s$ 25;30[22] | 500 | 11$^s$ 23;37 | | |
| | | | [23] | | |

* In **YLB** the values for collected years 21, 81, 101, 141, 161, 221, 281, 341, 401, 421, 461, and 481 are one minute less. [1] **B** 15° [2] +**F** 13′ [3] **L** 7$^s$ [4] **F** 18′ [5] +**F** 0′ **C** 55° [6] **YLB** 2′ [7] +**C** 0$^s$ [8] **Y** 44′ [9] **Y**+ 59′ [10] **CYLB** 25′ [11] **F** 28′ **C** 27° [12] **YL** 52′ **B** 52′ corrected to 51′ [13] **F** 3′ [14] **$C_1C_2$** 13′ [15] +**$C_1C_2$** 16′ [16] **C** 4′ [17] **$C_1$** 2° [18] **Y** 32′ [19] **C** 20′ **YL** 24′ [20] **Y** 37′ [21] **C** 53′ [22] **$C_1$** 10′ [23] **YLB** give an additional value 4$^s$ 16;21 for 600 single years.

**Table 22: Mean motion of Saturn (second part)**
*Sources*: **F** fols 55v–56r, **H** fols 42v–43r, **C** fol. 57v, $\mathbf{C_1}$ fol. 23v, $\mathbf{C_2}$ fol. 43v, **Y** fols 274v–275r, **L** fols 45v–46r, **B** pp. 86–87.

Mean Motion of Saturn in Days, Hours, and Between Longitudes

| days | days | hours | hours | hours | and their fractions | longitudes | in between longitudes of cities |
|---|---|---|---|---|---|---|---|
| | s ° ′ | | ° ′ | | ° ′ | | ′ ″ |
| 1 | $0^s$ 0; 0 | 1 | 0; 0 | 31 | 0; 3 | 71 | 0, 0 |
| 2 | $0^s$ 0; 2 | 2 | 0; 0 | 32 | 0; 3 | 72 | 0, 0 |
| 3 | $0^s$ 0; 4 | 3 | 0; 0 | 33 | 0; 3 | 73 | 0, 0 |
| 4 | $0^s$ 0; 6 | 4 | 0; 0 | 34 | 0; 3 | 74 | 0, 0 |
| 5 | $0^s$ 0; 8 | 5 | 0; 0 | 35 | 0; 3 | 75 | 0, 0 |
| 6 | $0^s$ 0;10 | 6 | 0; 1 | 36 | 0; 3 | 76 | 0, 0 |
| 7 | $0^s$ 0;12 | 7 | 0; 1 | 37 | 0; 3 | 77 | 0, 0 |
| 8 | $0^s$ 0;14 | 8 | 0; 1 | 38 | 0; 3 | 78 | 0, 0 |
| 9 | $0^s$ 0;16 | 9 | 0; 1 | 39 | 0; 3 | 79 | 0, 0 |
| 10 | $0^s$ 0;18 | 10 | 0; 1 | 40[1] | 0; 3 | 80 | 0, 0 |
| 11 | $0^s$ 0;20 | 11 | 0; 1 | 41 | 0; 3 | 81 | 0, 0 |
| 12 | $0^s$ 0;22 | 12 | 0; 1 | 42 | 0; 4 | 82 | 0, 0 |
| 13 | $0^s$ 0;24 | 13 | 0; 1 | 43 | 0; 4 | 83 | 0, 0 |
| 14 | $0^s$ 0;26 | 14 | 0; 1 | 44 | 0; 4 | 84 | 0, 0 |
| 15 | $0^s$ 0;28 | 15 | 0; 1 | 45 | 0; 4 | 85 | 0, 0 |
| 16 | $0^s$ 0;30 | 16 | 0; 1 | 46 | 0; 4 | 86 | 0, 0 |
| 17 | $0^s$ 0;32 | 17 | 0; 2[2] | 47 | 0; 4 | 87 | 0, 0 |
| 18 | $0^s$ 0;34[3] | 18 | 0; 2 | 48 | 0; 4 | 88 | 0, 0 |
| 19 | $0^s$ 0;36 | 19 | 0; 2 | 49 | 0; 4 | 89 | 0, 0 |
| 20 | $0^s$ 0;38 | 20 | 0; 2 | 50 | 0; 4 | 90 | 0, 0 |
| 21 | $0^s$ 0;40 | 21 | 0; 2 | 51 | 0; 4 | 91 | 0, 0 |
| 22 | $0^s$ 0;42 | 22 | 0; 2 | 52 | 0; 4 | 92 | 0, 0 |
| 23 | $0^s$ 0;44 | 23 | 0; 2 | 53 | 0; 4 | 93 | 0, 0 |
| 24 | $0^s$ 0;46 | 24 | 0; 2 | 54 | 0; 5 | 94 | 0, 0 |
| 25 | $0^s$ 0;48 | 25 | 0; 2 | 55 | 0; 5 | 95 | 0, 0 |
| 26 | $0^s$ 0;50 | 26 | 0; 2 | 56 | 0; 5 | 96 | 0, 0 |
| 27 | $0^s$ 0;52 | 27 | 0; 2 | 57 | 0; 5 | 97 | 0, 0 |
| 28 | $0^s$ 0;54[4] | 28 | 0; 2 | 58 | 0; 5 | 98 | 0, 0 |
| 29 | $0^s$ 0;56[5] | 29 | 0; 2 | 59 | 0; 5 | 99 | 0, 0 |
| 30 | $0^s$ 0;58 | 30 | 0; 3 | 60 | 0; 5 | 100 | 0, 0 |

**Y** omits the subtable for longitude corrections. [1] **C** 20 [2] **YLB** 1′ [3] **H** 37′ [4] **C** 56′ [5] **C** 54′.

**Table 23: Mean anomaly of Saturn (first part)**
*Sources*: **F** fols 56v–57r, **H** fols 43v–44r, **C** fol. 58r, **C$_1$** fol. 24r, **C$_2$** fol. 44r, **Y** fols 275v–276r, **L** fols 46v–47r, **B** pp. 88–89.

Mean Anomaly of Saturn in Years and Months

| years [1] | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 7$^s$ 6;13[2] | 1 | 11$^s$ 17;32 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 10$^s$ 26;56[3] | 2 | 11$^s$ 5; 4[4] | | |
| 41 | 2$^s$ 17;39 | 3 | 10$^s$ 22;36 | Urdībihisht | 0$^s$ 28;34 |
| 61 | 6$^s$ 8;22[5] | 4 | 10$^s$ 10; 9 | | |
| 81 | 9$^s$ 29; 5* | 5 | 9$^s$ 27;41 | Khurdād | 1$^s$ 27; 8 |
| 101 | 1$^s$ 19;47 | 6 | 9$^s$ 15;13 | | |
| 121 | 5$^s$ 10;30 | 7 | 9$^s$ 2;45 | Tīr | 2$^s$ 25;42 |
| 141 | 9$^s$ 1;13* | 8 | 8$^s$ 20;17 | | |
| 161 | 0$^s$ 21;56* | 9 | 8$^s$ 7;49 | Murdād | 3$^s$ 24;15* |
| 181 | 4$^s$ 12;39* | 10 | 7$^s$ 25;21 | | |
| 201 | 8$^s$ 3;21 | 11 | 7$^s$ 12;53* | Shahrīwar | 4$^s$ 22;49 |
| 221 | 11$^s$ 24; 4[6] | 12 | 7$^s$ 0;25* | | |
| 241 | 3$^s$ 14;47[7] | 13 | 6$^s$ 17;57[8] | Mihr | 5$^s$ 21;23[9] |
| 261 | 7$^s$ 5;30* | 14 | 6$^s$ 5;30 | | |
| 281 | 10$^s$ 26;12 | 15 | 5$^s$ 23; 2 | Ābān | 6$^s$ 19;57 |
| 301 | 2$^s$ 16;55 | 16 | 5$^s$ 10;34 | | |
| 321 | 6$^s$ 7;38 | 17 | 4$^s$ 28; 6 | Ādhar | 7$^s$ 18;31 |
| 341 | 9$^s$ 28;21* | 18 | 4$^s$ 15;38* | | *7$^s$ 23;17* |
| 361 | 1$^s$ 19; 4* | 19 | 4$^s$ 3;10* | Day | 8$^s$ 17; 5 |
| 381 | 5$^s$ 9;46 | 20 | 3$^s$ 20;43 | | *8$^s$ 21;51* |
| 401 | 9$^s$ 0;29 | single years | | Bahman | 9$^s$ 15;38* |
| 421 | 0$^s$ 21;12* | | | | *9$^s$ 20;24* |
| 441 | 4$^s$ 11;55* | 40 | 7$^s$ 11;26 | Isfandārmudh | 10$^s$ 14;12* |
| 461 | 8$^s$ 2;38* | 60 | 11$^s$ 2; 8 | | *10$^s$ 18;58* |
| 481 | 11$^s$ 23;20 | 80 | 2$^s$ 22;51 | | |
| 501 | 3$^s$ 14; 3 | 100 | 6$^s$ 13;34[10] | | |
| 521 | 7$^s$ 4;46* | 200 | 0$^s$ 27; 8 | | |
| 541 | 10$^s$ 25;29[11] | 300 | 7$^s$ 10;42 | | |
| 561 | 2$^s$ 16;11[12] | 400 | 1$^s$ 24;16 | | |
| 581 | 6$^s$ 6;54 | 500 | 8$^s$ 7;50 | | |
| | | | [13] | | |

**CC$_1$C$_2$L** do not include values for the early version of the Persian calendar; **B** writes these values with Hindu numerals. * In **YLB** the values for collected years 61, 81, 141–181, 241, 261, 341, 361, 421–461, and 521 are one minute smaller; in **YL** the values for extended years 11–13 and 18–19 and for the months Murdād, Bahman, and Isfandārmudh are one minute larger. [1] **C** omits all arguments in this column. [2] **L** 2$^s$ [3] **C** 16′ [4] **F** 3′ **Y** 5′ [5] **Y**+ 11′ [6] **Y** 7′ [7] +**H** 44° [8] +**F** 17′ [9] **C** 0$^s$ [10] **F** 7$^s$ [11] **YL** 28′ [12] **C$_1$C$_2$** 51′ [13] **YLB** add a value 2$^s$ 21;24° for 600 single years.

**Table 23: Mean anomaly of Saturn (second part)**
*Sources*: **F** fols 56v–57r, **H** fols 43v–44r, **C** fol. 58r, **$C_1$** fol. 24r, **$C_2$** fol. 44r, **Y** fols 275v–276r, **L** fols 46v–47r, **B** pp. 88–89.

| Mean Anomaly of Saturn in Days, Hours, and Between Longitudes of Cities | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes of cities |
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | $0^s$ 0; 0 | 1 | 0; 2 | 31 | 1;13* | 71 | additive | 0, 3 |
| 2 | $0^s$ 0;57 | 2 | 0; 5 | 32 | 1;16 | 72 | | 0, 2* |
| 3 | $0^s$ 1;54 | 3 | 0; 7 | 33 | 1;18* | 73 | | 0, 2* |
| 4 | $0^s$ 2;51 | 4 | 0; 9[1] | 34 | 1;21 | 74 | | 0, 2* |
| 5 | $0^s$ 3;49 | 5 | 0;12 | 35 | 1;23 | 75 | | 0, 2 |
| 6 | $0^s$ 4;46 | 6 | 0;14 | 36 | 1;26 | 76 | | 0, 2 |
| 7 | $0^s$ 5;43 | 7 | 0;16[2] | 37 | 1;28 | 77 | | 0, 2 |
| 8 | $0^s$ 6;40 | 8 | 0;19 | 38 | 1;30 | 78 | | 0, 2 |
| 9 | $0^s$ 7;37 | 9 | 0;21 | 39 | 1;33 | 79 | | 0, 2 |
| 10 | $0^s$ 8;34[3] | 10 | 0;24 | 40 | 1;35 | 80 | | 0, 2 |
| 11 | $0^s$ 9;31 | 11 | 0;26 | 41 | 1;37* | 81 | | 0, 1 |
| 12 | $0^s$ 10;28 | 12 | 0;29 | 42 | 1;39* | 82 | | 0, 1 |
| 13 | $0^s$ 11;26 | 13 | 0;31 | 43 | 1;42 | 83 | | 0, 1 |
| 14 | $0^s$ 12;23 | 14 | 0;33 | 44 | 1;44* | 84 | | 0, 1 |
| 15 | $0^s$ 13;20 | 15 | 0;36 | 45 | 1;47 | 85 | | 0, 1 |
| 16 | $0^s$ 14;17 | 16 | 0;38 | 46 | 1;49[4]* | 86 | | 0, 1[5] |
| 17 | $0^s$ 15;14 | 17 | 0;40 | 47 | 1;51* | 87 | | 0, 0 |
| 18 | $0^s$ 16;11 | 18 | 0;42* | 48 | 1;54 | 88 | | 0, 0 |
| 19 | $0^s$ 17; 8 | 19 | 0;45 | 49 | 1;56* | 89 | | 0, 0 |
| 20 | $0^s$ 18; 5[6] | 20 | 0;47* | 50 | 1;59 | 90 | | 0, 0 |
| 21 | $0^s$ 19; 3 | 21 | 0;50 | 51 | 2; 1 | 91 | subtractive | 0, 0 |
| 22 | $0^s$ 20; 0 | 22 | 0;52 | 52 | 2; 3* | 92 | | 0, 0 |
| 23 | $0^s$ 20;57 | 23 | 0;54* | 53 | 2; 6 | 93 | | 0, 0 |
| 24 | $0^s$ 21;54 | 24 | 0;57 | 54 | 2; 8* | 94 | | 0, 0* |
| 25 | $0^s$ 22;51 | 25 | 0;59* | 55 | 2;10* | 95 | | 0, 1[7] |
| 26 | $0^s$ 23;48 | 26 | 1; 2 | 56 | 2;13 | 96 | | 0, 1 |
| 27 | $0^s$ 24;45 | 27 | 1; 4 | 57 | 2;15* | 97 | | 0, 1 |
| 28 | $0^s$ 25;42 | 28 | 1; 6* | 58 | 2;18 | 98 | | 0, 1 |
| 29 | $0^s$ 26;40 | 29 | 1; 9 | 59 | 2;20 | 99 | | 0, 1 |
| 30 | $0^s$ 27;37 | 30 | 1;11 | 60 | 2;23 | 100 | | 0, 1* |

* In **YLB** the values for 4, 7, 18, 20, 23, 25, 28, 31, 33, 41, 42, 44, 46, 47, 49, 52, 54, 55, and 57 hours and for geographical longitudes 72–74, 94, and 100 are one second larger. [1] **B**+ 10′ corrected to 9′ [2] **B**+ 17′ corrected to 16′ [3] **Y** 36′ [4] +**$C_2$** 46–49 hours: dam. [5] **F** 0′ [6] **$CC_1C_2$** 6′ [7] **F** 0′.

**Table 24: First equation for Saturn (first half)**
*Sources*: **F** fols 57v-58r, **H** fols 44v–45r, **C** fols 58v–59r, **$C_1$** fols 24v–25r, **$C_2$** fols 44v and 37r, **Y** fols 276v–277r, **L** fols 47v–48r, **B** pp. 90–91.

First Equation for Saturn
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 0<br>° ′ | differences ′ | 1<br>° ′ | differences ′ | mean distance 19°<br>2<br>° ′ | differences ′ | 3<br>° ′ | differences ′ [1] | 4<br>° ′ | differences ′ | nearest distance 16°<br>5<br>° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 5;30[2] | 6 | 2;40 | 5 | 0;48 | 2 | 0;36 | 1 | 2; 7[3] | 5 | 5; 7 | 7 |
| 1 | 5;24 | 6 | 2;35 | 5 | 0;46 | 2 | 0;38 | 2 | 2;12 | 5 | 5;14 | 7 |
| 2 | 5;17[4] | 7 | 2;30 | 5 | 0;44 | 2 | 0;39 | 1 | 2;17 | 5 | 5;21 | 7 |
| 3 | 5;11 | 6 | 2;25 | 5 | 0;42 | 2 | 0;41 | 2 | 2;22[5] | 5 | 5;28 | 7 |
| 4 | 5; 5 | 6 | 2;21 | 4 | 0;41 | 1 | 0;43[6] | 2 | 2;27 | 5 | 5;35 | 7 |
| 5 | 4;59 | 6 | 2;16 | 5 | 0;39 | 2 | 0;44 | 1 | 2;32 | 5 | 5;42 | 7 |
| 6 | 4;53 | 6[7] | 2;12 | 4 | 0;38 | 1 | 0;46 | 2 | 2;37 | 5 | 5;49 | 7 |
| 7 | 4;47[8] | 6[9] | 2; 8 | 4 | 0;37[10] | 1 | 0;48 | 2 | 2;43 | 6 | 5;56 | 7 |
| 8 | 4;41 | 6 | 2; 3 | 5 | 0;35 | 2 | 0;50 | 2 | 2;48 | 5[11] | 6; 3 | 7 |
| 9 | 4;35 | 6 | 1;59 | 4 | 0;34 | 1 | 0;52[12]↓ | 2 | 2;54 | 6 | 6;10 | 7 |
| 10 | 4;29 | 6 | 1;55 | 4 | 0;33 | 1 | 0;54 | 2 | 3; 0 | 6 | 6;17 | 7 |
| 11 | 4;23 | 6 | 1;51 | 4 | 0;32 | 1 | 0;56 | 2 | 3; 6 | 6 | 6;24 | 7 |
| 12 | 4;17 | 6[13] | 1;47 | 4 | 0;31 | 1 | 0;59 | 3 | 3;12 | 6 | 6;31 | 7 |
| 13 | 4;11[14] | 6 | 1;43 | 4 | 0;30 | 1 | 1; 2 | 3 | 3;18 | 6 | 6;38 | 7 |
| 14 | 4; 5[15] | 6 | 1;39 | 4 | 0;29 | 1 | 1; 5 | 3 | 3;24 | 6 | 6;45 | 7 |
| 15 | 3;59 | 6 | 1;35 | 4 | 0;29 | 0 | 1; 8 | 3 | 3;30 | 6 | 6;52 | 7 |
| 16 | 3;54[16] | 5 | 1;31 | 4 | 0;28 | 1 | 1;11 | 3 | 3;36[17] | 6 | 7; 0 | 8 |
| 17 | 3;48 | 6 | 1;27 | 4 | 0;28 | 0 | 1;14 | 3 | 3;42 | 6 | 7; 8 | 8 |
| 18 | 3;42 | 6 | 1;23 | 4 | 0;28 | 0 | 1;17[18]↓ | 3 | 3;48 | 6 | 7;15 | 7 |
| 19 | 3;37 | 5 | 1;19 | 4 | 0;28 | 0 | 1;20[19] | 3 | 3;55 | 7[20] | 7;22 | 7 |
| 20 | 3;31 | 6 | 1;16[21] | 3 | 0;28 | 0 | 1;24[22]⊥ | 4 | 4; 1 | 6 | 7;29 | 7 |
| 21 | 3;26 | 5 | 1;13 | 3[23] | 0;28 | 0 | 1;28[24] | 4 | 4; 7 | 6 | 7;36 | 7 |
| 22 | 3;21 | 5 | 1;10 | 3 | 0;29 | 1 | 1;32 | 4 | 4;14 | 7 | 7;43 | 7 |
| 23 | 3;15[25] | 6 | 1; 7 | 3 | 0;29 | 0 | 1;36 | 4 | 4;20 | 6 | 7;50 | 7 |
| 24 | 3;10 | 5 | 1; 4 | 3 | 0;30 | 1 | 1;40 | 4 | 4;26 | 6 | 7;57[26] | 7 |
| 25 | 3; 5 | 5 | 1; 1 | 3 | 0;31 | 1 | 1;44 | 4 | 4;33 | 7[27] | 8; 4[28] | 7 |
| 26 | 3; 0 | 5 | 0;58 | 3 | 0;32 | 1 | 1;48 | 4 | 4;39 | 6 | 8;11 | 7 |
| 27 | 2;55 | 5 | 0;55 | 3 | 0;33 | 1 | 1;52 | 4[29] | 4;46 | 7 | 8;18 | 7 |
| 28 | 2;50 | 5 | 0;53 | 2 | 0;34 | 1 | 1;57⊥ | 5 | 4;53 | 7 | 8;25[30] | 7 |
| 29 | 2;45 | 5 | 0;50 | 3 | 0;35 | 1 | 2; 2 | 5 | 5; 0 | 7 | 8;32 | 7 |

[1] **L** om. entire column  [2] **L** 4′  [3] **C** 6′ **B** 4′  [4] **Y** 12′  [5] **$C_1$** 24′ **$C_2$** 27′  [6] **L** 48′  [7] **C** 7′
[8] **$C_1$** 46′  [9] **L** 7′  [10] **$C_1$** 36′  [11] **C** 6′  [12] **L** 3ˢ 9–28°: slide[+1]  [13] **L** 7′  [14] **Y** 51′  [15] **C** 17′
[16] **C** 14′  [17] **C** 37′  [18] +**C** 3ˢ 18–20°: slide[−1 col.]  [19] **C**+ 15′  [20] **C** 6′  [21] **C** 17′  [22] +**Y** 25′
[23] **L** 4′  [24] +**$C_1$** 29′  [25] **L** 25′  [26] **$C_2$** 17′  [27] **C** 6′  [28] **F** 3′  [29] +**C** 5′  [30] **C** 28′.

**Table 24: First equation for Saturn (second half)**
*Sources*: **F** fols 57v–58r, **H** fols 44v–45r, **C** fols 58v–59r, $\mathbf{C_1}$ fols 24v–25r, $\mathbf{C_2}$ fols 44v and 37r, **Y** fols 276v–277r, **L** fols 47v–48r, **B** pp. 90–91.

First Equation for Saturn
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 6 (° ′) | differences ′ | 7 (° ′) | differences ′ | mean distance 13° 8 (° ′) | differences ′ | 9 (° ′) | differences ′ | 10 (° ′) | differences ′ | furthest distance 16° 11 (° ′) | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 8;39 | 7 | 11;43 | 5 | 13;21 | 2 | 13;16 | 2 | 11;30 | 5 | 8;43 | 6 |
| 1 | 8;46 | 7 | 11;48 | 5 | 13;22 | 1 | 13;14 | 2 | 11;25 | 5 | 8;36 | 7 |
| 2 | 8;53 | 7 | 11;53 | 5 | 13;24 | 2 | 13;12 | 2 | 11;20 | 5 | 8;30 | 6 |
| 3 | 9; 0 | 7 | 11;58 | 5 | 13;25[1] | 1 | 13;10 | 2 | 11;15 | 5 | 8;24 | 6 |
| 4 | 9; 7[2] | 7 | 12; 3 | 5 | 13;26[3] | 1 | 13; 7 | 3 | 11;10 | 5 | 8;17 | 7 |
| 5 | 9;14 | 7 | 12; 8 | 5 | 13;27 | 1 | 13; 5 | 2 | 11; 5 | 5 | 8;11 | 6 |
| 6 | 9;21 | 7 | 12;12 | 4 | 13;28 | 1 | 13; 2 | 3 | 11; 0 | 5 | 8; 5 | 6 |
| 7 | 9;27 | 6 | 12;16 | 4 | 13;29 | 1 | 12;59 | 3 | 10;55 | 5 | 7;58 | 7 |
| 8 | 9;34 | 7 | 12;20 | 4 | 13;30 | 1 | 12;56 | 3 | 10;50 | 5 | 7;52 | 6 |
| 9 | 9;40 | 6 | 12;24 | 4 | 13;31 | 1 | 12;53 | 3 | 10;45 | 5 | 7;46 | 6 |
| 10 | 9;46 | 6 | 12;28 | 4 | 13;31 | 0 | 12;50 | 3 | 10;39 | 6 | 7;40 | 6 |
| 11 | 9;53 | 7 | 12;32 | 4 | 13;32 | 1 | 12;47 | 3 | 10;34 | 5 | 7;33 | 7 |
| 12 | 9;59 | 6 | 12;36 | 4 | 13;32 | 0 | 12;44 | 3 | 10;29 | 5[4] | 7;27[5] | 6 |
| 13 | 10; 5 | 6 | 12;40 | 4 | 13;32 | 0 | 12;41 | 3 | 10;23 | 6 | 7;20 | 7 |
| 14 | 10;12[6] | 7[7] | 12;43 | 3 | 13;32 | 0 | 12;37 | 4 | 10;18 | 5 | 7;14 | 6 |
| 15 | 10;18 | 6 | 12;46 | 3 | 13;32 | 0 | 12;33 | 4[8] | 10;12 | 6 | 7; 7 | 7 |
| 16 | 10;24 | 6 | 12;49 | 3 | 13;32 | 0 | 12;29 | 4[9] | 10; 6 | 6 | 7; 0 | 7 |
| 17 | 10;30 | 6 | 12;52 | 3 | 13;31 | 1 | 12;25 | 4[10] | 10; 1 | 5 | 6;53 | 7 |
| 18 | 10;36 | 6 | 12;55[11] | 3 | 13;31 | 0 | 12;21 | 4 | 9;55 | 6 | 6;46 | 7 |
| 19 | 10;42 | 6 | 12;58 | 3 | 13;30 | 1 | 12;17 | 4 | 9;49 | 6 | 6;40 | 6 |
| 20 | 10;48 | 6 | 13; 1 | 3 | 13;29 | 1 | 12;13 | 4 | 9;43 | 6 | 6;33[12] | 7 |
| 21 | 10;54 | 6 | 13; 4 | 3 | 13;28 | 1 | 12; 9 | 4 | 9;37 | 6 | 6;27[13] | 6[14]↓ |
| 22 | 11; 0 | 6 | 13; 6 | 2 | 13;27 | 1 | 12; 5 | 4 | 9;31 | 6 | 6;20 | 7 |
| 23 | 11; 6 | 6 | 13; 8 | 2 | 13;26 | 1 | 12; 1 | 4 | 9;25 | 6 | 6;14 | 6⊥ |
| 24 | 11;12 | 6 | 13;10 | 2 | 13;25 | 1 | 11;57 | 4 | 9;19 | 6 | 6; 8 | 6 |
| 25 | 11;17 | 5 | 13;12 | 2 | 13;23 | 2 | 11;52 | 5 | 9;13 | 6 | 6; 2 | 6 |
| 26 | 11;23 | 6 | 13;14 | 2 | 13;22 | 1[15]↓ | 11;48 | 4 | 9; 7 | 6 | 5;55 | 7 |
| 27 | 11;28 | 5 | 13;16 | 2 | 13;21 | 1⊥ | 11;44 | 4 | 9; 1 | 6 | 5;49 | 6 |
| 28 | 11;33 | 5 | 13;17 | 1 | 13;19 | 2 | 11;39 | 5 | 8;55 | 6 | 5;43 | 6 |
| 29 | 11;38 | 5 | 13;19 | 2 | 13;18 | 1 | 11;35 | 4 | 8;49 | 6 | 5;36 | 7 |

[1] **C** 26′ [2] **C** 50′ [3] **C** 25′ [4] **Y** 6′ [5] $\mathbf{C_1}$ 24′ [6] **L** 52′ [7] **C** 6′ [8] **L** 3′ [9] **Y** 5′ [10] **Y** om. (hence incorrectly read as 5′) [11] **L** 56′ [12] **C** 13′ [13] $\mathbf{C_1}$ 24′ [14] **C** $11^s$ 21–23°: block transposition [15] **Y** $8^s$ 26–27: om. (hence incorrectly read as 2′).

**Table 24: Second equation for Saturn (first half)**

*Sources*: **F** fols 58v–59r, **H** fols 45v–46r, **C** fols 59v–60r, $\mathbf{C_1}$ fols 25v–26r, $\mathbf{C_2}$ fols 37v–38r, **Y** fols 277v–278r, **L** fols 48v–49r, **B** pp. 92–93.

Second Equation for Saturn
to be equated and added to the centrum with the apogee

| degrees of the epicycle | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | **2** ° ′ | differences ′ | mean distance 3° **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0[1] | 7; 0 | 6 | 9;50 | 5 | 12; 4 | 3 | 13;11 | 0 | 12;40 | 3 | 10;25 | 6 |
| 1[2] | 7; 6 | 6 | 9;55 | 5 | 12; 8 | 4 | 13;12 | 1 | 12;37 | 3 | 10;19 | 6 |
| 2[3]↓ | 7;12 | 6 | 10; 0 | 5 | 12;12 | 4 | 13;12[4] | 0 | 12;34 | 3 | 10;13 | 6 |
| 3 | 7;18 | 6 | 10; 5 | 5 | 12;15 | 3 | 13;13 | 1 | 12;31 | 3 | 10; 7 | 6 |
| 4 | 7;24 | 6 | 10;10 | 5 | 12;19 | 4 | 13;13 | 0 | 12;28 | 3 | 10; 1 | 6 |
| 5 | 7;30 | 6 | 10;15 | 5 | 12;22 | 3 | 13;13 | 0 | 12;25 | 3 | 9;55 | 6 |
| 6 | 7;36 | 6 | 10;20[5] | 5 | 12;25 | 3 | 13;13 | 0 | 12;21 | 4 | 9;48 | 7 |
| 7 | 7;42 | 6 | 10;25[6] | 5 | 12;28 | 3 | 13;13 | 0 | 12;18 | 3 | 9;42 | 6 |
| 8 | 7;48 | 6 | 10;30 | 5 | 12;31 | 3 | 13;13 | 0 | 12;14 | 4 | 9;35 | 7 |
| 9 | 7;54 | 6 | 10;35 | 5 | 12;34 | 3 | 13;12 | 1 | 12;10 | 4 | 9;29 | 6 |
| 10 | 8; 0 | 6 | 10;40 | 5 | 12;37 | 3 | 13;12 | 0 | 12; 6 | 4 | 9;23 | 6 |
| 11 | 8; 6 | 6 | 10;45 | 5 | 12;40 | 3 | 13;11 | 1[7] | 12; 2 | 4 | 9;16 | 7 |
| 12 | 8;11[8] | 5 | 10;49 | 4 | 12;42 | 2 | 13;11 | 0 | 11;58 | 4 | 9; 9 | 7 |
| 13 | 8;17[9] | 6 | 10;54 | 5 | 12;45 | 3 | 13;10 | 1 | 11;54 | 4 | 9; 2 | 7 |
| 14 | 8;23[10] | 6 | 10;59 | 5 | 12;47 | 2 | 13; 9 | 1 | 11;50 | 4 | 8;55 | 7 |
| 15 | 8;28 | 5 | 11; 3 | 4 | 12;49 | 2 | 13; 8 | 1 | 11;45[11] | 5 | 8;48 | 7 |
| 16 | 8;34[12] | 6 | 11; 8 | 5 | 12;51 | 2 | 13; 7 | 1 | 11;41 | 4 | 8;41 | 7 |
| 17 | 8;40 | 6 | 11;13 | 5 | 12;53 | 2 | 13; 6 | 1 | 11;36 | 5 | 8;34 | 7 |
| 18 | 8;45 | 5 | 11;17 | 4 | 12;55 | 2 | 13; 5 | 1 | 11;31 | 5 | 8;27 | 7 |
| 19 | 8;51[13] | 6 | 11;22 | 5 | 12;57 | 2 | 13; 4 | 1 | 11;26 | 5 | 8;20 | 7 |
| 20 | 8;57[14] | 6 | 11;26 | 4 | 12;59 | 2 | 13; 2 | 2 | 11;21 | 5 | 8;13 | 7 |
| 21 | **eastern vis** 9; 2 | 5 | 11;30 | 4 | 13; 0 | 1 | 13; 0 | 2 | 11;16[15] | 5 | 8; 6 | 7 |
| 22 | 9; 8 | 6 | 11;34 | 4 | 13; 2 | 2[16] | 12;59 | 1 | 11;11 | 5 | 7;59 | 7 |
| 23 | 9;13 | 5 | 11;38 | 4 | 13; 4 | 2 | 12;57 | 2 | 11; 6 | 5 | 7;52 | 7 |
| 24 | 9;18 | 5[17] | 11;42 | 4 | 13; 5 | 1 | 12;55 | 2 | 11; 0 | 6 | 7;45 | 7 |
| 25 | 9;24 | 6 | 11;46 | 4 | 13; 7 | 2 | **stationary** 12;53[18] **retrograde** | 2 | 10;55[19] | 5 | 7;38 | 7 |
| 26 | 9;29[20] | 5 | 11;50 | 4 | 13; 8 | 1 | 12;50 | 3 | 10;49 | 6 | 7;31 | 7 |
| 27 | 9;34[21] | 5 | 11;53 | 3[22] | 13; 9 | 1 | 12;48 | 2 | 10;43 | 6 | 7;23 | 8 |
| 28 | 9;40 | 6 | 11;57[23] | 4 | 13;10 | 1 | 12;46 | 2 | 10;37 | 6 | 7;16 | 7 |
| 29⊥ | 9;45 | 5 | 12; 1 | 4 | 13;11 | 1 | 12;43 | 3 | 10;31 | 6 | 7; 8 | 8 |

**L** omits the values from all six columns of differences (but not the headers). [1] **L** om. (beginning of slide) [2] **L** om. [3] **L** arguments 2–29°: slide[−2] (arguments 28 and 29 are written in two otherwise empty rows at the bottom of the table) [4] **C** 13′ [5] **C** 25′ [6] **C** 20′ [7] **C** 0′ [8] $\mathbf{C_1}$ 14′ $\mathbf{C_2}$ 17′ [9] **C** 15′ [10] **C** 20′ [11] **L** 49′ [12] **C** 35′ [13] **C** 11′ [14] **C** 17′ [15] **C** 36′ [16] **B** corrected to 1′ in black [17] **Y** 6′ [18] **F** 13′ [19] **C** 15′ [20] **L** 26′ [21] **L** 35′ [22] **C** 4′ [23] $\mathbf{C_1}$ 56′.

**Table 24: Second equation for Saturn (second half)**
*Sources*: **F** fols 58v-59r, **H** fols 45v–46r, **C** fols 59v–60r, $\mathbf{C_1}$ fols 25v–26r, $\mathbf{C_2}$ fols 37v–38r, **Y** fols 277v–278r, **L** fols 48v–49r, **B** pp. 92–93.

Second Equation for Saturn
to be equated and added to the centrum with the apogee

| degrees of the epicycle | nearest distance 0° **6** ° ′ | differences ′ | **7** ° ′ | differences ′ | mean distance 27° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7; 0 | 8 | 3;35 | 6 | 1;20 | 3 | 0;49 | 1 | 1;56 | 4[1] | 4;10 | 5 |
| 1 | 6;52[2] | 8 | 3;29 | 6 | 1;17 | 3 | 0;49 | 0 | 1;59 | 3 | 4;15 | 5 |
| 2 | 6;44 | 8 | 3;23 | 6 | 1;14 | 3 | 0;50 | 1 | 2; 3 | 4 | 4;20 | 5 |
| 3 | 6;37 | 7 | 3;17 | 6 | 1;12 | 2 | 0;51 | 1 | 2; 7 | 4 | 4;26 | 6 |
| 4 | 6;29 | 8 | 3;11 | 6 | stationary 1;10 | 2 | 0;52 | 1 | 2;10 | 3 | 4;31 | 5 |
| 5 | 6;22 | 7 | 3; 5 | 6 | 1; 7 | 3 | 0;53 | 1 | 2;14 | 4 | 4;36 | 5 |
| 6 | 6;15 | 7 | 3; 0 | 5 | progressive 1; 5[3] | 2 | 0;55 | 2↓[4] | 2;18 | 4 | 4;42 | 6 |
| 7 | 6; 8 | 7 | 2;54 | 6 | 1; 3 | 2 | 0;56 | 1 | 2;22 | 4 | 4;47 | 5 |
| 8 | 6; 1 | 7 | 2;49 | 5 | 1; 1 | 2 | 0;58 | 2⊥ | 2;26 | 4 | western dis 4;52 | 5 |
| 9 | 5;54 | 7 | 2;44 | 5 | 1; 0 | 1 | 1; 0 | 2 | 2;30[5] | 4 | 4;58 | 6 |
| 10 | 5;47 | 7 | 2;39 | 5 | 0;58 | 2 | 1; 1 | 1 | 2;34 | 4 | 5; 3 | 5 |
| 11 | 5;40 | 7 | 2;34 | 5 | 0;56 | 2 | 1; 3 | 2 | 2;38 | 4 | 5; 9 | 6 |
| 12 | 5;33 | 7 | 2;29 | 5 | 0;55 | 1 | 1; 5 | 2 | 2;43 | 5 | 5;15 | 6 |
| 13 | 5;26 | 7 | 2;24 | 5 | 0;54 | 1 | 1; 7 | 2 | 2;47 | 4 | 5;20 | 5 |
| 14 | 5;19 | 7 | 2;19 | 5 | 0;53 | 1 | 1; 9 | 2 | 2;52 | 5 | 5;26 | 6 |
| 15 | 5;12 | 7 | 2;15 | 4 | 0;52 | 1 | 1;11 | 2 | 2;57 | 5 | 5;32 | 6 |
| 16 | 5; 5 | 7 | 2;10 | 5 | 0;51 | 1 | 1;13 | 2 | 3; 1 | 4 | 5;37 | 5 |
| 17 | 4;58[6] | 7 | 2; 6 | 4 | 0;50 | 1 | 1;15 | 2 | 3; 6 | 5 | 5;43 | 6 |
| 18 | 4;51[7] | 7 | 2; 2 | 4 | 0;49 | 1 | 1;18 | 3 | 3;11 | 5 | 5;49 | 6 |
| 19 | 4;44 | 7 | 1;58 | 4 | 0;49↓[8] | 0↓[9] | 1;20 | 2 | 3;15 | 4 | 5;54 | 5 |
| 20 | 4;37 | 7 | 1;54 | 4 | 0;48 | 1 | 1;23 | 3 | 3;20 | 5 | 6; 0 | 6 |
| 21 | 4;31 | 6 | 1;50 | 4 | 0;48⊥ | 0 | 1;26 | 3 | 3;25 | 5 | 6; 6 | 6 |
| 22 | 4;25 | 6[10] | 1;46 | 4[11] | 0;47 | 1⊥ | 1;29 | 3 | 3;30 | 5 | 6;12 | 6 |
| 23 | 4;18 | 7[12] | 1;42 | 4 | 0;47 | 0 | 1;32 | 3 | 3;35 | 5 | 6;18[13] | 6 |
| 24 | 4;12 | 6 | 1;39 | 3 | 0;47 | 0 | 1;35 | 3 | 3;40 | 5 | 6;24 | 6 |
| 25 | 4; 5 | 7 | 1;35 | 4[14] | 0;47 | 0 | 1;38 | 3 | 3;45 | 5 | 6;30 | 6 |
| 26 | 3;59 | 6 | 1;32 | 3 | 0;47 | 0 | 1;41 | 3 | 3;50 | 5 | 6;36 | 6 |
| 27 | 3;53 | 6 | 1;29 | 3 | 0;47 | 0 | 1;45 | 4[15] | 3;55[16] | 5 | 6;42 | 6 |
| 28 | 3;47 | 6 | 1;26 | 3 | 0;48 | 1[17] | 1;48 | 3 | 4; 0 | 5 | 6;48 | 6 |
| 29 | 3;41 | 6 | 1;23 | 3 | 0;48 | 0 | 1;52 | 4 | 4; 5 | 5 | 6;54 | 6 |

[1] **Y** 3′ [2] **C** 12′ [3] **C** 0′ [4] **C** 9ˢ 6–8°: block transposition [5] **C** 34′ [6] **C** 18′ [7] **C** 11′ [8] **L** 8ˢ 19–21°: slide[+1] [9] **L** 8ˢ 19–22°: slide[+1] (in correspondence with the equations) [10] **C** 7′ [11] **Y** 3′ [12] **C** 6′ [13] $\mathbf{C_2}$ 48′ [14] **F** 3′ **Y** om. (hence incorrectly read as 3′) [15] **F** 3′ [16] **C** 15′ [17] **C** 0′.

**Table 24: Variation of the nearest distance for Saturn**
*Sources*: **F** fol. 59v, **H** fol. 46v, **C** fol. 60v, **$C_1$** fol. 26v, **$C_2$** fol. 38v, **Y** fol. 278v, **L** fol. 49v, **B** p. 94.

Variation of the Nearest Distance for Saturn

| corrected centrum | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the furthest distance | 0; 1 | 0;11 | 0;20 | 0;23 | 0;17[1] | 0; 8 |
| 1 | | 0; 2 | 0;12 | 0;21 | 0;23 | 0;16 | 0; 7 |
| 2 | | 0; 2 | 0;12 | 0;21 | 0;23 | 0;16 | 0; 7 |
| 3 | | 0; 2 | 0;12 | 0;21 | 0;23 | 0;16 | 0; 7 |
| 4 | | 0; 3 | 0;13 | 0;22 | 0;23 | 0;15 | 0; 6 |
| 5 | | 0; 3 | 0;13 | 0;22 | 0;23 | 0;15 | 0; 6 |
| 6 | | 0; 3 | 0;13 | 0;22 | 0;23 | 0;15 | 0; 5 |
| 7 | | 0; 4 | 0;14 | 0;22 | 0;23 | 0;14 | 0; 5 |
| 8 | | 0; 4 | 0;14 | 0;22 | 0;22 | 0;14 | 0; 4 |
| 9 | | 0; 5 | 0;14 | 0;23 | 0;22 | 0;14 | 0; 4 |
| 10 | | 0; 5 | 0;15 | 0;23 | 0;22 | 0;13 | 0; 3 |
| 11 | | 0; 6 | 0;15 | 0;23 | 0;22 | 0;13 | 0; 3 |
| 12 | | 0; 6 | 0;15 | 0;23 | 0;22 | 0;13 | 0; 3 |
| 13 | | 0; 7 | 0;16 | 0;23 | 0;21 | 0;12 | 0; 2 |
| 14 | | 0; 7 | 0;16 | 0;23 | 0;21 | 0;12 | 0; 2 |
| 15 | | 0; 7 | 0;16 ↕[2] | 0;23 | 0;21 | 0;12 | 0; 2 |
| 16 | | 0; 8 | 0;17 | 0;23 | 0;20 | 0;11 | 0; 1 |
| 17 | | 0; 8 | 0;17 | 0;23 | 0;20 | 0;11 | 0; 1 |
| 18 | | 0; 8 | 0;17⊥ | 0;23 | 0;20 | 0;11 | 0; 0 |
| 19 | | 0; 9 | 0;18 | 0;23 | 0;20 | 0;11[3] | From the furthest distance |
| 20 | | 0; 9 | 0;18 ↕[4] | 0;23 | 0;19 | 0;10 | |
| 21 | | 0; 9 | 0;18⊥ | 0;23 | 0;19 | 0;10 | |
| 22 | | 0;10 | 0;19 | 0;23 | 0;19 | 0;10 | |
| 23 | | 0;10 | 0;19 | 0;23 | 0;19 | 0;10 | |
| 24 | | 0;10 | 0;19 | 0;23 | 0;19 | 0;10 | |
| 25 | | 0;10 | 0;19 | 0;23 | 0;18 | 0; 9 | |
| 26 | | 0;10 | 0;19 | 0;23 | 0;18 | 0; 9 | |
| 27 | [5] | 0;11 | 0;20 | 0;23 | 0;18 | 0; 9 | |
| 28 | 0; 0 | 0;11 | 0;20 | 0;23 | 0;17 | 0; 8 | |
| 29 | 0; 1 | 0;11 | 0;20 | 0;23 | 0;17 | 0; 8 | |

[1] **B** 16′ [2] **F** $4^s$ 15–18°: slide[+1] [3] **F** 10′ [4] **F** $4^s$ 20–21°: 19′ [5] **H** 0;0.

**Table 24: Interpolation minutes (nearest distance) for Saturn**

*Sources*: **F** fol. 59v, **H** fol. 47r, **C** fol. 60v, **C$_1$** fol. 26v, **C$_2$** fol. 38v, **Y** fol. 278v, **L** fol. 49v, **B** p. 94.

| Minutes of Proportions for Saturn | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **0** | **1** | **2** | **3** | **4** | **5** | |
| 0 | 0 | 27 | 49 | 60 | 55 | 33[1] | 30 |
| 1 | 1 | 28 | 49 | 60 | 55 | 32[2] | 29 |
| 2 | 2 | 29 | 50 | 60 | 54 | 31 | 28 |
| 3 | 3 | 29 | 50 | 60 | 54 | 30 | 27 |
| 4 | 4 | 30 | 51 | 60 | 53 | 29 | 26 |
| 5 | 5 | 31 | 51 | 60 | 53 | 28 | 25 |
| 6 | 6 | 32 | 52 | 60 | 52 | 27 | 24 |
| 7 | 7 | 33 | 52 | 60 | 51 | 26 | 23 |
| 8 | 8 | 34 | 53 | 60 | 51 | 25 | 22 |
| 9 | 8[3] | 34 | 53 | 60 | 50 | 24 | 21 |
| 10 | 9 | 35 | 54 | 60 | 49 | 23 | 20 |
| 11 | 10 | 36 | 54 | 60 | 49 | 22[4] | 19 |
| 12 | 11 | 37 | 55 | 60 | 48 | 21 | 18 |
| 13 | 12 | 38 | 55 | 60 | 47 | 20 | 17 |
| 14 | 13 | 38[5] | 56 | 60 | 47 | 19 | 16 |
| 15 | 14 | 39 | 56 | 59 | 46 | 17[6] | 15 |
| 16 | 15 | 40 | 56 | 59 | 45 | 16 | 14 |
| 17 | 16 | 40 | 57 | 59 | 45 | 15 | 13 |
| 18 | 17 | 41 | 57 | 59 | 44 | 14 | 12 |
| 19 | 18 | 42 | 57 | 59 | 43 | 13 | 11 |
| 20 | 19 | 42 | 58 | 58 | 42 | 12 | 10 |
| 21 | 19 | 43 | 58 | 58 | 41 | 10 | 9 |
| 22 | 20 | 44 | 58 | 58 | 41 | 9 | 8 |
| 23 | 21 | 44 | 59 | 57 | 40 | 8 | 7 |
| 24 | 22 | 45 | 59 | 57 | 39 | 7 | 6 |
| 25 | 23 | 46 | 59 | 57 | 38 | 6 | 5 |
| 26 | 24 | 46 | 59 | 56[7] | 37 | 5 | 4 |
| 27 | 25[8] | 47 | 59 | 56 | 36 | 3 | 3 |
| 28 | 25 | 48 | 60 | 56 | 35 | 2 | 2 |
| 29 | 26 | 48 | 60 | 55 | 34 | 1 | 1 |
| parts of the epicycle | **11** | **10** | **9** | **8** | **7** | **6** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **C** 32′ [2] **C** 3′ [3] **F** 9′ [4] **C$_2$** 24′ [5] **C$_1$C$_2$** 39′ [6] **C** 18′ corrected to 17′ **L** 18′ [7] **C$_1$** 57′ [8] **YLB** 24′.

**Table 24: Variation of the furthest distance for Saturn**
*Sources*: **F** fol. 60r, **H** fol. 47v, **C** fol. 61r, **$C_1$** fol. 27r, **$C_2$** fol. 39r, **Y** fol. 279r, **L** fol. 50r, **B** p. 95.

| Variation of the Furthest Distance for Saturn | | | | | | | |
|---|---|---|---|---|---|---|---|
| corrected centrum | **8** | **9** | **10** | **11** | **0** | **1** | **2** |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | ↓[1] From the nearest distance | 0; 4 | 0;14 | 0;18 | 0;20 | 0;17 | 0;10[2] |
| 1 | | 0; 5 | 0;14 | 0;18 | 0;20 | 0;17 | 0; 9 |
| 2 | | 0; 5 | 0;14 | 0;18 | 0;20 | 0;16 | 0; 9 |
| 3 | | 0; 5 | 0;14 | 0;19 | 0;20 | 0;16 | 0; 9 |
| 4 | | 0; 6 | 0;15 | 0;19 | 0;20 | 0;16 | 0; 8 |
| 5 | | 0; 6 | 0;15 | 0;19 | 0;20 | 0;16 | 0; 8 |
| 6 | | 0; 6 | 0;15 | 0;20[3] | 0;20 | 0;16 | 0; 8 |
| 7 | | 0; 7 | 0;15 | 0;20 | 0;20 | 0;16 | 0; 7 |
| 8 | | 0; 7 | 0;16 | 0;20 | 0;20 | 0;16 | 0; 7 |
| 9 | | 0; 7 | 0;16 | 0;20 | 0;19 | 0;15 | 0; 7 |
| 10 | | 0; 8 | 0;16 | 0;20 | 0;19 | 0;15 | 0; 6 |
| 11 | | 0; 8 | 0;16 | 0;20 | 0;19 | 0;15 | 0; 6 |
| 12 | | 0; 8 | 0;16 | 0;20 | 0;19 | 0;15 | 0; 6 |
| 13 | | 0; 9 | 0;16 | 0;20 | 0;19 | 0;14 | 0; 5 |
| 14 | | 0; 9 | 0;16 | 0;20 | 0;18[4] | 0;14 | 0; 5 |
| 15 | | 0; 9 | 0;17 | 0;20 | 0;18 | 0;14 | 0; 5 |
| 16 | | 0;10 | 0;17 | 0;20 | 0;18 | 0;14 | 0; 4 |
| 17 | | 0;10 | 0;17 | 0;20 | 0;18 | 0;13[5] | 0; 4 |
| 18[6]↓ | [7] | 0;10 | 0;17 | 0;20 | 0;18 | 0;13 | 0; 4 |
| 19 | 0; 0[8] | 0;11 | 0;17 | 0;20 | 0;18 | 0;13 | 0; 3 |
| 20 | 0; 1 | 0;11 | 0;17 | 0;20 | 0;18 | 0;13 | 0; 3 |
| 21 | 0; 1 | 0;11 | 0;17 | 0;20 | 0;18 | 0;12 | 0; 3 |
| 22 | 0; 2[9] | 0;12 | 0;17 | 0;20 | 0;17[10] | 0;12 | 0; 2 |
| 23 | 0; 2[11] | 0;12 | 0;17 | 0;20 | 0;17[12] | 0;12 | 0; 2 |
| 24 | 0; 2 | 0;12 | 0;17 | 0;20 | 0;17 | 0;12 | 0; 2 |
| 25 | 0; 3 | 0;12[13] | 0;18 | 0;20 | 0;17 | 0;11 | 0; 1 |
| 26 | 0; 3 | 0;13 | 0;18 | 0;20 | 0;17 | 0;11 | 0; 1 |
| 27 | 0; 3 | 0;13 | 0;18 | 0;20 | 0;17 | 0;11 | 0; 0 |
| 28⊥ | 0; 4 | 0;13 | 0;18 | 0;20 | 0;17 | 0;10 | From the nearest distance[15] |
| 29[14] | 0; 4⊥ | 0;13 | 0;18 | 0;20 | 0;17 | 0;10 | |

[1] **Y** 8ˢ 0–29°: slide[−1 col.] [2] **Y** 9′ [3] **B** 19′ [4] **L** 19′ [5] **F** 14′ [6] **L** arguments 18–28°: slide[+1] [7] +**C** 0;0 [8] +**H** om. [9] **Y**+ om. (hence read as 11′) [10] **L** 18′ [11] **Y**+ 11′ (instead of 12′) [12] **L** 18′ [13] **L** 13′ [14] **L** om. (end of slide) [15] **C** om. label, **Y** one value 0;0 in addition to the label, **L** two values 0;0 instead of the label.

**Table 24: Interpolation minutes (furthest distance) for Saturn**
*Sources*: **F** fol. 60r, **H** fol. 48r, **C** fol. 61r, **$C_1$** fol. 27r, **$C_2$** fol. 39r, **Y** fol. 279r, **L** fol. 50r, **B** p. 95.

| Minutes of Proportions for Saturn | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0 | 0 | 33 | 55 | 60 | 49 | 27 | 30 |
| 1 | 1 | 34 | 55 | 60 | 48 | 26 | 29 |
| 2 | 2 | 35 | 56[1]↓ | 60 | 48 | 25 | 28 |
| 3 | 3 | 36 | 56 | 59 | 47 | 24 | 27 |
| 4 | 5 | 37 | 56 | 59 | 46 | 24 | 26 |
| 5 | 6 | 38 | 57 | 59 | 46 | 23 | 25 |
| 6 | 7 | 39 | 57 | 59 | 45 | 22 | 24 |
| 7 | 8 | 40 | 57 | 59 | 44 | 21 | 23 |
| 8 | 9 | 41 | 58 | 58 | 44[2] | 20[3] | 22 |
| 9 | 10 | 41 | 58⊥ | 58 | 43 | 19 | 21 |
| 10 | 12 | 42[4] | 58 | 58 | 42 | 19 | 20 |
| 11 | 13 | 43[5] | 59 | 57 | 42 | 18 | 19 |
| 12 | 14 | 44 | 59 | 57 | 41 | 17 | 18 |
| 13 | 15 | 45 | 59 | 57 | 40 | 16 | 17 |
| 14 | 16 | 45 | 59 | 56 | 40 | 15 | 16 |
| 15 | 17 | 46 | 59 | 56 | 39 | 14 | 15 |
| 16 | 19 | 47 | 60 | 56 | 38 | 13 | 14 |
| 17 | 20 | 47 | 60 | 55 | 38 | 12 | 13 |
| 18 | 21 | 48 | 60 | 55 | 37 | 11 | 12 |
| 19 | 22 | 49 | 60 | 54 | 36 | 10 | 11 |
| 20 | 23 | 49 | 60 | 54 | 35[6] | 9 | 10 |
| 21 | 24 | 50 | 60 | 53 | 34 | 8 | 9 |
| 22 | 25 | 51 | 60 | 53[7] | 34 | 8 | 8 |
| 23 | 26 | 51 | 60 | 52 | 33 | 7 | 7 |
| 24 | 27 | 52 | 60 | 52 | 32 | 6 | 6 |
| 25 | 28 | 53 | 60 | 51 | 31 | 5 | 5 |
| 26 | 29 | 53 | 60 | 51 | 30 | 4 | 4 |
| 27 | 30 | 54 | 60 | 50 | 29 | 3 | 3 |
| 28 | 31 | 54 | 60 | 50 | 29 | 2 | 2 |
| 29 | 32 | 55 | 60 | 49 | 28[8] | 1 | 1 |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **Y** $8^s$ 2–9°: slide[−2] [2] **F** 43′ [3] **C** 21′ [4] **C** 43′ [5] **C** 44′ [6] **F** 33′ [7] **C** 52′ **Y** 13′ [8] **L** 23′.

**Table 25: Mean motion of Jupiter (first part)**

*Sources*: **F** fols 60v–61r, **H** fols 48v–49r, **C** fol. 61v, **C**$_1$ fol. 27v, **C**$_2$ fol. 39v, **Y** fols 279v–280r, **L** fols 50v–51r, **B** pp. 96–97.

Mean Motion of Jupiter in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 8$^s$ 15;25 | 1 | 1$^s$ 0;21 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 4$^s$ 22;17* | 2 | 2$^s$ 0;41 | | |
| 41 | 0$^s$ 29;10 | 3 | 3$^s$ 1; 2 | Urdībihisht | 0$^s$ 2;30 |
| 61 | 9$^s$ 6; 3 | 4 | 4$^s$ 1;23[1] | | |
| 81 | 5$^s$ 12;55[2] | 5 | 5$^s$ 1;43 | Khurdād | 0$^s$ 4;59 |
| 101 | 1$^s$ 19;48[3] | 6 | 6$^s$ 2; 4 | | |
| 121 | 9$^s$ 26;40* | 7 | 7$^s$ 2;24 | Tīr | 0$^s$ 7;29 |
| 141 | 6$^s$ 3;33 | 8 | 8$^s$ 2;45 | | |
| 161 | 2$^s$ 10;26 | 9 | 9$^s$ 3; 6 | Murdād | 0$^s$ 9;59 |
| 181 | 10$^s$ 17;18* | 10 | 10$^s$ 3;26 | | |
| 201 | 6$^s$ 24;11 | 11 | 11$^s$ 3;47[4] | Shahrīwar | 0$^s$ 12;28 |
| 221 | 3$^s$ 1; 4 | 12 | 0$^s$ 4; 7[5] | | |
| 241 | 11$^s$ 7;56[6] | 13 | 1$^s$ 4;28 | Mihr | 0$^s$ 14;58 |
| 261 | 7$^s$ 14;49 | 14 | 2$^s$ 4;49 | | |
| 281 | 3$^s$ 21;41* | 15 | 3$^s$ 5; 9[7] | Ābān | 0$^s$ 17;27[8] |
| 301 | 11$^s$ 28;34* | 16 | 4$^s$ 5;30 | | |
| 321 | 8$^s$ 5;27[9] | 17 | 5$^s$ 5;50[10] | Ādhar | 0$^s$ 19;57 |
| 341 | 4$^s$ 12;19* | 18 | 6$^s$ 6;11 | | *0$^s$ 20;22*[11] |
| 361 | 0$^s$ 19;12[12] | 19 | 7$^s$ 6;32 | Day | 0$^s$ 22;27[13] |
| 381 | 8$^s$ 26; 5 | 20 | 8$^s$ 6;53 | | *0$^s$ 22;52* |
| 401[14] | 5$^s$ 2;57[15] | single years | | Bahman | 0$^s$ 24;56[16] |
| 421 | 1$^s$ 9;50 | | | | *0$^s$ 25;22*[17] |
| 441 | 9$^s$ 16;43 | 40 | 4$^s$ 13;45 | Isfandārmudh | 0$^s$ 27;26 |
| 461 | 5$^s$ 23;35[18] | 60 | 0$^s$ 20;38 | | *0$^s$ 27;51* |
| 481 | 2$^s$ 0;28 | 80 | 8$^s$ 27;31 | | |
| 501 | 10$^s$ 7;20* | 100 | 5$^s$ 4;23[19] | | |
| 521 | 6$^s$ 14;13* | 200 | 10$^s$ 8;46 | | |
| 541 | 2$^s$ 21; 6 | 300 | 3$^s$ 13;10 | | |
| 561 | 10$^s$ 27;58* | 400 | 8$^s$ 17;33 | | |
| 581 | 7$^s$ 4;51[20] | 500 | 1$^s$ 21;56 | | |
| | | | [21] | | |

* In **YLB** the values for collected years 21, 81, 121, 181, 241, 281, 301, 341, 401, 461, 501, 521, and 561 are 1 minute larger. To the left of the collected years, headed *ṣḥ*, the main hand of **B** writes a column of corrected minutes that are identical to the ones in **FHCC**$_1$**C**$_2$ (cf. Section IV.5.3). [1] **C** 20′ [2] +**CC**$_1$ 15′ [3] **L** 9$^s$ [4] **FL** 46′ [5] **YL** 8′ [6] +**C** 16′ [7] **L** 49′ [8] **H** 26′ [9] **C** 26′ [10] **YLB** 51′ [11] **L** 32′ [12] **C** 59° [13] **B** 24′ [14] **C** 81 or 101 (فا) [15] +**C** 17′ **Y**+ 38′ [16] **F** 57′ [17] **YLB** 21′ [18] **B**+ 16′ [19] **Y** 13° [20] **F** 6° [21] **YLB** add a value 6$^s$ 26;19 for 600 single years.

**Table 25: Mean motion of Jupiter (second part)**

*Sources*: **F** fols 60v–61r, **H** fols 48v–49r, **C** fol. 61v, **C$_1$** fol. 27v, **C$_2$** fol. 39v, **Y** fols 279v–280r, **L** fols 50v–51r, **B** pp. 96–97.

Mean Motion of Jupiter in Days, Hours, and Between Longitudes

| days | days<br>s ° ′ | hours | hours<br>° ′ | hours | and their fractions<br>° ′ | longitudes | in between longitudes<br>′ ″ |
|---|---|---|---|---|---|---|---|
| 1 | 0$^s$ 0; 0 | 1 | 0; 0 | 31 | 0; 6 | 71[1]↓ | 0, 0 |
| 2 | 0$^s$ 0; 5 | 2 | 0; 0 | 32 | 0; 7[2] | 72 | 0, 0 |
| 3 | 0$^s$ 0;10 | 3 | 0; 1 | 33 | 0; 7 | 73 | 0, 0 |
| 4 | 0$^s$ 0;15 | 4 | 0; 1 | 34 | 0; 7 | 74 | 0, 0 |
| 5 | 0$^s$ 0;20 | 5 | 0; 1 | 35 | 0; 7 | 75 | 0, 0 |
| 6 | 0$^s$ 0;25 | 6 | 0; 1 | 36 | 0; 7 | 76 | 0, 0 |
| 7 | 0$^s$ 0;30 | 7 | 0; 1[3] | 37 | 0; 7 | 77 | 0, 0 |
| 8 | 0$^s$ 0;35 | 8 | 0; 2 | 38 | 0; 8 | 78 | 0, 0 |
| 9 | 0$^s$ 0;40[4] | 9 | 0; 2 | 39 | 0; 8 | 79 | 0, 0 |
| 10 | 0$^s$ 0;45 | 10 | 0; 2 | 40 | 0; 8 | 80 | 0, 0 |
| 11 | 0$^s$ 0;50 | 11 | 0; 2 | 41 | 0; 8 | 81 | 0, 0 |
| 12 | 0$^s$ 0;55 | 12 | 0; 2[5] | 42 | 0; 9 | 82 | 0, 0 |
| 13 | 0$^s$ 1; 0 | 13 | 0; 3 | 43 | 0; 9 | 83 | 0, 0 |
| 14 | 0$^s$ 1; 5 | 14 | 0; 3 | 44 | 0; 9 | 84 | 0, 0 |
| 15 | 0$^s$ 1;10 | 15 | 0; 3 | 45 | 0; 9 | 85 | 0, 0 |
| 16 | 0$^s$ 1;15 | 16 | 0; 3 | 46 | 0; 9 | 86 | 0, 0 |
| 17 | 0$^s$ 1;20 | 17 | 0; 3[6] | 47 | 0;10 | 87 | 0, 0 |
| 18 | 0$^s$ 1;25 | 18 | 0; 4 | 48 | 0;10 | 88 | 0, 0 |
| 19 | 0$^s$ 1;30 | 19 | 0; 4 | 49 | 0;10 | 89 | 0, 0 |
| 20 | 0$^s$ 1;35 | 20 | 0; 4 | 50 | 0;10 | 90 | 0, 0 |
| 21 | 0$^s$ 1;40 | 21 | 0; 4[7] | 51 | 0;10 | 91 | 0, 0 |
| 22 | 0$^s$ 1;45 | 22 | 0; 5[8] | 52 | 0;11 | 92 | 0, 0 |
| 23 | 0$^s$ 1;50 | 23 | 0; 5 | 53 | 0;11 | 93 | 0, 0 |
| 24 | 0$^s$ 1;55 | 24 | 0; 5 | 54 | 0;11 | 94 | 0, 0 |
| 25 | 0$^s$ 2; 0 | 25 | 0; 5 | 55 | 0;11 | 95 | 0, 0 |
| 26 | 0$^s$ 2; 5 | 26 | 0; 5 | 56 | 0;11 | 96 | 0, 0 |
| 27 | 0$^s$ 2;10 | 27 | 0; 6 | 57 | 0;12 | 97 | 0, 0 |
| 28 | 0$^s$ 2;15 | 28 | 0; 6 | 58 | 0;12 | 98 | 0, 0 |
| 29 | 0$^s$ 2;20 | 29 | 0; 6 | 59 | 0;12 | 99 | 0, 0 |
| 30 | 0$^s$ 2;25 | 30 | 0; 6 | 60 | 0;12 | 100⊥ | 0, 0 |

**Y** omits the entire subtable for longitude differences. [1] **+L** omits all arguments. [2] **Y** 6′ [3] **LB** 2′ [4] **L** 47′ [5] **YL** 3′ [6] **HYL** 4′ [7] **H** 5′ [8] **B** 4′.

**Table 26: Mean anomaly of Jupiter (first part)**
*Sources*: **F** fols 61v–62r, **H** fols 49v–50r, **C** fol. 62r, **$C_1$** fol. 28r, **$C_2$** fol. 40r, **Y** fols 280v–281r, **L** fols 51v–52r, **B** pp. 98–99.

| Mean Anomaly of Jupiter in Years and Months | | | | | |
|---|---|---|---|---|---|
| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 5$^s$ 29;29* | 1 | 10$^s$ 29;25 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 9$^s$ 17;52[1] | 2 | 9$^s$ 28;50 | | |
| 41 | 1$^s$ 6;15 | 3 | 8$^s$ 28;15 | Urdībihisht | 0$^s$ 27; 5 |
| 61 | 4$^s$ 24;37[2] | 4 | 7$^s$ 27;41 | | |
| 81 | 8$^s$ 13; 0* | 5 | 6$^s$ 27; 6 | Khurdād | 1$^s$ 24; 9 |
| 101 | 0$^s$ 1;23* | 6 | 5$^s$ 26;31 | | |
| 121 | 3$^s$ 19;46[3] | 7 | 4$^s$ 25;56 | Tīr | 2$^s$ 21;14 |
| 141 | 7$^s$ 8; 9 | 8 | 3$^s$ 25;21 | | |
| 161 | 10$^s$ 26;32 | 9 | 2$^s$ 24;46 | Murdād | 3$^s$ 18;18 |
| 181 | 2$^s$ 14;55[4] | 10 | 1$^s$ 24;11 | | |
| 201 | 6$^s$ 3;17* | 11 | 0$^s$ 23;36* | Shahrīwar | 4$^s$ 15;23 |
| 221 | 9$^s$ 21;40* | 12 | 11$^s$ 23; 1* | | |
| 241 | 1$^s$ 10; 3* | 13 | 10$^s$ 22;26* | Mihr | 5$^s$ 12;27 |
| 261 | 4$^s$ 28;26[5] | 14 | 9$^s$ 21;52 | | |
| 281 | 8$^s$ 16;49 | 15 | 8$^s$ 21;17 | Ābān | 6$^s$ 9;32 |
| 301 | 0$^s$ 5;12 | 16 | 7$^s$ 20;42 | | |
| 321 | 3$^s$ 23;34* | 17 | 6$^s$ 20; 7 | Ādhar | 7$^s$ 6;36 |
| 341 | 7$^s$ 11;57[6] | 18 | 5$^s$ 19;32* | | *7$^s$ 11; 7* |
| 361 | 11$^s$ 0;20* | 19 | 4$^s$ 18;57* | Day | 8$^s$ 3;41 |
| 381 | 2$^s$ 18;43* | 20 | 3$^s$ 18;23 | | *8$^s$ 8;12* |
| 401[7] | 6$^s$ 7; 6 | single years | | Bahman | 9$^s$ 0;45 |
| 421 | 9$^s$ 25;29 | | | | *9$^s$ 5;16* |
| 441 | 1$^s$ 13;51[8] | 40 | 7$^s$ 6;46 | Isfandārmudh | 9$^s$ 27;50[9] |
| 461 | 5$^s$ 2;14* | 60 | 10$^s$ 25; 9 | | *10$^s$ 2;21* |
| 481[10] ↓ | 8$^s$ 20;37* | 80 | 2$^s$ 13;31[11] | | |
| 501 | 0$^s$ 9; 0* | 100 | 6$^s$ 1;54 | | |
| 521 ⊥ | 3$^s$ 27;23[12] | 200 | 0$^s$ 3;48 | | |
| 541 | 7$^s$ 15;46 | 300 | 6$^s$ 5;43 | | |
| 561 | 11$^s$ 4; 9[13] | 400 | 0$^s$ 7;37 | | |
| 581 | 2$^s$ 22;31* | 500 | 6$^s$ 9;31 | | |
| | | | [14] | | |

* In **YLB** the values for collected years 1, 61–121, 201–261, 321–381, 441–501, and 581 are 1 minute larger; in **YL** (but not in **B**) the values for 11–13 and 18–19 extended years are 1 minute larger. To the left of the collected years, the main hand of **B** writes corrected minutes that are all three larger than the ones in **FHCC$_1$C$_2$** (cf. Section IV.5.3). [1] **L** 40′ **CC$_1$** 12′ [2] **L**+ 7$^s$ [3] **L**+ 57′ [4] **C** 15′ [5] +**C** 23° [6] +**C** 17′ **C$_1$C$_2$** 56′ **B**+ 12° (corrected from 11°, hence belonging to the corrected digit 0′ next to the table) [7] **C** 81 or 101 (فا) [8] +**C** 11′ [9] **L** 55′ [10] **C** arguments 481–521: slide[−3] [11] **L** 10$^s$ [12] **C** 28′ [13] **C$_1$C$_2$** 0$^s$ [14] **YLB** add a value 0$^s$ 11;25 for 600 single years.

**Table 26: Mean anomaly of Jupiter (second part)**
*Sources*: **F** fols 61v–62r, **H** fols 49v–50r, **C** fol. 62r, **$C_1$** fol. 28r, **$C_2$** fol. 40r, **Y** fols 280v–281r, **L** fols 51v–52r, **B** pp. 98–99.

Mean Anomaly of Jupiter in Days, Hours, and Between Longitudes

| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
|---|---|---|---|---|---|---|---|---|
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | 0ˢ 0; 0 | 1 | 0; 2 | 31 | 1;10 | 71[1]↓ | additive | 0, 3 |
| 2 | 0ˢ 0;54 | 2 | 0; 4* | 32 | 1;12 | 72 | | 0, 3[2]↓ |
| 3 | 0ˢ 1;48 | 3 | 0; 6* | 33 | 1;14 | 73 | | 0, 3⊥ |
| 4 | 0ˢ 2;42 | 4 | 0; 9 | 34 | 1;17 | 74 | | 0, 2 |
| 5 | 0ˢ 3;37 | 5 | 0;11 | 35 | 1;19 | 75 | | 0, 2 |
| 6 | 0ˢ 4;31 | 6 | 0;14 | 36 | 1;21 | 76 | | 0, 2 |
| 7 | 0ˢ 5;25 | 7 | 0;16 | 37 | 1;23 | 77 | | 0, 2 |
| 8 | 0ˢ 6;19 | 8 | 0;18 | 38 | 1;25* | 78 | | 0, 2 |
| 9 | 0ˢ 7;13 | 9 | 0;20 | 39 | 1;28 | 79 | | 0, 2 |
| 10 | 0ˢ 8; 7 | 10 | 0;23 | 40 | 1;30 | 80 | | 0, 2 |
| 11 | 0ˢ 9; 2 | 11 | 0;25 | 41 | 1;32* | 81 | | 0, 1 |
| 12 | 0ˢ 9;56 | 12 | 0;27[3] | 42 | 1;34* | 82 | | 0, 1 |
| 13 | 0ˢ 10;50 | 13 | 0;29 | 43 | 1;37 | 83 | | 0, 1 |
| 14 | 0ˢ 11;44 | 14 | 0;31* | 44 | 1;39 | 84 | | 0, 1 |
| 15 | 0ˢ 12;38 | 15 | 0;34[4] | 45 | 1;41* | 85 | | 0, 1 |
| 16 | 0ˢ 13;32 | 16 | 0;36 | 46 | 1;43* | 86 | | 0, 1 |
| 17 | 0ˢ 14;26 | 17 | 0;38 | 47 | 1;46 | 87 | | 0, 0 |
| 18 | 0ˢ 15;21 | 18 | 0;40* | 48 | 1;48 | 88 | | 0, 0 |
| 19 | 0ˢ 16;15 | 19 | 0;43 | 49 | 1;50* | 89 | | 0, 0 |
| 20 | 0ˢ 17; 9 | 20 | 0;45 | 50 | 1;52* | 90 | | 0, 0 |
| 21 | 0ˢ 18; 3 | 21 | 0;47 | 51 | 1;54* | 91 | subtractive | 0, 0 |
| 22 | 0ˢ 18;57 | 22 | 0;49* | 52 | 1;57 | 92 | | 0, 0 |
| 23 | 0ˢ 19;51 | 23 | 0;52 | 53 | 1;59[5] | 93 | | 0, 0 |
| 24 | 0ˢ 20;45 | 24 | 0;54 | 54 | 2; 2 | 94 | | 0, 1 |
| 25 | 0ˢ 21;40 | 25 | 0;56 | 55 | 2; 4 | 95 | | 0, 1 |
| 26 | 0ˢ 22;34 | 26 | 0;58* | 56 | 2; 6 | 96 | | 0, 1 |
| 27 | 0ˢ 23;28 | 27 | 1; 0* | 57 | 2; 8* | 97 | | 0, 1 |
| 28 | 0ˢ 24;22 | 28 | 1; 3 | 58 | 2;11 | 98⊥ | | 0, 1 |
| 29 | 0ˢ 25;16 | 29 | 1; 5 | 59 | 2;13 | 99[6]↓ | | 0, 1 |
| 30 | 0ˢ 26;10[7] | 30 | 1; 8 | 60 | 2;15 | 100⊥ | | 0, 2 |

* In **YLB** the values for 18, 22, 26–27, 38, 41–42, 45–46, 49–51, 53, and 57 hours are 1 minute larger; in **YL** (but not in **B**) also the values for 2–3 and 14 hours are 1 minute larger. In **F** all longitude corrections are given as zero. [1] **L** arguments 71–98°: slide[+2] (arguments 71 and 72 are written next to the column header) [2] **+HC$C_1$$C_2$** longitudes 72–73°: 2′ [3] **F** 22′ [4] **Y** 35′ [5] **B**+ 1° [6] **L** arguments 99–100°: om. (end of slide) [7] **L** 36°

**Table 27: First equation for Jupiter (first half)**
*Sources*: **F** fols 62v–63r, **H** fols 50v–51r, **C** fols 62v–63r, $\mathbf{C_1}$ fols 28v–29r, $\mathbf{C_2}$ fols 40v–41r, **Y** fols 281v–282r, **L** fols 52v–53r, **B** pp. 100–101.

First Equation for Jupiter
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | mean distance 15° **2** ° ′ | differences ′ | **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | nearest distance 12° **5** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 4;27 | 5 | 2;13 | 3 | 0;54 | 1 | 0;56 | 2 | 2;22 | 4 | 4;51 | 6 |
| 1 | 4;22 | 5 | 2; 9 | 4 | 0;53 | 1 | 0;57 | 1 | 2;26 | 4 | 4;56 | 5 |
| 2 | 4;17 | 5 | 2; 5 | 4 | 0;52 | 1 | 0;59 | 2 | 2;30 | 4 | 5; 2 | 6 |
| 3 | 4;12 | 5 | 2; 2 | 3[1] | 0;51 | 1 | 1; 1 | 2 | 2;35 | 5 | 5; 8[2] | 6 |
| 4 | 4; 7 | 5 | 1;58 | 4 | 0;50 | 1 | 1; 3 | 2 | 2;39 | 4 | 5;13 | 5 |
| 5 | 4; 2 | 5 | 1;55 | 3 | 0;49 | 1 | 1; 5 | 2 | 2;44 | 5 | 5;18 | 5 |
| 6 | 3;57 | 5 | 1;52 | 3[3] | 0;48 | 1 | 1; 7 | 2 | 2;49 | 5 | 5;24 | 6 |
| 7 | 3;52 | 5 | 1;48 | 4 | 0;47 | 1 | 1; 9 | 2 | 2;53 | 4 | 5;30 | 6 |
| 8 | 3;47 | 5 | 1;45 | 3 | 0;47 | 0 | 1;11 | 2 | 2;58 | 5 | 5;36 | 6 |
| 9 | 3;43 | 4 | 1;42 | 3 | 0;46 | 1 | 1;13 | 2 | 3; 3 | 5 | 5;42 | 6 |
| 10 | 3;38[4] | 5 | 1;39 | 3 | 0;46 | 0 | 1;15 | 2 | 3; 7 | 4 | 5;48 | 6 |
| 11 | 3;33 | 5 | 1;36 | 3 | 0;46 | 0 | 1;17 | 2 | 3;12 | 5 | 5;54 | 6 |
| 12 | 3;29[5] | 4 | 1;33 | 3 | 0;45 | 1 | 1;20 | 3 | 3;17 | 5 | 6; 0[6]↓ | 6[7] |
| 13 | 3;24[8] | 5 | 1;30 | 3 | 0;45 | 0 | 1;22 | 2 | 3;21 | 4 | 6; 6⊥ | 6 |
| 14 | 3;19 | 5 | 1;27 | 3 | 0;45 | 0 | 1;25 | 3 | 3;26 | 5 | 6;12[9] | 6 |
| 15 | 3;15 | 4 | 1;24[10] | 3 | 0;45 | 0 | 1;28 | 3 | 3;31 | 5 | 6;18 | 6 |
| 16 | 3;10 | 5 | 1;21 | 3 | 0;45 | 0 | 1;31 | 3 | 3;36 | 5 | 6;24 | 6 |
| 17 | 3; 5 | 5 | 1;18 | 3 | 0;45 | 0 | 1;34 | 3 | 3;41 | 5 | 6;30 | 6[11]↓ |
| 18 | 3; 1 | 4 | 1;16 | 2 | 0;45 | 0 | 1;37 | 3 | 3;46 | 5 | 6;36 | 6 |
| 19 | 2;56 | 5 | 1;14 | 2 | 0;46 | 1 | 1;40 | 3 | 3;51 | 5 | 6;42 | 6 |
| 20 | 2;52 | 4 | 1;12 | 2 | 0;46 | 0 | 1;44 | 4 | 3;56 | 5 | 6;47 | 5 |
| 21 | 2;48 | 4 | 1;10 | 2 | 0;46 | 0[12] | 1;48 | 4 | 4; 2 | 6 | 6;52[13] | 5 |
| 22 | 2;43 | 5 | 1; 8 | 2 | 0;47 | 1[14] | 1;51 | 3 | 4; 7 | 5 | 6;58 | 6 |
| 23 | 2;39 | 4 | 1; 6 | 2 | 0;47 | 0 | 1;55 | 4 | 4;12 | 5 | 7; 4 | 6⊥ |
| 24 | 2;36 | 3 | 1; 4 | 2 | 0;48 | 1 | 1;59 | 4 | 4;18 | 6 | 7; 9 | 5[15] |
| 25 | 2;32 | 4 | 1; 2 | 2 | 0;49 | 1 | 2; 2 | 3 | 4;24 | 6 | 7;15 | 6 |
| 26 | 2;28 | 4 | 1; 0 | 2 | 0;50 | 1 | 2; 6 | 4 | 4;29 | 5 | 7;20 | 5 |
| 27 | 2;24 | 4 | 0;59 | 1 | 0;51 | 1 | 2;10 | 4 | 4;35[16] | 6 | 7;25 | 5 |
| 28 | 2;20 | 4 | 0;57 | 2 | 0;52 | 1 | 2;14 | 4 | 4;40 | 5 | 7;31 | 6 |
| 29 | 2;16[17] | 4 | 0;55 | 2 | 0;54[18] | 2 | 2;18 | 4 | 4;45 | 5 | 7;36 | 5 |

[1] **C** 4′ [2] **H** 3′ [3] **L** 4′ [4] $\mathbf{C_2}$ 33′ [5] **C** 24′ [6] **C** 5ˢ 12–13°: slide[+3] [7] **C** 7′ [8] **C** 29′ [9] **C** om. minutes [10] **C** 27′ [11] **C** 5ˢ 17–23°: slide[+3] (since the values in **C** were copied in groups of three, it is probable that the actual miscopying started at argument 5ˢ 15°, and it may have continued up to argument 5ˢ 26°) $\mathbf{C_1}$ 5′ [12] **C** 1′ [13] **Y** 57′ [14] **C** 0′ [15] **Y** 6′ [16] **L** 34′ [17] **L** 46′ [18] **F** 52′.

**Table 27: First equation for Jupiter (second half)**
*Sources*: **F** fols 62v–63r, **H** fols 50v–51r, **C** fols 62v–63r, **$C_1$** fols 28v–29r, **$C_2$** fols 40v–41r, **Y** fols 281v–282r, **L** fols 52v–53r, **B** pp. 100–101.

First Equation for Jupiter
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 6 (° ′) | differences ′ | 7 (° ′) | differences ′ | mean distance 9° 8 (° ′) | differences ′ | 9 (° ′) | differences ′ | 10 (° ′) | differences ′ | furthest distance 12° 11 (° ′) | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 7;42 | 6 | 10; 1 | 3 | 11;12 | 1 | 10;56 | 2 | 9;24 | 4 | 7; 3 | 5 |
| 1 | 7;48 | 6 | 10; 5 | 4 | 11;13 | 1 | 10;54 | 2 | 9;21 | 3[1] | 6;58[2] | 5 |
| 2 | 7;53 | 5 | 10; 9 | 4 | 11;13 | 0[3] | 10;52 | 2 | 9;17 | 4 | 6;53 | 5 |
| 3 | 7;58 | 5 | 10;12 | 3 | 11;14 | 1 | 10;50 | 2 | 9;12 | 5[4]↓ | 6;47[5] | 6[6] |
| 4 | 8; 4[7] | 6[8] | 10;16 | 4 | 11;14 | 0 | 10;48 | 2 | 9; 8 | 4 | 6;42 | 5 |
| 5 | 8; 9 | 5 | 10;20 | 4 | 11;14 | 0 | 10;46 | 2 | 9; 4 | 4 | 6;37 | 5 |
| 6 | 8;14 | 5 | 10;23 | 3 | 11;15 | 1 | 10;44 | 2 | 8;59 | 5 | 6;32 | 5 |
| 7 | 8;19 | 5 | 10;26 | 3 | 11;15 | 0 | 10;42 | 2 | 8;55 | 4 | 6;27 | 5 |
| 8 | 8;24 | 5 | 10;29 | 3 | 11;15 | 0 | 10;39 | 3 | 8;50 | 5⊥ | 6;22 | 5 |
| 9 | 8;29 | 5 | 10;32 | 3 | 11;15 | 0 | 10;36 | 3 | 8;45 | 5 | 6;16 | 6 |
| 10 | 8;34 | 5 | 10;35 | 3 | 11;15 | 0 | 10;33 | 3 | 8;41 | 4 | 6;11[9] | 5 |
| 11 | 8;39 | 5 | 10;38 | 3 | 11;15 | 0 | 10;30 | 3 | 8;36 | 5 | 6; 6 | 5 |
| 12 | 8;43 | 4 | 10;40 | 2 | 11;15 | 0 | 10;27 | 3 | 8;31 | 5 | 6; 0 | 6 |
| 13 | 8;48 | 5[10] | 10;43 | 3 | 11;14 | 1 | 10;24 | 3 | 8;27 | 4 | 5;54 | 6 |
| 14 | 8;53 | 5 | 10;45 | 2 | 11;14 | 0 | 10;21 | 3 | 8;22 | 5 | 5;49 | 5 |
| 15 | 8;57 | 4 | 10;47 | 2 | 11;14 | 0 | 10;18 | 3 | 8;17 | 5 | 5;44 | 5 |
| 16 | 9; 2 | 5 | 10;49 | 2 | 11;13 | 1 | 10;15 | 3 | 8;13 | 4[11] | 5;38 | 6 |
| 17 | 9; 7 | 5 | 10;51 | 2 | 11;13 | 0 | 10;12 | 3[12]↓ | 8; 8 | 5 | 5;33 | 5 |
| 18 | 9;11 | 4 | 10;53 | 2 | 11;12 | 1 | 10; 8 | 4[13] | 8; 3[14] | 5 | 5;28 | 5 |
| 19 | 9;16[15] | 5 | 10;55 | 2 | 11;11 | 1 | 10; 5 | 3 | 7;58 | 5 | 5;23 | 5 |
| 20 | 9;21 | 5 | 10;57 | 2 | 11;10 | 1 | 10; 2 | 3 | 7;53 | 5 | 5;18 | 5 |
| 21 | 9;25[16] | 4 | 10;59 | 2 | 11; 9 | 1 | 9;58 | 4 | 7;48 | 5 | 5;13 | 5 |
| 22 | 9;30[17] | 5 | 11; 1 | 2 | 11; 8 | 1 | 9;55 | 3 | 7;43 | 5 | 5; 7 | 6 |
| 23 | 9;34[18] | 4 | 11; 3 | 2 | 11; 7 | 1 | 9;51 | 4 | 7;38 | 5 | 5; 2[19] | 5 |
| 24 | 9;38 | 4 | 11; 4 | 1 | 11; 6 | 1 | 9;47 | 4⊥ | 7;33 | 5 | 4;57 | 5 |
| 25 | 9;42 | 4 | 11; 6 | 2 | 11; 5[20] | 1[21] | 9;44[22] | 3 | 7;28 | 5 | 4;52 | 5 |
| 26 | 9;46 | 4 | 11; 8 | 2 | 11; 3 | 2[23] | 9;40 | 4 | 7;23 | 5 | 4;47 | 5 |
| 27 | 9;50 | 4 | 11; 9 | 1 | 11; 1 | 2 | 9;36 | 4 | 7;18 | 5 | 4;42 | 5 |
| 28 | 9;54 | 4 | 11;10 | 1 | 11; 0 | 1 | 9;32[24] | 4 | 7;13 | 5 | 4;37 | 5 |
| 29 | 9;58 | 4 | 11;11 | 1 | 10;58 | 2 | 9;28 | 4 | 7; 8[25] | 5 | 4;32 | 5 |

[1] **FL** 4′ [2] **$C_2$** 18′ [3] **C** 1′ [4] **L** $10^s$ 3–8°: slide[−1] [5] **F** 44′ [6] **$C_1$** 5′ [7] **F** 3′ [8] **L** 4′ [9] **$C_2$** 51′ [10] **L** 4′ [11] **C** 5′ [12] **L** $9^s$ 17–24°: slide[+1] [13] **+F** 3′ [14] **C** 8′ [15] **C** 26′ [16] **F** 15′ [17] **L** 32′ [18] **Y** 35′ [19] **Y** 3′ [20] **$CC_1C_2$** 4′ [21] **$CC_1C_2$** 2′ (in correspondence with the equation for $8^s$ 25°) [22] **Y** 45′ [23] **$CC_1C_2$** 1′ (in correspondence with the equation for $8^s$ 25°) [24] **Y** 30′ [25] **Y** 18′.

**Table 27: Second equation for Jupiter (first half)**
*Sources*: **F** fols 63v–64r, **H** fols 51v–52r, **C** fols 63v–64r, $\mathbf{C_1}$ fols 29v–30r, $\mathbf{C_2}$ fols 41v–42r, **Y** fols 282v–283r, **L** fols 53v–54r, **B** pp. 102–103.

Second Equation for Jupiter
to be equated and added to the centrum with the apogee

| degrees of the epicycle | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | **2** ° ′ | differences ′ | mean distance 6° **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 12; 0 | 10 | 16;42 | 9 | 20;38 | 7[1] | 22;51 | 2 | 22;23 | 4 | 18;34[2] | 10 |
| 1 | 12;10 | 10 | 16;51 | 9 | 20;44 | 6 | 22;53 | 2 | 22;19 | 4 | 18;23 | 11 |
| 2 | 12;20 | 10 | 17; 0 | 9 | 20;50 | 6 | 22;55 | 2 | 22;15 | 4 | 18;11 | 12 |
| 3 | 12;29 | 9 | 17; 8 | 8 | 20;56 | 6 | 22;57 | 2 | 22;10 | 5 | 18; 0 | 11 |
| 4 | 12;39 | 10 | 17;17 | 9 | 21; 2 | 6 | 22;59 | 2 | 22; 5 stationary | 5 | 17;48 | 12 |
| 5 | 12;49 | 10 | 17;26[3] | 9 | 21; 8 | 6 | 23; 0[4]↓ | 1 | 22; 0 | 5 | 17;36 | 12 |
| 6 | 12;58 | 9[5]↓ | 17;35 | 9 | 21;14 | 6 | 23; 1 | 1 | 21;54 | 6 | 17;24 | 12 |
| 7 | 13; 8 | 10 | 17;43 | 8 | 21;20 | 6 | 23; 2 | 1 | 21;48 retrograde | 6 | 17;12 | 12 |
| 8 | 13;18 | 10 | 17;52[6] | 9 | 21;26 | 6[7] | 23; 2 | 0 | 21;42 | 6[8] | 17; 0 | 12 |
| 9 | 13;27[9]↓ | 9 | 18; 0 | 8 | 21;31 | 5 | 23; 3 | 1 | 21;36 | 6 | 16;47 | 13 |
| 10 | 13;37 | 10 | 18; 9 | 9 | 21;36 | 5 | 23; 3 | 0 | 21;30 | 6 | 16;35 | 12[10] |
| 11 | 13;47⊥ | 10 | 18;17 | 8 | 21;41 | 5 | 23; 3 | 0 | 21;24 | 6 | 16;22 | 13[11] |
| 12 | 13;56 | 9 | 18;25 | 8 | 21;46 | 5 | 23; 3 | 0 | 21;17 | 7 | 16; 9 | 13 |
| 13 | 14; 6 | 10 | 18;33 | 8 | 21;51 | 5 | 23; 3 | 0 | 21;10 | 7 | 15;56 | 13 |
| 14 | 14;15[12] eastern vis | 9[13] | 18;41 | 8 | 21;56 | 5 | 23; 2 | 1 | 21; 2 | 8 | 15;42 | 14[14] |
| 15 | 14;24 | 9 | 18;49 | 8 | 22; 0 | 4 | 23; 2 | 0 | 20;54[15] | 8 | 15;29 | 13 |
| 16 | 14;34 | 10 | 18;57 | 8 | 22; 5 | 5 | 23; 1 | 1 | 20;47 | 7 | 15;16 | 13 |
| 17 | 14;43 | 9⊥ | 19; 5 | 8 | 22; 9 | 4 | 23; 0⊥ | 1 | 20;39 | 8 | 15; 3 | 13 |
| 18 | 14;52 | 9 | 19;12 | 7 | 22;13 | 4 | 22;59[16]↓ | 1 | 20;31 | 8[17]↓ | 14;49 | 14[18] |
| 19 | 15; 2 | 10 | 19;20 | 8 | 22;17 | 4 | 22;57[19] | 2 | 20;22 | 9 | 14;35 | 14 |
| 20 | 15;11 | 9 | 19;28 | 8 | 22;21 | 4[20] | 22;55 | 2 | 20;13 | 9 | 14;21 | 14 |
| 21 | 15;20 | 9 | 19;35 | 7 | 22;25[21] | 4 | 22;53 | 2 | 20; 4 | 9 | 14; 7 | 14 |
| 22 | 15;30 | 10 | 19;43 | 8 | 22;29[22] | 4 | 22;51 | 2 | 19;55 | 9 | 13;53 | 14 |
| 23 | 15;39 | 9 | 19;50 | 7 | 22;32[23] | 3 | 22;48 | 3 | 19;46 | 9 | 13;39 | 14 |
| 24 | 15;48 | 9 | 19;57 | 7 | 22;35 | 3 | 22;45 | 3 | 19;36 | 10 | 13;25 | 14 |
| 25 | 15;57 | 9 | 20; 4 | 7 | 22;38 | 3 | 22;42 | 3 | 19;26 | 10 | 13;11 | 14 |
| 26 | 16; 6 | 9 | 20;11 | 7 | 22;41 | 3 | 22;39 | 3 | 19;16 | 10⊥ | 12;57 | 14 |
| 27 | 16;15 | 9 | 20;17 | 6 | 22;44 | 3 | 22;35 | 4 | 19; 6 | 10 | 12;43 | 14 |
| 28 | 16;24 | 9 | 20;24 | 7 | 22;47 | 3 | 22;31 | 4 | 18;55[24] | 11 | 12;29 | 14 |
| 29 | 16;33 | 9 | 20;31 | 7 | 22;49 | 2 | 22;27⊥ | 4 | 18;44 | 11 | 12;15 | 14 |

[1] $\mathbf{C_1}$ 6′ [2] $\mathbf{C_1}$ 37′ [3] **C** 27′ [4] **Y** 3ˢ 5–17°: 21° (correct digit indicated at beginning and end of the range) [5] **L** 0ˢ 6–17°: 11′ whenever the correct value is 9′ [6] $\mathbf{C_2}$ 12′ [7] **L** dam. [8] $\mathbf{C_2}$ 7′ [9] $\mathbf{C_2}$ 0ˢ 9–11°: 12° [10] **L** dam. [11] **L** dam. [12] **C** 55′ [13] +$\mathbf{C_1 C_2}$ 10′ [14] **C** 12′ **L** dam. [15] **C** 55′ [16] **Y** 3ˢ 18–29°: 20° (correct digit indicated at beginning and end of the range) [17] $\mathbf{C_2}$ 4ˢ 18–26°: dam. [18] $\mathbf{C_1}$ 13′ [19] +**HL** 56′ [20] $\mathbf{C_1}$ 3′ [21] $\mathbf{C_1}$ units of minutes ill. $\mathbf{C_2}$ 26′ [22] **L** 25° [23] **Y** 30′ [24] **C** 19°.

**Table 27: Second equation for Jupiter (second half)**
*Sources*: **F** fols 63v–64r, **H** fols 51v–52r, **C** fols 63v–64r, **$C_1$** fols 29v–30r, **$C_2$** fols 41v–42r, **Y** fols 282v–283r, **L** fols 53v–54r, **B** pp. 102–103.

Second Equation for Jupiter
to be equated and added to the centrum with the apogee

| degrees of the epicycle | nearest distance 0° **6** ° ′ | differences ′ | **7** ° ′ | differences ′ | mean distance 24° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 12; 0 | 15 | 5;26 | 11 | 1;37 | 4 | 1; 9 | 2 | 3;22 | 6 | 7;18 | 9 |
| 1 | 11;45 | 15 | 5;16 | 10 | 1;33 | 4 | 1;11 | 2 | 3;29 | 7 | 7;27[1] | 9 |
| 2 | 11;31 | 14 | 5; 5 | 11 | 1;29 | 4 | 1;13 | 2 | 3;36 | 7 | 7;36 | 9 |
| 3 | 11;17 | 14 | 4;54 | 11 | 1;25 | 4 | 1;16[2]↓ | 3 | 3;43 | 7 | 7;45 | 9 |
| 4 | 11; 3 | 14 | 4;44 | 10 | 1;21 | 4 | 1;19 | 3 | 3;49 | 6 | 7;54 | 9 |
| 5 | 10;49 | 14 | 4;34 | 10 | 1;18 | 3 | 1;22⊥ | 3[3] | 3;56 | 7 | 8; 3 | 9 |
| 6 | 10;35 | 14 | 4;24 | 10 | 1;15 | 3 | 1;25 | 3 | 4; 3 | 7 | 8;12 | 9 |
| 7 | 10;21 | 14 | 4;14 | 10 | 1;12 | 3 | 1;28 | 3 | 4;10 | 7 | 8;21 | 9 |
| 8 | 10; 7 | 14 | 4; 5 | 9 | 1; 9 | 3 | 1;31 | 3 | 4;17 | 7 | 8;30 | 9 |
| 9 | 9;53[4] | 14 | 3;56[5] | 9 | 1; 7 | 2 | 1;35 | 4 | 4;25 | 8 | 8;40 | 10[6] |
| 10 | 9;39 | 14 | 3;47 | 9 | 1; 5 | 2 | 1;39 | 4 | 4;32 | 7 | 8;49 | 9 |
| 11 | 9;25 | 14 | 3;38 | 9 | 1; 3 | 2 | 1;43 | 4 | 4;40 | 8 | 8;58 | 9 |
| 12 | 9;11 | 14 | 3;29 | 9[7] | 1; 1 | 2 | 1;47[8] | 4 | 4;48 | 8 | 9; 8 | 10 |
| 13 | 8;57 | 14 | 3;21 | 8 | 1; 0 | 1 | 1;51 | 4 | 4;55 | 7 | 9;17 | 9 |
| 14 | 8;44 | 13 | 3;13 | 8 | 0;59 | 1 | 1;55 | 4[9] | 5; 3 | 8 | 9;26 | 9 |
| 15 | 8;31 | 13 | 3; 6 | 7 | 0;58 | 1 | 2; 0 | 5[10] | 5;11 | 8 | **western dis** 9;36 | 10 |
| 16 | 8;18 | 13 | 2;58[11] | 8 | 0;58 | 0[12] | 2; 4 | 4[13] | 5;19 | 8 | 9;45 | 9 |
| 17 | 8; 4 | 14[14] | 2;50 | 8 | 0;57 | 1 | 2; 9 | 5 | 5;27 | 8 | 9;54 | 9 |
| 18 | 7;51 | 13 | 2;43 | 7 | 0;57 | 0 | 2;14 | 5 | 5;35 | 8 | 10; 4 | 10 |
| 19 | 7;38 | 13 | 2;36[15] | 7 | 0;57 | 0 | 2;19 | 5 | 5;43 | 8 | 10;13 | 9 |
| 20 | 7;25 | 13 | 2;30 | 6 | 0;57 | 0 | 2;24 | 5 | 5;51 | 8 | 10;23 | 10 |
| 21 | 7;13 | 12[16]↓ | 2;24 | 6 | 0;57 | 0 | 2;29 | 5 | 6; 0 | 9 | 10;33 | 10 |
| 22 | 7; 0 | 13 | **stationary** 2;18 | 6 | 0;58 | 1 | 2;34 | 5 | 6; 8 | 8 | 10;42[17] | 9 |
| 23 | 6;48 | 12⊥ | 2;12 | 6 | 0;58 | 0 | 2;40 | 6 | 6;17 | 9 | 10;52 | 10 |
| 24 | 6;36 | 12 | 2; 6 | 6 | 0;59 | 1 | 2;46 | 6 | 6;25 | 8 | 11; 2 | 10 |
| 25 | 6;24 | 12 | **progressive** 2; 0 | 6 | 1; 0 | 1 | 2;52 | 6 | 6;34 | 9 | 11;11 | 9 |
| 26 | 6;12 | 12 | 1;55[18] | 5 | 1; 1 | 1 | 2;58 | 6 | 6;43 | 9 | 11;21 | 10 |
| 27 | 6; 0[19] | 12 | 1;50 | 5 | 1; 3 | 2 | 3; 4 | 6 | 6;52 | 9 | 11;31 | 10 |
| 28 | 5;49 | 11 | 1;45 | 5 | 1; 5 | 2 | 3;10 | 6 | 7; 0 | 8[20] | 11;40 | 9 |
| 29 | 5;37[21] | 12 | 1;41 | 4[22] | 1; 7 | 2 | 3;16 | 6 | 7; 9[23] | 9 | 11;50 | 10 |

[1] **C** 24′ [2] **C** 9ˢ 3–5°: slide[+1 col.] [3] **$C_1C_2$** 4′ [4] **$C_2$** 13′ [5] **C** 16′ [6] **L** 9′ [7] **L** 10′ [8] **$C_1$** 44′ [9] **F** 5′ [10] **$C_1$** 4′ [11] **$C_2$** 18′ [12] **$FC_1$** 1′ [13] **F** 5′ [14] **FC** 13′ [15] **C** 37′ [16] **C** 6ˢ 21–23°: block transposition [17] **H** 43′ [18] **H** 2° [19] **$C_1C_2$** 5° [20] **C** 9′ [21] **$C_1$** 34′ [22] **F** 5′ [23] **C** 19′.

**Table 27: Variation of the nearest distance for Jupiter**
*Sources*: **F** fol. 64v, **H** fol. 52v, **C** fol. 64v, $\mathbf{C_1}$ fol. 30v, $\mathbf{C_2}$ fol. 42v, **Y** fol. 283v, **L** fol. 54v, **B** p. 104.

Variation of the Nearest Distance for Jupiter

| corrected centrum | 2[1→] | 3 | 4 | 5[→1] | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the furthest distance | 0; 4 | 0;20 | 0;30 | 0;31 | 0;22 | 0; 7 |
| 1 | | 0; 4 | 0;20 | 0;30 | 0;31 | 0;22 | 0; 6 |
| 2 | | 0; 5 | 0;21 | 0;30 | 0;31 | 0;21 | 0; 6 |
| 3 | | 0; 5 | 0;21 | 0;30 | 0;30 | 0;21 | 0; 5 |
| 4 | | 0; 6 | 0;21 | 0;31 | 0;30 | 0;21 | 0; 5 |
| 5 | | 0; 6 | 0;22 | 0;31 | 0;30 | 0;20 | 0; 4 |
| 6 | | 0; 7 | 0;22 | 0;31 | 0;30 | 0;20 | 0; 4 |
| 7 | | 0; 7 | 0;22 | 0;31 | 0;29 | 0;19 | 0; 4 |
| 8 | | 0; 8 | 0;23 | 0;31 | 0;29 | 0;19 | 0; 3 |
| 9 | | 0; 8 | 0;23 | 0;31 | 0;29 | 0;18 | 0; 3 |
| 10 | | 0; 9 | 0;24 | 0;32 | 0;29 | 0;18 | 0; 2 |
| 11 | | 0; 9 | 0;24 | 0;32 | 0;28 | 0;17 | 0; 2 |
| 12 | | 0;10 | 0;25 | 0;32 | 0;28 | 0;17 | 0; 1 |
| 13 | | 0;10 | 0;25 | 0;32 | 0;28 | 0;16 | 0; 1 |
| 14 | | 0;11 | 0;26 | 0;32 | 0;28 | 0;16 | 0; 0 |
| 15 | | 0;11 | 0;26 | 0;32 | 0;27[2] | 0;15 | From the furthest distance |
| 16 | | 0;12 | 0;26 | 0;32 | 0;27 | 0;14[3] | |
| 17 | | 0;12 | 0;27[4] | 0;32 | 0;27 | 0;14 | |
| 18 | | 0;13 | 0;27 | 0;32 | 0;27 | 0;13 | |
| 19 | | 0;14 | 0;27 | 0;32 | 0;27 | 0;12 | |
| 20 | | 0;14 | 0;27 | 0;32 | 0;26 | 0;12 | |
| 21 | | 0;15 | 0;27 | 0;32 | 0;26 | 0;11 | |
| 22 | 0; 0 | 0;16 | 0;28 | 0;32 | 0;26 | 0;11 | |
| 23 | 0; 1 | 0;16 | 0;28 | 0;32 | 0;25 | 0;10 | |
| 24 | 0; 1 | 0;17 | 0;28 | 0;32 | 0;25 | 0;10 | |
| 25 | 0; 2 | 0;17 | 0;28 | 0;32 | 0;24 | 0; 9 | |
| 26 | 0; 2[5] | 0;18 | 0;29 | 0;32 | 0;24 | 0; 9 | |
| 27 | 0; 3 | 0;18 | 0;29 | 0;31 | 0;23 | 0; 8 | |
| 28 | 0; 4[6] | 0;19 | 0;29 | 0;31 | 0;23 | 0; 8 | |
| 29 | 0; 4 | 0;19 | 0;29 | 0;31 | 0;22 | 0; 7 | |

1–1 **C** 1 2 3 0 (in *abjad*, corrected in red)  2 $\mathbf{CC_1C_2}$ 28′  3 **H** 15′  4 $\mathbf{CC_2}$ 26′  5 **Y** 3′  6 **F** 3′.

**Table 27: Interpolation minutes (nearest distance) for Jupiter**
*Sources*: **F** fol. 64v, **H** fol. 53r, **C** fol. 64v, $\mathbf{C_1}$ fol. 30v, $\mathbf{C_2}$ fol. 42v, **Y** fol. 283v, **L** fol. 54v, **B** p. 104.

| Minutes of Proportions ⟨for Jupiter⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **0** | **1** | **2** | **3** | **4** | **5** | |
| 0[1]↓ | 0 | 26 | 47 | 59 | 56 | 36 | 30 |
| 1 | 1 | 27 | 47 | 59 | 56 | 35 | 29 |
| 2 | 2 | 27 | 48 | 59 | 55 | 34 | 28 |
| 3 | 2[2]↓ | 28 | 48 | 59 | 55 | 33 | 27 |
| 4 | 3 | 29 | 49 | 60 | 55 | 32 | 26 |
| 5 | 4 | 29 | 49 | 60 | 54 | 31 | 25 |
| 6 | 5 | 30 | 50 | 60 | 54 | 30 | 24 |
| 7 | 6 | 31 | 50 | 60 | 53 | 29 | 23 |
| 8 | 7 | 31 | 51 | 60 | 53 | 28 | 22 |
| 9 | 7 | 32 | 51 | 60 | 52 | 26 | 21 |
| 10 | 8⊥ | 33 | 52 | 60 | 51 | 25 | 20 |
| 11 | 9[3] | 33 | 52 | 60 | 51 | 24 | 19 |
| 12 | 10 | 34 | 53 | 60 | 50 | 23 | 18 |
| 13 | 11 | 35 | 53 | 60 | 49 | 22 | 17 |
| 14 | 12 | 36 | 54 | 60 | 49 | 20[4] | 16 |
| 15 | 13 | 36 | 54 | 59 | 48 | 19 | 15 |
| 16 | 14 | 37 | 55 | 59 | 47 | 18 | 14 |
| 17 | 15 | 38 | 55 | 59 | 47 | 16 | 13 |
| 18 | 16 | 39 | 56 | 59 | 46 | 15 | 12 |
| 19 | 17 | 40 | 56 | 59 | 45 | 14 | 11 |
| 20 | 18 | 40[5] | 57 | 59 | 44 | 13 | 10 |
| 21 | 18 | 41 | 57 | 58 | 43[6]↓ | 11 | 9 |
| 22 | 19 | 42 | 57 | 58 | 43 | 10 | 8 |
| 23 | 20 | 42 | 58 | 58 | 42 | 9 | 7 |
| 24 | 21 | 43 | 58 | 58 | 41⊥ | 8 | 6 |
| 25 | 22 | 44 | 58 | 58 | 40 | 7 | 5 |
| 26 | 23 | 44 | 58 | 57 | 39 | 5 | 4 |
| 27 | 23 | 45 | 58 | 57 | 38 | 4 | 3 |
| 28 | 24 | 46 | 59 | 57 | 38 | 3 | 2 |
| 29⊥ | 25 | 46 | 59 | 56 | 37 | 1 | 1 |
| parts of the epicycle | **11** | **10** | **9** | **8** | **7** | **6** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **Y** arguments 0, 1, 2, ...29: 1, 2, 3, ..., 30 [2] **C** $0^s$ 3–10°: slide[+1] [3] **C** 11′ (end of slide)
[4] **C** 22′ [5] $\mathbf{CC_1C_2}$ 41′ [6] **L** $4^s$ 21–24°: slide[−1].

**Table 27: Variation of the furthest distance for Jupiter**
*Sources*: **F** fol. 65r, **H** fol. 53v, **C** fol. 65r, **$C_1$** fol. 31r, **$C_2$** fol. 25r, **Y** fol. 284r, **L** fol. 55r, **B** p. 105.

Variation of the Furthest Distance for Jupiter

| corrected centrum | 8 | 9 | 10 | 11 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the nearest distance | 0; 8 | 0;21 | 0;28 | 0;29 | 0;23 | 0;11 |
| 1 | | 0; 9 | 0;21 | 0;28 | 0;29 | 0;23 | 0;11 |
| 2 | | 0; 9 | 0;22 | 0;28 | 0;29 | 0;22 | 0;10 |
| 3 | | 0;10 | 0;22 | 0;28 | 0;28 | 0;22 | 0;10 |
| 4 | | 0;10 | 0;22 | 0;29 | 0;28 | 0;22 | 0; 9 |
| 5 | | 0;11 | 0;23 | 0;29 | 0;28 | 0;21 | 0; 9 |
| 6 | | 0;11 | 0;23 | 0;29 | 0;28 | 0;21 | 0; 8 |
| 7 | | 0;12 | 0;23 | 0;29 | 0;28 | 0;21 | 0; 7 |
| 8 | | 0;12 | 0;24 | 0;29 | 0;28 | 0;20 | 0; 7 |
| 9 | | 0;13 | 0;24 | 0;29 | 0;27 | 0;20 | 0; 6 |
| 10 | | 0;13 | 0;24 | 0;29 | 0;27 | 0;20 | 0; 5 |
| 11 | | 0;14 | 0;25↓[1] | 0;29 | 0;27 | 0;19 | 0; 5 |
| 12 | | 0;14 | 0;25⊥ | 0;29 | 0;27 | 0;19 | 0; 4 |
| 13 | | 0;14[2] | 0;25 | 0;29 | 0;27 | 0;19 | 0; 4 |
| 14 | | 0;15 | 0;25 | 0;29 | 0;27 | 0;18 | 0; 3 |
| 15 | 0; 0 | 0;15 | 0;25 | 0;29 | 0;26↓[3] | 0;18 | 0; 3 |
| 16 | 0; 1 | 0;15 | 0;26 | 0;29 | 0;26 | 0;17 | 0; 2 |
| 17 | 0; 1 | 0;16 | 0;26 | 0;29 | 0;26⊥ | 0;17 | 0; 2 |
| 18 | 0; 1 | 0;16 | 0;26 | 0;29 | 0;26 | 0;16 | 0; 1[4] |
| 19 | 0; 2 | 0;17 | 0;26 | 0;29 | 0;26 | 0;16 | 0; 1 |
| 20 | 0; 2 | 0;17 | 0;26 | 0;29 | 0;26 | 0;15 | 0; 1 |
| 21 | 0; 3↓[5] | 0;18↓→[6] | 0;26[7] | 0;29 | 0;25 | 0;15 | 0; 0 |
| 22 | 0; 3⊥ | 0;18 | 0;27 | 0;29 | 0;25 | 0;15 | From the nearest distance |
| 23 | 0; 4↓[8] | 0;19 | 0;27⌟ | 0;29 | 0;25 | 0;14 | |
| 24 | 0; 4 | 0;19 | 0;27 | 0;29 | 0;25 | 0;14 | |
| 25 | 0; 5 | 0;19 | 0;27 | 0;29 | 0;25 | 0;14 | |
| 26 | 0; 5⊥ | 0;20 | 0;27 | 0;29 | 0;24 | 0;13 | |
| 27 | 0; 6↓[9] | 0;20 | 0;27 | 0;29 | 0;24 | 0;13 | |
| 28 | 0; 7[10] | 0;20 | 0;28 | 0;29 | 0;24 | 0;12 | |
| 29 | 0; 7⊥ | 0;21[11] | 0;28 | 0;29 | 0;23 | 0;12 | |

[1] **F** $10^s$ 11–12°: 24′ [2] **$CC_1C_2$** 15′ [3] **C** $0^s$ 15–17°: slide[+1 col.] **L** 27′ [4] **F** 2′ [5] **C** $8^s$ 21–22°: illegible corrections (presumably related to the slide in note 6) [6] **C** 9–$10^s$ 21–23°: slide[+1 col.] [7] **+HL** 27′ [8] **C** $8^s$ 23–26°: slide[−6] [9] **C** $8^s$ 27–29°: slide[−5] [10] **+F** 6′ [11] **$C_1$** 20′.

**Table 27: Interpolation minutes (furthest distance) for Jupiter**
*Sources*: **F** fol. 65r, **H** fol. 54r, **C** fol. 65r, **$C_1$** fol. 31r, **$C_2$** fol. 25r, **Y** fol. 284r, **L** fol. 55r, **B** p. 105.

| Minutes of Proportions ⟨for Jupiter⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0 | 0 | 36 | 56 | 59 | 47 | 26[1]↓ | 30 |
| 1 | 1 | 37 | 56 | 59 | 46 | 25 | 29 |
| 2 | 3[2] | 38 | 57 | 59 | 46 | 24 | 28 |
| 3 | 4[3] | 38 | 57 | 58 | 45 | 23 | 27 |
| 4 | 5 | 39 | 57 | 58 | 44 | 23 | 26 |
| 5 | 7 | 40 | 58 | 58 | 44 | 22 | 25 |
| 6 | 8 | 41 | 58 | 58 | 43 | 21 | 24 |
| 7 | 9 | 42 | 58 | 58 | 42 | 20 | 23 |
| 8 | 10 | 43 | 58 | 57 | 42 | 19 | 22 |
| 9 | 11 | 43 | 58[4] | 57 | 41 | 18 | 21 |
| 10 | 13 | 44 | 59 | 57 | 40 | 18 | 20 |
| 11 | 14 | 45 | 59 | 56 | 40 | 17 | 19 |
| 12 | 15 | 46 | 59 | 56[5] | 39 | 16 | 18 |
| 13 | 16 | 47 | 59 | 55 | 38 | 15 | 17 |
| 14 | 18 | 47 | 59 | 55 | 37 | 14 | 16 |
| 15 | 19 | 48[6] | 59 | 54 | 36 | 13 | 15 |
| 16 | 20[7] | 49[8] | 60 | 54 | 36 | 12 | 14 |
| 17 | 22 | 49 | 60 | 53 | 35 | 11 | 13 |
| 18 | 23 | 50 | 60 | 53 | 34 | 10 | 12 |
| 19 | 24 | 51[9] | 60 | 52 | 34[10]↓ | 9 | 11 |
| 20 | 25 | 51 | 60 | 52 | 33 | 8 | 10 |
| 21 | 26 | 52 | 60 | 51 | 32⊥ | 7 | 9 |
| 22 | 28[11] | 53 | 60 | 51 | 31 | 7 | 8 |
| 23 | 29 | 53 | 60 | 50 | 31 | 6 | 7 |
| 24 | 30 | 54 | 60 | 50 | 30 | 5 | 6 |
| 25 | 31 | 54 | 60 | 49 | 29 | 4 | 5 |
| 26 | 32 | 55 | 60 | 49 | 29 | 3⊥ | 4 |
| 27 | 33 | 55 | 59[12]↓ | 48 | 28 | 2 | 3 |
| 28 | 34 | 55 | 59 | 48 | 27 | 2 | 2 |
| 29 | 35 | 56 | 59⊥ | 47 | 27 | 1 | 1 |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **H** $11^s$ 0–26°: slide[+1] [2] **Y** 2′ [3] **Y** 3′ [4] **F** 59′ [5] **C** 57′ [6] **C** 43′ [7] **C** 22′ [8] **F** 48′
[9] **$CC_1C_2$** 50′ [10] **Y** $10^s$ 19–21°: slide[+1] **B** 33′ [11] **C** 23′ [12] **YL** $8^s$ 27–29°: 60′.

**Table 28: Mean motion of Mars (first part)**

*Sources*: **F** fols 65v–66r, **H** fols 54v–55r, **C** fol. 65v, $\mathbf{C_1}$ fol. 31v, $\mathbf{C_2}$ fol. 25v, **Y** fols 284v–285r, **L** fols 55v–56r, **B** pp. 106–107 (see Plate 9). The essentially different subtables for collected, extended and single years in **YLB** are separately edited on p. 168.

| Mean Motion of Mars in Years and Months | | | | | |
|---|---|---|---|---|---|
| years | collected ⟨years⟩<br>s ° ′ | years | extended ⟨years⟩<br>s ° ′ | months | months<br>s ° ′ |
| 1 | $8^s$ 12;34 | 1 | $6^s$ 11;17[1] | Farwardīn | $0^s$ 0; 0 |
| 21 | $3^s$ 28;22 | 2 | $0^s$ 22;35 | | |
| 41 | $11^s$ 14;10 | 3 | $7^s$ 3;52[2] | Urdībihisht | $0^s$ 15;43 |
| 61 | $6^s$ 29;59 | 4 | $1^s$ 15;10 | | |
| 81 | $2^s$ 15;47 | 5 | $7^s$ 26;27 | Khurdād | $1^s$ 1;27[3] |
| 101 | $10^s$ 1;35 | 6 | $2^s$ 7;44 | | |
| 121 | $5^s$ 17;24[4] | 7 | $8^s$ 19; 2 | Tīr | $1^s$ 17;10 |
| 141 | $1^s$ 3;12 | 8 | $3^s$ 0;19 | | |
| 161 | $8^s$ 19; 0 | 9 | $9^s$ 11;37 | Murdād | $2^s$ 2;53 |
| 181 | $4^s$ 4;49 | 10 | $3^s$ 22;54 | | |
| 201 | $11^s$ 20;37 | 11 | $10^s$ 4;12[5] | Shahrīwar | $2^s$ 18;37 |
| 221 | $7^s$ 6;25 | 12 | $4^s$ 15;29[6] | | |
| 241 | $2^s$ 22;13 | 13 | $10^s$ 26;46 | Mihr | $3^s$ 4;20 |
| 261 | $10^s$ 8; 2 | 14 | $5^s$ 8; 4 | | |
| 281 | $5^s$ 23;50 | 15 | $11^s$ 19;21 | Ābān | $3^s$ 20; 3 |
| 301 | $1^s$ 9;38 | 16 | $6^s$ 0;39 | | |
| 321 | $8^s$ 25;27 | 17 | $0^s$ 11;56 | Ādhar | $4^s$ 5;47 |
| 341 | $4^s$ 11;15 | 18 | $6^s$ 23;13 | | *$4^s$ 8;24* |
| 361 | $11^s$ 27; 3 | 19 | $1^s$ 4;31 | Day | $4^s$ 21;30 |
| 381 | $7^s$ 12;51 | 20 | $7^s$ 15;48 | | *$4^s$ 24; 7* |
| 401 | $2^s$ 28;40 | single years | | Bahman | $5^s$ 7;14[7] |
| 421 | $10^s$ 14;28 | | | | *$5^s$ 9;51* |
| 441 | $6^s$ 0;16 | 40 | $3^s$ 1;37 | Isfandārmudh | $5^s$ 22;57 |
| 461 | $1^s$ 16; 5 | 60 | $10^s$ 17;25 | | *$5^s$ 25;34* |
| 481 | $9^s$ 1;53 | 80 | $6^s$ 3;13 | | |
| 501[8] | $4^s$ 17;41 | 100 | $1^s$ 19; 1 | | |
| 521 | $0^s$ 3;29 | 200 | $3^s$ 8; 3 | | |
| 541 | $7^s$ 19;18 | 300 | $4^s$ 27; 4[9] | | |
| 561 | $3^s$ 5; 6 | 400 | $6^s$ 16; 6 | | |
| 581 | $10^s$ 20;54 | 500 | $8^s$ 5; 7 | | |

[1] $\mathbf{C_1C_2}$ 57′ [2] **H** 57′ [3] **F** 26′ [4] $\mathbf{C_1}$ 14° [5] **F** 52′ [6] **H** 23′ (the '3' apparently corrected by a different hand) [7] **YLB** 13′ [8] **C** 81 or 101 (ڡا) [9] **F** 20′.

**Table 28: Mean motion of Mars (second part)**
*Sources*: **F** fols 65v–66r, **H** fols 54v–55r, **C** fol. 65v, **$C_1$** fol. 31v, **$C_2$** fol. 25v, **Y** fols 284v–285r, **L** fols 55v–56r, **B** pp. 106–107.

Mean Motion of Mars in Days, Hours, and Between Longitudes

| days | days (s ° ′) | hours | hours (° ′) | hours | and their fractions (° ′) | longitudes | | in between longitudes (′ ″) |
|---|---|---|---|---|---|---|---|---|
| 1 | 0s 0; 0 | 1 | 0; 1 | 31 | 0;40* | 71 | additive | 0, 2[1]↓↓ |
| 2 | 0s 0;31 | 2 | 0; 3 | 32 | 0;42 | 72 | | 0, 2⊥ |
| 3 | 0s 1; 3 | 3 | 0; 4 | 33 | 0;43 | 73 | | 0, 1 |
| 4 | 0s 1;34[2] | 4 | 0; 5 | 34 | 0;44[3] | 74 | | 0, 1 |
| 5 | 0s 2; 6 | 5 | 0; 7 | 35 | 0;46 | 75 | | 0, 1 |
| 6 | 0s 2;37 | 6 | 0; 8 | 36 | 0;47 | 76 | | 0, 1 |
| 7 | 0s 3; 9 | 7 | 0; 9 | 37 | 0;48[4] | 77 | | 0, 1 |
| 8 | 0s 3;40 | 8 | 0;10 | 38 | 0;50 | 78 | | 0, 1 |
| 9 | 0s 4;12 | 9 | 0;12 | 39 | 0;51 | 79 | | 0, 1 |
| 10 | 0s 4;43 | 10 | 0;13 | 40 | 0;52 | 80 | | 0, 1 |
| 11 | 0s 5;14 | 11 | 0;14 | 41 | 0;54 | 81 | | 0, 1 |
| 12 | 0s 5;46 | 12 | 0;16 | 42 | 0;55 | 82 | | 0, 1 |
| 13 | 0s 6;17 | 13 | 0;17 | 43 | 0;56 | 83 | | 0, 1 |
| 14 | 0s 6;49 | 14 | 0;18 | 44 | 0;58 | 84 | | 0, 1 |
| 15 | 0s 7;20 | 15 | 0;20 | 45 | 0;59 | 85 | | 0, 0 |
| 16 | 0s 7;52 | 16 | 0;21 | 46 | 1; 0 | 86 | | 0, 0 |
| 17 | 0s 8;23 | 17 | 0;22 | 47 | 1; 2 | 87 | | 0, 0 |
| 18 | 0s 8;55 | 18 | 0;24 | 48 | 1; 3 | 88 | | 0, 0 |
| 19 | 0s 9;26 | 19 | 0;25 | 49 | 1; 4 | 89 | | 0, 0 |
| 20 | 0s 9;57 | 20 | 0;26 | 50 | 1; 5* | 90 | | 0, 0 |
| 21 | 0s 10;29 | 21 | 0;28 | 51 | 1; 7 | 91 | subtractive | 0, 0 |
| 22 | 0s 11; 0 | 22 | 0;29 | 52 | 1; 8 | 92 | | 0, 0 |
| 23 | 0s 11;32 | 23 | 0;30 | 53 | 1; 9 | 93 | | 0, 0 |
| 24 | 0s 12; 3 | 24 | 0;31 | 54 | 1;10* | 94 | | 0, 0 |
| 25 | 0s 12;35 | 25 | 0;32* | 55 | 1;11* | 95 | | 0, 0 |
| 26 | 0s 13; 6 | 26 | 0;34 | 56 | 1;13 | 96 | | 0, 1 |
| 27 | 0s 13;38 | 27 | 0;35 | 57 | 1;14* | 97 | | 0, 1 |
| 28 | 0s 14; 9 | 28 | 0;36* | 58 | 1;15* | 98 | | 0, 1 |
| 29 | 0s 14;40 | 29 | 0;38 | 59 | 1;17[5] | 99 | | 0, 1 |
| 30 | 0s 15;12[6] | 30 | 0;39 | 60 | 1;18* | 100 | | 0, 1⊥ |

*In **YLB** the values for 25, 28, 31, 34, 50, 54–55, 57–58, and 60 hours are 1 minute larger.
[1] **F** longitudes 71–100°: 0°0′ **$CC_1C_2$** longitudes 71–72°: 1′ [2] **L** 4′ [3] +**$C_1$** 45′ [4] **Y** 18′ [5] **F** 16′ [6] **F** 2′.

**Table 28a: Mean motion of Mars in sources YLB**

*Sources*: **Y** fol. 284v, **L** fol. 55v, **B** p. 106. The subtables for collected, extended and single years in these three witnesses are essentially different from the ones in the other five, which are edited on p. 166.

Mean Motion of Mars in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 8$^s$ 12;34 | 1 | 6$^s$ 11;17 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 3$^s$ 28;19[1] | 2 | 0$^s$ 22;35[2] | | |
| 41 | 11$^s$ 14; 4 | 3 | 7$^s$ 3;52 | Urdībihisht | 0$^s$ 15;43 |
| 61 | 6$^s$ 29;49[3] | 4 | 1$^s$ 15; 9 | | |
| 81 | 2$^s$ 15;34 | 5 | 7$^s$ 26;26 | Khurdād | 1$^s$ 1;27 |
| 101 | 10$^s$ 1;19 | 6 | 2$^s$ 7;43 | | |
| 121 | 5$^s$ 17; 4 | 7 | 8$^s$ 19; 1 | Tīr | 1$^s$ 17;10 |
| 141 | 1$^s$ 2;49 | 8 | 3$^s$ 0;18 | | |
| 161 | 8$^s$ 18;34 | 9 | 9$^s$ 11;35 | Murdād | 2$^s$ 2;53 |
| 181 | 4$^s$ 4;18 | 10 | 3$^s$ 22;52 | | |
| 201 | 11$^s$ 20; 3 | 11 | 10$^s$ 4;10 | Shahrīwar | 2$^s$ 18;37 |
| 221 | 7$^s$ 5;48 | 12 | 4$^s$ 15;27 | | |
| 241 | 2$^s$ 21;33 | 13 | 10$^s$ 26;44 | Mihr | 3$^s$ 4;20 |
| 261 | 10$^s$ 7;18 | 14 | 5$^s$ 8; 1 | | |
| 281 | 5$^s$ 23; 3 | 15 | 11$^s$ 19;19 | Ābān | 3$^s$ 20; 3 |
| 301 | 1$^s$ 8;48 | 16 | 6$^s$ 0;36 | | |
| 321 | 8$^s$ 24;33[4] | 17 | 0$^s$ 11;53 | Ādhar | 4$^s$ 5;47 |
| 341 | 4$^s$ 10;18 | 18 | 6$^s$ 23;10 | | *4$^s$ 8;24* |
| 361 | 11$^s$ 26; 3 | 19 | 1$^s$ 4;28 | Day | 4$^s$ 21;30 |
| 381 | 7$^s$ 11;48 | 20 | 7$^s$ 15;45 | | *4$^s$ 24; 7* |
| 401 | 2$^s$ 27;33 | single years | | Bahman | 5$^s$ 7;13 |
| 421 | 10$^s$ 13;18 | | | | *5$^s$ 9;51* |
| 441 | 5$^s$ 29; 3[5] | 40 | 3$^s$ 1;30 | Isfandārmudh | 5$^s$ 22;57 |
| 461 | 1$^s$ 14;48[6] | 60 | 10$^s$ 17;15 | | *5$^s$ 25;34* |
| 481 | 9$^s$ 0;33 | 80 | 6$^s$ 3; 0[7] | | |
| 501 | 4$^s$ 16;18 | 100 | 1$^s$ 18;45 | | |
| 521 | 0$^s$ 2; 3 | 200 | 3$^s$ 7;30 | | |
| 541 | 7$^s$ 17;48 | 300 | 4$^s$ 26;14[8] | | |
| 561 | 3$^s$ 3;33 | 400 | 6$^s$ 14;59[9] | | |
| 581 | 10$^s$ 19;18[10] | 500 | 8$^s$ 3;44[11] | | |
| | | 600 | 9$^s$ 22;29 | | |

In **B** the values for collected years are all two degrees smaller than the ones in **YL**. To the left of the subtables for collected and extended years, the main hand of **B** writes the minutes from **FHCC$_1$C$_2$**, for the collected years labeled *nuskha hadhihi al-daqāʾiq* 'copy of these minutes' (cf. Section IV.5.3).

[1] +**Y** 49′ [2] **Y** 37′ [3] +**Y** 45′ [4] **B**+ 38′ [5] **B**+ 24° (instead of 27°) [6] +**L** 20$^s$ [7] **YL** 2°
[8] **B** 54′ [9] **L** 19′ [10] +**L** 20°54′ (as in **FCC$_1$C$_2$**!) [11] **B** 0°.

**Table 29: Mean anomaly of Mars (first part)**
*Sources*: **F** fols 66v–67r, **H** fols 55v–56r, **C** fol. 66r, $\mathbf{C_1}$ fol. 32r, $\mathbf{C_2}$ fol. 26r, **Y** fols 285v–286r, **L** fols 56v–57r, **B** pp. 108–109. The essentially different subtables for collected, extended and single years in **YLB** are separately edited on p. 171.

Mean Anomaly of Mars in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | $4^s$ 27;22 | 1 | $5^s$ 18;28[1] | Farwardīn | $0^s$ 0; 0 |
| 21 | $9^s$ 6;49[2] | 2 | $11^s$ 6;57[3] | | |
| 41 | $1^s$ 16;16[4] | 3 | $4^s$ 25;25 | Urdībihisht | $0^s$ 13;51 |
| 61 | $5^s$ 25;43 | 4 | $10^s$ 13;53[5] | | |
| 81 | $10^s$ 5;10 | 5 | $4^s$ 2;22 | Khurdād | $0^s$ 27;42 |
| 101 | $2^s$ 14;38[6] | 6 | $9^s$ 20;50 | | |
| 121 | $6^s$ 24; 5 | 7 | $3^s$ 9;19 | Tīr | $1^s$ 11;33 |
| 141 | $11^s$ 3;32 | 8 | $8^s$ 27;47 | | |
| 161 | $3^s$ 12;59[7] | 9 | $2^s$ 16;15 | Murdād | $1^s$ 25;23[8] |
| 181 | $7^s$ 22;26 | 10 | $8^s$ 4;44 | | |
| 201 | $0^s$ 1;54 | 11 | $1^s$ 23;12 | Shahrīwar | $2^s$ 9;14 |
| 221 | $4^s$ 11;21 | 12 | $7^s$ 11;40 | | |
| 241 | $8^s$ 20;48 | 13 | $1^s$ 0; 9 | Mihr | $2^s$ 23; 5 |
| 261 | $1^s$ 0;15 | 14 | $6^s$ 18;37[9] | | |
| 281 | $5^s$ 9;42[10] | 15 | $0^s$ 7; 5 | Ābān | $3^s$ 6;56 |
| 301 | $9^s$ 19; 9 | 16 | $5^s$ 25;34 | | |
| 321 | $1^s$ 28;37 | 17 | $11^s$ 14; 2 | Ādhar | $3^s$ 20;47 |
| 341 | $6^s$ 8; 4 | 18 | $5^s$ 2;30 | | *$3^s$ 23; 5* |
| 361 | $10^s$ 17;31 | 19 | $10^s$ 20;59 | Day | $4^s$ 4;38 |
| 381 | $2^s$ 26;58[11] | 20 | $4^s$ 9;27[12] | | *$4^s$ 6;56*[13] |
| 401 | $7^s$ 6;25 | single years | | Bahman | $4^s$ 18;29[14] |
| 421 | $11^s$ 15;53 | | | | *$4^s$ 20;47* |
| 441 | $3^s$ 25;20 | 40 | $8^s$ 18;54 | Isfandārmudh | $5^s$ 2;20[15] |
| 461 | $8^s$ 4;47 | 60 | $0^s$ 28;22 | | *$5^s$ 4;38* |
| 481 | $0^s$ 14;14 | 80 | $5^s$ 7;49 | | |
| 501[16] | $4^s$ 23;41 | 100 | $9^s$ 17;16 | | |
| 521 | $9^s$ 3; 8 | 200 | $7^s$ 4;32 | | |
| 541 | $1^s$ 12;36 | 300 | $4^s$ 21;48 | | |
| 561 | $5^s$ 22; 3 | 400 | $2^s$ 9; 4 | | |
| 581 | $10^s$ 1;30 | 500 | $11^s$ 26;20 | | |

[1] **F** 38° [2] **F** 48′ [3] **F** 17′ [4] **C** 46′ [5] **C** 13′ [6] **C** 18′ [7] **FH** 13° **C** 19′ $\mathbf{C_1}$ 22° [8] **C** 33′ [9] $\mathbf{C_1}$ 34′ [10] **C** 45′ [11] **C** 18′ [12] $\mathbf{C_1}$ 24′ [13] **FB** 16′ [14] **YLB** 28′ [15] **YLB** 19′ [16] **C** 81 or 101 (ڡا).

**Table 29: Mean anomaly of Mars (second part)**
*Sources*: **F** fols 66v–67r, **H** fols 55v–56r, **C** fol. 66r, $\mathbf{C_1}$ fol. 32r, $\mathbf{C_2}$ fol. 26r, **Y** fols 285v–286r, **L** fols 56v–57r, **B** pp. 108–109.

| Mean Anomaly of Mars in Days, Hours, and Between Longitudes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | $0^s$ 0; 0 | 1 | 0; 1 | 31 | 0;36 | 71[1]↓ | additive | 0, 1[2]↓ |
| 2 | $0^s$ 0;28 | 2 | 0; 2 | 32 | 0;37 | 72 | | 0, 1 |
| 3 | $0^s$ 0;55 | 3 | 0; 3 | 33 | 0;39[3] | 73 | | 0, 1 |
| 4 | $0^s$ 1;23 | 4 | 0; 4* | 34 | 0;40* | 74 | | 0, 1 |
| 5 | $0^s$ 1;51[4] | 5 | 0; 5* | 35 | 0;41* | 75 | | 0, 1 |
| 6 | $0^s$ 2;18 | 6 | 0; 7 | 36 | 0;42 | 76 | | 0, 1 |
| 7 | $0^s$ 2;46 | 7 | 0; 8 | 37 | 0;43 | 77 | | 0, 1 |
| 8 | $0^s$ 3;14 | 8 | 0; 9 | 38 | 0;44 | 78 | | 0, 1 |
| 9 | $0^s$ 3;42 | 9 | 0;10 | 39 | 0;46* | 79 | | 0, 1 |
| 10 | $0^s$ 4; 9 | 10 | 0;12 | 40[5] | 0;47* | 80 | | 0, 1 |
| 11 | $0^s$ 4;37 | 11 | 0;13 | 41 | 0;48* | 81 | | 0, 1 |
| 12 | $0^s$ 5; 5 | 12 | 0;14 | 42 | 0;49* | 82 | | 0, 1 |
| 13 | $0^s$ 5;32 | 13 | 0;15 | 43 | 0;50 | 83 | | 0, 1 |
| 14 | $0^s$ 6; 0 | 14 | 0;16 | 44 | 0;51 | 84 | | 0, 0 |
| 15 | $0^s$ 6;28 | 15 | 0;17 | 45 | 0;53* | 85 | | 0, 0 |
| 16 | $0^s$ 6;55 | 16 | 0;18 | 46 | 0;54* | 86 | | 0, 0 |
| 17 | $0^s$ 7;23 | 17 | 0;19* | 47 | 0;55* | 87 | | 0, 0 |
| 18 | $0^s$ 7;51 | 18 | 0;20* | 48 | 0;56* | 88 | | 0, 0 |
| 19 | $0^s$ 8;18[6] | 19 | 0;22 | 49 | 0;57 | 89 | | 0, 0 |
| 20 | $0^s$ 8;46 | 20 | 0;23 | 50 | 0;58 | 90 | | 0, 0 |
| 21 | $0^s$ 9;14 | 21 | 0;24 | 51 | 1; 0* | 91 | subtractive | 0, 0 |
| 22 | $0^s$ 9;42 | 22 | 0;25 | 52 | 1; 1* | 92 | | 0, 0 |
| 23 | $0^s$ 10; 9 | 23 | 0;27 | 53 | 1; 2* | 93 | | 0, 0 |
| 24 | $0^s$ 10;37 | 24 | 0;28 | 54 | 1; 3* | 94 | | 0, 0 |
| 25 | $0^s$ 11; 5 | 25 | 0;29 | 55 | 1; 4* | 95 | | 0, 0 |
| 26 | $0^s$ 11;32 | 26 | 0;30[7] | 56 | 1; 5 | 96 | | 0, 0 |
| 27 | $0^s$ 12; 0 | 27 | 0;32[8] | 57 | 1; 6[9] | 97 | | 0, 1 |
| 28 | $0^s$ 12;28 | 28 | 0;33* | 58 | 1; 8* | 98⊥ | | 0, 1 |
| 29 | $0^s$ 12;55 | 29 | 0;34* | 59 | 1; 9* | 99[10] | | 0, 1 |
| 30 | $0^s$ 13;23 | 30 | 0;35 | 60 | 1;10* | 100[11] | | 0, 1⊥ |

* In **YLB** the values for 4–5 and 17–18 hours are 1 minute larger, and the values for 27–29, 33–35, 39–42, 45–48, 51–55, and 58–60 hours are 1 minute smaller. [1] **L** arguments 71–98°: slide[+2] (arguments 71 and 72° are written in the otherwise empty cells intended for the column headers) [2] **F** longitudes 71–100°: 0′0″ [3] +$\mathbf{C_1C_2}$ 38′ (as in **YLB**) [4] **F** 11′ [5] **C** 20 (written over 7?) [6] **YL** 19′ [7] **C** 32′ [8] +**C** 30′ [9] $\mathbf{FHC_1}$ 7′ [10] **L** 101 (end of slide) [11] **L** 102.

**Table 29a: Mean anomaly of Mars in sources YLB**

*Sources*: **Y** fol. 285v, **L** fol. 56v, **B** p. 108. The subtables for collected, extended and single years in these three witnesses are essentially different from the ones in the other five, which are edited on p. 169.

Mean Anomaly of Mars in Years and Months

| years | collected ⟨years⟩ s ° ′ |
|---|---|
| 1 | 4$^s$ 27;21 |
| 21 | 9$^s$ 6;51 |
| 41 | 1$^s$ 16;22 |
| 61 | 5$^s$ 25;52 |
| 81 | 10$^s$ 5;23 |
| 101 | 2$^s$ 14;53 |
| 121 | 6$^s$ 24;24 |
| 141 | 11$^s$ 3;54 |
| 161 | 3$^s$ 13;25[1] |
| 181 | 7$^s$ 22;55 |
| 201 | 0$^s$ 2;26 |
| 221 | 4$^s$ 11;56 |
| 241 | 8$^s$ 21;27 |
| 261 | 1$^s$ 0;57 |
| 281 | 5$^s$ 10;28 |
| 301 | 9$^s$ 19;59 |
| 321 | 1$^s$ 29;29 |
| 341 | 6$^s$ 9; 0 |
| 361 | 10$^s$ 18;30 |
| 381 | 2$^s$ 28; 1 |
| 401 | 7$^s$ 7;31 |
| 421 | 11$^s$ 17; 2 |
| 441 | 3$^s$ 26;32 |
| 461 | 8$^s$ 6; 3[4] |
| 481 | 0$^s$ 15;33 |
| 501 | 4$^s$ 25; 4 |
| 521 | 9$^s$ 4;34 |
| 541 | 1$^s$ 14; 5 |
| 561 | 5$^s$ 23;35 |
| 581 | 10$^s$ 3; 6 |

| years | extended ⟨years⟩ s ° ′ |
|---|---|
| 1 | 5$^s$ 18;29 |
| 2 | 11$^s$ 6;57 |
| 3 | 4$^s$ 25;26 |
| 4 | 10$^s$ 13;54 |
| 5 | 4$^s$ 2;23 |
| 6 | 9$^s$ 20;51 |
| 7 | 3$^s$ 9;20 |
| 8 | 8$^s$ 27;48 |
| 9 | 2$^s$ 16;17 |
| 10 | 8$^s$ 4;45 |
| 11 | 1$^s$ 23;14 |
| 12 | 7$^s$ 11;42[2] |
| 13 | 1$^s$ 0;11 |
| 14 | 6$^s$ 18;39 |
| 15 | 0$^s$ 7; 8 |
| 16 | 5$^s$ 25;36 |
| 17 | 11$^s$ 14; 5 |
| 18 | 5$^s$ 2;33 |
| 19 | 10$^s$ 21; 2 |
| 20 | 4$^s$ 9;31 |
| single years | |
| 40 | 8$^s$ 19; 1 |
| 60 | 0$^s$ 28;32[5] |
| 80 | 5$^s$ 8; 2 |
| 100 | 9$^s$ 17;33 |
| 200 | 7$^s$ 5; 5 |
| 300 | 4$^s$ 22;38 |
| 400 | 2$^s$ 10;10 |
| 500 | 11$^s$ 27;43 |
| 600 | 9$^s$ 15;16[6] |

| months | months s ° ′ |
|---|---|
| Farwardīn | 0$^s$ 0; 0 |
| Urdībihisht | 0$^s$ 13;51 |
| Khurdād | 0$^s$ 27;42 |
| Tīr | 1$^s$ 11;33 |
| Murdād | 1$^s$ 25;23 |
| Shahrīwar | 2$^s$ 9;14 |
| Mihr | 2$^s$ 23; 5 |
| Ābān | 3$^s$ 6;56 |
| Ādhar | 3$^s$ 20;47 |
| | *3$^s$ 23; 5* |
| Day | 4$^s$ 4;38 |
| | *4$^s$ 6;56*[3] |
| Bahman | 4$^s$ 18;28 |
| | *4$^s$ 20;47* |
| Isfandārmudh | 5$^s$ 2;19 |
| | *5$^s$ 4;38* |

In **B** the values for collected years are all two degrees larger than the ones in **YL**. To the left of the subtables for collected years, the main hand of **B** writes the minutes from **FHCC$_1$C$_2$**, labeled *nuskha* 'copy' (cf. Section IV.5.3). [1] **B** degrees ill. [2] **YB** 2$^s$ [3] **B** 16′ [4] +**Y** 28$^s$ [5] **B** 31′ [6] **Y** 0$^s$ **B** 15′.

**Table 30: First equation for Mars (first half)**

*Sources*: **F** fols 67v–68r, **H** fols 56v–57r, **C** fols 66v–67r, $\mathbf{C_1}$ fols 32v–33r, $\mathbf{C_2}$ fols 26v–27v, **Y** fols 286v–287r, **L** fols 57v–58r, **B** pp. 110–111.

First Equation for Mars
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 0 | differences ′ | mean distance 6° 1 | differences ′ | 2 | differences ′ | 3 | differences ′ | nearest distance 1° 4 | differences ′ | 5 | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | |
| 0 | 2;38[1] | 7[2] | 0;33 | 1 | 1;28 | 5 | 5;30 | 10 | 11;46 | 13 | 18; 8 | 11 |
| 1 | 2;32 | 6 | 0;32 | 1 | 1;34[3] | 6 | 5;41 | 11 | 12; 0 | 14 | 18;19 | 11 |
| 2 | 2;25[4] | 7 | 0;31 | 1 | 1;39 | 5 | 5;52 | 11 | 12;14 | 14 | 18;30 | 11 |
| 3 | 2;19 | 6 | 0;30 | 1 | 1;44 | 5 | 6; 3 | 11 | 12;27[5] | 13 | 18;40[6] | 10 |
| 4 | 2;13 | 6 | 0;30 | 0 | 1;50 | 6 | 6;15 | 12 | 12;40 | 13 | 18;50 | 10 |
| 5 | 2; 7[7] | 6 | 0;30 | 0 | 1;56 | 6 | 6;26 | 11 | 12;54 | 14 | 19; 0[8] | 10 |
| 6 | 2; 1[9] | 6 | 0;30 | 0 | 2; 2 | 6 | 6;37 | 11 | 13; 7 | 13 | 19;10 | 10 |
| 7 | 1;56 | 5[10] | 0;30 | 0 | 2; 8 | 6 | 6;49 | 12 | 13;21 | 14[11]↓ | 19;20 | 10 |
| 8 | 1;51 | 5 | 0;30 | 0 | 2;15 | 7 | 7; 1 | 12 | 13;34 | 13 | 19;30 | 10 |
| 9 | 1;46 | 5 | 0;31 | 1 | 2;22[12]↓ | 7 | 7;13[13] | 12 | 13;47 | 13 | 19;40 | 10 |
| 10 | 1;41 | 5 | 0;32 | 1 | 2;29 | 7 | 7;26 | 13 | 14; 0 | 13 | 19;49[14] | 9 |
| 11 | 1;36 | 5 | 0;33 | 1 | 2;36[15]⊥ | 7 | 7;38 | 12 | 14;14 | 14 | 19;59 | 10 |
| 12 | 1;31 | 5 | 0;34 | 1 | 2;43 | 7 | 7;50 | 12 | 14;27 | 13 | 20; 9 | 10 |
| 13 | 1;26 | 5 | 0;35 | 1 | 2;51 | 8 | 8; 3 | 13 | 14;40 | 13 | 20;18 | 9 |
| 14 | 1;21 | 5[16] | 0;36 | 1 | 2;59 | 8 | 8;15 | 12[17] | 14;53 | 13 | 20;27[18] | 9 |
| 15 | 1;17 | 4[19] | 0;38 | 2 | 3; 7 | 8 | 8;28 | 13 | 15; 6 | 13 | 20;36 | 9 |
| 16 | 1;13 | 4[20] | 0;40 | 2 | 3;15 | 8 | 8;41 | 13 | 15;19 | 13 | 20;45 | 9 |
| 17 | 1; 9 | 4 | 0;42 | 2 | 3;24 | 9 | 8;54 | 13 | 15;32 | 13 | 20;53 | 8 |
| 18 | 1; 5[21] | 4 | 0;44 | 2 | 3;33 | 9 | 9; 7[22] | 13 | 15;45 | 13 | 21; 1 | 8 |
| 19 | 1; 2 | 3 | 0;46[23] | 2 | 3;42 | 9 | 9;20 | 13 | 15;57 | 12 | 21; 9 | 8 |
| 20 | 0;58 | 4[24] | 0;49 | 3 | 3;51 | 9 | 9;33[25] | 13 | 16;10 | 13 | 21;17 | 8 |
| 21 | 0;55 | 3[26] | 0;52[27] | 3 | 4; 1 | 10 | 9;46 | 13 | 16;22 | 12⊥ | 21;24 | 7 |
| 22 | 0;52 | 3[28] | 0;55 | 3 | 4;11 | 10 | 10; 0 | 14 | 16;34 | 12 | 21;31 | 7 |
| 23 | 0;49 | 3[29] | 0;58[30] | 3 | 4;20 | 9 | 10;13 | 13 | 16;47 | 13 | 21;38 | 7 |
| 24 | 0;46[31] | 3 | 1; 2 | 4 | 4;30 | 10 | 10;26[32] | 13 | 16;59 | 12 | 21;45 | 7 |
| 25 | 0;43 | 3 | 1; 6[33] | 4 | 4;40 | 10 | 10;39 | 13 | 17;11 | 12 | 21;52 | 7[34] |
| 26 | 0;40 | 3 | 1;10 | 4 | 4;50 | 10 | 10;53[35] | 14 | 17;23 | 12 | 21;58 | 6 |
| 27 | 0;38 | 2 | 1;14 | 4 | 5; 0 | 10 | 11; 6 | 13 | 17;34 | 11 | 22; 4 | 6 |
| 28 | 0;36 | 2 | 1;18 | 4 | 5;10 | 10 | 11;20[36] | 14[37] | 17;45 | 11 | 22;10 | 6 |
| 29 | 0;34 | 2 | 1;23 | 5 | 5;20 | 10 | 11;33 | 13[38] | 17;57[39] | 12 | 22;16 | 6 |

[1] **C** 28′ [2] $\mathbf{C_1}$**YLB** 6′ [3] **YL** 37′ [4] **H** 26′ [5] **B** 26′ [6] **L** 50′ [7] **C** 50′ **Y** 1° [8] $\mathbf{C_1C_2}$ 5′ [9] **C** 1°56′ **Y** 9′ [10] **LB** 7′ [11] **F** $4^s$7–21°: slide[+2] [12] **C** $2^s$9–11° (minutes): slide[−1 col.] [13] **L** 15′ [14] $\mathbf{C_2}$ 9′ [15] **C**+ 18′ (for 38′) [16] **Y** 4′ [17] **F** 13′ [18] **B** 24′ [19] **Y** 5′ [20] **Y** om. (hence read as 5′) [21] **F** 8′ [22] **C** 6′ [23] $\mathbf{C_1C_2}$ 47′ [24] **F** 3′ [25] **C** 38′ [26] **Y** om. (hence read as 4′) **B** 4′ [27] $\mathbf{C_2}$ 55′ [28] **Y** om. (hence read as 4′) **LB** 4′ [29] **Y** om. (hence read as 4′) **B** 4′ [30] **Y** 18′ [31] **L** 47′ [32] **Y** 36′ [33] $\mathbf{C_1}$ 4′ [34] **C** 6′ [35] **L** 13′ [36] $\mathbf{CC_1C_2}$ 19′ [37] $\mathbf{CC_1C_2}$ 13′ (in correspondence with the equation for $3^s$28°) [38] $\mathbf{CC_1C_2}$ 14′ (idem) [39] **L** 55′ $\mathbf{C_1}$ 56′.

**Table 30: First equation for Mars (second half)**
*Sources*: **F** fols 67v–68r, **H** fols 56v–57r, **C** fols 66v–67r, **$C_1$** fols 32v–33r, **$C_2$** fols 26v–27v, **Y** fols 286v–287r, **L** fols 57v–58r, **B** pp. 110–111.

First Equation for Mars
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | mean distance 26° **6** | differences ′ | **7** | differences ′ | **8** | differences ′ | **9** | differences ′ | furthest distance 1° **10** | differences ′ | **11** | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | |
| 0 | 22;21 | 5 | 23;29 | 1 | 21;35 | 6 | 17;28 | 10 | 12;11 | 11 | 6;52 | 10 |
| 1 | 22;26 | 5 | 23;28[1] | 1 | 21;28 | 7 | 17;18[2] | 10 | 12; 0 | 11 | 6;42 | 10 |
| 2 | 22;32 | 6 | 23;27 | 1 | 21;22 | 6 | 17; 8 | 10 | 11;49 | 11 | 6;32 | 10 |
| 3 | 22;37 | 5 | 23;26[3] | 1 | 21;15 | 7 | 16;58 | 10 | 11;38[4] | 11 | 6;22 | 10 |
| 4 | 22;42 | 5 | 23;24 | 2[5] | 21; 8 | 7 | 16;48 | 10 | 11;27 | 11 | 6;12 | 10 |
| 5 | 22;46 | 4 | 23;22 | 2 | 21; 1 | 7 | 16;38 | 10 | 11;16 | 11 | 6; 2 | 10 |
| 6 | 22;50 | 4 | 23;20 | 2 | 20;54 | 7 | 16;28 | 10 | 11; 5[6]↓ | 11 | 5;53 | 9 |
| 7 | 22;54 | 4 | 23;17 | 3 | 20;46 | 8 | 16;18 | 10 | 10;54[7] | 11 | 5;44 | 9 |
| 8 | 22;58 | 4 | 23;14 | 3 | 20;39 | 7 | 16; 8 | 10 | 10;43⊥ | 11 | 5;35 | 9 |
| 9 | 23; 2 | 4 | 23;11 | 3 | 20;32 | 7 | 15;58 | 10 | 10;32 | 11 | 5;26 | 9 |
| 10 | 23; 5 | 3 | 23; 8 | 3 | 20;25 | 7 | 15;48 | 10 | 10;21 | 11 | 5;17 | 9 |
| 11 | 23; 8 | 3 | 23; 5[8] | 3 | 20;17 | 8 | 15;37 | 11 | 10;10 | 11 | 5; 8 | 9 |
| 12 | 23;11 | 3 | 23; 2 | 3[9] | 20; 9 | 8 | 15;26 | 11 | 9;59 | 11 | 4;59 | 9 |
| 13 | 23;14 | 3 | 22;58 | 4[10] | 20; 0 | 9 | 15;15 | 11 | 9;49 | 10 | 4;50 | 9 |
| 14 | 23;16 | 2 | 22;55 | 3[11] | 19;52[12] | 8 | 15; 4 | 11 | 9;38 | 11 | 4;41 | 9 |
| 15 | 23;18 | 2 | 22;51 | 4 | 19;44 | 8[13] | 14;54 | 10[14]↓ | 9;27 | 11 | 4;33 | 8 |
| 16 | 23;20 | 2 | 22;47 | 4 | 19;35 | 9 | 14;43 | 11 | 9;17 | 10[15] | 4;25 | 8 |
| 17 | 23;22 | 2 | 22;43 | 4 | 19;27 | 8 | 14;33 | 10 | 9; 6 | 11 | 4;16 | 9[16] |
| 18 | 23;24 | 2 | 22;39 | 4[17] | 19;19 | 8 | 14;22 | 11 | 8;56 | 10 | 4; 8 | 8 |
| 19 | 23;25 | 1 | 22;34 | 5 | 19;10 | 9 | 14;11 | 11 | 8;45 | 11 | 4; 0 | 8 |
| 20 | 23;26 | 1 | 22;29 | 5 | 19; 1 | 9[18] | 14; 1 | 10⊥ | 8;34 | 11 | 3;51 | 9 |
| 21 | 23;27 | 1 | 22;24 | 5 | 18;52[19] | 9[20] | 13;50 | 11 | 8;23 | 11 | 3;43 | 8 |
| 22 | 23;28 | 1 | 22;19 | 5 | 18;43 | 9 | 13;39 | 11 | 8;12 | 11 | 3;35 | 8 |
| 23 | 23;29 | 1 | 22;14 | 5 | 18;34 | 9 | 13;28 | 11 | 8; 2 | 10 | 3;28 | 7 |
| 24 | 23;30 | 1 | 22; 9 | 5 | 18;25 | 9 | 13;17 | 11 | 7;52 | 10 | 3;21 | 7 |
| 25 | 23;30 | 0 | 22; 4 | 5 | 18;16 | 9 | 13; 6 | 11 | 7;42 | 10 | 3;14 | 7 |
| 26 | 23;30 | 0 | 21;59 | 5 | 18; 7 | 9 | 12;55 | 11 | 7;32 | 10 | 3; 6 | 8 |
| 27 | 23;30 | 0 | 21;53 | 6 | 17;58 | 9 | 12;44 | 11 | 7;22 | 10 | 2;59 | 7 |
| 28 | 23;30 | 0 | 21;47 | 6 | 17;48 | 10 | 12;33 | 11 | 7;12 | 10 | 2;52 | 7 |
| 29 | 23;30 | 0 | 21;41 | 6 | 17;38 | 10 | 12;22 | 11 | 7; 2 | 10 | 2;45[21] | 7 |

[1] **C** 1′ [2] **C** 28′ [3] **C** 27′ [4] **C** 33′ [5] **LB** 1′ [6] **C** $10^s$ 6–8° (minutes): slide[+1 col.]
[7] **C**+ 58′ (for 18) [8] **$C_2$** 0′ [9] **L** 4′ [10] **F** 3′ [11] **LB** 4′ [12] **B** 12′ [13] **L** 9′ [14] **F** $9^s$ 15–20°: 11′
[15] **$CC_1C_2$** 11′ [16] **F** 8′ [17] **L** 5′ [18] **B** 8′ [19] **B** 12′ [20] **B** 8′ [21] **Y** 59′.

**Table 30: Second equation for Mars (first half)**

*Sources*: **F** fols 68v–69r, **H** fols 57v–58r, **C** fols 67v–68r, **$C_1$** fols 33v–34r, **$C_2$** fols 27v–28r, **Y** fols 287v–288r, **L** fols 58v–59r, **B** pp. 112–113, **r** = recomputation.

Second Equation for Mars
to be equated and added to the centrum with the apogee

| parts of the epicycle | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | **2** ° ′ | differences ′ | mean distance 21° **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ° ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 47; 0 | 24 | 58;51[1] | 23 | 70;13 | 22 | 80;22 | 18 | 87;21 | 8 | 84;25 | 26[2] |
| 1 | 47;24[3] | 24 | 59;15 | 24 | 70;35 | 22 | 80;40 | 18 | 87;29 | 8 | 83;57 | 28 |
| 2 | 47;48 | 24 | 59;38 | 23 | 70;57 | 22 | 80;58 | 18 | 87;36 | 7 | 83;26 | 31 |
| 3 | 48;12 | 24 | 60; 1 | 23 | 71;18[4] | 21[5] | 81;15 | 17 | 87;42 | 6 | 82;52[6] | 34[7] |
| 4 | 48;36 | 24 | 60;25 | 24 | 71;40 | 22[8] | 81;32 | 17 | 87;48 | 6 | 82;15[9] | 37[10] |
| 5 | 49; 0 | 24[11] | 60;48 | 23 | 72; 1 | 21[12] | 81;49 | 17 | 87;53 | 5 | 81;35 | 40 |
| 6 | 49;24 | 24 | 61;11 | 23 | 72;22[13] | 21 | 82; 6 | 17 | 87;58 | 5 | 80;52[14] | 43 |
| 7 | 49;48 | 24 | 61;34[15] | 23 | 72;44 | 21[16] | 82;23 | 17[17] | 88; 2 | 4 | 80; 7 stationary | 45 |
| 8 | 50;12 | 24 | 61;57 | 23 | 73; 5 | 21 | 82;40 | 17 | 88; 6 | 4 | 79;20 | 47 |
| 9 | 50;36 | 24 | 62;20 | 23 | 73;26[18] | 21 | 82;56 | 16 | 88; 8 | 2 | 78;30 | 50 |
| 10 | 50;59 | 23 | 62;43 | 23 | 73;47 | 21 | 83;12 | 16 | 88; 9 | 1 | 77;36 | 54 |
| 11 | 51;23 | 24 | 63; 6 | 23 | 74; 8 | 21 | 83;28[19] | 16 | 88;10 | 1 | 76;38 | 58 |
| 12 | 51;46[20] | 23[21] | 63;29 | 23 | 74;29[22] | 21 | 83;43 | 15 | 88; 9 | 1 | 75;35 | 1; 3[23] |
| 13 | 52;10 | 24 | 63;52 | 23 | 74;50 | 21[24]↓ | 83;58 | 15 | 88; 7 | 2 | 74;28 | 1; 7 |
| 14 | 52;34 | 24 | 64;15[25] | 23 | 75;11 | 21 | 84;13 | 15 | 88; 4 | 3 | 73;16[26] | 1;12 |
| 15 | 52;57[27] | 23[28] | 64;38 | 23 | 75;32 | 21 | 84;27 | 14 | 88; 0 | 4 | 72; 2 | 1;14[29] |
| 16 | 53;21 | 24 | 65; 1 | 23 | 75;52 | 20 | 84;41 | 14 | 87;55 | 5 | 70;45 | 1;17[30] |
| 17 | 53;45 | 24 | 65;24 | 23 | 76;12 | 20 | 84;55 | 14 | 87;50 | 5 | 69;24 | 1;21 |
| 18 | 54; 8 | 23 | 65;46 | 22 | 76;32 | 20 | 85; 9 | 14 | 87;45 | 5 | 68; 0 | 1;24 |
| 19 | 54;32 | 24 | 66; 9 | 23 | 76;52 | 20 | 85;23 | 14 | 87;39 | 6 | 66;33 retrograde | 1;27 |
| 20 | 54;56 | 24 | 66;31 | 22 | 77;12 | 20⊥ | 85;36 | 13 | 87;31 | 8[31] | 65; 1 | 1;32 |
| 21 | 55;19 | 23 | 66;53 | 22 | 77;32 | 20 | 85;49 | 13 | 87;21[32] | 10 | 63;25 | 1;36 |
| 22 | 55;43 | 24 | 67;16 | 23 | 77;52 | 20 | 86; 1 | 12 | 87; 8 | 13 | 61;45[33] | 1;40 |
| 23 | 56; 7 | 24 | 67;38 | 22 | 78;11 | 19 | 86;12 | 11 | 86;53 | 15 | 60; 1 | 1;44 |
| 24 | 56;30 | 23 | 68; 0 | 22[34] | 78;30 | 19 | 86;24[35] | 12 | 86;37 | 16 | 58;15 | 1;46[36] |
| 25 | 56;54[37] | 24[38] | 68;23 | 23 | 78;49 | 19 | 86;35 | 11 | 86;20 | 17 | 56;27 | 1;48 |
| 26 | 57;18[39] eastern vis | 24 | 68;45 | 22 | 79; 8 | 19 | 86;45 | 10 | 86; 1 | 19 | 54;37 | 1;50 |
| 27 | 57;41 | 23 | 69; 7 | 22[40] | 79;27 | 19 | 86;55 | 10 | 85;40[41] | 21 | 52;45 | 1;52 |
| 28 | 58; 5[42] | 24[43] | 69;29 | 22 | 79;46 | 19 | 87; 4 | 9 | 85;16[44] | 24 | 50;52[45] | 1;53 |
| 29 | 58;28 | 23 | 69;51 | 22 | 80; 4 | 18 | 87;13 | 9 | 84;51[46] | 25 | 48;57[47] | 1;55 |

[1] **C** 11′ [2] **C** 27′ [3] **$C_1$** 40′ [4] **L** 17′ [5] **L** 20′ (cf. note 4) [6] **C** 12′ [7] **F** 35′ [8] **F** 21′ **L** 23′ (cf. note 4) [9] **FC** 55′ [10] **FC** 36′ [11] **F** 26′ [12] **F** 22′ [13] **C** 24′ [14] **C** 12′ [15] **C** 24′ [16] **LBr** 22′ [17] **L** 16′ [18] **C** 27′ [19] **F** 23′ [20] **F** 11° [21] **B** 24′ [22] **F** 9′ [23] **C** 53′ [24] **C** 2^s 13–20°: slide[+3] [25] **L** 6′ [26] **C** 36′ [27] **$C_1$** 54′ [28] **B** 27′ [29] **C** 54′ [30] **F** 16′ **C** 57′ [31] **B** 3′ [32] **F** 28′ [33] **F** 41′ [34] **L** 23′ [35] **Y** 14′ [36] **FL** 47′ [37] **F** 14′ [38] **L** 23′ [39] **F** 58′ [40] **B** 23′ [41] **L** 45′ [42] **C** 40′ [43] **F** 23′ [44] **C** 56′ **L** 15°86′ (sic!) [45] **C** 12′ [46] **F** 11′ [47] **C** 17′.

**Table 30: Second equation for Mars (second half)**
*Sources*: **F** fols 68v–69r, **H** fols 57v–58r, **C** fols 67v–68r, **C$_1$** fols 33v–34r, **C$_2$** fols 27v–28r, **Y** fols 287v–288r, **L** fols 58v–59r, **B** pp. 112–113.

Second Equation for Mars
to be equated and added to the centrum with the apogee

| parts of the epicycle | nearest distance 0° **6** ° ′ | differences ° ′ | **7** ° ′ | differences ′ | mean distance 9° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 47; 0 | 1;57 | 9;35 | 28 | 6;39 | 8 | 13;38 | 18 | 23;47 | 22 | 35; 9 | 24 |
| 1 | 45; 3 | 1;57 | 9; 9 | 26 | 6;47 | 8 | 13;56 | 18 | 24; 9 | 22 | 35;32 | 23 |
| 2 | 43; 8 | 1;55 | 8;44[1] | 25 | 6;56 | 9 | 14;14 | 18 | 24;31 | 22 | 35;55 | 23 |
| 3 | 41;15 | 1;53 | 8;20 | 24 | 7; 5 | 9 | 14;33 | 19 | 24;53[2] | 22 | 36;19 western dis | 24 |
| 4 | 39;23 | 1;52 | 7;59 | 21 | 7;15 | 10 | 14;52 | 19 | 25;15 | 22 | 36;42 | 23 |
| 5 | 37;33 | 1;50[3] | 7;40[4] | 19 | 7;25 | 10 | 15;11 | 19 | 25;37 | 22 | 37; 6[5] | 24 |
| 6 | 35;45 | 1;48 | 7;23 | 17 | 7;36 | 11 | 15;30 | 19 | 26; 0 | 23 | 37;30 | 24 |
| 7 | 33;59 | 1;46 | 7; 7 | 16 | 7;48 | 12 | 15;49 | 19 | 26;22[6] | 22 | 37;53 | 23 |
| 8 | 32;15[7] | 1;44 | 6;52 | 15 | 7;59 | 11 | 16; 8 | 19 | 26;44 | 22 | 38;17 | 24 |
| 9 | 30;35[8]↓ stationary | 1;40 | 6;39[9] | 13 | 8;11 | 12 | 16;28 | 20 | 27; 7[10] | 23 | 38;41 | 24 |
| 10 | 28;59 | 1;36 | 6;29 | 10 | 8;24 | 13 | 16;48 | 20 | 27;29 | 22 | 39; 4 | 23 |
| 11 | 27;27[11]⊥ | 1;32 | 6;21 | 8 | 8;37 | 13 | 17; 8 | 20 | 27;51 | 22[12] | 39;28 | 24 |
| 12 | 26; 0 | 1;27 | 6;15 | 6 | 8;51 | 14 | 17;28 | 20 | 28;14 | 23 | 39;52 | 24 |
| 13 | 24;36[13] | 1;24 | 6;10 | 5 | 9; 5 | 14 | 17;48 | 20 | 28;36 | 22[14] | 40;15 | 23 |
| 14 | 23;15 | 1;21 | 6; 5 | 5 | 9;19 | 14 | 18; 8[15] | 20[16] | 28;59 | 23 | 40;39 | 24 |
| 15 | 21;58[17] | 1;17 | 6; 0 | 5 | 9;33 | 14 | 18;28 | 20[18] | 29;22 | 23 | 41; 3 | 24 |
| 16 | 20;44 | 1;14 | 5;56 | 4 | 9;47 | 14 | 18;49 | 21 | 29;45 | 23[19] | 41;26 | 23 |
| 17 | 19;32 | 1;12 | 5;53 | 3 | 10; 2 | 15 | 19;10 | 21 | 30; 8 | 23 | 41;50 | 24 |
| 18 | 18;25 | 1; 7 | 5;51 | 2 | 10;17 | 15 | 19;31 | 21 | 30;31 | 23 | 42;14 | 24 |
| 19 | 17;22[20] | 1; 3 | 5;50 | 1 | 10;32 | 15 | 19;52[21] | 21 | 30;54[22] | 23 | 42;37 | 23 |
| 20 | 16;24 | 58 | 5;51 | 1 | 10;48 | 16 | 20;13 | 21 | 31;17 | 23[23] | 43; 1 | 24 |
| 21 | 15;30 progressive | 54 | 5;52 | 1 | 11; 4[24] | 16 | 20;34 | 21 | 31;40 | 23 | 43;24 | 23 |
| 22 | 14;40 | 50 | 5;54 | 2 | 11;20 | 16 | 20;55[25] | 21 | 32; 3 | 23 | 43;48 | 24 |
| 23 | 13;53 | 47 | 5;58 | 4 | 11;37 | 17 | 21;16 | 21 | 32;26 | 23 | 44;12 | 24 |
| 24 | 13; 8 | 45 | 6; 2 | 4 | 11;54 | 17 | 21;38 | 22 | 32;49 | 23 | 44;36 | 24 |
| 25 | 12;25 | 43 | 6; 7[26] | 5[27] | 12;11 | 17 | 21;59 | 21 | 33;12 | 23 | 45; 0 | 24 |
| 26 | 11;45 | 40 | 6;12 | 5 | 12;28[28] | 17 | 22;20 | 21[29] | 33;35 | 23 | 45;24 | 24 |
| 27 | 11; 8 | 37 | 6;18 | 6 | 12;45 | 17 | 22;42 | 22[30] | 33;59 | 24[31] | 45;48 | 24 |
| 28 | 10;34 | 34 | 6;24 | 6 | 13; 2 | 17 | 23; 3 | 21 | 34;22 | 23 | 46;12 | 24 |
| 29 | 10; 3 | 31 | 6;31 | 7[32] | 13;20 | 18 | 23;25 | 22 | 34;45 | 23[33] | 46;36 | 24 |

[1] **F** 24′ [2] **CC$_2$** 13′ [3] **C** 7′ [4] **Y** 38′ [5] **C** 36° [6] **L** 32′ [7] **C** 55′ **L** 31° [8] **C** 6$^s$ 9–11° (minutes): slide[+3] [9] **C** 29′ [10] **C$_1$C$_2$** 26° [11] +**H** 26′ **C**+ 55′ (for 15′) **L** 26° [12] **C$_1$C$_2$** 23′ [13] **L** 32′ [14] **C$_1$C$_2$** 23′ [15] **L** 9′ [16] **L** 21′ [17] **F** 18′ **C** 38′ [18] **L** 21′ [19] **F** 22′ [20] **L** 16° [21] **CC$_2$** 12′ [22] **H** 14′ [23] **L** 24′ [24] **C** 22′ [25] **L** 56′ [26] **C** 2′ [27] **L** 3′ [28] **C** 23′ [29] **L** 22′ [30] **L** 21′ [31] **LB** 23′ [32] **L** 5′ [33] **Y** 24′.

**Table 30: Variation of the nearest distance for Mars**
*Sources*: **F** fol. 69v, **H** fol. 58v, **Y** fol. 288v, **L** fol. 59v, **B** p. 114. The essentially different table found in the three Cairo manuscripts is separately edited on p. 180.

| Variation of the Nearest Distance for Mars | | | | | | | |
|---|---|---|---|---|---|---|---|
| corrected centrum | **1** | **2** | **3** | **4** | **5** | **6** | **7** |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the furthest distance | 0;45 | 3;36 | 5;36 | 5;26 | 3;13 | 0;22 |
| 1 | | 0;51 | 3;42[1] | 5;38 | 5;24 | 3; 7 | 0;16 |
| 2 | | 0;56 | 3;47[2] | 5;41 | 5;21 | 3; 1 | 0;11 |
| 3 | | 1; 2 | 3;52 | 5;43 | 5;17 | 2;55 | 0; 5 |
| 4 | | 1; 7 | 3;56 | 5;46 | 5;13 | 2;49 | 0; 0 |
| 5 | | 1;12 | 4; 1 | 5;48 | 5;10 | 2;44[3] | From the furthest distance |
| 6 | | 1;17 | 4; 6 | 5;49 | 5; 6 | 2;38 | |
| 7 | | 1;22 | 4;11 | 5;49 | 5; 3 | 2;32 | |
| 8 | | 1;29 | 4;17 | 5;50 | 4;59 | 2;27 | |
| 9 | | 1;35 | 4;22 | 5;50 | 4;55 | 2;22 | |
| 10 | | 1;42[4] | 4;26 | 5;50 | 4;51 | 2;17 | |
| 11 | | 1;49 | 4;31 | 5;50 | 4;48 | 2;12 | |
| 12 | | 1;55 | 4;36 | 5;50 | 4;44 | 2; 7 | |
| 13 | | 2; 2 | 4;40 | 5;50 | 4;40 | 2; 2 | |
| 14 | | 2; 7 | 4;44 | 5;50 | 4;36 | 1;55 | |
| 15 | | 2;12 | 4;48 | 5;50 | 4;31 | 1;49 | |
| 16 | | 2;17 | 4;51 | 5;50 | 4;26 | 1;42 | |
| 17 | | 2;22 | 4;55[5] | 5;50 | 4;22 | 1;35 | |
| 18 | | 2;27 | 4;59 | 5;50 | 4;17 | 1;29 | |
| 19 | | 2;32 | 5; 3 | 5;49 | 4;11 | 1;23[6] | |
| 20 | | 2;38 | 5; 6 | 5;49 | 4; 6 | 1;17 | |
| 21 | | 2;44 | 5;10 | 5;48 | 4; 1 | 1;12 | |
| 22 | 0; 0 | 2;49 | 5;13 | 5;46 | 3;56 | 1; 7 | |
| 23 | 0; 5[7] | 2;55[8] | 5;17 | 5;43 | 3;52 | 1; 2 | |
| 24 | 0;11 | 3; 1 | 5;21 | 5;41 | 3;47 | 0;56 | |
| 25 | 0;16 | 3; 7 | 5;24 | 5;38 | 3;42 | 0;51 | |
| 26 | 0;22 | 3;13 | 5;26 | 5;36 | 3;36 | 0;45 | |
| 27 | 0;28 | 3;19 | 5;29 | 5;33 | 3;30 | 0;39 | |
| 28 | 0;33 | 3;24 | 5;31 | 5;31 | 3;24 | 0;33[9] | |
| 29 | 0;39 | 3;30 | 5;33 | 5;29 | 3;19 | 0;28 | |

[1] **B** 12′ [2] **H** 46′ [3] **F** 34′ [4] **F** 40′ [5] **F** 58′ [6] **F** 25′ **H** 22′ [7] **L** 8′ [8] **F** 3° [9] **LB** 35′.

**Table 30: Interpolation minutes (nearest distance) for Mars**

*Sources*: **F** fol. 69v, **H** fol. 59r, **C** fol. 68v, $\mathbf{C_1}$ fol. 34v, $\mathbf{C_2}$ fol. 28v, **Y** fol. 288v, **L** fol. 59v, **B** p. 114.

| Minutes of Proportions ⟨for Mars⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **0** | **1** | **2** | **3** | **4** | **5** | |
| 0 | 0 | 17 | 34 | 49 | 59 | 55[1] | 30 |
| 1 | 0 | 18 | 34 | 49 | 59 | 54 | 29 |
| 2 | 1 | 18 | 35 | 50 | 59 | 53 | 28 |
| 3 | 1[2] | 19 | 35 | 50 | 59 | 52 | 27 |
| 4 | 2 | 20 | 36 | 50 | 60 | 51 | 26 |
| 5 | 2 | 20 | 36 | 51 | 60 | 50 | 25 |
| 6 | 3 | 21 | 37 | 51 | 60 | 49 | 24 |
| 7 | 4 | 21 | 37[3] | 52 | 60 | 48 | 23 |
| 8 | 4 | 22 | 38 | 52 | 60 | 47 | 22 |
| 9 | 5 | 22 | 38[4] | 52 | 60 | 45 | 21 |
| 10 | 6 | 23 | 39 | 53 | 60 | 44 | 20 |
| 11 | 6 | 23 | 39 | 53 | 60 | 43 | 19 |
| 12 | 7 | 24 | 40 | 54 | 60 | 42 | 18 |
| 13 | 7 | 24 | 40 | 54 | 60 | 40 | 17 |
| 14 | 8 | 25 | 41 | 55 | 60 | 38[5] | 16 |
| 15 | 8 | 25 | 41 | 55 | 59 | 36 | 15 |
| 16 | 9 | 26 | 42 | 55 | 59 | 35 | 14 |
| 17 | 9 | 26 | 42 | 56 | 59 | 33 | 13 |
| 18 | 10 | 27 | 43 | 56 | 59 | 31 | 12 |
| 19 | 11 | 28 | 43 | 56 | 59 | 28 | 11 |
| 20 | 11 | 28 | 44 | 56 | 59 | 26 | 10 |
| 21 | 12 | 29 | 44 | 57 | 58 | 23 | 9 |
| 22 | 13 | 30 | 45 | 57 | 58 | 21 | 8 |
| 23 | 13 | 30 | 45 | 57 | 58 | 18 | 7 |
| 24 | 14 | 31 | 46 | 57[6] | 58 | 16 | 6 |
| 25 | 14 | 31 | 46 | 58 | 57 | 13 | 5 |
| 26 | 15 | 32 | 47 | 58 | 57 | 11 | 4 |
| 27 | 15 | 32 | 47 | 58 | 56[7] | 8 | 3 |
| 28 | 16 | 33 | 48 | 58 | 56 | 5 | 2 |
| 29 | 16 | 33 | 48 | 59 | 55 | 3 | 1 |
| parts of the epicycle | **11** | **10** | **9** | **8** | **7** | **6** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **H** 52′ [2] **C** 2′ [3] $\mathbf{C_1 C_2}$ 38′ [4] $\mathbf{C_2}$ 8′ [5] **C** 18′ [6] **C** 18′ $\mathbf{C_1 C_2}$ 58′ [7] **F** 57′.

**Table 30: Variation of the furthest distance for Mars**
*Sources*: **F** fol. 70r, **H** fol. 59v, **Y** fol. 289r, **L** fol. 60r, **B** p. 115. The essentially different table found in the three Cairo manuscripts is separately edited on p. 181.

Variation of the Furthest Distance for Mars

| corrected centrum | 7 | 8 | 9 | 10 | 11 | 0 | 1 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the nearest distance | 1;59 | 3;34 | 4;19 | 4;16 | 3;24 | 1;45 |
| 1 | | 2; 3 | 3;36 | 4;20 | 4;15 | 3;22 | 1;41 |
| 2 | | 2; 8 | 3;38 | 4;21 | 4;13 | 3;20 | 1;36 |
| 3 | | 2;12 | 3;40 | 4;21 | 4;12 | 3;18 | 1;31 |
| 4 | 0; 0 | 2;17 | 3;42 | 4;22 | 4;10 | 3;15 | 1;25 |
| 5 | 0; 4 | 2;21 | 3;45 | 4;23 | 4; 9 | 3;13 | 1;20 |
| 6 | 0; 9 | 2;26 | 3;47 | 4;23 | 4; 7 | 3;11 | 1;15 |
| 7 | 0;13 | 2;30 | 3;49 | 4;24 | 4; 6 | 3; 9 | 1;10 |
| 8 | 0;18 | 2;33 | 3;50 | 4;24 | 4; 5 | 3; 5 | 1; 6 |
| 9 | 0;23 | 2;36 | 3;52 | 4;24 | 4; 3 | 3; 2 | 1; 1 |
| 10 | 0;28 | 2;38 | 3;53 | 4;25 | 4; 2 | 2;58 | 0;57 |
| 11 | 0;34 | 2;41 | 3;55 | 4;25 | 4; 1 | 2;54 | 0;53 |
| 12 | 0;39 | 2;44 | 3;56 | 4;25 | 3;59 | 2;51 | 0;48 |
| 13 | 0;44 | 2;47 | 3;58 | 4;25 | 3;58 | 2;47 | 0;44 |
| 14 | 0;48 | 2;51 | 3;59 | 4;25 | 3;56 | 2;44 | 0;39 |
| 15 | 0;53 | 2;54 | 4; 1 | 4;25 | 3;55 | 2;41 | 0;34 |
| 16 | 0;57 | 2;58[1] | 4; 2 | 4;25 | 3;53 | 2;38 | 0;28 |
| 17 | 1; 1 | 3; 2 | 4; 3 | 4;24 | 3;52 | 2;36 | 0;23 |
| 18 | 1; 6 | 3; 5 | 4; 5 | 4;24 | 3;50 | 2;33 | 0;18 |
| 19 | 1;10 | 3; 9 | 4; 6 | 4;24 | 3;49 | 2;30 | 0;13 |
| 20 | 1;15 | 3;11 | 4; 7 | 4;23 | 3;47 | 2;26 | 0; 9 |
| 21 | 1;20 | 3;13 | 4; 9 | 4;23 | 3;45 | 2;21 | 0; 4[2] |
| 22 | 1;25 | 3;15 | 4;10 | 4;22 | 3;42 | 2;17 | 0; 0 |
| 23 | 1;31 | 3;18 | 4;12 | 4;21 | 3;40 | 2;12 | From the nearest distance |
| 24 | 1;36 | 3;20 | 4;13 | 4;21 | 3;38 | 2; 8 | |
| 25 | 1;41 | 3;22 | 4;15 | 4;20 | 3;36 | 2; 3 | |
| 26 | 1;45 | 3;24 | 4;16 | 4;19 | 3;34 | 1;59 | |
| 27 | 1;48 | 3;27 | 4;17 | 4;18 | 3;31 | 1;56 | |
| 28 | 1;52[3] | 3;29 | 4;17 | 4;17 | 3;29 | 1;52 | |
| 29 | 1;56 | 3;31 | 4;18 | 4;17 | 3;27 | 1;48 | |

[1] **Y** 57′ [2] **LB** 7′ [3] **L** 43′ (correction in a different hand after this part of the page was damaged).

**Table 30: Interpolation minutes (furthest distance) for Mars**
*Sources*: **F** fol. 70r, **H** fol. 60r, **C** fol. 69r, **$C_1$** fol. 35r, **$C_2$** fol. 29r, **Y** fol. 289r, **L** fol. 60r, **B** p. 115.

| Minutes of Proportions ⟨for Mars⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0 | 0 | 55 | 59 | 49 | 34 | 17 | 30 |
| 1 | 3 | 55 | 59 | 48 | 33 | 16 | 29 |
| 2 | 5 | 56 | 58 | 48 | 33 | 16 | 28 |
| 3 | 8 | 56 | 58 | 47 | 32 | 15 | 27 |
| 4 | 11 | 57 | 58 | 47 | 32 | 15 | 26 |
| 5 | 13 | 57 | 58 | 46 | 31 | 14 | 25 |
| 6 | 16 | 58 | 57 | 46 | 31 | 14 | 24 |
| 7 | 18 | 58 | 57 | 45 | 30 | 13 | 23 |
| 8 | 21 | 58 | 57 | 45 | 30 | 13 | 22 |
| 9 | 23 | 58 | 57 | 44 | 29 | 12 | 21 |
| 10 | 26 | 59 | 56 | 44[1] | 28 | 11 | 20 |
| 11 | 28 | 59 | 56 | 43 | 28 | 11 | 19 |
| 12 | 31 | 59 | 56 | 43 | 27 | 10 | 18 |
| 13 | 33 | 59 | 56 | 42 | 26[2] | 9 | 17 |
| 14 | 35 | 59[3] | 55 | 42 | 26 | 9 | 16 |
| 15 | 36 | 59 | 55 | 41 | 25 | 8 | 15 |
| 16 | 38 | 60 | 55[4] | 41 | 25 | 8 | 14 |
| 17 | 40 | 60 | 54 | 40 | 24 | 7 | 13 |
| 18 | 42 | 60 | 54 | 40[5]↓ | 24[6] | 7 | 12 |
| 19 | 43 | 60 | 53 | 39 | 23 | 6 | 11 |
| 20 | 44 | 60 | 53 | 39⊥ | 23 | 6 | 10 |
| 21 | 45 | 60 | 52 | 38 | 22 | 5 | 9 |
| 22 | 47 | 60 | 52 | 38 | 22 | 4 | 8 |
| 23 | 48 | 60 | 52 | 37 | 21 | 4 | 7 |
| 24 | 49 | 60 | 51 | 37 | 21 | 3 | 6 |
| 25 | 50 | 60 | 51[7] | 36 | 20 | 2 | 5 |
| 26 | 51 | 60 | 50 | 36 | 20 | 2 | 4 |
| 27 | 52 | 59[8]↓ | 50 | 35 | 19 | 1 | 3 |
| 28 | 53 | 59 | 50 | 35 | 18 | 1 | 2 |
| 29 | 54 | 59⊥ | 49 | 34[9] | 18 | 0 | 1 |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **F** 43′ [2] **C** 27′ [3] **Y** 60′ [4] **$CC_1C_2$** 54′ [5] **C** $9^s$ 18–20°: slide[−1 col.] (with the values for $9^s$ 19° and $9^s$ 20° corrected) [6] **Y** 23′ [7] **$C_1C_2$** 50′ [8] **$CC_1C_2$** $7^s$ 27–29°: 60′ [9] **L** 37′.

**Table 30a: Variation of the nearest distance for Mars in the Cairo manuscripts**
*Sources*: **C** fol. 68v, $\mathbf{C_1}$ fol. 34v, $\mathbf{C_2}$ fol. 28v. The essentially different table found in the other five sources is edited on p. 176.

Variation of the Nearest Distance for Mars

| corrected centrum | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the furthest distance | 1;14 | 4; 8 | 5;40 | 5;34 | 3;48 | 0;48[1] |
| 1 | | 1;21 | 4;13 | 5;42 | 5;32 | 3;43 | 0;41 |
| 2 | | 1;28 | 4;17 | 5;43 | 5;30 | 3;38 | 0;34 |
| 3 | | 1;34 | 4;21 | 5;44 | 5;27 | 3;33 | 0;27 |
| 4 | | 1;41 | 4;25 | 5;45 | 5;25 | 3;28 | 0;20 |
| 5 | | 1;47 | 4;30 | 5;45 | 5;22 | 3;23 | 0;14 |
| 6 | | 1;54 | 4;34 | 5;46[2]→ | 5;20 | 3;18 | 0; 7 |
| 7 | | 2; 0[3] | 4;38 | 5;47 | 5;17 | 3;13 | 0; 0 |
| 8 | | 2; 6 | 4;42 | 5;48 | 5;14⌟ | 3; 7 | From the furthest distance |
| 9 | | 2;13[4] | 4;45 | 5;49 | 5;11 | 3; 1 | |
| 10 | | 2;19 | 4;49 | 5;50 | 5; 9 | 2;56 | |
| 11 | | 2;25 | 4;53 | 5;50 | 5; 6 | 2;50 | |
| 12 | | 2;32 | 4;56 | 5;50 | 5; 3 | 2;44 | |
| 13 | | 2;38 | 5; 0 | 5;50 | 5; 0[5] | 2;38 | |
| 14 | | 2;44 | 5; 3 | 5;50 | 4;56 | 2;32 | |
| 15 | | 2;50 | 5; 6 | 5;50 | 4;53 | 2;25 | |
| 16 | | 2;56 | 5; 9 | 5;50 | 4;49 | 2;19 | |
| 17 | | 3; 1 | 5;11 | 5;49 | 4;45 | 2;13 | |
| 18 | | 3; 7 | 5;14 | 5;48 | 4;42 | 2; 6 | |
| 19 | 0; 0[6] | 3;13 | 5;17 | 5;47 | 4;38 | 2; 0 | |
| 20 | 0; 7 | 3;18 | 5;20 | 5;46 | 4;34 | 1;54 | |
| 21 | 0;14 | 3;23 | 5;22 | 5;45 | 4;30 | 1;47 | |
| 22 | 0;20 | 3;28 | 5;25 | 5;45 | 4;25 | 1;41 | |
| 23 | 0;27 | 3;33 | 5;27 | 5;44 | 4;21 | 1;34 | |
| 24 | 0;34 | 3;38 | 5;30 | 5;43 | 4;17 | 1;28 | |
| 25 | 0;41 | 3;43 | 5;32 | 5;42 | 4;13 | 1;21 | |
| 26 | 0;48 | 3;48 | 5;34 | 5;40 | 4; 8[7] | 1;14 | |
| 27 | 0;54 | 3;53 | 5;35 | 5;39 | 4; 3 | 1; 8 | |
| 28 | 1; 1 | 3;58 | 5;37 | 5;37 | 3;58 | 1; 1 | |
| 29 | 1; 8 | 4; 3 | 5;39 | 5;35 | 3;53 | 0;54 | |

[1] **C** 43′ [2] **C** 4–5[s] 6–8° (minutes): slide[+1 col.] [3] $\mathbf{C_2}$ 3′ [4] **C** 18′ [5] **C** 5′ [6] **C** 5′ [7] **C** 18′.

**Table 30a: Variation of the furthest distance for Mars in the Cairo manuscripts**
*Sources*: **C** fol. 69r, $\mathbf{C_1}$ fol. 35r, $\mathbf{C_2}$ fol. 29r. The essentially different table found in the other five sources is edited on p. 178.

Variation of the Furthest Distance for Mars

| corrected centrum | 7 | 8 | 9 | 10 | 11 | 0 | 1 |
|---|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the nearest distance | 1;35 | 3;19 | 4;19 | 4;14 | 3; 7 | 1;19 |
| 1 | | 1;39 | 3;22 | 4;20 | 4;13 | 3; 4 | 1;15 |
| 2 | | 1;43 | 3;25 | 4;21 | 4;12 | 3; 1 | 1;11 |
| 3 | | 1;47 | 3;27 | 4;22 | 4;10 | 2;58 | 1; 7 |
| 4 | | 1;50 | 3;30 | 4;22[1] | 4; 9 | 2;54 | 1; 2 |
| 5 | | 1;54 | 3;33 | 4;23 | 4; 7 | 2;51 | 0;58 |
| 6 | | 1;58 | 3;35 | 4;23 | 4; 6 | 2;48 | 0;54 |
| 7 | 0; 0[2] | 2; 2 | 3;38 | 4;24 | 4; 4 | 2;45 | 0;50 |
| 8 | 0; 4[3] | 2; 6 | 3;40 | 4;24 | 4; 2 | 2;42 | 0;46 |
| 9 | 0; 8 | 2; 9 | 3;43 | 4;24 | 4; 0 | 2;38 | 0;42 |
| 10 | 0;12 | 2;13 | 3;45 | 4;25 | 3;58 | 2;35 | 0;37 |
| 11 | 0;17 | 2;17 | 3;47 | 4;25 | 3;56 | 2;31 | 0;33 |
| 12 | 0;21 | 2;20 | 3;50 | 4;25 | 3;54 | 2;28 | 0;29 |
| 13 | 0;25 | 2;24 | 3;52 | 4;25 | 3;52 | 2;24 | 0;25 |
| 14 | 0;29 | 2;28 | 3;54 | 4;25 | 3;50 | 2;20 | 0;21 |
| 15 | 0;33 | 2;31[4] | 3;56 | 4;25 | 3;47 | 2;17 | 0;17 |
| 16 | 0;37 | 2;35[5] | 3;58 | 4;25 | 3;45 | 2;13 | 0;12 |
| 17 | 0;42 | 2;38 | 4; 0 | 4;24 | 3;43 | 2; 9 | 0; 8 |
| 18 | 0;46 | 2;42 | 4; 2 | 4;24 | 3;40 | 2; 6 | 0; 4 |
| 19 | 0;50 | 2;45 | 4; 4 | 4;24 | 3;38 | 2; 2 | 0; 0 |
| 20 | 0;54 | 2;48 | 4; 6 | 4;23 | 3;35 | 1;58 | From the nearest distance |
| 21 | 0;58 | 2;51 | 4; 7 | 4;23 | 3;33 | 1;54[6] | |
| 22 | 1; 2 | 2;54 | 4; 9 | 4;22 | 3;30 | 1;50 | |
| 23 | 1; 7 | 2;58 | 4;10 | 4;22 | 3;27 | 1;47 | |
| 24 | 1;11 | 3; 1 | 4;12 | 4;21 | 3;25 | 1;43 | |
| 25 | 1;15 | 3; 4 | 4;13 | 4;20 | 3;22 | 1;39 | |
| 26 | 1;19 | 3; 7 | 4;14 | 4;19 | 3;19 | 1;35 | |
| 27 | 1;23 | 3;10 | 4;15 | 4;18 | 3;16 | 1;31 | |
| 28 | 1;27 | 3;13 | 4;17 | 4;16 | 3;13 | 1;27 | |
| 29 | 1;31 | 3;16 | 4;18 | 4;15 | 3;10 | 1;23 | |

[1] **C** 23′ [2] **C** om. [3] **C** om. [4] **C** 35′ [5] **C** 31′ [6] **C** 55′.

**Table 30b: Correction of the true position of Mars (first part)**
*Sources*: **C** fols 69v–70r (see Plate 10), **$C_1$** fols 35v–36r, **$C_2$** fols 29v–30r, **L** fols 60v–61r, **B** pp. 116–117.

| Minutes of Proportions of the Correction | | | | Correction of the True Position of Mars | | | | | | | Minutes of Proportions of the Correction | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| epicycle | 0 | 1 | 2 | corrected centrum | 0 | 1 | 2 | 3 | 4 | 5 | epicycle | 3 | 4 | 5 |
| | ′ | ′ | ′ | | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | | ′ | ′ | ′ |
| 0 | 3 | 5 | 14 | 0 | 3;26[1] | 4;11 | 3;52[2] | 2;23 | 0; 5 | 2;15 | 0 | 22 | 28 | 30 |
| 1 | 3 | 6 | 14 | 1 | 3;28 | 4;12 | 3;50 | 2;19 | 0; 0 | 2;19 | 1 | 22 | 28 | 30 |
| 2 | 3 | 6 | 14 | 2 | 3;31 | 4;12 | 3;48 | 2;15 | 0; 5 | 2;23 | 2 | 22 | 28 | 30 |
| 3 | 3 | 6 | 15 | 3 | 3;33[3] | 4;12 | 3;46 | 2;11 | 0;10 | 2;27[4] | 3 | 22 | 28 | 31 |
| 4 | 2 | 7 | 15 | 4 | 3;36 | 4;12 | 3;44 | 2; 6 | 0;15 | 2;30 | 4 | 22 | 28 | 31 |
| 5 | 2 | 7 | 15 | 5 | 3;38 | 4;12 | 3;41 | 2; 2 | 0;20 | 2;34 | 5 | 23[5]↓ | 29 | 31 |
| 6 | 2 | 7 | 15 | 6 | 3;40 | 4;12 | 3;39[6] | 1;58 | 0;25 | 2;38 | 6 | 23 | 29 | 31 |
| 7 | 1 | 7[7] | 16 | 7 | 3;41 | 4;12 | 3;37 | 1;54 | 0;30 | 2;41 | 7 | 23 | 29 | 31 |
| 8 | 1 | 8 | 16 | 8 | 3;43 | 4;12 | 3;34 | 1;50 | 0;34 | 2;45 | 8 | 23⊥ | 29 | 32 |
| 9 | 1 | 8 | 16 | 9 | 3;45 | 4;12 | 3;32 | 1;45 | 0;39[8] | 2;49 | 9 | 24 | 29 | 32 |
| 10 | 1 | 8 | 16 | 10 | 3;47 | 4;12 | 3;29 | 1;40 | 0;44[9] | 2;52 | 10 | 24 | 29 | 32 |
| 11 | 0 | 8 | 17 | 11 | 3;49 | 4;12 | 3;27 | 1;36 | 0;49 | 2;56 | 11 | 24 | 29 | 32 |
| 12 | 0 | 9 | 17 | 12 | 3;51 | 4;12 | 3;24 | 1;32 | 0;54 | 2;59 | 12 | 24 | 30 | 33 |
| 13 | 0 | 9 | 17 | 13 | 3;52 | 4;11 | 3;21 | 1;27 | 0;59 | 3; 3 | 13 | 25 | 30 | 33 |
| 14 | 1 | 9 | 17 | 14 | 3;54 | 4;11 | 3;18 | 1;22 | 1; 3 | 3; 6 | 14 | 25 | 30 | 33 |
| 15 | 1 | 9 | 18 | 15 | 3;56 | 4;10 | 3;15 | 1;18 | 1; 8 | 3; 9 | 15 | 25 | 30 | 34 |
| 16 | 1 | 10 | 18 | 16 | 3;58 | 4;10 | 3;12 | 1;13 | 1;13 | 3;12 | 16 | 25 | 30 | 34 |
| 17 | 1 | 10 | 18 | 17 | 3;59 | 4; 9 | 3; 9 | 1; 8 | 1;18 | 3;15[10] | 17 | 25 | 30 | 35 |
| 18 | 2 | 10 | 19 | 18 | 4; 0 | 4; 8 | 3; 6 | 1; 3 | 1;22[11] | 3;18 | 18 | 26 | 30 | 35 |
| 19 | 2 | 11 | 19 | 19 | 4; 1 | 4; 7 | 3; 3 | 0;59 | 1;27 | 3;21 | 19 | 26 | 30 | 36 |
| 20 | 2 | 11 | 19 | 20 | 4; 2[12]↓ | 4; 6 | 2;59 | 0;54 | 1;32 | 3;24 | 20 | 26 | 30 | 36 |
| 21 | 3[13] | 11 | 19 | 21 | 4; 3 | 4; 5 | 2;56 | 0;49 | 1;36 | 3;27 | 21 | 26 | 30 | 37 |
| 22 | 3 | 11 | 20 | 22 | 4; 4 | 4; 4 | 2;52 | 0;44 | 1;40 | 3;29 | 22 | 26 | 30 | 38 |
| 23 | 3 | 12 | 20 | 23 | 4; 5 | 4; 3 | 2;49 | 0;39 | 1;45 | 3;32 | 23 | 27 | 30 | 38 |
| 24 | 3 | 12 | 20 | 24 | 4; 6 | 4; 1 | 2;45 | 0;34 | 1;50[14] | 3;34 | 24 | 27 | 30 | 39 |
| 25 | 4[15]↓ | 12 | 20 | 25 | 4; 7 | 4; 0 | 2;41 | 0;30 | 1;54 | 3;37 | 25 | 27 | 30 | 40 |
| 26 | 4⊥ | 13 | 21 | 26 | 4; 8 | 3;58 | 2;38[16] | 0;25 | 1;58 | 3;39 | 26 | 27 | 30 | 41 |
| 27 | 4 | 13 | 21 | 27 | 4; 9 | 3;57 | 2;34 | 0;20 | 2; 2 | 3;41 | 27 | 27 | 30 | 42 |
| 28 | 5 | 13 | 21 | 28 | 4;10⊥ | 3;55 | 2;30 | 0;15 | 2; 6 | 3;44 | 28 | 27 | 30 | 43 |
| 29 | 5 | 13 | 21 | 29 | 4;11 | 3;54 | 2;27 | 0;10 | 2;11 | 3;46 | 29 | 28 | 30 | 44 |

[1] **B** 16′ [2] **$C_2$** 12′ [3] **B** 38′ [4] **LB** 28′ [5] **C** $3^s$ 5–8°: 13′ [6] **B** 31′ [7] **C** 8′ [8] **C** 34′ [9] **C** 49′ [10] **C** 55′ [11] **B** 24′ [12] **$CC_1C_2$** $0^s$ 20–28°: slide[+1] [13] **L** 2′ [14] **L** 55′ [15] **LB** $0^s$ 25–26°: 3′ [16] **C** 18′.

**Table 30b: Correction of the true position of Mars (second part)**
*Sources*: **C** fols 69v–70r, $\mathbf{C_1}$ fols 35v–36r, $\mathbf{C_2}$ fols 29v–30r, **L** fols 60v–61r, **B** pp. 116–117.

| Minutes of Proportions of the Correction | | | | Correction of the True Position of Mars | | | | | | | Minutes of Proportions of the Correction | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| epicycle | 6 | 7 | 8 | corrected centrum | 6 | 7 | 8 | 9 | 10 | 11 | epicycle | 9 | 10 | 11 |
| | ′ | ′ | ′ | | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | | ′ | ′ | ′ |
| 0 | 45 | 39 | 30 | 0 | 3;48 | 4;12 | 3;31 | 2; 0[1] | 0; 4 | 1;53 | 0 | 27 | 20 | 12 |
| 1 | 46 | 38 | 30 | 1 | 3;50 | 4;12 | 3;28 | 1;57 | 0; 0 | 1;57 | 1 | 27 | 20 | 12 |
| 2 | 47 | 38 | 30 | 2 | 3;52 | 4;11 | 3;26 | 1;53 | 0; 4 | 2; 0[2] | 2 | 26 | 20 | 11 |
| 3 | 48 | 37 | 30 | 3 | 3;54 | 4;11 | 3;23 | 1;49 | 0; 8 | 2; 4 | 3 | 26 | 19 | 11 |
| 4 | 49 | 37 | 30 | 4 | 3;55 | 4;11 | 3;21 | 1;46 | 0;12 | 2; 8 | 4 | 26 | 19 | 11 |
| 5 | 51 | 36 | 30 | 5 | 3;57 | 4;10 | 3;18[3] | 1;42 | 0;16 | 2;11 | 5 | 26 | 19 | 11[4] |
| 6 | 52 | 35 | 30 | 6 | 3;58 | 4; 9 | 3;16 | 1;38 | 0;20 | 2;15[5] | 6 | 26 | 19 | 10 |
| 7 | 53 | 35 | 30 | 7 | 4; 0 | 4; 8 | 3;13 | 1;35 | 0;24 | 2;18 | 7 | 25 | 18 | 10 |
| 8 | 54 | 34 | 30 | 8 | 4; 1 | 4; 7 | 3;10 | 1;31 | 0;28 | 2;21 | 8 | 25 | 18 | 10 |
| 9 | 56 | 34 | 30 | 9 | 4; 3 | 4; 6 | 3; 8 | 1;27 | 0;32[6] | 2;24 | 9 | 25 | 18 | 9 |
| 10[7]↓ | 57 | 33 | 30 | 10 | 4; 4 | 4; 5 | 3; 5 | 1;24 | 0;36 | 2;28 | 10 | 25 | 17 | 9 |
| 11 | 59 | 33 | 30 | 11 | 4; 5 | 4; 4 | 3; 2 | 1;20 | 0;40 | 2;31 | 11 | 25 | 17 | 9 |
| 12 | 60 | 33 | 29 | 12 | 4; 6 | 4; 3 | 2;59 | 1;16 | 0;44 | 2;34 | 12 | 24 | 17 | 9 |
| 13 | 59 | 32 | 29 | 13 | 4; 7 | 4; 1 | 2;56 | 1;11 | 0;48 | 2;38[8] | 13 | 24 | 17 | 8 |
| 14 | 57[9] | 32 | 29 | 14 | 4; 8 | 4; 0 | 2;53 | 1; 7 | 0;52 | 2;41 | 14 | 24 | 16 | 8 |
| 15 | 56 | 32 | 29 | 15 | 4; 9 | 3;59 | 2;50 | 1; 4 | 0;56 | 2;44 | 15 | 24 | 16 | 8 |
| 16 | 54[10] | 32 | 29 | 16 | 4;10 | 3;58 | 2;47 | 1; 0 | 1; 0 | 2;47 | 16 | 23 | 16 | 8 |
| 17 | 53 | 31 | 29 | 17 | 4;10 | 3;56 | 2;44 | 0;56[11] | 1; 4[12]↓ | 2;50 | 17 | 23 | 16 | 7 |
| 18 | 52 | 31 | 29 | 18 | 4;11 | 3;54 | 2;41 | 0;52 | 1; 7 | 2;53 | 18 | 23 | 15 | 7 |
| 19 | 51 | 31 | 29 | 19 | 4;11[13] | 3;52 | 2;38 | 0;48 | 1;11 | 2;56 | 19 | 23 | 15 | 7 |
| 20 | 49 | 31 | 28[14] | 20 | 4;12 | 3;51 | 2;34 | 0;44 | 1;16⊥ | 2;59 | 20 | 22 | 15 | 7 |
| 21 | 48[15] | 31 | 28 | 21 | 4;12 | 3;49 | 2;31 | 0;40 | 1;20 | 3; 2 | 21 | 22 | 15 | 6 |
| 22 | 47 | 30 | 28 | 22 | 4;12 | 3;47 | 2;28[16] | 0;36 | 1;24 | 3; 5 | 22 | 22 | 14 | 6 |
| 23 | 46 | 30 | 28 | 23 | 4;12 | 3;45 | 2;24 | 0;32 | 1;27 | 3; 8 | 23 | 22 | 14 | 6 |
| 24 | 45 | 30 | 28 | 24 | 4;12 | 3;43[17] | 2;21 | 0;28 | 1;31 | 3;10 | 24 | 22 | 14 | 5 |
| 25 | 44 | 30 | 28 | 25 | 4;12 | 3;41 | 2;18 | 0;24 | 1;35 | 3;13 | 25 | 21 | 13 | 5 |
| 26 | 43 | 30 | 27 | 26 | 4;12 | 3;40 | 2;15 | 0;20 | 1;38 | 3;16 | 26 | 21 | 13 | 5 |
| 27 | 42 | 30 | 27 | 27 | 4;12 | 3;38 | 2;11 | 0;16 | 1;42 | 3;18 | 27 | 21 | 13 | 4 |
| 28 | 41 | 30 | 27 | 28 | 4;12 | 3;36 | 2; 8 | 0;12 | 1;46 | 3;21 | 28 | 21 | 13 | 4 |
| 29⊥ | 40 | 30 | 27 | 29 | 4;12 | 3;33 | 2; 4[18] | 0; 8 | 1;49 | 3;23 | 29 | 20 | 12 | 4 |

[1] **B** 1° [2] **C** 5′ [3] **C** 15′ [4] $\mathbf{CC_1C_2}$ 10′ [5] $\mathbf{C_1C_2}$ 14′ [6] **B** 22′ [7] **B** arguments 10–29°: om. [8] **L** 33′ [9] **C** 60′ [10] **B** 56′ [11] $\mathbf{CC_1C_2}$ 57′ [12] **C** $10^s$ 17–20° (minutes): slide[+1 col.] [13] $\mathbf{C_1C_2}$ 12′ [14] $\mathbf{CC_1C_2}$ 29′ [15] **B** 18′ [16] **C** 38′ [17] **B** 13′ [18] **B** 7′.

**Table 31: Mean motion of Venus (first part)**
*Sources*: **F** fol. 70v–71r, **H** fols 60v–61r, **C** fol. 70v, **$C_1$** fol. 36v, **$C_2$** fol. 30v, **Y** fols 289v–290r, **L** fol. 61v–62r, **B** pp. 118–119.

Mean Motion of Venus in Years and Months

| years | collected ⟨years⟩ s ° ′ |
|---|---|
| 1 | 1$^s$ 6;55 |
| 21 | 1$^s$ 2;10 |
| 41 | 0$^s$ 27;26 |
| 61 | 0$^s$ 22;41 |
| 81 | 0$^s$ 17;57[1] |
| 101 | 0$^s$ 13;12 |
| 121 | 0$^s$ 8;27[2] |
| 141 | 0$^s$ 3;43[4] |
| 161 | 11$^s$ 28;58[6] |
| 181 | 11$^s$ 24;14 |
| 201 | 11$^s$ 19;29 |
| 221 | 11$^s$ 14;45 |
| 241 | 11$^s$ 10; 0[7] |
| 261 | 11$^s$ 5;16 |
| 281 | 11$^s$ 0;31 |
| 301 | 10$^s$ 25;47 |
| 321 | 10$^s$ 21; 2 |
| 341 | 10$^s$ 16;18 |
| 361 | 10$^s$ 11;33[10] |
| 381[11] | 10$^s$ 6;49 |
| 401[14] | 10$^s$ 2; 4 |
| 421 | 9$^s$ 27;20 |
| 441 | 9$^s$ 22;35 |
| 461 | 9$^s$ 17;51 |
| 481 | 9$^s$ 13; 6 |
| 501 | 9$^s$ 8;21 |
| 521 | 9$^s$ 3;37 |
| 541 | 8$^s$ 28;52 |
| 561 | 8$^s$ 24; 8 |
| 581 | 8$^s$ 19;23 |

| years | extended ⟨years⟩ s ° ′ |
|---|---|
| 1 | 11$^s$ 29;46 |
| 2 | 11$^s$ 29;32 |
| 3 | 11$^s$ 29;17 |
| 4 | 11$^s$ 29; 3 |
| 5 | 11$^s$ 28;49 |
| 6 | 11$^s$ 28;35 |
| 7 | 11$^s$ 28;20[3] |
| 8 | 11$^s$ 28; 6[5] |
| 9 | 11$^s$ 27;52 |
| 10 | 11$^s$ 27;38 |
| 11 | 11$^s$ 27;24 |
| 12 | 11$^s$ 27; 9 |
| 13 | 11$^s$ 26;55[8] |
| 14 | 11$^s$ 26;41 |
| 15 | 11$^s$ 26;27 |
| 16 | 11$^s$ 26;12 |
| 17 | 11$^s$ 25;58[9] |
| 18 | 11$^s$ 25;44 |
| 19 | 11$^s$ 25;30 |
| 20 | 11$^s$ 25;15[12] |
| single years | |
| 40 | 11$^s$ 20;31 |
| 60 | 11$^s$ 15;46 |
| 80 | 11$^s$ 11; 2 |
| 100 | 11$^s$ 6;17 |
| 200 | 10$^s$ 12;35 |
| 300 | 9$^s$ 18;52 |
| 400 | 8$^s$ 25; 9[17] |
| 500 | 8$^s$ 1;27 |
| | [18] |

| months | months s ° ′ |
|---|---|
| Farwardīn | 0$^s$ 0; 0 |
| Urdībihisht | 0$^s$ 29;34 |
| Khurdād | 1$^s$ 29; 8 |
| Tīr | 2$^s$ 28;43 |
| Murdād | 3$^s$ 28;17 |
| Shahrīwar | 4$^s$ 27;51 |
| Mihr | 5$^s$ 27;25 |
| Ābān | 6$^s$ 26;59 |
| Ādhar | 7$^s$ 26;33 |
| | *8$^s$ 1;29* |
| Day | 8$^s$ 26; 8 |
| | *9$^s$ 1; 4*[13] |
| Bahman | 9$^s$ 25;42 |
| | *10$^s$ 0;38*[15] |
| Isfandārmudh | 10$^s$ 25;16 |
| | *11$^s$ 0;12*[16] |

To the left of the subtable for collected years, the main hand of **B** writes minutes that are two or three larger than the ones in the main table (cf. Section IV.5.3). [1] **YLB** 56′ **C** 17′ [2] **F** 26′ [3] **$C_1$** 21′ **$C_2$** 26′ [4] **L** 53′ [5] **$C_1$** 7′ (corrected) **$C_2$** 50′ [6] **B** 29° **C** 18′ [7] **$C_1$** 5′ [8] **C** 15′ [9] **C** 18′ [10] **$C_1$** 13′ [11] **C** ىعا [12] **F** 18′ [13] **YL** 3′ [14] **C** ىكا [15] **YL** 37′ [16] **H** 42′ [17] **F** 24° **YL** 10′ [18] **YLB** add a value 7$^s$ 7;44 for 600 single years.

**Table 31: Mean motion of Venus (second part)**
*Sources*: **F** fol. 70v–71r, **H** fols 60v–61r, **C** fol. 70v, $\mathbf{C_1}$ fol. 36v, $\mathbf{C_2}$ fol. 30v, **Y** fols 289v–290r, **L** fol. 61v–62r, **B** pp. 118–119.

| Mean Motion of Venus in Days, Hours, and Between Longitudes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | 0ˢ 0; 0 | 1 | 0; 2 | 31 | 1;16 | 71 | additive | 0, 3[1]↓ |
| 2 | 0ˢ 0;59 | 2 | 0; 5 | 32 | 1;19 | 72 | | 0, 3 |
| 3 | 0ˢ 1;58 | 3 | 0; 7 | 33 | 1;21 | 73 | | 0, 3 |
| 4 | 0ˢ 2;57 | 4 | 0;10 | 34 | 1;24 | 74 | | 0, 3 |
| 5 | 0ˢ 3;57[2] | 5 | 0;12 | 35 | 1;26 | 75 | | 0, 2 |
| 6 | 0ˢ 4;56 | 6 | 0;15 | 36 | 1;29 | 76 | | 0, 2 |
| 7 | 0ˢ 5;55[3] | 7 | 0;17 | 37 | 1;31 | 77 | | 0, 2 |
| 8 | 0ˢ 6;54 | 8 | 0;20 | 38 | 1;34 | 78 | | 0, 2 |
| 9 | 0ˢ 7;53 | 9 | 0;22 | 39 | 1;36 | 79 | | 0, 2 |
| 10 | 0ˢ 8;52[4] | 10 | 0;25 | 40 | 1;39 | 80 | | 0, 2 |
| 11 | 0ˢ 9;51[5] | 11 | 0;27 | 41 | 1;41 | 81 | | 0, 1 |
| 12 | 0ˢ 10;51 | 12 | 0;30 | 42 | 1;43 | 82 | | 0, 1 |
| 13 | 0ˢ 11;50 | 13 | 0;32 | 43 | 1;46 | 83 | | 0, 1 |
| 14 | 0ˢ 12;49 | 14 | 0;35 | 44 | 1;48 | 84 | | 0, 1 |
| 15 | 0ˢ 13;48 | 15 | 0;37 | 45 | 1;51 | 85 | | 0, 1 |
| 16 | 0ˢ 14;47 | 16 | 0;39 | 46 | 1;53 | 86 | | 0, 1 |
| 17 | 0ˢ 15;46 | 17 | 0;42 | 47 | 1;56 | 87 | | 0, 1 |
| 18 | 0ˢ 16;45 | 18 | 0;44 | 48 | 1;58 | 88 | | 0, 0 |
| 19 | 0ˢ 17;44[6] | 19 | 0;47 | 49 | 2; 1 | 89 | | 0, 0 |
| 20 | 0ˢ 18;44 | 20 | 0;49 | 50 | 2; 3 | 90 | | 0, 0 |
| 21 | 0ˢ 19;43 | 21 | 0;52 | 51 | 2; 6 | 91 | subtractive | 0, 0 |
| 22 | 0ˢ 20;42[7]↓ | 22 | 0;54 | 52 | 2; 8 | 92 | | 0, 0[8] |
| 23 | 0ˢ 21;41 | 23 | 0;57 | 53 | 2;11 | 93 | | 0, 1 |
| 24 | 0ˢ 22;40 | 24 | 0;59 | 54 | 2;13 | 94 | | 0, 1 |
| 25 | 0ˢ 23;39[9] | 25 | 1; 2 | 55 | 2;16 | 95 | | 0, 1 |
| 26 | 0ˢ 24;38 | 26 | 1; 4 | 56 | 2;18 | 96 | | 0, 1 |
| 27 | 0ˢ 25;38 | 27 | 1; 7 | 57 | 2;20 | 97 | | 0, 1 |
| 28 | 0ˢ 26;37[10] | 28 | 1; 9 | 58 | 2;23 | 98 | | 0, 1 |
| 29 | 0ˢ 27;36 | 29 | 1;11 | 59 | 2;25 | 99 | | 0, 1 |
| 30 | 0ˢ 28;35⊥ | 30 | 1;14[11] | 60 | 2;28[12] | 100 | | 0, 2[13]⊥ |

[1] **F** longitudes 71–100°: 0′0″ [2] $\mathbf{C_1}$ 56′ [3] **C** 21° [4] **F** 53′ [5] **F** 52′ [6] **YLB** 45′ [7] **C** 22–30 days: slide[−1] [8] **Y** 1′ [9] +**F** 24° [10] **C**+ 33′ (for 38′) [11] **YL** 13′ **B** 14′ corrected to 13′ [12] **C** 25′ [13] $\mathbf{CC_1C_2}$ 1′.

**Table 32: Mean anomaly of Venus (first part)**
*Sources*: **F** fol. 71v–72r, **H** fols 61v–62r, **C** fol. 71r, **$C_1$** fol. 37r, **$C_2$** fol. 31r, **Y** fols 290v–291r, **L** fol. 62v–63r, **B** pp. 120–121.

Mean Anomaly of Venus in Years and Months

| years | collected ⟨years⟩ s ° ′ | years | extended ⟨years⟩ s ° ′ | months | months s ° ′ |
|---|---|---|---|---|---|
| 1 | $4^s$ 1;58[1] | 1 | $7^s$ 15; 2[2] | Farwardīn | $0^s$ 0; 0 |
| 21[3] | $10^s$ 2;36 | 2 | $3^s$ 0; 4 | | |
| 41[4] | $4^s$ 3;14[5] | 3 | $10^s$ 15; 6[6] | Urdībihisht | $0^s$ 18;30 |
| 61 | $10^s$ 3;52[7] | 4 | $6^s$ 0; 8 | | |
| 81 | $4^s$ 4;30 | 5 | $1^s$ 15;10 | Khurdād | $1^s$ 6;59 |
| 101 | $10^s$ 5; 8[8] | 6 | $9^s$ 0;11 | | |
| 121 | $4^s$ 5;46[9] | 7 | $4^s$ 15;13[10] | Tīr | $1^s$ 25;29 |
| 141 | $10^s$ 6;25 | 8 | $0^s$ 0;15 | | |
| 161 | $4^s$ 7; 3 | 9 | $7^s$ 15;17 | Murdād | $2^s$ 13;59 |
| 181 | $10^s$ 7;41 | 10 | $3^s$ 0;19 | | |
| 201 | $4^s$ 8;19 | 11 | $10^s$ 15;21 | Shahrīwar | $3^s$ 2;29 |
| 221 | $10^s$ 8;57[11] | 12 | $6^s$ 0;23 | | |
| 241 | $4^s$ 9;35 | 13 | $1^s$ 15;25 | Mihr | $3^s$ 20;58 |
| 261 | $10^s$ 10;13 | 14 | $9^s$ 0;27 | | |
| 281 | $4^s$ 10;51 | 15 | $4^s$ 15;29 | Ābān | $4^s$ 9;28[12] |
| 301 | $10^s$ 11;29 | 16 | $0^s$ 0;30 | | |
| 321 | $4^s$ 12; 7[13] | 17 | $7^s$ 15;32 | Ādhar | $4^s$ 27;58 |
| 341 | $10^s$ 12;45[14] | 18 | $3^s$ 0;34 | | *$5^s$ 1; 3* |
| 361 | $4^s$ 13;23[15] | 19 | $10^s$ 15;36[16] | Day | $5^s$ 16;28 |
| 381 | $10^s$ 14; 1[17] | 20 | $6^s$ 0;38 | | *$5^s$ 19;33* |
| 401 | $4^s$ 14;40 | single years | | Bahman | $6^s$ 4;57 |
| 421 | $10^s$ 15;18 | | | | *$6^s$ 8; 2* |
| 441 | $4^s$ 15;56[18] | 40 | $0^s$ 1;16[19] | Isfandārmudh | $6^s$ 23;27 |
| 461 | $10^s$ 16;34 | 60 | $6^s$ 1;54 | | *$6^s$ 26;32* |
| 481 | $4^s$ 17;12 | 80 | $0^s$ 2;32 | | |
| 501 | $10^s$ 17;50 | 100 | $6^s$ 3;10 | | |
| 521 | $4^s$ 18;28 | 200 | $0^s$ 6;21 | | |
| 541 | $10^s$ 19; 6 | 300 | $6^s$ 9;31 | | |
| 561 | $4^s$ 19;44 | 400 | $0^s$ 12;42[20] | | |
| 581 | $10^s$ 20;22 | 500 | $6^s$ 15;52 | | |
| | | | [21] | | |

To the left of the subtable for collected years, the main hand of **B** writes minutes that are two larger than the ones in **FHC$C_1$$C_2$** (cf. Section IV.5.3). [1] **C** 18′ **B** 2° [2] **L** 35° [3] **C** 41 [4] **C** 21 [5] **C** 53° [6] **F** 16′ [7] **C** 12′ [8] **C** $3^s$ **YLB** 9′ [9] **YLB** 47′ [10] **L** 33′ [11] **C** 17′ [12] **C** 29° **$C_1$** 19° [13] **F** 11° [14] **C** $4^s$ 14°40′ (miscopied from argument 401) [15] **YLB** 24′ [16] **$C_1$** 16° [17] **C** 11′ **YLB** 2′ [18] **C** 16′ [19] **F** 56′ [20] **$C_1$** 45′ [21] **YLB** add a value $0^s$ 19°2′ for 600 single years.

**Table 32: Mean anomaly of Venus (second part)**
*Sources*: **F** fol. 71v–72r, **H** fols 61v–62r, **C** fol. 71r, $\mathbf{C_1}$ fol. 37r, $\mathbf{C_2}$ fol. 31r, **Y** fols 290v–291r, **L** fol. 62v–63r, **B** pp. 120–121.

Mean Anomaly of Venus in Days, Hours, and Between Longitudes

| days | days<br>s ° ′ | hours | hours<br>° ′ | hours | and their fractions<br>° ′ | longitudes | | in between longitudes<br>′ ″ |
|---|---|---|---|---|---|---|---|---|
| 1 | 0ˢ 0; 0 | 1 | 0; 2 | 31 | 0;47* | 71 | additive | 0, 2[1]↓ |
| 2 | 0ˢ 0;37 | 2 | 0; 3 | 32 | 0;49 | 72 | | 0, 2 |
| 3 | 0ˢ 1;14 | 3 | 0; 5 | 33 | 0;50* | 73 | | 0, 2 |
| 4 | 0ˢ 1;51 | 4 | 0; 6 | 34 | 0;52 | 74 | | 0, 2 |
| 5 | 0ˢ 2;28 | 5 | 0; 8 | 35 | 0;53* | 75 | | 0, 2 |
| 6 | 0ˢ 3; 5 | 6 | 0; 9 | 36 | 0;55 | 76 | | 0, 1 |
| 7 | 0ˢ 3;42 | 7 | 0;11 | 37 | 0;56* | 77 | | 0, 1 |
| 8 | 0ˢ 4;19[2] | 8 | 0;12 | 38 | 0;58* | 78 | | 0, 1 |
| 9 | 0ˢ 4;56[3] | 9 | 0;14 | 39 | 0;59* | 79 | | 0, 1 |
| 10 | 0ˢ 5;33 | 10 | 0;15 | 40 | 1; 1* | 80 | | 0, 1 |
| 11 | 0ˢ 6;10 | 11 | 0;17 | 41 | 1; 3 | 81 | | 0, 1 |
| 12 | 0ˢ 6;47[4] | 12 | 0;18* | 42 | 1; 4* | 82 | | 0, 1 |
| 13 | 0ˢ 7;24 | 13 | 0;20 | 43 | 1; 6 | 83 | | 0, 1 |
| 14 | 0ˢ 8; 1 | 14 | 0;22 | 44 | 1; 7* | 84 | | 0, 1 |
| 15 | 0ˢ 8;38 | 15 | 0;23 | 45 | 1; 9 | 85 | | 0, 1 |
| 16 | 0ˢ 9;15 | 16 | 0;25 | 46 | 1;11[5] | 86 | | 0, 0 |
| 17 | 0ˢ 9;52 | 17 | 0;26[6] | 47 | 1;12 | 87 | | 0, 0 |
| 18 | 0ˢ 10;29 | 18 | 0;28 | 48 | 1;14 | 88 | | 0, 0 |
| 19 | 0ˢ 11; 6 | 19 | 0;29 | 49 | 1;15* | 89 | | 0, 0 |
| 20 | 0ˢ 11;43 | 20 | 0;31 | 50 | 1;17 | 90 | | 0, 0 |
| 21 | 0ˢ 12;20 | 21 | 0;32 | 51 | 1;18* | 91 | subtractive | 0, 0 |
| 22 | 0ˢ 12;57 | 22 | 0;34 | 52 | 1;20 | 92 | | 0, 0 |
| 23 | 0ˢ 13;34 | 23 | 0;35 | 53 | 1;21* | 93 | | 0, 0 |
| 24 | 0ˢ 14;11 | 24 | 0;37 | 54 | 1;23 | 94 | | 0, 0 |
| 25 | 0ˢ 14;48 | 25 | 0;38* | 55 | 1;24* | 95 | | 0, 1 |
| 26 | 0ˢ 15;25 | 26 | 0;40 | 56 | 1;26 | 96 | | 0, 1 |
| 27 | 0ˢ 16; 2 | 27 | 0;41* | 57 | 1;27* | 97 | | 0, 1 |
| 28 | 0ˢ 16;39 | 28 | 0;43 | 58 | 1;29 | 98 | | 0, 1 |
| 29 | 0ˢ 17;16 | 29 | 0;44* | 59 | 1;30* | 99 | | 0, 1 |
| 30 | 0ˢ 17;53[7] | 30 | 0;46 | 60 | 1;32 | 100 | | 0, 1⊥ |

* In **YLB** the values for 25, 27, 31, 33, 37–39, 55, and 57 hours are 1 minute larger; in **YL** (but not in **B**) the values for 12, 29, 35, 40, 42, 44, 49, 51, 53, and 59 hours are also 1 minute larger.
[1] **F** longitudes 71–100°: 0′0″ [2] **L** 59′ [3] $\mathbf{CC_1C_2}$ 57′ [4] $\mathbf{C_1}$ 44′ [5] **L** 10′ [6] **C** 27′ [7] **F** 13′ **L** 43′.

**Table 33: First equation for Venus (first half)**
*Sources*: **F** fols 72v–73r, **H** fols 62v–63r, **C** fols 71v–72r, $\mathbf{C_1}$ fols 37v–38r, $\mathbf{C_2}$ fols 31v–32r, **Y** fols 291v–292r, **L** fols 63v–64r, **B** pp. 122–123.

First Equation for Venus
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 0 (° ′) | differences ′ | mean distance 11° 1 (° ′) | differences ′ | 2 (° ′) | differences ′ | 3 (° ′) | differences ′ | nearest distance 10° 4 (° ′) | differences ′ | 5 (° ′) | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0;31 | 1 | 0; 3 | 1 | 0; 7[1] | 1 | 0;41 | 1[2]→ | 1;39 | 3[3] | 2;42 | 2[4] |
| 1 | 0;29 | 2 | 0; 3 | 0 | 0; 7 | 0 | 0;43 | 2 | 1;41 | 2 | 2;44 | 2 |
| 2 | 0;28 | 1 | 0; 3 | 0[5] | 0; 8 | 1 | 0;45 | 2 | 1;43 | 2 | 2;46 | 2 |
| 3 | 0;27 | 1 | 0; 2 | 1[6] | 0; 9 | 1 | 0;46 | 1 | 1;45 | 2 | 2;48 | 2 |
| 4 | 0;26 | 1 | 0; 2 | 0 | 0;10 | 1 | 0;48 | 2 | 1;47 | 2 | 2;50 | 2 |
| 5 | 0;24 | 2 | 0; 2 | 0 | 0;11 | 1 | 0;50 | 2 | 1;49 | 2 | 2;52 | 2 |
| 6 | 0;23 | 1 | 0; 2 | 0 | 0;11 | 0 | 0;51 | 1 | 1;51 | 2 | 2;54 | 2 |
| 7 | 0;22 | 1 | 0; 1 | 1 | 0;12 | 1 | 0;53 | 2 | 1;54 | 3 | 2;56 | 2 |
| 8 | 0;21 | 1 | 0; 1 | 0 | 0;13 | 1 | 0;55 | 2 | 1;56 | 2 | 2;58 | 2 |
| 9 | 0;20 | 1 | 0; 1 | 0 | 0;14 | 1 | 0;57 | 2 | 1;58 | 2 | 2;59 | 1 |
| 10 | 0;19 | 1 | 0; 1 | 0 | 0;15 | 1 | 0;59 | 2 | 2; 0 | 2 | 3; 1 | 2 |
| 11 | 0;18 | 1 | 0; 1 | 0 | 0;16 | 1 | 1; 1 | 2 | 2; 2 | 2 | 3; 3 | 2 |
| 12 | 0;17 | 1 | 0; 1 | 0 | 0;17 | 1 | 1; 2 | 1 | 2; 4 | 2 | 3; 5 | 2 |
| 13 | 0;16 | 1 | 0; 1 | 0 | 0;18 | 1 | 1; 4 | 2 | 2; 6 | 2 | 3; 7 | 2 |
| 14 | 0;15 | 1 | 0; 1 | 0 | 0;19 | 1 | 1; 6 | 2 | 2; 9 | 3 | 3; 9 | 2 |
| 15 | 0;14 | 1 | 0; 1 | 0 | 0;20 | 1 | 1; 8 | 2 | 2;11 | 2 | 3;10 | 1 |
| 16 | 0;13 | 1 | 0; 1 | 0 | 0;22[7] | 2 | 1;10 | 2 | 2;13 | 2 | 3;12 | 2 |
| 17 | 0;12 | 1 | 0; 1 | 0 | 0;23 | 1 | 1;12 | 2 | 2;15 | 2 | 3;14 | 2 |
| 18 | 0;11 | 1 | 0; 2 | 1 | 0;24 | 1 | 1;14 | 2 | 2;17 | 2 | 3;15 | 1 |
| 19 | 0;10 | 1 | 0; 2 | 0 | 0;25 | 1 | 1;16 | 2 | 2;19 | 2 | 3;17 | 2 |
| 20 | 0; 9 | 1 | 0; 2 | 0 | 0;27 | 2 | 1;18 | 2 | 2;21 | 2 | 3;19[8] | 2[9] |
| 21 | 0; 9 | 0 | 0; 2 | 0 | 0;28 | 1 | 1;20 | 2 | 2;24 | 3 | 3;20 | 1 |
| 22 | 0; 8 | 1 | 0; 3 | 1 | 0;29 | 1 | 1;22 | 2 | 2;26 | 2 | 3;22 | 2 |
| 23 | 0; 7 | 1 | 0; 3 | 0 | 0;31[10] | 2 | 1;24 | 2 | 2;28 | 2 | 3;23 | 1 |
| 24 | 0; 7 | 0 | 0; 3 | 0 | 0;32 | 1 | 1;26 | 2 | 2;30 | 2 | 3;25 | 2[11]↓ |
| 25 | 0; 6 | 1[12] | 0; 4 | 1 | 0;34 | 2 | 1;28 | 2 | 2;32 | 2 | 3;26 | 1 |
| 26 | 0; 5 | 1 | 0; 4 | 0 | 0;35 | 1 | 1;30[13] | 2 | 2;34 | 2 | 3;28 | 2⊥ |
| 27 | 0; 5 | 0 | 0; 5 | 1 | 0;37 | 2 | 1;32 | 2 | 2;36[14] | 2 | 3;29 | 1 |
| 28 | 0; 4 | 1 | 0; 6 | 1 | 0;38 | 1 | 1;34 | 2 | 2;38 | 2 | 3;31 | 2 |
| 29 | 0; 4 | 0 | 0; 6 | 0 | 0;40 | 2 | 1;36 | 2 | 2;40 | 2⌟ | 3;32 | 1 |

[1] **C** 6′ [2] **F** 3′ **L** 3–4ˢ 0–29° (differences): slide[+1 col.] [3] +**F** 2′ [4] **F** 1′ [5] **L** 1′ [6] **L** 2′ [7] $\mathbf{C_2}$ 24′ [8] **C** 9′ [9] **C** 1′ [10] **L** 30′ [11] **C** 5ˢ 24–26°: block transposition [12] **Y** om. (hence read as 0′) [13] **F** 32′ [14] **C** 37′.

**Table 33: First equation for Venus (second half)**

*Sources*: **F** fols 72v–73r, **H** fols 62v–63r, **C** fols 71v–72r, $\mathbf{C_1}$ fols 37v–38r, $\mathbf{C_2}$ fols 31v–32r, **Y** fols 291v–292r, **L** fols 63v–64r, **B** pp. 122–123.

First Equation for Venus
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | **6** ° ′ | differences ′ | mean distance 9° **7** ° ′ | differences ′ | **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | furthest distance 10° **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 3;33 | 1 | 3;58[1] | 0 | 3;51 | 0 | 3;15 | 1 | 2;20 | 2 | 1;21 | 1[2]↓ |
| 1 | 3;35 | 2 | 3;58 | 0[3] | 3;50 | 1 | 3;13 | 2 | 2;18 | 2 | 1;19 | 2 |
| 2 | 3;36 | 1 | 3;58 | 0[4] | 3;49 | 1 | 3;11 | 2 | 2;16 | 2 | 1;17 | 2 |
| 3 | 3;37 | 1 | 3;59 | 1[5]↓ | 3;48 | 1 | 3;10 | 1 | 2;14 | 2 | 1;15 | 2 |
| 4 | 3;38 | 1[6] | 3;59 | 0 | 3;47 | 1 | 3; 8 | 2 | 2;12 | 2 | 1;13 | 2 |
| 5 | 3;40 | 2 | 3;59 | 0 | 3;46 | 1 | 3; 6 | 2 | 2;10 | 2 | 1;11 | 2 |
| 6 | 3;41 | 1[7] | 3;59 | 0 | 3;45 | 1 | 3; 5 | 1 | 2; 8 | 2 | 1; 9 | 2 |
| 7 | 3;42 | 1 | 3;59 | 0 | 3;44 | 1 | 3; 3 | 2 | 2; 6 | 2 | 1; 8 | 1 |
| 8 | 3;43 | 1 | 3;59 | 0 | 3;43 | 1 | 3; 1 | 2 | 2; 4 | 2 | 1; 6[8] | 2 |
| 9 | 3;44 | 1 | 3;59 | 0 | 3;42 | 1 | 3; 0 | 1 | 2; 2 | 2 | 1; 4 | 2 |
| 10 | 3;45 | 1 | 3;59 | 0 | 3;41 | 1 | 2;58 | 2 | 2; 0 | 2 | 1; 2 | 2 |
| 11 | 3;46 | 1 | 3;59 | 0 | 3;40 | 1 | 2;56 | 2 | 1;58 | 2 | 1; 0[9] | 2 |
| 12 | 3;47 | 1 | 3;59 | 0 | 3;39 | 1 | 2;54[10] | 2 | 1;56 | 2 | 0;59 | 1 |
| 13 | 3;48 | 1 | 3;59 | 0 | 3;38[11] | 1 | 2;52 | 2 | 1;54 | 2 | 0;57 | 2 |
| 14 | 3;49 | 1 | 3;58[12] | 1 | 3;37 | 1 | 2;51 | 1 | 1;52 | 2 | 0;55 | 2 |
| 15 | 3;49 | 0 | 3;58 | 0[13] | 3;36 | 1 | 2;49 | 2[14] | 1;50 | 2 | 0;54 | 1 |
| 16 | 3;50 | 1 | 3;58 | 0 | 3;34 | 2[15] | 2;47 | 2 | 1;48 | 2 | 0;52 | 2 |
| 17 | 3;51 | 1 | 3;58 | 0[16] | 3;33 | 1 | 2;45[17] | 2 | 1;46 | 2 | 0;50 | 2 |
| 18 | 3;52 | 1 | 3;57 | 1 | 3;32 | 1 | 2;43 | 2 | 1;44[18] | 2 | 0;49 | 1 |
| 19 | 3;53 | 1 | 3;57 | 0 | 3;31 | 1 | 2;41 | 2 | 1;42 | 2 | 0;47 | 2 |
| 20 | 3;53[19] | 0 | 3;57[20] | 0[21] | 3;29 | 2 | 2;39 | 2 | 1;40[22] | 2 | 0;45 | 2 |
| 21 | 3;54 | 1 | 3;56 | 1[23] | 3;28 | 1 | 2;38 | 1 | 1;38 | 2 | 0;44 | 1 |
| 22 | 3;54 | 0 | 3;56[24] | 0 | 3;26 | 2 | 2;36 | 2 | 1;36 | 2 | 0;42 | 2 |
| 23 | 3;55 | 1 | 3;55 | 1 | 3;25 | 1 | 2;34 | 2 | 1;34 | 2 | 0;41 | 1 |
| 24 | 3;56 | 1 | 3;55 | 0 | 3;24 | 1 | 2;32 | 2 | 1;32 | 2 | 0;39 | 2 |
| 25 | 3;56 | 0 | 3;54 | 1 | 3;22[25] | 2 | 2;30 | 2 | 1;30[26] | 2 | 0;38 | 1 |
| 26 | 3;57 | 1 | 3;53 | 1 | 3;21 | 1 | 2;28 | 2 | 1;28 | 2 | 0;36 | 2 |
| 27 | 3;57 | 0 | 3;53 | 0 | 3;19 | 2 | 2;26 | 2 | 1;26 | 2 | 0;35 | 1 |
| 28 | 3;57 | 0 | 3;52 | 1 | 3;18 | 1 | 2;24 | 2 | 1;24 | 2 | 0;34 | 1 |
| 29 | 3;58 | 1[27] | 3;51 | 1⊥ | 3;16 | 2 | 2;22 | 2 | 1;22 | 2 | 0;32 | 2⊥ |

[1] **Y** 18′ [2] **C** $11^s$ 0–29°: differences of the second equation (corrected only for $11^s$ 23–29°) [3] **Y** 2′ [4] $\mathbf{C_1}$ 1′ [5] $\mathbf{C_1}$ 0′ $\mathbf{C_2}$ $7^s$ 3–29°: om. [6] **L** 2′ [7] **Y** om. (causing all values up to $6^s$ 13° to be read as 2′) [8] $\mathbf{C_1 C_2}$ 7′ [9] $\mathbf{C_1}$ 1′ [10] **H** 51′ [11] **C** 18′ [12] **Y** 18′ [13] +**C** 1′ [14] **C** 1′ [15] **L** 1′ [16] +**C** 1′ [17] **L** 47′ [18] **Y** 47′ [19] **C** 54′ [20] $\mathbf{C_1}$ 17′ [21] +**C** 1′ [22] **F** 47′ **L** 46′ [23] +**C** 0′ [24] **Y** 55′ [25] **Y** 21′ [26] **L** 32′ [27] **C** 0′.

## Table 33: Second equation for Venus (first half)

*Sources*: **F** fols 73v–74r, **H** fols 63v–64r, **C** fols 72v–73r, **$C_1$** fols 38v–39r, **$C_2$** fols 32v–33r, **Y** fols 292v–293r, **L** fols 64v–65r, **B** pp. 124–125.

Second Equation for Venus
to be equated and added to the centrum with the apogee

| degrees of the epicycle | furthest distance 0° **0** ° ′ | differences ′ | **1** ° ′ | differences ′ | mean distance 23° **2** ° ′ | differences ′ | **3** ° ′ | differences ′ | **4** ° ′ | differences ′ | **5** ° ′ | differences ° ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 48; 0 | 26[1] | 60;30 | 24 | 72;38 | 23 | 83;43 | 20 | 92;14 | 13 | 91;39 | 22 |
| 1 | 48;26 | 26 | 60;55[2]↓ | 25 | 73; 2 | 24 | 84; 3 | 20 | 92;27 | 13 | 91;15 | 24 |
| 2 | 48;51 | 25 | 61;20 | 25 | 73;25 | 23 | 84;24 | 21 | 92;38 | 11 | 90;48 | 27[3] |
| 3 | 49;16 | 25 | 61;44 | 24 | 73;48 | 23 | 84;44 | 20 | 92;48 | 10 | 90;18 | 30[4] |
| 4 | 49;41[5] | 25 | 62; 9 | 25 | 74;11 | 23 | 85; 4 | 20 | 92;57 | 9 | 89;45[6] | 33 |
| 5 | 50; 6 | 25 | 62;34 | 25 | 74;34 | 23 | 85;24 | 20 | 93; 6 | 9 | 89; 8 | 37 |
| 6 | 50;31 | 25 | 62;58 | 24 | 74;57 | 23 | 85;44 | 20 | 93;14 | 8 | 88;28 | 40 |
| 7 | 50;56 | 25 | 63;23 | 25 | 75;20 | 23 | 86; 3 | 19 | 93;21 | 7 | 87;45 | 43 |
| 8 | 51;21 | 25 | 63;48 | 25 | 75;43 | 23 | 86;21 | 18[7] | 93;27 | 6 | 86;58[8] | 47 |
| 9 | 51;46[9] | 25 | 64;12 | 24 | 76; 6 | 23 | 86;40 | 19 | 93;33 | 6 | 86; 7 | 51[10] |
| 10 | 52;11 | 25 | 64;37 | 25 | 76;29 | 23 | 86;59 | 19 | 93;39 | 6 | 85;12 | 55[11] |
| 11 | 52;36 | 25 | 65; 1 | 24 | 76;52 | 23 | 87;17 | 18 | 93;45 | 6 | 84;12 | 1; 0[12] |
| 12 | 53; 1 | 25 | 65;25 | 24[13] | 77;14 | 22 | 87;35 | 18 | 93;50 | 5 | 83; 7[14] | 1; 5 |
| 13 | 53;26 | 25 | 65;50 | 25 | 77;37 | 23[15] | 87;53 | 18 | 93;54 | 4 | 81;58[16] | 1; 9 |
| 14 | 53;51 western vis | 25 | 66;14 | 24 | 77;59 | 22 | 88;11 | 18 | 93;57 | 3 | 80;44 | 1;14 |
| 15 | 54;16 | 25 | 66;38 | 24 | 78;21 | 22 | 88;29 | 18 | 93;59 | 2 | 79;24 stationary | 1;20 |
| 16 | 54;41 | 25 | 67; 3 | 25 | 78;43 | 22 | 88;46[17] | 17 | 93;59 | 0 | 77;58[18] | 1;26 |
| 17 | 55; 6[19] | 25 | 67;27 | 24 | 79; 5 | 22 | 89; 3 | 17 | 93;58 | 1 | 76;25 | 1;33 |
| 18 | 55;31 | 25 | 67;51 | 24 | 79;27 | 22 | 89;20 | 17 | 93;57 | 1[20] | 74;46 retrograde | 1;39 |
| 19 | 55;56 | 25 | 68;15 | 24 | 79;49 | 22 | 89;37 | 17 | 93;55 | 2 | 73; 2[21] | 1;44 |
| 20 | 56;21 | 25 | 68;39 | 24 | 80;11 | 22 | 89;53 | 16 | 93;51 | 4 | 71;12[22] | 1;50 |
| 21 | 56;46 | 25 | 69; 3 | 24 | 80;33 | 22 | 90; 9 | 16 | 93;46 | 5 | 69;15 | 1;57[23] |
| 22 | 57;11 | 25 | 69;27 | 24 | 80;55 | 22 | 90;24 | 15 | 93;39 | 7 | 67;11[24] | 2; 4 |
| 23 | 57;36 | 25 | 69;51 | 24 | 81;16 | 21 | 90;39 | 15 | 93;31 | 8 | 65; 2 | 2; 9 |
| 24 | 58; 1 | 25 | 70;15 | 24 | 81;38 | 22 | 90;53 | 14 | 93;21 | 10 | 62;47 | 2;15[25] |
| 25 | 58;26 | 25 | 70;39 | 24 | 82; 0 | 22 | 91; 7 | 14 | 93; 9 | 12 | 60;27 | 2;20 |
| 26 | 58;51[26] | 25 | 71; 3 | 24 | 82;21 | 21 | 91;21 | 14 | 92;55[27] | 14 | 58; 4 western dis | 2;23 |
| 27 | 59;16[28] | 25 | 71;27 | 24 | 82;42 | 21 | 91;35 | 14 | 92;39 | 16 | 55;38 | 2;26 |
| 28 | 59;41 | 25 | 71;51⊥ | 24 | 83; 3 | 21 | 91;48 | 13 | 92;21 | 18[29] | 53; 9 | 2;29 |
| 29 | 60; 6 | 25 | 72;15 | 24 | 83;23 | 20[30] | 92; 1 | 13 | 92; 1 | 20 | 50;36 | 2;33 |

**Y** writes differences larger than 1;0° as minutes rather than as degrees plus minutes. [1] **C** 27′ [2] **L** 1ˢ 1–28°: slide[+1] (partially corrected) [3] **F** 26′ [4] **F** 1′ [5] **L** 41°, om. minutes [6] **C** 80°9′ [7] **L** 17′ [8] **FC** 18′ [9] **F** 11° **L** 36′ [10] **C** 11′ [11] **C** 15′ [12] **C** 5′ [13] **F** 25′ [14] **Y** 84° [15] **C** 22′ [16] **C** 18′ [17] **L** 56′ [18] **$CC_1C_2$** 18′ [19] **Y** 56′ [20] **F** 2′ [21] **F** 72° **Y** 74° [22] **H** om. minutes [23] **C** 17′ [24] **F** 41′ **H** 65° [25] **$FC_1$** 14′ [26] **F** 59° [27] **C** 15′ [28] **F** 19° [29] **L** 13′ [30] **F** 21′.

**Table 33: Second equation for Venus (second half)**
*Sources*: **F** fols 73v–74r, **H** fols 63v–64r, **C** fols 72v–73r, $\mathbf{C_1}$ fols 38v–39r, $\mathbf{C_2}$ fols 32v–33r, **Y** fols 292v–293r, **L** fols 64v–65r, **B** pp. 124–125.

Second Equation for Venus
to be equated and added to the centrum with the apogee

| degrees of the epicycle | nearest distance 0° **6** ° ′ | differences ° ′ | **7** ° ′ | differences ′ | mean distance 7° **8** ° ′ | differences ′ | **9** ° ′ | differences ′ | **10** ° ′ | differences ′ | **11** ° ′ | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 48; 0 | 2;36 | 4;21[1] | 24 | 3;46 | 13 | 12;17[2]↓ | 20 | 23;22[3] | 24 | 35;30 | 25 |
| 1 | 45;24 | 2;36 | 3;59 | 22 | 3;59 | 13 | 12;37 | 20 | 23;45 | 23 | 35;54 | 24 |
| 2 | 42;51[4] eastern vis | 2;33 | 3;39 | 20 | 4;12 | 13 | 12;57⊥ | 20 | 24; 9 | 24 | 36;19[5] | 25 |
| 3 | 40;22 | 2;29 | 3;21 | 18 | 4;25 | 13 | 13;18 | 21 | 24;33 | 24 | 36;44 | 25 |
| 4 | 37;56[6] | 2;26 | 3; 5 | 16 | 4;39 | 14 | 13;39 | 21 | 24;57 | 24 | 37; 9 | 25 |
| 5 | 35;33[7] | 2;23 | 2;51 | 14 | 4;53 | 14 | 14; 0 | 21 | 25;21 | 24 | 37;34 | 25 |
| 6 | 33;13[8] | 2;20 | 2;39 | 12[9] | 5; 7 | 14 | 14;22 | 22 | 25;45 | 24 | 37;59 | 25 |
| 7 | 30;58 | 2;15[10] | 2;29 | 10 | 5;21 | 14 | 14;44 | 22 | 26; 9 | 24 | 38;24 | 25 |
| 8 | 28;49 | 2; 9 | 2;21 | 8 | 5;36 | 15 | 15; 5[11] | 21[12] | 26;33 | 24 | 38;49 | 25 |
| 9 | 26;45 | 2; 4[13] | 2;14 | 7 | 5;51 | 15 | 15;27 | 22 | 26;57 | 24 | 39;14 | 25 |
| 10 | 24;48 stationary | 1;57 | 2; 9 | 5 | 6; 7[14] | 16[15] | 15;49 | 22 | 27;21 | 24 | 39;39 | 25 |
| 11 | 22;58 | 1;50[16] | 2; 5 | 4 | 6;23 | 16 | 16;11 | 22 | 27;45 | 24 | 40; 4 | 25 |
| 12 | 21;14 | 1;44[17] | 2; 3 | 2 | 6;40 | 17 | 16;33 | 22 | 28; 9 | 24 | 40;29 | 25 |
| 13 | 19;35 progressive | 1;39 | 2; 2 | 1 | 6;57 | 17 | 16;55 | 22 | 28;33 | 24 | 40;54 | 25 |
| 14 | 18; 2 | 1;33 | 2; 1 | 1 | 7;14 | 17 | 17;17 | 22 | 28;57 | 24 | 41;19 eastern dis | 25 |
| 15 | 16;36[18] | 1;26 | 2; 1 | 0 | 7;31 | 17 | 17;39 | 22 | 29;22 | 25[19] | 41;44 | 25 |
| 16 | 15;16 | 1;20 | 2; 3 | 2 | 7;49 | 18 | 18; 1 | 22 | 29;46 | 24 | 42; 9 | 25 |
| 17 | 14; 2 | 1;14 | 2; 6 | 3 | 8; 7 | 18 | 18;23 | 22 | 30;10 | 24 | 42;34 | 25 |
| 18 | 12;53 | 1; 9 | 2;10 | 4 | 8;25 | 18 | 18;46 | 23 | 30;35 | 25[20] | 42;59 | 25 |
| 19 | 11;48 | 1; 5 | 2;15[21] | 5 | 8;43 | 18 | 19; 8 | 22 | 30;59 | 24 | 43;24 | 25 |
| 20 | 10;48 | 1; 0 | 2;21 | 6 | 9; 1[22] | 18 | 19;31 | 23 | 31;23 | 24[23] | 43;49 | 25 |
| 21 | 9;53 | 55[24] | 2;27[25] | 6 | 9;20 | 19 | 19;54 | 23 | 31;48 | 25[26] | 44;14 | 25 |
| 22 | 9; 2 | 51 | 2;33 | 6 | 9;39 | 19 | 20;17 | 23 | 32;12 | 24 | 44;39 | 25 |
| 23 | 8;15 | 47 | 2;39 | 6[27] | 9;57 | 18 | 20;40 | 23 | 32;37 | 25 | 45; 4 | 25 |
| 24 | 7;32[28] | 43 | 2;46 | 7 | 10;16 | 19 | 21; 3 | 23 | 33; 2 | 25 | 45;29 | 25 |
| 25 | 6;52 | 40 | 2;54 | 8 | 10;36 | 20 | 21;26 | 23 | 33;26 | 24[29] | 45;54 | 25 |
| 26 | 6;15 | 37 | 3; 3 | 9 | 10;56 | 20 | 21;49 | 23 | 33;51 | 25 | 46;19 | 25 |
| 27 | 5;42 | 33 | 3;12 | 9 | 11;16 | 20 | 22;12 | 23 | 34;16 | 25 | 46;44 | 25 |
| 28 | 5;12 | 30 | 3;22 | 10 | 11;36 | 20 | 22;35 | 23 | 34;40 | 24[30] | 47; 9 | 25 |
| 29 | 4;45 | 27[31] | 3;33 | 11[32] | 11;57 | 21 | 22;58[33] | 23 | 35; 5[34] | 25 | 47;34 | 25 |

**Y** writes differences larger than 1;0° as minutes rather than as degrees plus minutes. [1] **H** 3° [2] **C** $9^s$ 0–2° (minutes): slide[+1 col.] $\mathbf{C_1}$ 16′ [3] $\mathbf{C_1C_2}$ 42′ [4] **F** 45° [5] **L** 59′ [6] $\mathbf{C_1C_2}$ 36′ [7] **C** 13′ [8] **C** 33′ [9] **F** 14′ [10] $\mathbf{C_1}$ 16′ [11] $\mathbf{CC_1C_2}$ 6′ [12] **F** 23′ $\mathbf{CC_1C_2}$ 22′ [13] **F** 2′ [14] **C** 50′ [15] $\mathbf{C_1C_2}$ 15′ [16] **C** 30′ [17] **L** 14′ [18] **L** 35′ [19] **FY** 24′ [20] **F** 24′ [21] **C** 7′ [22] **H** 11′ [23] **F** 25′ [24] $\mathbf{C_1}$ 54′ [25] **C** 23′ [26] **F** 24′ [27] **C** 7′ [28] **H** 4° [29] **Y** 25′ [30] **Y** 25′ [31] **C** 26′ [32] **L** 10′ [33] **C** 18′ [34] **C** 0′.

**Table 33: Variation of the nearest distance for Venus**
*Sources*: **F** fol. 74v, **H** fol. 64v, **C** fol. 73v, **$C_1$** fol. 39v, **$C_2$** fol. 33v, **Y** fol. 293v, **L** fol. 65v, **B** p. 126.

| Variation of the Nearest Distance for Venus | | | | | | | |
|---|---|---|---|---|---|---|---|
| corrected centrum | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | ↓→[1] From the furthest distance | 0;17 | 0;43 | 1; 0 | 0;59 | 0;39 | 0;10 |
| 1 | | 0;18[2] | 0;44 | 1; 0 | 0;58 | 0;38 | 0; 9 |
| 2 | | 0;19 | 0;45⌋ | 1; 1 | 0;58 | 0;37 | 0; 8 |
| 3 | | 0;20 | 0;45↓[3] | 1; 1 | 0;58 | 0;36 | 0; 7 |
| 4 | | 0;21 | 0;46 | 1; 1 | 0;57 | 0;35 | 0; 6 |
| 5 | | 0;22 | 0;47 | 1; 1 | 0;57 | 0;34[4] | 0; 5 |
| 6 | | 0;23 | 0;48⊥ | 1; 1 | 0;56 | 0;34[5] | 0; 4 |
| 7 | | 0;24 | 0;49 | 1; 1 | 0;56 | 0;33 | 0; 3 |
| 8 | | 0;25 | 0;50 | 1; 2 | 0;55 | 0;32 | 0; 2 |
| 9 | | 0;26 | 0;50 | 1; 2 | 0;54 | 0;32 | 0; 1 |
| 10 | | 0;27 | 0;51 | 1; 2 | 0;54 | 0;31 | 0; 0 |
| 11 | ↓[6] | 0;28 | 0;52↓[7] | 1; 2 | 0;53 | 0;30 | [8] From the furthest distance |
| 12 | ↓[9] | 0;29 | 0;52 | 1; 2 | 0;52 | 0;29 | |
| 13 | ⊥ | 0;30 | 0;53 | 1; 2 | 0;52 | 0;28 | |
| 14 | 0; 0 | 0;31 | 0;54⊥ | 1; 2 | 0;51 | 0;27 | |
| 15 | 0; 1 | 0;32 | 0;54 | 1; 2 | 0;50 | 0;26 | |
| 16 | 0; 2 | 0;32 | 0;55 | 1; 2 | 0;50 | 0;25 | |
| 17 | 0; 3 | 0;33 | 0;56 | 1; 1 | 0;49 | 0;24 | |
| 18 | 0; 4 | 0;34[10] | 0;56 | 1; 1 | 0;48 | 0;23 | |
| 19 | 0; 5 | 0;34 | 0;57[11] | 1; 1 | 0;47 | 0;22 | |
| 20 | 0; 6 | 0;35 | 0;57 | 1; 1 | 0;46 | 0;21 | |
| 21 | 0; 7 | 0;36 | 0;58 | 1; 1 | 0;45 | 0;20 | |
| 22 | 0; 8 | 0;37 | 0;58 | 1; 1[12] | 0;45 | 0;19 | |
| 23 | 0; 9 | 0;38 | 0;58 | 1; 0 | 0;44 | 0;18 | |
| 24 | 0;10 | 0;39 | 0;59 | 1; 0 | 0;43 | 0;17 | |
| 25 | 0;11 | 0;40 | 0;59 | 1; 0 | 0;42 | 0;15 | |
| 26 | 0;12 | 0;41 | 0;59 | 1; 0 | 0;42 | 0;14 | |
| 27 | 0;13⊥ | 0;41 | 0;59 | 0;59 | 0;41 | 0;13 | |
| 28 | 0;14 | 0;42 | 1; 0 | 0;59 | 0;41 | 0;12 | |
| 29 | 0;15 | 0;42 | 1; 0 | 0;59 | 0;40 | 0;11 | |

[1] **C** 1–$3^s$ 0–2°: slide[−1 col.] (with the values for $3^s$ corrected) [2] **+L** 38′ [3] **Y** $3^s$ 3–6°: slide[+1] [4] **C** 35′ [5] **F** 33′ [6] **C** $1^s$ 11–13°: 0;0 [7] **F** $3^s$ 11–14°: slide[−1] [8] **F** 0;0 [9] **+L** $1^s$ 12–27°: slide[+2] [10] **F** 33′ [11] **C** 56′ [12] **Y** 0′.

**Table 33: Interpolation minutes (nearest distance) for Venus**

*Sources*: **F** fol. 74v, **H** fol. 65r, **C** fol. 73v, **$C_1$** fol. 39v, **$C_2$** fol. 33v, **Y** fol. 293v, **L** fol. 65v, **B** p. 126.

| Minutes of Proportions ⟨for Venus⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **0** | **1** | **2** | **3** | **4** | **5** | |
| 0 | 0 | 16 | 32 | 46 | 58 | 57 | 30 |
| 1 | 0 | 17 | 32 | 46 | 58 | 56 | 29 |
| 2 | 1 | 17 | 33 | 47 | 58 | 56 | 28 |
| 3 | 1 | 18 | 33 | 47 | 58 | 55 | 27 |
| 4 | 2 | 19 | 34 | 48 | 59 | 54 | 26 |
| 5 | 2 | 19 | 34 | 48 | 59 | 54 | 25 |
| 6 | 3 | 20 | 35 | 49 | 59 | 53 | 24 |
| 7 | 4[1] | 20 | 35 | 49 | 59 | 52 | 23 |
| 8 | 4 | 21 | 36 | 50 | 59 | 51 | 22 |
| 9 | 5 | 21[2] | 36 | 50 | 59[3] | 50 | 21 |
| 10 | 6 | 22 | 37 | 51 | 60 | 49 | 20 |
| 11 | 6 | 22 | 37 | 51 | 60 | 48 | 19 |
| 12 | 7 | 23 | 38 | 52 | 60 | 47 | 18 |
| 13 | 7 | 23 | 38 | 52 | 60 | 45 | 17 |
| 14 | 8 | 24 | 39 | 53 | 60 | 43 | 16 |
| 15 | 8 | 24 | 39 | 53 | 60 | 41 | 15 |
| 16 | 9 | 25 | 40 | 53 | 60 | 39 | 14 |
| 17 | 9 | 25 | 40 | 54[4] | 60 | 37 | 13 |
| 18 | 10 | 26 | 41 | 54 | 60 | 35 | 12 |
| 19 | 10 | 26 | 41 | 54 | 60 | 32 | 11 |
| 20 | 11 | 27 | 42 | 55 | 60 | 30 | 10 |
| 21 | 11 | 27 | 42 | 55 | 59 | 27 | 9 |
| 22 | 12 | 28 | 43 | 55 | 59 | 24 | 8 |
| 23 | 12 | 28 | 43 | 56 | 59 | 22 | 7 |
| 24 | 13 | 29 | 44 | 56 | 59 | 19 | 6 |
| 25 | 13 | 29 | 44 | 56 | 59 | 16 | 5 |
| 26 | 14 | 30 | 45 | 57 | 58 | 13 | 4 |
| 27 | 14 | 30 | 45 | 57 | 58 | 10 | 3 |
| 28 | 15 | 31 | 45[5] | 57 | 57 | 6 | 2 |
| 29 | 15 | 31 | 46 | 58 | 57 | 3 | 1 |
| parts of the epicycle | **11** | **10** | **9** | **8** | **7** | **6** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **Y** 3′ [2] **C** 22′ [3] **C** 19′ [4] **L** 14′ [5] **Y** 46′.

**Table 33: Variation of the furthest distance for Venus**

*Sources*: **F** fol. 75r, **H** fol. 65v, **C** fol. 74r, **$C_1$** fol. 40r, **$C_2$** fol. 34r, **Y** fol. 294r, **L** fol. 66r, **B** p. 127.

| Variation of the Furthest Distance for Venus | | | | | | | |
|---|---|---|---|---|---|---|---|
| corrected centrum | **7** | **8** | **9** | **10** | **11** | **0** | **1** |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the nearest distance | 0;20 | 0;46 | 0;58 | 0;57[1] | 0;42 | 0;13 |
| 1 | | 0;21 | 0;46 | 0;58 | 0;56 | 0;41 | 0;12 |
| 2 | | 0;22 | 0;47 | 0;59 | 0;56 | 0;40 | 0;11 |
| 3 | | 0;23 | 0;48 | 0;59 | 0;56 | 0;40 | 0;10 |
| 4 | | 0;24 | 0;48 | 0;59 | 0;56 | 0;39 | 0; 9 |
| 5 | | 0;25 | 0;49 | 0;59 | 0;55 | 0;38 | 0; 8 |
| 6 | | 0;26 | 0;49 | 1; 0[2]↓ | 0;55 | 0;38 | 0; 7 |
| 7 | | 0;27 | 0;50 | 1; 0 | 0;55 | 0;37[3] | 0; 6 |
| 8 | | 0;28 | 0;50 | 1; 0 | 0;54 | 0;36[4] | 0; 5 |
| 9 | | 0;29 | 0;51 | 1; 0 | 0;54 | 0;35 | 0; 4 |
| 10 | | 0;30 | 0;51 | 1; 0 | 0;54 | 0;34 | 0; 3 |
| 11 | 0; 0 | 0;31 | 0;52 | 1; 0 | 0;53 | 0;33 | 0; 2 |
| 12 | 0; 1 | 0;32 | 0;53 | 1; 0 | 0;53 | 0;32 | 0; 1 |
| 13 | 0; 2 | 0;33 | 0;53 | 1; 0 | 0;52 | 0;31 | 0; 0 |
| 14 | 0; 3 | 0;34 | 0;54 | 1; 0 | 0;51 | 0;30 | From the nearest distance |
| 15 | 0; 4 | 0;35 | 0;54 | 1; 0 | 0;51 | 0;29 | |
| 16 | 0; 5 | 0;36 | 0;54[5]↓ | 1; 0 | 0;50 | 0;28 | |
| 17 | 0; 6 | 0;37 | 0;55 | 1; 0 | 0;50 | 0;27 | |
| 18 | 0; 7 | 0;38 | 0;55 | 1; 0⊥ | 0;49 | 0;26[6] | |
| 19 | 0; 8 | 0;38 | 0;55 | 0;59 | 0;49[7]↓ | 0;25 | |
| 20 | 0; 9 | 0;39 | 0;56 | 0;59 | 0;48 | 0;24 | |
| 21 | 0;10 | 0;40 | 0;56 | 0;59 | 0;48⊥ | 0;23 | |
| 22 | 0;11 | 0;40 | 0;56 | 0;59[8] | 0;47 | 0;22 | |
| 23[9]↓ | 0;12 | 0;41 | 0;56⊥ | 0;58[10] | 0;46 | 0;21 | |
| 24 | 0;13 | 0;42 | 0;57[11] | 0;58 | 0;46 | 0;20 | |
| 25 | 0;14 | 0;42 | 0;57 | 0;58 | 0;45 | 0;19[12] | |
| 26 | 0;15 | 0;43 | 0;57 | 0;58 | 0;44[13] | 0;18 | |
| 27 | 0;16 | 0;44 | 0;57 | 0;57 | 0;44 | 0;16 | |
| 28⊥ | 0;18 | 0;44 | 0;58 | 0;57 | 0;43 | 0;15 | |
| 29 | 0;19 | 0;45 | 0;58 | 0;57 | 0;42[14] | 0;14 | |

[1] **L** minutes ill. [2] **Y** $10^{s}$ 6–18°: 0° [3] **C** 36′ [4] **C** 35′ [5] **Y** $9^{s}$ 16–23°: slide[+2] [6] **L** 27′ [7] **$CC_1C_2$** $11^{s}$ 19–21°: slide[+1] [8] **L** 58′ [9] **Y** arguments 23–28°: slide[+1] (with corrections in the same hand that are not in each case readable) [10] **F** 59′ [11] **C** 17′ [12] **B** 59′ [13] **H** 47′ **C** 45′ [14] **F** 43′.

**Table 33: Interpolation minutes (furthest distance) for Venus**

*Sources*: **F** fol. 75r, **H** fol. 66r, **C** fol. 74r, **$C_1$** fol. 40r, **$C_2$** fol. 34r, **Y** fol. 294r, **L** fol. 66r, **B** p. 127.

| Minutes of Proportions ⟨for Venus⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0 | 0 | 57 | 58 | 46 | 32 | 16 | 30 |
| 1 | 3 | 57 | 58 | 46 | 31 | 15 | 29 |
| 2 | 6 | 57 | 57 | 45 | 31 | 15 | 28 |
| 3 | 10 | 58 | 57 | 45[1]↓ | 30 | 14 | 27 |
| 4 | 13 | 58 | 57 | 45 | 30 | 14 | 26 |
| 5 | 16 | 59 | 56 | 44⊥ | 29 | 13 | 25 |
| 6 | 19 | 59 | 56 | 44 | 29 | 13 | 24 |
| 7 | 22 | 59 | 56 | 43 | 28[2] | 12 | 23 |
| 8 | 24 | 59[3]↓ | 55 | 43 | 28 | 12 | 22 |
| 9 | 27 | 59⊥ | 55 | 42 | 27 | 11 | 21 |
| 10 | 30 | 60 | 55 | 42 | 27 | 11 | 20 |
| 11 | 32 | 60 | 54 | 41 | 26 | 10 | 19 |
| 12 | 35 | 60 | 54 | 41 | 26 | 10 | 18 |
| 13 | 37 | 60 | 54 | 40 | 25 | 9 | 17 |
| 14 | 39 | 60 | 53 | 40 | 25 | 9 | 16 |
| 15 | 41 | 60 | 53 | 39 | 24 | 8 | 15 |
| 16 | 43 | 60 | 53 | 39 | 24 | 8 | 14 |
| 17 | 45 | 60 | 52 | 38 | 23 | 7 | 13 |
| 18 | 47 | 60 | 52 | 38 | 23 | 7 | 12 |
| 19 | 48 | 60 | 51 | 37 | 22 | 6 | 11 |
| 20 | 49 | 60 | 51 | 37 | 22 | 6 | 10 |
| 21 | 50 | 59 | 50 | 36 | 21 | 5 | 9 |
| 22 | 51 | 59 | 50 | 36 | 21 | 4 | 8 |
| 23 | 52 | 59 | 49 | 35 | 20 | 4 | 7 |
| 24 | 53 | 59 | 49 | 35 | 20 | 3 | 6 |
| 25 | 54 | 59 | 48 | 34 | 19 | 2 | 5 |
| 26 | 54 | 59 | 48 | 34 | 19 | 2 | 4 |
| 27 | 55 | 58 | 47[4]↓ | 33 | 18 | 1 | 3 |
| 28 | 56 | 58 | 47⊥ | 33 | 17 | 1 | 2 |
| 29 | 56[5] | 58 | 46 | 32 | 17 | 0 | 1 |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **C** 9ˢ 3–5° slide[+1 col.] [2] **C** 29′ [3] **Y** 7ˢ 8–9°: 60′ [4] **L** 8ˢ 27–28°: 46′ [5] **Y** 57′.

**Table 34: Mean motion of Mercury (first part)**
*Sources*: **F** fols 75v–76r, **H** fols 66v–67r, **C** fol. 74v, **C**$_1$ fol. 40v, **C**$_2$ fol. 34v, **Y** fols 294v–295r, **L** fols 66v–67r, **B** pp. 128–129.

Mean Motion of Mercury in Years and Months

| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
|---|---|---|---|---|---|
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | $1^{s}$ 26;55[1] | 1 | $11^{s}$ 29;46 | Farwardīn | $0^{s}$ 0; 0 |
| 21 | $1^{s}$ 22;10 | 2 | $11^{s}$ 29;32[2] | | |
| 41 | $1^{s}$ 17;26 | 3 | $11^{s}$ 29;17 | Urdībihisht | $0^{s}$ 29;34[3] |
| 61 | $1^{s}$ 12;41[4] | 4 | $11^{s}$ 29; 3 | | |
| 81[5]↕ | $1^{s}$ 7;57[6] | 5 | $11^{s}$ 28;49 | Khurdād | $1^{s}$ 29; 8 |
| 101 | $1^{s}$ 3;12 | 6 | $11^{s}$ 28;35 | | |
| 121 | $0^{s}$ 28;27[7] | 7 | $11^{s}$ 28;20 | Tīr | $2^{s}$ 28;43 |
| 141 | $0^{s}$ 23;43 | 8 | $11^{s}$ 28; 6 | | |
| 161 | $0^{s}$ 18;58[8] | 9 | $11^{s}$ 27;52 | Murdād | $3^{s}$ 28;17 |
| 181 | $0^{s}$ 14;14 | 10 | $11^{s}$ 27;38 | | |
| 201 | $0^{s}$ 9;29 | 11 | $11^{s}$ 27;24 | Shahrīwar | $4^{s}$ 27;51 |
| 221 | $0^{s}$ 4;45[9] | 12 | $11^{s}$ 27; 9 | | |
| 241 | $0^{s}$ 0; 0[10] | 13 | $11^{s}$ 26;55 | Mihr | $5^{s}$ 27;25 |
| 261 | $11^{s}$ 25;16 | 14 | $11^{s}$ 26;41 | | |
| 281 | $11^{s}$ 20;31 | 15 | $11^{s}$ 26;27 | Ābān | $6^{s}$ 26;59 |
| 301 | $11^{s}$ 15;47 | 16 | $11^{s}$ 26;12 | | |
| 321 | $11^{s}$ 11; 2 | 17 | $11^{s}$ 25;58 | Ādhar | $7^{s}$ 26;33[11] |
| 341 | $11^{s}$ 6;18 | 18 | $11^{s}$ 25;44 | | *$8^{s}$ 1;29* |
| 361 | $11^{s}$ 1;33[12] | 19 | $11^{s}$ 25;30 | Day | $8^{s}$ 26; 8[13] |
| 381 | $10^{s}$ 26;49 | 20 | $11^{s}$ 25;15 | | *$9^{s}$ 1; 3*[14] |
| 401 | $10^{s}$ 22; 4 | single years | | Bahman | $9^{s}$ 25;42 |
| 421 | $10^{s}$ 17;20 | | | | *$10^{s}$ 0;38*[15] |
| 441 | $10^{s}$ 12;35 | 40 | $11^{s}$ 20;31 | Isfandārmudh | $10^{s}$ 25;16 |
| 461 | $10^{s}$ 7;51[16] | 60 | $11^{s}$ 15;46[17] | | *$11^{s}$ 0;12* |
| 481 | $10^{s}$ 3; 6 | 80 | $11^{s}$ 11; 2[18] | | |
| 501 | $9^{s}$ 28;21 | 100 | $11^{s}$ 6;17 | | |
| 521 | $9^{s}$ 23;37 | 200 | $10^{s}$ 12;35[19] | | |
| 541⊥ | $9^{s}$ 18;52[20] | 300 | $9^{s}$ 18;52 | | |
| 561 | $9^{s}$ 14; 8[21] | 400 | $8^{s}$ 25; 9[22] | | |
| 581 | $9^{s}$ 9;23[23] | 500 | $8^{s}$ 1;27[24] | | |
| | | | [25] | | |

To the left of the subtable for collected years, the main hand of **B** writes minutes that are two or three larger than the ones in the main table (cf. Section IV.5.3). [1] **Y** 56′ **L** 15′ [2] **L** 39° [3] **C**$_1$ 37′ [4] **L** 17° [5] **B** arguments 81–541: slide[+1] (521 and 541 unclearly corrected) [6] **C** 17′ **YLB** 56′ [7] **C**$_1$ 24′ [8] **HC** 18′ **B** 19° [9] **H** 46′ [10] **Y** 31′ [11] **L** $2^{s}$ [12] **C**$_1$ 38′ [13] **Y** 9′ [14] **B** 4′ [15] **YL** 37′ [16] **C** 11′ [17] **Y** 16′ [18] **L** 7′ [19] **L** 15′ [20] **C** 12′ [21] **YLB** 7′ [22] **YL** 10′ [23] **FB** 24′ **C**$_1$ 13′ [24] **LB** 26′ [25] **YLB** add a value $7^{s}$ 7;44 for 600 single years.

**Table 34: Mean motion of Mercury (second part)**
*Sources*: **F** fols 75v–76r, **H** fols 66v–67r, **C** fol. 74v, $\mathbf{C_1}$ fol. 40v, $\mathbf{C_2}$ fol. 34v, **Y** fols 294v–295r, **L** fols 66v–67r, **B** pp. 128–129.

Mean Motion of Mercury in Days, Hours, and Between Longitudes

| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
|---|---|---|---|---|---|---|---|---|
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | 0s 0; 0 | 1 | 0; 2 | 31 | 1;16 | 71 | additive | 0, 3 |
| 2 | 0s 0;59 | 2 | 0; 5[1]↓ | 32 | 1;19 | 72 | | 0, 3 |
| 3 | 0s 1;58 | 3 | 0; 7 | 33 | 1;21 | 73 | | 0, 3 |
| 4 | 0s 2;57 | 4 | 0;10 | 34 | 1;24 | 74 | | 0, 3 |
| 5 | 0s 3;57 | 5 | 0;12 | 35 | 1;26[2] | 75 | | 0, 2 |
| 6 | 0s 4;56 | 6 | 0;15 | 36 | 1;29 | 76 | | 0, 2 |
| 7 | 0s 5;55 | 7 | 0;17 | 37 | 1;31 | 77 | | 0, 2 |
| 8 | 0s 6;54 | 8 | 0;20 | 38[3] | 1;34 | 78 | | 0, 2 |
| 9 | 0s 7;53[4] | 9 | 0;22 | 39 | 1;36 | 79 | | 0, 2 |
| 10 | 0s 8;52 | 10 | 0;25 | 40 | 1;39 | 80 | | 0, 2 |
| 11 | 0s 9;51 | 11 | 0;27 | 41 | 1;41 | 81 | | 0, 1 |
| 12 | 0s 10;51 | 12 | 0;30 | 42 | 1;43 | 82 | | 0, 1 |
| 13 | 0s 11;50 | 13 | 0;32 | 43 | 1;46 | 83 | | 0, 1 |
| 14 | 0s 12;49[5] | 14 | 0;35[6] | 44 | 1;48 | 84 | | 0, 1 |
| 15 | 0s 13;48 | 15 | 0;37 | 45 | 1;51 | 85 | | 0, 1 |
| 16[7]↓ | 0s 14;47 | 16 | 0;39 | 46 | 1;53 | 86 | | 0, 1 |
| 17 | 0s 15;46 | 17 | 0;42 | 47 | 1;56 | 87 | | 0, 1 |
| 18 | 0s 16;45 | 18 | 0;44 | 48 | 1;58 | 88 | | 0, 0 |
| 19 | 0s 17;44[8] | 19 | 0;47 | 49 | 2; 1 | 89 | | 0, 0 |
| 20 | 0s 18;44 | 20 | 0;49 | 50 | 2; 3 | 90 | | 0, 0 |
| 21 | 0s 19;43 | 21 | 0;52 | 51 | 2; 6 | 91 | subtractive | 0, 0 |
| 22 | 0s 20;42 | 22 | 0;54 | 52 | 2; 8 | 92 | | 0, 0 |
| 23 | 0s 21;41 | 23 | 0;57 | 53 | 2;11 | 93 | | 0, 1 |
| 24 | 0s 22;40 | 24 | 0;59 | 54 | 2;13 | 94 | | 0, 1 |
| 25 | 0s 23;39 | 25 | 1; 2 | 55 | 2;16 | 95 | | 0, 1 |
| 26 | 0s 24;38 | 26 | 1; 4 | 56 | 2;18 | 96 | | 0, 1 |
| 27⊥ | 0s 25;38 | 27 | 1; 7 | 57 | 2;20[9] | 97 | | 0, 1 |
| 28 | 0s 26;37 | 28 | 1; 9 | 58 | 2;23 | 98 | | 0, 1 |
| 29 | 0s 27;36 | 29 | 1;11[10]⊥ | 59 | 2;25 | 99 | | 0, 1 |
| 30 | 0s 28;35 | 30 | 1;14[11] | 60 | 2;28 | 100 | | 0, 2 |

[1] **L** 2–29 hours: slide[+1 row] (the value for 2 hours is inserted in between the lines for 1 and 2 hours, the space next to 30 hours is empty) [2] **B** 36′ [3] **L** 35 [4] **L** 58′ [5] **C** 59′ [6] +**B** 34′ [7] **C** arguments 16–27 days: slide[+1] [8] **YL** 45′ [9] **B** 21′ [10] **L**+ 13′ (instead of 14′) [11] **L** om. (end of slide) **Y** 13′ **B** 14′ written over scratched out original digit (apparently 13′).

**Table 35: Mean anomaly of Mercury (first part)**
*Sources*: **F** fols 76v–77r, **H** fols 67v–68r, **C** fol. 75r, **$C_1$** fol. 41r, **$C_2$** fol. 35r, **Y** fols 295v–296r, **L** fols 67v–68r, **B** pp. 130–131.

| Mean Anomaly of Mercury in Years and Months | | | | | |
|---|---|---|---|---|---|
| years | collected ⟨years⟩ | years | extended ⟨years⟩ | months | months |
| | s ° ′ | | s ° ′ | | s ° ′ |
| 1 | 5$^s$ 29; 3 | 1 | 1$^s$ 23;57 | Farwardīn | 0$^s$ 0; 0 |
| 21 | 5$^s$ 27;59[1] | 2 | 3$^s$ 17;54 | | |
| 41 | 5$^s$ 26;55[2] | 3 | 5$^s$ 11;50[3] | Urdībihisht | 3$^s$ 3;12 |
| 61 | 5$^s$ 25;50[4] | 4 | 7$^s$ 5;47 | | |
| 81 | 5$^s$ 24;46 | 5 | 8$^s$ 29;44 | Khurdād | 6$^s$ 6;24 |
| 101 | 5$^s$ 23;42 | 6 | 10$^s$ 23;41[5] | | |
| 121 | 5$^s$ 22;38 | 7 | 0$^s$ 17;37[6] | Tīr | 9$^s$ 9;36 |
| 141 | 5$^s$ 21;33 | 8 | 2$^s$ 11;34 | | |
| 161 | 5$^s$ 20;29 | 9 | 4$^s$ 5;31 | Murdād | 0$^s$ 12;48 |
| 181 | 5$^s$ 19;25 | 10 | 5$^s$ 29;28 | | |
| 201 | 5$^s$ 18;20[7] | 11 | 7$^s$ 23;25 | Shahrīwar | 3$^s$ 16; 0[8] |
| 221 | 5$^s$ 17;16 | 12 | 9$^s$ 17;22[9] | | |
| 241 | 5$^s$ 16;12 | 13 | 11$^s$ 11;18 | Mihr | 6$^s$ 19;12 |
| 261 | 5$^s$ 15; 8 | 14 | 1$^s$ 5;15 | | |
| 281 | 5$^s$ 14; 3 | 15 | 2$^s$ 29;12 | Ābān | 9$^s$ 22;24 |
| 301 | 5$^s$ 12;59[10] | 16 | 4$^s$ 23; 9 | | |
| 321 | 5$^s$ 11;55[11] | 17 | 6$^s$ 17; 5[12] | Ādhar | 0$^s$ 25;37 |
| 341 | 5$^s$ 10;50 | 18 | 8$^s$ 11; 2 | | *1$^s$ 11; 9* |
| 361 | 5$^s$ 9;46 | 19 | 10$^s$ 4;59 | Day | 3$^s$ 28;49 |
| 381 | 5$^s$ 8;42[13] | 20 | 11$^s$ 28;56 | | *4$^s$ 14;21* |
| 401 | 5$^s$ 7;38[14] | single years | | Bahman | 7$^s$ 2; 1[15] |
| 421 | 5$^s$ 6;33 | | | | *7$^s$ 17;33* |
| 441 | 5$^s$ 5;29 | 40 | 11$^s$ 27;51[16] | Isfandārmudh | 10$^s$ 5;13 |
| 461 | 5$^s$ 4;25 | 60 | 11$^s$ 26;47 | | *10$^s$ 20;45* |
| 481 | 5$^s$ 3;20 | 80 | 11$^s$ 25;43[17] | | |
| 501 | 5$^s$ 2;16 | 100 | 11$^s$ 24;39 | | |
| 521 | 5$^s$ 1;12 | 200 | 11$^s$ 19;17 | | |
| 541 | 5$^s$ 0; 8 | 300 | 11$^s$ 13;56[18] | | |
| 561 | 4$^s$ 29; 3[19] | 400 | 11$^s$ 8;34 | | |
| 581 | 4$^s$ 27;59 | 500 | 11$^s$ 3;13 | | |
| | | | [20] | | |

To the left of the subtable for collected years, the main hand of **B** writes minutes that are nine larger than the ones in **FHC$C_1C_2$** (cf. Section IV.5.3). [1] **C** 19′ **B** 28° [2] **CB** 27° **C** 15′ [3] **F** 51° [4] **YLB** 51′ [5] **Y** 4$^s$ [6] **YL** 38′ [7] **YLB** 21′ [8] **C** 5′ **Y** 15° (corrected to 16 in a different hand and ink) **L** 0$^s$ [9] **YL** 21′ [10] **C** 19′ **B** 13° [11] **C** 15′ **B** 12° [12] **$C_1C_2$** 6′ [13] **LB** 44′ [14] **F** 1° [15] **F** 6$^s$ **HL** 4$^s$ (**H** corrected to 3$^s$ by a different hand in black) [16] **F** 26° **C** 11′ [17] **$C_1C_2$** 24′ [18] **L** 18° [19] **F** 4′ [20] **YLB** add a value 10$^s$ 27;51 for 600 single years.

**Table 35: Mean anomaly of Mercury (second part)**
*Sources*: **F** fols 76v–77r, **H** fols 67v–68r, **C** fol. 75r, **$C_1$** fol. 41r, **$C_2$** fol. 35r, **Y** fols 295v–296r, **L** fols 67v–68r, **B** pp. 130–131.

| Mean Anomaly of Mercury in Days, Hours, and Between Longitudes | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| days | days | hours | hours | hours | and their fractions | longitudes | | in between longitudes |
| | s ° ′ | | ° ′ | | ° ′ | | | ′ ″ |
| 1 | $0^s$ 0; 0 | 1 | 0; 8 | 31 | 4; 1 | 71 | additive | 0,10 |
| 2 | $0^s$ 3; 6 | 2 | 0;16 | 32 | 4; 9 | 72 | | 0,10[1] |
| 3 | $0^s$ 6;13[2] | 3 | 0;23 | 33 | 4;17* | 73 | | 0, 9 |
| 4 | $0^s$ 9;19 | 4 | 0;31 | 34 | 4;24 | 74 | | 0, 9[3] |
| 5 | $0^s$ 12;26 | 5 | 0;39 | 35 | 4;32 | 75 | | 0, 8 |
| 6 | $0^s$ 15;32 | 6 | 0;47[4] | 36 | 4;40 | 76 | | 0, 7 |
| 7 | $0^s$ 18;38 | 7 | 0;54 | 37 | 4;48* | 77 | | 0, 7 |
| 8 | $0^s$ 21;45[5] | 8 | 1; 2 | 38 | 4;55 | 78 | | 0, 6 |
| 9 | $0^s$ 24;51 | 9 | 1;10 | 39 | 5; 3 | 79 | | 0, 6 |
| 10 | $0^s$ 27;58[6] | 10 | 1;18[7] | 40 | 5;11 | 80 | | 0, 5 |
| 11 | $1^s$ 1; 4 | 11 | 1;25[8] | 41 | 5;18 | 81 | | 0, 5 |
| 12 | $1^s$ 4;10 | 12 | 1;33 | 42 | 5;26 | 82 | | 0, 4 |
| 13 | $1^s$ 7;17[9] | 13 | 1;41 | 43 | 5;34 | 83 | | 0, 4 |
| 14 | $1^s$ 10;23[10] | 14 | 1;49 | 44 | 5;42 | 84 | | 0, 3 |
| 15 | $1^s$ 13;30[11] | 15 | 1;57 | 45 | 5;50 | 85 | | 0, 3 |
| 16 | $1^s$ 16;36 | 16 | 2; 4 | 46 | 5;57 | 86 | | 0, 2 |
| 17 | $1^s$ 19;42 | 17 | 2;12 | 47 | 6; 5 | 87 | | 0, 2 |
| 18 | $1^s$ 22;49 | 18 | 2;20 | 48 | 6;13 | 88 | | 0, 1 |
| 19 | $1^s$ 25;55 | 19 | 2;28 | 49 | 6;21 | 89 | | 0, 1 |
| 20 | $1^s$ 29; 2[12] | 20 | 2;35 | 50 | 6;28 | 90 | | 0, 0 |
| 21 | $2^s$ 2; 8 | 21 | 2;43 | 51 | 6;36 | 91 | subtractive | 0, 1 |
| 22 | $2^s$ 5;14 | 22 | 2;51 | 52 | 6;44 | 92 | | 0, 1 |
| 23 | $2^s$ 8;21 | 23 | 2;59 | 53 | 6;51* | 93 | | 0, 2 |
| 24 | $2^s$ 11;27 | 24 | 3; 6 | 54 | 6;59 | 94 | | 0, 2 |
| 25 | $2^s$ 14;34 | 25 | 3;14 | 55 | 7; 7 | 95 | | 0, 3 |
| 26 | $2^s$ 17;40[13] | 26 | 3;22 | 56 | 7;14* | 96 | | 0, 3 |
| 27 | $2^s$ 20;46 | 27 | 3;30[14] | 57 | 7;22[15] | 97 | | 0, 4 |
| 28 | $2^s$ 23;53[16] | 28 | 3;37[17] | 58 | 7;30[18] | 98 | | 0, 4 |
| 29 | $2^s$ 26;59[19] | 29 | 3;45 | 59 | 7;37* | 99 | | 0, 5 |
| 30 | $3^s$ 0; 6[20] | 30 | 3;53 | 60 | 7;45* | 100 | | 0, 5 |

* In **YLB** the values for 33 and 37 hours are 1 minute smaller and the values for 53, 56, and 59–60 hours are 1 minute larger. [1] **YLB** 9′ [2] **F** 12′ [3] **YLB** 8′ [4] **F** 46′ [5] **$C_1$** 44′ [6] **Y** 18′ [7] **Y** 58′ [8] **$C_1C_2$** 26′ [9] **F** 16′ [10] **$C_2$** 13° [11] **L** 32′ [12] **C** 1′ [13] **L** 16° [14] **B** 29′ [15] **YL** 23′ [16] **C** 13′ [17] **L** 27′ [18] **L** 6° [19] **C** 19′ [20] **F** 5′ **C** om. signs.

**Table 36: First equation for Mercury (first half)**

*Sources*: **F** fols 77v–78r, **H** fols 68v–69r, **C** fols 75v–76r, $\mathbf{C_1}$ fols 41v–42r, $\mathbf{C_2}$ fols 35v–36r, **Y** fols 296v–297r, **L** fols 68v–69r, **B** pp. 132–133.

First Equation for Mercury
to be added to the centrum and subtracted from the anomaly

| degrees of the centrum | 0 (° ′) | differences ′ | 1 (° ′) | differences ′ | mean distance 2° 2 (° ′) | differences ′ | 3 (° ′) | differences ′ | 4 (° ′) | differences ′ | nearest distance 0° 5 (° ′) | differences ′ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 2;43 | 2[1] | 1;35 | 2 | 0;59 | 1 | 1;19 | 2 | 2;28 | 3 | 4; 0 | 3 |
| 1 | 2;40 | 3 | 1;33 | 2 | 0;59 | 0 | 1;21 | 2 | 2;30 | 2 | 4; 3 | 3 |
| 2 | 2;37 | 3 | 1;31 | 2 | 0;59 | 0 | 1;23 | 2 | 2;33[2] | 3 | 4; 6 | 3 |
| 3 | 2;35 | 2 | 1;29 | 2 | 0;58 | 1 | 1;25 | 2 | 2;36 | 3 | 4; 9 | 3 |
| 4 | 2;32 | 3 | 1;27 | 2 | 0;58 | 0 | 1;26 | 1 | 2;39 | 3 | 4;13 | 4[3] |
| 5 | 2;29 | 3[4]↕ | 1;25 | 2 | 0;58 | 0 | 1;28 | 2 | 2;42 | 3 | 4;16 | 3 |
| 6 | 2;27 | 2 | 1;24 | 1 | 0;58 | 0 | 1;30 | 2 | 2;45 | 3 | 4;19 | 3 |
| 7 | 2;24 | 3 | 1;22 | 2 | 0;58 | 0 | 1;32 | 2 | 2;48 | 3 | 4;22 | 3 |
| 8 | 2;22 | 2 | 1;20 | 2 | 0;59 | 1 | 1;34 | 2 | 2;51 | 3 | 4;25 | 3 |
| 9 | 2;20 | 2 | 1;19 | 1 | 0;59 | 0 | 1;36 | 2 | 2;54 | 3 | 4;28[5] | 3 |
| 10 | 2;17 | 3 | 1;17 | 2 | 0;59 | 0[6] | 1;38 | 2 | 2;57 | 3 | 4;32 | 4[7] |
| 11 | 2;15 | 2 | 1;16 | 1 | 1; 0 | 1[8] | 1;40 | 2 | 3; 0 | 3 | 4;35 | 3[9] |
| 12 | 2;12 | 3 | 1;15 | 1 | 1; 0 | 0 | 1;42 | 2 | 3; 3 | 3 | 4;39 | 4[10] |
| 13 | 2;10 | 2⊥ | 1;13 | 2 | 1; 1 | 1 | 1;44 | 2 | 3; 6 | 3 | 4;42 | 3 |
| 14 | 2; 8 | 2 | 1;12 | 1 | 1; 1 | 0 | 1;46 | 2 | 3; 9 | 3 | 4;45 | 3 |
| 15 | 2; 6 | 2 | 1;11 | 1 | 1; 2 | 1 | 1;49 | 3 | 3;12 | 3 | 4;48 | 3 |
| 16 | 2; 3 | 3 | 1;10 | 1 | 1; 2 | 0 | 1;51 | 2 | 3;15 | 3 | 4;51 | 3 |
| 17 | 2; 1 | 2 | 1; 9 | 1 | 1; 3 | 1 | 1;53 | 2 | 3;18 | 3 | 4;54 | 3 |
| 18 | 1;59 | 2 | 1; 8 | 1 | 1; 4 | 1 | 1;56 | 3 | 3;21 | 3 | 4;57 | 3 |
| 19 | 1;56 | 3 | 1; 7 | 1 | 1; 5 | 1 | 1;58 | 2 | 3;25[11] | 4[12] | 5; 0 | 3 |
| 20 | 1;54 | 2 | 1; 6 | 1 | 1; 6 | 1 | 2; 0 | 2 | 3;28 | 3[13] | 5; 3 | 3 |
| 21 | 1;52 | 2 | 1; 5 | 1 | 1; 7 | 1 | 2; 3 | 3 | 3;32 | 4[14] | 5; 6 | 3 |
| 22 | 1;50 | 2 | 1; 4 | 1 | 1; 8 | 1 | 2; 5[15] | 2[16] | 3;35 | 3 | 5; 9 | 3 |
| 23 | 1;48 | 2 | 1; 3 | 1 | 1; 9 | 1 | 2; 8 | 3[17] | 3;38 | 3 | 5;12 | 3 |
| 24 | 1;46 | 2 | 1; 3 | 0 | 1;10 | 1 | 2;11 | 3 | 3;41 | 3 | 5;15 | 3 |
| 25 | 1;44 | 2 | 1; 2 | 1 | 1;11 | 1 | 2;13 | 2 | 3;44 | 3 | 5;18 | 3 |
| 26 | 1;42 | 2 | 1; 1 | 1 | 1;12 | 1 | 2;16 | 3 | 3;47 | 3 | 5;21 | 3 |
| 27 | 1;41 | 1 | 1; 1 | 0 | 1;14 | 2 | 2;19 | 3 | 3;51 | 4[18] | 5;24 | 3 |
| 28 | 1;39 | 2 | 1; 0 | 1 | 1;15 | 1 | 2;22 | 3 | 3;54 | 3 | 5;27 | 3 |
| 29 | 1;37[19] | 2 | 1; 0 | 0 | 1;17 | 2[20] | 2;25 | 3 | 3;57 | 3 | 5;30 | 3 |

[1] **YL** 3′ [2] **C** 23′ [3] $\mathbf{C_1}$ 3′ [4] **C** 0ˢ 5–13°: slide[+3] [5] **C** 18′ [6] **C** 1′ [7] $\mathbf{C_1C_2}$ 3′ [8] **C** 0′ [9] **L** 4′ [10] $\mathbf{C_1C_2}$ 3′ [11] **L** 24′ [12] **CL** 3′ (in correspondence with the equation for 4ˢ 19° in **L**) [13] **L** 4′ (in correspondence with the equation for 4ˢ 19°) [14] $\mathbf{CC_1C_2}$ 3′ [15] **C** 6′ [16] **C** 3′ (in correspondence with the equation for 3ˢ 22°) [17] **C** 2′ (in correspondence with the equation for 3ˢ 22°) [18] **C** 3′ [19] $\mathbf{C_1}$ 34′ [20] **C** dam.

**Table 36: First equation for Mercury (second half)**
*Sources*: **F** fols 77v–78r, **H** fols 68v–69r, **C** fols 75v–76r, $\mathbf{C_1}$ fols 41v–42r, $\mathbf{C_2}$ fols 35v–36r, **Y** fols 296v–297r, **L** fols 68v–69r, **B** pp. 132–133.

| First Equation for Mercury<br>to be added to the centrum and subtracted from the anomaly | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| degrees of the centrum | 6<br>° ′ | differences | mean distance 28°<br>7<br>° ′ | differences | 8<br>° ′ | differences | 9<br>° ′ | differences | 10<br>° ′ | differences | furthest distance 0°<br>11<br>° ′ | differences |
| 0 | 5;32 | 2 | 6;41 | 2 | 7; 1 | 0 | 6;25 | 2 | 5;17 | 3 | 4; 0 | 3 |
| 1 | 5;35 | 3 | 6;43 | 2 | 7; 0 | 1 | 6;23 | 2 | 5;15 | 2 | 3;57 | 3 |
| 2 | 5;38 | 3 | 6;45 | 2 | 7; 0 | 0 | 6;21 | 2 | 5;13 | 2 | 3;54 | 3 |
| 3 | 5;41 | 3 | 6;46 | 1 | 6;59 | 1 | 6;19 | 2 | 5;10 | 3 | 3;51 | 3 |
| 4 | 5;44 | 3 | 6;48 | 2 | 6;59 | 0 | 6;18 | 1 | 5; 8 | 2 | 3;48 | 3 |
| 5 | 5;47 | 3 | 6;49 | 1 | 6;58 | 1 | 6;16 | 2 | 5; 5 | 3 | 3;45 | 3 |
| 6 | 5;49 | 2 | 6;50 | 1 | 6;57 | 1 | 6;14 | 2 | 5; 2 | 3 | 3;43 | 2 |
| 7 | 5;52 | 3 | 6;51 | 1 | 6;57 | 0[1] | 6;12 | 2 | 5; 0 | 2 | 3;40 | 3 |
| 8 | 5;55 | 3 | 6;52 | 1 | 6;56 | 1 | 6;10 | 2 | 4;58[2] | 2 | 3;37 | 3 |
| 9 | 5;57 | 2 | 6;53 | 1 | 6;55 | 1 | 6; 8 | 2 | 4;55 | 3 | 3;35 | 2 |
| 10 | 6; 0 | 3 | 6;54 | 1 | 6;54 | 1 | 6; 6 | 2 | 4;53 | 2 | 3;32 | 3 |
| 11 | 6; 2 | 2 | 6;55 | 1 | 6;53[3] | 1 | 6; 4 | 2 | 4;50 | 3 | 3;30 | 2 |
| 12 | 6; 4 | 2 | 6;56 | 1 | 6;52 | 1 | 6; 1 | 3 | 4;47 | 3 | 3;28 | 2 |
| 13 | 6; 7 | 3 | 6;57 | 1 | 6;51 | 1 | 5;59 | 2 | 4;45 | 2 | 3;25 | 3 |
| 14 | 6; 9 | 2 | 6;58[4]↕ | 1 | 6;50 | 1 | 5;57 | 2 | 4;43 | 2 | 3;22 | 3 |
| 15 | 6;11 | 2 | 6;58 | 0 | 6;49 | 1[5] | 5;54 | 3 | 4;40 | 3 | 3;20 | 2 |
| 16 | 6;14 | 3 | 6;59 | 1 | 6;48 | 1 | 5;52 | 2 | 4;38 | 2 | 3;17 | 3[6] |
| 17 | 6;16 | 2 | 6;59 | 0 | 6;47 | 1 | 5;50 | 2 | 4;35 | 3 | 3;15 | 2[7] |
| 18 | 6;18 | 2 | 7; 0 | 1 | 6;45 | 2 | 5;48 | 2 | 4;32 | 3 | 3;13 | 2 |
| 19 | 6;20 | 2 | 7; 0 | 0 | 6;44 | 1 | 5;45 | 3 | 4;30 | 2 | 3;10 | 3 |
| 20 | 6;22 | 2 | 7; 1 | 1 | 6;43 | 1[8] | 5;43 | 2 | 4;28 | 2 | 3; 7 | 3 |
| 21 | 6;24[9] | 2 | 7; 1 | 0 | 6;41 | 2 | 5;40 | 3 | 4;25 | 3 | 3; 5 | 2 |
| 22 | 6;26 | 2 | 7; 1 | 0 | 6;40 | 1 | 5;38 | 2 | 4;23 | 2 | 3; 2 | 3 |
| 23 | 6;28 | 2 | 7; 2 | 1 | 6;38 | 2 | 5;36 | 2 | 4;20 | 3 | 3; 0 | 2 |
| 24 | 6;30 | 2 | 7; 2[10] | 0 | 6;36 | 2 | 5;33 | 3 | 4;17 | 3 | 2;58 | 2 |
| 25 | 6;32 | 2 | 7; 2 | 0 | 6;35 | 1 | 5;31 | 2 | 4;15 | 2 | 2;55 | 3 |
| 26 | 6;34 | 2 | 7; 2 | 0 | 6;33 | 2 | 5;28 | 3 | 4;12 | 3[11] | 2;52 | 3 |
| 27 | 6;35 | 1 | 7; 2 | 0 | 6;31 | 2 | 5;25 | 3 | 4; 9 | 3 | 2;50 | 2 |
| 28 | 6;37 | 2 | 7; 1 | 1 | 6;29 | 2 | 5;23 | 2 | 4; 6 | 3 | 2;47 | 3 |
| 29 | 6;39 | 2 | 7; 1⊥ | 0 | 6;27 | 2 | 5;20[12] | 3 | 4; 3 | 3 | 2;45 | 2 |

[1] **C** 1′ [2] **L** 18′ [3] **C** 43′ [4] **F** $7^s$ 14–29°: adjusts the equation values (but not the tabular differences) around the maximum (the values for $7^s$ 14–22° are all 1 minute smaller, the values for $7^s$ 20–29° equal to 7;0°) [5] **C** 2′ [6] **C** dam. [7] **C** dam. [8] **C** 2′ [9] $\mathbf{C_2}$ 26′ [10] +**Y** 3′ [11] **C** 2′ [12] **C** degrees dam.

**Table 36: Second equation for Mercury (first half)**

*Sources*: **F** fols 78v–79r, **H** fols 69v–70r, **C** fols 76v–77r, $\mathbf{C_1}$ fols 42v–43r, $\mathbf{C_2}$ fols 36v and 45r, **Y** fols 297v–298r, **L** fols 69v–70r, **B** pp. 134–135.

Second Equation for Mercury
to be equated and added to the centrum with the apogee

| degrees of the epicycle | furthest distance 0° **0** | differences | **1** | differences[1] | **2** | differences | mean distance 11° **3** | differences | **4** | differences | **5** | differences |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | | ° ′ | |
| 0 | 26; 0 | 17 | 34; 4 | 15 | 41;18 | 13 | 46;33 | 7 | 47;47 | 3 | 41;31 | 22 |
| 1 | 26;17 | 17 | 34;20 | 16 | 41;31 | 13 | 46;40 | 7 | 47;43 | 4 | 41; 8 | 23 |
| 2 | 26;33 | 16 | 34;35 | 15 | 41;44 | 13 | 46;47 | 7 | 47;38[2] | 5 | 40;44[3] | 24 |
| 3 | 26;49 | 16 | 34;50 | 15[4] | 41;56[5] | 12 | 46;54 | 7 | 47;33[6] | 5 | 40;20 | 24 |
| 4 | 27; 6 | 17 | 35; 6 | 16 | 42; 9 | 13 | 47; 1 | 7 | 47;27 | 6 | 39;55 | 25 |
| 5 | 27;22 | 16 | 35;21 | 15 | 42;21 | 12 | 47; 7 | 6 | 47;21 | 6 | 39;29[7] western dis | 26 |
| 6 | 27;38 | 16[8]↓ | 35;36 | 15 | 42;33 | 12 | 47;13 | 6 | 47;15 | 6 | 39; 3 | 26 |
| 7 | 27;55[9] | 17 | 35;51 | 15 | 42;45 | 12 | 47;19 | 6 | 47; 8 | 7 | 38;36 | 27 |
| 8 | 28;11 | 16 | 36; 6 | 15 | 42;57[10] | 12 | 47;24 | 5 | 47; 1 | 7 | 38; 9 | 27 |
| 9 | 28;27[11] | 16 | 36;21 | 15 | 43; 9 | 12 | 47;29 | 5 | 46;53 | 8 | 37;41 | 28 |
| 10 | 28;44 | 17 | 36;36 | 15 | 43;21 | 12 | 47;34 | 5 | 46;44 | 9 | 37;12 | 29 |
| 11 | 29; 0 | 16⊥ | 36;51 | 15 | 43;32 | 11 | 47;38[12] | 4 | 46;35 | 9 | 36;43 | 29 |
| 12 | 29;16 | 16 | 37; 6[13] | 15[14] | 43;43 | 11 | 47;42 | 4 | 46;25 | 10 | 36;13 | 30 |
| 13 | 29;32 | 16 | 37;21 | 15[15] | 43;54 | 11 | 47;46 | 4 | 46;14 | 11 | 35;43[16] | 30 |
| 14 | 29;48 | 16[17] | 37;36 | 15 | 44; 5 | 11 | 47;49 | 3 | 46; 2 | 12 | 35;12 | 31 |
| 15 | 30; 4 | 16 | 37;51 | 15 | 44;16 | 11 | 47;52[18] | 3 | 45;50 | 12 | 34;40 | 32 |
| 16 | 30;21 | 17 | 38; 5 | 14 | 44;27[19] | 11 | 47;55[20] | 3 | 45;37[21] | 13 | 34; 7 | 33 |
| 17 | 30;37 | 16 | 38;19 | 14[22] | 44;37 | 10 | 47;57 | 2 | 45;24 | 13 | 33;34[23] | 33 |
| 18 | 30;53[24] | 16 | 38;33 | 14 | 44;47[25] | 10 | 47;59 | 2 | 45;10 | 14 | 33; 1 | 33 |
| 19 | 31; 9 | 16 | 38;48 | 15[26] | 44;57 | 10 | 48; 0 | 1 | 44;55 | 15 | 32;27[27] | 34 |
| 20 | 31;25 | 16 | 39; 2[28] western vis | 14 | 45; 7[29] | 10 | 48; 1 | 1 | 44;40 | 15 | 31;53[30] | 34 |
| 21 | 31;41 | 16 | 39;16 | 14 | 45;16 | 9 | 48; 1 | 0 | 44;24 | 16 | 31;19 | 34 |
| 22 | 31;57[31] | 16 | 39;30 | 14 | 45;26 | 10[32]↓ | 48; 1 | 0 | 44; 7[33] | 17 | 30;45 | 34 |
| 23 | 32;13 | 16 | 39;44 | 14 | 45;36 | 10⊥ | 48; 1 | 0 | 43;50 | 17 | 30;10 | 35 |
| 24 | 32;29 | 16 | 39;58 | 14 | 45;45 | 9 | 48; 0 | 1 | 43;32[34] | 18 | 29;35 | 35 |
| 25 | 32;45 | 16 | 40;12 | 14 | 45;53 | 8 | 47;59 | 1 | 43;14 stationary | 18 | 29; 0 | 35 |
| 26 | 33; 1 | 16 | 40;26 | 14 | 46; 2 | 9 | 47;58 | 1 | 42;55 | 19 | 28;24 | 36 |
| 27 | 33;17[35] | 16 | 40;39 | 13 | 46;10 | 8 | 47;56 | 2 | 42;35 retrograde | 20 | 27;48 | 36 |
| 28 | 33;33 | 16 | 40;52[36] | 13 | 46;18 | 8 | 47;53 | 3 | 42;14 | 21 | 27;12 | 36 |
| 29 | 33;49 | 16 | 41; 5[37] | 13 | 46;26[38] | 8[39] | 47;50[40] | 3 | 41;53[41] | 21 | 26;36 | 36 |

[1] **L** om. column [2] **C** 33′ [3] **F** 41° [4] **+F** 14′ [5] **C** 16′ [6] **C** 38′ [7] **C** 59′ [8] **C** 0ˢ 6–11°: dam. [9] **C** 15′ [10] **C** 17′ [11] **H** 26′ **C** minutes dam. [12] **C** 28′ [13] **YL** 5′ [14] **+Y** 14′ (cf. note 13) [15] **+Y** 16′ (cf. note 13) [16] $\mathbf{C_2}$ minutes dam. [17] **F** 17′ [18] **C** 55′ [19] $\mathbf{C_1}$ 24′ [20] **C** 52′ [21] $\mathbf{C_1}$ 34′ [22] **+B** 15′ (correction in black) [23] $\mathbf{C_1}$ 37′ [24] **CL** 13′ [25] **C** 40′ $\mathbf{C_1}$ 46′ [26] $\mathbf{+CC_1C_2}$ 14′ **B** corrected in black [27] $\mathbf{C_1}$ 24′ [28] **H** om. minutes [29] **H** 4′ **C** 50′ [30] **C** 13′ [31] **C** 17′ [32] **Y** 2ˢ 22–23°: 9′ [33] **H** om. minutes [34] **C** 37′ [35] $\mathbf{C_1}$ 14′ [36] **C** 12′ [37] **C** degrees dam. [38] **H** 27′ **C** units of minutes dam. [39] **C** dam. [40] **C** degrees dam. [41] **H** 13′.

**Table 36: Second equation for Mercury (second half)**
*Sources*: **F** fols 78v–79r, **H** fols 69v–70r, **C** fols 76v–77r, **$C_1$** fols 42v–43r, **$C_2$** fols 36v and 45r, **Y** fols 297v–298r, **L** fols 69v–70r, **B** pp. 134–135.

Second Equation for Mercury
to be equated and added to the centrum with the apogee

| degrees of the epicycle | nearest distance 0° **6** ° ′ | differences | **7** ° ′ | differences | mean distance 19° **8** ° ′ | differences | **9** ° ′ | differences | **10** ° ′ | differences | **11** ° ′ | differences |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 26; 0 | 36 | 10;29 | 23 | 4;13 | 4[1] | 5;27 | 7 | 10;42 | 13 | 17;56 | 16 |
| 1 | 25;24 | 36 | 10; 7 | 22 | 4;10 | 3 | 5;34[2] | 7[3] | 10;55[4] | 13 | 18;11 | 15 |
| 2 | 24;48 | 36 | stationary 9;46 | 21 | 4; 7 | 3 | 5;42[5] | 8[6] | 11; 8 | 13 | 18;27 | 16 |
| 3 | 24;12 | 36 | 9;25[7] | 21 | 4; 4 | 3[8] | 5;50 | 8 | 11;21 | 13 | 18;43[9] | 16 |
| 4 | 23;36 | 36 | progressive 9; 5 | 20 | 4; 2 | 2 | 5;58 | 8 | 11;34 | 13 | 18;59 | 16 |
| 5 | 23; 0 | 36 | 8;46 | 19 | 4; 1 | 1 | 6; 7 | 9 | 11;48 | 14[10] | 19;15 | 16 |
| 6 | 22;25 | 35 | 8;28 | 18 | 4; 0 | 1 | 6;15 | 8 | 12; 2 | 14 | 19;31 | 16 |
| 7 | 21;50 | 35 | 8;10 | 18 | 3;59 | 1 | 6;24 | 9 | 12;16 | 14[11]↓ | 19;47 | 16 |
| 8 | 21;15 | 35 | 7;53 | 17 | 3;59 | 0 | 6;34 | 10 | 12;30 | 14 | 20; 3 | 16 |
| 9 | 20;41 | 34 | 7;36 | 17 | 3;59[12] | 0 | 6;44 | 10 | eastern dis 12;44 | 14 | 20;19 | 16 |
| 10 | 20; 7 | 34 | 7;20 | 16 | 3;59 | 0 | 6;53 | 9[13] | 12;58 | 14 | 20;35 | 16 |
| 11 | 19;33 | 34 | 7; 5 | 15 | 4; 0[14] | 1 | 7; 3 | 10 | 13;12 | 14⊥ | 20;51 | 16 |
| 12 | 18;59 | 34 | 6;50[15] | 15 | 4; 1 | 1 | 7;13 | 10 | 13;27 | 15 | 21; 7 | 16 |
| 13 | 18;26 | 33[16] | 6;36[17] | 14 | 4; 3 | 2 | 7;23 | 10 | 13;41 | 14 | 21;23 | 16 |
| 14 | 17;53[18] | 33 | 6;23 | 13 | 4; 5 | 2 | 7;33 | 10 | 13;55 | 14 | 21;39[19] | 16 |
| 15 | 17;20 | 33 | 6;10 | 13[20] | 4; 8 | 3 | 7;44 | 11 | 14; 9 | 14[21] | 21;56 | 17[22]↓ |
| 16 | 16;48 | 32 | 5;58[23] | 12 | 4;11 | 3 | 7;55 | 11 | 14;24 | 15 | 22;12 | 16 |
| 17 | 16;17 | 31 | 5;46 | 12 | 4;14 | 3 | 8; 6[24] | 11 | 14;39 | 15 | 22;28 | 16 |
| 18 | 15;47[25]↓ | 30 | 5;35 | 11 | 4;18 | 4 | 8;17 | 11 | 14;54 | 15 | 22;44 | 16 |
| 19 | 15;17 | 30 | 5;25 | 10 | 4;22 | 4 | 8;28 | 11 | 15; 9 | 15 | 23; 0 | 16 |
| 20 | 14;48 | 29 | 5;16 | 9 | 4;26 | 4 | 8;39 | 11 | 15;24 | 15 | 23;16[26] | 16 |
| 21 | 14;19[27] | 29 | 5; 7 | 9 | 4;31 | 5 | 8;51 | 12 | 15;39 | 15 | 23;33 | 17 |
| 22 | 13;51[28] | 28 | 4;59[29] | 8 | 4;36 | 5 | 9; 3 | 12 | 15;54 | 15 | 23;49 | 16 |
| 23 | 13;24⊥ | 27 | 4;52[30] | 7 | 4;41 | 5 | 9;15 | 12[31] | 16; 9 | 15 | 24; 5 | 16 |
| 24 | eastern vis 12;57[32] | 27[33] | 4;45 | 7 | 4;47 | 6 | 9;27 | 12 | 16;24 | 15 | 24;22 | 17 |
| 25 | 12;31 | 26 | 4;39 | 6 | 4;53 | 6 | 9;39 | 12 | 16;39 | 15 | 24;38[34] | 16 |
| 26 | 12; 5 | 26[35] | 4;33[36] | 6 | 4;59 | 6 | 9;51 | 12 | 16;54 | 15 | 24;54 | 16 |
| 27 | 11;40 | 25 | 4;27 | 6 | 5; 6 | 7 | 10; 4[37] | 13[38] | 17;10 | 16 | 25;11[39] | 17⊥ |
| 28 | 11;16[40] | 24 | 4;22 | 5 | 5;13 | 7 | 10;16 | 12[41] | 17;25 | 15 | 25;27 | 16 |
| 29 | 10;52[42] | 24 | 4;17[43] | 5 | 5;20 | 7 | 10;29[44] | 13 | 17;40[45] | 15 | 25;43 | 16 |

[1] **C** 3′ **$C_1$** dam. [2] **$CC_1C_2$** 35′ [3] **$CC_1C_2$** 8′ (cf. note 2) [4] **L** 15′ [5] **L** 12′ [6] **$CC_1C_2$** 7′ (cf. note 2) [7] **C** 29′ [8] **Y** 2′ [9] **C** 48′ [10] **$CC_1C_2$** 13′ [11] **C** $10^s$ 7–11°: dam. [12] **C** 0′ [13] **$C_1C_2$** 10′ [14] **C** 59′ [15] **C** 7′ [16] **C** 34′ [17] **L** 37′ [18] **C** 13′ [19] **Y** 59′ [20] **C** 18′ [21] **$C_1$** 15′ [22] **C** 16′ **L** $11^s$ 15–27°: 16′ [23] **C** 18′ [24] **C** 16′ [25] **C** $6^s$ 18–23°: slide[−3] [26] **C** 56′ [27] +**H** 18′ [28] +**H** 11′ **L** 59′ [29] **C** 19′ [30] **F** 57′ **C** 12′ [31] **L** 13′ [32] **C** 17′ [33] **L** 26′ [34] **C** 33′ [35] **$CC_1C_2$** 25′ [36] **C** 13′ [37] **$CC_1C_2$** 3′ [38] **$CC_1C_2$** 12′ (cf. note 37) [39] **C** 14′ [40] **H** 6′ [41] **$CC_1C_2$** 13′ (cf. note 37) [42] **C** 12′ **L** 42′ [43] **F** 18′ [44] **C** degrees dam. [45] **C** units of minutes dam.

## Table 36: Variation of the nearest distance for Mercury

*Sources*: **F** fol. 79v, **H** fol. 70v, **C** fol. 77v, **$C_1$** fol. 43v, **$C_2$** fol. 45v, **Y** fol. 298v, **L** fol. 70v, **B** p. 136.

| Variation of the Nearest Distance for Mercury | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| corrected centrum | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8[1] |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the furthest distance | 1; 6 | 1;58 | 1;41 | 1;19 | 1;33 | 1;58 | 1;27 |
| 1 | | 1; 9 | 1;58 | 1;40 | 1;19 | 1;34 | 1;58 | 1;25 |
| 2 | | 1;12 | 1;58 | 1;39 | 1;18 | 1;35 | 1;58 | 1;23 |
| 3 | | 1;15 | 1;59 | 1;38 | 1;18 | 1;36 | 1;59 | 1;20 |
| 4 | | 1;18 | 1;59 | 1;37 | 1;18 | 1;37 | 1;59 | 1;18 |
| 5 | | 1;20 | 1;59 | 1;36 | 1;18 | 1;38 | 1;59 | 1;15 |
| 6 | | 1;23 | 1;58 | 1;35 | 1;18 | 1;39 | 1;58 | 1;12 |
| 7 | | 1;25 | 1;58 | 1;34 | 1;19 | 1;40 | 1;58 | 1; 9 |
| 8 | | 1;27 | 1;58 | 1;33 | 1;19 | 1;41 | 1;58 | 1; 6 |
| 9 | [2] | 1;29 | 1;57 | 1;32 | 1;19 | 1;42 | 1;57 | 1; 2 |
| 10 | [3] | 1;32 | 1;57 | 1;31 | 1;20 | 1;43 | 1;57 | 0;59 |
| 11 | 0; 2 | 1;34 | 1;57 | 1;30 | 1;20 | 1;44 | 1;57 | 0;56 |
| 12 | 0; 5 | 1;36 | 1;56 | 1;29 | 1;20 | 1;45 | 1;56 | 0;53 |
| 13 | 0; 7 | 1;39 | 1;56 | 1;28 | 1;21 | 1;46 | 1;56 | 0;50 |
| 14 | 0;10 | 1;41 | 1;56 | 1;27 | 1;21 | 1;47 | 1;55 | 0;47[4] |
| 15 | 0;14 | 1;42 | 1;55 | 1;26 | 1;22 | 1;48 | 1;54 | 0;43 |
| 16 | 0;17 | 1;44 | 1;54 | 1;26 | 1;22 | 1;49 | 1;52 | 0;40 |
| 17 | 0;21 | 1;45 | 1;53 | 1;25 | 1;23 | 1;50 | 1;51 | 0;36 |
| 18 | 0;25 | 1;47 | 1;52 | 1;25 | 1;23 | 1;51 | 1;49 | 0;32 |
| 19 | 0;28[5] | 1;48 | 1;51 | 1;24 | 1;24 | 1;51 | 1;48 | 0;28 |
| 20 | 0;32 | 1;49 | 1;51 | 1;23 | 1;25 | 1;52 | 1;47 | 0;25 |
| 21 | 0;36 | 1;51 | 1;50 | 1;23 | 1;25 | 1;53 | 1;45 | 0;21 |
| 22 | 0;40 | 1;52 | 1;49 | 1;22 | 1;26 | 1;54 | 1;44 | 0;17 |
| 23 | 0;43 | 1;54 | 1;48 | 1;22 | 1;26[6] | 1;55 | 1;42 | 0;14 |
| 24 | 0;47 | 1;55 | 1;47 | 1;21 | 1;27 | 1;56 | 1;41 | 0;10 |
| 25 | 0;50 | 1;56 | 1;46 | 1;21 | 1;28 | 1;56 | 1;39 | 0; 7 |
| 26 | 0;53 | 1;56[7] | 1;45 | 1;20 | 1;29 | 1;56 | 1;36 | 0; 5 |
| 27 | 0;56 | 1;57 | 1;44 | 1;20 | 1;30 | 1;57 | 1;34 | 0; 2 |
| 28 | 0;59 | 1;57 | 1;43 | 1;20 | 1;31 | 1;57 | 1;32[8] | From the furthest distance[9] |
| 29 | 1; 2 | 1;57 | 1;42 | 1;19[10] | 1;32 | 1;57 | 1;29 | |

[1] **L** 60 س [2] **Y** 0;2 [3] **Y** 0;0 [4] **F** 46′ [5] **L** 23′ [6] **YL** 27′ [7] **L** 26′ [8] **C** 30′ [9] **HLB** 0;0 (**H** omits the label, **LB** in addition to the label) [10] **C** dam.

**Table 36: Interpolation minutes (nearest distance) for Mercury**
*Sources*: **F** fol. 79v, **H** fol. 71r, **C** fol. 77v, **$C_1$** fol. 43v, **$C_2$** fol. 45v, **Y** fol. 298v, **L** fol. 70v, **B** p. 136.

| Minutes of Proportions ⟨for Mercury⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **0** | **1** | **2** | **3** | **4** | **5** | |
| 0 | 0 | 22 | 42 | 56 | 59 | 42 | 30 |
| 1 | 1 | 23 | 42 | 56 | 59 | 41 | 29 |
| 2 | 2 | 24 | 43 | 57[1] | 59 | 40 | 28 |
| 3 | 2 | 24 | 43 | 57 | 58 | 39 | 27 |
| 4 | 3 | 25 | 44 | 57 | 58 | 38 | 26 |
| 5 | 4 | 26 | 44 | 58 | 58 | 37 | 25 |
| 6 | 4 | 26 | 45 | 58 | 58 | 36 | 24 |
| 7 | 5[2] | 27 | 45 | 58 | 58 | 35 | 23 |
| 8 | 6[3] | 28 | 46 | 58 | 57 | 33 | 22 |
| 9 | 6 | 28 | 46 | 58 | 57 | 32 | 21 |
| 10 | 7 | 29 | 47 | 59 | 57 | 31 | 20 |
| 11 | 8 | 30 | 47 | 59 | 56 | 29 | 19 |
| 12 | 9 | 30 | 48 | 59 | 56 | 28 | 18 |
| 13 | 10 | 31 | 48 | 59 | 55 | 27 | 17 |
| 14 | 11 | 32 | 49 | 59 | 54 | 25 | 16 |
| 15 | 11 | 32 | 49 | 59 | 54 | 24 | 15 |
| 16 | 12 | 33 | 50 | 60 | 53 | 22 | 14 |
| 17 | 13 | 34 | 50 | 60 | 52 | 21 | 13 |
| 18 | 13 | 34 | 51 | 60 | 52 | 19[4] | 12 |
| 19 | 14 | 35 | 51 | 60 | 51 | 17 | 11 |
| 20 | 15[5] | 36 | 52 | 60 | 50 | 16 | 10 |
| 21 | 15 | 36↓[6] | 52 | 60 | 50 | 14 | 9 |
| 22 | 16 | 37 | 53 | 60 | 49 | 12 | 8 |
| 23 | 17 | 38⊥ | 53 | 60 | 48 | 10 | 7 |
| 24 | 18 | 38 | 54 | 60 | 48 | 9 | 6 |
| 25 | 19 | 39 | 54 | 60 | 47 | 7 | 5 |
| 26 | 20 | 40 | 55 | 60 | 46 | 6 | 4 |
| 27 | 20 | 40 | 55 | 60 | 45 | 4 | 3 |
| 28 | 21 | 41 | 55 | 59↓[7] | 44 | 3 | 2 |
| 29 | 22 | 42 | 56 | 59⊥ | 43 | 1 | 1 |
| parts of the epicycle | **11** | **10** | **9** | **8** | **7** | **6** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **YL** 56′  [2] **C** 6′  [3] **C** 5′  [4] **C** 59′  [5] **F** 14′  [6] **C** $1^s$ 21–23°: shift[−1 col.]
[7] **$C_1C_2$** $3^s$ 28–29°: 60′.

**Table 36: Variation of the furthest distance for Mercury**
*Sources*: **F** fol. 80r, **H** fol. 71v, **C** fol. 78r, $\mathbf{C_1}$ fol. 44r, $\mathbf{C_2}$ fol. 46r, **Y** fol. 299r, **L** fol. 71r, **B** p. 137.

Variation of the Furthest Distance for Mercury

| corrected centrum | 8 | 9 | 10 | 11 | 0 | 1 |
|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0 | From the nearest distance | 0;12 | 2;10 | 3; 6 | 2;33 | 0;47 |
| 1 | | 0;16 | 2;13 | 3; 7 | 2;30 | 0;43 |
| 2 | | 0;21 | 2;16 | 3; 7 | 2;28 | 0;39 |
| 3 | | 0;26 | 2;20 | 3; 8 | 2;25 | 0;35 |
| 4 | | 0;31 | 2;23[1] | 3; 8 | 2;23 | 0;31 |
| 5 | | 0;35 | 2;25 | 3; 8 | 2;20 | 0;26 |
| 6 | | 0;39 | 2;28[2] | 3; 7 | 2;16 | 0;21 |
| 7 | | 0;43 | 2;30 | 3; 7 | 2;13 | 0;16 |
| 8 | | 0;47[3] | 2;33 | 3; 6[4] | 2;10[5] | 0;12 |
| 9 | | 0;51 | 2;35 | 3; 5 | 2; 6 | 0; 7 |
| 10 | | 0;56 | 2;37 | 3; 5 | 2; 3 | 0; 2 |
| 11 | | 1; 0 | 2;40 | 3; 4 | 2; 0 | [6] From the nearest distance |
| 12 | | 1; 4 | 2;42 | 3; 3 | 1;57 | |
| 13 | | 1; 8[7] | 2;45 | 3; 3 | 1;53 | |
| 14 | | 1;12 | 2;47 | 3; 2 | 1;50 | |
| 15 | | 1;16 | 2;49 | 3; 1 | 1;46 | |
| 16 | | 1;20 | 2;50 | 2;59 | 1;42 | |
| 17 | | 1;23 | 2;52 | 2;58 | 1;39 | |
| 18 | | 1;27 | 2;53 | 2;56 | 1;35 | |
| 19 | | 1;31 | 2;55 | 2;55 | 1;31 | |
| 20 | | 1;35 | 2;56 | 2;53 | 1;27[8] | |
| 21 | | 1;39 | 2;58 | 2;52 | 1;23 | |
| 22 | | 1;42 | 2;59 | 2;50 | 1;20 | |
| 23 | | 1;46 | 3; 1 | 2;49 | 1;16 | |
| 24 | | 1;50 | 3; 2 | 2;47 | 1;12 | |
| 25 | | 1;53 | 3; 3 | 2;45 | 1; 8 | |
| 26 | | 1;57 | 3; 3 | 2;42 | 1; 4 | |
| 27 | [9] | 2; 0 | 3; 4 | 2;40 | 1; 0 | |
| 28 | 0; 2 | 2; 3 | 3; 5 | 2;37 | 0;56[10] | |
| 29 | 0; 7 | 2; 6 | 3; 5 | 2;35 | 0;51[11] | |

[1] **C** 28′ [2] **C** 23′ [3] **L** 46′ [4] $\mathbf{C_2}$ minutes dam. [5] $\mathbf{C_1}$ 9′ $\mathbf{C_2}$ minutes dam. [6] $\mathbf{HCC_1C_2LB}$ 0;0 [7] **L** 3′ [8] **F** 26′ [9] $\mathbf{HC_1LB}$ 0;0 [10] **C** 16′ [11] **C** 11′.

**Table 36: Interpolation minutes (furthest distance) for Mercury**
*Sources*: **F** fol. 80r, **H** fol. 72r, **C** fol. 78r, **$C_1$** fol. 44r, **$C_2$** fol. 46r, **Y** fol. 299r, **L** fol. 71r, **B** p. 137.

| Minutes of Proportions ⟨for Mercury⟩ | | | | | | | |
|---|---|---|---|---|---|---|---|
| parts of the epicycle | to be multiplied by the variation and added to the second equation | | | | | | parts of the epicycle |
| | **6** | **7** | **8** | **9** | **10** | **11** | |
| 0 | 0 | 42 | 59 | 56 | 42 | 22 | 30 |
| 1 | 1 | 43 | 59 | 56 | 42 | 22 | 29 |
| 2 | 3 | 44 | 59 | 55[1] | 41 | 21 | 28 |
| 3 | 4 | 45 | 60 | 55 | 40 | 20 | 27 |
| 4 | 6 | 46 | 60 | 55 | 40 | 20 | 26 |
| 5 | 7 | 47 | 60 | 54 | 39 | 19 | 25 |
| 6 | 9 | 48 | 60 | 54 | 38 | 18 | 24 |
| 7 | 10 | 48 | 60 | 53 | 38 | 17[2] | 23 |
| 8 | 12 | 49 | 60 | 53 | 37 | 16 | 22 |
| 9 | 14 | 50[3] | 60 | 52 | 36 | 15 | 21 |
| 10 | 16 | 50 | 60 | 52 | 36 | 15[4] | 20[5] |
| 11 | 17 | 51 | 60 | 51 | 35 | 14 | 19[6] |
| 12 | 19 | 52 | 60 | 51 | 34 | 13 | 18 |
| 13 | 21 | 52 | 60 | 50 | 34 | 13 | 17 |
| 14 | 22 | 53 | 60 | 50 | 33 | 12 | 16 |
| 15 | 24 | 54 | 59 | 49 | 32 | 11 | 15 |
| 16 | 25 | 54 | 59 | 49 | 32 | 11 | 14 |
| 17 | 27 | 55 | 59 | 48 | 31 | 10 | 13 |
| 18 | 28 | 56 | 59 | 48 | 30 | 9 | 12 |
| 19 | 29 | 56 | 59 | 47 | 30 | 8 | 11 |
| 20 | 31 | 57 | 59 | 47 | 29 | 7 | 10 |
| 21 | 32 | 57 | 58 | 46 | 28 | 6 | 9 |
| 22 | 33 | 57[7] | 58 | 46 | 28 | 6[8]↓ | 8 |
| 23 | 35 | 58 | 58 | 45 | 27 | 5 | 7 |
| 24 | 36 | 58 | 58 | 45 | 26 | 4 | 6 |
| 25 | 37 | 58 | 58 | 44 | 26 | 4⊥ | 5 |
| 26 | 38 | 58 | 57 | 44 | 25 | 3 | 4 |
| 27 | 39 | 58 | 57 | 43 | 24 | 2 | 3 |
| 28 | 40 | 59 | 57 | 43 | 24 | 2 | 2 |
| 29 | 41 | 59 | 56[9] | 42[10] | 23[11] | 1 | 1 |
| parts of the epicycle | **5** | **4** | **3** | **2** | **1** | **0** | parts of the epicycle |
| | to be multiplied by the variation and subtracted from the second equation | | | | | | |

[1] **F** 56′ [2] **L** 16′ [3] **Y** 49′ [4] **F** 14′ [5] **H** 22 (corrected by a different hand) [6] **H** 23 (corrected by a different hand) [7] **C** 58′ [8] **F** 11$^s$ 22–25°: slide[+1] [9] **C** dam. [10] **C** dam. [11] **C** units dam.

**Table 37: Lunar latitude with values to minutes**

*Sources*: **F** fol. 80v, **H** fol. 72v, **C** fol. 78v, **$C_1$** fol. 44v, **$C_2$** fol. 46v (see Plate 11). **YLB** have a table with values to seconds and tabular differences which is here edited on p. 209.

| | Lunar Latitude | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | share of latitude | | latitude | share of latitude | | latitude | share of latitude | | latitude |
| north | **0** | **5** | ° ′ | **1** | **4** | ° ′ | **2** | **3** | ° ′ |
| south | **6** | **11** | | **7** | **10** | | **8** | **9** | |
| | 1 | 29 | 0; 5 | 1 | 29 | 2;35 | 1 | 29 | 4;23 |
| | 2 | 28 | 0;11 | 2 | 28 | 2;39 | 2 | 28 | 4;25 |
| | 3 | 27 | 0;16 | 3 | 27 | 2;44 | 3 | 27 | 4;27 |
| | 4 | 26 | 0;21 | 4 | 26 | 2;48 | 4 | 26 | 4;30 |
| | 5 | 25 | 0;26 | 5 | 25 | 2;52 | 5 | 25 | 4;32 |
| | 6 | 24 | 0;31 | 6 | 24 | 2;57[1] | 6 | 24 | 4;33 |
| | 7 | 23 | 0;37 | 7 | 23 | 3; 1 | 7 | 23 | 4;36 |
| | 8 | 22 | 0;42 | 8 | 22 | 3; 5 | 8 | 22 | 4;38 |
| | 9 | 21 | 0;47 | 9 | 21 | 3; 9 | 9 | 21 | 4;40 |
| | 10 | 20 | 0;52 | 10 | 20 | 3;13 | 10 | 20 | 4;42 |
| | 11 | 19 | 0;57 | 11 | 19 | 3;17 | 11 | 19 | 4;44 |
| | 12 | 18 | 1; 3 | 12 | 18 | 3;21 | 12 | 18 | 4;45 |
| | 13 | 17 | 1; 8 | 13 | 17 | 3;25 | 13 | 17 | 4;47 |
| | 14 | 16 | 1;13 | 14 | 16 | 3;29 | 14 | 16 | 4;48 |
| | 15 | 15 | 1;18 | 15 | 15 | 3;32 | 15 | 15 | 4;50 |
| | 16 | 14 | 1;23 | 16 | 14 | 3;36 | 16 | 14 | 4;51 |
| | 17 | 13 | 1;28 | 17 | 13 | 3;40 | 17 | 13 | 4;52 |
| | 18 | 12 | 1;33 | 18 | 12 | 3;43 | 18 | 12 | 4;54 |
| | 19 | 11 | 1;38 | 19 | 11 | 3;47 | 19 | 11 | 4;55[2] |
| | 20 | 10 | 1;43 | 20 | 10 | 3;50 | 20 | 10 | 4;55[3] |
| | 21 | 9 | 1;48 | 21 | 9 | 3;53 | 21 | 9 | 4;56 |
| | 22 | 8 | 1;53 | 22 | 8 | 3;57 | 22 | 8 | 4;57 |
| | 23 | 7 | 1;58 | 23 | 7 | 4; 0 | 23 | 7 | 4;58 |
| | 24 | 6 | 2; 2[4] | 24 | 6 | 4; 3 | 24 | 6 | 4;58 |
| | 25 | 5 | 2; 7 | 25 | 5 | 4; 6 | 25 | 5 | 4;59 |
| | 26 | 4 | 2;12 | 26 | 4 | 4; 9 | 26 | 4 | 4;59[5] |
| | 27 | 3 | 2;16 | 27 | 3 | 4;12 | 27 | 3 | 5; 0 |
| | 28 | 2 | 2;21 | 28 | 2 | 4;15 | 28 | 2 | 5; 0 |
| | 29 | 1 | 2;26 | 29 | 1 | 4;17 | 29 | 1 | 5; 0 |
| | 30 | 0 | 2;30[6] | 30 | 0 | 4;20[7] | 30 | 0 | 5; 0 |

**$CC_1C_2$** display this table with a double entry and arguments 1 to 180°; variants in the values for an argument *x* between 91 and 180° are here indicated under the symmetrical argument 180° − *x*.
[1] **$C_1$** 54′ [2] **$C_1C_2$** (for the symmetrical argument 101°): 54′ [3] **$C_1C_2$** 56′ [4] **F** 3′ [5] **$C_1$** (for the symmetrical argument 94°): 1′ [6] **C** dam. [7] **C** dam.

**Table 37a: Lunar latitude with values to seconds and tabular differences**

*Sources*: **Y** fol. 299v, **L** fol. 71v, **B** p. 138. **FCC$_1$C$_2$** have a table with values to minutes without tabular differences, which is here edited on p. 208.

Lunar Latitude

| share of latitude | | latitude | share of minutes | share of latitude | | latitude | share of minutes | share of latitude | | latitude | share of minutes |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | **5** | | | **1** | **4** | | | **2** | **3** | | |
| **6** | **11** | ° ′ ″ | ′ ″ | **7** | **10** | ° ′ ″ | ′ ″ | **8** | **9** | ° ′ ″ | ′ ″ |
| 1 | 29 | 0; 5,14 | 5,14 | 1[1]↓ | 29[2]↓ | 2;34,49 | 4,31 | 1 | 29 | 4;22,33 | 2,33 |
| 2 | 28 | 0;10,31 | 5,17 | 2 | 28 | 2;39,16 | 4,27[3] | 2 | 28 | 4;25, 3 | 2,30 |
| 3 | 27 | 0;15,44 | 5,13 | 3 | 27 | 2;43,42 | 4,26 | 3 | 27 | 4;27,28 | 2,25 |
| 4 | 26 | 0;20,57 | 5,13 | 4 | 26 | 2;48, 4 | 4,22 | 4 | 26 | 4;29,46[4] | 2,18 |
| 5 | 25 | 0;26,12 | 5,15 | 5 | 25 | 2;52,24 | 4,20 | 5 | 25 | 4;32, 2 | 2,16 |
| 6 | 24 | 0;31,27 | 5,15 | 6 | 24 | 2;56,39 | 4,15 | 6 | 24 | 4;34,11 | 2, 9 |
| 7 | 23 | 0;36,38 | 5,11 | 7 | 23 | 3; 0,51 | 4,12 | 7 | 23 | 4;36,17 | 2, 6 |
| 8 | 22 | 0;41,51 | 5,13 | 8 | 22 | 3; 5, 1 | 4,10 | 8 | 22 | 4;38,16 | 1,59 |
| 9 | 21 | 0;47, 3 | 5,12 | 9 | 21 | 3; 9, 6 | 4, 5 | 9 | 21 | 4;40,11 | 1,55 |
| 10 | 20 | 0;52,13 | 5,10 | 10 | 20 | 3;13, 9 | 4, 3 | 10 | 20 | 4;42, 0 | 1,49 |
| 11 | 19 | 0;57,23 | 5,10 | 11 | 19 | 3;17, 8 | 3,59 | 11 | 19 | 4;43,45 | 1,45 |
| 12 | 18 | 1; 2,31 | 5, 8 | 12 | 18 | 3;21, 3 | 3,55 | 12 | 18 | 4;45,24 | 1,39 |
| 13 | 17 | 1; 7,39 | 5, 8 | 13 | 17 | 3;24,53 | 3,50 | 13 | 17 | 4;46,58 | 1,34 |
| 14 | 16 | 1;12,45 | 5, 6 | 14 | 16 | 3;28,41 | 3,48 | 14 | 16 | 4;48,27 | 1,29 |
| 15 | 15 | 1;17,50 | 5, 5 | 15 | 15 | 3;32,25 | 3,44 | 15 | 15 | 4;49,51 | 1,24 |
| 16 | 14 | 1;22,54 | 5, 4 | 16 | 14 | 3;36, 5 | 3,40 | 16 | 14 | 4;51, 9[5] | 1,18[6] |
| 17 | 13 | 1;27,56 | 5, 2 | 17 | 13 | 3;39,41 | 3,36 | 17 | 13 | 4;52,23 | 1,14[7] |
| 18 | 12 | 1;32,54 | 4,58 | 18 | 12 | 3;43,12 | 3,31 | 18 | 12 | 4;53,30 | 1, 7 |
| 19 | 11 | 1;37,53 | 4,59 | 19 | 11 | 3;46,41 | 3,29 | 19 | 11 | 4;54,32 | 1, 2[8] |
| 20 | 10 | 1;42,51 | 4,58 | 20 | 10 | 3;50, 5 | 3,24 | 20 | 10 | 4;55,29 | 0,57 |
| 21 | 9 | 1;47,45 | 4,54 | 21 | 9 | 3;53,24 | 3,19 | 21 | 9 | 4;56,21 | 0,52[9] |
| 22 | 8 | 1;52,38 | 4,53 | 22 | 8 | 3;56,39 | 3,15 | 22 | 8 | 4;57, 8 | 0,47 |
| 23 | 7 | 1;57,30 | 4,52 | 23 | 7 | 3;59,50 | 3,11 | 23 | 7 | 4;57,48 | 0,40 |
| 24 | 6 | 2; 2,17 | 4,47 | 24 | 6 | 4; 2,57 | 3, 7 | 24 | 6 | 4;58,23 | 0,35 |
| 25 | 5 | 2; 7, 4 | 4,47 | 25 | 5 | 4; 5,59[10] | 3, 2 | 25 | 5 | 4;58,51 | 0,28 |
| 26 | 4 | 2;11,48 | 4,44 | 26 | 4 | 4; 8,56[11] | 2,57 | 26 | 4 | 4;59,16 | 0,25 |
| 27 | 3 | 2;16,29 | 4,41 | 27 | 3 | 4;11,48 | 2,52 | 27 | 3 | 4;59,35 | 0,19[12] |
| 28 | 2 | 2;21, 8 | 4,39 | 28 | 2 | 4;14,37 | 2,49 | 28 | 2 | 4;59,48 | 0,13 |
| 29 | 1 | 2;25,44 | 4,36 | 29 | 1 | 4;17,20 | 2,43 | 29 | 1 | 4;59,57 | 0, 9 |
| 30 | 0 | 2;30,18[13] | 4,34 | 30⊥ | 0⊥ | 4;20, 0 | 2,40 | 30 | 0 | 5; 0, 0 | 0, 3 |

[1] **L** arguments 1–30°: om. [2] **L** arguments 29, 28, 27, ..., 0°: om. [3] **L** 24″ [4] **Y** 16″ [5] **L** 3″ [6] **Y** 28″ [7] **Y** 15″ [8] **L** 12″ [9] **L** 12″ [10] **L** 49″ [11] **Y** 26″ [12] **LB** 49″ [13] **Y** 48″.

**Table 38: Latitude and first station of Saturn**

*Sources*: **F** fol. 81r, **H** fol. 73r, **C** fols 79r and 79v, **$C_1$** fols 45r and 45v, **$C_2$** -, **Y** fol. 300r, **L** fol. 72r, **B** p. 139. The table of the planetary latitudes in **$CC_1$** is also separately edited on p. 215.

Latitude of Saturn and its First Station

Seven degrees are added to its centrum, then it is used for the latitude and the station.

| | column of the argument | | north | south | minutes of shares | first station |
|---|---|---|---|---|---|---|
| | s ° | s ° | ° ′ | ° ′ | | s ° ′ |
| upper half | $0^s$ 6 | $11^s$ 24 | 2; 4 | 2; 2 | 60 | $3^s$ 22;47 |
| | $0^s$ 12 | $11^s$ 18 | 2; 5 | 2; 3 | 59 | $3^s$ 22;50 |
| | $0^s$ 18 | $11^s$ 12 | 2; 6 | 2; 3 | 57 | $3^s$ 22;52 |
| | $0^s$ 24 | $11^s$ 6 | 2; 7 | 2; 4 | 55 | $3^s$ 22;55 |
| | $1^s$ 0 | $11^s$ 0 | 2; 8 | 2; 5 | 52 | $3^s$ 22;57[1]↓ |
| | $1^s$ 6 | $10^s$ 24 | 2;10 | 2; 7 | 48 | $3^s$ 23; 1 |
| | $1^s$ 12 | $10^s$ 18 | 2;11 | 2; 8 | 44 | $3^s$ 23; 6[2] |
| | $1^s$ 18 | $10^s$ 12 | 2;12 | 2;10 | 40 | $3^s$ 23;13 |
| | $1^s$ 24 | $10^s$ 6 | 2;14 | 2;12 | 35 | $3^s$ 23;20 |
| | $2^s$ 0 | $10^s$ 0 | 2;16 | 2;15 | 30 | $3^s$ 23;28[3] |
| | $2^s$ 6 | $9^s$ 24 | 2;18 | 2;18 | 24 | $3^s$ 23;37 |
| | $2^s$ 12 | $9^s$ 18 | 2;21 | 2;21 | 18[4] | $3^s$ 23;45[5] |
| | $2^s$ 18 | $9^s$ 12 | 2;24 | 2;24 | 12 | $3^s$ 23;54 |
| | $2^s$ 24[6] | $9^s$ 6 | 2;27 | 2;27 | 6 | $3^s$ 24; 2[7] |
| lower half | $3^s$ 0 | $9^s$ 0 | 2;30 | 2;30 | 0 | $3^s$ 24;11 |
| | $3^s$ 6 | $8^s$ 24[8] | 2;33 | 2;33 | 6 | $3^s$ 24;19[9] |
| | $3^s$ 12 | $8^s$ 18 | 2;36 | 2;36 | 12 | $3^s$ 24;27 |
| | $3^s$ 18 | $8^s$ 12 | 2;39 | 2;39 | 18 | $3^s$ 24;34 |
| | $3^s$ 24 | $8^s$ 6 | 2;42 | 2;42 | 24 | $3^s$ 24;42 |
| | $4^s$ 0 | $8^s$ 0 | 2;45 | 2;45 | 30 | $3^s$ 24;50 |
| | $4^s$ 6 | $7^s$ 24 | 2;47 | 2;48 | 35 | $3^s$ 24;58 |
| | $4^s$ 12 | $7^s$ 18 | 2;50 | 2;51 | 40 | $3^s$ 25; 5 |
| | $4^s$ 18 | $7^s$ 12 | 2;53 | 2;54[10] | 44 | $3^s$ 25;10 |
| | $4^s$ 24 | $7^s$ 6 | 2;55 | 2;56 | 48 | $3^s$ 25;15 |
| | $5^s$ 0 | $7^s$ 0 | 2;57 | 2;58 | 52 | $3^s$ 25;19 |
| | $5^s$ 6 | $6^s$ 24 | 2;59 | 3; 0 | 55 | $3^s$ 25;23 |
| | $5^s$ 12 | $6^s$ 18 | 3; 0 | 3; 2 | 57[11] | $3^s$ 25;26 |
| | $5^s$ 18 | $6^s$ 12 | 3; 1 | 3; 3 | 59 | $3^s$ 25;28⊥ |
| | $5^s$ 24 | $6^s$ 6 | 3; 2 | 3; 4 | 60 | $3^s$ 25;29 |
| | $6^s$ 0 | $6^s$ 0 | 3; 2 | 3; 5 | 60 | $3^s$ 25;30[12] |

The first station is subtracted from a rotation, and the remainder is the second station.

The variants from the table in **$CC_1$** for the latitudes of all five planets and from the table for their stations have been included here. [1] **C** $1^s$ 0–$5^s$ 18°: slide[+1] [2] +**$C_1$** 4′ [3] +**B** 23′ [4] **Y** 13′ [5] **C**+ 14′ (instead of 54′) +**Y** 49′ [6] **C** (stations table): $3^s$ [7] +**L** 10′ [8] **C** (latitude table): om. '$8^s$' (causing this and the next two arguments to be read with '$9^s$') [9] **C**+ 26′ (instead of 27′) [10] **$C_1$** 57′ [11] **H** 7′ [12] **C** minutes dam.

**Table 39: Latitude and first station of Jupiter**
*Sources*: **F** fol. 81v, **H** fol. 73v, **C** fols 79r and 79v, **$C_1$** fols 45r and 45v, **$C_2$** -, **Y** fol. 300v, **L** fol. 72v, **B** p. 140. The table of the planetary latitudes in **$CC_1$** is also separately edited on p. 215.

Latitude of Jupiter and its First Station

Twelve degrees are added to the true centrum, then it is used for the latitude and the station.

| | column of the argument | | north | south | minutes of shares | first station |
|---|---|---|---|---|---|---|
| | s ° | s ° | ° ′ | ° ′ | | s ° ′ |
| upper half | $0^s$ 6 | $11^s$ 24 | 1; 7 | 1; 5 | 60 | $4^s$ 4; 7 |
| | $0^s$ 12 | $11^s$ 18 | 1; 8 | 1; 6 | 59 | $4^s$ 4;10 |
| | $0^s$ 18 | $11^s$ 12 | 1; 8 | 1; 6 | 57 | $4^s$ 4;12 |
| | $0^s$ 24 | $11^s$ 6 | 1; 9 | 1; 7 | 55 | $4^s$ 4;15 |
| | $1^s$ 0 | $11^s$ 0 | 1;10 | 1; 8 | 52 | $4^s$ 4;17 |
| | $1^s$ 6 | $10^s$ 24 | 1;11 | 1; 9 | 48 | $4^s$ 4;20 |
| | $1^s$ 12 | $10^s$ 18 | 1;12[1]↓ | 1;10 | 44 | $4^s$ 4;25 |
| | $1^s$ 18 | $10^s$ 12 | 1;13 | 1;11 | 40 | $4^s$ 4;32[2] |
| | $1^s$ 24 | $10^s$ 6 | 1;14[3]⊥ | 1;13[4] | 35 | $4^s$ 4;41 |
| | $2^s$ 0 | $10^s$ 0 | 1;16 | 1;16[5] | 30 | $4^s$ 4;52 |
| | $2^s$ 6 | $9^s$ 24 | 1;18 | 1;18 | 24 | $4^s$ 5; 2 |
| | $2^s$ 12 | $9^s$ 18 | 1;21 | 1;21 | 18 | $4^s$ 5;12 |
| | $2^s$ 18 | $9^s$ 12 | 1;24 | 1;24 | 12 | $4^s$ 5;22 |
| | $2^s$ 24 | $9^s$ 6 | 1;27 | 1;27 | 6 | $4^s$ 5;32 |
| lower half | $3^s$ 0 | $9^s$ 0 | 1;30 | 1;30 | 0 | $4^s$ 5;42 |
| | $3^s$ 6 | $8^s$ 24 | 1;33 | 1;33 | 6 | $4^s$ 5;51 |
| | $3^s$ 12 | $8^s$ 18 | 1;36 | 1;36 | 12 | $4^s$ 5;59 |
| | $3^s$ 18 | $8^s$ 12 | 1;39 | 1;39 | 18 | $4^s$ 6; 8 |
| | $3^s$ 24 | $8^s$ 6 | 1;42[6] | 1;42 | 24 | $4^s$ 6;16 |
| | $4^s$ 0 | $8^s$ 0 | 1;45[7] | 1;45 | 30 | $4^s$ 6;24 |
| | $4^s$ 6 | $7^s$ 24 | 1;47 | 1;48 | 35 | $4^s$ 6;32 |
| | $4^s$ 12 | $7^s$ 18 | 1;50 | 1;51 | 40 | $4^s$ 6;39 |
| | $4^s$ 18 | $7^s$ 12 | 1;52 | 1;54 | 44 | $4^s$ 6;46 |
| | $4^s$ 24 | $7^s$ 6 | 1;55 | 1;57 | 48 | $4^s$ 6;53 |
| | $5^s$ 0 | $7^s$ 0 | 1;58 | 2; 0 | 52 | $4^s$ 6;59 |
| | $5^s$ 6 | $6^s$ 24 | 2; 0 | 2; 3 | 55 | $4^s$ 7; 4[8] |
| | $5^s$ 12[9]↓ | $6^s$ 18 | 2; 2 | 2; 5 | 57 | $4^s$ 7; 7 |
| | $5^s$ 18⊥ | $6^s$ 12 | 2; 3 | 2; 6 | 59 | $4^s$ 7; 8 |
| | $5^s$ 24 | $6^s$ 6 | 2; 4[10] | 2; 7 | 60 | $4^s$ 7; 9 |
| | $6^s$ 0 | $6^s$ 0 | 2; 4[11] | 2; 8 | 60 | $4^s$ 7;10[12] |

The variants from the table in **$CC_1$** for the latitudes of all five planets and from the table for the stations have been included here (but the two variants in the arguments only in the table for Saturn).
[1] **C** $1^s$ 12–24°: slide[−1 col.] (i.e., copied from the column for the southern latitude) [2] **F** 30′ [3] **+LB** 15′ [4] **F** 12′ [5] **F** 15′ [6] **F** 45′ [7] **F** 46′ [8] **F** 3′ [9] **Y** arguments $5^s$ 12–18°: slide[+1] [10] **$CC_1$** 3′ [11] **C** 5′ [12] **C** signs and degrees dam.

**Table 40: Latitude and first station of Mars**

*Sources*: **F** fol. 82r, **H** fol. 74r, **C** fols 79r and 79v, **$C_1$** fols 45r and 45v, **$C_2$** -, **Y** fol. 301r, **L** fol. 73r, **B** p. 141. The table of the planetary latitudes in **$CC_1$** is also separately edited on p. 215.

Latitude of Mars and its First Station

| | | column of the argument | | north | south | minutes of shares | first station |
|---|---|---|---|---|---|---|---|
| | | s ° | s ° | ° ′ | ° ′ | | s ° ′ |
| Forty-seven degrees are added to its true centrum, then it is used for the latitude and the station. | upper half | $0^s$ 6 | $11^s$ 24 | 0; 8 | 0; 4 | 60 | $5^s$ 7;35 |
| | | $0^s$ 12 | $11^s$ 18 | 0; 9 | 0; 4 | 59 | $5^s$ 7;42 |
| | | $0^s$ 18 | $11^s$ 12 | 0;11 | 0; 5 | 57 | $5^s$ 7;51 |
| | | $0^s$ 24 | $11^s$ 6 | 0;13 | 0; 6 | 55 | $5^s$ 8; 2 |
| | | $1^s$ 0 | $11^s$ 0 | 0;14 | 0; 7 | 52[1] | $5^s$ 8;15 |
| | | $1^s$ 6 | $10^s$ 24 | 0;16 | 0; 9 | 48[2] | $5^s$ 8;32 |
| | | $1^s$ 12 | $10^s$ 18 | 0;18 | 0;12 | 44 | $5^s$ 8;53 |
| | | $1^s$ 18 | $10^s$ 12 | 0;21 | 0;15 | 40 | $5^s$ 9;19[3] |
| | | $1^s$ 24 | $10^s$ 6 | 0;24[4] | 0;18 | 35 | $5^s$ 9;48 |
| | | $2^s$ 0 | $10^s$ 0 | 0;28 | 0;22[5]↓ | 30 | $5^s$ 10;21 |
| | | $2^s$ 6 | $9^s$ 24 | 0;32[6] | 0;26[7] | 24 | $5^s$ 10;56 |
| | | $2^s$ 12 | $9^s$ 18 | 0;36 | 0;30⊥ | 18 | $5^s$ 11;32 |
| | | $2^s$ 18 | $9^s$ 12 | 0;41 | 0;36 | 12 | $5^s$ 12; 8 |
| | | $2^s$ 24 | $9^s$ 6 | 0;46 | 0;42 | 6 | $5^s$ 12;46 |
| | lower half | $3^s$ 0 | $9^s$ 0 | 0;52 | 0;49 | 0[8]↓ | $5^s$ 13;25 |
| | | $3^s$ 6 | $8^s$ 24 | 0;59 | 0;56 | 6 | $5^s$ 14; 2[9] |
| | | $3^s$ 12 | $8^s$ 18 | 1; 6 | 1; 4 | 12 | $5^s$ 14;37 |
| | | $3^s$ 18 | $8^s$ 12 | 1;14 | 1;13 | 18 | $5^s$ 15;11 |
| | | $3^s$ 24 | $8^s$ 6 | 1;23 | 1;24 | 24 | $5^s$ 15;42 |
| | | $4^s$ 0 | $8^s$ 0 | 1;34 | 1;37 | 30 | $5^s$ 16;11 |
| | | $4^s$ 6 | $7^s$ 24 | 1;46 | 1;53 | 35 | $5^s$ 16;39 |
| | | $4^s$ 12 | $7^s$ 18 | 2; 0 | 2;13[10] | 40 | $5^s$ 17; 6 |
| | | $4^s$ 18 | $7^s$ 12 | 2;16 | 2;35 | 44 | $5^s$ 17;32 |
| | | $4^s$ 24 | $7^s$ 6 | 2;34 | 3; 1 | 48 | $5^s$ 17;57[11] |
| | | $5^s$ 0 | $7^s$ 0 | 2;54 | 3;29 | 52⊥ | $5^s$ 18;21 |
| | | $5^s$ 6 | $6^s$ 24 | 3;15 | 4; 3 | 55 | $5^s$ 18;40[12] |
| | | $5^s$ 12 | $6^s$ 18 | 3;35 | 4;41 | 57 | $5^s$ 18;54 |
| | | $5^s$ 18 | $6^s$ 12 | 3;52 | 5;25 | 59 | $5^s$ 19; 5 |
| | | $5^s$ 24 | $6^s$ 6 | 4; 8 | 6;13[13] | 60 | $5^s$ 19;12 |
| | | $6^s$ 0 | $6^s$ 0 | 4;21[14] | 7; 7[15] | 60 | $5^s$ 19;15[16] |

The variants from the table in **$CC_1$** for the latitudes of all five planets and from the table for the stations have been included here (but the two variants in the arguments only in the table for Saturn).
[1] **B** 12 [2] **B** 43 [3] **F** 59′ [4] **C** 32′ [5] **C** $2^s$ 0–12° (minutes): slide[−1 col.] (i.e., the minutes 35, 29 (miscopied as 39) and 23 from the deviation of Venus were copied into the column for the southern latitude of Mars) [6] **LB** 22′ [7] **C+** 39′ (instead of 29′) [8] **Y** $3^s$ 0–$5^s$ 0°: slide[+2] [9] **F** 52′ [10] **$C_1$** 3′ [11] **CL** 17′ [12] **C** 45′ [13] **H** 53′ [14] **C** minutes dam. [15] **C** degrees dam. [16] **$CC_1$** 5′.

**Table 41: Latitude and first station of Venus**

*Sources*: **F** fol. 82v, **H** fol. 74v, **C** fols 79r and 79v, **$C_1$** fols 45r and 45v, **$C_2$** -, **Y** fol. 301v, **L** fol. 73v, **B** p. 142. The table of the planetary latitudes in **$CC_1$** is also separately edited on p. 215.

Latitude of Venus and its First Station

Forty-eight degrees are added to the true centrum, then it is used for the latitude and the station.

| | column of arguments | | deviation | slant | minutes of shares | first station |
|---|---|---|---|---|---|---|
| | s ° | s ° | ° ′ | ° ′ | | s ° ′ |
| upper half | 0ˢ 6 | 11ˢ 24 | 1; 2 | 0; 8 | 60 | 5ˢ 15;54[1] |
| | 0ˢ 12 | 11ˢ 18 | 1; 1 | 0;16 | 59 | 5ˢ 15;56 |
| | 0ˢ 18 | 11ˢ 12 | 1; 0 | 0;25 | 57 | 5ˢ 15;58 |
| | 0ˢ 24 | 11ˢ 6 | 0;59 | 0;33 | 55 | 5ˢ 16; 0 |
| | 1ˢ 0 | 11ˢ 0 | 0;57 | 0;41 | 52 | 5ˢ 16; 2 |
| | 1ˢ 6 | 10ˢ 24 | 0;55 | 0;49 | 48 | 5ˢ 16; 5 |
| | 1ˢ 12 | 10ˢ 18 | 0;51 | 0;57 | 44 | 5ˢ 16; 9[2] |
| | 1ˢ 18 | 10ˢ 12 | 0;46 | 1; 5 | 40 | 5ˢ 16;14 |
| | 1ˢ 24 | 10ˢ 6 | 0;41 | 1;13 | 35 | 5ˢ 16;21 |
| | 2ˢ 0 | 10ˢ 0 | 0;35 | 1;20[3]↓ | 30[4] | 5ˢ 16;29 |
| | 2ˢ 6 | 9ˢ 24 | 0;29[5] | 1;28 | 24 | 5ˢ 16;37 |
| | 2ˢ 12 | 9ˢ 18 | 0;23[6] | 1;35⊥ | 18 | 5ˢ 16;46 |
| | 2ˢ 18 | 9ˢ 12 | 0;16 | 1;42 | 12 | 5ˢ 16;54 |
| | 2ˢ 24 | 9ˢ 6 | 0; 8 | 1;50 | 6 | 5ˢ 17; 3 |
| lower half | 3ˢ 0 | 9ˢ 0 | 0; 0 | 1;57 | 0 | 5ˢ 17;11 |
| | 3ˢ 6 | 8ˢ 24 | 0;10 | 2; 3 | 6 | 5ˢ 17;18 |
| | 3ˢ 12 | 8ˢ 18 | 0;20 | 2; 9 | 12 | 5ˢ 17;25 |
| | 3ˢ 18 | 8ˢ 12 | 0;32 | 2;15 | 18 | 5ˢ 17;32 |
| | 3ˢ 24 | 8ˢ 6 | 0;45 | 2;20 | 24 | 5ˢ 17;39 |
| | 4ˢ 0 | 8ˢ 0 | 1; 0 | 2;24 | 30 | 5ˢ 17;46 |
| | 4ˢ 6 | 7ˢ 24 | 1;19 | 2;27 | 35 | 5ˢ 17;53 |
| | 4ˢ 12 | 7ˢ 18 | 1;40 | 2;29 | 40 | 5ˢ 17;59 |
| | 4ˢ 18 | 7ˢ 12 | 2; 5 | 2;30 | 44 | 5ˢ 18; 3 |
| | 4ˢ 24 | 7ˢ 6 | 2;32 | 2;28 | 48 | 5ˢ 18; 7 |
| | 5ˢ 0 | 7ˢ 0 | 3; 3 | 2;22 | 52 | 5ˢ 18;10[7]↓ |
| | 5ˢ 6 | 6ˢ 24 | 3;37 | 2;12[8] | 55 | 5ˢ 18;13 |
| | 5ˢ 12 | 6ˢ 18 | 4;14[9] | 1;55 | 57 | 5ˢ 18;15⊥ |
| | 5ˢ 18 | 6ˢ 12 | 4;53 | 1;27 | 59[10] | 5ˢ 18;17[11] |
| | 5ˢ 24 | 6ˢ 6 | 5;36 | 0;48[12] | 60 | 5ˢ 18;19 |
| | 6ˢ 0 | 6ˢ 0 | 6;22 | 0; 0[13] | 60 | 5ˢ 18;20 |

The variants from the table in **$CC_1$** for the latitudes of all five planets and from the table for the stations have been included here (but the two variants in the arguments only in the table for Saturn). [1] **Y** 16′ [2] **L** 26° [3] **C** 2ˢ 0–12° (minutes): slide[+2 cols] (i.e., the minutes 22, 26 and 30 from the southern latitude of Mars were copied into the column for the slant of Venus) [4] **L** 32 [5] **C** 28′ [6] **C** 22′ [7] **C** 5ˢ 0–12°: slide[+1 col.] (i.e., the minutes 21, 45 (mistake for 40) and 14 (54 without dot on *nūn*) were copied from the column for Mars) [8] **C** 15′ [9] **$C_1$** 11′ [10] **CB** 19 [11] **C** 15′ [12] **$C_1$L** 1° [13] **$C_1$** 1°.

**Table 42: Latitude and first station of Mercury**

*Sources*: **F** fol. 83r, **H** fol. 75r, **C** fols 79r and 79v, $\mathbf{C_1}$ fols 45r and 45v, $\mathbf{C_2}$ -, **Y** fol. 302r, **L** fol. 74r, **B** p. 143. The table of the planetary latitudes in $\mathbf{CC_1}$ is also separately edited on p. 215.

Latitude of Mercury and its First Station

| Sixteen degrees are added to the true centrum, then it is used for the latitude and the station. | | column of arguments | | deviation | slant | minutes of shares | first station |
|---|---|---|---|---|---|---|---|
| | | s ° | s ° | ° ′ | ° ′ | | s ° ′ |
| | upper half | $0^s$ 6 | $11^s$ 24 | 1;45 | 0;11 | 60 | $4^s$ 27;10 |
| | | $0^s$ 12 | $11^s$ 18 | 1;44 | 0;22 | 59 | $4^s$ 27; 5 |
| | | $0^s$ 18 | $11^s$ 12 | 1;43 | 0;33[1] | 57 | $4^s$ 26;57 |
| | | $0^s$ 24 | $11^s$ 6 | 1;40 | 0;44 | 55 | $4^s$ 26;48 |
| | | $1^s$ 0 | $11^s$ 0 | 1;36 | 0;55 | 52 | $4^s$ 26;36 |
| | | $1^s$ 6 | $10^s$ 24 | 1;30 | 1; 6 | 48 | $4^s$ 26;22 |
| | | $1^s$ 12 | $10^s$ 18 | 1;23 | 1;16 | 44 | $4^s$ 26; 8 |
| | | $1^s$ 18 | $10^s$ 12 | 1;16 | 1;26 | 40 | $4^s$ 25;52 |
| | | $1^s$ 24 | $10^s$ 6 | 1; 8 | 1;35[2] | 35 | $4^s$ 25;36 |
| | | $2^s$ 0 | $10^s$ 0 | 0;59 | 1;44 | 30 | $4^s$ 25;18 |
| | | $2^s$ 6 | $9^s$ 24 | 0;49 | 1;52[3] | 24 | $4^s$ 25; 4 |
| | | $2^s$ 12 | $9^s$ 18 | 0;38 | 2; 0 | 18 | $4^s$ 24;53 |
| | | $2^s$ 18 | $9^s$ 12 | 0;26 | 2; 7[4] | 12 | $4^s$ 24;45 |
| | | $2^s$ 24 | $9^s$ 6 | 0;13 | 2;14 | 6 | $4^s$ 24;40 |
| | lower half | $3^s$ 0 | $9^s$ 0 | 0; 0 | 2;20 | 0 | $4^s$ 24;38 |
| | | $3^s$ 6 | $8^s$ 24 | 0;15[5]↓ | 2;25 | 6 | $4^s$ 24;36 |
| | | $3^s$ 12 | $8^s$ 18 | 0;31 | 2;28 | 12 | $4^s$ 24;34[6] |
| | | $3^s$ 18 | $8^s$ 12 | 0;48 | 2;29 | 18 | $4^s$ 24;33 |
| | | $3^s$ 24 | $8^s$ 6 | 1; 6[7] | 2;30 | 24 | $4^s$ 24;31 |
| | | $4^s$ 0 | $8^s$ 0 | 1;25 | 2;29 | 30 | $4^s$ 24;30 |
| | | $4^s$ 6 | $7^s$ 24 | 1;45⊥ | 2;26 | 35 | $4^s$ 24;31 |
| | | $4^s$ 12 | $7^s$ 18 | 2; 6 | 2;20 | 40 | $4^s$ 24;32 |
| | | $4^s$ 18 | $7^s$ 12 | 2;27 | 2;11 | 44 | $4^s$ 24;33 |
| | | $4^s$ 24 | $7^s$ 6 | 2;47 | 2; 0 | 48 | $4^s$ 24;34 |
| | | $5^s$ 0 | $7^s$ 0 | 3; 7 | 1;46 | 52 | $4^s$ 24;35 |
| | | $5^s$ 6 | $6^s$ 24 | 3;26 | 1;29 | 55 | $4^s$ 24;36 |
| | | $5^s$ 12 | $6^s$ 18 | 3;42 | 1;10 | 57 | $4^s$ 24;37 |
| | | $5^s$ 18 | $6^s$ 12 | 3;55[8] | 0;48 | 59 | $4^s$ 24;38 |
| | | $5^s$ 24 | $6^s$ 6 | 4; 2 | 0;24 | 60 | $4^s$ 24;39 |
| | | $6^s$ 0 | $6^s$ 0 | 4; 5[9] | 0; 0[10] | 60 | $4^s$ 24;40[11] |

The variants from the table in $\mathbf{CC_1}$ for the latitudes of all five planets and from the table for the stations have been included here (but the two variants in the arguments only in the table for Saturn).
[1] **C** 23′ [2] **F** 36′ [3] **L** 12′ [4] **Y** 14′ [5] $\mathbf{CC_1}$ $3^s$ 6–$4^s$ 6° (degrees): slide[+3] [6] **C** 37′ [7] **C**+ 1°25′ [8] **B** 50′ [9] **C** dam. [10] **C** minutes dam. [11] **F** 46′.

**Table 38–42a: Latitudes of the planets in the Cairo manuscripts**
*Sources*: **C** fol. 79r, **C₁** fol. 45r, **C₂** -. All variants are also included in the preceding editions of the joint tables for planetary latitudes and stations as included in all other manuscripts.

Latitudes of the Planets

| | column of the argument | | Saturn | | Jupiter | | Mars | | Venus | | Mercury | | shares of the latitude |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | north | south | north | south | north | south | devia-tion | slant | devia-tion | slant | |
| | s ° | s ° | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | |
| upper half | 0$^s$ 6 | 11$^s$ 24 | 2; 4 | 2; 2 | 1; 7 | 1; 5 | 0; 8 | 0; 4 | 1; 2 | 0; 8 | 1;45 | 0;11 | 60 |
| | 0$^s$ 12 | 11$^s$ 18 | 2; 5 | 2; 3 | 1; 8 | 1; 6 | 0; 9 | 0; 4 | 1; 1 | 0;16 | 1;44 | 0;22 | 59 |
| | 0$^s$ 18 | 11$^s$ 12 | 2; 6 | 2; 3 | 1; 8 | 1; 6 | 0;11 | 0; 5 | 1; 0 | 0;25 | 1;43 | 0;33[1] | 57 |
| | 0$^s$ 24 | 11$^s$ 6 | 2; 7 | 2; 4 | 1; 9 | 1; 7 | 0;13 | 0; 6 | 0;59 | 0;33 | 1;40 | 0;44 | 55 |
| | 1$^s$ 0 | 11$^s$ 0 | 2; 8 | 2; 5 | 1;10 | 1; 8 | 0;14 | 0; 7 | 0;57 | 0;41 | 1;36 | 0;55 | 52 |
| | 1$^s$ 6 | 10$^s$ 24 | 2;10 | 2; 7 | 1;11 | 1; 9 | 0;16 | 0; 9 | 0;55 | 0;49 | 1;30 | 1; 6 | 48 |
| | 1$^s$ 12 | 10$^s$ 18 | 2;11 | 2; 8 | 1;12[2]↓ | 1;10 | 0;18 | 0;12 | 0;51 | 0;57 | 1;23 | 1;16 | 44 |
| | 1$^s$ 18 | 10$^s$ 12 | 2;12 | 2;10 | 1;13 | 1;11 | 0;21 | 0;15 | 0;46 | 1; 5 | 1;16 | 1;26 | 40 |
| | 1$^s$ 24 | 10$^s$ 6 | 2;14 | 2;12 | 1;14⊥ | 1;13 | 0;24[3] | 0;18 | 0;41 | 1;13 | 1; 8 | 1;35 | 35 |
| | 2$^s$ 0 | 10$^s$ 0 | 2;16 | 2;15 | 1;16 | 1;16 | 0;28 | 0;22[4]↓ | 0;35 | 1;20[5]↓ | 0;59 | 1;44 | 30 |
| | 2$^s$ 6 | 9$^s$ 24 | 2;18 | 2;18 | 1;18 | 1;18 | 0;32 | 0;26[6] | 0;29[7] | 1;28 | 0;49 | 1;52 | 24 |
| | 2$^s$ 12 | 9$^s$ 18 | 2;21 | 2;21 | 1;21 | 1;21 | 0;36 | 0;30⊥ | 0;23[8] | 1;35⊥ | 0;38 | 2; 0 | 18 |
| | 2$^s$ 18 | 9$^s$ 12 | 2;24 | 2;24 | 1;24 | 1;24 | 0;41 | 0;36 | 0;16 | 1;42 | 0;26 | 2; 7 | 12 |
| | 2$^s$ 24 | 9$^s$ 6 | 2;27 | 2;27 | 1;27 | 1;27 | 0;46 | 0;42 | 0; 8 | 1;50 | 0;13 | 2;14 | 6 |
| | 3$^s$ 0 | 9$^s$ 0 | 2;30 | 2;30 | 1;30 | 1;30 | 0;52 | 0;49 | 0; 0 | 1;57 | 0; 0 | 2;20 | 0 |
| lower half | 3$^s$ 6 | 8$^s$ 24[9] | 2;33 | 2;33 | 1;33 | 1;33 | 0;59 | 0;56 | 0;10 | 2; 3 | 0;15[10]↓ | 2;25 | 6 |
| | 3$^s$ 12 | 8$^s$ 18 | 2;36 | 2;36 | 1;36 | 1;36 | 1; 6 | 1; 4 | 0;20 | 2; 9 | 0;31 | 2;28 | 12 |
| | 3$^s$ 18 | 8$^s$ 12 | 2;39 | 2;39 | 1;39 | 1;39 | 1;14 | 1;13 | 0;32 | 2;15 | 0;48 | 2;29 | 18 |
| | 3$^s$ 24 | 8$^s$ 6 | 2;42 | 2;42 | 1;42 | 1;42 | 1;23 | 1;24 | 0;45 | 2;20 | 1; 6[11] | 2;30 | 24 |
| | 4$^s$ 0 | 8$^s$ 0 | 2;45 | 2;45 | 1;45 | 1;45 | 1;34 | 1;37 | 1; 0 | 2;24 | 1;25 | 2;29 | 30 |
| | 4$^s$ 6 | 7$^s$ 24 | 2;47 | 2;48 | 1;47 | 1;48 | 1;46 | 1;53 | 1;19 | 2;27 | 1;45⊥ | 2;26 | 35 |
| | 4$^s$ 12 | 7$^s$ 18 | 2;50 | 2;51 | 1;50 | 1;51 | 2; 0 | 2;13[12] | 1;40 | 2;29 | 2; 6 | 2;20 | 40 |
| | 4$^s$ 18 | 7$^s$ 12 | 2;53 | 2;54[13] | 1;52 | 1;54 | 2;16 | 2;35 | 2; 5 | 2;30 | 2;27 | 2;11 | 44 |
| | 4$^s$ 24 | 7$^s$ 6 | 2;55 | 2;56 | 1;55 | 1;57 | 2;34 | 3; 1 | 2;32 | 2;28 | 2;47 | 2; 0 | 48 |
| | 5$^s$ 0 | 7$^s$ 0 | 2;57 | 2;58 | 1;58 | 2; 0 | 2;54 | 3;29 | 3; 3 | 2;22 | 3; 7 | 1;46 | 52 |
| | 5$^s$ 6 | 6$^s$ 24 | 2;59 | 3; 0 | 2; 0 | 2; 3 | 3;15 | 4; 3 | 3;37 | 2;12[14] | 3;26 | 1;29 | 55 |
| | 5$^s$ 12 | 6$^s$ 18 | 3; 0 | 3; 2 | 2; 2 | 2; 5 | 3;35 | 4;41 | 4;14[15] | 1;55 | 3;42 | 1;10 | 57 |
| | 5$^s$ 18 | 6$^s$ 12 | 3; 1 | 3; 3 | 2; 3 | 2; 6 | 3;52 | 5;25 | 4;53 | 1;27 | 3;55 | 0;48 | 59 |
| | 5$^s$ 24 | 6$^s$ 6 | 3; 2 | 3; 4 | 2; 4[16] | 2; 7 | 4; 8 | 6;13 | 5;36 | 0;48[17]↓ | 4; 2 | 0;24 | 60 |
| | 6$^s$ 0 | 6$^s$ 0 | 3; 2 | 3; 5 | 2; 4[18] | 2; 8 | 4;21 | 7; 7 | 6;22 | 0; 0⊥ | 4; 5 | 0; 0 | 60 |

Nearly half of the values for 6$^s$ 0° in **C** are illegible due to damage to the bottom of the page. [1] **C** 23′ [2] **C** 1$^s$ 12–24°: slide[−1 col.] [3] **C** 32′ [4] **C** 2$^s$ 0–12°: slide[−1 col.] [5] **C** 2$^s$ 0–12° (minutes): slide[+2 col.] [6] **C**+ 39′ (instead of 29′) [7] **C** 28′ [8] **C** 22′ [9] **C** om. ‘8$^s$’ (causing this and the next two arguments to be read with ‘9$^s$’) [10] **CC₁** 3$^s$ 6°–4$^s$ 6° (degrees): slide[+3] [11] **C**+ 1°25′ [12] **C₁** 3′ [13] **C₁** 57′ [14] **C** 15′ [15] **C₁** 11′ [16] **CC₁** 3′ [17] **C₁** 5$^s$ 24°–6$^s$ 0°: 1° [18] **C** 5′.

## Table 43: First and second declination (first third)

*Sources*: **F** fols 83v–84r, **H** fols 75v–76v, **C** fols 80r and 81r, **C₁** fols 11r and 12r, **C₂** -, **Y** fols 302v–303v, **L₁** = **L** fols 74v–75v, **L₂** = **L** fols 32v and 33r, **B** pp. 144–146. In **CC₁** the first and the second declination were tabulated separately. **L** includes both the combined (**L₁**) and the separate (**L₂**) tables, but in the separate table for the second declination no tabular values were filled in.

| First and Second Declination | | | | | |
|---|---|---|---|---|---|
| argu- ment | | first declination | differ- ences | second declination | differ- ences |
| **0** | **5** | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| **6** | **11** | | | | |
| 1 | 29 | 0;24, 0 | 24, 0 | 0;26,11 | 26,11[1] |
| 2 | 28 | 0;48, 0 | 24, 0 | 0;52,22 | 26,11 |
| 3 | 27 | 1;11,59 | 23,59 | 1;18,31 | 26, 9 |
| 4 | 26 | 1;35,57 | 23,58 | 1;44,39 | 26, 8 |
| 5 | 25 | 1;59,54 | 23,57 | 2;10,45 | 26, 6 |
| 6 | 24 | 2;23,49 | 23,55 | 2;36,46 | 26, 1 |
| 7 | 23 | 2;47,40 | 23,51 | 3; 2,44 | 25,58 |
| 8 | 22 | 3;11,30[2] | 23,50 | 3;28,36 | 25,52 |
| 9 | 21 | 3;35,18 | 23,48 | 3;54,25[3] | 25,49 |
| 10 | 20 | 3;59, 2 | 23,44 | 4;20, 6 | 25,41 |
| 11 | 19 | 4;22,42 | 23,40 | 4;45,42 | 25,36 |
| 12 | 18 | 4;46,17 | 23,35 | 5;11,10 | 25,28 |
| 13 | 17 | 5; 9,48[4] | 23,31 | 5;36,31 | 25,21 |
| 14 | 16 | 5;33,16 | 23,28 | 6; 1,44 | 25,13 |
| 15 | 15 | 5;56,37 | 23,21 | 6;26,47 | 25, 3 |
| 16 | 14 | 6;19,53[5] | 23,16 | 6;51,41 | 24,54[6] |
| 17 | 13 | 6;43, 2 | 23, 9 | 7;16,25 | 24,44 |
| 18 | 12 | 7; 6, 5 | 23, 3 | 7;40,58 | 24,33 |
| 19 | 11 | 7;29, 2 | 22,57 | 8; 5,22 | 24,24 |
| 20 | 10 | 7;51,53[7] | 22,51 | 8;29,31 | 24, 9 |
| 21 | 9 | 8;14,36 | 22,43 | 8;53,31[8] | 24, 0 |
| 22 | 8 | 8;37,11 | 22,35 | 9;17,15 | 23,44 |
| 23 | 7 | 8;59,36 | 22,25 | 9;40,47 | 23,32 |
| 24 | 6 | 9;21,55 | 22,19 | 10; 4, 7 | 23,20 |
| 25 | 5 | 9;44, 4 | 22, 9 | 10;27,11 | 23, 4[9] |
| 26 | 4 | 10; 6, 3 | 21,59 | 10;50, 3 | 22,52 |
| 27 | 3 | 10;27,54[10] | 21,51 | 11;12,38 | 22,35 |
| 28 | 2 | 10;49,33 | 21,39 | 11;34,56[11] | 22,18[12] |
| 29 | 1 | 11;11, 3[13] | 21,30 | 11;57, 2 | 22, 6 |
| 30 | 0 | 11;32,22[14] | 21,19 | 12;18,47[15] | 21,45[16] |

In **L₁** the two argument columns are slid down by two rows. [1] **F** 41″ [2] **L₂** 51′ [3] **F** 35″ **C** 14′ [4] **F** 4° [5] **L₂** 59′ [6] **L₁** 14″ [7] **L₂** 11′ [8] **CY** 13′ [9] **F** 3″ [10] **F** 53″ **L₁** 14″ [11] **H** 16″ **C** 36″ [12] **L₁** 12″ [13] **F** 4″ [14] **L₂** 22′ [15] **L₁** om. minutes [16] **L₁** 48″.

**Table 43: First and second declination (second third)**

*Sources*: **F** fols 83v–84r, **H** fols 75v–76v, **C** fols 80r and 81r, **C**$_1$ fols 11r and 12r, **C**$_2$ -, **Y** fols 302v–303v, **L**$_1$ = **L** fols 74v–75v, **L**$_2$ = **L** fols 32v and 33r, **B** pp. 144–146. In **CC**$_1$ the first and the second declination were tabulated separately. **L** includes both the combined and the separated tables, but in the separate table for the second declinations no tabular values were filled in.

| First and Second Declination | | | | | |
|---|---|---|---|---|---|
| argu-ment | | first declination | differ-ences | second declination | differ-ences |
| **1** | **4** | | | | |
| **7** | **10** | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 1 | 29 | 11;53,28 | 21, 6 | 12;40,17 | 21,30 |
| 2 | 28 | 12;14,25 | 20,57[1] | 13; 1,35 | 21,18 |
| 3[2]↓ | 27 | 12;35, 9[3] | 20,44[4] | 13;22,26 | 20,51[5] |
| 4 | 26 | 12;55,40 | 20,31 | 13;43, 5 | 20,39 |
| 5 | 25 | 13;15,59[6] | 20,19 | 14; 3,27 | 20,22 |
| 6 | 24 | 13;36, 4 | 20, 5 | 14;23,27[7] | 20, 0 |
| 7 | 23 | 13;55,55 | 19,51[8] | 14;43,12[9] | 19,45 |
| 8 | 22 | 14;15,35 | 19,40 | 15; 2,40 | 19,28 |
| 9 | 21 | 14;34,59 | 19,24 | 15;21,42 | 19, 2[10] |
| 10 | 20 | 14;54, 6 | 19, 7 | 15;40,28 | 18,46 |
| 11 | 19 | 15;13, 1 | 18,55 | 15;58,57 | 18,29 |
| 12 | 18 | 15;31,41 | 18,40 | 16;17, 0 | 18, 3 |
| 13 | 17 | 15;50, 2 | 18,21 | 16;34,47 | 17,47 |
| 14 | 16 | 16; 8, 9 | 18, 7 | 16;52,14 | 17,27 |
| 15 | 15 | 16;26, 1 | 17,52[11] | 17; 9,18 | 17, 4 |
| 16 | 14 | 16;43,34 | 17,33 | 17;26, 0 | 16,42 |
| 17 | 13 | 17; 0,48 | 17,14 | 17;42,24 | 16,24[12] |
| 18 | 12 | 17;17,49 | 17, 1 | 17;58,29 | 16, 5 |
| 19 | 11 | 17;34,29 | 16,40 | 18;14, 7[13] | 15,38 |
| 20 | 10 | 17;50,50 | 16,21 | 18;29,25 | 15,18[14] |
| 21 | 9 | 18; 6,53 | 16, 3 | 18;44,24 | 14,59 |
| 22 | 8 | 18;22,38 | 15,45[15] | 18;59, 3 | 14,39 |
| 23 | 7 | 18;38, 2[16] | 15,24 | 19;13,14 | 14,11 |
| 24 | 6 | 18;53, 5 | 15, 3 | 19;27, 5 | 13,51 |
| 25 | 5 | 19; 7,50[17] | 14,45[18] | 19;40,36 | 13,31 |
| 26 | 4 | 19;22,15[19] | 14,25[20] | 19;53,47 | 13,11 |
| 27 | 3 | 19;36,19 | 14, 4 | 20; 6,34[21] | 12,47 |
| 28⊥ | 2 | 19;50, 0 | 13,41[22]↓ | 20;18,54[23] | 12,20 |
| 29 | 1 | 20; 3,20 | 13,20 | 20;30,55 | 12, 1 |
| 30 | 0 | 20;16,20 | 13, 0⊥ | 20;42,34[24] | 11,39 |

[1] **L**$_2$ 56″ [2] **C** arguments 3–28° (first declination): slide[+1] [3] **Y** 11″ [4] **Y** 46″ [5] **F** 9″ [6] **C** 53° [7] **F** 24′ **C**$_1$ 28′ [8] **L**$_1$ 11″ [9] **C**$_1$ 52″ [10] **F** 14″ **L**$_1$ 42′ [11] **L**$_1$ 12″ [12] **H** 27″ [13] **L**$_1$**B** 2″ [14] **FHC**$_1$ 22″ **C** 32″ [15] **L**$_2$ 16′ [16] **L**$_1$ 40″ [17] **H** 4′ [18] **L**$_2$ 15′ [19] **L**$_1$**B** 55″ [20] **CC**$_1$ 24″ **L**$_2$ 15′ [21] **C** 19° [22] **L**$_1$ 1$^s$ 28–30°: 15′ [23] **L**$_1$ 57″ [24] **F** 40′ **L**$_1$ 37″.

**Table 43: First and second declination (last third)**

*Sources*: **F** fols 83v–84r, **H** fols 75v–76v, **C** fols 80r and 81r, $\mathbf{C_1}$ fols 11r and 12r, $\mathbf{C_2}$ -, **Y** fols 302v–303v, $\mathbf{L_1}$ = **L** fols 74v–75v, $\mathbf{L_2}$ = **L** fols 32v and 33r, **B** pp. 144–146. In $\mathbf{CC_1}$ the first and the second declination were tabulated separately. **L** includes both the combined and the separated tables, but in the separate table for the second declinations no tabular values were filled in.

| First and Second Declination | | | | | |
|---|---|---|---|---|---|
| argu-ment | | first declination | differ-ences | second declination | differ-ences |
| **2** | **3** | | | | |
| **8** | **9** | ° ′ ″ | ′ ″ | ° ′ ″ | ′ ″ |
| 1 | 29 | 20;28,57[1]↓ | 12,37 | 20;53,54 | 11,20 |
| 2 | 28 | 20;41,11 | 12,14 | 21; 4,48 | 10,54 |
| 3 | 27 | 20;53, 1⊥ | 11,50 | 21;15,16 | 10,28 |
| 4 | 26 | 21; 4,29 | 11,28 | 21;25,22 | 10, 6 |
| 5 | 25 | 21;15,35 | 11, 6 | 21;35, 8 | 9,46 |
| 6 | 24 | 21;26,17 | 10,42 | 21;44,32 | 9,24 |
| 7 | 23 | 21;36,34 | 10,17 | 21;53,35 | 9, 3 |
| 8 | 22 | 21;46,27 | 9,53[2] | 22; 2,14 | 8,39 |
| 9 | 21 | 21;55,54 | 9,27 | 22;10,25 | 8,11 |
| 10 | 20 | 22; 4,59 | 9, 5 | 22;18,15 | 7,50 |
| 11 | 19 | 22;13,37 | 8,38 | 22;25,42 | 7,27 |
| 12 | 18 | 22;21,53 | 8,16 | 22;32,48 | 7, 6 |
| 13 | 17 | 22;29,42 | 7,49 | 22;39,32 | 6,44 |
| 14 | 16 | 22;37, 5 | 7,23 | 22;45,52[3] | 6,20 |
| 15 | 15 | 22;44, 1 | 6,56 | 22;51,51 | 5,59 |
| 16 | 14 | 22;50,32 | 6,31 | 22;57,27 | 5,36 |
| 17 | 13 | 22;56,35 | 6, 3 | 23; 2,38 | 5,11 |
| 18 | 12 | 23; 2,14 | 5,39 | 23; 7,23 | 4,45[4] |
| 19 | 11 | 23; 7,27 | 5,13[5] | 23;11,47[6] | 4,24 |
| 20 | 10 | 23;12,14 | 4,47[7] | 23;15,48 | 4, 1 |
| 21 | 9 | 23;16,32 | 4,18 | 23;19,27 | 3,39 |
| 22 | 8 | 23;20,25 | 3,53 | 23;22,42 | 3,15 |
| 23 | 7 | 23;23,50 | 3,25 | 23;25,35 | 2,53 |
| 24 | 6 | 23;26,47 | 2,57 | 23;28, 4 | 2,29 |
| 25 | 5 | 23;29,18 | 2,31 | 23;30,10 | 2, 6 |
| 26 | 4 | 23;31,22 | 2, 4[8] | 23;31,52 | 1,42 |
| 27 | 3 | 23;32,58[9] | 1,36 | 23;33,15 | 1,23 |
| 28 | 2 | 23;34, 5 | 1, 7[10] | 23;34,13 | 0,58 |
| 29 | 1 | 23;34,47 | 0,42 | 23;34,47[11] | 0,34 |
| 30 | 0 | 23;35, 0 | 0,13 | 23;35, 0 | 0,13 |

[1] $\mathbf{L_2}$ $2^s$ 1–3°: 21° [2] $\mathbf{L_1}$ 13″ [3] **C** 44′12″ [4] **F** 44″ [5] **C** 33″ [6] **F** 46″ [7] **H** 46″ (corrected in black by a different hand) [8] $\mathbf{L_1}$ 30″ [9] $\mathbf{L_1}$ 18″ [10] $\mathbf{L_1}$ 50″ [11] **H** om. minutes.

**Table 44: Tangent of first declination**
*Sources*: **F** fol. 84r, **H** fol. 77r, **C** fol. 80v, $\mathbf{C_1}$ fol. 11v, $\mathbf{C_2}$ -, **Y** fol. 304r, **L** fol. 76r, **B** p. 147, **r** = recomputation.

| Tangent of the First Declination | | | | | |
|---|---|---|---|---|---|
| ecliptic degrees | tangent of declination p ′ ″ | ecliptic degrees | tangent of declination p ′ ″ | ecliptic degrees | tangent of declination p ′ ″ |
| 1 | 0;25, 8[1] | 31 | 12;38, 7 | 61 | 22;24,50[2] |
| 2 | 0;50,17 | 32 | 13; 1, 2[3] | 62 | 22;39,26 |
| 3 | 1;15,25[4] | 33 | 13;23,49 | 63 | 22;53,34 |
| 4 | 1;40,31 | 34 | 13;46,22[5] | 64 | 23; 7,19 |
| 5 | 2; 5,37[6] | 35 | 14; 8,49 | 65 | 23;20,45 |
| 6 | 2;30,42[7] | 36 | 14;31, 5 | 66 | 23;33,43[8] |
| 7 | 2;55,44 | 37 | 14;53, 6 | 67 | 23;46, 9 |
| 8 | 3;20,44[9] | 38 | 15;15, 1[10] | 68 | 23;58, 7 |
| 9 | 3;45,46[11] | 39 | 15;36,41 | 69 | 24; 9,32 |
| 10 | 4;10,42 | 40 | 15;58, 2 | 70 | 24;20,36[12] |
| 11 | 4;35,38 | 41 | 16;19,18 | 71 | 24;31,12 |
| 12 | 5; 0,31 | 42 | 16;40,21 | 72 | 24;41,21 |
| 13 | 5;25,20[13] | 43 | 17; 1, 2 | 73 | 24;50,57 |
| 14 | 5;50, 7[14] | 44 | 17;21,34 | 74 | 25; 0, 0 |
| 15 | 6;14,47[15] | 45 | 17;41,55 | 75 | 25; 8,30 |
| 16 | 6;39,26 | 46 | 18; 1,54 | 76 | 25;16,30 |
| 17 | 7; 4, 2 | 47 | 18;21,32 | 77 | 25;23,56 |
| 18 | 7;28,30 | 48 | 18;41, 4[16] | 78 | 25;30,53 |
| 19 | 7;52,56 | 49 | 19; 0,19[17] | 79 | 25;37,24 |
| 20 | 8;17,17 | 50 | 19;19,10 | 80 | 25;43,20 |
| 21 | 8;41,34 | 51 | 19;37,44 | 81 | 25;48,42[18] |
| 22 | 9; 5,45 | 52 | 19;56, 4 | 82 | 25;53,32 |
| 23 | 9;29,46 | 53 | 20;14, 0 | 83 | 25;57,47[19] |
| 24 | 9;53,42 | 54 | 20;31,32 | 84 | 26; 1,28 |
| 25 | 10;17,34 | 55 | 20;48,49 | 85 | 26; 4,35 |
| 26 | 10;41,20 | 56 | 21; 5,48[20] | 86 | 26; 7,10 |
| 27 | 11; 4,59 | 57 | 21;22,22 | 87 | 26; 9, 9 |
| 28 | 11;28,26 | 58 | 21;38,30 | 88 | 26;10,33 |
| 29 | 11;51,48 | 59 | 21;54,16[21] | 89 | 26;11,26 |
| 30 | 12;15, 3 | 60 | 22; 9,47 | 90 | 26;11,40[22] |

[1] **L** 55′ [2] **Y** 23′ **YLB** 44″ [3] **LB** 7″ [4] **C** 55′ [5] **Y** 16′ [6] **F** 40′ **C** 33″ [7] **F** 44″ [8] **C** 34′ [9] **HYLBr** 46″ [10] **F** 5′ [11] **Y** 16″ [12] **Y** 9′ [13] **L** 55′ [14] **F** 6° **LB** 15″ [15] **F** 46″ [16] **F** 3″ [17] **L** 18ᵖ [18] **H** 12″ [19] **LB** 46″ [20] **Y** 18″ [21] **F** 56″ [22] **YLB** 41″.

**Table 45: Right ascension with less accurate values (first half)**

*Sources*: **F** fol. 84v, **H** fols 77v–78r (in one block of nine and one block of three signs), **C** fol. 81v, **C$_1$** fol. 12v, **C$_2$** -. **YLB** include the more accurate table here edited on pp. 222–223.

| Ascensions of the Zodiacal Signs at the Equator | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Aries | Taurus | Gemini | Cancer | Leo | Virgo |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 0;55 | 28;51 | 58;50 | 91; 5 | 123;15 | 153; 4 |
| 2 | 1;50 | 29;49 | 59;53 | 92;11 | 124;17[1] | 154; 1 |
| 3 | 2;45 | 30;46 | 60;56 | 93;16 | 125;19[2] | 154;58 |
| 4 | 3;40 | 31;44 | 61;59 | 94;21 | 126;21 | 155;54 |
| 5 | 4;35 | 32;42 | 63; 2 | 95;27 | 127;23 | 156;51 |
| 6 | 5;30 | 33;40 | 64; 5 | 96;32 | 128;25 | 157;48 |
| 7 | 6;25 | 34;39 | 65; 9 | 97;38 | 129;26 | 158;44 |
| 8 | 7;20 | 35;37 | 66;13 | 98;43 | 130;27 | 159;40 |
| 9 | 8;15 | 36;36 | 67;17 | 99;49[3] | 131;28 | 160;36 |
| 10 | 9;11 | 37;35 | 68;21 | 100;54 | 132;28 | 161;32 |
| 11 | 10; 6 | 38;33 | 69;25 | 102; 0 | 133;29 | 162;29 |
| 12 | 11; 1 | 39;32 | 70;29 | 103; 5 | 134;30 | 163;25 |
| 13 | 11;57 | 40;32 | 71;33 | 104; 9 | 135;30 | 164;21 |
| 14 | 12;52 | 41;31 | 72;38 | 105;14 | 136;29 | 165;16 |
| 15 | 13;48 | 42;31 | 73;42 | 106;18 | 137;29 | 166;12 |
| 16 | 14;44 | 43;31 | 74;46 | 107;22 | 138;29 | 167; 8 |
| 17 | 15;39 | 44;30 | 75;51 | 108;27[4] | 139;28 | 168; 3 |
| 18 | 16;35 | 45;30 | 76;55 | 109;31 | 140;28 | 168;59 |
| 19 | 17;31 | 46;31 | 78; 0 | 110;35 | 141;27 | 169;54 |
| 20 | 18;28 | 47;32 | 79; 6 | 111;39 | 142;25 | 170;49 |
| 21 | 19;24 | 48;32 | 80;11 | 112;43 | 143;24 | 171;45 |
| 22 | 20;20 | 49;33 | 81;17 | 113;47 | 144;23 | 172;40 |
| 23 | 21;16 | 50;34 | 82;22 | 114;51 | 145;21 | 173;35 |
| 24 | 22;12 | 51;35 | 83;28 | 115;55 | 146;20 | 174;30 |
| 25 | 23; 9 | 52;37 | 84;33 | 116;58 | 147;18 | 175;25[5] |
| 26 | 24; 6[6] | 53;39 | 85;39 | 118; 1 | 148;16 | 176;20 |
| 27 | 25; 2 | 54;41 | 86;44 | 119; 4 | 149;14 | 177;15 |
| 28 | 25;59 | 55;43 | 87;49 | 120; 7 | 150;11 | 178;10 |
| 29 | 26;56 | 56;45 | 88;55 | 121;10 | 151; 9 | 179; 5 |
| 30 | 27;53 | 57;47 | 90; 0 | 122;13 | 152; 7 | 180; 0[7] |

**CC$_1$** indicate the zodiacal signs with *abjad* numerals. [1] **C$_1$** 16′ [2] **C$_1$** 29′ [3] **F** 59′ [4] **CC$_1$** 26′ [5] **F** 24′ [6] **C** minutes ill. [7] **C** degrees dam.

**Table 45: Right ascension with less accurate values (second half)**
*Sources*: **F** fol. 84v, **H** fols 77v–78r (in one block of nine and one block of three signs), **C** fol. 81v, **C**$_1$ fol. 12v, **C**$_2$ -. **YLB** include the more accurate table here edited on pp. 222–223.

| Ascensions of the Zodiacal Signs at the Equator | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Libra | Scorpio | Sagittarius | Capricorn | Aquarius | Pisces |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 180;55 | 208;51 [1] | 238;50 | 271; 5 | 303;15 | 333; 4 |
| 2 | 181;50 | 209;49 | 239;53 | 272;11 | 304;17 | 334; 1 |
| 3 | 182;45 | 210;46 | 240;56 | 273;16 | 305;19 | 334;58 [2] |
| 4 | 183;40 | 211;44 | 241;59 | 274;21 | 306;21 | 335;54 |
| 5 | 184;35 | 212;42 | 243; 2 [3] | 275;27 | 307;23 | 336;51 |
| 6 | 185;30 | 213;40 | 244; 5 | 276;32 | 308;25 | 337;48 |
| 7 | 186;25 | 214;39 | 245; 9 | 277;38 | 309;26 | 338;44 |
| 8 | 187;20 | 215;37 | 246;13 | 278;43 | 310;27 | 339;40 |
| 9 | 188;15 | 216;36 | 247;17 | 279;49 | 311;28 | 340;36 |
| 10 | 189;11 | 217;35 | 248;21 | 280;54 | 312;28 | 341;32 |
| 11 | 190; 6 [4] | 218;33 | 249;25 | 282; 0 | 313;29 | 342;29 |
| 12 | 191; 1 | 219;32 | 250;29 [5] | 283; 5 | 314;30 [6] | 343;25 |
| 13 | 191;57 | 220;32 | 251;33 | 284; 9 | 315;30 | 344;21 |
| 14 | 192;52 | 221;31 | 252;38 | 285;14 | 316;29 | 345;16 |
| 15 | 193;48 | 222;31 | 253;42 [7] | 286;18 | 317;29 | 346;12 |
| 16 | 194;44 | 223;31 | 254;46 | 287;22 | 318;29 | 347; 8 |
| 17 | 195;39 | 224;30 | 255;51 | 288;27 [8] | 319;28 | 348; 3 |
| 18 | 196;35 | 225;30 | 256;55 | 289;31 | 320;28 | 348;59 |
| 19 | 197;31 | 226;31 | 258; 0 | 290;35 | 321;27 | 349;54 |
| 20 | 198;28 | 227;32 | 259; 6 | 291;39 | 322;25 | 350;49 |
| 21 | 199;24 [9] | 228;32 | 260;11 | 292;43 | 323;24 | 351;45 |
| 22 | 200;20 | 229;33 | 261;17 | 293;47 | 324;23 | 352;40 |
| 23 | 201;16 | 230;34 | 262;22 | 294;51 | 325;21 | 353;35 |
| 24 | 202;12 | 231;35 | 263;28 | 295;55 | 326;20 [10] | 354;30 |
| 25 | 203; 9 | 232;37 | 264;33 | 296;58 | 327;18 | 355;25 [11] |
| 26 | 204; 6 | 233;39 | 265;39 | 298; 1 [12] | 328;16 | 356;20 [13] |
| 27 | 205; 2 | 234;41 | 266;44 [14] | 299; 4 [15] | 329;14 | 357;15 |
| 28 | 205;59 | 235;43 | 267;49 | 300; 7 | 330;11 | 358;10 |
| 29 | 206;56 | 236;45 | 268;55 | 301;10 | 331; 9 | 359; 5 |
| 30 | 207;53 | 237;47 [16] | 270; 0 | 302;13 [17] | 332; 7 [18] | 360; 0 |

**CC**$_1$ indicate the zodiacal signs with *abjad* numerals. [1] **C** 11′ [2] **F** 53′ [3] **C** 246° [4] **C** 187° [5] **C** 19′ [6] **C** om. minutes [7] **C** 218° [8] **CC**$_1$ 26′ (consistent with the deviation for $3^s$ 17°) [9] **C** 149° [10] **C** 30′ [11] **F** 24′ [12] **C** 295° [13] **C** 307° (سز) [14] **C**$_1$ 41′ [15] **C** 249° [16] **H** 15′ [17] **H** 3′ [18] **C** 50′.

**Table 45a: Right ascension with more accurate values (first half)**
*Sources*: **Y** fols 304v–305r, **L** fols 76v–77r, **B** pp. 148–149. **FHCC$_1$** include the less accurate right ascension table here edited on pp. 220–221.

| Ascensions of the Zodiacal Signs at the Equator | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Aries | Taurus | Gemini | Cancer | Leo | Virgo |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 0;55 | 28;50 | 58;50 | 91; 6 | 123;15 | 153; 4 |
| 2 | 1;50 | 29;48 | 59;53 | 92;11[1] | 124;17 | 154; 1 |
| 3 | 2;45 | 30;46 | 60;56 | 93;17 | 125;19 | 154;58 |
| 4 | 3;40 | 31;43 | 61;59 | 94;22 | 126;21 | 155;55 |
| 5 | 4;35 | 32;41 | 63; 2 | 95;28 | 127;23 | 156;52 |
| 6 | 5;30 | 33;40 | 64; 5 | 96;33 | 128;24 | 157;48 |
| 7 | 6;25 | 34;38 | 65; 9 | 97;38 | 129;26 | 158;45[2] |
| 8 | 7;20 | 35;36 | 66;13 | 98;43[3] | 130;27 | 159;41[4] |
| 9 | 8;16 | 36;35 | 67;17 | 99;48 | 131;28 | 160;37 |
| 10 | 9;11 | 37;34 | 68;21 | 100;53 | 132;29 | 161;33 |
| 11 | 10; 6 | 38;33 | 69;25 | 101;58 | 133;30 | 162;29 |
| 12 | 11; 1 | 39;32 | 70;29 | 103; 3 | 134;30 | 163;24 |
| 13 | 11;57 | 40;31 | 71;33 | 104; 8 | 135;30 | 164;21 |
| 14 | 12;52 | 41;30 | 72;37 | 105;13 | 136;30 | 165;17 |
| 15 | 13;48 | 42;30 | 73;42 | 106;18 | 137;30 | 166;12 |
| 16 | 14;43 | 43;30 | 74;47 | 107;23 | 138;30 | 167; 8 |
| 17 | 15;39 | 44;30 | 75;52 | 108;27[5] | 139;29 | 168; 3 |
| 18 | 16;35 | 45;30 | 76;57 | 109;31 | 140;28 | 168;59 |
| 19 | 17;31 | 46;31 | 78; 2 | 110;35 | 141;27 | 169;54 |
| 20 | 18;27 | 47;31 | 79; 7 | 111;39 | 142;26 | 170;49 |
| 21 | 19;23 | 48;32 | 80;12[6] | 112;43 | 143;25 | 171;44 |
| 22 | 20;19 | 49;33 | 81;17 | 113;47 | 144;24[7] | 172;40 |
| 23 | 21;15 | 50;34 | 82;22 | 114;51 | 145;22 | 173;35 |
| 24 | 22;12 | 51;36 | 83;27 | 115;55 | 146;20 | 174;30 |
| 25 | 23; 8 | 52;37 | 84;32 | 116;58 | 147;19 | 175;25 |
| 26 | 24; 5 | 53;39 | 85;38[8] | 118; 1 | 148;17 | 176;20 |
| 27 | 25; 2 | 54;41 | 86;43 | 119; 4[9] | 149;14 | 177;15 |
| 28 | 25;59 | 55;43 | 87;49 | 120; 7 | 150;12 | 178;10 |
| 29 | 26;56 | 56;45 | 88;54 | 121;10 | 151;10 | 179; 5 |
| 30 | 27;53 | 57;47 | 90; 0 | 122;13[10] | 152; 7 | 180; 0 |

[1] **B** 14′ [2] **Y** 118° [3] **B** 13′ [4] **Y** 11′ [5] **B** 24′ [6] **B** 52′ [7] **L** 25′ [8] **Y** 33′ [9] **Y** 6′ [10] **B** 3′.

**Table 45a: Right ascension with more accurate values (second half)**
*Sources*: **Y** fols 304v–305r, **L** fols 76v–77r, **B** pp. 148–149. **FHCC$_1$** include the less accurate right ascension table here edited on pp. 220–221.

| Ascensions of the Zodiacal Signs at the Equator | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Libra | Scorpio | Sagittarius | Capricorn | Aquarius | Pisces |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 180;55 | 208;50 | 238;50 | 271; 6 | 303;15 | 333; 4 |
| 2 | 181;50 | 209;48 | 239;53 | 272;11 | 304;17 | 334; 1 |
| 3 | 182;45 | 210;46 | 240;56 | 273;17 | 305;19 | 334;58 |
| 4 | 183;40 | 211;43[1] | 241;59 | 274;22 | 306;21 | 335;55 |
| 5 | 184;35 | 212;41 | 243; 2 | 275;28 | 307;23 | 336;52 |
| 6 | 185;30 | 213;40 | 244; 5 | 276;33 | 308;24 | 337;48 |
| 7 | 186;25 | 214;38 | 245; 9 | 277;38 | 309;26 | 338;45 |
| 8 | 187;20 | 215;36 | 246;13 | 278;43 | 310;27 | 339;41 |
| 9 | 188;16 | 216;35 | 247;17 | 279;48 | 311;28 | 340;37 |
| 10 | 189;11 | 217;34 | 248;21 | 280;53 | 312;29 | 341;33 |
| 11 | 190; 6 | 218;33 | 249;25 | 281;58 | 313;29 | 342;29 |
| 12 | 191; 1 | 219;32 | 250;29 | 283; 3 | 314;30 | 343;24[2] |
| 13 | 191;57 | 220;31 | 251;33 | 284; 8 | 315;30 | 344;21 |
| 14 | 192;52 | 221;30 | 252;37[3] | 285;13 | 316;30 | 345;17 |
| 15 | 193;48 | 222;30 | 253;42[4] | 286;18 | 317;30 | 346;12 |
| 16 | 194;43 | 223;30 | 254;47 | 287;23 | 318;30 | 347; 8 |
| 17 | 195;39 | 224;30 | 255;52 | 288;27 | 319;29 | 348; 3 |
| 18 | 196;35 | 225;30 | 256;57 | 289;31 | 320;28 | 348;59[5] |
| 19 | 197;31 | 226;31 | 258; 2 | 290;35 | 321;27[6] | 349;54 |
| 20 | 198;27 | 227;31 | 259; 7 | 291;39 | 322;26 | 350;49 |
| 21 | 199;23 | 228;32 | 260;12 | 292;43 | 323;25 | 351;44 |
| 22 | 200;19 | 229;33 | 261;16[7] | 293;47 | 324;24 | 352;40 |
| 23 | 201;15 | 230;34 | 262;22 | 294;51 | 325;22 | 353;35 |
| 24 | 202;12 | 231;36 | 263;27 | 295;55 | 326;20 | 354;30 |
| 25 | 203; 8 | 232;37 | 264;32 | 296;58 | 327;19 | 355;25 |
| 26 | 204; 5 | 233;39 | 265;38 | 298; 1 | 328;17 | 356;20 |
| 27 | 205; 2 | 234;41 | 266;43 | 299; 4 | 329;14 | 357;15 |
| 28 | 205;59 | 235;43 | 267;49 | 300; 7 | 330;12 | 358;10 |
| 29 | 206;56 | 236;45 | 268;54 | 301;10 | 331;10 | 359; 5 |
| 30 | 207;53 | 237;47 | 270; 0 | 302;13 | 332; 7 | 360; 0 |

[1] **Y** 44′ [2] **L** 27′ [3] **Y** 34′ [4] **L** 213° [5] **L** 349° [6] **L** 26′ [7] **LB** 17′.

**Table 46: Oblique ascension for 36° (first half)**
*Source*: **F** fol. 85r, **H** fols 78v–79r (in one block of nine and one block of three signs). **CYLB** include the table for 35;30° that is here edited on pp. 226–227.

| Ascensions of the Zodiacal Signs for Latitude 36° | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Aries | Taurus | Gemini | Cancer | Leo | Virgo |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 0;37 | 20; 3 | 43; 6 | 72;36 | 107;53 | 144;48 |
| 2 | 1;15 | 20;45 | 43;59 | 73;44 | 109; 6 | 146; 2 |
| 3 | 1;52 | 21;27 | 44;52 | 74;50 | 110;20 | 147;15 |
| 4 | 2;30 | 22; 9 | 45;44 | 75;56 | 111;34 | 148;28 |
| 5 | 3; 7 | 22;51[1] | 46;37 | 77; 4 | 112;47 | 149;41 |
| 6 | 3;45 | 23;33 | 47;30 | 78;10 | 114; 1 | 150;55 |
| 7 | 4;23 | 24;17 | 48;26 | 79;20 | 115;15 | 152; 8 |
| 8 | 5; 0 | 24;59 | 49;22 | 80;28 | 116;29 | 153;21 |
| 9 | 5;38 | 25;43 | 50;18 | 81;38 | 117;44 | 154;34 |
| 10 | 6;17 | 26;27 | 51;13 | 82;47 | 118;57 | 155;46 |
| 11 | 6;54 | 27; 9 | 52; 9 | 83;56 | 120;11 | 157; 0 |
| 12 | 7;32 | 27;53 | 53; 5 | 85; 5 | 121;25 | 158;13 |
| 13 | 8;11 | 28;39 | 54; 3 | 86;15 | 122;39 | 159;26 |
| 14 | 8;49 | 29;23 | 55; 2 | 87;26 | 123;53 | 160;38 |
| 15 | 9;28 | 30; 9 | 56; 0 | 88;36 | 125; 7 | 161;52 |
| 16 | 10; 6 | 30;55 | 56;58[2] | 89;46 | 126;21 | 163; 5 |
| 17 | 10;44 | 31;39 | 57;57 | 90;57 | 127;35 | 164;17 |
| 18 | 11;23 | 32;25 | 58;55 | 92; 7 | 128;49 | 165;30 |
| 19 | 12; 2 | 33;13 | 59;56 | 93;19 | 130; 3 | 166;42 |
| 20 | 12;42 | 34; 1 | 60;59 | 94;31 | 131;17 | 167;55 |
| 21 | 13;22 | 34;48 | 62; 0 | 95;44 | 132;31 | 169; 8 |
| 22 | 14; 1 | 35;35 | 63; 2 | 96;56 | 133;45 | 170;20 |
| 23 | 14;40 | 36;23 | 64; 4 | 98; 8 | 134;59 | 171;33 |
| 24 | 15;19 | 37;11 | 65; 6 | 99;20 | 136;13 | 172;45 |
| 25 | 15;59 | 38; 1 | 66;10 | 100;33 | 137;27 | 173;57 |
| 26 | 16;40 | 38;52 | 67;14 | 101;46 | 138;41 | 175;10 |
| 27 | 17;19 | 39;42 | 68;18 | 103; 0 | 139;55 | 176;22 |
| 28 | 18; 0 | 40;32 | 69;22 | 104;13 | 141; 7 | 177;35 |
| 29 | 18;40 | 41;23 | 70;26 | 105;26 | 142;21 | 178;47[3] |
| 30 | 19;21 | 42;13 | 71;30 | 106;39 | 143;35 | 180; 0 |

[1] **F** 11′ [2] **F** 16° [3] **H** om. minutes.

**Table 46: Oblique ascension for 36° (second half)**
*Source*: **F** fol. 85r, **H** fols 78v–79r (in one block of nine and one block of three signs). **CYLB** include the table for 35;30° that is here edited on pp. 226–227.

| Ascensions of the Zodiacal Signs for Latitude 36° | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Libra | Scorpio | Sagittarius | Capricorn | Aquarius | Pisces |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 181;13 | 217;39 | 254;34 | 289;34 | 318;37 | 341;20 |
| 2 | 182;25 | 218;53 | 255;47 | 290;38 | 319;28 | 342; 0 |
| 3 | 183;38 | 220; 5 | 257; 0 | 291;42 | 320;18 | 342;41 |
| 4 | 184;50 | 221;19 | 258;14 | 292;46 | 321; 8 | 343;20 |
| 5 | 186; 3 | 222;33 | 259;27 | 293;50 | 321;59 | 344; 1 |
| 6 | 187;15 | 223;46[1] | 260;40 | 294;54 | 322;49 | 344;41 |
| 7 | 188;27 | 225; 1 | 261;52 | 295;56 | 323;37 | 345;20 |
| 8 | 189;40 | 226;15 | 263; 4 | 296;58 | 324;25 | 345;59 |
| 9 | 190;52 | 227;29 | 264;16 | 298; 0 | 325;12 | 346;38 |
| 10 | 192; 5 | 228;43 | 265;29 | 299; 1 | 325;59 | 347;18 |
| 11 | 193;18 | 229;56[2] | 266;41 | 300; 4 | 326;47 | 347;58 |
| 12 | 194;30 | 231;11 | 267;53 | 301; 5 | 327;35 | 348;37 |
| 13 | 195;43 | 232;25 | 269; 3 | 302; 3 | 328;21 | 349;16 |
| 14 | 196;55 | 233;39 | 270;14 | 303; 2 | 329; 5 | 349;54 |
| 15 | 198; 8 | 234;53 | 271;24 | 304; 0 | 329;51[3] | 350;32 |
| 16 | 199;22 | 236; 7 | 272;34 | 304;58 | 330;37[4] | 351;11 |
| 17 | 200;34 | 237;21 | 273;45 | 305;57 | 331;21 | 351;49 |
| 18 | 201;47 | 238;35 | 274;55 | 306;55 | 332; 7 | 352;28 |
| 19 | 203; 0 | 239;49 | 276; 4 | 307;51 | 332;51 | 353; 6 |
| 20 | 204;14 | 241; 3 | 277;13 | 308;47 | 333;33 | 353;43 |
| 21 | 205;26 | 242;16 | 278;22 | 309;42 | 334;17 | 354;22 |
| 22 | 206;39 | 243;31 | 279;32 | 310;38 | 335; 1 | 355; 0 |
| 23 | 207;52 | 244;45 | 280;40 | 311;34[5] | 335;43 | 355;37 |
| 24 | 209; 5 | 245;59 | 281;50 | 312;30 | 336;27 | 356;15 |
| 25 | 210;19 | 247;13 | 282;56 | 313;23 | 337; 9 | 356;53 |
| 26 | 211;32 | 248;26 | 284; 4 | 314;16 | 337;51 | 357;30 |
| 27 | 212;45[6] | 249;40 | 285;10 | 315; 8 | 338;33 | 358; 8 |
| 28 | 213;58 | 250;54 | 286;16 | 316; 1 | 339;15 | 358;44[7] |
| 29 | 215;12 | 252; 7 | 287;24 | 316;54 | 339;57 | 359;23 |
| 30 | 216;25 | 253;21 | 288;30 | 317;47 | 340;39 | 360; 0 |

[1] **H** 47′ [2] **H** 57′ [3] **F** 57′ [4] **F** 31′ [5] **F** 54′ [6] **H** 15′ [7] **H** 45′.

**Table 46a: Oblique ascension for 35°30′ (first half)**

*Sources*: **C** fols 82r–82v (together with the equation of daylight edited on p. 229), $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fols 306r–306v, **L** fols 78r–78v, **B** pp. 151–152 (see Plate 12). **FH** have an oblique ascension table for 36° that is here edited on pp. 224–225.

Ascensions of the Zodiacal Signs for Latitude 35°30′

| equal degrees | Aries | Taurus | Gemini | Cancer | Leo | Virgo |
|---|---|---|---|---|---|---|
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 0;38 | 20;12 | 43;22 | 72;57 | 108;14 | 145; 3 |
| 2 | 1;16 | 20;54 | 44;15 | 74; 2 | 109;28 | 146;16 |
| 3 | 1;53[1] | 21;35 | 45; 7 | 75; 9 | 110;41 | 147;29 |
| 4 | 2;31 | 22;17 | 46; 0 | 76;15 | 111;55 | 148;40 |
| 5 | 3; 9 | 22;59 | 46;53 | 77;23 | 113; 8 | 149;53 |
| 6 | 3;47 | 23;42 | 47;48 | 78;31 | 114;23 | 151; 7 |
| 7 | 4;25 | 24;26 | 48;43 | 79;39 | 115;37 | 152;20 |
| 8 | 5; 4 | 25; 9 | 49;39 | 80;49 | 116;50 | 153;32 |
| 9 | 5;42 | 25;53 | 50;34 | 81;57 | 118; 4 | 154;45[2] |
| 10 | 6;20 | 26;36 | 51;29 | 83; 6 | 119;18[3] | 155;57 |
| 11 | 6;58[4] | 27;21 | 52;26 | 84;17 | 120;32[5] | 157;10 |
| 12 | 7;37 | 28; 6 | 53;24[6] | 85;27 | 121;46[7] | 158;23 |
| 13 | 8;15[8] | 28;50 | 54;21 | 86;36 | 122;59 | 159;36 |
| 14 | 8;54 | 29;35 | 55;19 | 87;46 | 124;13 | 160;47 |
| 15 | 9;32 | 30;20 | 56;17 | 88;57 | 125;26[9] | 162; 0 |
| 16 | 10;11 | 31; 7[10] | 57;16 | 90; 7 | 126;39 | 163;12 |
| 17 | 10;50 | 31;53 | 58;16 | 91;19 | 127;54 | 164;25 |
| 18 | 11;29 | 32;40 | 59;17 | 92;32 | 129;10 | 165;37 |
| 19 | 12; 8 | 33;26 | 60;17 | 93;42 | 130;23 | 166;48 |
| 20 | 12;47 | 34;13 | 61;18 | 94;54[11] | 131;34[12] | 168; 0 |
| 21 | 13;27[13] | 35; 2 | 62;19 | 96; 6 | 132;49 | 169;12 |
| 22 | 14; 6 | 35;50 | 63;21 | 97;17 | 134; 3 | 170;24 |
| 23 | 14;46 | 36;39[14] | 64;23 | 98;29 | 135;16 | 171;35 |
| 24 | 15;25 | 37;27 | 65;25 | 99;42 | 136;30 | 172;47 |
| 25 | 16; 5 | 38;16 | 66;27 | 100;53[15] | 137;43 | 173;59 |
| 26 | 16;46 | 39; 7 | 67;31 | 102; 6 | 138;55 | 175;11 |
| 27 | 17;27 | 39;57 | 68;35 | 103;21 | 140; 9 | 176;23 |
| 28 | 18; 8 | 40;48 | 69;40 | 104;35 | 141;24 | 177;36[16] |
| 29 | 18;49 | 41;38 | 70;45 | 105;48 | 142;36[17] | 178;48 |
| 30 | 19;30 | 42;29 | 71;51 | 107; 1[18] | 143;50[19] | 180; 0[20] |

**C** indicates the zodiacal signs with *abjad* numerals. [1] **CL** 13′ [2] **C** 35′ [3] **C** 15′ [4] **B** 53′ [5] **C** 30′ [6] **CY** 13° **C** 27′ [7] **C** 45′ [8] **C** 14′ [9] **F** 27′ [10] **C** 50′ [11] **C** 13′ [12] **LB** 36′ [13] **Y** 26′ [14] **C** 9′ [15] **C** 6;13 [16] **B** 16′ [17] **B** 16′. [18] **C** dam. [19] **C** dam. [20] **C** dam.

**Table 46a: Oblique ascension for 35°30′ (second half)**

*Sources*: **C** fols 82r–82v (together with the equation of daylight edited on p. 229), $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fols 306r–306v, **L** fols 78r–78v, **B** pp. 151–152. **FH** have an oblique ascension table for 36° that is here edited on pp. 224–225.

| Ascensions of the Zodiacal Signs for Latitude 35°30′ | | | | | | |
|---|---|---|---|---|---|---|
| equal degrees | Libra | Scorpio | Sagittarius | Capricorn | Aquarius | Pisces |
| | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 1 | 181;12 | 217;24 | 254;12 | 289;15 | 318;22 | 341;11 |
| 2 | 182;24 | 218;36 | 255;25 | 290;20 | 319;12 | 341;52 |
| 3 | 183;37 | 219;51 | 256;39 | 291;25 | 320; 3 | 342;33[1] |
| 4 | 184;49 | 221; 5 | 257;54 | 292;29 | 320;53[2] | 343;14 |
| 5 | 186; 1 | 222;17 | 259; 7 | 293;33 | 321;44[3] | 343;55[4] |
| 6 | 187;13 | 223;30 | 260;18 | 294;35 | 322;33 | 344;35 |
| 7 | 188;25 | 224;44 | 261;31 | 295;37 | 323;21 | 345;14 |
| 8 | 189;36 | 225;57 | 262;43 | 296;39 | 324;10 | 345;54 |
| 9 | 190;48 | 227;11 | 263;54 | 297;41 | 324;58[5] | 346;33 |
| 10 | 192; 0 | 228;26 | 265; 6 | 298;42 | 325;46[6] | 347;13 |
| 11 | 193;12 | 229;37 | 266;18 | 299;43 | 326;34 | 347;52 |
| 12 | 194;23[7] | 230;50 | 267;28 | 300;43 | 327;20 | 348;31 |
| 13 | 195;35 | 232; 6[8] | 268;41[9] | 301;44 | 328; 7 | 349;10 |
| 14 | 196;48 | 233;21 | 269;53 | 302;44 | 328;53 | 349;49[10] |
| 15 | 198; 0 | 234;34[11] | 271; 3 | 303;43 | 329;40 | 350;28 |
| 16 | 199;13 | 235;47 | 272;14 | 304;41 | 330;25 | 351; 6 |
| 17 | 200;24 | 237; 1 | 273;24 | 305;39 | 331;10 | 351;45 |
| 18 | 201;37 | 238;14 | 274;33 | 306;36 | 331;54 | 352;23[12] |
| 19 | 202;50 | 239;28 | 275;43 | 307;34 | 332;39 | 353; 2[13] |
| 20 | 204; 3 | 240;42 | 276;54[14] | 308;31 | 333;24 | 353;40 |
| 21 | 205;15[15] | 241;56 | 278; 3 | 309;26 | 334; 7 | 354;18[16] |
| 22 | 206;28 | 243;10 | 279;11 | 310;21 | 334;51 | 354;56 |
| 23 | 207;40 | 244;23 | 280;21 | 311;17 | 335;32[17] | 355;35 |
| 24 | 208;53 | 245;37 | 281;29 | 312;12 | 336;18 | 356;13 |
| 25 | 210; 7 | 246;52 | 282;37 | 313; 7 | 337; 1 | 356;51 |
| 26 | 211;20 | 248; 5 | 283;45 | 314; 0[18] | 337;43 | 357;29 |
| 27 | 212;31 | 249;19[19] | 284;51 | 314;53 | 338;25 | 358; 7[20] |
| 28 | 213;44[21] | 250;32 | 285;58 | 315;45 | 339; 6 | 358;44[22] |
| 29 | 214;57 | 251;46 | 287; 3 | 316;38[23] | 339;48 | 359;22 |
| 30 | 216;10[24] | 252;59[25] | 288; 9 | 317;31[26] | 340;30 | 360; 0 |

**C** indicates the zodiacal signs with *abjad* numerals. [1] **C** 13′ [2] **CB** 13′ [3] **B** 14′ [4] **C** 14′ [5] **CL** 18′ [6] **C** 44′ [7] **C** 28′ [8] **C** 5′ [9] **C** 11′ [10] **C** 19′ [11] **C** 35′ [12] **CY** 312° [13] **CY** 313° [14] **C** 15′ [15] **L** 18′ [16] **Y** 28′ [17] **C** 34′ [18] **C** 354° [19] **C** 59′ [20] **CY** 318° [21] **C** 47′ [22] **CYL** 318° [23] **C** 33′ [24] **C** degrees dam. [25] **C** dam. [26] **C** degrees dam. **Y** 30′.

## Table 47: Maximum equation of daylight for latitudes 16° to 45°
## Table 48: Equation of daylight for latitude 36°

*Sources*: **F** fol. 85v (without title, column headers and tabular values in the table for the maximum equation of daylight), **H** fol. 78r (only the equation of daylight for latitude 36°). **CYLB** have a table for the equation of daylight for latitude 35;30° which is here edited on p. 229.

| ⟨Maximum Equation of Daylight⟩ | | |
|---|---|---|
| ⟨latitude⟩ | ⟨maximum equation⟩ | ⟨differences⟩ |
| | ° ′ ″ | ° ′ |
| 16 | | |
| 17 | | |
| 18 | | |
| 19 | | |
| 20 | | |
| 21 | | |
| 22 | | |
| 23 | | |
| 24 | | |
| 25 | | |
| 26 | | |
| 27 | | |
| 28 | | |
| 29 | | |
| 30 | | |
| 31 | | |
| 32 | | |
| 33 | | |
| 34 | | |
| 35 | | |
| 36 | | |
| 37 | | |
| 38 | | |
| 39 | | |
| 40 | | |
| 41 | | |
| 42 | | |
| 43 | | |
| 44 | | |
| 45 | | |

| Equation of Daylight for Latitude 36° | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ecliptic | | equation | ecliptic | | equation | ecliptic | | equation |
| **0** | **5** | ° ′ | **1** | **4** | ° ′ | **2** | **3** | ° ′ |
| **6** | **11** | | **7** | **10** | | **8** | **9** | |
| 1 | 29 | 0;18 | 1 | 29 | 8;48 | 1 | 29 | 15;44 |
| 2 | 28 | 0;35 | 2 | 28 | 9; 4 | 2 | 28 | 15;54 |
| 3 | 27 | 0;53 | 3 | 27 | 9;19 | 3 | 27 | 16; 4[1] |
| 4 | 26 | 1;10 | 4 | 26 | 9;35 | 4 | 26 | 16;15 |
| 5 | 25 | 1;28 | 5 | 25 | 9;51 | 5 | 25 | 16;25 |
| 6 | 24 | 1;45 | 6 | 24 | 10; 7 | 6 | 24 | 16;35 |
| 7 | 23 | 2; 2 | 7 | 23 | 10;22 | 7 | 23 | 16;43 |
| 8 | 22 | 2;20 | 8 | 22 | 10;38 | 8 | 22 | 16;51 |
| 9 | 21 | 2;37 | 9 | 21 | 10;53 | 9 | 21 | 16;59 |
| 10 | 20 | 2;54 | 10 | 20 | 11; 8 | 10 | 20 | 17; 8 |
| 11 | 19 | 3;12 | 11 | 19 | 11;24 | 11 | 19 | 17;16 |
| 12 | 18 | 3;29 | 12 | 18 | 11;39 | 12 | 18 | 17;24 |
| 13 | 17 | 3;46 | 13 | 17 | 11;53 | 13 | 17 | 17;30 |
| 14 | 16 | 4; 3 | 14 | 16 | 12; 8 | 14 | 16 | 17;36 |
| 15 | 15 | 4;20 | 15 | 15 | 12;22 | 15 | 15 | 17;42 |
| 16 | 14 | 4;38 | 16 | 14 | 12;36[2] | 16 | 14 | 17;48 |
| 17 | 13 | 4;55 | 17 | 13 | 12;51 | 17 | 13 | 17;54 |
| 18 | 12 | 5;12[3] | 18 | 12 | 13; 5 | 18 | 12 | 18; 0 |
| 19 | 11 | 5;29 | 19 | 11 | 13;18 | 19 | 11 | 18; 4 |
| 20 | 10 | 5;46 | 20 | 10 | 13;31 | 20 | 10 | 18; 7 |
| 21 | 9 | 6; 2 | 21 | 9 | 13;44 | 21 | 9 | 18;11 |
| 22 | 8 | 6;19 | 22 | 8 | 13;58[4] | 22 | 8 | 18;15 |
| 23 | 7 | 6;36 | 23 | 7 | 14;11 | 23 | 7 | 18;18 |
| 24 | 6 | 6;53[5] | 24 | 6 | 14;24 | 24 | 6 | 18;22 |
| 25 | 5 | 7;10 | 25 | 5 | 14;36 | 25 | 5 | 18;23 |
| 26 | 4 | 7;26 | 26 | 4 | 14;47 | 26 | 4 | 18;25 |
| 27[6] | 3 | 7;43 | 27 | 3 | 14;59 | 27 | 3 | 18;26 |
| 28 | 2 | 7;59 | 28 | 2 | 15;11 | 28 | 2 | 18;27 |
| 29 | 1 | 8;16 | 29 | 1 | 15;22 | 29 | 1 | 18;29 |
| 30 | 0 | 8;32 | 30 | 0 | 15;34 | 30 | 0 | 18;30 |

[1] **F** 54′ [2] **F** 35′ [3] **F** 4° [4] **F** 18′ [5] **H** 13′ [6] **H** 28.

**Table 48a: Equation of daylight for latitude 35°30′**

*Sources*: **C** fols 82r–82v (together with the oblique ascension, with symmetrical values for every degree of the ecliptic), $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 305v, **L** fol. 77v, **B** p. 150. **FH** have a table for the equation of daylight for latitude 36° which is here edited on p. 228.

Equation of Daylight for Latitude 35°30′

| ecliptic | | equation | ecliptic | | equation | ecliptic | | equation |
|---|---|---|---|---|---|---|---|---|
| **0** | **5** | ° ′ | **1** | **4** | ° ′ | **2** | **3** | ° ′ |
| **6** | **11** | | **7** | **10** | | **8** | **9** | |
| 1 | 29 | 0;17 | 1 | 29 | 8;36 | 1 | 29 | 15;25[1] |
| 2 | 28 | 0;34 | 2 | 28 | 8;51 | 2 | 28 | 15;35[2] |
| 3 | 27 | 0;52[3] | 3 | 27 | 9; 8 | 3 | 27 | 15;46 |
| 4 | 26 | 1; 9[4] | 4 | 26 | 9;24 | 4 | 26 | 15;57 |
| 5 | 25 | 1;26 | 5 | 25 | 9;39 | 5 | 25 | 16; 7 |
| 6 | 24 | 1;43 | 6 | 24 | 9;54 | 6 | 24 | 16;15 |
| 7 | 23 | 2; 0 | 7 | 23 | 10; 9 | 7 | 23 | 16;24 |
| 8 | 22 | 2;16 | 8 | 22 | 10;24[5] | 8 | 22 | 16;32 |
| 9 | 21 | 2;33 | 9 | 21 | 10;39[6] | 9 | 21 | 16;40 |
| 10 | 20 | 2;50 | 10 | 20 | 10;55[7] | 10 | 20 | 16;49[8] |
| 11 | 19 | 3; 7[9] | 11 | 19 | 11; 8 | 11 | 19 | 16;56 |
| 12 | 18 | 3;23 | 12 | 18 | 11;22 | 12 | 18 | 17; 2 |
| 13 | 17 | 3;40 | 13 | 17 | 11;38[10] | 13 | 17 | 17;10 |
| 14 | 16 | 3;57 | 14 | 16 | 11;53[11] | 14 | 16 | 17;17 |
| 15 | 15 | 4;14 | 15 | 15 | 12; 7[12] | 15 | 15 | 17;23[13] |
| 16 | 14 | 4;31 | 16 | 14 | 12;20 | 16 | 14 | 17;29 |
| 17 | 13 | 4;47 | 17 | 13 | 12;34[14] | 17 | 13 | 17;34 |
| 18 | 12 | 5; 4 | 18 | 12 | 12;47[15] | 18 | 12 | 17;38 |
| 19 | 11 | 5;21 | 19 | 11 | 13; 2 | 19 | 11 | 17;43 |
| 20 | 10 | 5;38[16] | 20 | 10 | 13;16 | 20 | 10 | 17;48[17] |
| 21 | 9 | 5;54 | 21 | 9 | 13;27[18] | 21 | 9 | 17;52[19]↓ |
| 22 | 8 | 6;11 | 22 | 8 | 13;40 | 22 | 8 | 17;55 |
| 23 | 7 | 6;27 | 23 | 7 | 13;52[20] | 23 | 7 | 17;59⊥ |
| 24 | 6 | 6;44 | 24 | 6 | 14; 5 | 24 | 6 | 18; 2 |
| 25 | 5 | 7; 1 | 25 | 5 | 14;18 | 25 | 5 | 18; 5 |
| 26 | 4 | 7;17 | 26 | 4 | 14;29 | 26 | 4 | 18; 7 |
| 27 | 3 | 7;32 | 27 | 3 | 14;41 | 27 | 3 | 18; 8 |
| 28 | 2 | 7;48 | 28 | 2 | 14;52 | 28 | 2 | 18; 9 |
| 29 | 1 | 8; 4 | 29 | 1 | 15; 4 | 29 | 1 | 18; 9 |
| 30 | 0 | 8;20 | 30 | 0 | 15;15 | 30 | 0 | 18; 9 |

Variants for arguments larger than 90° in **C** are indicated with their argument under the symmetrical values for arguments 1–90°. Due to damage to the bottom of the page, in **C** most of the values in the row for 30° are illegible. [1] **C** 25° (causing also the next three values to be read incorrectly) [2] **Y** 36′ [3] **CL** 12′ [4] **Y** 21′ [5] **L** 29′ [6] **C** $4^s$ 21°: 29′ [7] **C** 11°15′ [8] **C** 19′ [9] **C** $11^s$ 19°: 50′ [10] **Y** 33′ [11] **C** 33′ **C** $4^s$ 16°: 18′ [12] **C** 50′ [13] **C** $3^s$ 15°: 33′ [14] **Y** 35′ [15] **C** $4^s$ 12°: 14° (causing also the values for $4^s$ 13–15° to be read incorrectly) [16] **C** 33′ [17] **C** 18′ [18] **C** $7^s$ 21°: 24′ [19] **C** $2^s$ 21–23°: 18° [20] **CL** 12′.

**Table 49: Hourly motion and diameters of the Sun and Moon**
*Sources*: **F** fol. 86r, **H** fol. 79r, **C** fol. 83r, **$C_1$** fol. 46r, **$C_2$** -, **Y** fol. 307r, **L** fols 79r–79v, **B** p. 153, **r** = recomputation / reconstruction.

Table of the Hourly Path of the Two Luminaries and their Diameters

| solar anomaly or true anomaly of the Moon | | solar hours | lunar hours | solar diameter | lunar diameter | diameter of the shadow |
|---|---|---|---|---|---|---|
| | | ′ ″ | ′ ″ | ′ ″ | ′ ″ | ′ ″ |
| [1] $0^s$ 0 | $0^s$ 0 | 2,23 | 30,17 | 31,28 | 29,40 | 77, 8 |
| $0^s$ 6 | $11^s$ 24 | 2,23 | 30,19 | 31,30 | 29,42 | 77,13 |
| $0^s$ 12 | $11^s$ 18 | 2,23 | 30,21 | 31,33 | 29,44 | 77,18 |
| $0^s$ 18 | $11^s$ 12 | 2,23 | 30,24 | 31,36 | 29,47 | 77,26 |
| $0^s$ 24 | $11^s$ 6 | 2,23 | 30,28 | 31,38 | 29,51 | 77,37 |
| $1^s$ 0 | $11^s$ 0 | 2,24 | 30,34 | 31,41 | 29,57 | 77,52 |
| $1^s$ 6 | $10^s$ 24 | 2,24 | 30,42 | 31,45 | 30, 5[2] | 78,13 |
| $1^s$ 12[3]↓ | $10^s$ 18 | 2,24 | 30,51[4] | 31,50 | 30,13 | 78,34[5] |
| $1^s$ 18 | $10^s$ 12 | 2,25 | 31, 1 | 31,54 | 30,24 | 79, 2 |
| $1^s$ 24 | $10^s$ 6 | 2,25 | 31,12 | 31,59 | 30,34 | 79,28 |
| $2^s$ 0 | $10^s$ 0 | 2,25 | 31,24 | 32, 3 | 30,46 | 80, 0 |
| $2^s$ 6 | $9^s$ 24 | 2,26 | 31,38 | 32, 8 | 31, 0 | 80,36[6] |
| $2^s$ 12 | $9^s$ 18 | 2,26 | 31,53 | 32,14 | 31,14 | 81,12 |
| $2^s$ 18 | $9^s$ 12 | 2,27 | 32, 8[7] | 32,21 | 31,29 | 81,51[8] |
| $2^s$ 24 | $9^s$ 6 | 2,27 | 32,24 | 32,27[9] | 31,45 | 82,33[10] |
| $3^s$ 0 | $9^s$ 0 | 2,28 | 32,41 | 32,34 | 32, 1 | 83,15[11]↓ |
| $3^s$ 6 | $8^s$ 24 | 2,28 | 32,59 | 32,40 | 32,19 | 84, 1 |
| $3^s$ 12 | $8^s$ 18 | 2,29 | 33,17[12] | 32,47 | 32,37 | 84,48 |
| $3^s$ 18 | $8^s$ 12 | 2,29 | 33,36 | 32,53 | 32,55 | 85,35⊥ |
| $3^s$ 24 | $8^s$ 6 | 2,30 | 33,55[13] | 33, 0 | 33,14 | 86,24[14]↓ |
| $4^s$ 0 | $8^s$ 0 | 2,30 | 34,14 | 33, 7[15] | 33,32 | 87,11[16] |
| $4^s$ 6⊥ | $7^s$ 24 | 2,31 | 34,32[17] | 33,14 | 33,50 | 87,58 |
| $4^s$ 12[18]↓ | $7^s$ 18 | 2,31 | 34,49 | 33,20 | 34, 7 | 88,42 |
| $4^s$ 18 | $7^s$ 12 | 2,32 | 35, 4 | 33,25 | 34,27[19] | 89,19[20] |
| $4^s$ 24⊥ | $7^s$ 6 | 2,32 | 35,18 | 33,30 | 34,35 | 89,55 |
| $5^s$ 0 | $7^s$ 0 | 2,33[21] | 35,31 | 33,34 | 34,48 | 90,29 |
| $5^s$ 6 | $6^s$ 24 | 2,33 | 35,42 | 33,37 | 34,59 | 90,57 |
| $5^s$ 12 | $6^s$ 18 | 2,33 | 35,51 | 33,39 | 35, 7[22] | 91,18 |
| $5^s$ 18 | $6^s$ 12 | 2,33 | 35,58[23] | 33,40 | 35,14 | 91,36 |
| $5^s$ 24 | $6^s$ 6 | 2,33 | 36, 2 | 33,40 | 35,18 | 91,47⊥ |
| $6^s$ 0 | $6^s$ 0 | 2,33 | 36, 4[24] | 33,40 | 35,20 | 91,52 |

[1] **YLB** omit the row for $0^s\,0°$; in **$CC_1$** the values from this row were written above the main tabular frame by the main hand. [2] **C** 0″ [3] **C** arguments $1^s\,12°$–$4^s\,6°$: slide[+3] [4] **F** 41″ [5] **$C_1$** 39″ [6] **F** 81′ [7] **Y** 3″ [8] **F** 82′ [9] **$C_1$** 24″ [10] **C** 13″ [11] **Y** $3^s\,0°$–18°: minutes one less [12] **Y** 57″ [13] **CL** 15′ [14] **Y** $3^s\,24°$–$5^s\,24°$: minutes two less [15] **C** 50″ [16] +**C** om. minutes (causing them to be read as 86) [17] **H** 34″ [18] **C** arguments $4^s\,12°$–24°: slide[−1] [19] **r** 21′ (computed from the hourly lunar motion or back-computed from the diameter of the shadow) [20] +**C** om. minutes (causing them to be read as 88) [21] **B** 32″ [22] **C** om. minutes (causing these and the minutes of the next two values to be read as 34), 50″ [23] **C** 36′18″ [24] **C** seconds dam.

**Table 49$^{bis}$: Conjunction and opposition (first part)**

*Sources*: **Y** fols 311v–313v (see Plate 13); **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| elongation | lunar velocity 11;50 | | elongation | lunar velocity 12;0 | | elongation | lunar velocity 12;10 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4,59 | 2;11,48 | 1 | 0; 4,55 | 2; 9,50 | 1 | 0; 4,51 | 2; 7,55 [1] |
| 2 | 0; 9,58 | 4;23,36 | 2 | 0; 9,50 | 4;19,40 | 2 | 0; 9,42 | 4;15,50 |
| 3 | 0;14,57 | 6;35,24 | 3 | 0;14,45 | 6;29,30 | 3 | 0;14,33 | 6;23,45 |
| 4 | 0;19,56 | 8;47,12 [2] | 4 | 0;19,40 | 8;39,20 | 4 | 0;19,24 [3] | 8;31,40 |
| 5 | 0;24,55 | 10;59, 0 | 5 | 0;24,35 [4] | 10;49,10 | 5 | 0;24,15 | 10;39,35 |
| 6 | 0;29,54 | 13;10,48 | 6 | 0;29,30 | 12;59, 0 | 6 | 0;29, 6 | 12;47,30 |
| 7 | 0;34,53 | 15;22,36 | 7 | 0;34,25 | 15; 8,50 | 7 | 0;33,57 | 14;55,25 |
| 8 | 0;39,52 | 17;34,24 | 8 | 0;39,20 | 17;18,40 | 8 | 0;38,48 | 17; 3,20 |
| 9 | 0;44,51 | 19;46,12 | 9 | 0;44,15 | 19;28,30 | 9 | 0;43,39 | 19;11,15 |
| 10 | 0;49,50 | 21;58, 0 | 10 | 0;49,10 | 21;38,20 | 10 | 0;48,30 | 21;19,10 |
| 11 | 0;54,49 | 24; 9,48 | 11 | 0;54, 5 | 23;48,10 | 11 | 0;53,21 | 23;27, 5 |
| 12 | 0;59,48 | 26;21,36 [5] | 12 | 0;59, 0 | 25;58, 0 | 12 | 0;58,12 [6] | 25;35, 0 |
| 13 | 1; 4,47 | 28;33,24 | 13 | 1; 3,55 | 28; 7,50 | 13 | 1; 3, 3 | 27;42,55 [7] |
| 14 | 1; 9,46 | 30;45,12 | 14 | 1; 8,50 | 30;17,40 | 14 | 1; 7,54 | 29;50,50 |
| 15 | 1;14,45 | 32;57, 0 [8] | 15 | 1;13,45 | 32;27,30 | 15 | 1;12,45 | 31;58,45 |
| 18 | 1;29,42 | 39;32,24 | 18 | 1;28,30 | 38;57, 0 | 18 | 1;27,18 | 38;22,30 |
| 21 | 1;44,39 | 46; 7,48 | 21 | 1;43,15 [9] | 45;26,30 | 21 | 1;41,51 | 44;46,15 |
| 24 | 1;59,36 | 52;43,12 | 24 | 1;58, 0 | 51;56, 0 | 24 | 1;56,24 | 51;10, 0 |
| 27 | 2;14,33 | 59;18,36 | 27 | 2;12,45 | 58;25,30 | 27 | 2;10,57 | 57;33,45 |
| 30 | 2;29,30 | 65;54, 0 | 30 | 2;27,30 | 64;55, 0 [10] | 30 | 2;25,30 | 63;57,30 |
| 33 | 2;44,27 | 72;29,24 [11] | 33 | 2;42,15 | 71;24,30 | 33 | 2;40, 3 | 70;21,15 |
| 36 | 2;59,24 | 79; 4,48 [12] | 36 | 2;57, 0 | 77;54, 0 | 36 | 2;54,36 | 76;45, 0 |
| 39 | 3;14,21 | 85;40,12 | 39 | 3;11,45 | 84;23,30 | 39 | 3; 9, 9 | 83; 8,45 |
| 42 | 3;29,18 | 92;15,36 [13] | 42 | 3;26,30 | 90;53, 0 | 42 | 3;23,42 | 89;32,30 [14] |
| 45 | 3;44,15 | 98;51, 0 | 45 | 3;41,15 | 97;22,30 | 45 | 3;38,15 | 95;56,15 |
| 48 | 3;59,12 | 105;26,24 | 48 | 3;56, 0 | 103;52, 0 | 48 | 3;52,48 | 102;20, 0 |
| 51 | 4;14, 9 [15] | 112; 1,48 | 51 | 4;10,45 | 110;21,30 | 51 | 4; 7,21 [16] | 108;43,45 |
| 54 | 4;29, 6 | 118;37,12 | 54 | 4;25,30 | 116;51, 0 | 54 | 4;21,54 | 115; 7,30 [17] |
| 57 | 4;44, 3 | 125;12,36 | 57 | 4;40,15 | 123;20,30 | 57 | 4;36,27 | 121;31,15 |
| 60 | 4;59, 0 | 131;48, 0 | 60 | 4;55, 0 | 129;50, 0 | 60 | 4;51, 0 | 127;55, 0 |

**Y** gives the arguments as 1, 2, 3, ..., 30 in one column on every page. **D** gives the arguments as 1, 2, 3, ..., 30 for every column (but writes only the first and last argument in the second and third column on each page). [1] **D** 15″ [2] **Y** 42′ [3] **Y** 49′ [4] **D** 34″ [5] **Y** 51′ [6] **D** 53′ [7] **D** 45″ [8] **D** 17′ [9] **P** 55″ [10] **Y** 54′ [11] **Y** 39′ [12] **Y** 71° [13] **P** 16′ [14] **B** 84° [15] **D** 54′ [16] **P** 5′ [17] **P** 6′.

**Table 49^bis^: Conjunction and opposition (second part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| elongation | lunar velocity 12;20 | | elongation | lunar velocity 12;30 | | elongation | lunar velocity 12;40 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4,47 | 2; 6, 4 | 1 | 0; 4,43 | 2; 4,15[1] | 1 | 0; 4,39 | 2; 2,30 |
| 2 | 0; 9,34 | 4;12, 8 | 2 | 0; 9,26 | 4; 8,30[2]↕ | 2 | 0; 9,18 | 4; 5, 0 |
| 3 | 0;14,21 | 6;18,12 | 3 | 0;14, 9 | 6;12,45 | 3 | 0;13,57 | 6; 7,30 |
| 4 | 0;19, 8 | 8;24,16 | 4 | 0;18,52 | 8;17, 0 | 4 | 0;18,36 | 8;10, 0 |
| 5 | 0;23,55[3] | 10;30,20 | 5 | 0;23,35 | 10;21,15 | 5 | 0;23,15[4] | 10;12,30 |
| 6 | 0;28,42 | 12;36,24 | 6 | 0;28,18[5] | 12;25,30 | 6 | 0;27,54 | 12;15, 0 |
| 7 | 0;33,29 | 14;42,28 | 7 | 0;33, 1 | 14;29,45 | 7 | 0;32,33 | 14;17,30 |
| 8 | 0;38,16 | 16;48,32 | 8 | 0;37,44 | 16;34, 0 | 8 | 0;37,12 | 16;20, 0 |
| 9 | 0;43, 3 | 18;54,36 | 9 | 0;42,27 | 18;38,15 | 9 | 0;41,51 | 18;22,30 |
| 10 | 0;47,50 | 21; 0,40 | 10 | 0;47,10 | 20;42,30⊥ | 10 | 0;46,30 | 20;25, 0[6] |
| 11 | 0;52,37 | 23; 6,44 | 11 | 0;51,53 | 22;46,45 | 11 | 0;51, 9 | 22;27,30 |
| 12 | 0;57,24 | 25;12,48 | 12 | 0;56,36 | 24;51, 0 | 12 | 0;55,48 | 24;30, 0 |
| 13 | 1; 2,11 | 27;18,52 | 13 | 1; 1,19 | 26;55,15 | 13 | 1; 0,27 | 26;32,30 |
| 14 | 1; 6,58 | 29;24,56 | 14 | 1; 6, 2 | 28;59,30 | 14 | 1; 5, 6 | 28;35, 0 |
| 15 | 1;11,45[7] | 31;31, 0 | 15 | 1;10,45 | 31; 3,45[8] | 15 | 1; 9,45 | 30;37,30 |
| 18 | 1;26, 6 | 37;49,12 | 18 | 1;24,54 | 37;16,30 | 18 | 1;23,42 | 36;45, 0 |
| 21 | 1;40,27 | 44; 7,24[9] | 21 | 1;39, 3 | 43;29,15 | 21 | 1;37,39 | 42;52,30 |
| 24 | 1;54,48[10] | 50;25,36[11] | 24 | 1;53,12 | 49;42, 0 | 24 | 1;51,36 | 49; 0, 0 |
| 27 | 2; 9, 9 | 56;43,48[12] | 27 | 2; 7,21 | 55;54,45[13]↕ | 27 | 2; 5,33 | 55; 7,30 |
| 30 | 2;23,30 | 63; 2, 0 | 30 | 2;21,30 | 62; 7,30[14] | 30 | 2;19,30 | 61;15, 0 |
| 33 | 2;37,51 | 69;20,12 | 33 | 2;35,39 | 68;20,15 | 33 | 2;33,27 | 67;22,30[15] |
| 36 | 2;52,12 | 75;38,24 | 36 | 2;49,48 | 74;33, 0 | 36 | 2;47,24 | 73;30, 0 |
| 39 | 3; 6,33 | 81;56,36 | 39 | 3; 3,57 | 80;45,45⊥ | 39 | 3; 1,21 | 79;37,30 |
| 42 | 3;20,54 | 88;14,48 | 42 | 3;18, 6 | 86;58,30 | 42 | 3;15,18 | 85;45, 0 |
| 45 | 3;35,15 | 94;33, 0 | 45 | 3;32,15 | 93;11,15 | 45 | 3;29,15 | 91;52,30 |
| 48 | 3;49,36 | 100;51,12 | 48 | 3;46,24[16] | 99;24, 0 | 48 | 3;43,12 | 98; 0, 0 |
| 51 | 4; 3,57[17] | 107; 9,24 | 51 | 4; 0,33 | 105;36,45[18]↕ | 51 | 3;57, 9 | 104; 7,30 |
| 54 | 4;18,18 | 113;27,36[19] | 54 | 4;14,42[20] | 111;49,30 | 54 | 4;11, 6 | 110;15, 0 |
| 57 | 4;32,39[21] | 119;45,48 | 57 | 4;28,51 | 118; 2,15 | 57 | 4;25, 3 | 116;22,30 |
| 60 | 4;47, 0 | 126; 4, 0 | 60 | 4;43, 0 | 124;15, 0⊥ | 60 | 4;39, 0 | 122;30, 0 |

[1] **Y** 0″ (beginning of slide) [2] **Y** 2–10° (seconds): slide[−1] [3] **Y** 15″ **P** 35″ [4] **P** 55″ [5] **Y** 48″ [6] **Y** 24′ [7] **Y** 49″ [8] **D** 4′ [9] **P** 4′ [10] **P** 43″ [11] **YP** 20′ [12] **Y** 48′ [13] **Y** 27–39° (seconds): slide[+1] [14] +**P** 2′ [15] **Y** 60° **P** 7″ [16] **Y** 14″ [17] **P** 17″ [18] **D** 51–60° (degrees): om. the dots on *qāf* [19] **Y** 43° (?) [20] **Y** 15′ [21] **B** 52′.

**Table 49$^{bis}$: Conjunction and opposition (third part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| elongation | lunar velocity 12;50 | | elongation | lunar velocity 13;0 | | elongation | lunar velocity 13;10 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4,36 | 2; 0,49 | 1 | 0; 4,32 | 1;59, 8[1] | 1 | 0; 4,29 | 1;57,32 |
| 2 | 0; 9,12 | 4; 1,38 | 2 | 0; 9, 4 | 3;58,16 | 2 | 0; 8,58 | 3;55, 4 |
| 3 | 0;13,48 | 6; 2,27 | 3 | 0;13,36 | 5;57,24 | 3 | 0;13,27 | 5;52,36 |
| 4 | 0;18,24 | 8; 3,16 | 4 | 0;18, 8[2] | 7;56,32 | 4 | 0;17,56 | 7;50, 8 |
| 5 | 0;23, 0 | 10; 4, 5 | 5 | 0;22,40 | 9;55,40 | 5 | 0;22,25 | 9;47,40 |
| 6 | 0;27,36 | 12; 4,54 | 6 | 0;27,12 | 11;54,48 | 6 | 0;26,54 | 11;45,12 |
| 7 | 0;32,12 | 14; 5,43 | 7 | 0;31,44 | 13;53,56[3] | 7 | 0;31,23 | 13;42,44 |
| 8 | 0;36,48 | 16; 6,32 | 8 | 0;36,16 | 15;53, 4 | 8 | 0;35,52 | 15;40,16 |
| 9 | 0;41,24 | 18; 7,21 | 9 | 0;40,48 | 17;52,12 | 9 | 0;40,21 | 17;37,48 |
| 10 | 0;46, 0 | 20; 8,10 | 10 | 0;45,20 | 19;51,20 | 10 | 0;44,50 | 19;35,20 |
| 11 | 0;50,36 | 22; 8,59 | 11 | 0;49,52 | 21;50,28 | 11 | 0;49,19 | 21;32,52 |
| 12 | 0;55,12 | 24; 9,48 | 12 | 0;54,24[4] | 23;49,36 | 12 | 0;53,48 | 23;30,24 |
| 13 | 0;59,48 | 26;10,37 | 13 | 0;58,56 | 25;48,44 | 13 | 0;58,17 | 25;27,56 |
| 14 | 1; 4,24 | 28;11,26 | 14 | 1; 3,28 | 27;47,52[5] | 14 | 1; 2,46 | 27;25,28 |
| 15 | 1; 9, 0 | 30;12,15 | 15 | 1; 8, 0 | 29;47, 0 | 15 | 1; 7,15 | 29;23, 0 |
| 18 | 1;22,48 | 36;14,42 | 18 | 1;21,36 | 35;44,24 | 18 | 1;20,42 | 35;15,36 |
| 21 | 1;36,36 | 42;17, 9[6] | 21 | 1;35,12 | 41;41,48 | 21 | 1;34, 9 | 41; 8,12[7] |
| 24 | 1;50,24 | 48;19,36 | 24 | 1;48,48 | 47;39,12 | 24 | 1;47,36 | 47; 0,48 |
| 27 | 2; 4,12 | 54;22, 3 | 27 | 2; 2,24 | 53;36,36 | 27 | 2; 1, 3 | 52;53,24 |
| 30 | 2;18, 0 | 60;24,30 | 30 | 2;16, 0 | 59;34, 0 | 30 | 2;14,30 | 58;46, 0[8] |
| 33 | 2;31,48 | 66;26,57 | 33 | 2;29,36 | 65;31,24 | 33 | 2;27,57 | 64;38,36 |
| 36 | 2;45,36 | 72;29,24 | 36 | 2;43,12 | 71;28,48 | 36 | 2;41,24 | 70;31,12 |
| 39 | 2;59,24 | 78;31,51 | 39 | 2;56,48[9] | 77;26,12 | 39 | 2;54,51 | 76;23,48[10] |
| 42 | 3;13,12 | 84;34,18 | 42 | 3;10,24 | 83;23,36 | 42 | 3; 8,18 | 82;16,24 |
| 45 | 3;27, 0 | 90;36,45 | 45 | 3;24, 0 | 89;21, 0 | 45 | 3;21,45 | 88; 9, 0 |
| 48 | 3;40,48 | 96;39,12 | 48 | 3;37,36 | 95;18,24 | 48 | 3;35,12 | 94; 1,36 |
| 51 | 3;54,36 | 102;41,39 | 51 | 3;51,12 | 101;15,48 | 51 | 3;48,39 | 99;54,12 |
| 54 | 4; 8,24 | 108;44, 6 | 54 | 4; 4,48[11] | 107;13,12 | 54 | 4; 2, 6 | 105;46,48 |
| 57 | 4;22,12 | 114;46,33 | 57 | 4;18,24 | 113;10,36 | 57 | 4;15,33 | 111;39,24 |
| 60 | 4;36, 0 | 120;49, 0 | 60 | 4;32, 0 | 119; 8, 0 | 60 | 4;29, 0 | 117;32, 0[12] |

[1] **B** 3″ [2] **B** 3″ **D** 13′ [3] **P** 57″ [4] **P** 27″ [5] **Y** 26° [6] **Y** 16′ [7] **Y** 2″ [8] **Y** 53° [9] **Y** 57′ [10] **Y** 77° [11] **B** 18″ [12] **Y** 112°.

**Table 49bis: Conjunction and opposition (fourth part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| elongation | lunar velocity 13;20 | | elongation | lunar velocity 13;30 | | elongation | lunar velocity 13;40 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4,25 | 1;55,57[1] | 1 | 0; 4,22 | 1;54,26 | 1 | 0; 4,19 | 1;52,57 |
| 2 | 0; 8,50 | 3;51,54 | 2 | 0; 8,44 | 3;48,52 | 2 | 0; 8,38 | 3;45,54 |
| 3 | 0;13,15[2] | 5;47,51 | 3 | 0;13, 6 | 5;43,18 | 3 | 0;12,57 | 5;38,51 |
| 4 | 0;17,40 | 7;43,48 | 4 | 0;17,28[3] | 7;37,44 | 4 | 0;17,16 | 7;31,48 |
| 5 | 0;22, 5 | 9;39,45 | 5 | 0;21,50 | 9;32,10 | 5 | 0;21,35 | 9;24,45 |
| 6 | 0;26,30 | 11;35,42 | 6 | 0;26,12 | 11;26,36 | 6 | 0;25,54 | 11;17,42 |
| 7 | 0;30,55 | 13;31,39 | 7 | 0;30,34[4] | 13;21, 2 | 7 | 0;30,13 | 13;10,39 |
| 8 | 0;35,20 | 15;27,36[5] | 8 | 0;34,56 | 15;15,28 | 8 | 0;34,32 | 15; 3,36 |
| 9 | 0;39,45 | 17;23,33 | 9 | 0;39,18 | 17; 9,54 | 9 | 0;38,51 | 16;56,33 |
| 10 | 0;44,10 | 19;19,30 | 10 | 0;43,40 | 19; 4,20 | 10 | 0;43,10 | 18;49,30 |
| 11 | 0;48,35 | 21;15,27 | 11 | 0;48, 2 | 20;58,46 | 11 | 0;47,29 | 20;42,27 |
| 12 | 0;53, 0 | 23;11,24 | 12 | 0;52,24 | 22;53,12 | 12 | 0;51,48 | 22;35,24 |
| 13 | 0;57,25 | 25; 7,21 | 13 | 0;56,46 | 24;47,38 | 13 | 0;56, 7 | 24;28,21 |
| 14 | 1; 1,50 | 27; 3,18 | 14 | 1; 1, 8 | 26;42, 4 | 14 | 1; 0,26 | 26;21,18 |
| 15 | 1; 6,15 | 28;59,15 | 15 | 1; 5,30 | 28;36,30 | 15 | 1; 4,45[6] | 28;14,15 |
| 18 | 1;19,30 | 34;47, 6[7] | 18 | 1;18,36 | 34;19,48 | 18 | 1;17,42 | 33;53, 6 |
| 21 | 1;32,45 | 40;34,57 | 21 | 1;31,42 | 40; 3, 6 | 21 | 1;30,39 | 39;31,57 |
| 24 | 1;46, 0 | 46;22,48 | 24 | 1;44,48 | 45;46,24 | 24 | 1;43,36 | 45;10,48 |
| 27 | 1;59,15 | 52;10,39 | 27 | 1;57,54 | 51;29,42 | 27 | 1;56,33 | 50;49,39 |
| 30 | 2;12,30 | 57;58,30 | 30 | 2;11, 0 | 57;13, 0 | 30 | 2; 9,30 | 56;28,30 |
| 33 | 2;25,45 | 63;46,21 | 33 | 2;24, 6 | 62;56,18 | 33 | 2;22,27 | 62; 7,21 |
| 36 | 2;39, 0 | 69;34,12 | 36 | 2;37,12 | 68;39,36 | 36 | 2;35,24 | 67;46,12[8] |
| 39 | 2;52,15 | 75;22, 3 | 39 | 2;50,18 | 74;22,54 | 39 | 2;48,21 | 73;25, 3 |
| 42 | 3; 5,30 | 81; 9,54 | 42 | 3; 3,24 | 80; 6,12 | 42 | 3; 1,18 | 79; 3,54 |
| 45 | 3;18,45 | 86;57,45 | 45 | 3;16,30 | 85;49,30 | 45 | 3;14,15 | 84;42,45[9] |
| 48 | 3;32, 0 | 92;45,36[10] | 48 | 3;29,36 | 91;32,48 | 48 | 3;27,12 | 90;21,36 |
| 51 | 3;45,15 | 98;33,27 | 51 | 3;42,42 | 97;16, 6 | 51 | 3;40, 9 | 96; 0,27 |
| 54 | 3;58,30 | 104;21,18 | 54 | 3;55,48 | 102;59,24 | 54 | 3;53, 6 | 101;39,18 |
| 57 | 4;11,45 | 110; 9, 9 | 57 | 4; 8,54 | 108;42,42 | 57 | 4; 6, 3 | 107;18, 9[11] |
| 60 | 4;25, 0 | 115;57, 0[12] | 60 | 4;22, 0 | 114;26, 0 | 60 | 4;19, 0 | 112;57, 0 |

[1] **Y** 54″ [2] **P** 18′ [3] **Y** 23″ [4] **D** 24″ [5] **P** 26″ [6] **D** 44″ [7] **Y** 5″ [8] **Y** 16′17″ [9] **Y** 44″ [10] **D** 37″ [11] **Y** 87″ [12] **Y** 111°.

**Table 49[bis]: Conjunction and opposition (fifth part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

Conjunction and Opposition

| elongation | lunar velocity 13;50 | | elongation | lunar velocity 14;0 | | elongation | lunar velocity 14;10 | |
|---|---|---|---|---|---|---|---|---|
| | part of the distance | hours of the distance | | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4,16 | 1;51,30 | 1 | 0; 4,13 | 1;50, 5 | 1 | 0; 4,10 | 1;48,42 |
| 2 | 0; 8,32 | 3;43, 0 | 2 | 0; 8,26 | 3;40,10 | 2 | 0; 8,20 | 3;37,24 |
| 3 | 0;12,48 | 5;34,30 | 3 | 0;12,39 | 5;30,15 | 3 | 0;12,30 | 5;26, 6 |
| 4 | 0;17, 4 | 7;26, 0 | 4 | 0;16,52 | 7;20,20 | 4 | 0;16,40 | 7;14,48 |
| 5 | 0;21,20 | 9;17,30 | 5 | 0;21, 5 | 9;10,25 | 5 | 0;20,50 | 9; 3,30 |
| 6 | 0;25,36 | 11; 9, 0 | 6 | 0;25,18 | 11; 0,30 | 6 | 0;25, 0 | 10;52,12 |
| 7 | 0;29,52 | 13; 0,30 | 7 | 0;29,31 | 12;50,35 | 7 | 0;29,10 | 12;40,54 |
| 8 | 0;34, 8 | 14;52, 0 | 8 | 0;33,44 | 14;40,40 | 8 | 0;33,20 | 14;29,36 |
| 9 | 0;38,24 | 16;43,30 | 9 | 0;37,57 | 16;30,45 | 9 | 0;37,30 | 16;18,18 |
| 10 | 0;42,40 | 18;35, 0 | 10 | 0;42,10 | 18;20,50 | 10 | 0;41,40 | 18; 7, 0 |
| 11 | 0;46,56 | 20;26,30 | 11 | 0;46,23 | 20;10,55 | 11 | 0;45,50 | 19;55,42[1] |
| 12 | 0;51,12 | 22;18, 0 | 12 | 0;50,36 | 22; 1, 0 | 12 | 0;50, 0 | 21;44,24 |
| 13 | 0;55,28 | 24; 9,30 | 13 | 0;54,49[2] | 23;51, 5 | 13 | 0;54,10 | 23;33, 6 |
| 14 | 0;59,44 | 26; 1, 0 | 14 | 0;59, 2 | 25;41,10 | 14 | 0;58,20 | 25;21,48 |
| 15 | 1; 4, 0 | 27;52,30 | 15 | 1; 3,15 | 27;31,15 | 15 | 1; 2,30 | 27;10,30 |
| 18 | 1;16,48 | 33;27, 0[3] | 18 | 1;15,54 | 33; 1,30 | 18 | 1;15, 0 | 32;36,36 |
| 21 | 1;29,36 | 39; 1,30 | 21 | 1;28,33 | 38;31,45 | 21 | 1;27,30 | 38; 2,42 |
| 24 | 1;42,24 | 44;36, 0 | 24 | 1;41,12 | 44; 2, 0 | 24 | 1;40, 0 | 43;28,48 |
| 27 | 1;55,12 | 50;10,30 | 27 | 1;53,51 | 49;32,15 | 27 | 1;52,30 | 48;54,54[4] |
| 30 | 2; 8, 0 | 55;45, 0 | 30 | 2; 6,30 | 55; 2,30 | 30 | 2; 5, 0 | 54;21, 0 |
| 33 | 2;20,48 | 61;19,30 | 33 | 2;19, 9[5] | 60;32,45 | 33 | 2;17,30 | 59;47, 6 |
| 36 | 2;33,36[6] | 66;54, 0[7]↓ | 36 | 2;31,48 | 66; 3, 0 | 36 | 2;30, 0 | 65;13,12 |
| 39 | 2;46,24 | 72;28,30 | 39 | 2;44,27 | 71;33,15 | 39 | 2;42,30 | 70;39,18 |
| 42 | 2;59,12 | 78; 3, 0 | 42 | 2;57, 6 | 77; 3,30 | 42 | 2;55, 0 | 76; 5,24 |
| 45 | 3;12, 0 | 83;37,30[8] | 45 | 3; 9,45[9] | 82;33,45 | 45 | 3; 7,30 | 81;31,30 |
| 48 | 3;24,48 | 89;12, 0 | 48 | 3;22,24 | 88; 4, 0 | 48 | 3;20, 0[10] | 86;57,36[11] |
| 51 | 3;37,36[12] | 94;46,30 | 51 | 3;35, 3 | 93;34,15 | 51 | 3;32,30 | 92;23,42 |
| 54 | 3;50,24 | 100;21, 0 | 54 | 3;47,42 | 99; 4,30 | 54 | 3;45, 0 | 97;49,48 |
| 57 | 4; 3,12 | 105;55,30 | 57 | 4; 0,21 | 104;34,45 | 57 | 3;57,30 | 103;15,54 |
| 60 | 4;16, 0 | 111;30, 0⊥ | 60 | 4;13, 0 | 110; 5, 0[13] | 60 | 4;10, 0 | 108;42, 0 |

**P** omits the column for lunar velocity 14;0. [1] **Y** 2″ [2] **Y** 14′ [3] **Y** 23°24′ [4] **P** 14″ [5] **D** 19″ [6] **B** 38′ corrected to 33′ (?) [7] **Y** 36–60° (minutes): miscopied from the 'part of the distance' for lunar velocity 14;0 [8] **Y** + 11′ (instead of 9′) [9] **D** om. degrees (causing also the next two values to be misread) [10] **P** 25′ [11] **D** 17′ [12] **D** 37″ [13] **Y** 7′.

**Table 49^bis^: Conjunction and opposition (sixth part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | |
|---|---|---|---|---|---|
| elongation | lunar velocity 14;20 | | elongation | lunar velocity 14;30 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4, 7 | 1;47,22 | 1 | 0; 4, 4 | 1;46, 2 |
| 2 | 0; 8,14 | 3;34,44[1] | 2 | 0; 8, 8 | 3;32, 4[2] |
| 3 | 0;12,21 | 5;22, 6 | 3 | 0;12,12 | 5;18, 6 |
| 4 | 0;16,28 | 7; 9,28 | 4 | 0;16,16 | 7; 4, 8 |
| 5 | 0;20,35 | 8;56,50 | 5 | 0;20,20 | 8;50,10 |
| 6 | 0;24,42 | 10;44,12 | 6 | 0;24,24 | 10;36,12 |
| 7 | 0;28,49 | 12;31,34 | 7 | 0;28,28 | 12;22,14 |
| 8 | 0;32,56 | 14;18,56 | 8 | 0;32,32 | 14; 8,16 |
| 9 | 0;37, 3 | 16; 6,18 | 9 | 0;36,36 | 15;54,18 |
| 10 | 0;41,10 | 17;53,40 | 10 | 0;40,40 | 17;40,20 |
| 11 | 0;45,17 | 19;41, 2 | 11 | 0;44,44[3] | 19;26,22 |
| 12 | 0;49,24 | 21;28,24 | 12 | 0;48,48 | 21;12,24 |
| 13 | 0;53,31 | 23;15,46[4] | 13 | 0;52,52 | 22;58,26 |
| 14 | 0;57,38 | 25; 3, 8 | 14 | 0;56,56 | 24;44,28 |
| 15 | 1; 1,45 | 26;50,30[5]↓ | 15 | 1; 1, 0 | 26;30,30 |
| 18 | 1;14, 6 | 32;12,36 | 18 | 1;13,12 | 31;48,36 |
| 21 | 1;26,27 | 37;34,42 | 21 | 1;25,24 | 37; 6,42 |
| 24 | 1;38,48 | 42;56,48⊥ | 24 | 1;37,36 | 42;24,48 |
| 27 | 1;51, 9 | 48;18,54 | 27 | 1;49,48 | 47;42,54 |
| 30 | 2; 3,30 | 53;41, 0 | 30 | 2; 2, 0 | 53; 1, 0[6] |
| 33 | 2;15,51 | 59; 3, 6 | 33 | 2;14,12 | 58;19, 6 |
| 36 | 2;28,12 | 64;25,12[7] | 36 | 2;26,24 | 63;37,12 |
| 39 | 2;40,33 | 69;47,18 | 39 | 2;38,36 | 68;55,18 |
| 42 | 2;52,54 | 75; 9,24 | 42 | 2;50,48 | 74;13,24 |
| 45 | 3; 5,15 | 80;31,30 | 45 | 3; 3, 0 | 79;31,30 |
| 48 | 3;17,36 | 85;53,36 | 48 | 3;15,12 | 84;49,36 |
| 51 | 3;29,57 | 91;15,42 | 51 | 3;27,24 | 90; 7,42 |
| 54 | 3;42,18 | 96;37,48 | 54 | 3;39,36 | 95;25,48 |
| 57 | 3;54,39 | 101;59,54 | 57 | 3;51,48 | 100;43,54[8] |
| 60 | 4; 7, 0 | 107;22, 0[9] | 60 | 4; 4, 0 | 106; 2, 0 |

[1] **Y** 45″ **D** 42″ [2] **Y** 3″ [3] **Y** 45′45″ [4] **Y** 25′47″ [5] **Y** 15–24° (seconds): slide[+1] [6] **Y** 47′ [7] **D** 26′ [8] **Y** 48′ [9] **Y** 87°.

**Table 49$^{bis}$: Conjunction and opposition (seventh part)**

*Sources*: **Y** fols 311v–313v; **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, Bibliothèque nationale de France, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, Bibliothèque nationale de France, arabe 2486, fols 124v–127r (without the column for lunar velocity 14;0).

| Conjunction and Opposition | | | | | |
|---|---|---|---|---|---|
| elongation | lunar velocity 14;40 | | elongation | lunar velocity 14;50 | |
| | part of the distance | hours of the distance | | part of the distance | hours of the distance |
| | ° ′ ″ | h m s | | ° ′ ″ | h m s |
| 1 | 0; 4, 1 | 1;44,45 | 1 | 0; 3,59 | 1;43,31[1] |
| 2 | 0; 8, 2 | 3;29,30 | 2 | 0; 7,58 | 3;27, 2 |
| 3 | 0;12, 3 | 5;14,15 | 3 | 0;11,57 | 5;10,33 |
| 4 | 0;16, 4 | 6;59, 0 | 4 | 0;15,56 | 6;54, 4 |
| 5 | 0;20, 5 | 8;43,45 | 5 | 0;19,55 | 8;37,35 |
| 6 | 0;24, 6 | 10;28,30 | 6 | 0;23,54 | 10;21, 6 |
| 7 | 0;28, 7 | 12;13,15 | 7 | 0;27,53 | 12; 4,37 |
| 8 | 0;32, 8 | 13;58, 0 | 8 | 0;31,52 | 13;48, 8 |
| 9 | 0;36, 9 | 15;42,45 | 9 | 0;35,51 | 15;31,39 |
| 10 | 0;40,10 | 17;27,30 | 10 | 0;39,50 | 17;15,10 |
| 11 | 0;44,11 | 19;12,15 | 11 | 0;43,49 | 18;58,41 |
| 12 | 0;48,12 | 20;57, 0[2] | 12 | 0;47,48 | 20;42,12 |
| 13 | 0;52,13 | 22;41,45 | 13 | 0;51,47 | 22;25,43 |
| 14 | 0;56,14 | 24;26,30 | 14 | 0;55,46 | 24; 9,14 |
| 15 | 1; 0,15 | 26;11,15 | 15 | 0;59,45 | 25;52,45 |
| 18 | 1;12,18 | 31;25,30 | 18 | 1;11,42 | 31; 3,18 |
| 21 | 1;24,21 | 36;39,45 | 21 | 1;23,39 | 36;13,51 |
| 24 | 1;36,24 | 41;54, 0 | 24 | 1;35,36 | 41;24,24 |
| 27 | 1;48,27[3] | 47; 8,15 | 27 | 1;47,33 | 46;34,57 |
| 30 | 2; 0,30 | 52;22,30 | 30 | 1;59,30 | 51;45,30 |
| 33 | 2;12,33 | 57;36,45 | 33 | 2;11,27 | 56;56, 3[4] |
| 36 | 2;24,36 | 62;51, 0[5] | 36 | 2;23,24 | 62; 6,36 |
| 39 | 2;36,39 | 68; 5,15[6] | 39 | 2;35,21 | 67;17, 9 |
| 42 | 2;48,42 | 73;19,30 | 42 | 2;47,18 | 72;27,42 |
| 45 | 3; 0,45 | 78;33,45 | 45 | 2;59,15 | 77;38,15 |
| 48 | 3;12,48 | 83;48, 0 | 48 | 3;11,12 | 82;48,48 |
| 51 | 3;24,51 | 89; 2,15 | 51 | 3;23, 9 | 87;59,21[7] |
| 54 | 3;36,54 | 94;16,30 | 54 | 3;35, 6 | 93; 9,54[8] |
| 57 | 3;48,57 | 99;30,45 | 57 | 3;47, 3 | 98;20,27[9] |
| 60 | 4; 1, 0 | 104;45, 0 | 60 | 3;59, 0 | 103;31, 0 |

**B** omits the arguments for the column for lunar velocity 14;50. **Y** adds a column for lunar velocity 15;0 without tabular values. [1] **Y** 0″ [2] **Y** 56′ **B** 17′ [3] **B** 26″ [4] **P** 57° [5] **Y** 11′ [6] **Y** 0′ [7] **P** 29″ [8] **D** 10′ [9] **D** 26″.

## Table 50: Lunar distance from the Earth

*Sources*: **F** fol. 86v, **H** fols 79v–80r, **C** fol. 83v, $\mathbf{C_1}$ 46v, $\mathbf{C_2}$ -, **Y** fol. 307v, **L** fol. 79v, **B** pp. 154–155.

Distance of the Moon from the Earth
at the times of conjunctions, oppositions and the visibility of the lunar crescent

| epicycle | | double elongation 360° 0° | 355° 5° | 350° 10° | 345° 15° | 340° 20° | 335° 25° | 330° 30° | 325° 35° | 320° 40° | 315° 45° |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ | ° ′ |
| 0$^s$ 0 | 0$^s$ 0 | 65;15 | 65;12[1] | 65; 4[2] | 64;50 | 64;30[3] | 64; 5[4] | 63;36 | 63; 2[5] | 62;23 | 61;39 |
| 0$^s$ 6 | 11$^s$ 24 | 65;14 | 65;11 | 65; 3 | 64;49 | 64;29 | 64; 4[6] | 63;36 | 63; 2 | 62;23 | 61;39 |
| 0$^s$ 12 | 11$^s$ 18 | 65;10 | 65; 7 | 64;59 | 64;45[7] | 64;26 | 64; 1 | 63;32 | 62;59 | 62;21 | 61;37 |
| 0$^s$ 18 | 11$^s$ 12 | 65; 4 | 65; 1 | 64;53 | 64;39 | 64;20 | 63;55[8] | 63;27 | 62;53 | 62;15 | 61;32 |
| 0$^s$ 24 | 11$^s$ 6 | 64;53 | 64;50 | 64;42 | 64;28 | 64; 9 | 63;45 | 63;17 | 62;43 | 62; 6 | 61;23 |
| 1$^s$ 0 | 11$^s$ 0 | 64;39 | 64;36 | 64;28 | 64;14[9] | 63;55 | 63;31 | 63; 3 | 62;30 | 61;53 | 61;10 |
| 1$^s$ 6 | 10$^s$ 24 | 64;22 | 64;19[10] | 64;11 | 63;58 | 63;39 | 63;14 | 62;46 | 62;14 | 61;37 | 60;54[11] |
| 1$^s$ 12 | 10$^s$ 18 | 64; 2 | 63;59 | 63;51 | 63;38 | 63;18 | 62;54 | 62;26 | 61;54[12] | 61;16 | 60;34 |
| 1$^s$ 18 | 10$^s$ 12 | 63;40 | 63;37 | 63;29 | 63;16 | 62;56 | 62;32 | 62; 5 | 61;32 | 60;54 | 60;12 |
| 1$^s$ 24 | 10$^s$ 6 | 63;15 | 63;12 | 63; 4 | 62;51 | 62;31 | 62; 7 | 61;40 | 61; 7 | 60;30 | 59;47[13] |
| 2$^s$ 0 | 10$^s$ 0 | 62;48 | 62;45 | 62;37 | 62;24 | 62; 4 | 61;40 | 61;12 | 60;39 | 60; 1 | 59;19 |
| 2$^s$ 6 | 9$^s$ 24 | 62;19 | 62;16 | 62; 8 | 61;55[14] | 61;35 | 61;11 | 60;43 | 60;10 | 59;32 | 58;49[15] |
| 2$^s$ 12 | 9$^s$ 18 | 61;48 | 61;45 | 61;37 | 61;24 | 61; 4 | 60;40 | 60;11 | 59;38[16] | 59; 0 | 58;17[17] |
| 2$^s$ 18 | 9$^s$ 12 | 61;17 | 61;14 | 61; 6 | 60;53 | 60;33 | 60; 8 | 59;40 | 59; 7 | 58;29 | 57;45 |
| 2$^s$ 24 | 9$^s$ 6 | 60;45[18] | 60;42 | 60;34[19] | 60;21 | 60; 0 | 59;36 | 59; 7 | 58;33 | 57;54[20] | 57;11 |
| 3$^s$ 0 | 9$^s$ 0 | 60;12[21] | 60; 9 | 60; 1 | 59;48 | 59;27 | 59; 2 | 58;33[22] | 57;59[23] | 57;20 | 56;36 |
| 3$^s$ 6 | 8$^s$ 24 | 59;39 | 59;36 | 59;28 | 59;15 | 58;54 | 58;28 | 57;59 | 57;25 | 56;46 | 56; 1 |
| 3$^s$ 12 | 8$^s$ 18 | 59; 7 | 59; 4 | 58;56[24] | 58;43 | 58;21 | 57;56 | 57;26[25] | 56;51 | 56;11 | 55;27 |
| 3$^s$ 18 | 8$^s$ 12 | 58;34 | 58;31 | 58;23[26] | 58;10 | 57;48 | 57;22 | 56;53 | 56;18[27] | 55;38 | 54;53[28] |
| 3$^s$ 24 | 8$^s$ 6 | 58; 3 | 58; 0 | 57;52[29] | 57;39 | 57;17 | 56;51 | 56;21 | 55;46 | 55; 6 | 54;21[30] |
| 4$^s$ 0 | 8$^s$ 0 | 57;32 | 57;29 | 57;21 | 57; 8 | 56;46[31] | 56;20 | 55;50 | 55;15 | 54;35[32] | 53;49 |
| 4$^s$ 6 | 7$^s$ 24 | 57; 5[33] | 57; 2 | 56;54 | 56;41 | 56;19 | 55;53 | 55;22 | 54;47 | 54; 6 | 53;21 |
| 4$^s$ 12 | 7$^s$ 18 | 56;38 | 56;35 | 56;27 | 56;14 | 55;52[34] | 55;26[35] | 54;55 | 54;10[36] | 53;40 | 52;54 |
| 4$^s$ 18 | 7$^s$ 12 | 56;14 | 56;11 | 56; 3 | 55;50 | 55;28[37] | 55; 2[38] | 54;32 | 53;56 | 53;16 | 52;30 |
| 4$^s$ 24 | 7$^s$ 6 | 55;52 | 55;49 | 55;41 | 55;28 | 55; 5[39] | 54;40 | 54;10 | 53;34 | 52;53[40] | 52; 8 |
| 5$^s$ 0 | 7$^s$ 0 | 55;33 | 55;30[41] | 55;22 | 55; 8 | 54;47 | 54;21 | 53;51 | 53;16 | 52;35 | 51;50 |
| 5$^s$ 6 | 6$^s$ 24 | 55;17[42] | 55;14 | 55; 6 | 54;52 | 54;31 | 54; 5 | 53;35 | 53; 1 | 52;20 | 51;35 |
| 5$^s$ 12 | 6$^s$ 18 | 55; 4 | 55; 1 | 54;53 | 54;39 | 54;18 | 53;53 | 53;23[43] | 52;49 | 52; 9[44] | 51;24[45] |
| 5$^s$ 18 | 6$^s$ 12 | 54;54 | 54;51 | 54;43 | 54;29 | 54; 8 | 53;43[46] | 53;14[47] | 52;41 | 51;59 | 51;15 |
| 5$^s$ 24 | 6$^s$ 6 | 54;48 | 54;45 | 54;37 | 54;23 | 54; 3 | 53;38 | 53; 8 | 52;35 | 51;55[48] | 51;10 |
| 6$^s$ 0 | 6$^s$ 0 | 54;45 | 54;42 | 54;34 | 54;20[49] | 54; 0[50] | 53;35[51] | 53; 6[52] | 52;32 | 51;53 | 51; 9 |

The last three columns in **H** and the last two columns in **BL** appear on a separate page with repeated argument columns. [1] $\mathbf{C_1}$ 14′ [2] $\mathbf{C_1}$ 50′ [3] **L** 4′ [4] $\mathbf{C_1}$ 63° [5] $\mathbf{C_1}$ 23′ [6] **F** 3′ [7] **C** 35′ [8] **F** 64° [9] **C** 63° [10] **F** 11′ **Y** 15′ [11] **C** 12′ [12] **C** om. degrees [13] **C** om. degrees [14] **F** 62° [15] **Y** 47′ [16] **C** 19°18′ [17] **FC** 18° [18] $\mathbf{CC_1}$ 42′ [19] **C** 24′ [20] **C** 18°14′ [21] $\mathbf{CC_1}$ 15′ **Y** 17′ [22] **F** 53° [23] **C** 17°39′ [24] **C** 18°43′ [25] **C** 17°27′ [26] **C** 10′ (?) [27] **F** 55° [28] **L** 57° [29] **H** 32′ [30] **L** 49′ [31] $\mathbf{C_1}$ 43′ [32] $\mathbf{C_1}$ 34′ [33] **C** 17°0′ [34] **C** 14°12′ $\mathbf{C_1}$ 56°9′ **Y** 15° [35] **C** 14°27′ [36] **YLB** 17′ [37] **C** 14° $\mathbf{C_1}$ 38′ **Y** 15° [38] **C** 14° [39] **C** 14° $\mathbf{C_1}$ 9′ **Y** 15° [40] **FC** 12° **CL** 13′ [41] **H** 32′ [42] **F** 16′ [43] **CY** 13° [44] **Y** 19′ [45] **C** 27′ [46] **F** 43°53′ **CL** 48′ [47] **YC** 13° [48] **C** 11°14′ $\mathbf{C_1}$ 54′ [49] **C** minutes dam. [50] **C** dam. [51] **L** 15′ [52] **CY** 13°.

**Tables 51–53: Solar parallax / magnitude of eclipses / prorogations (*tasyīrs*)**

*Sources*: **F** fol. 87r, **H** fols 80r and 81r, **C** fol. 84r, **$C_1$** fol. 47r, **$C_2$** -, **Y** fols 307r and 308v, **L** fols 80r and 81r (for the table of prorogations: only a tabular frame and the title), **B** pp. 155 and 157.

| Solar Parallax | | Equation of Eclipse Digits | | Prorogations (*tasyīrs*) | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | sun | | one zodiacal sign in a solar year | | | | 13 zodiacal signs in a solar year | | | |
| complement of the altitude | par-allax | eclipse digits | corrected digits | | | | | | | | |
| | ′ ″ | | digits ′ | months | ° ′ | days | ° ′ | months | s ° ′ | days | s ° ′ |
| 3 | 0,10 | 1 | 0;20 | | | 1 | 0; 5 | | | 1 | $0^s$ 1; 4 |
| 6[1] | 0,20 | 2 | 1; 0 | 1 | 2;28 | 2 | 0;10 | 1 | $1^s$ 2; 3 | 2 | $0^s$ 2; 8 |
| 9 | 0,29 | 3 | 1;50 | | | 3 | 0;15 | | | 3 | $0^s$ 3;12[2] |
| 12 | 0,38 | 4 | 2;40 | 2 | 4;56 | 4 | 0;20[3] | 2 | $2^s$ 4; 6 | 4 | $0^s$ 4;16 |
| 15 | 0,47 | 5 | 3;40 | | | 5 | 0;25[4] | | | 5 | $0^s$ 5;21 |
| 18 | 0,56 | 6 | 4;40[5] | 3 | 7;24 | 6 | 0;30[6] | 3 | $3^s$ 6;10[7] | 6 | $0^s$ 6;25 |
| 21 | 1, 4 | 7 | 5;50 | | | 7 | 0;34 | | | 7 | $0^s$ 7;29 |
| 24[8] | 1,13 | 8 | 7; 0 | 4 | 9;51 | 8 | 0;39 | 4 | $4^s$ 8;13 | 8 | $0^s$ 8;33 |
| 27 | 1,21 | 9 | 8;20 | | | 9 | 0;44 | | | 9 | $0^s$ 9;37 |
| 30 | 1,29 | 10 | 9;40 | 5 | 12;19 | 10 | 0;49 | 5 | $5^s$ 10;16[9] | 10 | $0^s$ 10;41[10] |
| 33 | 1,37 | 11 | 10;50 | | | 11 | 0;54 | | | 11 | $0^s$ 11;45 |
| 36 | 1,45 | 12 | 12; 0 | 6 | 14;47 | 12 | 0;59 | 6 | $6^s$ 12;19[11] | 12 | $0^s$ 12;49[12] |
| 39 | 1,53 | moon | | | | 13 | 1; 4 | | | 13 | $0^s$ 13;53 |
| 42 | 2, 0 | eclipse digits | corrected digits | 7 | 17;16 | 14 | 1; 9 | 7 | $7^s$ 14;23[13] | 14 | $0^s$ 14;58 |
| 45 | 2, 7 | | | | | 15 | 1;14 | | | 15 | $0^s$ 16; 2 |
| 48 | 2,14 | | digits ′ | 8 | 19;44 | 16 | 1;19[14] | 8 | $8^s$ 16;26 | 16 | $0^s$ 17; 6 |
| 51 | 2,21 | | | | | 17 | 1;24[15] | | | 17 | $0^s$ 18;10 |
| 54[16] | 2,27 | | | 9 | 22;12 | 18 | 1;29[17] | 9 | $9^s$ 18;29 | 18 | $0^s$ 19;14[18] |
| 57 | 2,32 | 1 | 0;30 | | | 19 | 1;34[19] | | | 19 | $0^s$ 20;18 |
| 60[20] | 2,37 | 2 | 1;10 | 10 | 24;39 | 20 | 1;39 | 10 | $10^s$ 20;32[21] | 20 | $0^s$ 21;22 |
| 63 | 2,41 | 3 | 2; 0 | | | 21 | 1;43 | | | 21 | $0^s$ 22;26 |
| 66 | 2,44 | 4 | 3;10 | 11 | 27; 7 | 22 | 1;48 | 11 | $11^s$ 22;36[22] | 22 | $0^s$ 23;30 |
| 69 | 2,47 | 5 | 4;20 | | | 23 | 1;53 | | | 23 | $0^s$ 24;34 |
| 72 | 2,50 | 6 | 5;30 | 12 | 29;35 | 24 | 1;58 | 12 | $12^s$ 24;39 | 24 | $0^s$ 25;39 |
| 75 | 2,53 | 7 | 6;40 | | | 25 | 2; 3 | | | 25 | $0^s$ 26;43 |
| 78 | 2,56 | 8 | 8; 0 | 12;5[23] | 30; 0 | 26 | 2; 8 | 12;5[24] | $13^s$ 0; 0 | 26 | $0^s$ 27;47[25] |
| 81 | 2,58[26] | 9 | 9;10 | | | 27 | 2;13 | | | 27 | $0^s$ 28;51 |
| 84 | 2,59 | 10 | 10;20 | | | 28 | 2;18 | | | 28 | $0^s$ 29;55 |
| 87 | 3, 0 | 11 | 11;20 | | | 29 | 2;23 | | | 29 | $1^s$ 0;59 |
| 90 | 3, 0 | 12 | 12; 0 | | | 30 | 2;28 | | | 30 | $1^s$ 2; 3[27] |

[1] **$C_1$** 7 [2] **C** om. [3] **C** 24′ [4] **C** om. [5] **Y** 17′ [6] **C** 9°11′ (from 4 months) [7] **C** 20′ [8] **C** 25 [9] **C** $4^s$ 6′ [10] **C** 11° [11] **F** $5^s$ [12] **Y** 2° [13] **F** $6^s$ [14] **C** 27°7′ (from 11 months) [15] **C** om. [16] **H** 55 [17] **C** 29°35′ (from 12 months) [18] **Y** 12′ [19] **C** 50° [20] **C** 17 [21] **C** 27° [22] **C** $1^s$ minutes ill. [23] **C$C_1$Y** 12 [24] **C$C_1$Y** 12 [25] **B** 26° [26] **L** 59′ [27] **B** 1°.

**Table 54: Geographical table (first column)**
*Sources*: **F** fol. 87v, **H** fol. 80v (see Plate 14), **C** fol. 84v, $\mathbf{C_1}$ fol. 47v, $\mathbf{C_2}$ -, **Y** fol. 308r, **L** fol. 132r (after the colophon), **B** p. 156. The main set of tables in **L** includes on fol. 80v an empty frame for the geographical table with only the title and headers filled in.

The Longitude⟨s⟩ of the Localities from the Fortunate Isles
and Their Latitude⟨s⟩ from the Equator

| | the names of the localities | | | longitude | latitude |
|---|---|---|---|---|---|
| *no.* | *Arabic* | *transliteration* | K&K | ° ′ | ° ′ |
| **First Climate** | | | | | |
| A1 | (ال)حبشة [1] | Ḥabasha | Habasha | 51;40 | 19;30[2] |
| A2 | (ال)نوبة [3] | Nūba | Dunqula | 63; 0 | 14;30 |
| A3 | صنعا [4] | Ṣanʿā | Sana | 73;30 | 14;30 |
| A4 | عدن | ʿAdan | Aden | 75; 0 | 13; 0 |
| A5 | عمان | ʿUmān | Oman | 94;30[5] | 19;45[6] |
| A6 | سوري [7] [8] | Sūrī | Suri | 135;15 | 5;15 |
| **Second Climate** | | | | | |
| A7 | مدينة [9] [10] | Madīna | Madina | 75;20 | 25; 0 |
| A8 | مكّة [11] | Makka | Mecca | 77;10 | 21;40 |
| A9 | (ال)يمامة[12] | Yamāma | Yamama | 81;45 | 21;30 |
| A10 | هجر[13] | Hajar | Hajar | 83; 0[14] | 24;15[15] |
| A11 | البحرين | al-Baḥrayn | Bahrayn | 84;20 | 25;45 |
| A12 | النيرون[16] | al-Nīrūn | Nirun | 102;20[17] | 23;30 |
| A13 | منصورة[18] | Manṣūra | Mansura al-Sind | 103; 0[19] | 22; 0 |
| **Third Climate**[20] | | | | | |
| A14 | اسكندريّة | Iskandariyya | Alexandria | 60;30[21] | 30;20 |
| A15 | (ال)رملة | al-Ramla | Ramla | 65;40 | 32;40 |
| A16 | بيت المقدس | Bayt al-Maqdis | Jerusalem | 66;30[22] | 32; 0 |
| A17 | قيساريّة[23] | Qaysāriyya | Qaysariya Sham | 68;30[24] | 33;15 |
| A18 | (ال)طبريّة | Ṭabariyya | Tiberias | 68;45[25]↓ | 32; 0 |
| A19 | دمشق | Dimashq | Damascus | 70; 0 | 33; 0 |
| A20 | فسطاط[26] | Fusṭāṭ | Fustat | 73; 0 | 31; 0 |
| A21 | (ال)كوفة | Kūfa | Kufa | 79;30 | 31;50[27] |
| A22 | بغداد | Baghdād | Baghdad | 75; 0[28] | 33; 0[29] |
| A23 | واسط | Wāsiṭ | Wasit | 81;30[30] | 32;20 |
| A24 | (ال)بصرة | al-Baṣra | Basra | 84; 0[31] | 31; 0[32] |
| A25 | (ال)اهواز | al-Ahwāz | Ahwaz | 85; 0 | 30; 0[33] |
| A26 | سينيز[34] | Sīnīz | Siniz | 86;45 | 30; 0[35] |
| A27 | جنّابا | Jannābā | Jannaba | 87;20⊥ | 30; 0[36] |
| A28 | شيراز[37] | Shīrāz | Shiraz | 88; 0[38] | 32; 0[39] |
| A29 | بسا[40] | Basā | Fasa | 88;15[41] | 33;30 |
| A30 | جور[42] | Jūr | Jur / Hur | 88;30[43] | 31;30 |

See p. 243 for the apparatus of this table.

**Table 54: Geographical table (second column)**

*Sources*: **F** fol. 87v, **H** fol. 80v, **C** fol. 84v, $\mathbf{C_1}$ fol. 47v, $\mathbf{C_2}$ -, **Y** fol. 308r, **L** fol. 132r (after the colophon), **B** p. 156.

| The Longitude⟨s⟩ of the Localities from the Fortunate Isles and Their Latitude⟨s⟩ from the Equator | | | | | |
|---|---|---|---|---|---|
| | the names of the localities | | | longitude | latitude |
| *no.* | *Arabic* | *transliteration* | K&K | ° ′ | ° ′ |
| B1 | سابور[1] | Sābūr | Sabur | 88;40 | 30; 0[2] |
| B2 | اصطخر | Iṣṭakhr | Istakhr | 89; 0 | 32; 0[3] |
| B3 | سيراف[4] | Sīrāf | Siraf | 89;30 | 29;30 |
| B4 | سيرجان[5] | Sīrjān | Sirjan | 93; 0 | 32;30 |
| B5[6] | جيرفت[7] | Jīruft | Jiruft | 98; 0 | 31;45 |
| B6 | (ال)محمديّة | Muḥammadiyya | Muhamadiyya | 100; 0[8]↓ | 31;45 |
| B7 | كرمان | Kirmān | Kirman | 100; 0[9] | 30; 0 |
| B8 | كابل | Kābul | Kabul | 110; 0⊥ | 28; 0 |
| **Fourth Climate** | | | | | |
| B9 | عمّوريّة | ʿAmmūriyya | Ammuriya | 63; 0 | 38; 0[10] |
| B10 | طرسوس[11] | Ṭarsūs | Tarsus | 67;40 | 37;15[12] |
| B11 | (ال)مصيصة[13] | Maṣṣīṣa | Massisa | 69;40 | 36; 0 |
| B12 | طرابلس[14] | Ṭarābulus | Tripoli Sham | 70;30 | 34; 0 |
| 15 | | | | | |
| B13 | حلب | Ḥalab | Aleppo | 71; 0 | 35;50 |
| B14 | حمص | Ḥimṣ | Homs | 71; 0 | 33;40 |
| B15[16]↓ | (ال)رقّة | al-Raqqa | Raqqa | 73;15 | 36; 0 |
| B16⊥ | آمد | Āmid | Amid | 75;15[17] | 38; 0 |
| B17 | حرّان[18] | Ḥarrān | Harran | 77; 0 | 37; 0[19] |
| B18 | نصيبين[20] | Naṣībīn | Nisibin | 77;50 | 36; 0 |
| B19[21] | (ال)موصل | al-Mawṣil | Mosul | 78; 0[22] | 36;30 |
| B20[21] | بلد[23] | Balad | Balad | 78;45[24] | 36;20 |
| B21[25] | انطاكية | Anṭākiya | Antioch | 79; 0[26] | 35;30 |
| B22 | سرّ من رأى[27] | Surra man raʾā | Samarra | 80; 0 | 34; 0[28] |
| B23 | شهرزور[29] | Shahrazūr | Shahrazur | 80;20[30] | 37;15[31] |
| B24 | حلوان | Ḥulwān | Hulwan | 81;45 | 34; 0 |
| B25 | نهاوند | Nihāwand | Nahawand | 82; 0 | 36;10[32] |
| B26 | همدان[33] | Hamadān | Hamadan | 83; 0 | 36;10 |
| B27 | قمّ[34] | Qumm | Qumm | 80;15[35] | 34; 0 |
| B28 | اصفهان[36] | Iṣfahān | Isfahan | 84;40[37] | 32; 0[38] |
| B29 | (ال)ريّ | al-Rayy | Rayy | 85; 0[39] | 35;30[40] |
| B30 | قزوين[41] | Qazwīn | Qazwin | 85; 0[42] | 37; 0[43] |
| 44 | | | | | |

See p. 243 for the apparatus of this table.

**Table 54: Geographical table (third column)**
*Sources*: **F** fol. 87v, **H** fol. 80v, **C** fol. 84v, $\mathbf{C_1}$ fol. 47v, $\mathbf{C_2}$ -, **Y** fol. 308r, **L** fol. 132r (after the colophon), **B** p. 156.

The Longitude⟨s⟩ of the Localities from the Fortunate Isles and Their Latitude⟨s⟩ from the Equator

| | the names of the localities | | | longitude | latitude |
|---|---|---|---|---|---|
| *no.* | *Arabic* | *transliteration* | K&K | ° ′ | ° ′ |
| C1 | الديلم | al-Daylam | Daylam | 85; 0 | 38; 0 |
| C2 | دنباوند[1] | Dunbāwand | Dunbawand | 85;30 | 36;15 |
| C3 | سالوس[2] | Sālūs | Shalus | 85;45[3] | 37;50 |
| C4 | رويان[4] | Rūyān | Ruyan | 86;35[5] | 37;10 |
| C5 | آمل | Āmul | Amul | 87;20 | 37;45 |
| C6 | سارية | Sāriya | Sariya | 87;50[6] | 38; 0 |
| C7 | قومس | Qūmis | Qumis | 88;15[7] | 36;25[8] |
| C8 | استاراباد[9] | Astārābād | Astarabad | 89;50 | 38;45 |
| C9 | جرجان | Jurjān | Jurjan | 90; 0 | 36;50 |
| C10 | طوس | Ṭūs | Tus | 92; 0 | 37; 0 |
| C11[10] | سرخس[11] | Sarakhs | Sarakhs | 93;20 | 36; 0[12] |
| C12 | مرو | Marw | Marv | 94;20 | 37;30 |
| C13 | مرو الرود[13] | Marw al-Rūdh | Marv Rud | 95; 0 | 38;50[14] |
| C14 | بخارا | Bukhārā | Bukhara | 97;20[15] | 36;50 |
| C15[16] | بلخ | Balkh | Balkh | 98;30 | 38;40[17] |
| C16 | سمرقند | Samarqand | Samarqand | 99;30[18] | 36;30 |
| **Fifth Climate** | | | | | |
| C17 | رومية[19] | Rūmiya | Rome | 45;25[20] | 41;50 |
| C18 | ملطية[21] | Malaṭya | Malatiya | 71; 0 | 39; 0 |
| C19 | خلاط[22] | Khilāt | Akhlat | 74;50[23] | 39;50 |
| C20 | ارزن[24] | Arzan | Arzan | 76; 0 | 39;15[25] |
| C21 | بردعة[26] | Bardaʿa | Bardhaah | 84; 0[27] | 43; 0 |
| C22 | خوارزم | Khwārizm | Khwarizm | 101;50[28] | 42;10[29] |
| C23 | اسبيجاب[30] | Isbījāb | Isfijab | 108;10[31] | 39;50 |
| C24 | طراز[32] | Ṭarāz | Taraz | 110;30[33] | 40;25 |
| **Sixth Climate** | | | | | |
| C25 | قسطنطينيّة[34] | Qusṭantīniyya | Constantinople | 59;50 | 45; 0 |
| C26 | هرقلة[35] | Hiraqla | Heraqla | 63;20[36] | 46;30[37] |
| C27 | جرزان[38] | Jurzān | Jurzan | 81; 0[39] | 44; 0 |
| C28 | (ال)خزر[40] | Khazar | Khazar | 83; 0[41] | 45; 0 |
| **Seventh Climate** | | | | | |
| C29 | انقرة[42] | Anqara | Ankara | 68; 0 | 48; 0 |
| C30 | مدينة ياجوج | madīna Yājūj | Majuj | 172;30[43] | 63; 0[44] |
| [45] | | | | | |

See p. 243 for the apparatus of this table.

**Table 54: Geographical table (apparatus)**
*Sources*: **F** fol. 87v, **H** fol. 80v, **C** fol. 84v, **$C_1$** fol. 47v, **$C_2$** -, **Y** fol. 308r, **L** fol. 132r (after the colophon), **B** p. 156. Elements between parentheses appear only in part of the indicated sources.

First column (p. 240):

**L** omits all definite article from place names. [1] **YL** add. مدينة [2] **F** 0′ [3] **YLB** add. مدينة [4] **YLB** add. (من) (ال)يمن [5] **L** 97° [6] **$C_1$** 15′ [7] **H** سواري (?) [8] **YLB** add. (من) سرنديب [9] **YLB** add. النبي [10] **YL** add. عليه السلام [11] **Y** add. حرسها الله [12] **Y** البهامه [13] **L** هجره (?) **B** هحد (?) [14] **F** 20′ **C** 33° [15] **$C_1$** 27° [16] **H** بيرون, **YLB** (ال)سروز [17] **FCY** om. dots on *qāf*, **HL** 82° [18] **YL** (ال)منصوريه [19] **FCLB** om. dots on *qāf*, **H** 83° [20] **F** add. الى العموريّة 'up to 'Ammūriyya' [21] **F** 50′ **C** degrees سل [22] **L** 0′ [23] **F** ساربه, **H** قمساريه [24] **L** 15′ [25] **L** A18–A27 (except A23): 0′ [26] **H** ill., **$C_1$** قسطاط, **L** قسطاد مصر, **B** السطاط [27] **L** 0′ [28] **+$CC_1$** 80° [29] **Y** 15′ **B** 25′ [30] **L+** 20′ [31] **L+** 85° [32] **Y** 5′ [33] **C** 32° [34] **$FC_1$** سسس, **H** شينيز, **Y** ششتر, **L** شنىر [35] **C** 33° [36] **C** 31° [37] **$CC_1$** سيراف [38] **C** om. minutes [39] **F** 30° [40] **H** بيشا, **L** فسا [41] **C** om. minutes [42] **$FCC_1YL$** حور [43] **F** 108° **C** dam.

Second column (p. 241):

[1] **H** نيسابور, **Y** بسابور, **L** شابور, **B** بسابور [2] **$C_1L$** 32° [3] **F** 30° [4] **H** شيراف, **B** سراف [5] **H** شيرجان, **C** سرحان [6] **$CC_1$** om. entire entry [7] **H** جيرقت [8] **C** B6–B8: om. dots on *qāf* [9] **+B** om. dots on *qāf* [10] **$CC_1L$** 28° [11] **L** طرطوس [12] **L** 36° [13] **FB** (ال)مصصه [14] **Y** اطرابلس, **L** طرابلوس [15] **$CC_1$** insert B21 here. [16] **L** inserts B15 and B16 at the end of the column. [17] **$C_1$** 73° [18] **H** خزان [19] **L** 36° **B** 30′ [20] **$C_1B$** نصيبين, **L** نصبيه [21] **$C_1$** places B20 before B19. [22] **L** 23° [23] **L** بلده (?) [24] **L** 28° [25] **$CC_1$** place B21 between B12 and B13, roughly in correspondence with the corrected longitude, and here insert Takrīt تكريت with longitude 79;40 and latitude 34;30. [26] **$CC_1$** 69° [27] **H** سر من راق, **C** سر من را, **$C_1$** سر مرى, **B** سر مر راى [28] **L** 35° [29] **H** شهريير, **$C_1B$** سهرور, **L** شهرى زور [30] **Y** 0′ [31] **$C_1$** 34° [32] **H** 14′ [33] **YL** همذان [34] **H** قشم (?) [35] **F** om. degrees, **L** 55′ [36] **F** اصبهان [37] **C** 0′ (corrected to 40?) [38] **F** 34° [39] **F** 84° **C** 83° **$C_1B$** 81° **Y** 83;15 [40] **L** 36°0′ [41] **H** قروين, **$C_1$** قروىں [42] **C** dam. [43] **$C_1$** 34° [44] **L** appends B15 and B16 at the end of the column.

Third column (p. 242):

[1] **$FCC_1$** دباوند, **H** دباولد, **LB** دماوند [2] **H** شالوش [3] **L** 86° [4] **F** ووبان, **L** روبال [5] **CB** 87° **L** 30′ [6] **F** 88°, om. minutes [7] **L** 55′ [8] **$CC_1$** 20′ [9] **H** استاراباذ, **BL** استراباذ [10] **L** inserts C11 and C15 at the end of the column. [11] **H** سرخا [12] **C** 40′ [13] **HC** مروورود, **$C_1LB$** مروروذ, **Y** مرود [14] **F** 15′ [15] **H** 96° [16] **L** inserts C11 and C15 at the end of the column. [17] **F** 5′ [18] **H** 49° **B** 69° (سط) [19] **Y** add. الكبيرة, **L** add. كبير, **B** add. الكبره [20] **F** 9′ **H** 20′ (with the 5 added above it, apparently by the main hand) **L** 95° [21] **L** ملاطه [22] **L** اخلاط [23] **$CC_1$** 78° [24] **L** اىردن, **B** اررت [25] **F** 59° **B** 55′ [26] **H** بردعد, **L** برد ع [27] **F** 85° [28] **C** om. dots on *qāf*, 30′ **Y** 104° **L** 81° [29] **L** 15′ [30] **H** اسبجاب, **L** االسحاب [31] **CYB** om. dots on *qāf*, **L** 88° [32] **H** طرار [33] **F** 50′ **C** om. dots on *qāf*, **L** degrees فى [34] **H** قسطنطين, **C** قسطىطه, **Y** قطنطية, **L** قسزطىة (?), **B** قسطىطىة [35] **H** هرقل [36] **YL** 60° [37] **F** 47° **Y** 16° [38] **H** جرطان [39] **Y** 21° [40] **H** جرم (?), **$CC_1$** حره [41] **F** 88° [42] **H** انفره, **L** عره [43] **F** 54° **L** 42° [44] **L** 68° [45] **L** appends C11 and C15 at the end of the column.

**Table 55: Star table (constellations and stars, first page)**

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fols 310v–311r, **L** fols 81v–82r (only title and headers), **B** pp. 158–159. **CC₁** contain a table with a somewhat different collection of stars, which is here edited on pp. 248–251; stars marked with an asterisk do not appear in the table in **CC₁**.

The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird

| *no.* | the northern constellations | the stars |
|---|---|---|
| A1* | Ursa Minor | the tail of the Smaller Bear, which is the Little Goat *incr* |
| A2* | | the brighter one of the two calves *incr* |
| A3* | Ursa Major | ⟨the star⟩ on the back of the Greater Bear *incr* |
| A4* | | ⟨the star⟩ at the end of its tail *incr* |
| A5 | Boötes | 'the *simāk* armed with a lance' *incr* |
| A6 | Corona Borealis | the bright ⟨star⟩ of Corona Borealis |
| A7 | Lyra | 'the falling eagle' |
| A8 | 'the Hen' (Cygnus) | ⟨the star⟩ on the beak of the hen *incr* |
| A9 | | ⟨the star⟩ on the tail of the hen *incr* |
| A10 | 'the Woman on the Throne' (Cassiopeia) | the front of the throne, which is 'the hand dyed (with henna)' |
| A11* | | ⟨the star⟩ on the chest of the woman *incr* |
| A12* | | ⟨the star⟩ on the knee of the woman *incr* |
| A13 | 'the Carrier of the Head | the right hand of who holds the demon *fatal* |
| A14 | of the Demon' (Perseus) | the luminous ⟨star⟩ on its right side *fatal* |
| A15 | | the bright ⟨star⟩ in the Head of the Demon *fatal* |
| A16 | 'the Rein-Holder' (Auriga) | the shoulder of the charioteer, which is *al-ʿayyūq* |
| A17 | | his right shoulder |
| A18 | 'the Snake Carrier' (Ophiuchus) | ⟨the star⟩ on the head of the snake carrier *incr* |
| A19 | 'the Flying Eagle' (Aquila) | 'the Flying Eagle' |
| A20 | 'the Second Horse' (Pegasus) | the shoulder of the horse *fatal* |

20 stars; in the first ⟨magnitude⟩ 3, in the second ⟨magnitude⟩ 11,
in the third ⟨magnitude⟩ 5, nebulous 1.

| *no.* | the ecliptic constellations | the stars |
|---|---|---|
| A21 | Aries | the brighter one of the two marks |
| A22 | Taurus | the eye of the bull, which is Aldebaran *fatal* |
| A23 | 'the Twins' (Gemini) | the head of the foremost twin |
| A24 | | the head of the rearmost twin |

**Table 55: Star table (constellations and stars, second page)**
*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fols 310v–311r, **L** fols 81v–82r (only title and headers), **B** pp. 158–159.

The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird

| *no.* | the ecliptic constellations | the stars |
|---|---|---|
| B1 | 'the Crab' (Cancer) | the chest of the crab,which is 'the Manger' *fatal* |
| B2 | Leo ('the Lion') | ⟨the star⟩ on the shoulder of the lion *fatal* |
| B3 | | the royal ⟨star⟩, which is the 'Heart of the Lion' |
| B4 | | ⟨the star⟩ at the end of its tail, which is *al-ṣarfa* |
| B5 | 'the Ear of Corn' (Virgo) | 'the unarmed *simāk*' |
| B6 | 'the Balance' (Libra) | the southern scale of the balance *incr* |
| B7 | | the northern scale of the balance *incr* |
| B8 | Scorpius | ⟨the star⟩ between the eyes of the scorpion *incr* |
| B9 | | the heart of the scorpion *fatal* |
| B10 | | ⟨the star that⟩ follows the sting *fatal* |
| B11 | 'the Bow' (Sagittarius) | ⟨the star⟩ on the eye of the archer *fatal* |
| B12 | 'the Bucket' (Aquarius) | the mouth of the southern fish |

16 stars; in the first ⟨magnitude⟩ 5, in the second ⟨magnitude⟩ 6,
in the third ⟨magnitude⟩ 2, nebulous 3.

| *no.* | the southern constellations | the stars |
|---|---|---|
| B13 | 'the Giant' (Orion) | the head of the giant *fatal* |
| B14 | | his right shoulder *fatal* |
| B15 | | his left shoulder |
| B16* | | the foremost of the belt *incr* |
| B17 | | the middle of it |
| B18* | | the last of it *incr* |
| B19 | | his left foot |
| B20 | 'the River' (Eridanus) | the luminous ⟨star⟩ at the end of the river |
| B21 | 'the Greater Dog' (Canis Maior) | the southern Dog-Star |
| B22 | 'the Smaller Dog' (Canis Minor) | the northern Dog-Star |
| B23 | Centaurus | ⟨the star⟩ on ⟨the foot of⟩ the centaur |
| B24 | Argo ('the Ship') | the end of the steering-oar of the ship, which is Suhayl |

12 stars; ⟨of⟩ the first ⟨magnitude⟩ 7, ⟨of⟩ the second ⟨magnitude⟩ 4, nebulous 1.

And the total of these stars is 48 stars: in the first magnitude 15,
in the second 21[1], in the third 7, nebulous 5.

[1] **F** 29.

**Table 55: Star table (coordinates, first page)**

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fols 310v–311r, **B** pp. 158–159, **A** = *Almagest* (according to Kunitzsch, *Der Sternkatalog*, with the longitudes increased by 12°), **E** = al-Battānī, *al-Zīj al-Ṣābi'* (as contained in the manuscript Escorial, RBMSL, árabe 908, fols 226v–237r, with the longitudes increased by 50′). Variants from the star table in **CC**$_1$ have been included here.

| The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird | | | | | | | |
|---|---|---|---|---|---|---|---|
| Northern constellations | | | | | | | |
| *no.* | long. | lat. | magn. | dir. | temp. | *Baily* | *identification* |
| A1* | 72;10 | 66; 0 | 2[1] | N | S Ma | 1 | α Umi |
| A2* | 119;10[2] | 72;10[3] | 2 | N | S Ma | 6 | β UMi |
| A3* | 119;40[4] | 49; 0[5] | 2 | N | Ma | 24 | α UMa |
| A4* | 161;50[6] | 54; 0 | 2 | N | Ma | 35 | η UMa |
| A5 | 189; 0 | 31;30 | 1 | N | J Ma | 110 | α Boo (Arcturus) |
| A6 | 206;40 | 45;30[7] | 2[8] | N | V Me | 111 | α CrB |
| A7 | 269;20[9] | 62; 0[10] | 1 | N | V Me | 149 | α Lyr (Vega) |
| A8 | 284;10[11] | 49;40[12] | 3 | N | V Me | 159 | β Cyg (Cygnus) |
| A9 | 321;10[13] | 60; 0 | 2 | N | V Me [14] | 163 | α Cyg (Deneb) |
| A10 | 19;50[15] | 51;20[16] | 3 | N | S V | 189 | β Cas |
| A11* | 22;50[17] | 46;45 | 3 | N | S V [18] | 179 | α Cas |
| A12* | 32;40 | 45;30 | 3 | N | S V [19] | 182 | δ Cas |
| A13 | 38;40 | 40;20[20] | neb.[21] | N | Ma Me | 191 | $\chi^h$ Per |
| A14 | 46;50 | 30; 0 | 2[22] | N | Ma Me | 197 | α Per (Mirfak/Algenib) |
| A15 | 41;40[23] | 23; 0 | 2↓[24] | N | Ma Me | 202 | β Per (Algol) |
| A16 | 67; 0 | 22;30 | 1 | N | Ma Me | 222 | α Aur (Capella) |
| A17 | 74;50[25] | 20; 0 | 2 | N | Ma Me | 223 | β Aur |
| A18 | 246;50[26] | 36; 0 | 3[27] | N | S [28] | 234 | α Oph |
| A19 | 285;50[29] | 29;10 | 2[30] | N | Ma J [31] | 288 | α Aql (Altair) |
| A20 | 344;10 | 31; 0 | 2⊥[32] | N | Ma Me [33] | 317 | β Peg (Pegasus) |
| Ecliptic constellations | | | | | | | |
| *no.* | long. | lat. | magn. | dir. | temp. | *Baily* | *identification* |
| A21 | 18;30[34] | 7;20 | 3↓[35] | N | Ma S | 362 | γ Ari |
| A22 | 54;40[36] | 5;10 | 1 | S | Ma | 393 | α Tau |
| A23 | 95;20[37] | 9;20[38] | 2 | N | Me [39] | 424 | α Gem (Castor) |
| A24 | 98;40 | 6;15 | 2⊥ | N | Ma | 425 | β Gem (Pollux) |

[1] **AE** 3 [2] **F** 159° [3] **B** 77° corrected to 72°, **A** 50′ [4] **F** 159° [5] **Y** 10′ [6] **F** 7′ [7] **AE** 44° [8] **A** $2^+$ [9] **F** 35′ [10] **F** 42° [11] **A** 286°30′ [$9^s$ 4;30] **CC**$_1$ 0′ [12] **AE** 20′ [13] **F** 4′ [14] **F** Moon V [15] **Y** 49° **CC**$_1$ 40′ [16] **ACC**$_1$ 40′ [17] **B** minutes ill. (40 corrected to 50?), **E** 52° [52;0] [18] **F** om. [19] **B** Ma V [20] **ACC**$_1$ 30′ [21] **B** om. [22] **B** om. [23] **F** 46°50′ (repeated from the previous entry) [24] **B** A15–A20: slide[−2] (extending two rows below the other rows for the northern constellations) [25] **F** 24° [26] **F** 286°7′ **CC**$_1$ 40′ [27] +**F** ⊆, **ACC**$_1$ $3^+$ [28] **F** Ma, **Y** S Me [29] **F** 7′ [30] +**ACC**$_1$ $2^+$ **E** $3^+$ [31] **F** Ma [32] +**AECC**$_1$ $2^-$ [33] **F** Ma [34] **A** 40′ [35] **B** B1–B4: directions written out in full instead of the magnitudes, **ACC**$_1$ $3^-$ **E** $4^-$ [36] **F** 14° [37] **F** 94° [38] **A** 40′ [39] **F** Ma, **H** V.

**Table 55: Star table (coordinates, second page)**
*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fols 310v–311r, **L** fols 81v–82r (only title and headers), **B** pp. 158–159, **A** = *Almagest*, **E** = al-Battānī, *al-Zīj al-Ṣābi'*. Variants from the star table in **$CC_1$** have been included here.

The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird

Ecliptic constellations (continued)

| *no.* | long. | lat. | magn. | dir. | temp. | *Baily* | *identification* |
|---|---|---|---|---|---|---|---|
| B1 | 112;20[1] | 0;40 | neb. | N | Ma Me | 449 | M44 (Praesepe) |
| B2 | 134;10 | 8;30 | 2 | N | S[2] | 467 | γ Leo |
| B3 | 135;10[3] | 0;10 | 1 | N | Ma J | 469 | α Leo (Regulus) |
| B4 | 156;30[4] | 11;50 | 1[5] | N | S V | 488 | β Leo (Denebola) |
| B5 | 188;40 | 2; 0[6] | 1 | S | V[7] | 510 | α Vir (Spica Virginis) |
| B6 | 210; 0[8] | 0;20[9] | 2 | N | S | 529 | α Lib |
| B7 | 214;10[10] | 8;50[11] | 2 | N | S | 531 | β Lib |
| B8 | 227;20[12] | 1;20 | 3 | S[13] | S | 546 | β Sco |
| B9 | 234;40[14] | 4; 0 | 2 | S | Ma | 553 | α Sco (Antares) |
| B10 | 253;10[15] | 13;20[16] | neb. | S | Moon Ma[17] | 567 | M7 |
| B11 | 267;10 | 0;45 | neb.[18] | N | sun[19] Ma[20] | 577 | $\nu^{1,2}$ Sgr |
| B12 | 319; 0[21] | 20;20[22] | 1 | S | S Me | 670 | α PsA (Fomalhaut) |

Southern constellations

| *no.* | long. | lat. | magn. | dir. | temp. | *Baily* | *identification* |
|---|---|---|---|---|---|---|---|
| B13 | 69; 0[23] | 16;30[24] | neb. | S | Ma Me | 734 | λ Ori |
| B14 | 74; 0 | 17; 0 | 1[25] | S | S | 735 | α Ori (Betelgeuse) |
| B15 | 66; 0[26] | 17;30 | 2[27] | S | S J | 736 | γ Ori |
| B16* | 67;20 | 25;21[28] | 2 | S | S J | 759 | δ Ori |
| B17 | 69;20 | 24;50 | 2 | S | S J | 760 | ε Ori |
| B18* | 70;10 | 25;20[29] | 2 | S | S J | 761 | ζ Ori |
| B19 | 61;50 | 31;30 | 1[30] | S | S J | 768 | β Ori |
| B20 | 12;10 | 53;30 | 1 | S | J | 805 | θ Eri |
| B21 | 89;40[31] | 39;10 | 1[32] | S | J | 818 | α CMa (Sirius) |
| B22 | 101;10[33] | 16;10 | 1[34] | S | Me | 848 | α CMi (Procyon) |
| B23 | 200;20 | 41;10[35] | 1 | S | V Ma | 969 | α Cen |
| B24 | 89;10[36] | 75; 0 | 1 | S | S J | 892 | α Car (Canopus) |

[1] **B** 4° (?) [2] **H** S V [3] **$ACC_1$** 134°30′ [**A** 4$^s$ 2;30] **E** 134°50′ [134;0] [4] **B** 136° **E** 155° [154;40] [5] **A** $1^-$ [6] **F** 12° [7] **F** V S [8] **H** 200;10 (?) [9] **A** 40′ **E** 2°0′ [10] **F** 40′ **E** 213°50′ [213;0] [11] **E** 20′ [12] **A** 228° [7$^s$ 6;20] **E** 228°30′ [227;40] [13] **A** N [14] **E** 20′ [233;30] [15] **E** 252°50′ [252;0] [16] **A** 15′ [17] **F** S Ma, **H** sun Ma [18] **E** 3 [19] **F** add. ا [20] **H** Moon Ma [21] **F** 5′ **YB** degrees سط [22] **A** 23°0′ [23] **H** 319° (copied from the previous entry) [24] **A** 13°50′ [25] **AE** $1^-$ [26] **F** 8′ [27] **AE** $2^+$ [28] **A** 24°10′ **E** 24°2′ [29] **AE** 40′ [30] **F** 2 [31] **F** 109° [32] **Y** $1^+$ [33] **F** 20′ [34] **B** لحک (for $1^+$?) [35] **F** tens of degrees ill. [36] **F** 29° **B** 109° (?).

**Table 55a: Star table in the Cairo manuscripts (star names, first column)**
*Sources*: **C** fol. 85r, $\mathbf{C_1}$ fol. 48r (see Plate 15), $\mathbf{C_2}$ -. **FHYB** contain a table with a somewhat different collection of stars, which is here edited on pp. 244–247 (the table in **L** has only a title and headers); stars marked with an asterisk do not appear in the table in **FHYB**.

| The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird | |
|---|---|
| *no.* | the stars |
| C1 | 'the *simāk* armed with a lance' (Arcturus) |
| C2 | the bright ⟨star⟩ of Corona Borealis |
| C3 | 'the falling eagle' (Vega) |
| C4 | the beak of the hen |
| C5* | the chest of the hen |
| C6 | its tail, which is *al-ridf* (Deneb) |
| C7 | the hump of the camel, 'the hand dyed (with henna)' |
| C8 | the wrist of the carrier of the head of the demon *fatal* |
| C9 | the side of the carrier (Mirfak/Algenib) *fatal* |
| C10 | the bright ⟨star⟩ of the head of the demon (Algol) *fatal* |
| C11 | the ⟨left⟩ shoulder of the charioteer, *al-ʿayyūq* (Capella) |
| C12 | his right shoulder |
| C13* | the right foot of the charioteer |
| C14 | the head of the snake carrier |
| C15* | the ⟨right⟩ lower leg of the snake carrier |
| C16 | 'the flying eagle' (Altair) |
| C17* | the tail of the dolphin |
| C18* | the northern ⟨star⟩ of the second horse |
| C19 | the northern ⟨star⟩ of the first horse, the shoulder of the horse |
| C20 | the brighter of the two marks |
| C21 | the eye of the bull, Aldebaran |
| C22* | the eastern horn of the bull |
| C23* | the western horn of the bull |
| C24* | the chest of the bull |
| C25 | the head of the foremost twin (Castor) |
| C26 | the head of the rearmost twin (Pollux) |
| C27* | the foot of the rearmost twin |
| C28* | the knee of the foremost twin |
| C29 | the chest of the crab, the 'manger' (Praesepe) *fatal* |
| C30 | the shoulder of the lion *fatal* |

**C** om. the indications *fatal*.

**Table 55a: Star table in the Cairo manuscripts (star names, second column)**
*Sources*: **C** fol. 85r, $\mathbf{C_1}$ fol. 48r, $\mathbf{C_2}$ -.

| The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird | |
|---|---|
| *no.* | the stars |
| C31 | heart of the lion (Regulus) |
| C32 | the tail of the lion, *al-ṣarfa* (Denebola) |
| C33 | 'the unarmed *simāk*' (Spica Virginis) |
| C34 | the northern scale of the balance |
| C35 | the southern scale |
| C36 | between the eyes of the scorpion |
| C37 | the heart of the scorpion (Antares) *fatal* |
| C38 | ⟨the star⟩ that follows the sting *fatal* |
| C39* | the head of the arrow |
| C40* | the southern end of the bow |
| C41* | the northern end |
| C42* | the right foot |
| C43* | the middle between his shoulders |
| C44* | the northern ⟨star⟩ at the root of the tail |
| C45 | the eye of the archer *fatal* |
| C46* | the tail of the goat |
| C47 | the end of the mouth of the water, in the fish (Fomalhaut) |
| C48* | his right shoulder |
| C49* | his right lower leg |
| C50 | the head of the giant *fatal* |
| C51 | his right shoulder (Betelgeuse) *fatal* |
| C52 | his left shoulder |
| C53 | the middle of the belt |
| C54 | his left foot |
| C55 | the end of the river |
| C56 | the southern *Shiʿrā* (Sirius) |
| C57* | the foremost of the two bright ⟨stars⟩ ... (?) |
| C58 | the northern *Shiʿrā* (Procyon) |
| C59 | the foot of the centaur |
| C60 | Suhayl (Canopus) |

Sixty stars; in the first degree (*qadr*) 15, in the second ⟨degree⟩ 18, in the third ⟨degree⟩ 22, nebulous 5.

**C** om. the indications *fatal* except for stars C37, C38 and C45.

**Table 55a: Star table in the Cairo manuscripts (coordinates, first column)**

*Sources*: **C** fol. 85r, **$C_1$** fol. 48r (see Plate 15), **$C_2$** -, **A** = *Almagest* (according to Kunitzsch, *Der Sternkatalog*, with the longitudes increased by 12°), **E** = al-Battānī, *al-Zīj al-Ṣābi'* (as contained in the manuscript Escorial, RBMSL, árabe 908, fols 226v–237r, with the longitudes increased by 50′).

| The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird | | | | | | | |
|---|---|---|---|---|---|---|---|
| *no.* | long. | lat. | dir. | magn. | temp. | *Baily* | *identification* |
| C1 | $6^s$ 9; 0 | 31;30 | N | 1 | J Ma [1] | 110 | α Boo |
| C2 | $6^s$26;40 | 45;30 | N | 2 | V Me | 111 | α CrB |
| C3 | $8^s$29;20 | 62; 0 | N | 1 | V Me | 149 | α Lyr |
| C4 | $9^s$14;10 [2] | 49;40 | N | $3^+$ | V Me | 159 | β Cyg |
| C5* | $10^s$10;30 | 57;20 [3] | N | 3 | V Me | 162 | γ Cyg |
| C6 | $10^s$21;10 | 60; 0 [4] | N | 2 | V Me | 163 | α Cyg |
| C7 | $0^s$19;40 [5] | 51;40 [6] | N | 3 | S V | 189 | β Cas |
| C8 | $1^s$ 8;40 | 40;30 [7] | N | neb. | Ma Me | 191 | $\chi^h$ Per |
| C9 | $1^s$16;50 | 30; 0 | N | 2 | Ma Me | 197 | α Per |
| C10 | $1^s$11;40 | 23; 0 | N | 2 | Ma Me | 202 | β Per |
| C11 | $2^s$ 7; 0 | 22;30 | N | 1 | Ma Me | 222 | α Aur |
| C12 | $2^s$14;50 [8] | 20; 0 | N | 2 | Ma Me | 223 | β Aur |
| C13* | $2^s$ 8; 0 [9] | 5; 0 | N | $3^+$ | Ma Me | 230 | β Tau |
| C14 | $8^s$ 6;40 [10] | 36; 0 [11] | N | $3^+$ | S | 234 | α Oph |
| C15* | $8^s$ 5;40 | 2;40 [12] | N | $3^+$ [13] | S | 246 | ξ Oph |
| C16 | $9^s$15;50 [14] | 29;10 | N | $2^+$ | Ma J | 288 | α Aql |
| C17* | $9^s$29;40 | 29;10 | N | $3^-$ | S Ma | 301 | ε Del |
| C18* | $11^s$29;20 [15] | 26; 0 | N | $2^-$ | Ma Me | 315 | α And |
| C19 | $11^s$14;10 | 31; 0 | N | $2^-$ | Ma Me [16] | 317 | β Peg |
| C20 | $0^s$18;30 | 7;20 | N | $3^-$ | Ma S [17] | 362 | γ Ari |
| C21 | $1^s$24;40 | 5;10 | S | 1 | Ma | 393 | α Tau |
| C22* | $2^s$ 9;40 | 2;50 [18] | N [19] | 3 | Ma | 398 | ζ Tau |
| C23* | $2^s$ 7;40 [20] | 5; 0 | N | 3 [21] | Ma | 400 | β Tau |
| C24* | $1^s$15;40 [22] | 8; 0 | S | 3 | Ma [23] | 385 | λ Tau |
| C25 | $3^s$ 5;20 | 9;20 [24] | N | 2 | Me [25] | 424 | α Gem |
| C26 | $3^s$ 8;40 | 6;15 | N | 2 | Ma [26] | 425 | β Gem |
| C27* | $2^s$24; 0 | 7;30 | S | 3 | Me | 440 | γ Gem |
| C28* | $3^s$ 0;30 [27] | 2;30 | S | 3 | Me | 434 | ζ Gem |
| C29 | $3^s$22;20 | 0;40 | N | neb. | Ma Me | 449 | M44 |
| C30 | $4^s$14;10 | 8;30 | N | 2 | S Me [28] | 467 | γ Leo |

[1] **C** J Me [2] **$C_1$** 0′ [3] **C** 17° [4] **C** 40°20′ [5] **FHYB** 50′ [6] **FHYB** 20′ **C** 11° [7] **FHYB** 20′ [8] **C** $7^s$ **$C_1$** 54° [9] **A** 67°40′ [$1^s$25;40] **E** [67;10] (i.e., in agreement with Kūshyār) [10] **FHYB** 50′ [**F**: 7′] [11] **C** 37° [12] **$C_1$** 10′ **AE** 15′ [13] **A** $4^+$ [14] **C** 55° [15] **A** 50′ **E** [318;30 for 358;30] (the latter in agreement with Kūshyār) [16] **C** Ma S [17] **C** Ma [18] **A** 30′ [19] **AE** South [20] **C** 4° [21] **A** $3^+$ **E** 3 neb. [22] **E** 20′ [44;30 (possibly corrected to 50′ with a dot)] [23] **C** Me [24] **$CC_1$** 9′ [25] **C** Ma [26] **C** Me [27] **A** 15′ [28] **FHYB** S.

**Table 55a: Star table in the Cairo manuscripts (coordinates, second column)**
*Sources*: **C** fol. 85r, **$C_1$** fol. 48r, **$C_2$** -, **A** = *Almagest*, **E** = al-Battānī, *al-Zīj al-Ṣābi'*.

| The Fixed Stars and their Positions at the Beginning of the Year 301 Yazdigird | | | | | | | |
|---|---|---|---|---|---|---|---|
| *no.* | long. | lat. | dir. | magn. | temp. | *Baily* | *identification* |
| C31 | $4^{s}$14;30[1] | 0;10 | N | 1 | J Ma | 469 | α Leo |
| C32 | $5^{s}$ 6;30 | 11;50 | N | 1 *sīn* | S V | 488 | β Leo |
| C33 | $6^{s}$ 8;40 | 2; 0 | S | 1 | V Me[2] | 510 | α Vir |
| C34 | $7^{s}$ 4;10 | 8;50 | N | 2 | S Me[3] | 531 | β Lib |
| C35 | $7^{s}$ 0; 0 | 0;20 | N | 2 | S Me[4] | 529 | α Lib |
| C36 | $7^{s}$17;20 | 5; 0[5] | S | 3 | S Ma[6] | 546 | β Sco |
| C37 | $7^{s}$24;40 | 4; 0 | S | 2 | Ma J[7] | 553 | α Sco |
| C38 | $8^{s}$13;10 | 13;20 | S | neb. | Moon Ma | 567 | M7 |
| C39* | $8^{s}$16;30 | 6;20[8] | S | 3 | Ma Moon | 570 | γ Sgr |
| C40* | $8^{s}$20; 0 | 10;50[9] | S | 3[10] | J Me | 572 | ε Sgr |
| C41* | $8^{s}$21; 0 | 1;30 | S | 3 | J Me | 573 | λ Sgr |
| C42* | $8^{s}$18;20[11] | 13; 0 | S | 3 | J Me | 594 | η Sgr |
| C43* | $8^{s}$29;20[12] | 4;30 | S | 3[13] | J Me | 590 | τ Sgr |
| C44* | $9^{s}$ 4;50 | 6;30 | S[14] | 2[15] | V S | 583? | υ Sgr? |
| C45 | 8s27;10 | 0;45 | N | neb. | sun Ma | 577 | $\nu^{1,2}$ Sgr |
| C46* | $10^{s}$ 9;50[16] | 2;30[17] | S | 3 | V Ma | 623 | γ Cap |
| C47 | $10^{s}$19; 0[18] | 20;20[19] | S | 1 | S Me[20] | 670 | α PsA |
| C48* | $10^{s}$18;20 | 11; 0 | N | 3 | S Me | 630 | α Aqr |
| C49* | $10^{s}$23;40 | 7;30 | S[21] | 3 | S Me | 646 | δ Aqr |
| C50 | $2^{s}$ 9; 0[22] | 16;30 | S | neb. | Ma Me | 734 | λ Ori |
| C51 | $2^{s}$14; 0[23] | 17; 0 | S | 1 | S J[24] | 735 | α Ori |
| C52 | $2^{s}$ 6; 0 | 17;30 | S | 2 | S J | 736 | γ Ori |
| C53 | $2^{s}$ 9;20 | 24;50 | S | 2 | S J | 760 | ε Ori |
| C54 | $2^{s}$ 1;50 | 31;30 | S | 1 | S J | 768 | β Ori |
| C55 | $0^{s}$12;10[25] | 53;30[26] | S | 1 | J | 805 | θ Eri |
| C56 | $2^{s}$29;40 | 39;10 | S | 1 | J Ma[27] | 818 | α CMa |
| C57* | $2^{s}$ 8; 0 | 57;40[28] | S | 2 | V[29] | 845 | α Col |
| C58 | $3^{s}$11;10[30] | 16;10 | S | 1 | Me Ma[31] | 848 | α CMi |
| C59 | $6^{s}$20;20[32] | 41;10 | S | 1 | V Ma | 969 | α Cen |
| C60 | $2^{s}$29;10[33] | 75; 0 | S | 1 | S J | 892 | α Car |

[1] **FHYB** 15°10′ [2] **F** V S, **HYB** V [3] **FHYB** S [4] **FHYB** S [5] **FHYB** 1°20′ [6] **FHYB** S [7] **FHYB** Ma [8] **C$C_1$** 9′ [9] **C** 30′ [10] **E** 4 [11] **A** 40′ [12] **A** 40′ [13] **A** 4+ [14] **A** North [15] **A** 4 [16] **A** 6° [$9^{s}$24;50 (misread by al-Battānī as 27°?)] [17] **A** 10′ [18] **C** 9° [19] **A** 23°0′ [20] **C** S Ma [21] **E** North (as all surrounding stars) [22] **$C_1$** 21° [23] **$C_1$** 54° [24] **FHYB** S [25] **$C_1$** 8′ [26] **C** 13° [27] **FHYB** J [28] **C** 17° [29] **$C_1$** ill. [30] **C** signs dam. [31] **FHYB** Me [32] **C** signs and degrees dam. [33] **C** signs and degrees dam.

# Part III

# Texts

As the tables themselves have been edited in Part II in transliterated form, this part presents an edition of all textual elements of Book II of the *Jāmiʿ Zīj*, including the introductory section with table of contents, the section on displaced equations and the colophons, as well as the textual elements (paratexts) of all tables. For every table the title, sometimes a subtitle, the name of the argument (in some cases a horizontal as well as a vertical argument), the column and subcolumn headers, and if present any labels are given under separate headers. I have included the explanatory texts and comments found in the tables, between their columns or in the margins. With very few exceptions I have omitted occasional marginal notes by users of the manuscripts, but I have included all textual elements copied in the main hands and especially those that appear to have become part of the manuscript tradition. In those cases where a translation of a textual element is not given in the edition of the table in Part I, it is provided here. A range of common names and headers are represented by standard translations or symbols in the edition of the tables; a full list of such names and symbols can be found below.

I have used the conventions for transcribing and editing that are generally used for Arabic texts in the project *Ptolemaeus Arabus et Latinus*. In particular, the writing has been standardised according to the modern usage of standard Arabic. This implies that *hamza* and *madda* are inserted even if the manuscripts omit them, *alif maqsūra* is standardised as ى, a final long *ī* is always written as ي, words from roots with final radical *wāw* or *yāʾ* are standardised according to modern grammar, and numbers are likewise written in standardised form.[1] *Shadda* is always written except on solar letters after the article *al-* and in the relative pronouns *alladhī*, *allatī*, etc. Vowels are added only occasionally, especially if they appear in the manuscripts or to make clear that a verb form is passive.

Text in the Judaeo-Arabic manuscript **H** is transcribed in exactly the same way as that in the manuscripts in Arabic script. In general, the transliteration of Arabic text and numbers into Hebrew characters is as expected. In order to produce the letters ث *thāʾ*, غ *ghayn*, ض *ḍād*, ظ *ẓāʾ* and خ *khāʾ* respectively the letters ת (for Arabic ت *tāʾ*), ג (ج *jīm*), צ (ص *ṣād*), ט (ط *ṭāʾ*) and כ (ك *kāf*) are supplied with a dot on top. Further dots appear to be used to indicate the abbreviation of words. Several somewhat different shapes for the ligature *alif-lām* (ﭏ) can be found in **H**, namely  and, more rarely,  and . These are used not only for the definite article *al-*, but for almost every occurrence of the letter combination *alif-lām*, for example also in such words as *aṭwāl* or *maṭāliʿ*. *Shadda*s are written occasionally in **H** and words or phrases in Arabic script are sometimes found in amongst the Hebrew script.

[1] On the irregularities in the use of numbers in the manuscripts of Kūshyār's *Jāmiʿ Zīj*, see also Bagheri, *az-Zīj al-Jāmiʿ*, pp. xxxix–xl.

Trivial mistakes in subcolumn headers (such as the confusion of degrees, minutes and seconds) are not included in the apparatuses; nor is the occasional inclusion or omission of the definite article *al-* before subcolumn headers. However, in the general editions of the planetary tables the occurrence of forms with and without article in different manuscripts is indicated by placing *al-* between parentheses, as in (ال)سنون. Occasionally also the inclusion or omission of *wa-* or the appearance of *fa-* instead of *wa-* in single witnesses may be omitted from the apparatuses.

In edited Arabic text, angular brackets ⟨...⟩ indicate letters or words apparently mistakenly omitted by the scribe. In translations these brackets contain words that need to be added to make the sentence fully comprehensible. Additional explanations may be inserted between parentheses, often introduced by 'i.e.'. A question mark indicates an uncertain reading. Texts that offer only minor variants to previously translated texts, that contain obvious errors, or otherwise do not make sense, are not translated.

# List of symbols and other standard vocabulary used in the tables and apparatuses

(Note that the Arabic terms for fractional and integer sexagesimal digits are partially the same and that the context of their occurrence needs to be consulted to determine what they stand for.)

| | | | |
|---|---|---|---|
| r | rotation(s), full circle(s) | *dawr*, pl. adwār | دور، أدوار |
| s | zodiacal sign(s) | *burj*, pl. *burūj* | برج، بروج |
| ° | degree(s) | *daraja*, pl. *daraj* | درجة، درج |
| p | part(s) | *juzʾ*, pl. *ajzāʾ* | جزء، أجزاء |
| f | feet or fingers | *aqdām* or *aṣābiʿ* | أصابع or أقدام |

*Sexagesimal fractional positions (positions beyond seconds appear only in Table 12)*:

| | | | |
|---|---|---|---|
| ′ | minute(s) | *daqīqa*, pl. *daqāʾiq* | دقيقة، دقائق |
| ″ | second(s) | *thānī*, pl. *thawānī* | ثاني، ثواني |
| ‴ | third(s) | *thālith*, pl. *thawālith* | ثالث، ثوالث |
| iv | fourth(s) | *rābiʿ*, pl. *rawābiʿ* | رابع، روابع |
| v | fifth(s) | *khāmis*, pl. *khawāmis* | خامس، خوامس |
| vi | sixth(s) | *sādis*, pl. *sawādis* | سادس، سوادس |

*Sexagesimal integer positions (only in the chronological tables)*:

| | | | |
|---|---|---|---|
| (1) | first position | *awwal* | أوّل |
| (2) | second position | *thānī* | ثاني |
| (3) | third position | *thālith* | ثالث |
| (4) | fourth position | *rābiʿ* | رابع |

*Names of the planets and their motions*:

| | | |
|---|---|---|
| planet(s), star(s) | *kawkab*, pl. *kawākib* | كوكب، كواكب |
| sun | *al-shams* | الشمس |
| moon | *al-qamar* | القمر |
| Saturn | *zuḥal* | زحل |
| Jupiter | *al-mushtarī* | المشتري |
| Mars | *al-mirrīkh* | المرّيخ |
| Venus | *al-zuhara* | الزهرة |
| Mercury | *uṭārid* | عطارد |
| mean motion / position | *al-wasaṭ* | الوسط |
| (mean) anomaly | *al-khāṣṣa* | الخاصّة |
| double elongation | *al-buʿd al-muḍāʿaf* | البعد المضاعف |
| (lunar) node(s) | *al-jawzahar* | الجوزهر |

# Sigla

Manuscripts containing Book II of Kūshyār's *Jāmiʿ Zīj*

| | |
|---|---|
| **B** | Berlin, Staatsbibliothek Preußischer Kulturbesitz, Or. quart. 101/1 |
| **C** | Cairo, Dār al-kutub, *mīqāt* 400 |
| $\mathbf{C_1}$ | Cairo, Dār al-kutub, *mīqāt* 188/2 |
| $\mathbf{C_2}$ | Cairo, Dār al-kutub, *mīqāt* 691 |
| **F** | Istanbul, Süleymaniye Kütüphanesi, Fatih 3418/1 |
| **H** | Ahuan Islamic Art, MS 40 (Judaeo-Arabic) |
| **L** | Leiden, Universiteitsbibliotheek, Or. 8 |
| **Y** | Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3 |

Other manuscripts and works

| | |
|---|---|
| **D** | Paris, Bibliothèque nationale de France, arabe 5968: *Dustūr al-munajjimīn* (*c.* 1110) |
| **E** | Escorial, RBMSL, árabe 908: al-Battānī, *Ṣābiʾ Zīj* ($7^{th}$ c. Hijra) |
| **N** | Nallino, *Al-Battānī sive Albatenii*, vol. II. |
| **A** | Ptolemy, *Almagest* |
| **T** | Ptolemy, *Tetrabiblos* |
| **r** | recomputed or reconstructed tabular value |

Symbols and notations

| | |
|---|---|
| 〈…〉 | Arabic text that was omitted by the author or the scribe; corresponding additions to the translation |
| [...] | Arabic text that should be deleted |
| (...) | Clarifications in the translation of an Arabic text |
| $^{1-}$Arabic text$^{-1}$ | An Arabic phrase of at least two words with a variant |
| $^{1-1}$ Variant | Corresponding apparatus entry for such a phrase |
| Arabic text $^{:1}$ | Indication of a variant for the entire preceding Arabic phrase |
| $^{1:}$ Variant | Corresponding apparatus entry for such a phrase |

## Opening sentences and tables of contents

The tables of contents of Book II of the *Jāmiʿ Zīj* in the seven manuscripts that include it show significant differences. These are not always due to the differences between the tables discussed in Section I.7, and occasionally the tables of contents are even in disagreement with the actual tables. In several of the manuscripts, the table numbers appear to have been inserted in red somewhat arbitrarily after the titles of the tables had been written. No pair of manuscripts have identical table numbers throughout, and manuscripts **C** and **L** show omissions of several numbers and a jump. For these reasons each table of contents is here transcribed separately, with all *abjad* numbers written exactly as in the sources. An overview of the tables in all manuscripts with their numbers in the tables of contents is given in Table A on pp. 39–43.

Fatih Manuscript *Source*: **F** fol 37v.

لله الحمد والمنّة

بسم الله الرحمن الرحيم

قال الكيا أبو الحسن كوشيار بن لبّان: قد فرغت بقوّة الله ومنّه من المقالة الأولى من الزيج الجامع في الحساب والرسائل وبدأت أشرع بمعونة الله وتوفيقه في المقالة الثانية في الجداول التي تقدّمت رسائلها ووجه العمل بها في المقالة الأولى.

**المقالة الثانية من الزيج الجامع في الجداول، ستّة وخمسون نوعًا**

**ا** في أيّام السنين السريانيّة **ب** أيّام السنين العربيّة **ج** أيّام السنين الفارسيّة **د** مدخل السنين السريانيّة **ه** مدخل السنين العربيّة **و** مدخل السنين الفارسيّة **ز** مدخل صوم النصارى **ح** الجيب **ط** السهم **ي** الظلّ الأوّل **يا** الظلّ الثاني **يب** مقدّمات الأوساط **يج** وسط الشمس **يد** حركة الأوجات **يه** تعديل الأيّام بلياليها **يو** تعديل الشمس **يز** وسط القمر **يح** خاصّة القمر **يط** البعد المضعّف **ك** تعديل القمر **كا** وسط الجوزهر **كب** وسط زحل **كج** خاصّة زحل **كد**[1] تعديل زحل **كه** وسط المشتري **كو** خاصّة المشتري **كز** تعاديل المشتري **كح** وسط المرّيخ **كط** خاصّة المرّيخ **ل** تعاديل المرّيخ **لا** وسط الزهرة **لب** خاصّة الزهرة **لج** تعاديل الزهرة **لد**[2] وسط عطارد **له** خاصّة عطارد **لو** تعاديل عطارد **لز** عرض القمر **لح** عرض زحل ومقامه الأوّل **لط** عرض المشتري ومقامه الأوّل **م** عرض المرّيخ ومقامه الأوّل **ما** عرض الزهرة ومقامها الأوّل **مب** عرض عطارد ومقامه الأوّل **مج** الميل الأوّل والثاني

[1] **F** كو [2] **F** لو

**مد** ظلّ الميل الأوّل **مه** مطالع البروج بخطّ الاستواء **مو** مطالع البروج لعرض لو **مز** تعديل نهار أوّل السرطان من عرض لو إلى عرض مو **مح** تعديل النهار لعرض لو **مط** مسير ساعة النيّرين وقطريهما **ن** بعد القمر من الأرض **نا** اختلاف منظر الشمس **نب** تعديل أصابع الكسوفين **نج** التسييرات **ند** طول[1] البلدان **نه** الكواكب الثابتة **نو** معرفة التعاديل الأصليّة من هذه التعاديل.

Judaeo-Arabic Manuscript *Source*: **H** fols 24r–25r.

**المقاله الثانيه في الجداول، ثلاثه وخمسون نوعًا**

**ا** أيّام السنين السريانيّه **ب** أيّام السنين العربيّه **ج** أيّام السنين الفارسيّه **د** مدخل السنين السريانيّه **ه** مدخل السنين العربيّه **و** مدخل السنين الفارسيّه **ز** الجيب **ح** السهم **ط** الظلّ الأوّل **ي** الظلّ الثاني **يا** مقدّمات الأوساط **يب** وسط الشمس **يج** حركة الأوجات **يد** تعديل الأيّام بلياليها **يه** تعديل الشمس **يو** وسط القمر **يز** خاصّة القمر **يح** البعد المضاعف **يط** تعديل القمر **ك** وسط الجوزهر **كا** وسط زحل **كب** خاصّة زحل **كج** تعديل زحل **كد** وسط المشتري **كه** خاصّة المشتري **كو** تعديل المشتري **كز** وسط المرّيخ **كح** خاصّة المرّيخ **كط** تعديل المرّيخ **ل** وسط الزهرة **لا** خاصّة الزهرة **لب** تعديل الزهرة **لج** وسط عطارد **لد** خاصّة عطارد **له** تعديل عطارد **لو** عرض القمر **لز** عرض زحل ومقامه الأوّل **لح** عرض المشتري ومقامه الأوّل **لط** عرض المرّيخ ومقامه الأوّل **م** عرض الزهرة ومقامه (كذا) الأوّل **ما** عرض عطارد ومقامه الأوّل **مب** الميل الأوّل والثاني **مج** ظلّ الميل الأوّل **مد** مطالع البروج بخطّ الاستواء **مه** تعديل النهار لعرض لو **مو** مطالع البروج لعرض لو **مز** مسير ساعات النيّرين وقطريهما **مح** بعد القمر من الأرض **مط** اختلاف منظر الشمس **ن** تعديل أصابع الكسوفين **نا** طول البلدان **نب** التسييرات **نج** الكواكب الثابتة.

الجملة ثلاثة وخمسون نوعًا.

Cairo, *mīqāt* 400 *Source*: **C** fol. 42v.

بسم الله الرحمن الرحيم

**المقالة الثانية في الجداول، أربعة وأربعون نوعًا**

**ا** أيّام السنة السريانيّة **ب** مدخل السنين والشهور السريانيّة **ج** أيّام السنين العربيّة

[1] **F** اطرل

د مدخل السنين والشهور العربيّة ه أيّام السنين الفارسيّة و مدخل السنين والشهور الفارسيّة ز الجيب ح السهم ط الظلّ الأوّل ى الظلّ الثاني يا مقدّمات الأوساط يب وسط الشمس يج حركات الأوجات يد تعديل الأيّام بلياليها يه تعديل الشمس يو أوساط القمر يز تعاديل القمر يز (كذا) وسط الجوزهر يح أوساط زحل يط تعاديل زحل ك أوساط المشتري كا تعاديل المشتري كب أوساط المرّيخ كج تعاديل المرّيخ كد أوساط الزهرة كه تعاديل الزهرة كو أوساط عطارد كز تعاديل عطارد كح عرض القمر كط عروض الكواكب ل مقامات الكواكب لا الميل الأوّل ⟨...⟩ ظلّ الميل لب الميل الثاني لج مطالع البروج بخطّ الاستواء لد مطالع البروج وتعديل نهار لعرض له ل له مسير ساعة النيّرين وقطريهما لو بعد القمر من الأرض لز اختلاف منظر الشمس لح تعديل أصابع الكسوفين ⟨...⟩ التسييرات لط طول البلدان مح الكواكب الثابتة مد في التعاديل الأصليّة التي هي قبل تقريبها.
والحمد لله ربّ العالمين.

Cairo, *mīqāt* 691 *Source*: **$C_2$** fol. 2v (see Plate 2a).

بسم الله الرحمن الرحيم اللّهمّ عونك

**المقالة الثانية في الجداول، أربعة وأربعون نوعًا**

ا في أيّام السنة السريانيّة ب مدخل السنين والشهور السريانيّة ج أيّام السنين العربيّة د مدخل السنين والشهور العربيّة ه أيّام السنين الفارسيّة و مدخل الشهور الفارسيّة وسنيها ز الجيب ح السهم ط الظلّ الأوّل ى الظلّ الثاني يا مقدّمات الأوساط يب وسط الشمس يج حركات الأوجات يد تعديل الأيّام بلياليها يه تعديل الشمس يو أوساط القمر يز تعاديل القمر يح وسط الجوزهر يط أوساط زحل ك تعاديل زحل كا أوساط المشتري كب تعاديل المشتري كج أوساط المرّيخ كد تعاديل المرّيخ كه أوساط الزهرة كو تعاديل الزهرة كز أوساط عطارد كح تعاديل عطارد كط عرض القمر ل عروض الكواكب لا مقامات الكواكب لب الميل الأوّل لج ظلّ الميل لد الميل الثاني له مطالع البروج بخطّ الاستواء لو مطالع البروج وتعديل نهار لعرض له ل لز مسير ساعة النيّرين وقطريهما لح بعد القمر من الأرض لط اختلاف منظر الشمس م تعديل أصابع الكسوفين ما التسييرات مب طول البلدان مج الكواكب الثابتة مد في التعاديل الأصليّة التي هي قبل تقريبها.
والحمد لله ربّ العالمين وصلّى الله على محمّد النبيّ وآله أجمعين.

Yeni Cami Manuscript *Source*: **Y** fol. 257r.

## المقالة الثانية في الجداول، وهي ثلاث وخمسين نوعًا

| | | |
|---|---|---|
| **ا** أيّام السنين السريانيّة | **ب** أيّام السنين العربيّة | **ج** أيّام السنين الفارسيّة |
| **د** مدخل السنين السريانيّة | **ه** مدخل السنين العربيّة | **و** مدخل السنين الفارسيّة [1] |
| **ز** الجيب | **ح** السهم | **ط** الظلّ الأوّل |
| **ى** الظلّ الثاني | **يا** مقدّمات الأوساط | **يب** وسط الشمس |
| **يج** حركة الأوجات | **يد** تعديل الأيّام بلياليها | **يه** تعديل الشمس |
| **يو** وسط القمر | **يز** خاصّة القمر | **يح** البعد المضاعف |
| **يط** تعاديل القمر | **ك** وسط الجوزهر | **كا** وسط زحل |
| **كب** خاصّة زحل | **كج** تعاديل زحل | **كد** وسط المشتري |
| **كه** خاصّة المشتري | **كو** تعاديل المشتري | **كز** وسط المرّيخ |
| **كح** خاصّة المرّيخ | **كط** تعاديل المرّيخ | **ل** وسط الزهرة |
| **لا** خاصّة الزهرة | **لب** تعاديل الزهرة | **لج** وسط عطارد |
| **لد** خاصّة عطارد | **له** تعديل عطارد | **لو** عرض القمر |
| **لز** عرض زحل ومقامه الأوّل | **لح** عرض المشتري ومقامه الأوّل | **لط** عرض المرّيخ ومقامه الأوّل |
| **م** عرض الزهرة ومقامه الأوّل | **ما** عرض عطارد ومقامه الأوّل | **مب** الميل الأوّل والثاني |
| **مج** ظلّ الميل الأوّل | **مد** مطالع البروج بخطّ الاستواء | **مه** تعديل النهار لعرض له ل |
| **مو** مطالع البروج لعرض له ل | **مز** مسير ساعة النيّرين وقطريهما | **مح** بعد القمر من الأرض |
| **مط** اختلاف منظر الشمس | **ن** تعديل أصابع الكسوفين | **نا** طول البلدان |
| **نب** التسييرات | **نج** الكواكب الثابتة | [**ند**] والحمد لله وحده |

Leiden Manuscript *Source*: **L** fol. 21r.

## المقالة الثانية، وهي خمسة و ثلثون نوعًا سوى معرفة التعاديل الأصليّة

هذه المقالة فيما يحتاج إليه من الجداول في أبواب المقالة الأولى بترتيبها.

**ا** أيّام السنين الروميّة **ب** مدخل السنين الروميّة **ج** أيّام السنين العربيّة **د** مدخل السنين العربيّة **ه** أيّام السنين الفارسيّة **و** مدخل صوم النصارى **ز** الجيب [2] **ح** الظلّ الأوّل **ط** الظلّ بالأقدام **ى** الميل الأوّل **يا** الميل الثاني **يب** ظلّ الميل الأوّل **يج** مطالع البروج بخطّ الاستواء وتعديل نهار أوّل السرطان من عرض يو إلى عرض مه **يد** مطالع البروج لعرض لو وتعديل نهاره **يو** مقدّمات الأوساط **يز** حركة الأوجات

[1] **Y** omits an entry for the Lent table. [2] **L** omits an entry for the versed sine table.

يح وسط الشمس يط تعديل الأيّام بلياليها ك تعديل الشمس كا أوساط القمر وتعاديله كب وسط الجوزهر كج أوساط زحل وتعاديله كد أوساط المشتري وتعاديله كه أوساط المرّيخ وتعاديله كو أوساط الزهرة وتعاديلها 〈...〉 أوساط عطارد وتعاديله كز عرض القمر كح عروض الكواكب كط الاجتماع والاستقبال ل بعد القمر من مركز الأرض لا اختلاف منظر الشمس لب تعديل أصابع كسوف الشمس وأصابع خسوف القمر لج طول البلدان لد التسييرات الوسطى والصغرى له الكواكب الثابتة. معرفة التعاديل الأصليّة من هذه التعاديل المكتوبة. تمّ والحمد لله المنعم.

Berlin Manuscript *Source*: **B** pp. 34–35.

... فإنّا نختم المقالة 〈الأولى〉 بهذا الباب. وتتلوه المقالة الثانية في الجدول. والله أعلم بالصواب.

**بسم الله الرحمن الرحيم**

**وهي ثلاثة وخمسون نوعًا.** ا أيّام السنين السريانيّة ب أيّام السنين العربيّة ج أيّام السنين الفارسيّة د مدخل السنين السريانيّة ه مدخل السنين العربيّة و مدخل السنين الفارسيّة ز الجيب ح السهم ط الظلّ الأوّل ي الظلّ الثاني يا مقدّمات الأوساط يب وسط الشمس يج حركة الأوجات يد تعديل الأيّام بلياليها يه تعديل الشمس يو وسط القمر يز خاصّة القمر يح البعد المضاعف يط تعاديل القمر ك وسط الجوزهر كا وسط زحل كب خاصّة زحل كج تعاديل زحل كد وسط المشتري كه خاصّة المشتري كو تعاديل المشتري كز وسط المرّيخ كح خاصّة المرّيخ كط تعاديل المرّيخ ل وسط الزهرة لا خاصّة الزهرة لب تعاديل الزهرة لج وسط عطارد لد خاصّة عطارد له تعاديل عطارد لو عرض القمر لز عرض زحل ومقامه الأوّل لح عرض المشتري ومقامه الأوّل لط عرض المرّيخ ومقامه الأوّل م عرض الزهرة ومقامها الأوّل ما عرض عطارد ومقامه الأوّل مب الميل الأوّل والثاني مج ظلّ الميل الأوّل مد مطالع البروج بخطّ الاستواء مه تعديل النهار لعرض له ل[1] مو مطالع البروج لعرض له ل[2] مز مسير ساعة النيّرين وقطريهما مح بعد القمر من الأرض مط اختلاف منظر الشمس ن تعديل أصابع الكسوفين نا طول البلدان نب التسييرات نج الكواكب الثابتة.

[1] **B** omits the latitude. [2] **B** omits the latitude.

## General editions

Certain types of tables (especially, the mean motions and the six different types of tables for the planetary equations) appear repeatedly in the *Jāmiʿ Zīj*. To avoid unnecessary repetitions in the edition of their textual elements, and in order to make similarities and differences between such tables visible, I have combined the editions of these tables in what I call a 'general edition'. The sections on the individual mean motion and equation tables only provide variants in the titles of the tables and indicate which explanatory texts and marginal notes are found in and around the tables. For all other variants, the reader is referred to the general editions. Also for the names of the zodiacal signs and the months in the three main calendars used in the *Jāmiʿ Zīj*, I have provided a general edition covering all their occurrences (in the case of the Persian month names, this involves a total of more than 100 witnesses).

The apparatuses of the general editions use the following abbreviations:

| | |
|---|---|
| *Calendrical tables* | |
| CalDays | Tables 1–3, Days of years and months |
| Madkhal | Tables 4–6, *Notae* of years and months |
| *Mean motion tables* | |
| Sun | Table 13, Solar mean motion |
| Moon | Table 17, Lunar mean motion |
| LunAnom | Table 18, Lunar mean anomaly |
| Elong | Table 19, Double elongation |
| Node | Table 21, Mean motion of the lunar node |
| Sat | Table 22, Saturn mean motion |
| SatAnom | Table 23, Saturn mean anomaly |
| Jup | Table 25, Jupiter mean motion |
| JupAnom | Table 26, Jupiter mean anomaly |
| Mars | Table 28, Mars mean motion |
| MarsAnom | Table 29, Mars mean anomaly |
| Ven | Table 31, Venus mean motion |
| VenAnom | Table 32, Venus mean anomaly |
| Mer | Table 34, Mercury mean motion |
| MerAnom | Table 35, Mercury mean anomaly |
| *Planetary equations* | |
| Sat | Table 24, equation tables for Saturn |
| Jup | Table 27, equation tables for Jupiter |
| Mars | Table 30, equation tables for Mars |
| Ven | Table 33, equation tables for Venus |
| Mer | Table 36, equation tables for Mercury |
| *General* | |
| all | all tables concerned |
| others | all tables not explicitly mentioned before |

For example, in the apparatus of the general edition of the mean motion tables, '**F**-Sun+Moon' indicates the tables for the solar and lunar mean motion in **F**. In the general edition of the planetary equations, '**L**-Sat+Mer' stands for the tables for Saturn and Mercury in **L**.

Furthermore, 'Book I' refers to the listings of month names in the section on the three main calendars, namely Chapter I.2 in **CHC$_2$LB** and Chapter I.1.2 in **Y** (note that this section is missing from **F** and that **C$_1$** includes only tables from Book II and no text from Book I).

The scribes of the eight manuscripts of Kūshyār's *Jāmiʿ Zīj* were obviously much more familiar with the Syrian and Arabic month names than with the Persian ones. For the edition of the Persian month names, specific editing rules are therefore given on p. 268.

## General edition: Zodiacal signs

*Sources*: **FHYLB**: tables for right ascension (45/45a) and oblique ascension (46/46a),[a] **B**: table for the equation of time (15a).

| | |
|---|---|
| Aries | الحمل |
| Taurus | الثور |
| Gemini | الجوزاء |
| Cancer | السرطان |
| Leo | الأسد |
| Virgo | السنبلة |
| Libra | الميزان |
| Scorpio | العقرب |
| Sagittarius | القوس |
| Capricorn | الجدي |
| Aquarius | الدلو |
| Pisces | الحوت |

[a] **Y** add. برج before each zodiacal sign in both ascension tables.

## General edition: Syrian / Byzantine month names

*Sources*: Manuscripts **F**, **H**, **C**, $\mathbf{C_2}$, **Y**, **L** and **B**. CalDays = Table 1, Madkhal = Table 4, Book I = Chapter I.2 in **H** (fol. 4r–4v), **C** (fol. 3v), $\mathbf{C_2}$ (fol. 5v), **L** (fol. 3r) and **B** (p. 7), and Chapter I.1.2 in **Y** (fols 232v–233r).

In Chapter I.2 (Chapter I.1.2 in **Y**), Kūshyār explains that the 'Syrian' (*suryānī*) and the 'Byzantine' or 'Greek' (*rūmī*) calendar only differ in their month names (Tishrīn, Kānūn, etc. vs January, February, March, etc.) and in the year beginning (October vs January). Only **L** uses the indication 'Byzantine' consistently (although, like all other manuscripts, it lists the Syrian month names); it also does not mention that the two calendars are basically identical. The other sources generally write 'Syrian', with the sole exception of the *notae* tables in **YB**.

| | | |
|---|---|---|
| Tishrīn al-awwal | تشرين الأوّل[1] | |
| Tishrīn al-ākhir | تشرين الآخر[2] | **CLB**: تشرين الثاني[3] |
| Kānūn al-awwal | كانون الأوّل[4] | |
| Kānūn al-ākhir | كانون الآخر[5] | **CLB**: كانون الثاني[6] |
| Shubāṭ | شباط | |
| Ādhār | آذار | |
| Nīsān | نيسان | |
| Ayyār | ايّار | |
| Ḥazīrān | حزيران[7] | |
| Tammūz | تمّوز | |
| Āb | آب[8] | |
| Aylūl | ايلول[9] | |

[1] **F**-Madkhal, **H**-CalDays+Madkhal, **L**-CalDays+Madkhal: om.; **C**-Madkhal: '1' (Hindu numeral) [2] **F**-Madkhal, **H**-CalDays+Madkhal: om.; **HY**-Book I: الثاني [3] **C**-Madkhal: '2' (Hindu numeral); **L**-CalDays+Madkhal: om. [4] **F**-Madkhal, **H**-CalDays+Madkhal, **L**-CalDays+Madkhal: om.; **C**-Madkhal: '1' (Hindu numeral) [5] **F**-Madkhal, **H**-CalDays+Madkhal: om.; **HY**-Book I: الثاني [6] **C**-Madkhal: '2' (Hindu numeral); **L**-CalDays+Madkhal: om.; **B**-CalDays: الآخر [7] **C**-Madkhal: حران (the month names in **C**-Madkhal are all written sloppily due to lack of space in the column header cells) [8] **Y**-Book I+Madkhal: اب [9] **C** ابلون

## General edition: Arabic month names

*Sources*: Manuscripts **F**, **H**, **C**, **C₂**, **Y**, **L** and **B**. CalDays = Table 2, Madkhal = Table 5, Book I = Chapter I.2 in **C** (fols 3v–4r), **H** (fol. 4v), **C₂** (fols 5v–6r) and **L** (fol. 3r–3v), and Chapter I.1.2 in **Y** (fol. 233r). Chapter I.2 in **B** (pp. 7–8) states only that the Arabic month names are well-known and does not list them.

| | | |
|---|---|---|
| (al-)Muḥarram | (ال)محرّم | |
| Ṣafar | صفر | |
| Rabīʿ al-awwal | ربيع الأوّل[1] | |
| Rabīʿ al-ākhir | ربيع الآخر[2] | |
| Jumādā l-ūlā | جمادى الأولى[3] | **YL**: جمدي[4] الأوّل[5] |
| Jumādā l-ākhira | جمادى الآخرة[6] | **YL**: جمدي[7] الآخر[8], **B**: جمادى الآخر |
| Rajab | رجب | |
| Shaʿbān | شعبان | |
| Ramaḍan | رمضان | |
| Shawwāl | شوّال | |
| Dhū l-qaʿda | ذو[9] القعدة | |
| Dhū l-ḥijja | ذو[10] الحجّة | |

[1] **H**-CalDays, **YL**-Madkhal: om. [2] **C** الثاني; **H**-CalDays, **YL**-Madkhal: om. [3] **H**-CalDays: om.; **C**-CalDays+Madkhal: الأوّل [4] **L**-CalDays+Madkhal: جمادى [5] **Y**-CalDays+Madkhal, **L**-Madkhal: om. [6] **F**-Madkhal, **C**-CalDays+Madkhal: الآخر; **H**-CalDays: om. [7] **L**-CalDays+Madkhal: جمادى [8] **Y**-CalDays+Madkhal, **L**-Madkhal: om. [9] **Y**-CalDays+Madkhal: om. [10] **C**-CalDays+Madkhal: ذى, **Y**-CalDays+Madkhal: om.

## General edition: Persian month names

*Sources*: Manuscripts **F**, **H**, **C**, **$C_1$**, **$C_2$**, **Y**, **L** and **B**. CalDays = Table 3, Madkhal = Table 6, abbreviations for mean motion tables as given on p. 264, Book I = Chapter I.2 in **C** (fol. 4r–4v), **H** (fol. 5r), **$C_2$** (fol. 6r–6v) and **L** (fol. 3v), and Chapter I.1.2 in **Y** (fol. 233r–233v). Chapter I.2 in **B** (pp. 8–9) only mentions 'Isfandārmund' (sic!) three times.

### *Editing rules*

Occasional incorrect omissions or additions of diacritical dots are in general not noted in the apparatus. If a particular source systematically omits or adds diacritical dots in a name on at least two thirds of the occurrences, this form will be given as the main variant for the source; occasional correct writings of the name will then be considered coincidental and are as a rule not listed in the apparatus. For example, سر is given as the main variant for the Persian month name Tīr in **$CC_2L$**, and the correct writing تير in **$C_2$**-MerAnom is considered coincidental. In all variant forms of the month names, the diacritical dots are written as they appear in the source concerned, with correct diacritical dots given if they appear for at least one of the occurrences.

### *General variants*

**FY** do not usually add *māh* ماه (Persian for 'month') after the month names.[a]

**H** generally writes *māh* after all month names in Book I and in the chronological tables, but abbreviates or omits it in the mean motion tables if the table cells do not leave enough room, especially for Farwardīn, Urdibihisht, Khurdād, Murdād, Shahrīr, and Isfandār.[b]

**$CC_1C_2LB$** generally add *māh* after the month names.[c]

**$CC_1C_2$** always write ديماه together, **C** mostly writes اردبهشتماه together, and also in other incidental cases the Cairo manuscripts connect *māh* with the month names.

[a] **F**-Sun: add. *māh* after all month names; **F**-Moon: add. *māh* after Tīr and Day; **F**-LunAnom+VenAnom: add. *māh* after Day; **F**-Sat+SatAnom+Jup+Mer: add. *māh* after Day and Bahman; **F**-JupAnom: add. *māh* except after Farwardīn, Urdībihisht, Khurdād, and Isfandārmudh; **F**-Ven: add. *māh* after Farwardīn, Day and Bahman; **Y**-Madkhal+Moon+LunAnom+Jup+JupAnom+Ven+Mer: add. *māh* after Mihr [b] **H**-CalDays+Madkhal: om. *māh* after Isfandārmudh; **H**-LunAnom+Elong: om. *māh* after Adhar; **H**-Node: om. *māh* after Bahman; **H**-JupAnom: om. *māh* after Ābān; **H**-Mars: om. *māh* after Mihr. Especially in the solar and lunar tables the omission or abbreviation of *māh* appears to be indicated by a dot above the last letter, making it impossible to distinguish between *dāl* and *dhāl* as the last letter of Khurdād and Murdād. [c] **C**-Madkhal om. *māh* after Farwardīn, Urdībihisht, and Khurdād; **L**-CalDays+Mars: om. *māh* after all month names; **L**-Moon: om. *māh* after Urdībihisht; **L**-Node: om. *māh* after Mihr; **B**-LunAnom: om. *māh* after Shahrīr; **B**-Elong (except Farwardīn)+Node+Sat+VenAnom: om. *māh* after all month names.

*Edition of the month names*

| | | |
|---|---|---|
| Farwardīn | فروردين[1] | **C** [2]فرودن |
| Urdībihisht | اردیبهشت[3] | **HC$C_1C_2$** [4]اردبهشت, **B** [5]اربهشت |
| Khurdād | خرداد[6] | **H** [7]كرداد, **C$C_2$** [8]حرداد |
| Tīr | تير[9] | **C$C_2$L** [10]سر, **$C_1$** [11]نير |
| Murdād | مرداد[12] | **HB** [13]مرداذ |
| Shahrīwar | شهريور | **HC$C_1C_2$LB** [14]شهرير |
| Mihr | مهر[15] | |
| Ābān | آبان | **F** [16]اابان, **C$C_2$L** [17]ابان |
| Ādhar | آذر[18] | **F** [19]ااذر, **C$C_2$** [20]ادر |
| Day | دي[21] | **H** [22]ذي, **C$C_1C_2$** [23]ديماه (written together) |
| Bahman | بهمن[24] | |
| Isfandārmudh | اسفندارمد[25] | **HC$C_1$Y** [26]اسفندار, **B** [27]اسفندارمند (sic!) |

[1] **H**-CalDays: פרורדימאה, **H**-Sun: פרורד; **$C_1$**-Moon: فوردن, **$C_1$**-MarsAnom: فرورب, **$C_1$**-MerAnom: فروردن; **L**-Moon: ill. [2] **C**-Book I: فروردن [3] **Y**-Moon+Mars+Mer: اردبهشت; **L**-Node: partially unclear, **L**-Mars: اربهشت [4] **H**-Sun+Moon: ארדבהש, **H**-LunAnom+Elong: ארדבה; **C**-Book I: اردشت, **C**-Sun: final part ill., **C**-others: often contracted to اردبهت; **$C_1$**: several scribal variations, esp. final ت in shape of ز or ن or connected with ماه, and dot on ر; **$C_2$**-Book I: dam. [5] **B**-CalDays+Madkhal: اردبهشت, **B**-Sun: اردسهشب [6] **$C_1$**-LunAnom+Sat+Jup+MarsAnom+Mer: حرداد; **Y**-LunAnom اردیهشب; **LB**: also with initial ح and final ض [7] **H**-Book I: خرداد, **H**-MarsAnom: כרדאדד [8] **C**-Book I+Jup, **$C_2$**-Book I+MerAnom: خرداد, **C**-Moon+JupAnom+Mars+Mer+MerAnom: حراداد, **C**-MarsAnom+VenAnom: خراد, **C**-Ven: خراداد [9] **H**-Sun: תירמא, **H**-LunAnom: תירמ [10] **C**-CalDays+Madkhal, **L**-CalDays: تر; **$C_2$**-Mer: بير, **$C_2$**-MerAnom: تير [11] **$C_1$**-Sun+Mars+MarsAnom: سر, **$C_1$**-Moon: unclear [12] **H**-Book I: مرداذ, **H**-Moon: מרדאדמ; **C**-MerAnom: مرداه; **L**: also مرداذ [13] **B**-Elong+Node+Sat+VenAnom+Mer+MerAnom: مرداد [14] **H** שהרירמ; **C**-Madkhal, **$C_2$**-Node+JupAnom: سهربن; **$C_1$**-Moon+LunAnom: شهريور, **$C_1$**-Node: شهيرمر, **$C_1$**-Jup+JupAnom: شهربر; **$C_2$**-Book I: dam.; **L**-Jup: شهر [15] **C**-LunAnom: مهربر; **$C_1$**-Moon: محهر (?); **$C_2$**-Book I: dam. [16] **F**-CalDays: ابان [17] **C**-Book I+Mer, **L**-Book I+Sun+Moon: ابان [18] **H**-CalDays: אֹדרממאה, **H**-LunAnom+Elong: اذار, **H**-Jup+JupAnom: ادر; **Y**-Node: ااذر, **Y**-Sat: اذرر [19] **F**-CalDays+LunAnom: آدر [20] **C**-CalDays: ادربماه (دىماه corrected to ادر ماه) [21] **Y** (repeatedly): ذي [22] **H**-CalDays+Madkhal+Node+Sat+SatAnom+JupAnom+Mars: دي [23] **C**-Mars: unclear (ه ابان ?); **$C_1$**-LunAnom+SatAnom+MarsAnom+Mer: ذيماه, **$C_1$**-JupAnom: دى ماه; **$C_2$**-Jup+JupAnom+Mars+MarsAnom: دي ماه [24] **H**-Moon+LunAnom: בהמןמא; **Y**-MarsAnom: اذر [25] **$C_2$**-Jup, **L**-Book I+SatAnom+JupAnom: اسفندارمذ [26] **H**-Book I+CalDays+Madkhal: اسفندرمذ; **C**-Book I+Moon: اسفندارمد, **C**-Madkhal: اسفنداد ماه, **C**-MerAnom: اسفندبا ماه; **Y**-Book I: اسفندارمذ ماه; **L** ار often reads as لر [27] **B** ار often reads as لر, **B**-CalDays+Moon+LunAnom+Elong+Node+Sat+Mars: اسفندارمذ, **B**-Jup: اسفندارمد

### Table 1: Days of Syrian years and months

*Sources*: **F** fol. 38r, **H** fol. 25v, **C** fol. 43r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 257v, **L** fol. 21v, **B** p. 36.

Title: أيّام[1] السنين السريانيّة[2] [3-]تُؤخذ[4] بالسنين والشهور التامّة[-3]

Subtable headers: أيّام[5] السنين[6] المجموعة / أيّام[7] السنين[8] المبسوطة / أيّام[9] الشهور

Arguments: (ال)سنون / (ال)سنون / (ال)شهور

Subcolumn headers: رابع / ثالث / ثاني / أوّل

Label for the values for a leap year: [a] ‘⟨in a⟩ leap ⟨year⟩’ كبيسة[10]

For the Syrian month names, see the general edition on p. 266.

*Explanatory texts*

**A. On how to use the table for intercalary years**

*Sources*: **F** in between the subtables for extended years and months (vertically), **L** in the left margin (vertically).

إذا[11] كانت السنة الناقصة كبيسة، أخذنا أيّام شباط[12] وما بعده من الشهور بزيادة يوم [13] [14-]على ما أُثبت بالحمرة[-14].

When the incomplete year is ⟨a⟩ leap ⟨year⟩, we take the days of Shubāṭ (February) and the months after it with an increase of a (**L** add. ‘single’) day, according to what was recorded in red.

*Sources*: **CB** in between the subtables for extended years and months (vertically).

إذا[15] كانت السنة [16-]التي تليها[-16] سنة كبيسة[17]، أخذنا من الشهور[18] حروف الكبيسة.

When the year that follows it is an intercalary year, we take from the ⟨subtable for⟩ months the letters of (i.e., corresponding to) the intercalary ⟨year⟩.

[a] Not in **YLB**.

[1] **Y** om. [2] **L** الروميّة ‘Byzantine’ [3–3] **H** بالسنة والشهر التامّ [4] **FCB** بوخذ, **YL** يوخذ [5] **YLB** om. [6] **L** السنون [7] **YLB** om. [8] **L** السنون [9] **YLB** om. [10] **C** أوّل ‘first ⟨position⟩’ (of the values for a leap year) [11] **L** إن [12] **L** الشباط [13] **L** add. واحد [14–14] **L** om. [15] **C** إن [16–16] **C** يلها [17] **CB** الكبيسة [18] **B** السنين

### B. On the numbers of days between the Syrian, Arabic and Persian eras

*Sources*: **FHCYLB** left bottom corner (under the subtable for months, **FHCB** vertically).[a]

This era is earlier than the Arabic ⟨one⟩ by

[1-]هذا التأريخ[-1] أقدم من العربيّ بأيّام[2]

| | | | | |
|---|---|---|---|---|
| 1 | 34 | 38 | 20 | |
| fourth | third | second | first | ⟨position⟩ |

| | | | |
|---|---|---|---|
| آ | لد | لح | ك |
| رابع | ثالث | ثاني | أوّل |

days, and it is earlier than the Persian ⟨one⟩ by

وهو[3] أقدم من الفارسيّ[4] بأيّام[5]

| | | | | |
|---|---|---|---|---|
| 1 | 35 | 38 | 44 | |
| fourth | third | second | first | ⟨position⟩ |

| | | | |
|---|---|---|---|
| آ | له | لح | مد |
| رابع | ثالث | ثاني | أوّل |

days.

### C. On the number of days between the Byzantine era and the Flood

*Source*: **B** in between the subtables for collected and extended years (vertically).

بين تأريخ الروم وتأريخ الطوفان ١٠١٩٢٧٣ يومًا، يكون ألفي وسبعمائة واثنى وتسعين سنة ومائة و ثلاثة وتسعين يومًا.

Between the Byzantine era and the era of the Flood there are 1,019,273 days, that is, 2792 years and 193 days.

## Table 2: Days of Arabic years and months

*Sources*: **F** fol. 38v, **H** fol. 27r (after the Arabic *notae*), **C** fol. 44r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 258v, **L** fol. 22v, **B** p. 37.

Title: [6-]أيّام السنين[-6] العربيّة [7-]تُؤخذ[8] بالسنين[9] والشهور التامّة[-7]

Subtable headers: أيّام[10] السنين[11] المجموعة / أيّام[12] السنين[13] المبسوطة / أيّام[14] الشهور

Arguments: (ال)سنون / (ال)سنون[15] / (ال)شهور

Subcolumn headers: رابع / ثالث / ثاني / أوّل

For the Arabic month names, see the general edition on p. 267.

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[a] **H** writes the position indicators 'first', 'second', 'third' and 'fourth' vertically with respect to the note (with those for the Persian epoch upside down on the page). **C** exchanges the two numbers and gives the position indicators for the digits only once and vertically (i.e., upside down on the page). **Y** places the sexagesimal digits and the position indicators in cells.

[1–1] **HCYB** om. [2] **Y** om. [3] **HCYL** و, **B** om. [4] **C** العربيّ (overwritten with الفارسيّ in a different hand) [5] **Y** om. [6–6] **Y** السنون [7–7] **H** بالسنة والشهر التامّ [8] **FY** بوخذ, **LB** يوخذ [9] **Y** بالسنون [10] **YLB** om. [11] **YL** السنون [12] **YLB** om. [13] **YL** السنون [14] **YLB** om. [15] **Y** المبسوطة

### *Explanatory texts*

**A. On the number of days between the Arabic and Persian eras**

*Sources*: **FHCYLB** left bottom corner (under the subtable for months, **HCB** vertically).[a]

This era is earlier than the Persian ⟨one⟩ by — [1-]هذا[2] التأريخ[-1] أقدم من الفارسيّ بأيّام[3]

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 0 | 24 | | كد | ح | آ |
| third | second | first | ⟨position⟩ | أوّل | ثاني | ثالث |

days.

## Table 3: Days of Persian years and months

*Sources*: **F** fol. 39r, **H** fol. 27v, **C** fol. 45r, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -, **Y** fol. 259v, **L** fol. 23v, **B** p. 38.

Title: [4-]أيّام السنين[-4] الفارسيّة [5-]تُؤخذ[6] بالسنين[7] والشهور التامّة[-5]

Subtable headers:
[8-]أيّام السنين[-8] المجموعة / [9-]أيّام السنين[-9] المبسوطة[10] [11:] / أيّام[12] الشهور

Arguments: (ال)سنون / (ال)سنون / (ال)شهور

Subcolumn headers: رابع / ثالث / ثاني / أوّل

Label for the alternative values in the margin: [b]
**F**: ‘marginal gloss’ — الحاشية
**H**: ‘if the epagomenal days are in Isfandārmudh māh’ — إن كانت مشرقه (كذا) في اسفندارمذ ماه
**B**: ‘if the epagomenal days are in Ābān māh’ — إن كانت المسترقة في آبان ماه

For the Persian month names, see the general edition on pp. 268–69.

### *Explanatory texts*

**A. On the use of the values for the earlier / later versions of the Persian calendar**

*Sources*: **FYB** left bottom corner (under the subtable for months).

الحاشية[13] على أنّ المسترقة في اسفندارمذ[14] ماه[15] .

⟨The values in⟩ the marginal gloss (**YB**: ‘red’) are for the epagomenal days in Isfandārmudh māh.

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[a] **HLB** write the position indicators ‘first’, ‘second’ and ‘third’ vertically with respect to the note, and **Y** writes them above the digits. [b] **YL** do not include alternative values, **C** omits the label.

[1–1] **HCYB** om. [2] **L** هذ [3] **CY** om. [4–4] **Y** السنون [5–5] **H** بالسنة والشهر التامّ [6] **FY** بوخذ, **LB** يوخذ [7] **C** السنين [8–8] **Y** om., **L** السنون, **B** السنين [9–9] **Y** om., **L** السنون [10] **H** المبسوط [11:] **B** om. entire header [12] **YLB** om. [13] **YB** الحمرة [14] **B** اسفندارمند [15] **Y** places ماه before اسفندارمذ

**B. On the number of days between the Persian and Hijra eras**
*Source*: **B** between the subtables for collected and extended years (vertically).

بين تأريخ الفرس والهجرة ٣٦٢٤ يومًا.

Between the Persian era and the Hijra there are 3624 days.

## Table 4: *Notae* of Syrian years and months

*Sources*: **F** fol. 39v, **H** fol. 26r, **C** fol. 43v, $C_1$ -, $C_2$ -, **Y** fol. 258r, **L** fol. 22r, **B** p. 39.

Title: مدخل[1] الشهور[2] السريانيّة[3] يُؤخذ[4] بالسنة[5] [6] الناقصة[7]

Argument (vertical): السنون السريانيّة[8]

Argument (horizontal): الشهور السريانيّة [9:]

For the Syrian month names, see the general edition on p. 266.

*Explanatory texts*

**A. On the use of this table**
*Sources*: **CB** left margin (vertically), **Y** under the title.

إذا قُسمت[10] سنته[11] على كح، سقطت غرس[12] [13-]غرفح غسو غسمد[14] غسعب[15]، [-13] سنة[16] واستؤنف[17] بالباقي[18] من السنين.

When its year is divided by 28, 1260, 1288, 1316, 1344, 1372 ⟨etc.⟩ years drop out, and ⟨the operation⟩ is continued with the remainder of the years.

## Table 5: *Notae* of Arabic years and months

*Sources*: **F** fol. 40r, **H** fol. 26v (before the Arabic days), **C** fol. 44v, $C_1$ -, $C_2$ -, **Y** fol. 259r, **L** fol. 23r, **B** p. 40.

Title: [a] مدخل السنين[19] [20] العربيّة يُؤخذ[21] بالسنة الناقصة[22] والشهر[23] الناقص[24]

Subcolumn headers: السنون[25] / الأيّام // الشهور / الأيّام[26]

For the Arabic month names, see the general edition on p. 267.

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[a] **H** places an additional title الشهور above the subtable for months.

[1] **Y** om. [2] **C** السنين والشهور, **YLB** السنين [3] **YLB** الروميّة 'Byzantine' [4] **H** om. [5] **Y** بالسنين [6] **CLB** add. والشهر, **Y** add. والشهور [7] **H** التي تريد, **C** الناقص, **YLB** بعينه 'themselves' [8] **YLB** om. [9:] **CYLB** om. entirely [10] **B** قسم [11] **C** سنوه (?) [12] **Y** عسرس 'twenty' [13–13] **Y** om., **B** gives these four numbers at the end of the note in tabular form and upside down [14] **B** غسمب '1342' [15] **C** عب '1400' (or '1002') [16] **C** om. [17] **Y** واستؤنفت [18] **B** الباقي [19] **Y** السنون [20] **C** add. والشهور [21] **H** om. [22] **HCYLB** om. [23] **Y** والشهور [24] **H** الذي تريد, **Y** الناقصة, **LB** بعينه 'themselves' [25] **CB** السنين [26] **FB** om. **H** أيّام المدخل

## Table 6: *Notae* of Persian years and months

*Sources*: **F** fol. 40v, **H** fol. 28r, **C** fol. 45v, **C₁** -, **C₂** -, **Y** fol. 260r, **L** -, **B** p. 41.

Title: [a] مدخل الشهور الفارسيّة[1] يُؤخذ[2] بالسنة الناقصة[3]

Argument (vertical): الشهور

Argument (horizontal): [b] السنون السبعة

For the Persian month names, see the general edition on pp. 268–69.

*Explanatory texts*

**A. On the use of the values for the earlier / later version of the Persian calendar**

*Sources*: **F** left bottom, **H** left margin (vertically, partially hardly legible due to wear to the page), **Y** under the table, **B** right margin (vertically).

الحمرة على أنّ المسترقة[4] في اسفندارمذ[5] ماه.

The ⟨values in⟩ red are for the epagomenal days in Isfandārmudh māh.

*Source*: **C** left margin (vertically).

الحمرة إن كانت المسترقة في آبان ماه.

The ⟨values in⟩ red ⟨are to be used⟩ if the epagomenal days are in Ābān māh.

**B. On the use of the table**

*Source*: **B** left margin (vertically).[c]

إذا قُسمت[6] سنة على زَ، سقطته ٢١ واستؤنف بالباقي.

When a year is divided by 7, ⟨multiples of⟩ 21 drop out (?), and ⟨the operation⟩ is continued with the remainder.

## Table 7: *Notae* of the Christian Lent

*Sources*: **F** fol. 41r, **H** -, **C** -, **C₁** -, **C₂** -, **Y** fol. 260v, **L** fol. 24r (see Plate 3), **B** -.

Title: مدخل صوم النصارى، السواد من شباط والحمرة[7] من آذار

Argument (vertical): سنو[8] الطول

Argument (horizontal): سنو[9] العرض

---

[a] **H** second part of the title in black under the first part, in the cell of the horizontal argument. [b] Only in **HCB**. [c] This note, in the main hand of manuscript **B**, is obviously similar to explanatory note A to Table 4, but the role played by '21' is unclear to me.

[1] **Y** الفارسيّ [2] **H** om. [3] **H** التي نريد [4] **H** ill. [5] **B** اسفندارمند [6] **B** قسم [7] **F** الحمرة [8] **YL** سني
[9] **YL** سني

**Table 8: Sine**

*Sources*: **F** fol. 41v, **H** fol. 28v, **C** fol. 46r, **$C_1$** fol. 13r, **$C_2$** -, **Y** fol. 261r, **L** -, **B** p. 42.

Title: جدول[1] الجيب

Argument: القوس

Column headers: الجيب / (ال)تفاضل

Subcolumn headers: درج / دقائق / ثواني // درج / دقائق / ثواني

*Explanatory texts*

**A. On the sine of arcs larger than 90°**

*Source*: **H** under the title.

أيّ قوس أكثر من تسعين فجيبها جيب تمامها من مائة وثمانين.

For any arc more than 90 ⟨degrees⟩, its sine is ⟨equal to⟩ the sine of its supplement (lit. 'completion') to 180 ⟨degrees⟩.

**Table 8a: Sine for fractions of a degree**

*Sources*: **L** fols 24v–31v (see Plate 4), **B** pp. 49–63, **D** = *Dustūr al-munajjimīn*, MS Paris, BnF, arabe 5968, fols 29r–36r.

Title: [a] [2] الجيب [3]

**Table 9: Versed sine**

*Sources*: **F** fol. 42r–42v, **H** fol. 29r–29v, **C** fols 46v–47r, **$C_1$** fols 13v–14r, **$C_2$** -, **Y** fols 261v–262r, **L** fols 33v–34r, **B** pp. 43–44.

Title: [4] جدول[5] السهم

Argument: القوس

Column headers: السهم / (ال)تفاضل

Subcolumn headers: درج / دقائق / ثواني // درج / دقائق / ثواني

[a] **B** gives the title only on the first page of the table in a different hand.

[1] **HCC$_1$B** om. [2] **L** (last two pages): add. تمامت 'completion', **B** add. جدول [3] **D** (first page): add. لكوشيار 'by Kūshyār' [4] **Y** (second page): add. تتمّة 'completion', **L** (second page): add. تمامت 'completion' [5] **HCC$_1$LB** om.

## Table 10: First tangent

*Sources*: **F** fol. 43r, **H** fol. 30r, **C** fol. 47v (see Plate 5), **$C_1$** fol. 14v, **$C_2$** -, **Y** fol. 262v, **$L_1$** = **L** fol. 34v, **$L_2$** = **L** fol. 32r, **B** p. 45.

Title: الظلّ الأوّل [1-]وهو المعكوس لحساب الأبواب[-1] [2]

Argument: القوس

Column headers: الظلّ / (ال)تفاضل[3]

Subcolumn headers: أجزاء[4] / دقائق / ثواني // أجزاء[5] / دقائق / ثواني

*Explanatory texts*

**A. Addition to the title**

*Sources*: **$CC_1$** above the title, diagonally and cut off (cf. the variants in the title).

وهو[6] المعكوس لحساب[7] ⟨الأبواب⟩.

And it is the reversed ⟨shadow⟩ for the calculation ⟨of the elementary quantities⟩.

**B1. On calculations with tangents of arcs larger than 45°** (in **$FCC_1$**)

*Sources*: **F** top left (next to the values for arguments 31 to 45°, vertically), **$CC_1$** bottom margin.[a]

[8] أيّ جيب أو ظلّ قوس كانا[9] أقلّ من خمسة وأربعين نريد[10] أن نضرب[11] أحدهما[12] في ظلّ[13] قوس أكثر من خمسة وأربعين، قسمناه على ظلّ تمام القوس. [14-]فإن أردنا أن نقسم[-14] أحدهما على ظلّ قوس أكثر من خمسة وأربعين، ضربناه في[15] ظلّ تمام القوس. فلنحفظ ذلك [16-]إن شاء الله[-16].

Any sine or tangent of an arc less than 45 ⟨degrees⟩ which we want to multiply by the tangent of an arc more than 45 ⟨degrees⟩, we divide by the tangent of the complement of the arc. And if we want to divide one of the two by the tangent of an arc more than 45 degrees, we multiply it by the tangent of the complement of the arc. And we should bear this in mind, if God wills (**C**: 'if the exalted God wills', **$C_1$**: 'finished').

[a] In **C** some words in the bottom line are illegible due to wear and some further letters were cut off. Furthermore, **C** writes مه for خمسة وأربعين on the second and third occurrences.

[1–1] **H** om., **$CC_1L_2$** على أنّ المقياس ستّون جزءًا 'for the case that ⟨the length of⟩ the gnomon is 60 parts', **$L_1B$** لحساب الأبواب وهو المعكوس 'for the calculation of the elementary quantities, and it is the reversed ⟨shadow⟩' [2] **$L_1$** add. لحساب 'for (the) calculation' (cf. the previous variant and explanatory text A below) [3] **$L_2$** حصّة درجة [4] **$HYL_1B$** درج [5] **$HYL_1L_2B$** درج [6] **$C_1$** هو [7] **C** بحساب [8] **$CC_1$** add. في الزيج الثاني 'in the second ⟨copy of this⟩ *zīj*' [9] **$CC_1$** om. [10] **$C_1$** تريد [11] **C** ضرب [12] **C** أهده [13] **C** الظلّ [14–14] **C** dam. [15] **C** بغ (?) [16–16] **C** إن شاء الله تعال, **$C_1$** انتهى

**B2. On calculations with tangents of arcs larger than 45°** (in **FL$_2$**)
*Sources*: **L$_2$** bottom left (under the values for arguments 31 to 45°, vertically), **F** bottom left (under the values for 31 to 45°, only the last third of the note as found in **L$_2$**, but cf. explanatory text B1).

[1-]أيّ عدد نريد أن نضربه[2] في ظلّ قوس والقوس أكثر من خمسة وأربعين، قسمنا العدد على ظلّ تمام القوس. وأيّ عدد نريد أن نقسمه على ظلّ قوس[3] والقوس أكثر من خمسة وأربعين، ضربنا العدد في ظلّ تمام القوس.[-1] فأمّا[4] إن أردنا أن نضرب ظلّ قوس في ظلّ قوس وكليهما[5] أكثر من خمسة وأربعين أو نقسم[6] ظلّ قوس أكثر من خمسة وأربعين على عدد [7] عدلنا[8] عن استعمال الظلّ فيه. [9]

⟨For⟩ any number we want to multiply by the tangent of an arc that is more than 45 ⟨degrees⟩, we divide the number by the tangent of the complement of the arc. And ⟨for⟩ any number that we want to divide by the tangent of an arc that is more than 45 ⟨degrees⟩, we multiply the number by the tangent of the complement of the arc. As for (**F**: 'From another copy of this *zīj*:') if we want to multiply the tangent of an arc by the tangent of ⟨another⟩ arc and each of the two ⟨arcs⟩ is more than 45 ⟨degrees⟩ or ⟨if⟩ we ⟨want to⟩ divide the tangent of an arc more than 45 ⟨degrees⟩ by a⟨ny⟩ number, we refrain from using the tangent in this ⟨case⟩. (**F** add. 'Success is with God.')

**C. On the relation between the first and second tangents**
*Sources*: **CC$_1$** between the first and second columns (vertically).

هذا[10] الظلّ لكلّ قوس هو الظلّ الثاني لتمام القوس.

For each arc this tangent is the second tangent of the complement of the arc.

**D. Multiplication by a tangent is equal to division by the cotangent**
*Sources*: **CC$_1$** between the second and third columns, vertically.

كلّ عدد فسواء ضُرب في ظلّ قوس أو قُسم على ظلّ تمام القوس.

⟨For⟩ each number it is the same whether it is multiplied by the tangent of an arc or divided by the tangent of the complement of the arc.

---

[1–1] **F** om. (the contents in a somewhat different wording is included in text B1) [2] **L$_2$** يضربه [3] **L$_2$** القوس [4] **F** من نسخة أخرى من نسخ هذا الزيج [5] **F** كلهما [6] **L$_2$** يقسم [7] **F** add. أكثر من خمسة وأربعين [8] **L$_2$** om. [9] **F** add. وبالله التوفيق [10] **C$_1$** هو

## Table 11: Second tangent

*Sources*: **F** fol. 43v, **H** fol. 30v, **C** fol. 48r, $\mathbf{C_1}$ fol. 15r, $\mathbf{C_2}$ -, **Y** fol. 263r, $\mathbf{L_1}$ = **L** fol. 35r, $\mathbf{L_2}$ = **L** fol. 32r (only the table for gnomon length 7 feet), **B** p. 46.

Title: الظلّ الثاني [1-]وهو المستوي لمعرفة ظلّ أنصاف النهار[-1]

Argument: 'noon altitude' ارتفاع نصف النهار [2:]

Column header: الظلّ[3]

Subcolumn headers: أصابع / دقائق // أقدام / دقائق [4:]

### *Explanatory texts*

### A. Addition to the title

*Sources*: **C** above the title (vertically and cut off), $\mathbf{C_1}$ above the title (diagonally, and possibly in a different hand).

وهو المستوي لمعرفة ⟨ظلّ⟩ أنصاف[5] ⟨النهار⟩.

And it is the straight ⟨shadow⟩ for knowing the noon⟨ shadow⟩s.

### B1. On the beginning of the midday prayer

*Source*: $\mathbf{CC_1}$ between the second and third columns (vertically).

إذا زاد[6] على ظلّ نصف النهار شيء[7]، كان أوّل الظهر.

When ⟨the shadow⟩ has increased over the midday shadow by some amount (lit. when some amount exceeds the shadow at noon), it is the beginning of the midday prayer.

### B2. On the beginning of the afternoon prayer

*Source*: **H** in the left margin (vertically, hardly legible), $\mathbf{CC_1}$ between the first and second columns (vertically).

إذا[8] زاد[9] على[10] ظلّ نصف النهار مثل[11] أجزاء المقياس، كان أوّل العصر.

When the shadow has increased over the midday shadow by the length of the gnomon (lit. 'when the equivalent of the parts of the gnomon exceeds ⟨the length of⟩ the shadow at noon'), it is the beginning of the afternoon prayer.

---

[1–1] **H** om., $\mathbf{CC_1}$ على أنّ المقياس اثنا عشر إصبعًا أو سبعة أقدام ($\mathbf{C_1}$ om. أنّ), $\mathbf{L_2}$ على أنّ المقياس سبعة أجزاء [2:] **C** القوس, $\mathbf{C_1}$ قوس الارتفاع, $\mathbf{YL_2}$ الارتفاع [3] **C** (in all three columns): الظلّ / التفاضل [4:] **C** (in each subcolumn): أجزاء / دقائق / ثواني, $\mathbf{C_1}$ (in each subcolumn): أصابع / أقدام (sic!), **Y** adds in each column an empty subcolumn headed ثواني, $\mathbf{L_2}$ (in all three columns) أقدام / دقائق [5] **C** cut off [6] **C** زيد [7] $\mathbf{CC_1}$ شيئًا [8] $\mathbf{CC_1}$ وإذا [9] **H** letter *zā'* unclear (written over another letter?), **C** زيد [10] **H** om. [11] **H** appears to have بمثل

**B3. On beginning and end of the afternoon prayer**

*Source*: **L2** in the left margin (vertically), **L3** = **L** fol. 126v (additional table, in the left margin, vertically), **D** = *Dustūr al-munajjimīn*, MS Paris, BnF, arabe 5968, fol. 41r (to the right of the table, vertically).

إذا زاد[1] ظلّ نصف النهار بمثل أجزاء الشخص، كان أوّل [2] العصر. وإذا زاد بمثليه[3]، كان آخر[4] [5] العصر. [6]

When ⟨the shadow⟩ exceeds the midday shadow by [the equivalent of] the length (lit. 'parts') of the gnomon, it is the beginning of the afternoon prayer. And when ⟨the shadow⟩ exceeds ⟨it⟩ by twice ⟨this amount⟩, it is the end of the afternoon prayer.

[1] **L2** كان [2] **D** add. وقت [3] **L3** بمثل [4] **L3** احرا [5] **D** add. وقت [6] **L3** add. فافهم إن شاء الله رحمه تعال

## Table 12: Preliminaries of the mean motions

*Sources*: **F** fol. 44r (see Plate 6), **H** fol. 31r, **C** fol. 48v, **$C_1$** fol. 15v, **$C_2$** -, **Y** fol. 263v (see Plate 7), **L** fol. 35v, **B** p. 47.

Title: مقدّمات الأوساط[1]

Header of the subtable for Byzantine epoch positions: [a] الأصول الروميّة بالرقّة

Header of the subtable for Arabic epoch positions: [b] الأصول العربيّة بالرقّة

Header of the subtable for mean motions in 20 Syrian years:

حركات الكواكب في عشرين سنة سريانيّة على ما أثبته[2] البتّاني[3] في زيجه[4] الرقّيّ[5]

Header of the subtable for mean positions at the Yazdigird epoch:

[6-]الأصول[7] الفارسيّة[8] لنصف نهار[-6] أوّل[9] يوم من تأريخ يزدجرد من[10] أصل البتّاني[11] [12-]على طول[-12]
[13-]الرقّة وهو[14] عح به[-13]

Header of the subtable for daily mean motions: [c] حركات[15] الكواكب[16] ليوم[17] واحد

Planets / planetary mean motions as indicated in the first subcolumns:

الشمس / القمر / خاصّة القمر [18:] / جوزهر القمر [19:] /
زحل / المشتري / المرّيخ / خاصّة[20] الزهرة / خاصّة[20] عطارد

Subcolumn headers: الكواكب //
أدوار[21] / بروج / درج / دقائق / ثواني / ثوالث / روابع / خوامس / سوادس

---

[a] Only in **F**. [b] Only in **F**. [c] Due to lack of space, **L** writes this heading on the right side of the subtable vertically, **B** on the left side of the table vertically.

[1] **L** أوساط [2] **B** أثبتها [3] **$C_1$** البياني [4] **L** زيج [5] **Y** للرقّة [6–6] **L** أصول نصف النهار [7] **$CC_1$** الأصل [8] **$HCC_1YLB$** om. [9] **LB** om. [10] **$HCC_1YLB$** على ما في [11] **C** البانى, **$C_1$** البياني, **L** بتاني [12–12] **$HCC_1YLB$** لطول [13–13] **$CC_1$** عح به وهو طول الرقّة [14] **Y** om. [15] **$CC_1B$** حركة [16] **$HCC_1YB$** الأوساط [17] **$HCC_1YLB$** في يوم [18:] **$HCC_1YB$** الخاصّة [19:] **F** (subtable for epoch positions): جوزهره, **$HCC_1YB$** الجوزهر [20] **H** vertically in the margin in black, **$CC_1YB$** om. (in **B** added in a different hand). [21] Only in the subtable for the mean motions in 20 Syrian years in **$HCC_1$**.

*Layout of the table*

This table consists of three (**HCC$_1$YLB**) or four (**F**) subtables. In all sources the single page that the table takes up is divided into four quarters, with ample space in between. In manuscripts **HCC$_1$YLB** we find the mean positions at the Yazdigird epoch in the upper right quarter, the mean motions in 20 Syrian years in the upper left quarter, and the daily mean motions in the lower right quarter. The lower left quarter is filled with explanatory texts (especially F1); in **L** it has a tabular frame similar to the other quarters, which, however, was left empty. In all six sources the space between the two subtables on the right is filled with a list of apogee positions (explanatory text A1). Only **F** has an entirely different arrangement: it has the mean motions in 20 Syrian years in the upper left quarter (as do the other sources), but gives the Yazdigird epoch positions in the lower right quarter and the daily mean motions in the lower left quarter. **F** fills the upper right quarter with al-Battānī's mean positions for the Byzantine and Hijra epochs, which do not appear in any of the other sources. In the edition, I have followed the manuscript of what Mohammad Bagheri and I consider to be closest to the original version of the *Jāmiʿ Zīj*, namely **F**.

Some insignificant differences between the manuscripts have not been included in the apparatuses for the table and the text. For example, **YLB** add a column of zodiacal signs for the daily mean motions, which contains $0^s$ for every planet. Furthermore, **FCC$_1$** add a column of sevenths (headed سوابع) in the subtable for daily mean motions; in **CC$_1$** this column is filled with zeroes, while in **F** the phrase الجداول صحيحة, 'the columns are correct', is written through it vertically.

*Explanatory texts*

In all manuscripts, the spaces between the subtables of this table and the margins contain explanations of various aspects related to mean motions in general and the calculation of Kūshyār's mean motion tables from those of al-Battānī in particular. Most of these explanations are written in the main hands of the manuscripts, and especially in manuscript **F** they are abundant. At least part of the explanatory texts may be assumed to have been contained in, or associated with, the original version of Kūshyār's *zīj*.

### A1. Apogee positions for the Yazdigird epoch

*Sources*: **F** left margin (vertically), **HCC$_1$YLB** in the empty space between the two subtables in the right half of the table (**L** vertically). [a]

Header: [b] مواضع الأوجات لأوّل [1-]يوم من[-1] تأريخ[2] ملك[3] يزدجرد [4] [5]

Positions of the apogees for the first day of the era of the reign of Yazdigird

| | | | |
|---|---|---|---|
| Sun [6] | 2$^s$ 18;31 [7] | Mars | 4$^s$ 3;15 |
| Saturn | 8$^s$ 0;45 [8] | [9-]Venus | 2$^s$ 18;31 [-9] |
| Jupiter | 5$^s$ 10;45 | Mercury | 6$^s$ 17;45 [10] |

### A2. Apogee positions for the Seleucid year 1191

*Source*: **F** second of four texts in the vertical space in the middle of the table (upside down); **F′** = copy of this list found next to the table of apogee motion (Table 14) in **F**; **E** = al-Battānī's apogee values as found in MS Escorial, RBMSL, árabe 908, fols 73v and 117v (cf. Nallino, *al-Battānī sive Albatenii*, vol. III, pp. 107–08 and 172–73 (edition) and vol. II, pp. 72 and 114 (Latin translation)).

مواضع الأوجات [11-]على ما كتبها البتّاني في زيجه[-11] لأوّل سنة غقصا[12] لذي القرنين، وهو أوّل سنة رمط ليزدجرد بالتقريب [13].

Positions of the apogees according to what al-Battānī wrote in his zīj for the beginning of the year 1191 Two-Horned (i.e., Alexander), which is approximately the beginning of the year 249 Yazdigird.

| | | | |
|---|---|---|---|
| Sun and Venus | 2$^s$ 22;14 [14] | Mars | 4$^s$ 6;58 [15] |
| Saturn | 8$^s$ 4;28 | Venus | 2$^s$ 22;14 [16] |
| Jupiter | 5$^s$ 14;28 | Mercury | 6$^s$ 21;28 [17] |

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[a] The copy of this list found next to the table of apogee motion (Table 14) in **F** is identical with the present one. The header and values in **H** are hardly legible due to wear to the page. [b] **C$_1$** omits the entire heading.

[1–1] **H** om. [2] **L** om. [3] **HCYB** om. [4] **Y** add. بن شهريار ('ibn Shahriyār') [5] **F** (around the corner, i.e., in the upper margin, upside down) add. كذى ذكره البتّاني في زيجه ('Thus al-Battānī stated ⟨the apogee positions⟩ in his zīj'), **H** add. على أرصاد البتّاني ('according to the observations of al-Battānī'), **L** adds under the list وذلك على ما موجب ما يذكره البتّاني ('And that is according to what al-Battānī states'). [6] **F** add. والزهرة ('and Venus'), **Y** places the sun between Mars and Venus. [7] **B** add. 2″ (furthermore, next to the listing of apogee positions, **B** writes 2$^s$ 8;31,22° in red and below that, separated by a horizontal line, 2$^s$ 8;31,2° in black, apparently both in the main hand). [8] **F** 15′ [9–9] **F** combines the entry for Venus with the sun. [10] **CC$_1$** 44″ [11–11] **F′** om. [12] **F′** عصا [13] **F′** add. على ما كتبه البتّاني في زيجه [14] **E** 15′ **F′** 7$^s$ [15] **F** 18′ [16] **F** 12$^s$ 20° **F′** 28′ [17] **F** 15°

**B1. Comment on the apogee longitude of Mars according to Theon** (in **F**)
*Source*: **F** middle column, uppermost text of four.

أوج المرّيخ على ما ذكره ثيون الإسكندرانيّ في القانون موضع قلب الأسد لأوّل يوم من ملك يزدجرد د ي لو، وهو أقرب إلى الصواب.

The apogee of Mars according to what Theon of Alexandria stated in the Canon (i.e., the *Handy Tables*) is ⟨equal to⟩ the position of Regulus (*qalb al-asad*, 'Heart of the Lion') on the first day of the reign of Yazdigird, ⟨namely⟩ $4^s$ 10;36°, and it is nearer to what is correct.

**B2. Comment on the apogee longitude of Mars according to Theon** (in **CC₁YB**)
*Sources*: **C** middle of right margin (vertically); **C₁** right margin (vertically); **YB** following the apogee positions in A1.

[1] أوج المرّيخ [2-]على ما يذكر[-2] ثيون[3] [4-]في القانون[-4] [5-]د ي لو[-5] [6]، وهو موضع[7] قلب الأسد [8-]والأقرب[9] إلى الصواب[-8].

The apogee of Mars according to what Theon states in the Canon is $4^s$ 10;36°, which is the position of Regulus and nearest to what is correct.

**C. Number of days in 20 Syrian years**
*Sources*: **H** lower left quarter (vertically); **C** lower left quarter, under explanatory text F1 (with the first four words repeated in red before F1); **C₁** lower left quarter, under F1 (vertically); **Y** lower left quarter, before F1 and G (vertically); **B** in a separate cell in the lower left quarter (vertically). In **H** the position indicators 'first', 'second' and 'third' are written in red above the sexagesimal digits. In **CC₁B** the note takes up three lines with the position indicators written above the sexagesimal digits.

أيّام عشرين سنة[10] سريانيّة أوّل[11] ب، ثاني ا، ثالث[11] مه.

The ⟨number of⟩ days of 20 Syrian years is: ⟨in⟩ the first ⟨sexagesimal position⟩ 2, the second 1, the third 45 (i.e., $2 \cdot 60^2 + 1 \cdot 60 + 45 = 7305$).

[1] **C₁** add. خاصّة in red (scribal mistake for حاشية ?) [2–2] **Y** برأي [3] **C** ثاون [4–4] **Y** om. [5–5] **C** om. [6] **C₁** add. خاصّة in red (cf. note 1) [7] **B** مواضع [8–8] **Y** om. [9] **C₁** والأقرّ [10] **H** om. [11] **CC₁** exchange أوّل and ثالث

**D. Calculation of daily mean motions from the motions in 20 Syrian years**

*Sources*: **F** middle of left column, upper of two texts; **L** left margin (vertically).

هذه [1-]حركات الأدوار لعشرين سنة سريانيّة[-1] على ما وجدها البتّاني بأرصاده بالرقّة. فإذا كانت معلومة وقسمناها[2] على أيّام عشرين سنة بعينها[3]، وهي ٧٣٠٥، فتحصل[4] حركة [5-]كلّ كوكب[-5] في يوم [6] على ما كتبنا[7] في [8-]جدول حركاتها ليوم واحد[-8].

These are the motions of the revolutions for 20 Syrian years according to what al-Battānī found in his observations in Raqqa. And when they are known and we divide them by the days of the very same 20 years, namely 7305, then there results the motion of each planet in a day according to what we wrote in the table of their motions in a single day.

**E. Conversion of mean positions from one epoch to another**

*Sources*: **F** middle of right column (where all other sources give the apogee positions), **L** lower half of right margin (vertically, in red).

هذه الأصول هي [9-]لنصف نهار[-9] أوّل يوم من سنة ظلا[10] لذي القرنين. فإذا ضربنا[11] ما بين هذا اليوم وبين تأريخ يزدجرد من الأيّام، وهي ۴۴۹۱، في حركة اليوم وزدنا المبلغ على هذا الأصل، حصلت الأصول ليزدجرد على ما كتبناه[12] في [13-]جدول الأصول الفارسيّة[-13]. وكذلك إن ضربنا ما[14] بين الهجرة ويزدجرد [15-]من الأيّام[-15]، وهي ٣۶٢٣[16]، في حركة اليوم وزدنا المبلغ على الأصول الهجريّة، [17-]حصلت الأصول ليزدجرد على ما كتبناه في الجدول[-17].

These epoch positions are for midday of the first day of the year 931 Two-Horned (i.e., Alexander). And when (**L** add. 'they are known and') we multiply the days between this day and the Yazdigird epoch, namely 4491, by the motion in a day and add the outcome to these epoch position⟨s⟩, the epoch positions for ⟨the⟩ Yazdigird ⟨epoch⟩ result according to what we have written in the table of the Persian epoch positions. And similarly if we multiply the days between the Hijra and Yazdigird ⟨epochs⟩, namely 3623 (L: '3624', correct: 3625), by the motion in a day and add the outcome to the Hijra epoch positions, the epoch positions for ⟨the⟩ Yazdigird ⟨epoch⟩ result according to what we have written in the table.

[1–1] **L** الحركات [2] **L** قسمناها [3] **L** وورومه [4] **L** فحصل [5–5] **L** الكوكب [6] **L** add. واحد [7] **F** ill. (أثبتناه ?) [8–8] **L** الجداول [9–9] **F** لنهار [10] **L** أحد وثلثين وتسع مائة [11] **L** كانت معلومة وضربنا [12] **L** كتبا [13–13] **L** الجدول [14] **L** الأيّام [15–15] **L** om. [16] **L** ٣۶٢۴ [17–17] **L** om.

### F1. Relations between mean positions and mean anomalies

*Sources*: **F** middle of left column, lower of two texts (vertically); **H** lower left quarter, immediately following explanatory text C (vertically); **C** lower left quarter (vertically); **C$_1$** lower left quarter (separated by horizontal lines); **Y** lower left quarter, between explanatory texts B1 and G (vertically); **L** right margin, upper of two texts (vertically); **B** lower left quarter, immediately followed by G (vertically).

إذا نُقص وسط الشمس من وسط القمر وضوعف الباقي، كان البعد[1] المضاعف. إذا[2] نُقص أوساط[3] الكواكب العلويّة من وسط الشمس، كان ما بقي خاصّة العلويّات[4]. ووسط[5] كلّ واحد من الزهرة وعطارد مثل[6] وسط الشمس. [7-]ولذلك ذكرنا أوساط العلويّات في المقدّمة دون خاصّاتها وذكرنا خاصّتي الزهرة وعطارد دون وسطهما[-7].

When the solar mean position is subtracted from the lunar mean position and the remainder is doubled, ⟨the result⟩ is the double elongation. When the mean positions of the superior planets are subtracted from the solar mean position, what remains is the anomaly of the superior planets. The mean position of both Venus and Mercury is equal to the solar mean position. And therefore we have stated the mean positions of the superior planets in the preliminaries rather than their anomalies and we have stated the anomalies of Venus and Mercury rather than their mean positions.

### F2. Relation between solar mean position and solar mean anomaly

*Source*: **F** middle column, third of four texts (diagonally).

وإذا نُقص أوج الشمس من وسطها، كان الباقي خاصّتها.

And when the solar apogee is subtracted from its mean position, the remainder is its anomaly.

[1] **L** بعد  [2] **HCC$_1$YLB** وإذا  [3] **HC$_1$YLB** وسط, **C** أيّام  [4] **HCC$_1$YLB** الكوكب  [5] **F** ثمّ (؟) وسط  [6] **CC$_1$** هو  [7–7] **H** والمثبت في هذه المقدّمة خاصّتهما. والأصول الموضوعة في الجداول من بعد هي على طول تسعين من جزائر الخالدات. والله وليّ التوفيق ‘and what is recorded in these preliminaries is their anomaly. The epoch positions written down in the tables hereafter are for longitude ninety from the Fortunate Isles. And God is the grantor of success.’, **CC$_1$YB** ولذلك ذكرنا الخاصّة في المقدّمة ‘and therefore we have stated the anomaly in the preliminaries’, **L** om.

**G1. On the base meridian** (in **F**)
*Source*: **F** middle column, fourth of four texts (vertically).

هذه الأصول هي لطول الرقّة الذى هو عج ىه. وما وُضع من الأصول من بعد في جميع الجداول هي لطول ص من الجزائر الخالدات في المغرب، وبينها وبين ساحل البحر عشر درجات من دور الفلك. والحمد لله شكرًا على نعمه.

These epoch positions are for the longitude of Raqqa, which is 73;15°. The epoch positions that have been written down hereafter in all the tables are for longitude 90° from the Fortunate Isles in the west, and between it and the coast of the sea are ten degrees of the revolution of the sphere. And praise be to God, thanking him for his blessings.

**G2. On the base meridian** (in **YB**)
*Sources*: **YB** lower left quarter, appended to F1 (vertically).

والأصول الموضوعة في الجداول من بعد هي على طول تسعين[1] من جزائر الخالدات في المغرب، وبينه وبين ساحل البحر عشر[2] درجات من دور الفلك. [3]

The epoch positions written down in the tables hereafter are for longitude 90° from the Fortunate Isles in the west, and between it and the coast of the sea are ten degrees of the revolution of the sphere. (**Y** add. 'Success is with God.')

[1] **B** ص [2] **Y** عشرة [3] **Y** add. وبالله التوفيق.

## General edition: Mean motion tables

Title (first part, on the example of the solar mean motion):

**F**: [1] وسط الشمس في السنين والشهور[2]

**HCC$_1$C$_2$YLB**: وسط الشمس [3]

Title (second part, on the example of the solar mean motion):

**F**: [4] وسط الشمس في الأيّام والساعات وما بين الأطوال[5]

**HY**: [6] وسط الشمس [7] [8:]

**L**: [a] تمامت[9] وسط الشمس[10]

**B**: [b] تمام وسط الشمس

**CC$_1$C$_2$**: [c] -

Subtable headers:

| | | | |
|---|---|---|---|
| collected years | المجموعة | **HC$_1$C$_2$**: | في السنين[11] المجموعة |
| | | **C**: | في [12] المجموعة |
| extended years | المبسوطة | **H**: | في السنين المبسوطة |
| | | **CC$_1$C$_2$**: | في [13] المبسوطة |
| ‘single’ years | سنون مفردة [14:] | **HCC$_1$C$_2$**: | السنون المفردة |
| | | **L**: | سنون[15] المفردة[16] [17:] |
| months | الشهور [18] | **HCC$_1$C$_2$**: | في الشهور |
| days | الأيّام | **HCC$_1$**: | في[19] الأيّام |
| hours and their fractions | الساعات وكسورها | **HC$_1$C$_2$**: | في[20] الساعات وكسورها[21] |
| | | **C**: | في الساعات والكسور[22] |

[a] **L**-Node: not applicable (the table covers a single page) [b] **B**-Node: not applicable (the table covers a single page) [c] Not applicable (the mean motion tables in **CC$_1$C$_2$** were compressed to a single page).

[1] **F**-Sun+Sat: add. جدول [2] **F**-LunAnom: وفي الشهور [3] **Y**-Moon (first of the four pages occupied by this table): add. في السنين, **Y**-Moon (second page): add. في الشهور والأيّام, **Y**-LunAnom: add. في السنين والشهور [4] **F**-Sun: add. جدول [5] **F**-SatAnom: أطوال البلدان [6] **Y**-Node+Sat+Jup: add. تمام, **Y**-JupAnom: add. بقيّة [7] **Y**-Moon (third of the four pages occupied by this table): add. في الساعات وكسورها, **Y**-LunAnom: add. في الأيّام والساعات [8:] **H**-Elong: المضاعف (om. البعد); **Y**-Moon (fourth page, which also contains the subtable for longitude differences): تعديل الأيّام ولياليها [9] **L**-Moon+Sat+SatAnom+Jup+JupAnom+Ven+VenAnom: تمام [10] **L**-Moon: الخسوف (with القمر added above it in red) [11] **C$_2$**-Sat+JupAnom+VenAnom: om. [12] **C**-Sun: add. السنين [13] **C**-Sun, **C$_1$**-Elong, **C$_2$**-SatAnom+Mer: add. السنين [14:] **B**-Sun: السنون المفردة [15] **L**-Sun: السنون [16] **L**-LunAnom: مفردة [17:] **L**-Node: om. entire heading [18] **Y**-Ven: add. الفارسيّة [19] **C$_1$**-LunAnom: om. [20] **C$_1$**-LunAnom: om. [21] **C$_1$**-Jup: وكسوها, **C$_2$**-Sat+JupAnom: والكسور [22] **C**-LunAnom+Sat+JupAnom+Mars+MarsAnom: وكسورها

| | | | |
|---|---|---|---|
| differences between longitudes | فيما بين الأطوال[1] | **H**: | في ما بين [2–]الطولين للبلدان[–2] |
| | | **C**: | فيما[3] بين أطوال[4] البلدان |
| | | **$C_1C_2$**: | في[5] ما بين طول البلدان |
| | | **Y**: | ما بين الأطوال[6] [7:] |
| | | **LB**: | [8] ما بين طول[9] البلدان |

Arguments:

| | | |
|---|---|---|
| collected years | (ال)سنون | |
| extended and 'single' years | (ال)سنون | |
| months | (ال)شهور | |
| days | (ال)أيّام[10] | |
| hours (first column) | (ال)ساعات[11] | |
| hours (second column) | (ال)ساعات | **$C_1$**: كسور الساعات |
| longitude(s) | (ال)أطوال | **$CC_1C_2$**: الطول (twice) |

Subcolumn headers:

| | | |
|---|---|---|
| signs | بروج | |
| degrees | درج | |
| minutes | دقائق | |
| seconds | ثواني | (only for the Sun and the Moon) |

Labels in the subtable for longitude differences:

*Sources*: **F**-Sun+Moon+LunAnom+Elong+Mer: horizontally next to the values for arguments 71 and 90°; **H**-Sun+Moon+LunAnom+Elong+JupAnom+Mars+VenAnom+Mer: vertically to the right of the subtable starting at (or next to) arguments 71 and 90°; **C**-Moon+LunAnom+Elong+Node: vertically between the digits of the values for arguments 73 or 74° and those for arguments 88, 89 or 90°; **$C_1C_2$**: vertically between the digits of the values starting at arguments 71 and 91°;[a] **Y**-Sun+Moon+LunAnom+Elong+SatAnom: vertically to the right of the subtable at the beginning and the end of the ranges for which the labels are valid;[b] **L**-Sun+LunAnom+JupAnom+Mer: written vertically to the right of the subtable, just above and just below the value for argument 90°; **L**-Sat+VenAnom+MerAnom, **B**: written vertically to the right of the subtable, with the words expanded over the whole range for which they are valid.[c]

| | |
|---|---|
| 'additive' / 'subtractive' | زائد / ناقص |

For the names of the Persian months, see the general edition on pp. 268–69.

[a] **$C_1$**-Node: ناقص starts at 90°, **$C_1$**-Sat: زائد between the digits of the values for arguments 74–76°, **$C_1$**-JupAnom+Mars+Mer+MerAnom: om. [b] **Y**-Sun: between arguments and tabular values [c] **B**-Moon: written horizontally next to arguments 71 and 90° (as in **F**), **B**-Node: om.

[1] **F**-Sun+Sat+SatAnom: أطوال البلدان [2–2] **H**-Sun: طول البلدان [3] **C**-Moon: ما, **C**-LunAnom: في ما [4] **C**-Moon+LunAnom+Node: طول [5] **$C_1$**-Sun+Moon+LunAnom: om. [6] **Y**-LunAnom: أطوال البلدان [7:] **Y**-Sat+Jup: omit the entire subtable for longitude differences. [8] **L**-LunAnom+Jup: add. في [9] **L**-Node: om. [10] **C**-JupAnom: في الأيّام [11] **Y**-Sun: من (؟) ساعات

*Explanatory texts*

**A. On the use of the values for the earlier version of the Persian calendar**

*Sources*: **FYLB** bottom left (under the subtable for months, **F** only for the sun); **H** left margin (vertically, only for the sun). Note that **CC$_1$C$_2$** do not include values for the early version of the Persian calendar.

**F**-Sun: الحمرة هي الوسط المركّب مع المسترقة في جميع الكواكب

The ⟨values in⟩ red are the mean motion⟨s⟩ set up with the epagomenal days ⟨included⟩ for all the planets.

**H**-Sun: الحمرة على أنّ المشترقة (كذا) في اسفندارمذ ماه لجميع الكواكب

The ⟨values in⟩ red are for ⟨the case that⟩ the epagomenal days are in Isfandārmudh māh for all the planets.

**YLB**: [a] الحمرة هي[1] الوسط[2] المركّبة[3] المسترقة[4] في جميع الكواكب

**B. On the displacements of the mean motions**

*Sources*: **LB** in the right margin next to the epoch value of the table for collected years (**L**-Moon: left margin; **LB**-MarsAnom, **B**-Sun+Moon+Node: om.).

*On the example of the solar mean motion*: الأصل[5] بنقصان[6] درجتين

The epoch value ⟨of the solar mean motion⟩, with a decrease of two degrees

*On the example of the double elongation*: الأصل بعينه

The epoch value ⟨of the double elongation⟩, exactly as is

*On the example of the mean anomaly of Saturn*: الأصل بزيادة[7] سبعة درجات

The epoch value ⟨of the anomaly of Saturn⟩, with an increase of 7 degrees

The displacements are given as follows (a minus sign indicates a decrease, a plus sign an increase, 'none' no adjustment): Sun −2°, Moon −8°, LunAnom −14°, Elong none, Node none, Sat −14°, SatAnom +7°, Jup −18°, JupAnom +6°, Mars −59°, ⟨MarsAnom +12°,⟩ Ven −50°, VenAnom +2°, Mer −30°[8], MerAnom +4 °.

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[a] **Y**-Node+all planets: om.; **L**-Sat+Jup+JupAnom+MarsAnom+Ven: om.; **B**-Sun: المكتوب بالحمرة هو وسط المركّب للمسترقة في جميع الكواكب. Since the text appears to be distorted in both versions, I have not translated it.

[1] **L**-Moon+Elong: om. [2] **Y**-Elong: البعد [3] **L**-LunAnom: المركّب [4] **Y**-LunAnom: om. [5] **L**-Sun: om. [6] **L**-LunAnom: نقصان, **B**-LunAnom+Sat+Jup+Mars: ينقصان [7] **L**-SatAnom+JupAnom: برباد; **B**-SatAnom+JupAnom: يزاد [8] **L** +4° (miscopied from the mean anomaly of Mercury)

## *Marginal notes*

**C. Apogee position for the Yazdigird epoch** (on the example of the Sun)

*Sources*: **CC$_1$**-Sun, **CC$_1$C$_2$**-Sat+Jup+Mars+Ven+Mer: under the title, directly above the subtable header في الشهور.

⟨The longitude of⟩ the ⟨solar⟩ apogee is 2$^s$ 18;31° — الأوج ب يح لا

The longitudes of the planetary apogees are given as follows: Saturn 8$^s$ 0;45°, Jupiter 5$^s$ 10;45°, Mars 4$^s$ 3;15°, Venus 2$^s$ 18;31, Mercury 6$^s$ 17;44, all in full agreement with the listings in Chapter I.4.4 (cf. Bagheri, *az-Zīj al-Jāmiʿ*, p. 37 (translation) and Arabic p. 24; see also Table 12).

**D. Longitude of the ascending node for the Yazdigird epoch** (on the example of Saturn)

*Sources*: **C**-JupAnom+MarsAnom, **C$_1$C$_2$**-SatAnom+JupAnom+MarsAnom+VenAnom+MerAnom: under the title, above the subtable header في الشهور.

⟨The longitude of⟩ the node ⟨of Saturn⟩ is 3$^s$ 10;45° — الجوزهر ج ي مه

The longitudes of the other planetary nodes are given as follows: Jupiter 3$^s$ 0;45°, Mars 1$^s$ 3;15°, Venus 11$^s$ 18;31, Mercury 9$^s$ 17;44, all in full agreement with the listing in Chapter I.4.4 (see Bagheri, *az-Zīj al-Jāmiʿ*, p. 37 (translation) and Arabic pp. 24–25).

**E. Simple table for apogee positions and motions** (on the example of the Sun)

*Sources*: **CC$_1$C$_2$** outer margin of the tables of mean longitude (except for the Moon and Venus, vertically), with arguments in red (here overlined / bold). **C$_2$** does not contain the solar mean motion table. See Plate 8 for this type of table for the Sun in **C$_1$**.[a]

الأوج    شا ب كج ا    تا ب كد لا    ثا ب كو ا    ثكا ب كو لط

ثما ب كو لز[1]    ثسا ب كو نه    ثفا ب كز لج    خا ب كز لا

ا ب ج د ه و ز ح ط ي يا يب يج يد يه يو يز يح يط ك

ا ب ج د ه ه و ز ح ط ي يا يب يج يد يد يه يو يز يح

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| The ⟨solar⟩ apogee | **301** | 2$^s$ 23; 1° | **401** | 2$^s$ 24;31° | **501** | 2$^s$ 26; 1° | **521** | 2$^s$ 26;19° |
| | **541** | 2$^s$ 26;37° | **561** | 2$^s$ 26;55° | **581** | 2$^s$ 27;13° | **601** | 2$^s$ 27;31° |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 14 | 15 | 16 | 17 | 18 |

[a] Due to lack of space, the line with epoch positions is here distributed over two lines. In **C**, the table for the Sun has 7$^s$ instead of 2$^s$ for all eight apogee positions in the first line; furthermore, the arguments from 521 Yazdigird onwards are misplaced after the corresponding apogee positions and the last apogee position is repeated. In all five tables, **C** omits the second value ه '5' in the bottom row and does not place the other values precisely under their corresponding arguments.

[1] لو **C**

Venus does not need a table of its own since its apogee longitude is equal to that of the Sun. The apogee motions in extended years 1, 2, …, 20 are basically identical in all five tables (the values 5′ and 14′ occur twice in order to arrive at a total apogee motion of 18′ in 20 Persian years). However, the copyist of **C** apparently was not aware that he was copying a table here: he omitted the second occurrence of the value 5′ in each table and also once that of 14′, and in most cases did not line up the arguments and values as they should have been.

The values for the indicated Yazdigird years for all the planets are as follows:

| year | Sun | Saturn | Jupiter | Mars | Mercury |
|---|---|---|---|---|---|
| 301 | 2$^s$ 23; 1[1]↓ | 8$^s$ 5;15[2] | 5$^s$ 15;15 | 4$^s$ 7;45[3] | 6$^s$ 22;14[4] |
| 401 | 2$^s$ 24;31 | 8$^s$ 6;45 | 5$^s$ 16;45 | 4$^s$ 9;15 | 6$^s$ 23,44 |
| 501 | 2$^s$ 26; 1 | 8$^s$ 8;15 | 5$^s$ 18;15 | 4$^s$ 10;45 | 6$^s$ 25;14 |
| 521 | 2$^s$ 26;19 | 8$^s$ 8;33 | 5$^s$ 18;33 | 4$^s$ 11; 3 | 6$^s$ 25;32 |
| 541 | 2$^s$ 26;37[5] | 8$^s$ 8;51[6] | 5$^s$ 18;51[7] | 4$^s$ 11;21 | 6$^s$ 25;50 |
| 561 | 2$^s$ 26;55 | 8$^s$ 9; 9 | 5$^s$ 19; 9 | 4$^s$ 11;39 | 6$^s$ 26; 8 |
| 581 | 2$^s$ 27;13 | 8$^s$ 9;27 | 5$^s$ 19;27 | 4$^s$ 11;57[8] | 6$^s$ 26;26[9] |
| 601[10] | 2$^s$ 27;31⊥ | 8$^s$ 9;45 | 5$^s$ 19;45 | 4$^s$ 12;15[11] | 6$^s$ 26;44[12] |

[1] 301–601 Yazdigird: **C** 7$^s$ [2] **C**$_1$ 14′ [3] **C**$_1$ 4° [4] **C** 54′ [5] **C** 36′ [6] **C** 11′ [7] **C** 11′ [8] **C** 17′ [9] **C** 56′ [10] **C**$_2$-Mer: 521 [11] **C**$_1$ 14′ [12] **C** minutes cut off

## Table 13: Solar mean motion

*Sources*: **F** fol. 44v–45r, **H** fols 31v–32r, **C** fol. 50r, $\mathbf{C_1}$ fol. 16r (see Plate 8), $\mathbf{C_2}$ -, **Y** fol. 264r–264v, **L** fol. 36r–36v, **B** pp. 48 and 67.

Title (first part): [1] وسط الشمس [2]

Title (second part): [a] [3] وسط الشمس [4]

See further the general edition of all mean motion tables on pp. 287–88.

### *Explanatory texts and marginal notes*

For text A (in **FHYLB**), text B (in **L**), and notes C and E (in $\mathbf{CC_1C_2}$), see the general edition on pp. 289–91.

#### F. Apogee longitude for 331 Yazdigird

*Source*: **L** above the title on the first page (partially cut off at the top).

الأوج لأوّل شلا ٮ كح كح.

The apogee for the beginning of ⟨the year⟩ 331 ⟨Yazdigird⟩ is $2^s$ 23;28°.

#### G. Adjustment for the base meridian of the *Fākhir Zīj*

*Source*: **B** right margin near the bottom of the first page (vertically, in black ink, cut off at end).

يزاد في المجموعة م ثواني فما بلغ فهو زيج الفاخ⟨ر⟩.

40 seconds are added ⟨to the values⟩ in ⟨the subtable of⟩ collected ⟨years⟩, and what results are ⟨the mean positions from⟩ the *Fākh⟨ir⟩ Zīj*.

#### H. Adjustment of mean motions in the *Shāhī Zīj* to a different longitude (in Persian)

*Source*: **B** in the left margin of the second page (vertically).

زيج شاهى بر طول صب ل است، وطول بلد فه لز. طول بلد از طول نشابور، کی فه لز است، نقصان کرديم؛ بماند چندين و نج. بر پانزده قسمت کرديم، آمد چندين ٦ کز لب. اين ساعات ما بين الطولين است. نصيب اين از حرکت، از حرکت أوساط بر زيج شاهى مى‌بايد افزود تا با طول فه لز آيد.

The *Shāhī Zīj* is for ⟨the geographical⟩ longitude 92;30° ⟨of Nishapur⟩, and the longitude of a ⟨given⟩ locality is 85;37°. We subtracted the longitude of the locality, which is 85;37°, from the longitude of Nishapur; the remainder was 6;53°. We divided it by fifteen, the result was $0^h\,27^m\,32^s$. This is the ⟨time difference in⟩ hours between the two longitudes. The motion corresponding to this ⟨amount⟩ should be added to the mean motions in the *Shāhī Zīj* so that they become for the longitude 85;37°.

[a] In $\mathbf{CC_1}$ the table occupies only a single page.

[1] **F** add. جدول [2] **F** add. في السنين والشهور [3] **F** add. جدول, **L** add. تمامت, **B** add. تمام [4] **F** add. في الأيّام والساعات وما بين الأطوال

**Table 14: Apogee motion**

*Sources*: **F** fol. 45v, **H** fol. 32v, **C** fol. 50v, **$C_1$** fol. 16v, **$C_2$** -, **Y** fol. 265r, **L** fol. 37r, **B** p. 68.

Title: حركة الأوجات

Column titles:

**F**: في السنين آحاد وعشرات ومائين // في الشهور والأيّام

**$HCC_1$**: في السنين والشهور والأيّام

**YLB**: في السنين والشهور

Subtable titles (only in **F**): آحاد / عشرات / مائين // شهور / أيّام

Arguments: آحاد السنين[1] / عشرات[2] [3] / مائين[4] [5] / (ال)شهور / (ال)أيّام

Subcolumn headers: درج / دقائق / ثواني // دقائق / ثواني

Titles of the three lists of apogee positions (only in **F**): [a]

مواضع الأوجات لأوّل يوم من ملك يزدجرد بن شهريار بالفارسيّة
على موجب ما ذكره البتّاني في زيجه
مواضع الأوجات لأوّل سنة غقصا[6] لذي القرنين، وهو أوّل سنة رمط
ليزدجرد بالتقريب، على ما كتبه البتّاني في زيجه
مواضع الأوجات لأوّل سنة شلا ليزدجرد

*Explanatory texts*

**A. Method for calculating the apogee motion directly from the number of years**

*Sources*: **$CC_1$** to the left of the table (vertically), **L** in the left margin (vertically), **B** in the right margin (vertically, in red).

يُنقص من سني[7] يزدجرد عشرها وما بقي فهو[8] دقائق حركة[9] الأوجات في تلك السنين.

From the Yazdigird years a tenth of them is subtracted, and what remains are the minutes of the motion of the apogees in those years.

**B. On the apogee motion underlying the table**

*Source*: **$C_1$** between title and subtable title.

في كلّ ٦٦ سنة وثمانية أشهر درجة.

In each 66 years and 8 months ⟨the apogee motion amounts to⟩ a degree.

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[a] The translations of these titles are included in the edition of the tables on p. 119.

[1] **L** الشمس [2] **H** عشارات [3] **$HCC_1$** add. السنين [4] **L** مبين [5] **$HCC_1$** add. السنين [6] **F** عصا [7] **$C_1$** سنين [8] **C** فهي [9] **$C_1$** حركات

## Table 15: Equation of time for every 6 degrees (with interpolation constants)

*Sources*: **F** fol. 46r, **Y** fol. 265r.

Title: تعديل الأيّام بلياليها

Subtitle (only in **F**): هذا على أنّ الأوج في كد من الجوزاء، يؤخذ بوسط الشمس ويُنقص من الساعات المفروضة أبدًا

Argument: وسط الشمس

Column headers: [a] التعديل / حصّة الدرجة[1]

Subcolumn headers: [b] بروج / درج // دقائق / ثواني // ثواني / ثوالث

*Explanatory texts*

**A. On values for single degrees**

*Source*: **F** to the left of the table (vertically).

هذا الوسط بزيادة ستّ درج ستّ درج. فإن أردت ما يصيب درجة واحدة لتأخذ الوسط بزيادة درجة درجة، تضيف ما أُثبت من حصّة الدرجة الواحدة.

This mean motion is ⟨tabulated⟩ with steps of six degrees. If you want ⟨values⟩ for a single degree, so that you can take the mean motion with steps of a degree, you ⟨repeatedly⟩ add the share of a single degree that was recorded ⟨in the table⟩.

## Table 15a: Equation of time for every degree

*Sources*: **H** fols 32v–33r (in three blocks of four signs each), **C** fols 50v–51r (in three blocks of four signs each), **C₁** fols 16v–17r (in three blocks of four signs each), **L** fol. 37v, **B** p. 69 (gives the names of the zodiacal signs instead of *abjad* numbers from 0 to 11).

Title: [2] تعديل الأيّام بلياليها [3]

Argument: وسط الشمس

Column headers: [c] دقائق / ثواني

[a] **F** has no column header for the equation. [b] **F** has no subcolumn headers for the argument. [c] **L** alternately writes only دقائق or only ثواني in each of the twelve columns for the signs.

[1] **Y** درجة [2] **CC₁** (second page): add. تمام [3] **H** add. على أنّ الأوج في أربعة وعشرين من الجوزاء. يؤخذ بوسط الشمس وينقص من وقت تقويم النيّرين أبدًا 'for the apogee in 24 ⟨degrees⟩ of Gemini. It is taken with the mean solar motion and it is always subtracted from the time of the true position of the two luminaries.' (cf. explanatory text A and the subtitle of Table 15 in **F**)

*Explanatory texts*

**A. On the use of the values in the table**

*Sources*: **CC₁** to the right of the block for signs 0–3 (vertically); **L** to the right of the table (vertically); **B** to the left of the table (vertically).

هذه [-1]دقائق وثواني من ساعة[-1]، يُؤخذ بوسط الشمس ويُنقص من [-2]الساعات المفروضة[3]،[-2] لتقويم النيّرين أبدًا[4]. [5]

These are minutes and seconds of an hour; they are taken ⟨from the table⟩ with the solar mean motion ⟨as the argument⟩ and they are always subtracted from the assumed (**C₁**: 'written down') time in order to calculate the true position⟨s⟩ of the two luminaries (i.e., the Sun and the Moon).

**B. On the solar apogee**

*Sources*: **CC₁** under the title on the second page, between the blocks for signs 4–7 and 8–11.

(And) the ⟨solar⟩ apogee is in 24° Gemini. (و)الأوج في كد من الجوزاء.

## Table 15b: Equation of time expressed in lunar mean motion

*Sources*: **Y** fol. 268r, **L** fol. 39v, **B** p. 75. This subtable is part of the table for the lunar mean motion (Table 17).

Title (only in **Y**): تعديل الأيّام ولياليها [6:]

Argument: وسط الشمس

Column header: النقصان

Subcolumn headers: [a] بروج / درج // دقائق / ثواني

## Table 16: Solar equation

*Sources*: **F** fols 46v–48r, **H** fols 33v–35r, **C** fols 51v–52v, **C₁** fols 17v–18v, **C₂** -, **Y** fols 265v–266r, **L** fol. 38r–38v, **B** pp. 70–73.

Title: [7] تعديل الشمس

Argument: درجات[8] الخاصّة [9:]

[a] Only **Y** gives the subcolumn headers for the argument.

[1–1] **L** الدقائق والثواني من الساعات [2–2] **L** ساعات مفروضة [3] **C₁** الموضوعة [4] **C** added in a different hand [5] **L** add. والله أعلم 'and God knows best' [6:] Rather than a title, **B** gives an explanation ما يُنقص من وسط القمر أو تقويمه بحسب تعديل الأيّام ('What is subtracted from the lunar mean motion or its true position in accordance with the equation of time') in the top and left margins (written around the corner of the table), possibly in a different hand. [7] **F** add. جدول [8] **H** om., **YLB** أجزاء [9:] **CC₁** خاصّة الشمس

Column header: (ال)تفاضل[1]

Subcolumn headers: درج / دقائق / ثواني // دقائق / ثواني

Labels for distances:[a] البعد الأوسط [2:] / البعد الأقرب [3:] / البعد الأوسط / البعد الأبعد

Labels 'increasing' / 'decreasing' (only in **F**): زائد / ناقص

## Table 17: Lunar mean motion

*Sources*: **F** fols 48v–49r, **H** fols 35v–36r, **C** fol. 53r, **$C_1$** fol. 19r, **$C_2$** -, **Y** fols 266v–268r (spread out over four pages), **L** fol. 39r–39v, **B** pp. 74–75. See Table 15b for the subtable for the equation of time expressed in lunar mean motion that is part of this table in manuscripts **YLB**.

Title (first part): وسط القمر [4]

Title (second part):[b] [5] وسط القمر[6] [7] [8:]

See further the general edition of all mean motion tables on pp. 287–88.

### *Explanatory texts and marginal notes*

For text A (in **YLB**) and text B (in **L**), see the general edition on pp. 289–91.

## Table 18: Lunar mean anomaly

*Sources*: **F** fols 49v–50r, **H** fols 36v–37r, **C** fol. 53v, **$C_1$** fol. 19v, **$C_2$** -, **Y** fols 268v–269r, **L** fols 40v–41r, **B** pp. 76–77.

Title (first part): خاصّة القمر [9]

Title (second part):[c] [10] خاصّة القمر [11]

See further the general edition of all mean motion tables on pp. 287–88.

### *Explanatory texts and marginal notes*

For text A (in **YLB**) and text B (in **LB**), see the general edition on pp. 289–91.

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[a] **$FCC_1$** at the top of the columns (with the exact number of degrees indicated), **HYB** vertically (**YB** in red) approximately next to the tabular values concerned, **L** om. [b] In **$CC_1$** the table occupies only a single page. [c] In **$CC_1$** the table occupies only a single page.

[1] **YLB** حصّة درجة (**L**, first page): (حصّة الدرجة) [2:] **$CC_1$** om. [3:] **B** om. [4] **F** add. في السنين والشهور, **Y** (first of four pages): add. في السنين, **Y** (second page): add. في الشهور والأيّام [5] **LB** add. تمام [6] **L** الخسوف (corrected to القمر in red above the title) [7] **F** add. في الأيّام والساعات وما بين الأطوال, **Y** (third of four pages): add. في الساعات وكسورها [8:] **Y** (fourth page): تعديل الأيّام ولياليها [9] **F** add. في السنين وفي الشهور, **Y** add. في السنين والشهور [10] **L** add. تمامت, **B** add. تمام [11] **F** add. في الأيّام والساعات وما بين الأطوال, **Y** add. في الأيّام والساعات

## Table 19: Double elongation

*Sources*: **F** fols 50v–51r, **H** fols 37v–38r, **C** fol. 54r, **C$_1$** fol. 20r, **C$_2$** -, **Y** fols 269v–270r, **L** fols 41v–42r, **B** pp. 78–79.

Title (first part): البعد المضاعف [1] [2:]

Title (second part): [a] البعد[3] المضاعف [4] [5:]

See further the general edition of all mean motion tables on pp. 287–88.

### *Explanatory texts and marginal notes*

For text A (in **YLB**) and text B (in **LB**), see the general edition on pp. 289–91.

## Table 20: Lunar first equation

*Sources*: **F** fols 51v–52r, **H** fols 38v–39r, **C** fols 54v–55r, **C$_1$** fols 20v–21r, **C$_2$** -, **Y** fols 270v–271r, **L** fols 42v–43r, **B** pp. 80–81.

Title (first page): التعديل[6] الأوّل للقمر

Title (second page): التعديل[7] الأوّل للقمر [8:]

Argument: درجات[9] المضاعف

Column header for the tabular differences: [b]
- **F**: ‘differences ⟨in⟩ minutes’ تفاضل دقائق[10] [11]
- **CC$_1$**: ‘the differences’ التفاضل
- **YLB**: ‘share of ⟨the⟩ degree’ حصّة درجة[12] [13]

Subcolumn headers: درج / دقائق

Labels (mean, nearest and furthest distance):
*Sources*: **F** at the top of the columns (accompanied by numbers of degrees); **HY** om. entirely; **CC$_1$LB** vertically in the right margins: ‘furthest distance (of the eccentric)’ (starting next to argument $0^s 0°$) and ‘nearest distance’ (starting next to argument $6^s 0°$); **C$_1$** furthermore has ‘mean distance’ vertically next to the values for arguments $8^s 6$–$9°$ and زائد ‘increasing’ vertically next to the values for arguments $9^s 9$–$11°$; **B** ‘mean distance’ vertically next to the columns for the signs starting at $3^s 5°$ (instead of 24°, corrected in a different hand) and $8^s 6°$.

البعد الأبعد [14] / البعد الأوسط / البعد الأقرب / البعد الأوسط

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[a] In **CC$_1$** the table occupies only a single page. [b] **H** does not provide tabular differences.

[1] **F** add. في السنين والشهور [2:] **LB** مضاعف القمر [3] **H** om. [4] **F** add. في الأيّام والساعات وما بين الأطوال [5:] **L** تمامت مضاعف القمر, **B** تمام مضاعف القمر [6] **L** تعديل [7] **B** تمام تعديل [8:] **L** تمامت تعديل القمر [9] **HCC$_1$** om., **YLB** أجزاء [10] **F** (signs 0–5): حصّة دقيقة [11] **F** (signs 2–5): add. ثواني [12] **L** الدرجة [13] **Y** (signs 6–11): add. ق as abbreviation for دقائق [14] **LB** add. من فلك الأوج

## Table 20: Lunar second equation

*Sources*: **F** fols 52v–53r, **H** fols 39v–40r, **C** fols 55v–56r, $\mathbf{C_1}$ fols 21v–22r, $\mathbf{C_2}$ -, **Y** fols 271v–272r, **L** fols 43v–44r, **B** pp. 82–83.

Title (first page): التعديل الثاني للقمر

Title (second page): [1] التعديل[2] الثاني للقمر

Argument: أجزاء[3] التدوير

Column header for the tabular differences: [a]

**F**: ‘differences ⟨in⟩ minutes’ تفاضل دقائق

$\mathbf{CC_1}$: ‘the differences’ التفاضل

**YLB**: ‘share of ⟨the⟩ degree’ حصّة[4] درجة [5]

Subcolumn headers: درج / دقائق

Labels (mean, nearest and furthest distance):

*Sources*: **F** at the top of the columns (accompanied by numbers of degrees); **H** om. entirely; **Y** between the degrees and minutes of the equation values (first ‘mean distance’ incorrectly next to the values for arguments $3^s$ 10–12°); **CLB** vertically in the right margins: ‘furthest distance (of the epicycle)’ (starting next to argument $0^s$ 0°) and ‘nearest distance’ (starting next to argument $6^s$ 0°); **B** furthermore has ‘mean distance’ vertically between the degrees and minutes of the equation values for signs 3 and 8 starting at the degrees indicated in **F** (with additions زائدة (?) and حصّة الدرجة الناقصة in a different hand); $\mathbf{C_1}$ ‘nearest distance’ (vertically in the right margin starting next to argument $6^s$ 0°) and ‘mean distance’ (vertically next to the values for arguments $8^s$ 24–27°).

البعد الأبعد [6] / البعد الأوسط / البعد الأقرب / البعد الأوسط

## Table 20: Lunar variation of the nearest distance

*Sources*: **F** fol. 53v, **H** fol. 40v, **C** fol. 56v, $\mathbf{C_1}$ fol. 22v, $\mathbf{C_2}$ -, **Y** fol. 272v, **L** fol. 44v, **B** p. 84.

Title: اختلاف البعد الأقرب للقمر[7]

Argument: [b] أجزاء[8] المضاعف

Subcolumn headers: [c] درج / دقائق

Label for the undefined part of the table: [d] موضع الاجتماع والاستقبال [9] [10:]

[a] **H** does not provide tabular differences. [b] **FY** also at the bottom (**F** at bottom left upside down), **C** (top left): om. [c] **Y** also at the bottom of the table above signs 11–6, with دقائق written upside down. [d] $\mathbf{HC_1LB}$ values zero instead of a label.

[1] **B** add. تمام [2] **L** تمامت تعديل [3] $\mathbf{HCC_1}$ om. [4] **L** (signs 4–7 and 10–11): خصه [5] **Y** add. ق as abbreviation for دقائق [6] **LB** add. من فلك التدوير [7] **YLB** om. [8] **F** (bottom): om., $\mathbf{HCC_1Y}$ om. [9] **F** add. صفر ‘zero’ [10:] **C** من البعد الأبعد in red in addition to values zero; **Y** adds zeroes in the cells for degrees.

## Table 20: Lunar interpolation minutes

*Sources*: **F** fol. 54r, **H** fol. 41r, **C** fol. 56v, **$C_1$** fol. 22v, **$C_2$** -, **Y** fol. 273r, **L** fol. 44v, **B** p. 84.

Title: دقائق النسب [1]

Subtitle (top of the table):[a] يُضرب في الاختلاف ويُزاد على التعديل الثاني [2:]

Subtitle (bottom of the table):[b] يُضرب في الاختلاف ويُنقص من التعديل الثاني [3:]

Argument:[c] أجزاء[4] التدوير [5]

Subcolumn headers:[d] دقائق

## Table 21: Mean motion of the lunar node

*Sources*: **F** fols 54v–55r, **H** fols 41v–42r, **C** fol. 57r, **$C_1$** fol. 23r, **$C_2$** fol. 43r, **Y** fols 273v–274r, **L** fol. 45r, **B** p. 85.

Title (first part): وسط الجوزهر[6] [7]

Title (second part):[e] [8] وسط الجوزهر [9]

See further the general edition of all mean motion tables on pp. 287–88.

### *Explanatory texts and marginal notes*

For text A (in **LB**) and text B (in **L**), see the general edition on pp. 289–91.

[a] **H** to the right of the table (vertically, starting at the top). [b] **F** upside down, **H** to the left of the table (vertically, starting at the bottom), **$CC_1$** under the table (**C** partially cut off) [c] **FY** also at bottom right and left [d] **$FHCC_1Y$** om. (note that the subcolumn headers are not needed since the title of this table already indicates that its values are 'minutes'; apparently **LB** have added the headers mainly for calligraphic reasons, namely to keep all lines aligned with those of the adjacent variation table). [e] In **$CC_1C_2LB$** the table occupies only a single page.

[1] **F** add. للقمر [2:] **LB** يضرب ويزاد (**L** above the title) [3:] **LB** (under the table): يضرب وينقص [4] **$HCC_1Y$** om. [5] **C** add. للزيادة 'for addition' at the top right and للنقصان 'for subtraction' at the top left; **$C_1$** places these additions next to the table at the top right and bottom left. [6] **L** الجوزهره [7] **F** add. في السنين والشهور [8] **Y** add. تمام [9] **F** add. في الأيّام والساعات وما بين الأطوال

## General edition: Planetary first equation

Title (on the example of Saturn):

| | |
|---|---|
| **F**: | التعديل الأوّل لزحل يزاد على المركز وينقص من الخاصّة |
| **HCC₁C₂YLB**: | [1] التعديل [2] الأوّل لزحل [3] [4:] |

| | | | |
|---|---|---|---|
| Argument: ‘degrees of the centrum’ | درجات المركز | **HCC₁C₂**: | المركز |
| | | **YLB**: | أجزاء المراكز[5] |
| Column header: ‘differences minutes’ | تفاضل دقائق[6] | **H**: | - |
| | | **CC₁C₂**: | التفاضل[7] |
| | | **YLB**: | حصّة درجة [8] |

Subcolumn headers: درج / دقائق

Labels (mean, nearest and furthest distance):

*Sources*: **F** at the top of the columns (accompanied by numbers of degrees), **H** horizontally between the tabular values, **CC₁C₂** vertically to the right of the columns,[a] **YB** vertically between the digits of the tabular values (**B** partially in a different hand).[b] In addition, **B** also gives the labels plus degrees at the heads of the columns in the tables for Mars, Venus and Mercury (for Mars and partially for Mercury in a different hand), as in **F**.[c]

| | |
|---|---|
| mean distance (1) | [9:] البعد الأوسط |
| nearest distance | [10:] البعد الأقرب |
| mean distance (2) | [11:] البعد الأوسط |
| furthest distance | [13:] [12] البعد الأبعد |

[a] **C** includes only occasional labels in the tables for Jupiter, Venus and Mercury. [b] **Y** omits the labels in the table for Venus. [c] **L** omits all labels with the single exception of البعد الأوسط on the first page of the table for Mercury. **Y**-Mars+Mer, **L**-Mars+Ven+Mer, **B**-all add incorrectly placed labels البعد الأبعد من فلك الأوج vertically in the right margin (**Y**: between the digits of the tabular values, **Y**-Mar: om. فلك; **L**-Ven: التدوير for الأوج; **B**-Mars+Mer: in a different hand).

[1] **B**-all (second page): add. تمام [2] **L**-Sat+Jup+Mars (second page): تمام تعديل, **L**-Ven (second page): تمامت تعديل [3] **C**-Sat (vertically on both pages), **L**-Sat (at top of first page in small script): add. يزاد على المركز وينقص من الخاصّة (to some further tables in **C** this was added in a different hand); **C₁**-Sat (both pages, vertically), **C₂**-Sat (both pages, skew, cut off, on second page without والوسط): add. يزاد على المركز والوسط وينقص من الخاصّة [4:] **L**-Mer (first page): تعديل الأوّل للعطارد, **L**-Mer (second page): تمامت تعديل عطارد [5] **Y**-Sat (first page), **L**-Sat, **B**-Sat+Mer (second page): المركز; **L**-Ven (first page): التدوير [6] **F**-Mer (2nd page): om. [7] **C₁**-Sat: تفاضل [8] **Y**-Sat (first page): add. ق [9:] **F**-Mars: om., **B**-Mars+Ven: الأوسط in the column header. [10:] **F**-Mars, **C₁**-Mars: om.; **B**-Mars+Mer: الأقرب in the column header, **B**-Ven: الأبعد الأقرب in the column header. [11:] **C**-Mer: om.; **B**-Mars+Mer: الأوسط in the column header, **B**-Ven: الأبعد الأقرب in the column header. [12] **H**-Ven, **YB**-Sat: الأقرب (in **YB** corrected to الأبعد, in **B** in the margin). [13:] **C**-Mer: om., **B**-Mars+Mer: الأبعد in the column header, **B**-Ven: البعد in the column header.

The below table indicates the positions of the labels. These are the degrees mentioned in the column headers in manuscripts **F** and **B**, which in general correspond to the arguments of the tabular values just below the horizontal labels in **H**. Variants are included in the apparatus whenever the degrees in the column headers or the position of a horizontal label differ from that given in the table or whenever a vertical label does not start at the given position or one row higher. In the latter cases the variant indicates the entire range of arguments over which the vertical label extends. An asterisk before a variant indicates that the misplacement may be due to lack of space due to another label or the beginning or end of a column. In order to allow for easier verification of the positions, the displacements are provided in an additional row of the table.[a]

| planet | Saturn | Jupiter | Mars | Venus [1] | Mercury |
|---|---|---|---|---|---|
| mean (1) | $2^s$ 19 [2] | $2^s$ 15 [3] | $1^s$ 6 [4] | $1^s$ 11 [5] | $2^s$ 2 [6] |
| nearest | $5^s$ 16 [7] | $5^s$ 12 | $4^s$ 1 [8] | $4^s$ 10 [9] | $5^s$ 0 [10] |
| mean (2) | $8^s$ 13 [11] | $8^s$ 9 | $6^s$ 26 [12] | $7^s$ 9 [13] | $7^s$ 28 [14] |
| furthest | $11^s$ 16 [15] | $11^s$ 12 | $10^s$ 1 [16] | $10^s$ 10 [17] | $11^s$ 0 [18] |
| displacement | 14° | 18° | $1^s$ 29° | $1^s$ 20° | $1^s$ 0° |

## General edition: Planetary second equation

Title (on the example of Saturn):

**F**: التعديل الثاني لزحل يعدّل ويزاد على المركز مع الأوج

**$HCC_1C_2$YLB**: [19] التعديل[20] الثاني لزحل [21:]

Argument: 'degrees of the epicycle' درجات[22] التدوير — **$HCC_1C_2$**: التدوير; **YLB**: أجزاء التدوير

Column header: 'differences minutes' تفاضل دقائق[23] [24] — **H**: -; **$CC_1C_2$**: التفاضل; **YLB**: حصّة درجة

Subcolumn headers: درج / دقائق

[a] Misplaced labels in **B** that are not in the main hand are not included in the apparatus.

[1] **Y** omits all four labels for Venus [2] **$C_1$** 17–21° **B** 20–23° [3] **$C_1$** 12–16° **Y** 16–20° [4] **F** om. [5] **H** 12° **C** 13–15° **$C_1C_2$** 12–17° [6] **C** 3–6° **Y** 4–7° [7] **Y** 17–19° [8] **F$C_1$** om. **Y** 2–5° [9] **H** 11° **C** 11–13° [10] **Y** 1–3° [11] **Y** 15–18° **B** 14–19° [12] **F** 24° **Y** 26–28° **B** 26° in header, om. from table [13] **F** om. degrees, **$C_1$** 7–12° **$C_2$** 7–11° [14] **$C_1$** *24–29° **$C_2$** *24–28° **Y** 26–28° **B** *25–28° [15] **Y** 18–20° [16] **Y** 2–5° [17] **H** 11° **C** 16–18° [18] **Y** 2–4° [19] **B**-Jup (first page): add. أوّل, **B**-all (second page): add. تمام [20] **L**-Sat+Mars (second page): تمام تعديل, **L**-Jup+Ven (second page): تمامت تعديل [21:] **L**-Mer (first page): تعديل ثاني للعطارد, **L**-Mer (second page): تمامت تعديل ثاني للعطارد [22] **F**-Mars: أجزاء [23] **F**-Mars ($5^s$ and $6^s$): درج ودقائق, **F**-Mer: om. [24] **F**-Ven ($5^s$ and $6^s$): add. ودرج

## Labels (distances and planetary phases):

*Sources*: **F** distance labels at the top of the columns for the signs with the numbers of degrees added in red,[a] phases horizontally between two tabular values; **H** furthest and nearest distance in the right margin, all others horizontally between the tabular values;[b] **C** only occasional labels vertically to the right of the columns;[c] $\mathbf{C_1C_2}$ vertically to the right of the columns;[d] **YLB** vertically between the digits of the equation values;[e] **B**-Mars+Ven+Mer add labels for the distances together with their degrees at the heads of the columns for the signs (as in **F**, partially in different hands).[f]

| label | masculine form | feminine form (Venus)[1] |
|---|---|---|
| furthest distance | 2: البعد الأبعد | |
| mean distance (1) | 3: البعد الأوسط | |
| nearest distance | 4: البعد الأقرب | |
| mean distance (2) | 5: البعد الأوسط | |
| stationary (1) | مقيم[6] | مقيمة |
| retrograde | راجع[7] | راجعة |
| stationary (2) | مقيم[8] | مقيمة |
| progressive | مستقيم | مستقيمة[9] |
| western visibility | يرى[10] مغرب[11] | ترى[12] مغربة |

[a] **F**-Mars: om. all four distance labels. [b] **H**-Sat: stationary (1) and retrograde in the right margin with their positions indicated by small circles between the tabular values. [c] Later users of **C** added several labels in a faint, sometimes hardly discernible red, partially vertically to the right of the columns and partially horizontally between the tabular values. At the top of the columns, مستقيم and (را)جع were indicated in the same faint red. Most labels in **C** are so badly placed that I have decided not to include their positions in the apparatus to the table on the next page. [d] $\mathbf{C_1}$-Mer (second page): om. all labels. [e] **L** includes only the labels for the furthest distance along the tables for Mars, Venus and Mercury, and all labels on the second page of the table for Mercury. **B** writes the labels for the furthest and nearest distances in the margin and those for the phases of Venus and Mercury horizontally between the tabular values. [f] In all sources the labels are generally written without diacritical dots. These omissions are not indicated in the apparatus, but incorrectly added diacritical dots are. Most witnesses use for Venus the feminine forms of the labels for the planetary phases as indicated in the third column, but $\mathbf{C_1}$ does this only for the first station and for progressive motion.

[1] $\mathbf{C_1}$-Ven: masculine forms except for مقيمة and مستقيمة on the second page. 2: **Y**-except Sat: om.; **L**-Mars+Venus+Mer, **B**-all (in both sources in the right margin): البعد الأبعد من فلك التدوير; **B**-Mars (at head of column): الأبعد 3: **B**-Mars (at head of column): الأوسط 4: **H**-Jup, **B**-Ven (at head of column): om.; **B**-Mars+Mer (at head of column): الأقرب 5: **B**-Mars+Mer (at head of column): الأوسط [6] **H**-Sat: add. לאלראגע (لالراجع; the label and the addition are both written in the margin in a different hand) [7] **B**-Mer: ع (in a different hand) [8] **B**-Mer: مسقم [9] **B**-Ven: مستقم [10] $\mathbf{C_1}$-Mer: يري [11] **F**-Mer: مغربا [12] **C**-Ven: يرى, $\mathbf{C_1}$-Ven: يري, **Y**-Ven: تري

| label | masculine form | feminine form (Venus)[1] |
|---|---|---|
| western disappearance | يخفى[2] مغرب[3] | تخفى[4] مغربة[5] |
| eastern visibility | يرى[6] مشرق[7] [8]: | ترى مشرقة[9] |
| eastern disappearance | يخفى مشرق[10] | تخفى مشرقة [11]: |

The below table indicates the positions of the labels for distances and phases. For the distance labels these are the degrees mentioned in the column headers in manuscripts **F** and **B**. For the phases the table presents the arguments of the tabular values just below the horizontal labels found in sources **F**, **H** and **B**. Variants are included in the apparatus whenever the position of a horizontal label differs from that given in the table and whenever a vertical label does not start at the given position or one row higher. An asterisk before a variant indicates that the misplacement may be due to lack of space due to another label or the beginning or end of a column.

| planet | Saturn | Jupiter | Mars[12] | Venus | Mercury[13] |
|---|---|---|---|---|---|
| furthest distance | $0^s$ 0 | $0^s$ 0[14] | $0^s$ 0[15] | $0^s$ 0[16] | $0^s$ 0[17] |
| mean distance (1) | $3^s$ 3[18] | $3^s$ 6 | $3^s$ 21[19] | $3^s$ 23[20] | $3^s$ 11[21] |
| nearest distance | $6^s$ 0 | $6^s$ 0[22] | $6^s$ 0 | $6^s$ 0[23] | $6^s$ 0 |
| mean distance (2) | $8^s$ 27[24] | $8^s$ 24[25] | $8^s$ 9[26] | $8^s$ 7[27] | $8^s$ 19[28] |
| stationary (1) | $3^s$ 24[29] | $4^s$ 5 | $5^s$ 8[30] | $5^s$ 16[31] | $4^s$ 26[32] |
| retrograde | $3^s$ 26[33] | $4^s$ 8 | $5^s$ 20[34] | $5^s$ 19[35] | $4^s$ 28 |
| stationary (2) | $8^s$ 4 | $7^s$ 22[36] | $6^s$ 10[37] | $6^s$ 11 | $7^s$ 2[38] |
| progressive | $8^s$ 6 | $7^s$ 25 | $6^s$ 22[39] | $6^s$ 14[40] | $7^s$ 4[41] |
| western visibility | — | — | — | $0^s$ 15 | $1^s$ 21 |
| western disappearance | $11^s$ 8[42] | $11^s$ 15[43] | $11^s$ 4 | $5^s$ 27[44] | $5^s$ 6[45] |
| eastern visibility | $0^s$ 21[46] | $0^s$ 15[47] | $0^s$ 26[48] | $6^s$ 3[49] | $6^s$ 24[50] |
| eastern disappearance | — | — | — | $11^s$ 15 | $10^s$ 9[51] |

[1] $\mathbf{C_1}$-Ven: masculine forms [2] **C**-Jup, $\mathbf{C_2}$-Sat+Jup+Mars: حفى; $\mathbf{C_1}$-Ven+Mer: يخفي, $\mathbf{C_1}$-Sat+Jup+Mars: خفي [3] **F**-all: مغربا, **Y**-Jup+Mars: في المغرب [4] $\mathbf{C_1}$-Ven: يخفي, **Y**-Ven: ىحفى [5] **Y**-Ven: مغرب [6] $\mathbf{C_1}$-Jup+Mars+Ven, $\mathbf{C_2}$-Jup, **Y**-Sat+Mars: يري; **L**-Mer: ىحمى (sic!) [7] **F**-all: مشرقا, **Y**-Jup: في المسرق, **L**-Mer: بمسرق [8] **C**-Sat: unclear; $\mathbf{C_1C_2}$-Sat: om. [9] **Y**-Ven: مشرق [10] **F**-Mer: مشرقا, **L**-Mer: بمسرق, **B**-Mer: قه [11] $\mathbf{C_1}$-Ven: يخفي مشرق, **Y**-Ven: يحفى مشرقيه [12] **F** omits all four distances for Mars. [13] $\mathbf{C_1}$ omits all labels from the second half of the Mercury table. [14] **Y** om. [15] **Y** om. [16] **Y** om. [17] **Y** om. [18] **F** om. **Y** 5–8° [19] $\mathbf{C_1C_2}$ 18–22° [20] **F** $2^s$ $\mathbf{C_1}$ 24–28° **Y** 25–27° [21] $\mathbf{C_1C_2}$ 21–24° **Y** 12–15° **B** 22–25° (label next to the tabular values, in a different hand) [22] **H** om. [23] **B** om. at head of column [24] $\mathbf{C_1}$ 22–26° $\mathbf{C_2}$ 25–28° [25] **H** $9^s$ $\mathbf{C_1}$ 18–22° $\mathbf{C_2}$ 18–23° **Y** 25–28° **B** 17–21° [26] **Y** 10–13° [27] **YB** 8–11° (**B** in a different hand?) [28] **B** correctly positioned label crossed out and wrongly replaced at 8–12° by a different hand. [29] **F** 25° [30] $\mathbf{C_1C_2}$ 15–16° **B** 9–11° [31] **B** 17° [32] **B** 25° [33] $\mathbf{C_1C_2}$ 27–29° [34] $\mathbf{C_1C_2}$ 17–18° **B** 21–23° [35] **Y** 20–21° [36] **B** 16–17° (corrected to 22–23° in a different hand) [37] $\mathbf{C_1C_2}$ 12–13° [38] $\mathbf{C_2}$ 2–3° **B** 3° [39] $\mathbf{C_1C_2}$ 14–17° [40] **B** 15° [41] $\mathbf{C_2}$ 5–8° **B** 5° [42] **H** 10° $\mathbf{C_1C_2}$ 9–12° **Y** 9–11° [43] **Y** 13–16° [44] $\mathbf{C_2}$ 25–28° **Y** 28–29° [45] **B** 7° [46] $\mathbf{C_1C_2}$ om. **Y** 22–24° [47] **H** 16° **Y** 16–18° [48] **F** 27° [49] $\mathbf{C_1}$ *5–9° $\mathbf{C_2}$ *5–8° **Y** *6–8° [50] **L** 20–22° [51] **Y** 10–12°

**General edition: Variation of the nearest distance**

Title (on the example of Saturn): اختلاف[1] البعد[2] الأقرب لزحل[3]

Argument: 'corrected centrum' المركز المعدّل[4] **YLB**: أجزاء[5] المركز[6] المعدّل[7]

Subcolumn headers: درج / دقائق

Labels (at beginning and end of the table): [a] من[8] البعد الأبعد

**General edition: Interpolation minutes (nearest distance)**

Title: دقائق النسب[9] [10]

Subtitles:

at the top of the table: [b] يضرب[11] في الاختلاف[12] ويزاد على التعديل الثاني

at the bottom of the table: [c] يضرب في الاختلاف[13] وينقص من التعديل الثاني

Argument: [d]

'parts of the epicycle' أجزاء[14] التدوير[15] **H**: التدوير

$\mathbf{CC_1C_2}$: التدوير للزيادة (top right), التدوير للنقصان (top left)

Subcolumn headers: [e] دقائق

---

[a] **H**-Mer (end): om.; **C**-Mer (beginning and end): om.; **Y**-Mer: zeroes instead of labels, **Y**-others: labels in addition to zeroes; **L**-except Mer: empty cells instead of labels. [b] **H** to the right of the table (vertically, starting at the top), $\mathbf{CC_1C_2}$, **L**-Mars+Ven: om. (**C**-Jup+Mars: vertically on the right side of the table in a different hand), **B**-Mars+Mer: above the title, **B**-Ven: above the title in a different hand [c] **F**: upside down, **Y**: horizontally; **H** to the left of the table (vertically, starting at the bottom); $\mathbf{CC_1C_2}$: om. (**C**-Jup+Mars: vertically to the left of the table in a different hand); **L**-Jup, **B**-all: under the table in the shortened form يضرب وينقص [d] **FY**: four occurrences (top and bottom, left and right); $\mathbf{HCC_1C_2LB}$: two occurrences (top left and top right); **L**-Sat (top left): om. [e] Only in **L**-Mars+Ven and **B**-Mars+Ven+Mer (**L**-Mars: both درج and دقائق in each column, **B**-Mars: درج and دقائق alternately).

[1] **H**-Ven+Mer: om. [2] **L**-Ven: بعد [3] **YLB** om. (**L**-Ven: زهرة) [4] **C**-Ven: om. [5] **Y**-Mars: om. [6] **Y**-Jup, **L**-Sat+Ven+Mer, **B**-Sat: المراكز [7] **Y**-Sat+Jup+Mer, **L**-Sat+Mer, **B**-Sat: om. [8] **Y**-Ven (beginning and end): om. [9] **L**-Ven+Mer: النسبة [10] **F**-Sat: add. لزحل [11] **L**-Mer: نصرة [12] **H**-all: اختلاف البعد الأقرب, **Y**-Sat: الاختلا [13] **H**-all: اختلاف البعد الأقرب [14] **Y**-Mer (all four occurrences), **L**-Jup (top left): om. [15] **Y**-Sat (top right): المركز

**General edition: Variation of the furthest distance**

Title (on the example of Saturn): اختلاف[1] البعد[2] الأبعد لزحل[3]

Argument: 'corrected centrum' المركز المعدّل **YLB**: أجزاء[4] المركز[5] المعدّل[6]

Subcolumn headers: [a] درج / دقائق

Labels (at beginning and end of the table): [b] من البعد[7] الأقرب[8]

**General edition: Interpolation minutes (furthest distance)**

Title: دقائق النسب[9] [10]

Subtitles:

at the top of the table: [c] يضرب[11] في الاختلاف[12] ويزاد على التعديل الثاني[13]

at the bottom of the table: [d] يضرب في الاختلاف[14] وينقص من التعديل الثاني

Argument: [e]

'parts of the epicycle' أجزاء[15] التدوير **H**: التدوير

**CC$_1$C$_2$**: التدوير للزيادة (top right), التدوير للنقصان (top left)

Subcolumn headers: [f] دقائق

[a] **H**-Mer: in Arabic script [b] **C**-Sat (end), **L**-Jup+Mars (beginning and end), **B**-Mars (beginning): om.; **Y**-Mars: zeroes instead of labels, **Y**-others: labels in addition to zeroes; **L**-Sat (end): zeroes instead of label [c] **H**-all: to the right of the table (vertically, starting at the top), **CC$_1$C$_2$**, **L**-Jup+Mars+Ven, **B**-Mars: om., **B**-Jup: shortened form يضرب ويزاد to the right of the table, **B**-Ven: above the title (in a different hand), **B**-Mer: above the title [d] **F** upside down, **Y** straight up, **H** to the left of the table (vertically, starting at the bottom), **CC$_1$C$_2$L** om., **B** shortened form يضرب وينقص (**B**-Ven: ينقص) under the table [e] **FY**: 4 occurrences (top and bottom, left and right); **HCC$_1$C$_2$LB**: 2 occurrences (top left and top right) [f] Only in **L**-Jup+Mars+Ven and **B**-except Sat (**B**-Mars: درج and دقائق alternately).

[1] **H**-Ven+Mer: om. [2] **L**-Ven: بعد [3] **YLB** om. [4] **Y**-Mars: om. [5] **Y**-Sat+Jup, **L**-Ven, **B**-Sat: المراكز; **L**-Sat: التدوير [6] **Y**-except Mars, **LB**-Sat: om. [7] **L**-Sat (beginning): بعد, **L**-Sat (end)+Jup+Mars: om. [8] **C**-Ven+Mer (beginning), **Y**-Sat (end): الأبعد [9] **L**-Ven+Mer: النسبة [10] **F**-Sat: add. لزحل [11] **L**-Mer: بصره [12] **H**-all: اختلاف البعد الأبعد [13] **Y**-Sat: om. [14] **H**-all: اختلاف البعد الأبعد [15] **Y**-Jup (bottom left and right): om.

## Tables 22–36: Planetary mean motions and equations

For complete overviews of the variants in the titles and headers and for the explanatory texts and marginal notes that appear for more than one table in several manuscripts, see the general edition of all mean motion tables on pp. 287–88 and the general edition of all planetary equation tables on pp. 300–05. This section only provides the titles of all tables with their variants and indicates which explanatory texts and marginal notes appear in which manuscripts.

### Table 22: Mean motion of Saturn

*Sources*: **F** fols 55v–56r, **H** fols 42v–43r, **C** fol. 57v, $\mathbf{C_1}$ fol. 23v, $\mathbf{C_2}$ fol. 43v, **Y** fols 274v–275r, **L** fols 45v–46r, **B** pp. 86–87.

Title: [1] وسط زحل [2]

*Explanatory texts and marginal notes*

For text A (in **B**), text B (in **LB**) and notes C and E (in $\mathbf{CC_1C_2}$), see the general edition on pp. 289–91.

### Table 23: Mean anomaly of Saturn

*Sources*: **F** fols 56v–57r, **H** fols 43v–44r, **C** fol. 58r, $\mathbf{C_1}$ fol. 24r, $\mathbf{C_2}$ fol. 44r, **Y** fols 275v–276r, **L** fols 46v–47r, **B** pp. 88–89.

Title: [3] خاصّة زحل [4]

*Explanatory texts and marginal notes*

For texts A and B (in **LB**) and note D (in $\mathbf{C_1C_2}$), see the general edition on pp. 289–91.

### Table 24: First equation for Saturn

*Sources*: **F** fols 57v-58r, **H** fols 44v–45r, **C** fols 58v–59r, $\mathbf{C_1}$ fols 24v–25r, $\mathbf{C_2}$ fols 44v and 37r, **Y** fols 276v–277r, **L** fols 47v–48r, **B** pp. 90–91.

Title: [5] التعديل[6] الأوّل لزحل [7]

[1] **F** (first page): add. جدول, **YLB** (second page): add. تمام [2] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال [3] **LB** (second page): add. تمام [4] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين أطوال البلدان [5] **B** (second page): add. تمام [6] **L** (second page): تمام تعديل [7] $\mathbf{FCC_1C_2}$ add. يزاد على المركز وينقص من الخاصّة ($\mathbf{CC_1}$ vertically, $\mathbf{C_2}$ diagonally and cut off; $\mathbf{C_1C_2}$ add. والوسط after المركز)

### Table 24: Second equation for Saturn

*Sources*: **F** fols 58v-59r, **H** fols 45v-46r, **C** fols 59v–60r, $\mathbf{C_1}$ fols 25v–26r, $\mathbf{C_2}$ fols 37v–38r, **Y** fols 277v–278r, **L** fols 48v–49r, **B** pp. 92–93.

Title: [1] التعديل [2] الثاني لزحل [3]

### Table 24: Variation of the nearest distance for Saturn

*Sources*: **F** fol. 59v, **H** fol. 46v, **C** fol. 60v, $\mathbf{C_1}$ fol. 26v, $\mathbf{C_2}$ fol. 38v, **Y** fol. 278v, **L** fol. 49v, **B** p. 94.

Title: اختلاف البعد الأقرب لزحل [4]

### Table 24: Interpolation minutes (nearest distance) for Saturn

*Sources*: **F** fol. 59v, **H** fol. 47r, **C** fol. 60v, $\mathbf{C_1}$ fol. 26v, $\mathbf{C_2}$ fol. 38v, **Y** fol. 278v, **L** fol. 49v, **B** p. 94.

Title: دقائق النسب [5]

### Table 24: Variation of the furthest distance for Saturn

*Sources*: **F** fol. 60r, **H** fol. 47v, **C** fol. 61r, $\mathbf{C_1}$ fol. 27r, $\mathbf{C_2}$ fol. 39r, **Y** fol. 279r, **L** fol. 50r, **B** p. 95.

Title: اختلاف البعد الأبعد لزحل [6]

### Table 24: Interpolation minutes (furthest distance) for Saturn

*Sources*: **F** fol. 60r, **H** fol. 48r, **C** fol. 61r, $\mathbf{C_1}$ fol. 27r, $\mathbf{C_2}$ fol. 39r, **Y** fol. 279r, **L** fol. 50r, **B** p. 95.

Title: دقائق النسب [7]

### Table 25: Mean motion of Jupiter

*Sources*: **F** fols 60v–61r, **H** fols 48v–49r, **C** fol. 61v, $\mathbf{C_1}$ fol. 27v, $\mathbf{C_2}$ fol. 39v, **Y** fols 279v–280r, **L** fols 50v–51r, **B** pp. 96–97.

Title: [8] وسط المشتري [9]

*Explanatory texts and marginal notes*

For text A (in **B**), text B (in **LB**), and notes C and E (in $\mathbf{CC_1C_2}$), see the general edition on pp. 289–91.

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[1] **B** (second page): add. تمام [2] **L** (second page): تمام تعديل [3] **F** add. يعدّل ويزاد على المركز مع الأوج [4] **YLB** om. [5] **F** add. لزحل [6] **YLB** om. [7] **F** add. لزحل [8] **YLB** (second page): add. تمام [9] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال

## Table 26: Mean anomaly of Jupiter

*Sources*: **F** fols 61v–62r, **H** fols 49v–50r, **C** fol. 62r, $\mathbf{C_1}$ fol. 28r, $\mathbf{C_2}$ fol. 40r, **Y** fols 280v–281r, **L** fols 51v–52r, **B** pp. 98–99.

Title: [1] خاصّة المشتري [2]

### *Explanatory texts and marginal notes*

For text A (in **B**), text B (in **LB**), and note D (in $\mathbf{CC_1C_2}$), see the general edition on pp. 289–91.

## Table 27: First equation for Jupiter

*Sources*: **F** fols 62v–63r, **H** fols 50v-51r, **C** fols 62v–63r, $\mathbf{C_1}$ fols 28v–29r, $\mathbf{C_2}$ fols 40v–41r, **Y** fols 281v–282r, **L** fols 52v–53r, **B** pp. 100–101.

Title: [3] التعديل[4] الأوّل للمشتري [5]

## Table 27: Second equation for Jupiter

*Sources*: **F** fols 63v–64r, **H** fols 51v–52r, **C** fols 63v–64r, $\mathbf{C_1}$ fols 29v–30r, $\mathbf{C_2}$ fols 41v–42r, **Y** fols 282v–283r, **L** fols 53v–54r, **B** pp. 102–103.

Title: [6] التعديل[7] الثاني للمشتري [8]

## Table 27: Variation of the nearest distance for Jupiter

*Sources*: **F** fol. 64v, **H** fol. 52v, **C** fol. 64v, $\mathbf{C_1}$ fol. 30v, $\mathbf{C_2}$ fol. 42v, **Y** fol. 283v, **L** fol. 54v, **B** p. 104.

Title: اختلاف البعد الأقرب للمشتري[9]

## Table 27: Interpolation minutes (nearest distance) for Jupiter

*Sources*: **F** fol. 64v, **H** fol. 53r, **C** fol. 64v, $\mathbf{C_1}$ fol. 30v, $\mathbf{C_2}$ fol. 42v, **Y** fol. 283v, **L** fol. 54v, **B** p. 104.

Title: دقائق النسب

## Table 27: Variation of the furthest distance for Jupiter

*Sources*: **F** fol. 65r, **H** fol. 53v, **C** fol. 65r, $\mathbf{C_1}$ fol. 31r, $\mathbf{C_2}$ fol. 25r, **Y** fol. 284r, **L** fol. 55r, **B** p. 105.

Title: اختلاف البعد الأبعد للمشتري[10]

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[1] **Y** (second page): add. بقيّة, **LB** (second page): add. تمام [2] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال [3] **B** (second page): add. تمام [4] **L** (second page): تمام تعديل [5] **F** add. يزاد على المركز وينقص من الخاصّة [6] **B** (first page): add. أوّل, **B** (second page): add. تمام [7] **L** (second page): تمامت تعديل [8] **F** add. يعدّل ويزاد على المركز مع الأوج [9] **YLB** om. [10] **YLB** om.

## Table 27: Interpolation minutes (furthest distance) for Jupiter

*Sources*: **F** fol. 65r, **H** fol. 54r, **C** fol. 65r, **$C_1$** fol. 31r, **$C_2$** fol. 25r, **Y** fol. 284r, **L** fol. 55r, **B** p. 105.

Title: دقائق النسب

## Table 28: Mean motion of Mars

*Sources*: **F** fols 65v–66r, **H** fols 54v–55r, **C** fol. 65v, **$C_1$** fol. 31v, **$C_2$** fol. 25v, **Y** fols 284v–285r, **L** fols 55v–56r, **B** pp. 106–107 (see Plate 9).

Title: [1] وسط المرّيخ [2]

### *Explanatory texts and marginal notes*

For texts A and B (in **LB**) and notes C and E (in **C$C_1C_2$**), see the general edition on pp. 289–91.

## Table 29: Mean anomaly of Mars

*Sources*: **F** fols 66v–67r, **H** fols 55v–56r, **C** fol. 66r, **$C_1$** fol. 32r, **$C_2$** fol. 26r, **Y** fols 285v–286r, **L** fols 56v–57r, **B** pp. 108–109.

Title: [3] خاصّة المرّيخ [4]

### *Explanatory texts and marginal notes*

For text A (in **B**) and note D (in **C$C_1C_2$**), see the general edition on pp. 289–91.

## Table 30: First equation for Mars

*Sources*: **F** fols 67v–68r, **H** fols 56v–57r, **C** fols 66v–67r, **$C_1$** fols 32v–33r, **$C_2$** fols 26v–27v, **Y** fols 286v–287r, **L** fols 57v–58r, **B** pp. 110–111.

Title: [5] التعديل[6] الأوّل للمرّيخ [7]

## Table 30: Second equation for Mars

*Sources*: **F** fols 68v–69r, **H** fols 57v–58r, **C** fols 67v–68r, **$C_1$** fols 33v–34r, **$C_2$** fols 27v–28r, **Y** fols 287v–288r, **L** fols 58v–59r, **B** pp. 112–113.

Title: [8] التعديل[9] الثاني للمرّيخ [10]

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[1] **L** (second page): add. تمامت, **B** add. تمام
[2] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال
[3] **L** (second page): add. تمامت, **B** (second page): add. تمام
[4] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال
[5] **B** (second page): add. تمام
[6] **L** (second page): تمام تعديل
[7] **F** add. يزاد على المركز وينقص من الخاصّة
[8] **B** (second page): add. تمام
[9] **L** (second page): تمام تعديل
[10] **F** add. يعدّل ويزاد على المركز مع الأوج

## Tables 30 and 30a: Variation of the nearest distance for Mars

*Sources*: **F** fol. 69v, **H** fol. 58v, **C** fol. 68v, $\mathbf{C_1}$ fol. 34v, $\mathbf{C_2}$ fol. 28v, **Y** fol. 288v, **L** fol. 59v, **B** p. 114.

Title: اختلاف البعد الأقرب للمرّيخ[1]

## Table 30: Interpolation minutes (nearest distance) for Mars

*Sources*: **F** fol. 69v, **H** fol. 59r, **C** fol. 68v, $\mathbf{C_1}$ fol. 34v, $\mathbf{C_2}$ fol. 28v, **Y** fol. 288v, **L** fol. 59v, **B** p. 114.

Title: دقائق النسب

## Tables 30 and 30a: Variation of the furthest distance for Mars

*Sources*: **F** fol. 70r, **H** fol. 59v, **C** fol. 69r, $\mathbf{C_1}$ fol. 35r, $\mathbf{C_2}$ fol. 29r, **Y** fol. 289r, **L** fol. 60r, **B** p. 115.

Title: اختلاف البعد الأبعد للمرّيخ[2]

## Table 30: Interpolation minutes (furthest distance) for Mars

*Sources*: **F** fol. 70r, **H** fol. 60r, **C** fol. 69r, $\mathbf{C_1}$ fol. 35r, $\mathbf{C_2}$ fol. 29r, **Y** fol. 289r, **L** fol. 60r, **B** p. 115.

Title: دقائق النسب

## Table 30b: Correction of the true position of Mars

*Sources*: **C** fols 69v–70r (see Plate 10), $\mathbf{C_1}$ fols 35v–36r, $\mathbf{C_2}$ fols 29v–30r, **L** fols 60v–61r, **B** pp. 116–117.

Title: دقائق النسب[3] الإصلاح[4] // إصلاح تقويم المرّيخ

Arguments: [5] التدوير // المركز المعدّل

Subcolumn headers: دقائق // درج / دقائق

## Table 31: Mean motion of Venus

*Sources*: **F** fols 70v–71r, **H** fols 60v–61r, **C** fol. 70v, $\mathbf{C_1}$ fol. 36v, $\mathbf{C_2}$ fol. 30v, **Y** fols 289v–290r, **L** fols 61v–62r, **B** pp. 118–119.

Title: [6] وسط الزهرة [7]

### *Explanatory texts and marginal notes*

For text A (in **B**), text B (in **LB**), and note C (in $\mathbf{CC_1C_2}$), see the general edition on pp. 289–91.

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[1] **YLB** om. [2] **YLB** om. [3] $\mathbf{C_1C_2}$ نسب [4] **C** (first page) om., **C** (second page) written as a column header in a separate cell underneath the title [5] **B** (first page, first column) add. أجزاء [6] **LB** (second page): add. تمام [7] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال

## Table 32: Mean anomaly of Venus

*Sources*: **F** fols 71v–72r, **H** fols 61v-62r, **C** fol. 71r, **$C_1$** fol. 37r, **$C_2$** fol. 31r, **Y** fols 290v–291r, **L** fols 62v–63r, **B** pp. 120–121.

Title: [1] خاصّة الزهرة [2]

*Explanatory texts and marginal notes*

For texts A and B (in **LB**) and note D (in **$C_1C_2$**), see the general edition on pp. 289–91.

## Table 33: First equation for Venus

*Sources*: **F** fols 72v–73r, **H** fols 62v–63r, **C** fols 71v–72r, **$C_1$** fols 37v–38r, **$C_2$** fols 31v–32r, **Y** fols 291v–292r, **L** fols 63v–64r, **B** pp. 122–123.

Title: [3] التعديل[4] الأوّل للزهرة [5]

## Table 33: Second equation for Venus

*Sources*: **F** fols 73v–74r, **H** fols 63v–64r, **C** fols 72v–73r, **$C_1$** fols 38v–39r, **$C_2$** fols 32v–33r, **Y** fols 292v–293r, **L** fols 64v–65r, **B** pp. 124–125.

Title: [6] التعديل[7] الثاني للزهرة [8]

## Table 33: Variation of the nearest distance for Venus

*Sources*: **F** fol. 74v, **H** fol. 64v, **C** fol. 73v, **$C_1$** fol. 39v, **$C_2$** fol. 33v, **Y** fol. 293v, **L** fol. 65v, **B** p. 126.

Title: اختلاف[9] البعد[10] الأقرب للزهرة[11]

## Table 33: Interpolation minutes (nearest distance) for Venus

*Sources*: **F** fol. 74v, **H** fol. 65r, **C** fol. 73v, **$C_1$** fol. 39v, **$C_2$** fol. 33v, **Y** fol. 293v, **L** fol. 65v, **B** p. 126.

Title: دقائق النسب[12]

## Table 33: Variation of the furthest distance for Venus

*Sources*: **F** fol. 75r, **H** fol. 65v, **C** fol. 74r, **$C_1$** fol. 40r, **$C_2$** fol. 34r, **Y** fol. 294r, **L** fol. 66r, **B** p. 127.

Title: اختلاف[13] البعد[14] الأبعد للزهرة[15]

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[1] **LB** (second page): add. تمام [2] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال [3] **B** (second page): add. تمام [4] **L** (second page): تمامت تعديل [5] **F** add. يزاد على المركز وينقص من الخاصّة [6] **B** (second page): add. تمام [7] **L** (second page): تمامت تعديل [8] **F** add. يعدّل ويزاد على المركز مع الأوج [9] **H** om. [10] **L** بعد [11] **YB** om., **L** زهره [12] **L** السسه [13] **H** om. [14] **L** بعد [15] **YLB** om.

## Table 33: Interpolation minutes (furthest distance) for Venus

*Sources*: **F** fol. 75r, **H** fol. 66r, **C** fol. 74r, **$C_1$** fol. 40r, **$C_2$** fol. 34r, **Y** fol. 294r, **L** fol. 66r, **B** p. 127.

Title: دقائق النسب[1]

## Table 34: Mean motion of Mercury

*Sources*: **F** fols 75v–76r, **H** fols 66v–67r, **C** fol. 74v, **$C_1$** fol. 40v, **$C_2$** fol. 34v, **Y** fols 294v–295r, **L** fols 66v–67r, **B** pp. 128–129.

Title: [2] وسط عطارد [3]

*Explanatory texts and marginal notes*

For texts A and B (in **LB**) and notes C and E (in **$CC_1C_2$**), see the general edition on pp. 289–91.

## Table 35: Mean anomaly of Mercury

*Sources*: **F** fols 76v–77r, **H** fols 67v–68r, **C** fol. 75r, **$C_1$** fol. 41r, **$C_2$** fol. 35r, **Y** fols 295v–296r, **L** fols 67v–68r, **B** pp. 130–131.

Title: [4] خاصّة عطارد [5]

*Explanatory texts and marginal notes*

For texts A and B (in **LB**) and note D (in **$C_1C_2$**), see the general edition on pp. 289–91.

## Table 36: First equation for Mercury

*Sources*: **F** fols 77v–78r, **H** fols 68v–69r, **C** fols 75v–76r, **$C_1$** fols 41v–42r, **$C_2$** fols 35v–36r, **Y** fols 296v–297r, **L** fols 68v–69r, **B** pp. 132–133.

Title: [6] التعديل الأوّل لعطارد [7] [8:]

## Table 36: Second equation for Mercury

*Sources*: **F** fols 78v–79r, **H** fols 69v–70r, **C** fols 76v–77r, **$C_1$** fols 42v–43r, **$C_2$** fols 36v and 45r, **Y** fols 297v–298r, **L** fols 69v–70r, **B** pp. 134–135.

Title: [9] التعديل الثاني لعطارد [10] [11:]

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[1] **L** النسبة [2] **L** (second page): add. تمامت, **B** (second page): add. تمام [3] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال [4] **L** (second page): add. تمامت, **B** (second page): add. تمام [5] **F** (first page): add. في السنين والشهور, **F** (second page): add. في الأيّام والساعات وما بين الأطوال [6] **B** (second page): add. تمام [7] **F** add. يزاد على المركز وينقص من الخاصّة [8:] **L** (first page): تعديل الأوّل للعطارد, **L** (second page): تمامت تعديل عطارد [9] **B** (second page): add. تمام [10] **F** add. يعدّل ويزاد على المركز مع الأوج [11:] **L** (first page): تعديل ثاني للعطارد, **L** (second page): تمامت تعديل ثاني للعطارد

### Table 36: Variation of the nearest distance for Mercury

*Sources*: **F** fol. 79v, **H** fol. 70v, **C** fol. 77v, **$C_1$** fol. 43v, **$C_2$** fol. 45v, **Y** fol. 298v, **L** fol. 70v, **B** p. 136.

Title: اختلاف[1] البعد الأقرب لعطارد[2]

### Table 36: Interpolation minutes (nearest distance) for Mercury

*Sources*: **F** fol. 79v, **H** fol. 71r, **C** fol. 77v, **$C_1$** fol. 43v, **$C_2$** fol. 45v, **Y** fol. 298v, **L** fol. 70v, **B** p. 136.

Title: دقائق النسب[3]

### Table 36: Variation of the furthest distance for Mercury

*Sources*: **F** fol. 80r, **H** fol. 71v, **C** fol. 78r, **$C_1$** fol. 44r, **$C_2$** fol. 46r, **Y** fol. 299r, **L** fol. 71r, **B** p. 137.

Title: اختلاف[4] البعد الأبعد لعطارد[5]

### Table 36: Interpolation minutes (furthest distance) for Mercury

*Sources*: **F** fol. 80r, **H** fol. 72r, **C** fol. 78r, **$C_1$** fol. 44r, **$C_2$** fol. 46r, **Y** fol. 299r, **L** fol. 71r, **B** p. 137.

Title: دقائق النسب[6]

[1] **H** om. [2] **YLB** om. [3] **L** السسه [4] **H** om. [5] **YLB** om. [6] **L** السبة

## Table 37: Lunar latitude with values to minutes

*Sources*: **F** fol. 80v, **H** fol. 72v, **C** fol. 78v, $\mathbf{C_1}$ fol. 44v, $\mathbf{C_2}$ fol. 46v (see Plate 11). The table in **FH** has three columns with quadruple entries, while the table in $\mathbf{C_1C_2}$ has six columns with double entries.

| | |
|---|---|
| Title: | عرض القمر |
| Argument: | حصّة العرض |
| Column header: | العرض |
| Subcolumn headers: | درج / دقائق |
| Labels 'north' / 'south': [a] | شمال / جنوب |

*Marginal notes*

$\mathbf{C_1C_2}$ indicate under each column whether (for each of the two signs covered by the column) the latitude is 'northern' (ش for شماليّ) or 'southern' (ح for جنوبيّ), whether it is 'ascending' (i.e., moving towards the north, ص for صاعد) or 'descending' (ه for هابط), and whether it is 'increasing' (in absolute value, i.e., moving away from the ecliptic, ر for زائد) or 'decreasing' (ن for ناقص).[b]

## Table 37a: Lunar latitude with values to seconds and tabular differences

*Sources*: **Y** fol. 299v, **L** fol. 71v, **B** p. 138.

| | |
|---|---|
| Title: | عرض القمر [1:] |
| Argument: | حصّة العرض |
| Column headers: | العرض / حصّة دقيقة |
| Subcolumn headers: | درج / دقائق / ثواني // دقائق / ثواني |
| Labels 'north' / 'south': [c] | شمال / جنوب |

[a] Only in **FH**. [b] It cannot be verified whether **C** had this same set of symbols, since the entire bottom margin of the page was cut off. [c] Only in **L** (in the right margin, somewhat too high).

[1:] **L** عرض قمر ومقامت الأوّل (analogous to the following tables for the latitudes and first stations of the five planets)

## Tables 38–40: Latitude and first station of the superior planets

*Sources*: **F** fols 81r–81v, **H** fols 73r–74r, **Y** fols 300r–301r, **L** fols 72r–73r, **B** pp. 139–141.

| | |
|---|---|
| Title: | عرض زحل[1] ومقامه[2] الأوّل |
| Argument: | سطر[3] العدد |
| Column headers: | شمال[4] / جنوب[5] / دقائق[6] الحصص[7] / المقام الأوّل |
| Subcolumn headers: | بروج / درج / دقائق |
| Labels for the upper and lower half: [a] | النصف الأعلى / النصف الأسفل |

*Explanatory texts*

**A. On the correction for the displacements**

*Sources*: **FHB**-all, **LY**-Sat: right margin (vertically).

يُزاد على مركزه[8] المعدّل [9-]سبع درجات[-9]، ثمّ يُستعمل في العرض والمقام.

Seven degrees is added to the true centrum, then it is used for the latitude and the station.

**B. On the calculation of the second station**

*Sources*: **FYLB**-Sat: left margin (vertically).

يُنقص المقام الأوّل من الدور[10]، فيكون ما يبقى المقام الثاني.

The first station is subtracted from the full circle (i.e., 360°), and what remains is the second station.

[a] **HYL**: om., **FB** (except **B**-Sat) also separate the halves by a black horizontal line.

[1] Jup: المشتري, Mars: المرّيخ (**L** omits the definite article *al-*) [2] **L**-all: ومقامت [3] **HY**-all: سطرا [4] **F**-Sat: الشمال [5] **F**-Sat: الجنوب [6] **F**-Mars: درجات [7] **Y**-Jup: العرض, **Y**-Mars: حصص العرض, **L**-Sat+Mars: حضيض [8] **F**-Jup, **Y**-Sat المركز [9–9] Jup: اثنا عشرة درجة ('twelve degrees', **H** اثنى عشر درجة), Mars: سبع وأربعين درجة ('forty-seven degrees', **F** تسع and **H** سبعة for سبع) [10] **B** التدوير

## Tables 41–42: Latitude and first station of the inferior planets

*Sources*: **F** fols 82v–83r, **H** fols 74v–75r, **Y** fols 301v–302r, **L** fols 73v–74r, **B** pp. 142–143.

Title: عرض الزهرة[1] ومقامها[2] الأوّل

Argument: سطر[3] العدد

Column headers: الميل[4] / الانحراف[5] / دقائق[6] الحصص[7] / المقام الأوّل

Subcolumn headers: بروج / درج / دقائق

Labels for the upper and lower half: [a] النصف الأعلى / النصف الأسفل

*Explanatory texts*

**A. On the correction for the displacements**

*Sources*: **FHL**, **B**-Ven: right margin (vertically).

يُزاد على المركز[8] المعدّل [9-]ثمانية وأربعين[-9] درجة، ثمّ يُستعمل [10-]في العرض[-10] والمقام.

Forty-eight degrees are added to the true centrum, then it is used for (**L**-Mer: 'with') the latitude and the station.

**B. On the calculation of the second station**

*Sources*: **L**-Mer: left margin (vertically).

يُنقص المقام الأوّل من الدور، فيكون ما يبقى المقام الثاني.

The first station is subtracted from the full circle, and what remains is the second station.

## Table 38–42a: Latitudes of the planets in the Cairo manuscripts

*Sources*: **C** fol. 79r, $\mathbf{C_1}$ fol. 45r, $\mathbf{C_2}$ -.

Title: عروض الكواكب

Argument: سطر العدد

Column headers: زحل / المشتري / المرّيخ / الزهرة / عطارد // حصص[11] العرض

Subcolumn headers: شمال / جنوب // الميل / الانحراف // درج / دقائق

[a] **HYL** om. **FB** also separate the halves by a black horizontal line.

[1] Mer: عطارد, **L**-Ven: رهره [2] Mer, **YB**-Ven: مقامه; **L**-all: مقامت [3] **HY**-all: سطرا [4] **L**-Ven: شمال, **L**-Mer: om. [5] **L**-Ven: جنوب, **L**-Mer: om. [6] **Y**-Mer: om. [7] **Y**-all: حصص العرض, **L**-Ven: حصص, **L**-Mer: حضيض العرض [8] **H**-all, **L**-Ven: مركزه [9–9] Mer: ستّ وعشرين ('twenty-six', **H** ستّة for ستّ) [10–10] **L**-Mer: بالعرض [11] $\mathbf{C_1}$ حصّة

Displacements (above the names of the planets): [a]
7 / 12 / 47 / 48 / 26 — ر / ب / مز / مح / كو

Labels for the upper and lower half: [b] — النصف الأعلى / النصف الأسفل

*Explanatory texts*

**A. On the values of the displacements at the top of the columns**
*Sources*: **C$C_1$** right margin (vertically).

الأعداد التي على رؤوس[1] الكواكب هي التي تُزاد على المركز المعدّل الحاصل[2] عند التقويم

The numbers at the headers of ⟨the columns for⟩ the planets are the ones that are added to the corrected centrum resulting when ⟨you calculate⟩ the true position.

**Table 38–42b: First stations of the planets in the Cairo manuscripts**
*Sources*: **C** fol. 79v, **$C_1$** fol. 45v, **$C_2$** -.

Title: مقامات الكواكب [3:]

Argument: سطر[4] العدد

Column headers: زحل / المشتري / المرّيخ / الزهرة / عطارد

Subcolumn headers: بروج / درج / دقائق

Displacements (above the names of the planets):
7 / 12 / 47 / 48 / 26 — ر / ب / مز / مح / كو

*Explanatory texts*

**A. On the values of the displacements at the top of the columns**
*Sources*: **C$C_1$** right margin (vertically).

الأعداد التي على رؤوس[5] الكواكب هي[6] التي تُزاد على المركز المعدّل الحاصل عند التقويم

The numbers at the headers of ⟨the columns for⟩ the planets are the ones that are added to the corrected centrum resulting when ⟨you calculate⟩ the true position.

---

[a] **C** has nonsensical numbers in a different hand. [b] In **C** both the labels and a black horizontal line separating the two halves are in a different hand.

[1] **C** روس [2] **C** om. [3:] **C** ظلّ الميقام (؟) الأوّل للكواكب (beginning apparently miscopied from Table 44, ظلّ crossed out in black, correct title repeated in the top right corner of the table as المقام الأوّل للكواكب, marked with بيان ‘clarification’) [4] **$C_1$** سطور [5] **C** روس [6] **C** om.

## Table 43: First and second declination

*Sources*: **F** fols 83v–84r, **H** fols 75v–76v, **Y** fols 302v–303v, $\mathbf{L_1}$ = **L** fols 74v–75v, **B** pp. 144–146.

| | |
|---|---|
| Title: | [1] الميل[2] الأوّل والثاني[3] |
| Argument: | [4] العدد |
| Column headers: | الميل الأوّل[5] / التفاضل // الميل الثاني[6] / التفاضل |
| Subcolumn headers: | درج / دقائق / ثواني |
| Labels 'north' / 'south': [a] | شمال / جنوب |

## Table 43a: First declination

*Sources*: **C** fol. 80r, $\mathbf{C_1}$ fol. 11r, $\mathbf{C_2}$ -, $\mathbf{L_2}$ = **L** fol. 32v.

| | |
|---|---|
| Title: | الميل الأوّل[7] |
| Argument: | العدد[8] |
| Column headers: | الميل / التفاضل[9] |
| Subcolumn headers: | درج / دقائق / ثواني |
| Labels 'north' / 'south': [b] | شمال / جنوب |

## Table 43b: Second declination

*Sources*: **C** fol. 81r, $\mathbf{C_1}$ fol. 12r, $\mathbf{C_2}$ -, $\mathbf{L_2}$ = **L** fol. 33r (without tabular values).

| | |
|---|---|
| Title: | الميل الثاني[10] |
| Argument: | العدد[11] |
| Column headers: | الميل / التفاضل[12] |
| Subcolumn headers: | درج / دقائق / ثواني |
| Labels 'north' / 'south': [c] | شمال / جنوب |

[a] Only in **B** (in the right margin); in **H** added by a different hand. [b] Only in $\mathbf{L_2}$ (in the right margin). [c] Only in $\mathbf{L_2}$ (in the right margin).

[1] **Y** (second and third pages): add. تمام [2] $\mathbf{L_1}$ ميل [3] **H** has a separate title الميل الثاني on each page. [4] **HY** add. سطرا, $\mathbf{L_1}$ add. سطر [5] **H** om. [6] **H** om. [7] **C** add. للشمس [8] $\mathbf{L_2}$ أجزاء فلك البروج [9] $\mathbf{L_2}$ حصّة درجة [10] **C** add. للكواكب [11] **C** om., $\mathbf{L_2}$ أجزاء فلك البروج [12] $\mathbf{L_2}$ حصّة درجة

## Table 44: Tangent of first declination

*Sources*: **F** fol. 84r, **H** fol. 77r, **C** fol. 80v, **$C_1$** fol. 11v, **$C_2$** -, **Y** fol. 304r, **L** fol. 76r, **B** p. 147.

Title: ظلّ الميل الأوّل[1] [2]

Argument: درجات فلك البروج [3:]

Column header: [a] ظلّ الميل

Subcolumn headers: أجزاء[4] / دقائق / ثواني

*Explanatory texts*

### A. On calculating the equation of daylight

*Sources*: **F** between the tables for the first and second declination and the tangent of declination (vertically), **LB** in the right margin (vertically).

يُضرب ظلّ[5] ميل الدرجة في ظلّ عرض البلد[6] منحطًّا فما بلغ فهو جيب تعديل النهار.

The tangent of the declination of the ⟨given⟩ degree is multiplied by the tangent of the latitude of the city divided by 60 (lit. 'lowered'), and what results is the sine of the equation of daylight.

## Tables 45 and 45a: Right ascension

*Sources*: **F** fol. 84v, **H** fols 77v–78r, **C** fol. 81v, **$C_1$** fol. 12v, **$C_2$** -. **Y** fols 304v–305r, **L** fols 76v–77r, **B** pp. 148–149. The table in **YLB** has less accurate values than the one in **$FCC_1$**.

Title: [7] مطالع البروج[8] بخطّ الاستواء [9:]

Argument (vertical): درج[10] السواء

Argument (horizontal): [b] الحمل / الثور / الجوزاء / ... / الحوت

Subcolumn headers: درج / دقائق

---

[a] **B** (all three columns): om. [b] **$CC_1$** write the signs as abjad numbers from 0 to 11, **Y** adds برج 'sign' before every zodiacal sign.

[1] **$C_1$** om. [2] **C** add. وهو ضل (كذا) ميل الشمس 'and it is the tangent of the solar declination' [3:] **H** أجزاء السواء [4] **$HCC_1YL$** درج [5] **F** vertically in the top margin, labelled صح [6] **L** البار [7] **Y** (second page): add. تمام [8] **Y** (second page): البروج البروج, **L** (both pages): بروج [9:] **H** (second page): تمام المطالع [10] **$CC_1LB$** درجات

## Tables 46 and 46a: Oblique ascension

*Sources*: **F** fol. 85r, **H** fols 78v–79r, **Y** fols 306r–306v, **L** fols 78r–78v, **B** pp. 151–152 (see Plate 12). The table in **FH** is for latitude 36;0°, the table in **YLB** for latitude 35;30°. For the combined table for the oblique ascension and equation of daylight for latitude 35;30° in **C**, see under Table 46b.

Title: [1] مطالع البروج[2] لعرض لو[3] [4:]

Argument (vertical): درج[5] السواء

Argument (horizontal): [a] الحمل / الثور / الجوزاء / ... / الحوت

Subcolumn headers: درج / دقائق

## Table 46b: Oblique ascension and equation of daylight

*Sources*: **C** fols 82r–82v, $\mathbf{C_1}$ -, $\mathbf{C_2}$ -.

Title: [6] مطالع البروج وتعديل النهار لعرض له ل

Argument: درجات السواء

Column header: تعديل نهار (كذا)

Subcolumn headers: درج / دقائق

## Table 47: Maximum equation of daylight for latitudes 16° to 45°

*Source*: **F** fol. 85v (without title, column headers and tabular values). This table is not found in any of the other manuscripts but is included in the table of contents of **L**.

Title: [b]

تعديل نهار أوّل السرطان من عرض تو إلى عرض مه[7]

Equation of daylight at the beginning of Cancer from latitude 16° to latitude 45°

Subcolumn headers: درج / دقائق / ثواني

[a] **Y** add. برج 'sign' before every zodiacal sign. [b] According to the tables of contents in **F** fol. 37v and **L** fol. 21r.

[1] **Y** (second page): add. تمام [2] **L** (both pages): بروج [3] **YLB** له ل [4:] **H** (second page): تمام المطالع [5] **LB** درجات [6] **C** (second page): add. تمام [7] **F** (table of contents): مو

## Tables 48 and 48a: Equation of daylight

*Sources*: **F** fol. 85v, **H** fol. 78r, **Y** fol. 305v, **L** fol. 77v, **B** p. 150. The table in **FH** is for latitude 36;0°, the table in **YLB** for latitude 35;30°. For the combined table for the oblique ascension and equation of daylight for latitude 35;30° in **C**, see under Table 46b.

Title: تعديل النهار لعرض لو[1]

Argument: فلك[2] البروج

Column header: التعديل

Subcolumn headers: درج / دقائق

*Explanatory texts*

**A. On finding the equation of equal hours**

*Sources*: **LB** in the right margin (vertically).

يُضرب في ثمان دقائق، فيكون تعديل الساعات المستوية.

⟨The equation of daylight⟩ is multiplied by 8 minutes, and the result is the equation of the equal hours.

**B. On finding the equation of seasonal hours**

*Sources*: **LB** in the left margin (vertically).

يُضرب في عشرة دقائق، فيكون تعديل أجزاء الساعات الزمانيّة.

⟨The equation of daylight⟩ is multiplied by 10 minutes, and the result is the equation of the degrees (lit. 'parts') of seasonal hours.

## Table 49: Hourly motion and diameters of the Sun and Moon

*Sources*: **F** fol. 86r, **H** fol. 79r, **C** fol. 83r, **C₁** fol. 46r, **C₂** -, **Y** fol. 307r, **L** fol. 79r, **B** p. 153.

Title: [3] مسير ساعات[4] النيّرين[5] وقطريهما

Argument: خاصّة[6] الشمس أو[7] [8-]الخاصّة المعدّلة للقمر[-8] [9]

Column headers: ساعات[10] الشمس / ساعات[11] القمر //
قطر الشمس / قطر القمر / قطر الظلّ

Subcolumn headers: -[12] // دقائق[13] / ثواني[14]

---

[1] **YLB** له ل  [2] **H** om.  [3] **F** add. جدول  [4] **F** للساعات, **HCC₁YB** ساعة  [5] **L** سرس  [6] **CC₁** حصة  [7] **C** om., **Y** و, **L** اول, **B** ادل (?)  [8–8] **H** تدوير القمر, **Y** خاصّة القمر المعدّلة  [9] **C** add. وهي التدوير  [10] **HCC₁YLB** ساعة  [11] **HCC₁YLB** ساعة  [12] **Y** add. بروج and درج for the arguments  [13] **C** (all five occurrences): درج  [14] **C** (all five occurrences): دقائق

## Table 49[bis]: Conjunctions and oppositions

*Sources*: **Y** fols 311v–313v (see Plate 13); **B** pp. 211–214, 206–205 and 217; **D** = *Dustūr al-munajjimīn*, Paris, BnF, arabe 5968, fols 112r–115r; **P** = *Baghdādī Zīj*, Paris, BnF, arabe 2486, fols 124v–127r.

Title: [a] [1] الاجتماع والاستقبال [2]

Horizontal argument: [-3]بهت القمر[3-] يا ن / يب ج / يب ي / ...

Vertical argument: البعد بين النيّرين

Column headers: جزء البعد[4] / ساعات البعد

Subcolumn headers: درج / دقائق / ثواني // ساعات[5] / دقائق / ثواني

### *Explanatory text*

**A. On finding the time of true syzygy**

*Sources*: **B** p. 211, left margin (vertically); **D** fol. 112r, left margin (vertically), with the last two sentences of the instructions found in **YL** appended (cf. pp. 464–65).

قُسم [-6]وسط يوم الشمس[6-] على بهت يوم القمر منحطًّا فحصل تزايد جزء[7] البعد بالتقريب. ونقص من بهت[8] ساعة القمر [-9]وسط ساعة الشمس[9-] وقسم البعد[10] على الباقي فحصل تزايد[11] ساعات البعد بالتقريب.

The daily solar mean motion is divided by the daily lunar velocity divided by 60 (lit. 'lowered'), and there results the increase of the part of the distance approximately. From the hourly velocity of the moon, the hourly solar mean motion is subtracted and the distance is divided by the remainder, and there results the increase of the hours of the distance approximately.

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[a] **B** omits the title on pp. 214 (lunar velocities 13;20–13;40) and 206 (13;50–14;10).

[1] **P** (all pages): add. جدول; **B** (last page): add. تمام, **D** (last page): add. باقي [2] **D** (first page): add. لكوشيار [3–3] **P** البهت [4] **Y** (fol. 312v, lunar velocities 13;10–13;40): قمر [5] **YP** درج [6–6] **D** مسير الشمس الأوسط, corrected to بعد الشمس in a later hand. [7] **D** om. [8] **D** مسير [9–9] **D** مسير ساعة وسط الشمس [10] **D** om. [11] **D** om.

## Table 50: Lunar distance from the Earth

*Sources*: **F** fol. 86v, **H** fols 79v–80r, **C** fol. 83v, **$C_1$** fol. 46v, **$C_2$** -, **Y** fol. 307v, **L** fols 79v–80r, **B** pp. 154–155. **H** places the last three columns of the table on the second page with repeated columns for the arguments; **LB** do the same with the last two columns.

Title: [1] بعد القمر[2] من الأرض في أوقات[3] الاجتماعات والاستقبالات ورؤية[4] الأهلّة [5:]

Argument (vertical): التدوير

Argument (horizontal): المضاعف

Subcolumn headers: -[6] // درج / دقائق

## Table 51: Solar parallax

*Sources*: **F** fol. 87r, **H** fol. 80r, **C** fol. 84r, **$C_1$** fol. 47r, **$C_2$** -, **Y** fol. 307r, **L** fol. 80r, **B** p. 155.

Title: اختلاف منظر الشمس[7]

Argument: تمام الارتفاع

Column header: اختلاف المنظر

Subcolumn headers: دقائق / ثواني

## Table 52: Magnitude of eclipses

*Sources*: **F** fol. 87r, **H** fol. 80r, **C** fol. 84r, **$C_1$** fol. 47r, **$C_2$** -, **Y** fol. 307r, **L** fol. 80r, **B** pp. 155.

Title: تعديل أصابع الكسوفين[8]

Subtitles: [a] الشمس[9] // القمر[10]

Arguments: أصابع الكسوف // أصابع الخسوف[11]

Column headers: الأصابع[12] المعدّلة[13] (sun) // الأصابع[14] المعدّلة (moon)

Subcolumn headers: أصابع / دقائق[15]

[a] **H** vertically to the left of the table in Hebrew script (possibly in a different hand) and vertically to the right of the table in Arabic script, extended over the entire heights of the two subtables; **$CC_1$LB** vertically to the right of the table.

[1] **YLB** add. جدول [2] **L** قمر [3] **$CC_1$** وقت [4] **LB** رؤية (without و) as the first word on the second page. [5:] **H** (second page): تمام بعد القمر من الأرض [6] **Y** adds بروج and درج for the arguments. [7] **$CC_1$** write الشمس vertically to the left of the (sub)column headers. [8] **C** الكسوف (vertically to the left of the (sub)column headers), **$C_1$** om., **LB** الخسوفين [9] **FY** om. [10] **Y** om. [11] **C** الكسوف [12] **YLB** أصابع [13] **Y** معدّلة [14] **L** أصابع [15] **B** (moon): أصابع

## Table 53: Prorogations (*tasyīr*s)

*Sources*: **F** fol. 87r, **H** fol. 81r, **C** fol. 84r, **$C_1$** fol. 47r, **$C_2$** -, **Y** fol. 308v, **L** fol. 81r (table left entirely empty), **B** p. 157.

Title: التسييرات

Column headers: في السنة الشمسيّة[1] برج واحد // في السنة[2] الشمسيّة[3] ثلاثة عشر برجًا

Arguments: شهور / أيّام // شهور / أيّام

Subcolumn headers: درج[4] / دقائق[5] // بروج / درج / دقائق

## Table 54: Geographical table

*Sources*: **F** fol. 87v, **H** fol. 80v (see Plate 14), **C** fol. 84v, **$C_1$** fol. 47v, **$C_2$** -, **Y** fol. 308r, **L** fol. 132r (after the colophon), **L′** = **L** fol. 80v (only title and column headings), **B** p. 156.

Title: [6] طول[7] البلدان[8] من الجزائر[9] الخالدات وعرضها[10] من[11] خطّ الاستواء [12]

Argument: أسماء[13] البلدان

Column headers: الطول / العرض

Subcolumn headers: [a] درج / دقائق

Labels for the climates: [b] الإقليم الأوّل / الإقليم الثاني / الإقليم الثالث [14] / الإقليم الرابع / الإقليم الخامس / الإقليم[15] السادس / الإقليم[16] السابع[17]

For the Arabic names of the localities, together with transliterations and the names under which they appear in Kennedy and Kennedy, *Geographical Coordinates*, see the edition of the table on pp. 240–43.

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[a] **$HCC_1$** om. [b] **$FHCC_1B$** vertically alongside the columns of the table, **Y** in red between the rows of the table, **L** om. **$C_1$** includes الإقليم only for the first climate, **Y** writes إقليم without article for the first, second and third climates. **L′** writes إقليم الثالث in the header cell of the second column. **HB** indicate the exact beginning of the climates by a dot. Incidental small misplacements of the labels have not been indicated in the apparatus to the table.

[1] **$CC_1$** (above the rest of the column header): شمسيّة [2] **H** السنين [3] **$CC_1$** (above the rest of the column header): شمسيّة [4] **$C_1$** دقائق [5] **$C_1$** ثواني [6] **YLL′** add. جدول [7] **Y** أطوال [8] **L′** بلدان [9] **HCYLL′B** جزائر [10] **$HCC_1L'$** عروضها [11] **L** عن [12] **L** add. في قديم الدهر 'in ancient times' [13] **$HCC_1YL'B$** om., **L′** repeats البلدان in a separate cell. [14] **F** add. إلى العموريّة 'up to 'Ammūriyya' [15] **$FC_1$** om. [16] **$FCC_1$** om. [17] **C** cut off at the bottom of the page.

**Table 55: Star table (titles and headers)**

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fol. 310v–311r, **L** fols 81v–82r (without star names and coordinates), **B** pp. 158–159. **CC₁** have a different table for the same epoch, which is here edited as Table 55a.

Title: [a] الكواكب الثابتة ومواضعها لأوّل سنة شا ليزدجرد بن شهريار

Column headers: الكواكب / الطول / العرض / العظم[1] / الجهة / المزاجات

Subcolumn headers: [b] درج / دقائق

Subtitles: [c] الصور الشماليّة / صور المنطقة / الصور الجنوبيّة

For the names of constellations and individual stars and the listings of numbers of stars (northern, ecliptical, southern, and total), see the edition of the table below.

*Explanatory text*

**A. On the indications given for certain stars**

*Source*: **Y** fol. 310v (vertically in an otherwise empty column to the right of the constellation names), **B** p. 158 (vertically in the column with constellation names).

المعلّم عليه بهمزة الحمرة هو الذي يجوز أن يُعرف[2] في طوالع المواليد.

⟨A star⟩ marked with a red hamza is one that may be determined in the ⟨calculation of⟩ ascendants of nativities.

**Table 55a: Star table in the Cairo manuscripts (titles and headers)**

*Sources*: **C** fol. 85r, **C₁** fol. 48r (see Plate 15). **FHYB** have a different table for the same epoch, which is here edited as Table 55.

Title: [d] الكواكب الثابتة لأوّل سنة شا ليزدجرد

Column headers: الكواكب / الطول / العرض / الجهة / الأقدار / المزاج

See below for the edition of the star names and the listing of the numbers of stars.

[a] For variants in the title, see the edition of the table below. **H** divides the title into two parts: the main title الكواكب الثابتة is written in red, and the second part, أوّل سنة شا ليزدجرد, is written smaller and in black above it. [b] **FH** om. [c] See the edition of the table below for variants in the subtitles. [d] For variants in the title, see the edition of the table below.

[1] **H** الأقدار [2] **B** تعدل

*Comments*

For ease of reference I include in the below edition of the names of stars and constellations the numbers for the stars that were introduced in Part II. An asterisk indicates that a star is not included in the other of the two different star tables found in the manuscripts of the *Jāmiʿ Zīj*. For the corresponding Baily numbers and identifications, see the editions of the coordinates in the two tables on pp. 246–47 and 250–51.

Misplacements of the names of the constellations in Table 55 (never by more than one line) are not indicated in the apparatus. **H** omits the names of the constellations and writes the subtitles in the empty column used for the names of the constellations in the other manuscripts. Only **Y** repeats the column and subcolumn headers for the ecliptical and southern constellations (in addition to writing them at the top of the columns).

**F** writes out all directions in full; **H** writes out גנוב 'southern' in full for southern stars in the ecliptic belt. In all other manuscripts, and in all other cases in **H**, the directions are indicated by the abbreviations ش or س (ש in **H**) for شمال 'northern' and ح (ג in **H**) for جنوب 'southern'. Rather than repetitions of the same abbreviation, **Y** writes the directions vertically in full. **B** does this in addition and next to the abbreviations.

In the column of temperaments, the names of the planets are abbreviated as follows in the manuscripts: س for شمس 'Sun', ر for قمر 'Moon', ل for زحل 'Saturn', ے for مشتري 'Jupiter', خ (in **F** incidentally نخ) for مرّيخ 'Mars', ه for زهرة 'Venus', and د for عطارد 'Mercury'. The Judaeo-Arabic manuscript **H** uses the corresponding Hebrew letters with the exception of Mars, for which the Arabic خ is used. Fatal or malefic stars are indicated by the word قاطع 'cutter' or abbreviated by the letter ق 'q', benefic ones by the abbreviation ء (looking more like د in **Y**, possibly for *zāʾid* 'increasing'; see furter the commentary in Section IV.14, pp. 515–16).

Lack of space occasionally made it impossible for the scribes to write the numbers of stars of each magnitude horizontally immediately under the last star of each group or at the end of the table. In Table 55, **F** writes the summation for the southern stars vertically in the five empty lines for Orion (الجبّار) in the column for the names of the constellations, headed by جملة الصور الجنوبيّة 'total of the southern constellations'. **Y** writes the grand total in the left margin of the first page of the table (fol. 310v), **B** vertically in the left margin of the second page of the table (p. 159), while **H** leaves out all summations entirely. In the table in the Cairo manuscripts, which does not have an explicit division in northern, ecliptic and southern stars, only $\mathbf{C_1}$ gives the totals for the whole table in the bottom margin of the second page.

**Table 55: Star table (constellations and star names, first page)**

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fol. 310v–311r, **L** fols 81v–82r (left empty), **B** pp. 158–159. Due to space limitations, the table is here broken differently over the two pages.

| | | |
|---|---|---|
| | [1] الكواكب الثابتة ومواضعها[2] لأوّل[3] سنة[4] شا̅ [5-]ليزدجرد بن شهريار[-5] | |
| no. | الكواكب | [6] الصور[7] الشماليّة |
| A1* | ذنب الدبّ الأصغر وهو الجدي ء | من الدبّ الأصغر |
| A2* | الأنور من الفرقدين ء | |
| A3* | الذي[8] على ظهر الدبّ[9] الأكبر ء | من الدبّ الأكبر |
| A4* | الذي[10] على طرف ذنبه ء | |
| A5 | السماك[11] الرامح ء[12] | من صورة العوّاء |
| A6 | المنير من الفكّة | من صورة الفكّة[13] |
| A7 | النسر الواقع | من [14] اللورا[15] |
| A8 | [17-]الذي على[-17] منقار [18] الدجاجة ء | من صورة[16] الدجاجة |
| A9 | [19-]الذي على[-19] ذنب الدجاجة ء[20] | |
| A10 | صدر الكرسي وهو الكفّ[21] الخضيب [22:] [23] | من ذات الكرسي |
| A11* | [24-]الذي في[-24] صدر المرأة ء | |
| A12* | الذي[25] على ركبة المرأة ء | |
| A13 | اليد[26] اليمنى من ممسك الغول ق | من حامل رأس الغول |
| A14 | المضيء[27] الذي في شقّه الأيمن ق | |
| A15 | النيّر الذي[28] في رأس الغول ق[29] | |
| A16 | [30-]كتف ذي العنان وهو[-30] العيّوق[31] [32] | من صورة ذي العنان |
| A17 | كتفه اليمنى [33:] [34] | |
| A18 | [36-]الذي على[-36] رأس الحواء ء[37] | من صورة الحواء[35] |
| A19 | النسر الطائر [40] | من النسر[38] الطائر [39:] |
| A20 | منكب الفرس[42] ق | من صورة[41] الفرس الثاني |

[43] ك̅ كوكبًا، في الأوّل ح̅، في[44] الثاني نا̅، في[45] الثالث ه̅[46] السحابيّ[47] ا̅[48].

| | ⟨الكواكب⟩ | [49] صور[50] المنطقة |
|---|---|---|
| A21 | الأنور من الشرطين [51] | من صورة الحمل |
| A22 | عين الثور وهو الدبران ق[52] | من صورة الثور |
| A23 | رأس التوأم المقدّم | من صورة التوأمين |
| A24 | رأس التوأم المؤخّر [53] | |
| B1 | صدر السرطان وهو المعلف [54:] ق | من صورة السرطان |
| B2 | [55-]الذي على[-55] منكب الأسد[56] ق | من صورة الأسد |

See p. 330 for the apparatus of this table.

## Table 55: Star table (constellations and star names, second page)

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fols 310v–311r, **L** fols 81v–82r (left empty), **B** pp. 158–159.

[1] الكواكب الثابتة ومواضعها[2] لأوّل[3] سنة شا [4-]ليزدجرد بن شهريار[-4]

| no. | الكواكب | صور المنطقة [5:] |
|---|---|---|
| B3 | الملكيّ[6] [7-]الذي هو[-7] قلب الأسد | ⟨من صورة الأسد⟩ |
| B4 | [8-]الذي على طرف ذنبه وهو[-8] الصرفة | |
| B5 | السماك الأعزل | من صورة السنبلة |
| B6 | كفّة الميزان الجنوبيّة ء | من صورة الميزان |
| B7 | [9-]كفّة الميزان[-9] الشماليّة ء | |
| B8 | الذي[10] بين عيني العقرب ء [11] | من صورة العقرب |
| B9 | قلب العقرب ق[12] | |
| B10 | تالي[13] الشولة[14] ق | |
| B11 | [15-]الذي على[-15] عين الرامي ق | من صورة القوس |
| B12 | فم الحوت الجنوبيّ[16] [17] | من صورة الدلو |

تو كوكبًا، في الأوّل ه، في الثاني و، في الثالث ب، السحابيّ ح. [18:]

| | ⟨الكواكب⟩ | [19] الصور[20] الجنوبيّة |
|---|---|---|
| B13 | رأس الجبّار ق | |
| B14 | كتفه اليمنى ق[21] | |
| B15 | كتفه اليسرى | |
| B16* | المقدّم من المنطقة ء | |
| B17 | الأوسط منها | |
| B18* | الأخير منها [22:] ء | |
| B19 | رجله اليسرى | [23] |
| B20 | [24-]المضيء في[-24] آخر النهر | من صورة النهر |
| B21 | الشعرى اليمانيّة | من الكلب الأكبر |
| B22 | الشعرى الشآميّة | من الكلب الأصغر |
| B23 | [25-]الذي على[-25] رجل[26] قنطورس | من صورة قنطورس |
| B24 | طرف سكّان السفينة وهو سهيل [28:] | من صورة[27] السفينة |

[29] نب[30] كوكبًا، [31] الأوّل ز، [32] الثاني د، السحابيّ[33] ا[34]

فجميع هذه[35] الكواكب مح[36] كوكبًا[37]، في العظم الأوّل نه،
وفي[38] الثاني كا[39]، وفي[40] الثالث ز، السحابيّ ه.

See p. 330 for the apparatus of this table.

**Table 55a: Star table in the Cairo manuscripts (star names)**
*Sources*: **C** fol. 85r, **$C_1$** fol. 48r.

| الكواكب الثابتة لأوّل سنة شا ليزدجرد[1] | | | |
|---|---|---|---|
| **no.** | **الكواكب** | **no.** | **الكواكب** |
| C1 | السماك[2] الرامح | C31 | قلب الأسد |
| C2 | النيّر من الفكّة | C32 | ذنب الأسد الصرفة[3] |
| C3 | النسر الواقع | C33 | السماك[4] الأعزل |
| C4 | منقار الدجاجة | C34 | الكفّة الشماليّة من الميزان |
| C5* | صدر الدجاجة | C35 | الكفّة الجنوبيّة |
| C6 | ذنبها وهو الردف | C36 | بين عيني العقرب[5] |
| C7 | سنام الناقة كفّ الخضيب[6] | C37 | قلب العقرب[7] **ق** |
| C8 | معصم[8] حامل رأس الغول **ق** | C38 | تالي[9] الشولة[10] **ق** |
| C9 | جنب الحامل **ق** | C39* | زجّ السهم |
| C10 | النيّر من رأس الغول **ق** | C40* | الطرف الجنوبيّ من القوس |
| C11 | منكب العنان[11] العيّوق | C41* | الطرف الشماليّ |
| C12 | منكبه الأيمن | C42* | رجل الأسد [12:] |
| C13* | رجل العنان[13] الأيمن | C43* | المتوسّط بين كتفيه[14] |
| C14 | رأس الحواء | C44* | الشماليّ من أصل الذنب |
| C15* | ساق الحواء | C45 | عين الرامي **ق** |
| C16 | النسر الطائر | C46* | ذنب الجدي |
| C17* | ذنب الدلفين | C47 | آخر مصبّ الماء في الحوت |
| C18* | الشماليّ من الفرغ الثاني | C48* | كتفه[15] اليمنى |
| C19 | الشماليّ من الفرغ الأوّل [16] | C49* | ساقه اليمنى |
| C20 | أنور الشرطين | C50 | رأس الجبّار **ق**[17] |
| C21 | عين الثور الدبران | C51 | منكبه اليمنى[18] **ق**[19] |
| C22* | قرن الثور الشرقيّ | C52 | منكبه اليسرى[20] |
| C23* | قرن الثور الغربيّ | C53 | الوسط من المنطقة |
| C24* | صدر الثور | C54 | رجله اليسرى |
| C25 | رأس التوأم المقدّم | C55 | آخر النهر |
| C26 | رأس التوأم المؤخّر | C56 | الشعرى اليمانيّة |
| C27* | رجل التوأم المؤخّر | C57* | المقدّم من [21–]المضيئين نقده (؟)[–21] |
| C28* | ركبة التوأم المقدّم | C58 | الشعرى الشآميّة |
| C29 | صدر السرطان المعلف[22] **ق** | C59 | رجل قنطورس |
| C30 | منكب[23] الأسد **ق** | C60 | سهيل |

ستّون كوكبًا، في القدر الأوّل يه، وفي الثاني يح، وفي الثالث كب، والسحابيّة ه. [24:]

See p. 330 for the apparatus of this table.

## Table 55: Star table (apparatus)

*Sources*: **F** fol. 88r–88v, **H** fols 81v–82r, **Y** fol. 310v–311r, **B** pp. 158–159.

### First page (p. 327)

**F** om. صورة from all constellation names except العوّاء. **H** does not include the names of the constellations, the indications ء, and the numbers of stars for each section. التوأم 'the Twins' (Gemini) is written as التؤم in manuscripts **HY** and in the two star names in **F**. [1] **Y** add. جدول [2] **H** om. [3] **H** أوّل [4] **B** om. [5–5] **H** ليزدجرد, **YB** يزدجرديّة [6] **H** add. من [7] **B** الصورة [8] **H** om. [9] **B** دبّ [10] **H** om. [11] **Y** السمال [12] **Y** om. [13] **F** اللورا [14] **Y** add. ممسك [15] **F** الفورا, **YB** اللوزه [16] **FB** om. [17–17] **H** om. [18] **Y** add. عين [19–19] **H** om. [20] **Y** om. [21] **Y** كفّ [22:] **H** كفّ الخضيب, **B** كفّ الخضيب وهو صدر الكرسي [23] **F** add. ء [24–24] **H** على [25] **H** om. [26] **HYB** يد [27] **H** om. [28] **H** om. [29] **B** om. [30–30] **H** om. [31] **F** عيّوق [32] **B** add. ء [33:] **H** كتف اليمنى من ذي العنان [34] **F** add. ء [35] **F** العوا [36–36] **H** om., **Y** الذي [37] **F** om. [38] **B** نسر [39:] **F** om. [40] **B** add. ء [41] **FB** om. [42] **F** الموس [43] **B** add. فذلك [44] **B** وفي [45] **B** وفي [46] **F** om. [47] **F** سحابيّ [48] **B** om. [49] **H** add. من [50] **B** صورة [51] **Y** add. ء [52] **B** om. [53] **Y** add. ء [54:] **H** المعلف من السرطان, **B** المعلف من صدر السرطان [55–55] **H** om., **Y** الذي [56] **B** الفرس

### Second page (p. 328)

**F** omits صورة from all constellation names except الجبّار. **H** does not include the names of the constellations, the indications ء and the numbers of stars for each section and for the whole table. **Y** omits all indications قاطع and ء on this page. [1] **Y** add. تتمّة [2] **H** om. [3] **H** أوّل [4–4] **H** ليزدجرد, **YB** يزدجرديّة [5:] **FH** om., **B** الصورة الشماليّة [6] **H** om. [7–7] **H** om., **Y** الذي يسمّى, **B** يسمّى [8–8] **H** ذنب الأسد, **B** om. [9–9] **HBY** كفّته [10] **H** om., **B** at the end of the preceding line [11] **F** add. ق [12] **B** ء [13] **Y** باني [14] correct: الشوكة [15–15] **H** om. [16] **B** الجنوبيّة [17] **B** add. ء [18:] **FH** om. entirely, **B** om. all numbers [19] **H** add. من [20] **B** الصورة [21] **F** places قاطع in the following line. [22:] **H** المتأخّر [23] **B** add. الصورة الجنوبيّة [24–24] **H** om. [25–25] **H** om. [26] **F** om. [27] **B** طرف [28:] **H** سهيل, **B** السهيل وهو طرف سكّان السفينة [29] **F** add. جملة الصور الجنوبيّة [30] **B** om. [31] **Y** add. في [32] **Y** add. في [33] **Y** سحابيّ [34] **F** om. [35] **Y** om. [36] **F** سح [37] **B** om. [38] **B** في [39] **F** كط [40] **B** في.

## Table 55a: Star table in the Cairo manuscripts (apparatus)

*Sources*: **C** fol. 85r, $\mathbf{C_1}$ fol. 48r.

**C** om. all indications قاطع from the first column. $\mathbf{C_1}$ partially writes التوأم 'the Twins' (Gemini) as التؤم. [1] **C** يزدجرد [2] $\mathbf{CC_1}$ السمال [3] **C** om. [4] **C** السمال [5] $\mathbf{C_1}$ الفقرب [6] $\mathbf{C_1}$ الحد, and المفنيز (?) on the next line [7] $\mathbf{C_1}$ الفراب [8] $\mathbf{C_1}$ معصمه [9] **C** بابى [10] correct: الشوكة [11] **C** العنار, $\mathbf{C_1}$ العناز (?) [12:] **C** الرجل الالسر [13] **C** العنار, $\mathbf{C_1}$ العناز [14] $\mathbf{C_1}$ كفيه [15] $\mathbf{C_1}$ كيفيه [16] $\mathbf{CC_1}$ add. منكب الفرس (**C** in the next line, $\mathbf{C_1}$ in the line of star no. C21) [17] **C** om. [18] **C** السمس [19] **C** om. [20] $\mathbf{C_1}$ اليسار [21–21] $\mathbf{C_1}$ om. [22] $\mathbf{C_1}$ places this word in the next line. [23] **C** dam. [24:] **C** omits the numbers of stars entirely.

## 56: On finding the original equations

*Sources*: **F** fols 89r–90r, **$C_1$** fols 48v–49v, **B** p. 220 (incomplete). **L** lists this section in its table of contents of Book II, but does not actually include it.[a]

### القول في معرفة التعاديل الأصليّة من الجداول [1:]

### Statement on finding the original equations from the tables

**الشمس.** [2] أيّ[3] درجة أردنا تعديلها نقصنا منها درجتين وأخذنا[4] ما بإزاء الباقي من التعديل. فإن كان الباقي فيما بين شنح إلى قعح، نقصنا التعديل من درجتين. وإن كان الباقي فيما بين قعح إلى شنح، نقصنا من التعديل درجتين. فما بقي فهو تعديل الدرجة.

**The Sun.** (**$C_1$** add. 'Its mean motion is ⟨tabulated⟩ with a decrease of two degrees.') From whichever degree whose equation we want ⟨to know⟩, we subtract two degrees and we take the equation that is opposite the remainder. If the remainder is between 358 and 178, we subtract the equation from two degrees. And if the remainder is between 178 and 358, we subtract two degrees from the equation. What remains is the ⟨original⟩ equation of the degree.

**القمر.** [5] أيّ درجة أردنا تعديلها الأوّل [6-]أخذنا ما بإزائها من التعديل[-6]. فإن كانت الدرجة أقلّ من قف، نقصنا من التعديل أربع عشرة درجة. وإن كانت الدرجة أكثر من قف، نقصنا التعديل من أربع عشرة[7] درجة. فما بقي فهو تعديل الدرجة. وأيّ درجة أردنا تعديلها الثاني أخذنا ما بإزائها من التعديل. فإن كانت الدرجة أقلّ من قف، نقصنا التعديل من ثمان درج. وإن كانت الدرجة أكثر من قف، نقصنا من التعديل ثمان درج. فما بقي فهو تعديل الدرجة. والاختلاف[8] ودقائق النسب موافق للأصل.

**The Moon.** (**$C_1$** add. 'Its mean motion is ⟨tabulated⟩ with a decrease of eight degrees and its ⟨mean⟩ anomaly with a decrease of fourteen degrees.') For whichever degree whose first equation we want ⟨to know⟩, we take the equation that is opposite it. If the degree is less than 180, we subtract fourteen

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[a] The edition primarily follows **F**. General variants: **$C_1$** writes أيّة rather than أيّ before درجة. **$C_1$** writes out all *abjad* numbers except the limits of the ranges of the type شنح إلى قعح and the numbers in its two examples. **B** writes out the *abjad* numbers '180' قف as مائة وثمانين. Further trivial variations in the writing of the numbers have also been omitted from the apparatus.

[1:] **$C_1$** في التعاديل الأصليّة التي هي قبل تقريبها 'On the original equations that were before their approximation' (in the sense of 'before they were replaced by something that approximates them'), **B** معرفة التعاديل الأصليّة من هذه التعاديل المكتوبة [2] **$C_1$** add. وسطه بنقصان درجتين [3] **F**+ أيّة
[4] **B** وأخذناها [5] **$C_1$** add. وسطه بنقصان ثمان درج وخاصّته بنقصان أربعة عشر درجة [6–6] **F** om. [7] **F** om.
[8] **B** الاختلاف

degrees from the equation. And if the degree is more than 180, we subtract the equation from fourteen degrees. What remains is the ⟨first⟩ equation of the degree. And for whichever degree whose second equation we want, we take the equation that is opposite it. If the degree is less than 180, we subtract the equation from eight degrees. And if the degree is more than 180, we subtract eight from the equation. What remains is the ⟨second⟩ equation of the degree. The variation and the interpolation minutes agree with the original.

**زحل.** [1] أيّ درجة أردنا تعديلها الأوّل نقصنا منها أربع عشرة درجة وأخذنا تعديل الباقي. فإن كان الباقي فيما بين شمو إلى قسو، نقصنا التعديل من سبع درج. وإن كان الباقي فيما بين قسو إلى شمو، نقصنا من التعديل سبع درج. [2] وأيّ درجة أردنا تعديلها الثاني أخذنا ما بإزائها من التعديل. فإن كانت الدرجة أقلّ من قف، نقصنا من التعديل سبع درج. وإن كانت الدرجة[3] أكثر من قف، نقصنا التعديل من سبع درج. وأيّ درجة أردنا اختلافها نقصنا منها سبع درجات وأخذنا ما بإزاء الباقي من الاختلاف. فإن كان من البعد الأقرب، فهو الموسوم في الأصل بالزائد[4]. وإن كان من البعد الأبعد، فهو الموسوم في الأصل بالناقص. [5]

**Saturn.** (**$C_1$** add. 'Its mean motion is ⟨tabulated⟩ with a decrease of fourteen degrees and its ⟨mean⟩ anomaly with an increase of seven degrees.') From whichever degree whose first equation we want ⟨to know⟩, we subtract fourteen degrees and we take ⟨from the table⟩ the equation of the remainder. If the remainder is between 346 and 166, we subtract the equation from seven degrees. And if the remainder is between 166 and 346, we subtract seven degrees from the equation. (**$C_1$** add. 'What remains is the original equation for the assumed degree. **Example of it.** The first equation opposite the full circle (i.e., 360°) is zero. And in this manuscript opposite $11^s 16^\circ$ ⟨we find the equation as⟩ $7°0'$. When ⟨this equation⟩ is subtracted from seven degrees, there remains 0°, which is the first equation opposite the full circle.') And for whichever degree whose second equation we want ⟨to know⟩, we take the equation opposite it. If the degree is less than 180, we subtract from the equation seven degrees. And if the degree is more than 180, we subtract the equation from seven degrees. And from whichever degree whose variation we want ⟨to know⟩, we subtract seven degrees and we take the variation opposite the remainder. If it is from ⟨the table for⟩ the nearest distance, it is what is

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[1] **$C_1$** add. وسطه بنقصان أربعة عشر درجة وخاصّته بزيادة سبع درجات [2] **$C_1$** add. فما بقي فهو تعديل الأصل للدرجة المفروضة. **مثاله.** إنّ التعديل الأوّل بإزاء الدور صفر. وفي هذه النسخة بإزاء يا يو ز ح. فإذا نُقص من سبعة يبقى ح، وهو التعديل الأوّل بإزاء الدور. [3] **F** om. [4] **$C_1$** بالزيادة [5] **$C_1$** add. **مثاله.** إنّ اختلاف البعد الأبعد موضوع بإزاء ح ح من المركز المعدّل، وفي هذه النسخة بإزاء يا كح.

marked in the original by ‘additive’. And if it is from ⟨the table for⟩ the furthest distance, it is what is marked in the original by ‘subtractive’. (**$C_1$** add. ‘**Example of it.** The variation at the furthest distance is written down opposite a true centrum of $0^s 0°$, but in this manuscript opposite $11^s 23°$.’)

**المشتري.** [1] أيّ درجة أردنا تعديلها الأوّل نقصنا منها ثماني عشرة درجة وأخذنا تعديل الباقي. [2]فإن كان الباقي[2] فيما بين شمـ إلى قسـ، نقصنا التعديل من ستّ درجات. وإن كان الباقي فيما بين قسـ إلى شمـ، نقصنا من التعديل ستّ درجات. وأيّ درجة أردنا تعديلها الثاني أخذنا ما بإزائها من التعديل. فإن كانت الدرجة أقلّ من قف[3]، نقصنا من التعديل نـ. وإن كانت الدرجة أكثر من قف، نقصنا التعديل من نـ. وأيّ درجة أردنا اختلافها نقصنا منها نـ وأخذنا ما [4]بإزاء الباقي[4] من الاختلاف.

**المرّيخ.** [5] [6]أيّ درجة[6] أردنا تعديلها الأوّل نقصنا منها نط درجة وأخذنا تعديل الباقي. فإن كان الباقي فيما بين شا إلى قكا، نقصنا التعديل من نـ. وإن كان الباقي فيما بين قكا إلى شا، نقصنا من التعديل نـ. وأيّ درجة أردنا تعديلها الثاني أخذنا ما بإزائها من التعديل. فإن كانت الدرجة أقلّ من قف، نقصنا من التعديل مر درجة. وإن كانت الدرجة[7] أكثر من قف، نقصنا التعديل من مر درجة. وأيّ درجة أردنا اختلافها، نقصنا منها مر وأخذنا ما بإزاء الباقي من الاختلاف.

**الزهرة[8].** [9] أيّ درجة أردنا تعديلها الأوّل نقصنا منها ن وأخذنا تعديل الباقي. فإن كان الباقي فيما بين شى إلى قل، نقصنا التعديل من درجتين. وإن كان الباقي فيما بين قل إلى شى، نقصنا من التعديل درجتين. وأيّ درجة أردنا تعديلها الثاني، أخذنا ما بإزائها [10]من التعديل[10]. فإن كانت الدرجة أقلّ من قف، نقصنا [11]من التعديل[11] مح درجة. وإن كانت الدرجة[12] أكثر من قف، نقصنا [13]التعديل من[13] مح درجة. وأيّ[14] درجة أردنا اختلافها نقصنا منها مح وأخذنا ما بإزاء الباقي من الاختلاف.

**عطارد.** [15] أيّ درجة أردنا تعديلها الأوّل نقصنا منها ل درجة وأخذنا تعديل الباقي. فإن كان الباقي فيما بين شل إلى قن، نقصنا التعديل من أربع درجات. وإن كان الباقي فيما بين[16] قن

[1] **$C_1$** add. وسطه بنقصان ثمنية عشر درجة وخاصّته بزيادة ستّ درجات [2–2] **B** om. [3] The text in **B** breaks off after مائة. [4–4] **F** بإزائها للباقي [5] **$C_1$** add. وسطه بنقصان تسعة وخمسين درجة وخاصّته بزيادة اثني عشر درجة [6–6] **$C_1$** om. [7] **F** om. [8] **$C_1$** الزهرت [9] **$C_1$** add. وسطها بنقصان خمسين درجة وخاصّتها بزيادة درجتين [10–10] **F** om. [11–11] **F** التعديل من [12] **F** om. [13–13] **F** من التعديل [14] **$C_1$** وارية (sic!) [15] **$C_1$** add. وسطه بنقصان ثلثين درجة وخاصّته بزيادة أربع درجات

إلى شل، نقصنا من التعديل أربع درجات. وأيّ درجة أردنا تعديلها الثاني أخذنا ما بإزائها من التعديل. فإن كانت الدرجة أقلّ من قف، نقصنا من التعديل كو. وإن كانت الدرجة أكثر من قف، نقصنا التعديل من كو. وأيّ درجة أردنا اختلافها نقصنا منها كو وأخذنا ما بإزاء الباقي من الاختلاف.

## Colophons

*Sources*: **F** fol. 89r (before the section on the original equations), **H** fol. 82r (under the second part of the star table, followed by a eulogy).

هذا آخر الجداول، وبه تُختم[1] المقالة الثانية والجزء الأوّل [2] العمليّ من الزيج الجامع. ويتلوه في الجزء الثاني المقالة الثالثة والرابعة في الهيئة[3] والبرهان، وهو الجزء العلميّ من الصناعة. [4]

This is the end of the tables, and with it is concluded (**H**: 'we conclude') the second treatise and the first part of the *Jāmiʿ Zīj* on operations. It is followed in the second part by the third and fourth treatises on models (*hay'a*) and proof, and it is the part on knowledge of the art. (**H** adds: 'Praise be to God, Lord of the Worlds, and God is sufficient for us, and truly an excellent Guardian.')

*Source*: **F** fol. 90r (after the section on the original equations).

واتّفق الفراغ من نسخ النصف الأوّل في الجزء العمليّ من الزيج الجامع لصاحبه محمود بن أحمد بن الحسين المعلّميّ في الليلة الحادية والعشرين من ذي القعدة سنة ثمه للهجرة في مسكن حريث بسمرقند. والحمد لله حمدًا كثيرًا وصلوته على نبيّه المصطفى بشيرًا ونذيرًا.

The copying of the manuscripts of the first half of the *Jāmiʿ Zīj* on the part on operations was finished by its owner Maḥmūd ibn Aḥmad ibn al-Ḥusayn al-Muʿallimī in the night of 21 Dhū l-qaʿda of the year 545 Hijra in the house of Ḥarīth in Samarqand. Much praise be to God and may his blessing be on his chosen prophet, bringer of glad tidings and warner.

*Source*: **Y** fol. 318v (under the last additional table for ascensions of year transfers).

تمّت المقالة الثاني في جداول.

⟨Here⟩ ends the second treatise on tables.

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[16] **F** om. [1] **H** نختم [2] **F** add. من [3] **H** אהיה [4] **H** add. والحمد لله ربّ العالمين وحسبنا الله ونعم الوكيل

# Part IV

# Commentary

## IV.1. Introductory remarks

The main purpose of this commentary to the tables in Kūshyār's *Jāmiʿ Zīj* is to explain certain features of the tables that are not found in other comparable astronomical handbooks and to give some indication of, on the one hand, the sources that Kūshyār used in compiling his own *zīj* and, on the other, the influence he had on later work by Islamic astronomers. Most sections of this commentary start with a short bibliography including general treatments of the topics concerned as well as specific studies that have a bearing on Kūshyār's tables. These bibliographies also include references to the relevant sections of Mohammad Bagheri's edition, translation and commentary of Books I and IV in *az-Zīj al-Jāmiʿ*. In many cases I have compared Kūshyār's tables with tables in a range of earlier and later *zīj*es in order to establish possible relationships between them. For more information on these *zīj*es and their authors, see the historical background in Section I.1 and the Quick reference guide on pp. 521–27.

For basic explanations of the planetary models and related issues such as eclipses and planetary stations, the reader is in the first place referred to studies of Ptolemy's *Almagest*, in particular Pedersen's *A Survey of the Almagest* and Neugebauer's *HAMA* (*A History of Ancient Mathematical Astronomy*). References to the Greek edition of the *Almagest* in Heiberg, *Syntaxis mathematica* or the English translation in Toomer, *Ptolemy's Almagest* have not generally been supplied, but Pedersen and Neugebauer will lead the interested reader directly to the relevant chapters. For most topics, references are provided to Glen Van Brummelen's doctoral dissertation *Mathematical Tables in Ptolemy's Almagest*, because it provides thorough analyses of the tables that ultimately lay at the basis of Kūshyār's work (mostly through al-Battānī).

I have investigated the values of the parameters and the methods of computation of many of the tables edited in this book. For this I have used, often tacitly, methods that were developed by Glen Van Brummelen and myself in the early 1990s. For the underlying concepts and full descriptions of these methods the reader is referred to Van Brummelen, *Mathematical Tables*, Chapter 3, pp. 28–45 and van Dalen, *Ancient and Mediaeval Astronomical Tables*, Chapters 1 and 2. Numerous examples of applications of these methods are also included in van Dalen, *Islamic Astronomical Tables*, a reprint volume of my relevant articles on the topic. Some of the most often used technical concepts are explained in the Quick reference guide (pp. 528–31).

For the research presented in this book, I have made extensive use of the computer programmes which I originally wrote in the 1990s. I have since adapted them for use in Windows 7 and later, although they maintain their DOS outlook with command line structure and no mouse control. These programmes include:

- TA (Table Analysis), a programme for the input, output and analysis of spherical astronomical and planetary tables in the Ptolemaic and Islamic traditions. It incorporates least squares for the estimation of the parameter values underlying these tables.
- MM (Mean Motions), a similar programme for mean motion tables. It implements the Least Number of Errors Criterion introduced in my PhD dissertation for the estimation of the daily mean motions on the basis of which these tables were computed.
- ZijManager, a Windows programme developed in Delphi by Rafał Ziołkowski (Warsaw) on the basis of the source code of TA and MM. In addition to providing a Windows interface, this programme's most powerful feature is the possibility of entering and editing multiple variants of the same table in parallel columns.
- CALH (Calendars–Historical), a programme that converts dates in nearly 20 variants of 10 calendars frequently used in Islamic astronomical sources. In its development stage in the early 1990s, CALH was extensively tested and the results verified to agree with Spuler and Mayr, *Wüstenfeld-Mahler'sche Vergleichungs-Tabellen* and Schram, *Kalendariographische und chronologische Tafeln* for all calendars found in these two works. The definitions of several other calendars in CALH, such as the Malikī calendar with its year beginning on the day of the vernal equinox and the lunisolar Chinese-Uighur calendar, were taken directly from descriptions in *zīj*es.
- SCTR (Sexagesimal Calculator-TRigonometric), for carrying out sexagesimal calculations and conversions to and from decimals. The trigonometric functions are implemented by means of sexagesimal arithmetic and reach the full accuracy of 14 sexagesimal digits to which all results are shown.
- Historical Horoscopes was originally written to recompute the horoscopes in an astrological work by al-Battānī (see Kennedy et al., 'Al-Battānī's Astrological History'), but later developed into a very useful tool for computing planetary positions on the basis of the tables in *zīj*es. The calculations are carried out using a parameter file for each *zīj*, which specifies the base locality and the epoch and then, for every planet, the eccentricity, epicycle radius, apogee longitude, and the daily mean motions in longitude and anomaly plus the corresponding epoch values. The programme uses these parameters to calculate the planetary positions on the basis of 'idealised equations', i.e., the equations are calculated directly from the parameters using the modern formulas. For the equation of anomaly either an exact calculation or Ptolemaic

interpolation may be used. In addition to these methods of calculation, the *DISHAS* database (see below) will also provide the possibility of computing planetary positions (and hence ephemerides or almanacs) on the basis of actual historical tables.

- KaK ('Kennedy and Kennedy') displays the full set of geographical data included in Kennedy and Kennedy, *Geographical Coordinates*, partially corrected and supplemented with several new sources, and allows more flexible searches of the data than are possible with the book.

Most of the above computer programmes are available from my personal website http://www.bennovandalen.de/. Others are still under development or have not yet been implemented with complete input and output checks to prevent crashes, but I will be happy to make these available on request as well.

Exact planetary positions for the medieval period based on the modern algorithms by Steve Moshier were calculated by means of the powerful software Alcyone Ephemeris, developed by Rainer Lange with the assistance of Noel Swerdlow.

It is impossible to provide here full recomputations or reconstructions of all 55 tables in Book II of Kūshyār's *Jāmiʿ Zīj*. Only in occasional, particularly illustrative cases have I given the results of recomputations or listed the errors in the tables. In due time, all the mathematically computed tables from the *zīj* will be made available in *DISHAS* (*Digital Information System for the History of Astral Sciences*), a collaborative project hosted at the Observatoire de Paris and led by Matthieu Husson (Observatoire de Paris; project ALFA: Shaping a European scientific scene: Alfonsine Astronomy), Clemency Montelle (University of Canterbury in Christchurch, New Zealand; project HAMSI: History of Astronomical and Mathematical Sciences in India) and the present author (project Ptolemaeus Arabus et Latinus). This database will be equipped with tools that allow the user not only to consult historical astronomical tables but also to carry out various types of analysis of these tables and to create editions and apparatuses according to his or her own specifications.

## IV.2. Chronology

**Bibliography**: Ginzel, *Handbuch der mathematischen und technischen Chronologie*; Taqizadeh, *Various Eras and Calendars I* and *Various Eras and Calendars II*; the *EI²* articles 'Taʾrīkh I. Dates and Eras in the Islamic World' by F. C. de Blois and B. van Dalen (the latter was reprinted as van Dalen, 'Dates and Eras'); Bagheri, 'Kūshyār ibn Labbān's Account' (an extract from Bagheri, *az-Zīj al-Jāmiʿ*); Bagheri, 'Mabḥath-i taqwīm' (with an edition of the chronological section from the Persian translation of the *Jāmiʿ Zīj* on pp. 30–43, an edition of the chronological tables from Book II of the Arabic versions of the *zīj* on pp. 59–65, and some comments on the Lent table on pp. 57–58); Bagheri, *az-Zīj al-Jāmiʿ*, Section I.1, pp. 6–14 (translation), 15–24 (commentary) and Arabic pp. 7–14. All date conversions in this book were carried out by means of my programme CALH (see Section IV.1).

Most *zīj*es start with a chapter on chronology, in which instructions and tables are given for the conversion of dates between the calendars that were most commonly used in the Islamic Middle Ages. Kūshyār provides such tables only for the three standard calendars: the so-called Syrian or Byzantine one, running entirely parallel with the Julian calendar, the purely lunar Arabic calendar, and the Persian calendar with its constant solar year of 365 days. He gives the numbers of days between the epochs of these three calendars and several important other eras in Book I, but no detailed information on, for example, the Coptic or Hebrew calendar. Most *zīj*es also give data on the feasts celebrated in connection with the calendars described. This information may consist of a simple list of the dates of festivals or, as in Kūshyār's case, a table for the calculation of the beginning of Lent, the seven-week fasting period preceding Easter, and therewith also of the date of Easter itself.

### IV.2.1. The intercalation system of the Persian calendar

The Persian calendar, used by the Zoroastrians for more than two millennia, goes back to Achaemenid times.[1] It was adopted from the Egyptian calendar in the early fifth century BC, with a further adjustment in order to fix certain feasts on given dates in the time of Xerxes I (r. 486–465 BC). The Egyptian origin can, for example, still be recognized in the fact that the month beginnings in the Yazdigird calendar, named after the last Persian king (r. AD 632–652), still partially coincide with those in the Egyptian calendar used by Ptolemy. Since the year length in both calendars is exactly 365 days, namely twelve months of 30 days plus 5 epagomenal days, and since the epagomenal days in the Persian calendar (called *al-mustaraqa* 'the stolen ⟨days⟩' or *Andarja*) appeared after the

[1] For the following account, I have mainly relied on the re-evaluation and analysis of available historical information on the Persian calendar in de Blois, 'The Persian Calendar'. See, however, Bagheri, *az-Zīj al-Jāmiʿ*, pp. 17–18 for a further discussion and partial refutation of de Blois' arguments.

eighth month Ābān in Islamic times, the use of several types of intercalation in the Persian calendar has been hypothesised both by Islamic authors and by modern historians. Kūshyār and al-Bīrūnī mention the insertion of an intercalary month once every 120 years, on which occasion the epagomenal days would have been moved to the next month. This intercalation would have been abandoned in Islamic times, and the epagomenal days moved back to the end of the year in the early eleventh century. De Blois has shown that most likely this system of intercalation was fictitious and that only a single reform of the Persian calendar was carried out in Sasanian times, when the religious New Year, together with the preceding five epagomenal days, was transferred to the beginning of the ninth month in order to conform with the vernal equinox. In the early eleventh century, the epagomenal days were then moved back to the end of the year, but this change was not adopted by all Zoroastrians.

In most of the extant manuscripts of the *Jāmiʿ Zīj*, Kūshyār's chronological tables as well as his mean motion tables provide values for both versions of the Yazdigird calendar, namely the old version with the epagomenal days after Ābān and the later version with the epagomenal days at the end of the year. One of these sets of values is written in black, like all other tabular values, and the other in red (italics in the edition of the tables). Only the Cairo manuscripts do not include the alternative values in the mean motion tables, possibly due to lack of space or because they were copied in a place where, or at a time when, the old system had become irrelevant.

### IV.2.2. Days of years and months

Tables 1–3 are used for date conversions between the three most common calendars in Islamic *zīj*es, namely the Syrian or Byzantine calendar with the Seleucid epoch,[2] the Arabic calendar with the Hijra epoch, and the Persian calendar reckoned from the beginning of the reign of the last Persian king, Yazdigird III.

Each of the three tables is structured in the same way: three columns give the accumulated numbers of days that correspond to groups of years (*al-sinūn al-majmūʿa*), extended years (*al-sinūn al-mabsūṭa*) and months (*shuhūr*) in the three calendars. The groups consist of 28 years for the Syrian and Persian calendars and 30 years for the Arabic calendar. For the Syrian and Persian calendars this implies that the corresponding dates in each cycle fall on the

[2] In Islamic sources, the Seleucid era was incorrectly associated with Alexander the Great and hence referred to as the era of al-Iskandar or *Dhū l-qarnayn*, the Two-Horned; see the *EI*[2] article 'al-Iskandar' by W. Montgomery Watt. The names Syrian (*suryānī*), mostly used by Kūshyār, and Byzantine (*rūmī*), exclusively used by al-Battānī, refer to the same calendar, namely a version of the Julian calendar that starts its year count from the Seleucid epoch in 312 BC: cf. the introductory paragraph to the general edition of the Syrian / Byzantine month names on p. 266.

same weekday (in the Persian calendar they in fact already do this after seven years; cf. also the commentary on Table 6 in Section IV.2.3), and for the Arabic calendar that the same pattern of intercalation repeats itself in each cycle. The table for the Syrian calendar starts from 924 Alexander (AD 612/613), and the tables for the Hijra and Yazdigird calendars from their respective epochs in AD 622 and AD 632. The numbers of days between the three epochs, as given in Chapter I.1.4 of the *Jāmiʿ Zīj*[3] and also as explanatory notes to the tables, are: Alexander to Hijra: 1,34,38,20 = 340,700 days, Alexander to Yazdigird: 1,35,38,44 = 344,324 days, and Hijra to Persian: 1,0,24 = 3624 days. This implies that Kūshyār started the Syrian year with the month October rather than adopting the less common September beginning from al-Battānī, and that he used the astronomical variant of the Arabic calendar (epoch Thursday, 15 July 622, based on the conjunction of the Sun and the Moon) rather than the civil version (epoch Friday, 16 July 622, based on the first sighting of the lunar crescent after new moon). The arguments of the subtables for collected years are given in Hindu numerals, as was common for ranges including numbers larger than 360;[4] the arguments of the subtables for extended years are given in *abjad* notation. Some of the manuscripts indicate the numbers of days of every month in the subtable for months.

The use of the tables is as follows:[5] first, the number of days since the epoch of the calendar in which a date is given is found from the table concerned by adding the appropriate values from the subtables for collected years, extended years and months plus the day of the current month. Then, the number of days between the epoch of the given calendar and that of the desired one (see above) is added or subtracted depending on which is earlier, yielding the number of days since the epoch of the desired calendar. Finally, the date in the desired calendar is found from the table concerned by the inverse operation of the first step.

As indicated in the headings, the tables are to be used with completed years and months. For example, the number of days that have passed between the epoch of the Syrian calendar and Ādhār 990 Alexander is found by adding the tabular values for 980 collected years, 9 extended years, and the month Shubāṭ. The sexagesimal digits of the numbers of days, which are all integer numbers, are labelled *awwal* ('first'), *thānī* ('second'), *thālith* ('third') and *rābiʿ* ('fourth'). These refer, respectively, to units and to multiples of 60, $60^2$ and $60^3$.

[3] Bagheri, *az-Zīj al-Jāmiʿ*, p. 11 (translation) and Arabic p. 12.
[4] cf. p. 76.
[5] cf. Chapter I.1.4 in Bagheri, *az-Zīj al-Jāmiʿ*, p. 11 (translation) and Arabic p. 12.

Table 1: Days of Syrian years and months

This table has the basic structure explained above, but has some peculiar features related to the intercalation of the Syrian calendar, whose months run parallel with the Julian one. In the subtable for extended years, the Arabic letter *kāf*, short for *kabīsa* ('intercalary', 'ly' for 'leap year' in the edition) indicates that the years 3, 7, …, 27 in every cycle of 28 years are leap years (these correspond to every fourth year in the Julian calendar). Only in manuscripts **FHC** does the subtable for months provide a second set of units from the month Shubāṭ (February) onwards. These give the accumulated numbers of days in a leap year and are hence always one larger than the units for an ordinary year. Two different formulations of explanatory text A in manuscripts **FL** and **CB** (see p. 270) point explicitly to the alternative sets of values.

The numbers of days between the Seleucid epoch of the Syrian calendar on the one hand and the epochs of the Arabic and Persian calendars on the other (explanatory text B) have already been addressed in the general commentary on these tables. In explanatory text C, apparently paraphrased from Chapter I.1.1 of the *Jāmiʿ Zīj*,[6] the equivalence of the number of days between the Seleucid epoch and the epoch of the Flood is valid for ordinary Persian (or Egyptian) years of 365 days. It places the date of the Flood on Friday, 18 February 3102 BC. This epoch is given in numerous *zīj*es, while another common date for it is one day earlier. It is related to the astrological concepts of great conjunctions and world years, long periods of time after which all planets return to the same positions.[7]

Whereas the chronological tables in all other manuscripts do not show any traces of perusal, the ones in **C** display several marginal notes, most of which were probably made by students of the *zīj* in the early eighteenth century practising their skills in calendar conversion.

Table 2: Days of Arabic years and months

This table also has the basic structure explained above. In all manuscripts except **F**, the leap years in the 30-year cycle of the Arabic calendar are indicated by the letter *kāf* (for *kabīsa* 'intercalary', 'ly' for 'leap year' in the edition) in the subtable for extended years: in each cycle the years 2, 5, 7, 10, 13, 15, 18, 21, 24, 26, 29 have 355 instead of 354 days. The same intercalation scheme was also used by al-Battānī and many other authors of *zīj*es.

Only manuscript **C** contains several marginal notes, most likely made by students in the early eighteenth century. Their calculations include one of a year transfer during the Hijra year 1127 (AD 1714/5).

6 Bagheri, *az-Zīj al-Jāmiʿ*, p. 6 (translation) and Arabic p. 7.
7 See, for example, Kennedy, 'Ramifications', pp. 24–26.

### Table 3: Days of Persian years and months

This table also has the basic structure explained above. Four of the six manuscripts provide values for both the old version of the Yazdigird calendar, which inserted the five epagomenal days after the eighth month Ābān, and the later version, which was introduced in the time of Kūshyār and which placed the epagomenal days back at the end of the year (cf. Section IV.2.1). In manuscripts **FH** the values for the old version are given in the table itself and those for the new version in the margin, while in manuscripts **CB** it is the other way around. Manuscript **Y** indicates in explanatory text A that the values in red are for the late version of the Yazdigird calendar, but in fact omits these values. Manuscripts **Y** and **B** repeat the units of four of the last five values for months in red under the second sexagesimal position (i.e., multiples of 60), which does not appear to have any particular significance.

## IV.2.3. *Notae* of years and months

The three tables of this type allow the user to determine the *nota* (Arabic *madkhal*, pl. *madākhil*, lit. 'entrance'; also referred to as *ʿalāma*, lit. 'sign' or 'token', and also called 'feria'), i.e., the day of the week on which a given date in one of the three common calendars falls. Strictly speaking the *notae* are not necessary for the conversion of a date from one of the three calendars into another, but in practice the uncertainty in a Hijra date (civil vs. astronomical calendar, exact day of sighting of the lunar crescent) and the possibility of scribal mistakes made a check of a conversion by means of weekdays almost obligatory.

The *notae* repeat themselves after a period that depends on the cycle of each calendar and the number of days of the week. Since the Syrian or Byzantine calendar inserts a leap day every four years, its *notae* recur after $4 \times 7 = 28$ years. The length of the intercalary cycle of the Arabic calendar is 30 years, a period that comprises $30 \times 354 + 11 = 10{,}631$ days. Since this number is not divisible by 7, the *notae* for the Arabic calendar recur only after $30 \times 7 = 210$ years. The Persian calendar has a constant year length of 365 days (which is not divisible by 7) and hence dates return to the same days of the week after seven years. For each of the three calendars the *notae* are tabulated for the years of one full cycle and for the twelve months of the year. In order to find a *nota* in a given year, the tables should be entered with the remainder of the division of the year by the length of the cycle concerned (for example, for the Syrian year 1321 one enters the table with $1321 - 47 \times 28 = 5$). For the Syrian and Persian calendars the tables have a double argument, i.e., they can be entered by feeding in the year vertically and the month horizontally. Because of the large period of recurrence of the *notae* in the Arabic calendar, the Arabic table is split up in a subtable for the remainder of the years divided by 210 and a subtable for months.

Whereas Tables 1–3 had *completed* years and months as their arguments, Tables 4–6 must be entered with the current ('incomplete' *nāqiṣ(a)*, or in some manuscripts 'with itself'/'with the very same' *bi-ʿaynihi*) year and month. Thus, the tabular value at the intersection of the row labelled '1' and the column labelled 'Tishrīn I' gives the day of the week on which the first day of the month Tishrīn I fell in the Seleucid years 1, 29, 57, …, namely '2' for Monday. Similarly, the first day of the month Farwardīn of the Yazdigird years 1, 8, 15, … fell on a Tuesday ('3'). The *notae* indicated in the subtable for Arabic years are the weekdays of the first day of Muḥarram. For example, 1 Muḥarram of the years 1, 211, 421, … Hijra fell on a Thursday ('5'), confirming that the astronomical version of the calendar is used. To find the first day of another month, we add the value given in the subtable for months: for example, the first day of Ramaḍān in the year 22 Hijra (astronomical) fell on day $6 + 5 = 11 \equiv 4$, i.e., a Wednesday.[8]

As one might expect given the simplicity of the mathematics, the tables for the *notae* in the manuscripts of Kūshyār's *Jāmiʿ Zīj* are generally correct. All deviations are incidental scribal errors that occur only in individual manuscripts.

### Table 4: *Notae* of Syrian years and months

Manuscripts **CYB** include an explanatory text (A) that may stem from Kūshyār himself. It explains that one needs to subtract a multiple of 28 from the given Syrian year and then enter the table with the remainder. The years given as examples of multiples of 28 (1260, 1288, …, 1372 Alexander) correspond to the Julian years 948/9, 976/7, …, 1060/1 and hence cover Kūshyār's lifetime.

### Table 5: *Notae* of Arabic years and months

The table in manuscript **F** has an unusually large number of mistakes, mostly confusions of the *abjad* numbers for '3' and '4'.

### Table 6: *Notae* of Persian years and months

In this table, the extant manuscripts give values for both the early and the later version of the Yazdigird calendar. However, while in Table 3 only manuscripts **FH** give the values for the early version as the main variants in black (and **YL** do not give values for the early version at all), in this table **YB** join **FH,** and only **C** gives the values for the later version as the main variants and the values for the early version as alternatives in red (note that **L** omits this entire table).

[8] See Chapter I.1.5 in Bagheri, *az-Zīj al-Jāmiʿ*, pp. 11–12 (translation) and Arabic p. 12. For further information on *notae* and their calculation, see van Dalen, 'Dates and Eras'.

## IV.2.4. Computation of Easter

### Table 7: *Notae* of the Christian Lent

**Bibliography**: Ginzel, *Handbuch der mathematischen und technischen Chronologie*, vol. III, pp. 134–143 and 210–225; Mosshammer, *The Easter Computus*; Saliba, 'Easter Computation', esp. pp. 197–98 and 209–11; Bagheri, 'Mabḥath-i taqwīm', pp. 57–58 and 65 (with an edition of Kūshyār's Lent table and explanations of its errors); Bagheri, *az-Zīj al-Jāmiʿ*, pp. 12–13 (translation) and 22 (commentary) and Arabic p. 13. The Lent table in manuscript **L** is reproduced in Plate 3.

Many *zīj*es include tables for calculating the date of the beginning of Lent, the 48-days period of fasting preceding Easter, or in some cases the date of Easter itself. Saliba gives an overview of tables of this kind as found in one Syriac and 15 Arabic sources. He analyses the widely distributed table known as the *Chronicon* and, among several further types, Kūshyār's table for finding the date of the beginning of Lent.[9] Saliba found 'no single method which accounts for the variants between ⟨Kūshyār's table⟩ and the *Chronicon*'.[10] I will here make a new attempt to understand the method of computation of the table and the errors that it contains by looking in the first place at the properties of the table itself rather than at the Easter theory on which it is supposed to be based. Copies of Kūshyār's Lent table can also be found in the *Ashrafī Zīj* by Sayf-i munajjim-i Yazdī al-Kamālī (Shiraz, early fourteenth century) and in the *Jadīd Zīj* by Ibn al-Shāṭir (Damascus, middle of the fourteenth century); since these versions were edited and compared by Saliba and do not provide relevant additional information, I have not included them in my own edition.

The type of tables for Lent and Easter given by Kūshyār has a double argument with the solar component (the 28-year cycle that we have already seen for the Syrian calendar) as the vertical argument and the lunar component (a 19-year schematic lunar cycle, for which see below) as the horizontal one. Kūshyār's table must be entered with the remainder of the division of the Seleucid year by 28 as the 'length' (*ṭūl*, i.e., vertical) argument. Note that the years 3, 7, 11, ..., 27 in each cycle are leap years, corresponding to Julian years that are multiples of four. The text of Book I of the *Jāmiʿ Zīj* prescribes that the 'width' (*ʿarḍ*, i.e., horizontal) argument is to be taken as the remainder of the division by 19 of the Seleucid year plus 5.[11] For example, for the Seleucid

[9] Saliba, 'Easter Computation'. For Kūshyār's table see the discussion on pp. 197–98 and the transcriptions of the tables in manuscript **F** and two dependent sources on pp. 209–11.

[10] Ibid., p. 198.

[11] Kūshyār's explanations of the calculation of the beginning of Lent and the use of the Lent table can be found in the chapter of Book I of the *Jāmiʿ Zīj* that describes the feasts in the three commonly used calendars. This is Chapter I.6 in **C** (with the paragraph on the Great Lent on fol. 7r), **C**$_2$ (fol. 9v) and **B** (p. 12); Chapter I.1.6 in **Y** (fols 234v–235r), Cairo, *mīqāt* Muṣṭafā Fāḍil 213/1 (fol. 5v) and Leiden, Universiteitsbibliotheek, Or. 523 (Persian

year 311 Alexander (corresponding to the Julian year 1 BC) the horizontal argument becomes $(311 + 5) \bmod 19 = 12$. The number at the intersection of the row and column determined by the two remainders is the date of the first day of Lent, belonging to February if it is written in black and to March if it is written in red. (For an Easter table of the same type, the black dates would be in April and the red ones in March, or the other way around.)

As explained by Saliba,[12] the dates of Easter in medieval Syriac and Arabic sources were calculated by means of a schematic lunar cycle of 19 years. The position of a year in this cycle is called its 'Golden Number'. In order to keep pace with the Syrian (i.e., Julian) solar calendar, each lunar cycle in this scheme contains 12 common years of 12 lunar months and seven embolismic years of 13 lunar months. The embolismic months are inserted in the years with Golden Numbers III, VI, VIII, XI, XIV, XVII, and XIX, but the Julian or Syriac years in which the intercalation takes place may not follow the same pattern, because they depend both on the beginning of the lunar years (in different traditions the first lunar month could fall in September, January or April) and on the place within the schematic lunar years at which the embolismic months are inserted. The lengths of the lunar months are fixed in such a way that, with only one exception over the course of every cycle (see below), a common lunar year is 11 days shorter than the corresponding Julian year and an embolismic year is 19 days longer. This was made possible by adding an additional day to those lunar months in the course of which a leap day is inserted in the Syrian or Julian calendar (i.e., the lunar months that contain 29 Shubāṭ/February). Since the resulting lunar cycle is one day longer than 19 Julian years, once in every cycle the lunar year is made one day shorter, which in Latin sources is called the *saltus lunae*. This jump was most conveniently inserted at the end of the lunar cycle, but in certain traditions at other places as well. Once the date of a particular phase in the lunar cycle (e.g., new moon or full moon) for any year is known, the dates of these phases in any other year can simply be calculated by repeatedly subtracting 11 days or adding 19 days in agreement with the arrangement of the embolismic years within the lunar cycle and by applying the *saltus lunae* where appropriate.

translation, fol. 6v), and Chapter I.5 in **H** (fol. 7v) and **L** (fol. 4v). The chronological chapter is entirely missing from **F**. The chapter is translated and edited in Bagheri, *az-Zīj al-Jāmiʿ*, pp. 12–13 and Arabic p. 13. Whereas for most feasts in the three calendars simply the calendar date is given, the passage headed *al-ṣawm al-kabīr* describes one or more of a total of three methods to find the beginning of the Great Lent: namely, a computational rule (in $\mathbf{CC_2LB}$), a criterion relating the beginning of Lent to the new moon between two given dates (in all eight manuscripts except **L**), and instructions for using the Lent table (only in **L**, although the table is also included in **Y**).

12 Saliba, 'Easter Computation', pp. 181–85.

Easter is now defined as the Sunday following the *Paschal full moon*, which is the first full moon (the fourteenth day of a schematic lunar month) falling on or after 21 March. If the Paschal full moon falls on a Sunday, Easter is the next Sunday. The beginning of fasting (the Great Lent) precedes Easter by 48 days and is thus always on a Monday.

From the above explanation it follows that all values in any column of an Easter table with the position of the year in the 28-year cycle of the Julian or Syrian calendar as the vertical argument and the position in the 19-year schematic lunar cycle as the horizontal one are calculated from the same date of the Paschal full moon: the differences between the values within a column only stem from the weekday on which the Paschal full moon falls. As an example, the reader may refer to one of the relatively few Easter tables in Islamic sources (most *zīj*es contain a table for the beginning of the Great Lent rather than an Easter table), namely the one in the *Tāj al-azyāj* by Muḥyī l-Dīn al-Maghribī (Damascus, 1258).[13]

The values in the first column range from 3 to 9 April, from which it follows that the Paschal full moon from which these Easter dates were found occurred on 2 April. Al-Maghribī's table is used by adding 17 to the current Seleucid year before casting out multiples of 28. This implies that the first row in the table corresponds to Alexander years that equal 12 modulo 28 (e.g., 1412, 1440, 1468, etc.). In these years, 2 April fell on a Tuesday, so that Easter, the following Sunday, was on 7 April, confirmed by the '7' in the table. In a similar way, all Easter dates in the first column can easily be verified. The Easter dates in the second column of the table range from 23 to 29 March, so that the Paschal full moon for this column fell on 22 March, preceding the Paschal full moon of the first column by eleven days.

It follows that the entire Easter table is determined by the dates of Paschal full moon for the 19 years of the lunar cycle. In the case of al-Maghribī's table, these dates (expressed as days counted from the last day of February, i.e., 1 stands for 1 March and 32 for 1 April) are:

33, 22, **41**, 30, **49**, 38, 27, **46**, 35, 24, **43**, 32, 21, **40**, 29, **48**, 36*, 25, **44**.
Differences: −11], −11, +19, −11, +19, −11, −11, +19, −11, −11, +19, −11, −11, +19, −11, +19, −12*, −11, +19, [−11

From these dates we can recognize the basic properties of the schematic lunar cycle: the date of the Paschal full moon either shifts backwards by 11 days

[13] cf. Saliba, 'Easter Computation', pp. 189–90 and Dorce, *El Taŷ al-azyāŷ*, pp. 29–30 and 163–64. This table is not to be confused with the 'Maghribī Table', also discussed by Saliba, from the same author's later *zīj*, the *Adwār al-anwār*. The table from the *Tāj al-azyāj* is found in Escorial, RBMSL, árabe 932, fol. 66r, but is missing from Dublin, Chester Beatty, Arabic 4129, which has the Lent table on fol. 33r. According to Dorce, the manuscript of the *Tāj al-azyāj* in the possession of the Arabic Department of Barcelona University likewise contains only the Lent table (fol. 49r).

(when the lunar year is common) or moves forward by 19 days (when the lunar year is embolismic, indicated above in boldface). The only exception to this rule is the date 36 (i.e., 5 April) for the seventeenth year of the cycle (indicated by an asterisk), which precedes the date for the sixteenth year (17 April) by 12 instead of 11 days. In other words, the *saltus lunae* in al-Maghribī's system takes place in the sixteenth year of the lunar cycle and hence, since 10 needs to be added to the Seleucid years before their place in the lunar cycle is found by casting out multiples of 19, in years 6, 25, 44, … Alexander.

An Easter table has only seven essentially different rows, namely one for each of the seven days of the week. Each of these seven rows appears four times in the table. A smaller Easter table could therefore be designed by means of a subtable giving the day of the week of, for example, the last day of February for each of the 28 years of the cycle of the Syrian calendar (i.e., basically a table of *notae* similar to Table 4 in Kūshyār's *zīj*), and then a subtable with the 19 years of the lunar cycle and the seven days of the week as a double argument.[14] Note that each row of an Easter table will contain at most five different possible Easter dates, namely the Sundays in the period from 22 March to 25 April for the Syrian year concerned.

It is a straightforward matter to calculate a table for the beginning of Lent from an Easter table, namely by moving all dates 48 days backwards, taking into account the presence of an additional day at the end of February (Shubāṭ) in Syrian leap years, i.e., years 3, 7, 11, … Alexander. As Dorce indicates, this is what al-Maghribī appears to have done in the *Tāj al-azyāj*. The resulting Lent table has a peculiarity that the Easter table does not possess: instead of four occurrences of each of seven consecutive dates, as a result of the presence of a leap day in intercalary years each column in the derived Lent table contains eight consecutive dates: namely three occurrences of the earliest, only one occurrence of the latest (always in a leap year), and four occurrences of the remaining six dates.

Since most *zīj*es contain only a Lent table and not an Easter table, in some cases we may need to consider the possibility that the Lent table was computed directly rather than by shifting calculated Easter dates backwards by 48 days. Instead of Paschal full moons, one could use for this a different, theoretical phase of the lunar month that precedes the Paschal full moons by approximately 48 days, and hence falls some time between the last quarter and new moon. I will refer to this phase as the 'Lent moon'. Instead of calculating the dates of these phases directly from the lunar cycle, one might also derive them from the Paschal full moons by subtracting 48 days. This implies that,

[14] A table of this type is in fact contained in the Leiden manuscript of the Persian translation of Kūshyār's *Jāmiʿ Zīj*, but there is no reason to assume that it stems from Kūshyār (cf. p. 35).

when expressed as dates of February, they will be 20 (namely 48 days minus the length of February in an ordinary year) less than the Paschal full moons expressed as dates of March. For the *Tāj al-azyāj* the dates of the Lent moons would thus become:

13, 2, **21**, 10, **29**, 18, 7, **26**, 15, 4, **23**, 12, 1, **20**, 9, **28**, 16*, 5, **24**.

For each year in the lunar cycle, the beginning of Lent is now defined as the Monday following the day of the Lent moon. Since the leap day may fall between the Lent moon and the beginning of Lent, a Lent table has 14 essentially different rows: seven for each possible weekday of, for example, the last day of January (Kānūn II) in an ordinary year (these rows all appear in the table three times) and seven for a leap year (these rows all appear only once). If we recompute the Lent table in the *Tāj al-azyāj* in this way, we find systematic differences from al-Maghribī's table of exactly one week whenever the Paschal full moon falls on a Sunday in a leap year (or, equivalently, when Easter falls on the latest of the seven possible dates within its column) and the corresponding Lent moon falls in February (which is always the case in leap years). In leap years the Lent moons found above precede the Paschal full moons by 49 instead of 48 days because we disregarded the leap day. Consequently, when the Paschal full moon falls on a Sunday and hence precedes Easter by seven days, the Lent moon falls on the Sunday preceding Easter by 56 days, and Lent will be found as the following Monday, which precedes Easter by 55 instead of 48 days. This happens two to four times for every Syrian leap year and exactly once in every year of the lunar cycle. Of course, this problem cannot be remedied by subtracting 47 days instead of 48 from the dates of the Paschal full moons, because then a similar problem occurs in all ordinary years, leading to 57 differences of exactly one week instead of only 19. Similarly, we may not expect to find better dates for the Lent moons directly from al-Maghribī's table either, since adjusting the date of any Lent moon by a day will shift the beginning of Lent by a week for four rows in the column concerned; this creates more new errors than it corrects existing ones. It follows that it is relatively easy to judge from a Lent table whether it was calculated directly from a set of Lent moons or indirectly from an Easter table.

We will now look at Kūshyār's Lent table in more detail. As mentioned above, all dates in the table should be Mondays. It thus follows that the values '1' in rows 14 and 25 and columns 3, 6, 11, 14 and 19 (underlined in the edition) are wrong (in the years corresponding to these two rows 1 March is a Sunday) and should all be corrected to '2'. The same errors also appear in the copies of Kūshyār's Lent table in the *Ashrafī Zīj* and the *Jadīd Zīj*.[15] Our above

[15] As noted by Saliba, two whole columns and occasional values in other columns were invisible on a microfilm of Paris, BnF, suppl. persan 1488 (fol. 28v) of the *Ashrafī Zīj* (colour

discussion of the only 14 essentially different rows in a Lent table presents a likely clue as to the origin of these errors: apparently, Kūshyār mistakenly copied the values for a leap year (found in the third row of his table), instead of the ones for an ordinary year (found in the eighth row), into rows 14 and 25.[16]

Since Saliba found no single method that allowed him to derive Kūshyār's Lent table from an Easter table and hence from Paschal full moons, we will now test whether Kūshyār might have calculated his table from Lent moons. By comparing the corresponding columns of Kūshyār's table and the *Chronicon* as edited by Saliba,[17] we note that the majority of the differences are indeed of the type that one would expect if Lent moons were used for the computation: in every column we find exactly one difference of one week that occurs in one of the seven rows for leap years. Furthermore, each of the columns in Kūshyār's table contains only seven consecutive dates for the beginning of Lent rather than the eight that we would expect if the table were calculated from an Easter table (cf. above).

Similar to our determination of the Paschal full moons underlying al-Maghribī's Easter table, we can now find the positions of the Lent moons from Kūshyār's Lent table as the dates immediately preceding the range of seven dates for the beginning of Lent contained in any particular column. We thus find the following set of Lent moons (expressed as dates in February):

18, 7, **26**, 15, 4, **23**, 12, 1, **20**, 8*, **28***, 17, 6, **24***, 13, 2, **21**, 10, **29**.

Differences: −11], −11, +19, −11, −11, +19, −11, −11, +19, −12*, +20*, −11, −11, +18*, −11, −11, +19, −11, +19, [−11

We note that Kūshyār's Lent moons follow the usual pattern of the schematic lunar cycle with the exception of the three dates marked with an asterisk. The third of these, for column 14, is 18 instead of 19 days later than the previous one, which indicates that the *saltus lunae* takes place in the fourteenth year of the lunar cycle. Furthermore, the dates of the Lent moons for columns 10 and 11 are respectively 12 (instead of 11) days earlier and 20 (instead of 19) days later than the preceding dates in the cycle. A plausible explanation for these deviations is that the tenth column was mistakenly calculated from a Lent moon on 8 instead of 9 February.[18] This mistake produces exactly the

scans of this manuscript are now available online through https://archivesetmanuscrits.bnf.fr/). These values all concern dates in Ādhār and should have been written in red, as indicated at the top of the table. However, in the recently discovered second manuscript of the *Ashrafī Zīj* (Qum, Gulpāyagānī Library, MS 64731, fol. 31v), the red numbers are present and the incorrect values '1' stemming from Kūshyār clearly recognizable.

[16] This explanation was already offered in Bagheri, 'Mabḥath-i taqwīm', p. 57.

[17] Saliba, 'Easter Computation', p. 200.

[18] Bagheri, 'Mabḥath-i taqwīm', pp. 57–58, explains these errors by assuming that Kūshyār, in the computational rule explained below, subtracted 1 from the resulting product of 19 and the remainder of the Seleucid year plus 5 although this product did not yet exceed 250.

four additional errors of one week with regard to the *Chronicon* that Kūshyār's table displays in the tenth column, namely for the Seleucid years in which 8 February falls on a Sunday. For these arguments the incorrect Lent moon leads to a Lent date of 9 February (likewise underlined in the edition) rather than the correct 16 February. The remaining differences between Kūshyār and the *Chronicon* result from the different positions of the *saltus lunae* in the two tables: in the *Chronicon* the jump takes place in the last year of the cycle, corresponding to the twelfth column in Kūshyār's table, whereas Kūshyār inserts it in the thirteenth year of his cycle.

From the above we may conclude that:

1) Kūshyār did not compute his Lent table from a set of Paschal full moons or a table of Easter dates, but from a set of what I have called 'Lent moons', which, analogously to the Paschal full moons, define a phase in the schematic lunar cycle by means of a single date for each of the 19 years in the cycle. This can be recognized from a very typical error pattern with respect to a Lent table computed directly from Easter dates, namely a single deviation of exactly one week in each of the 19 columns of the table, appearing only in the seven rows for leap years.
2) In this process Kūshyār made two mistakes. First, he calculated the Lent dates for horizontal argument 13 on the basis of a Lent moon on 8 instead of 9 February, leading to four dates for the beginning of Lent in the column concerned that are one week too early. Second, he filled the rows for vertical arguments 14 and 25 with the dates for a leap year instead of for an ordinary year, leading to ten occurrences of the date 1 March instead of the correct 2 March. The only further deviations from the standard Lent table as found in the *Chronicon* are caused by a different position of the *saltus lunae* within the 19-year lunar cycle.

Kūshyār's table for the beginning of Lent has some more uncertainties, which only further research on the calculation of Easter in Arabic sources may be able to clarify.

*Epoch*. Saliba takes the epoch of a Lent or Easter table to be a year corresponding to the first column and first row in the table. Because of the periodicity of the computation of Easter, epoch candidates for any table will be a series of years differing by 532. For Kūshyār the epoch must be a Seleucid year $Y$ such that $Y \bmod 28 = 1$ and $(Y + 5) \bmod 19 = 1$. Solutions are the Seleucid years $281 + n \times 532$, of which 1345 Alexander (AD 1033/34) is closest to Kūshyār's time, but some years after the completion of the *Jāmiʿ Zīj*. It is unclear to me whether the epochs defined in this way had any practical significance.

*Golden Numbers*. The *Chronicon* and several Easter and Lent tables based on it show that the first column in these tables does not necessarily

correspond to Golden Number I, the first year of the 19-year lunar cycle. Since, according to the main traditions of the computation of Easter, the Julian year 1 BC, and hence the Seleucid year 311 Alexander, have Golden Number I, in Kūshyār's table the twelfth column (found as (311 + 5) mod 19) must correspond to Golden Number I, whereas in the *Chronicon* it is column 19. It is unclear to me why the columns of Lent tables were shifted differently in different sources. An answer to this question may be related to the one above on the significance of the epoch.

*Embolismic years*. Under the above assumption that the twelfth column of Kūshyār's Lent table corresponds to years with Golden Number I, the following years are embolismic years (in which the Lent moon jumps 19 days forward instead of 11 days backwards): III, VI, VIII, XI, XIV, XVII, XIX. This is the standard set of embolismic years that is also used in the *Chronicon*.

*Saltus lunae*. As we have seen above, the *saltus lunae* in Kūshyār's scheme for Lent computation takes place in the fourteenth column, i.e., in years with Golden Number III. This deviates from the common custom of inserting the jump in the last year of the lunar cycle, but adheres to the other common custom of inserting it in an embolismic year (in this case the first embolismic year of the lunar cycle). In the *Chronicon* the *saltus lunae* appears at the end of the tabulated lunar cycle and therewith in years with Golden Number II and in a common year.

Manuscripts **$\mathbf{CC_2LB}$** of the *Jāmiʿ Zīj* contain a computational rule for determining the beginning of Lent.[19] The same rule is also included in the early twelfth-century Ismāʿīlī *zīj Dustūr al-munajjimīn*, which copies numerous sections from Kūshyār, and in the two later *zīj*es that contain Kūshyār's Lent table, namely the *Ashrafī Zīj* and the *Jadīd Zīj* (see above). The following is a paraphrase of the translations of this rule by Saliba and Bagheri.[20] Note that the remainder of the division of a multiple of *n* by *n* is taken as *n* rather than 0, as indicated explicitly in the four manuscripts.

> Add 5 to the current Seleucid year and cast out multiples of 19. Multiply the remainder by 19. If the product is larger than 250, reduce it by 1. Cast out multiples of 30. The remainder, expressed as a day in February, is the beginning of Lent if it falls on a Monday; otherwise, the beginning of Lent is the Monday following it.

This rule is in fact in agreement with the Lent table. The initial addition of 5 to the Seleucid year is the same as the one needed for the use of the table.

[19] cf. footnote 11.

[20] Saliba, 'Easter Computation', p. 192 and Bagheri, *az-Zīj al-Jāmiʿ*, p. 12.

The remainder of the division by 19 thus corresponds to the column number in the table. The multiplication by 19 and casting out of multiples of 30 implements a repeated addition of 19 or subtraction of 11, exactly as in the scheme of the lunar cycle. It is then to be expected that the subtraction of 1 when the multiplication by 19 reaches a certain limit should implement the occurrence of the *saltus lunae*. We have seen above that Kūshyār inserts it in the years corresponding to the fourteenth column of his Lent table. This is exactly where the multiplication by 19 exceeds 250 (namely, $14 \times 19 = 262$). This implies that the rule is correct as intended and also correctly reproduces the Lent table in the *Jāmiʿ Zīj* with the exception of the 14 erroneous values whose cause I have explained above.

In all manuscripts of the *Jāmiʿ Zīj* except **L**, in the *Dustūr al-munajjimīn* and in the *Jadīd Zīj* the above rule is accompanied by another one stating that the beginning of Lent is on the Monday nearest to a conjunction of Sun and Moon between 2 February and 8 March. The correctness of this rule was discussed extensively by Saliba.[21]

The Lent table in **F** does not carry any explanatory or marginal notes. **L** has the *abjad* number 1321 (غشكا) written in the right margin next to the row for vertical argument 5 as well as in the cell for horizontal argument 15, apparently in the same hand and ink as the table itself (see Plate 3). Note that, in order to find the beginning of the Great Lent for the Syrian year 1321, 5 must be added and then multiples of 19 cast out to find the horizontal argument, and multiples of 28 must be cast out to obtain the vertical argument. Because $(1321 + 5) \bmod 19 = 15$ and $1321 \bmod 28 = 5$, the two indications of the year 1321 correctly mark the row and column from which the beginning of Lent will be found (namely as 20 Shubāṭ 1321 Alexander, which is 20 February 1010). Easter was 48 days later, i.e., on 9 Nīsān 1321 Alexander (9 April 1010). Since this year fell within Kūshyār's lifetime, the indications on the table may stem from him originally and may indicate a time at which he was still working on his *zīj*.[22]

[21] Saliba, 'Easter Computation', pp. 191–92.

[22] For the dates of Kūshyār's life, cf. Section I.2 and Bagheri, *az-Zīj al-Jāmiʿ*, p. xiii.

## IV.3. Trigonometry

**Bibliography**: Suter, 'Das Buch der Auffindung'; Schoy, *Über den Gnomonschatten*; Schoy, 'Beiträge zur arabischen Trigonometrie'; Schoy, *Die trigonometrischen Lehren*; Kennedy, 'The History of Trigonometry'; Kennedy, *The Exhaustive Treatise*, esp. Chapters 7–12, vol. I, pp. 68–110 and the commentary in vol. II, pp. 25–51; Hamadanizadeh, 'The Trigonometric Tables'; Berggren, *Episodes in the Mathematics*, Chapter 5, pp. 127–56; Berggren, 'Spherical Trigonometry'; Debarnot, 'Trigonometry'; Van Brummelen, *The Mathematics of the Heavens*, Chapter 4, esp. pp. 137–66.

Table 8: Sine

Kūshyār's sine table for degrees of arc found in manuscripts **FHCC$_1$** contains only a single error of +1″, namely for 23°. The correct value is found in the otherwise basically identical table in manuscripts **YB** (and also in the sine table with values for minutes in **LB**). The tabular differences of the sine table are in full agreement with the sine values: all deviations from correct differences are due to scribal errors. The tabular differences for 23° and 24° in **YB** are in accordance with the correct sine value in these manuscripts. Another copy of this table—with an explicit attribution to Kūshyār, the correct value for the sine of 23°, but the incorrect tabular differences from **FHCC$_1$**—is included in Escorial, RBMSL, árabe 927, fol. 52v of the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr.[23]

Kūshyār's sine table is clearly different from al-Battānī's, which has values for every 30′ of arc and at least 13 errors of ±1″ in values for whole degrees, which cannot be attributed to scribal errors.[24] Because of the lack of errors in Kūshyār's table, relations to the sine tables in other *zīj*es are difficult to prove. An auxiliary table in the Persian *Mufrad Zīj* by al-Ṭabarī (Amul, *c.* 1100), which has no heading but displays the sine and the cotangents (lit. 'shadows') for gnomon lengths 12 and 7, most likely comes from Kūshyār, because it contains the single error in the sine and several errors in the cotangent tables that are also found in the *Jāmiʿ Zīj*.[25] The three most important *zīj*es contemporaneous with Kūshyār (Abū al-Wafāʾ's *al-Majisṭī*, Ibn Yūnus's *Ḥākimī Zīj* and

[23] See the facsimile edition Sezgin, *The Verified Astronomical Tables*, p. 101.

[24] Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 55–56 (with commentary on pp. 220–21). Note that al-Battānī seems to have followed Ptolemy's table of chords in the *Almagest* in choosing the increment of the arguments of his sine table, although the two tables are not otherwise related. In his edition Nallino mostly corrected doubtful sine values in the Escorial manuscript of al-Battānī's *zīj* to exact values, ignoring the palaeographical likeliness of the corrections. For a critical apparatus of the table that also takes into account the Arsenal manuscript of the Castilian translation and relevant traditions of the *Toledan Tables*, see van Dalen and Pedersen, 'Re-editing the Tables', pp. 414–15.

[25] Cambridge, University Library, Browne O.1, fol. 130v.

al-Bīrūnī's *al-Qānūn al-Masʿūdī*) contained accurate sine tables to at least sexagesimal thirds.[26]

Table 8a: Sine for fractions of a degree

Besides the standard type of sine table with values to seconds for every integer degree, two manuscripts of the *Jāmiʿ Zīj* contain a sine table that presents the sine for every integer degree at the top of 90 columns distributed over 15 pages, together with additive values for 1, 2, 3, …, 15, 18, 21, …, 60 minutes in each of the columns (Plate 4 shows the first page of this table in the Leiden manuscript). Thus the sine for, for example, 23;33° can be obtained by taking the sine of 23° (23;26,38) from the top of the column headed 23 and adding the value found in this column for argument 33′ (0;31,45); the result 23;58,23 is accurate to seconds. According to the instructions for the use of this table in **L**, Chapter I.7 (fol. 5v, see below), the same additive values with the sexagesimal point shifted may also be used for further fractions. For example, to calculate the sine of 23;33,33°, add 0;0,31,45 to the above result in order to obtain 23;58,55 (exact: 23;58,54,18,…).

The sine values for integer degrees at the top of the columns in this table are all correct to the precision of seconds to which they are given. I have checked the additive values for fractions by adding them to the sines of the integer arguments at the top of the columns and recomputing the resulting sines of arguments with integer numbers of minutes. I thus found that the additive values are mostly correct; only 838 out of the 2700 additive values (i.e., 31%) contain an error of ±1″ and three more values contain an error of ±2″. I have made no further attempt to establish the method by which Kūshyār calculated this table. The clustering of errors in the additive values for certain integer arguments points to the use of rounding/truncation or linear interpolation or occasionally even to the use of a sine value for the integer argument different from the one given at the top of the columns. The fact that the additive values for 60′, which one would expect to be identical to the tabular differences of the values for integer degrees, actually differ from them in 29 out of 90 cases, may also hint at a particular method of computation.

**L** contains the sine table for fractions of a degree but not the one with values for integer degrees only (Table 8). The 15 pages of the sine table appear in between the table for the beginning of the Great Lent and the table of the versed sine, in agreement with the table of contents on fol. 21r. The explanatory text on the use of the sine table in **L**, Chapter I.7, is specifically directed towards the use of this type of table and hence deviates significantly from the

[26] See the commentary on Table 8a and especially footnote 32 for references and further discussion.

corresponding chapter in the other manuscripts of the *Jāmiʿ Zīj*.[27] The text reads as follows:[28]

إنّا، لسهولة العمل في أخذ الجيب والقوس، وضعنا جيب أجزاء القوس وجيب كسورها في الجدول. فوضعنا في عرض الصفحة أجزاء القوس وتحتها جيوبها، ووضعنا في طول الصفحة كسور الجزء الواحد وبإزائها جيوبها تحت الجزء الصحيح. فإذا كانت الكسور دقائق كانت جيوبها دقائق وثواني وإذا كانت الكسور ثواني كانت جيوبها ثواني وثوالث وعلى هذا الرسم. فإذا أردنا جيب قوس، أخذنا الجيب الذي تحت الجزء الصحيح وزدنا عليه جيب الكسور من الجدول الذي تحته على الرسم المذكور. وإن أردنا قوس جيب، طلبنا في جيوب الأجزاء الصحاح مثل الجيب الذي معنا أو ما هو أقرب إليه ممّا هو أقلّ منه وأخذنا ما فوقه من الأجزاء، وما بقي من الجيب معنا دخلنا به في جيب الكسور وأخذنا ما بإزائه من الكسور على الرسم المذكور فنضيفها إلى الأجزاء الصحاح.

For the simplification of the operation for taking the sine and the arc⟨sine⟩, we laid down in the table the sine of degrees (*ajzāʾ*) of arc and the sine of their parts (*kusūr*). Thus we laid down in the width of the page the degrees of arc and under them their sines, and we laid down in the length of the page the parts of one degree and opposite them their sines under the whole degree. If the parts are minutes, their sines are minutes and seconds; if the parts are seconds, their sines are seconds and thirds, and ⟨so on⟩ according to this pattern. When we want the sine of an arc, we take the sine [that is] under the whole degree and add to it from the table the sine of the parts that are under it according to the above-mentioned pattern. And if we want the arc of a sine, we search among the sines of the whole degrees for [the equal of] the sine that we have or the nearest smaller one and take the degrees that are ⟨written⟩ above it; and with what remains of the sine that we have (after subtracting the sine of the whole degree) we enter ⟨the column of⟩ the sine of the parts and take the parts opposite ⟨the remainder⟩ according to the above-mentioned pattern, and then we add them to the whole degrees.

**B** contains both types of sine table. The table with values for integer degrees is found in between the table for *notae* of Persian years and the table of the versed sine, in agreement with the table of contents on p. 35. The table with values for fractions of a degree (pp. 49–63), together with a table for prorogations (pp. 64–66), is inserted in between the two halves of the solar mean motion table (pp. 48 and 67); furthermore, part 14 of the table (p. 61) comes before part 13 (p. 62). The hand in which these nine folios were written appears to be the same as in the rest of the manuscript; however, the title *al-jayb* appears only on the first page of the table in a different hand. The earlier folio numbering of pp. 49–66 ranges from 139 to 147 and does not correspond to the

[27] cf. Bagheri, *az-Zīj al-Jāmiʿ*, pp. 27 (translation), 30 (commentary) and Arabic p. 17. Bagheri did not edit and translate the deviating text from **L**, which is found on fol. 5v.

[28] I have omitted the last four lines of the chapter, which explain how to find the sine and cosine of arcs between 90 and 360° from those of arcs smaller than 90°.

numbering of the surrounding pages (cf. the description of manuscript **B** in Section I.4). It is thus clear that the sine and prorogation tables did not originally belong in this place in the manuscript and were moved there only later on.[29] The correspondence of page and folio numbers described on pp. 26–27 suggests that the nine sheets were originally inserted somewhere after page 216, close to the end of the first part of the manuscript, in the midst of the spurious tables appended to the *Jāmiʿ Zīj*. The instructions for the use of the sine table are missing from **B**, so we have no means to check to which of the two tables they refer. We may conclude that **L** is the only manuscript of the *Jāmiʿ Zīj* of which the sine table for fractions of a degree was an original part, and that it was probably not an original part of the manuscript from which the texts and tables in **B** were copied.

A copy of Kūshyār's table, explicitly attributed to him on its first page, is found in the unique Paris manuscript of the *Dustūr al-munajjimīn* and has been included in my edition with siglum **D**.[30] Alternative (and incorrect) digits for the position of the seconds in the sine values for integer degrees at the top of the columns are indicated in this source for nine of these values. These digits probably stem from a collation with al-Battānī's sine table, although not all of his deviating values are indicated in the *Dustūr*. The *Dustūr* also copies three explanatory sections on the sine from the *Jāmiʿ Zīj*, including the one entitled *Fī jayb al-qaws wa-qaws al-jayb min al-jadwal*.[31] However, the version of this section that the *Dustūr* includes is the one found in **FHCC$_2$Y** dealing with the sine table with values for integer degrees only (Table 8), rather than the one from **L** relating to the sine table for fractions of a degree included in **LB** and the *Dustūr al-munajjimīn itself.*

I am not aware of similar types of sine tables in earlier or contemporary Islamic sources. In their *zīj*es Abū l-Wafāʾ and al-Bīrūnī included sine tables with values to four sexagesimal places for every quarter of a degree.[32] Ibn

[29] The sheet with parts 13 and 14 of the sine table was reversed already before the folios were moved, since the original folio number on this sheet appears at the top right of the verso, as in the case of all other folios containing the table.

[30] I edited and investigated this table as part of my work for the DFG project 'Der Dustūr al-munaǧǧimīn als Quelle für die Geschichte der Ismaʿiliyya und ihre astronomisch-astrologischen Vorstellungen' (cf. p. xv).

[31] This is Chapter I.2.3 in Bagheri's edition; see Bagheri, *az-Zīj al-Jāmiʿ*, p. 27 (translation) and Arabic p. 17.

[32] For Abū l-Wafāʾ, see Carra de Vaux, 'L'Almageste d'Abû'lwéfa Albûzdjâni'; Schoy, 'Beiträge zur arabischen Trigonometrie', pp. 392–94; van Dalen, *Ancient and Mediaeval Astronomical Tables*, Chapter 4, pp. 158 and 168, etc.; Van Brummelen, *The Mathematics of the Heavens*, pp. 142–44, and Moussa, 'Mathematical Methods', pp. 6–24. An extract from Abū l-Wafāʾ's sine table is most likely included in Paris, BnF, arabe 2486, fols 224v–225v. From numerous numerical examples in the unique manuscript of *al-Majisṭī* (Paris, BnF, arabe 2494) it becomes clear that Abū l-Wafāʾ's table originally had values for every quarter of a degree. For al-Bīrūnī,

Yūnus tabulated the sine to four sexagesimal places for every 10 minutes of arc in his *Ḥākimī Zīj* and to five sexagesimal places for every minute of arc in a separate work *Kitāb al-jayb li-daqīqa fa-daqīqa*.[33] Kūshyār could have taken his accurate values for integer degrees from Abū l-Wafāʾ, but the arrangement with additive values for the above peculiar range of fractions of a degree is most likely his own contribution. Kūshyār's table may have influenced the thirteenth-century *Shāmil Zīj*, anonymous in all manuscripts but possibly written by the well-known philosopher Athīr al-Dīn al-Abharī. The sine table in this work displays additive values for 2, 4, 6, ..., 60 minutes to be added to the sine or versed sine values for integer degrees given at the top of the columns. The planetary equations and the geographical table in the *Shāmil Zīj* also show that the *Jāmiʿ Zīj* was one of its main sources.[34]

Table 9: Versed sine

The versed sine Vers (in Latin called *sagitta* after the Arabic word *sahm* for 'arrow') is calculated as

$$\text{Vers}\, x = R - \text{Cos}\, x = R - \text{Sin}\,(90° - x),$$

where $\text{Sin}\, x = -\text{Sin}\,(-x)$ for $x < 0°$ and all three functions are expressed in a base circle with radius $R$, commonly taken to be 60 in Islamic sources. The versed sine is usually tabulated for arguments 1 to 180°.

The versed sine as included in the manuscripts **FHCC$_1$** contains only two errors of ±1″, namely for arguments 67 and 113. The errors are symmetrical (i.e., they occur in two values that can be directly calculated from each other because $\text{Vers}(180°–x) = 2R – \text{Vers}(x)$ and correspond to the single error in the

see Schoy, 'Beiträge zur arabischen Trigonometrie', pp. 396–398; Schoy, *Die trigonometrischen Lehren*, and the table in al-Bīrūnī, *al-Qānunu'l-Masʿūdī*, vol. I, pp. 308–25. Since it is almost certain that al-Bīrūnī reproduced Abū l-Wafāʾ's tangent table (unpublished result by the present author) and because his sine table differs only insignificantly from that for integer degrees in the *zīj* of Ibn Maḥfūẓ al-Baghdādī and from eight sine values for non-integer degrees quoted by Abū l-Wafāʾ in the text of *al-Majisṭī*, it is highly probable that al-Bīrūnī also copied Abū l-Wafā's sine table while changing the radius of the basis circle from 60 to 1. In fact, al-Bīrūnī quotes from *al-Majisṭī* at several places in his works and he may even have received a copy directly from Abū l-Wafāʾ, since the two scholars are known to have been in contact with each other concerning the observation of lunar eclipses for the determination of the difference in geographical longitude between Baghdad and Khwarazm (cf. Ali, *The Determination of the Coordinates*, pp. 214–15).

[33] Suter, 'Das Buch der Auffindung'; Schoy, 'Beiträge zur arabischen Trigonometrie', pp. 364–91 and 394–95; King, *The Astronomical Works*, pp. 77–89; Van Brummelen, *The Mathematics of the Heavens*, pp. 141–42, and the tables in Leiden, Universiteitsbibliotheek, Or. 143, pp. 216–221 and Berlin, SBPK, Landberg 1038 (Ahlwardt no. 5752), fols 2v–48r.

[34] See van Dalen, 'The *Zīj-i Nāṣirī*', pp. 845 and 857, and van Dalen, 'The Geographical Table'.

sine table. As in the case of the sine table, the error and the corresponding tabular differences were corrected in the version of the table contained in manuscripts **YLB**. In none of the six manuscripts does the table of the versed sine show any sign of perusal in the form of marginal notes, etc.

### Table 10: First tangent

All manuscripts of Book II of the *Jāmiʿ Zīj* except **C**$_2$ contain Kūshyār's table of the tangent for radius of the base circle 60. **L** contains multiple copies of several tables from the *Jāmiʿ Zīj*, possibly assembled from different versions of the *zīj*. On fols 32r–33r, in between the sine for fractions of a degree and the versed sine, we find tables for the first and second tangent and for the first and second declination, the latter with the values omitted. Another copy of both tangent tables follows after the versed sine. The table of contents on fol. 21r in fact places the (co)tangent and declination tables, together with the other spherical-astronomical tables, immediately after the sine. However, the table of contents does not include the versed sine or the additional copies of the (co)tangent and the declination. Furthermore, the right and oblique ascension tables in **L** are found after the planetary tables, like the spherical-astronomical tables in all other manuscripts. It seems clear that some of the tables are mixed up in **L**, and especially that some tables from a manuscript close to **F** and **H** were added to the tables also found in **YB** (cf. footnote 71 on pp. 58–59). Since the tangent table on fol. 32r of **L** differs from the one on fol. 34v and from the table in **Y** with regard to its range of arguments and the term used for tabular differences (*ḥiṣṣa daraja* instead of *tafāḍul*, which is not found in any other manuscript), and since the cotangent table on fol. 32r omits the column for the shadow of a gnomon of twelve fingers, I consider it more likely that the tables on fols 34v–35r are Kūshyār's original ones and hence I refer to these by siglum **L**$_1$ and to the tables on fol. 32r by siglum **L**$_2$.

In all eight witnesses the first tangent $R \cdot \tan x$ is tabulated for $R = 60$ with values to seconds and tabular differences. In **FL**$_2$ it is tabulated only up to argument 45°, in **YL**$_1$**B** up to 60°, and in **HCC**$_1$ it is extended up to argument 90° (for the table in **C**, see Plate 5). A copy of this table, explicitly attributed to Kūshyār but without the tabular differences, is also found in the *Dustūr al-munajjimīn* (Paris, Bibliothèque nationale de France, arabe 5968, fol. 41r). It tabulates the tangent up to argument 45°, with 30 values in the first column and 15 in the second as in manuscripts **FL**$_2$, and differs from my edition only in occasional scribal mistakes.

Around half of the values up to argument 60° contain errors of ±1″. The values for arguments 61–90° in **HCC**$_1$ have increasingly large errors, which reach ±4″ for arguments up to 75° and maxima of +227″ and −926″ for arguments 88 and 89°. Such errors are typical of tangent and cotangent tables in medieval sources. In practice the tangent values were calculated by dividing two values from a sine table by each other:

Table C: Recomputation of Kūshyār's tangent values for arguments 61–90°, with errors (i.e., differences from an exact recomputation) in the third and seventh columns and the differences from tangent values reconstructed from Kūshyār's sine table in the fourth and eighth columns. The errors and differences are expressed in seconds.

| arg | tangent | errors | diffs | | arg | tangent | errors | diffs |
|---|---|---|---|---|---|---|---|---|
| 61 | 108;14,34 | | | | 76 | 240;38,49 | | −2 |
| 62 | 112;50,37 | | | | 77 | 259;53,27 | +8 | |
| 63 | 117;45,23 | −1 | | | 78 | 282;16,39 | −1 | |
| 64 | 123; 1, 8 | +2 | | | 79 | 308;40,14 | −10 | |
| 65 | 128;40,10 | −3 | | | 80 | 340;16,34 | −3 | |
| 66 | 134;45,45 | +1 | | | 81 | 378;49,26 | −4 | |
| 67 | 141;21, 4 | | +7 | | 82 | 426;54,58 | −22 | −42 |
| 68 | 148;30,20 | +1 | | | 83 | 488;39,24 | −15 | |
| 69 | 156;18,22 | +3 | | | 84 | 570;52, 0 | +17 | +1 |
| 70 | 164;50,59 | +4 | | | 85 | 685;47,23 | −48 | −1 |
| 71 | 174;15, 8 | −2 | +1 | | 86 | 858; 3,45 | +81 | −3 |
| 72 | 184;39,36 | −4 | | | 87 | 1144;52, 9 | +3 | +161 |
| 73 | 196;15, 8 | +4 | | | 88 | 1718;14,18 | +227 | |
| 74 | 209;14,39 | −3 | | | 89 | 3437; 8,26 | −926 | |
| 75 | 223;55,22 | −1 | | | 90 | 216000; 0, 0 | - | - |

$$\operatorname{Tan} x = \frac{60 \cdot \operatorname{Sin} x}{\operatorname{Sin}(90^\circ - x)}.$$

When $x$ approaches 90°, $\operatorname{Sin}(90^\circ - x)$ approaches 0. If $\operatorname{Sin}(90^\circ - x)$ is taken from a table with a fixed number of sexagesimal digits, this implies that the relative error in $\operatorname{Sin}(90^\circ - x)$ increases due to the rounding of its exact value to the precision of the table (note that the rounding error has the same order of magnitude for the entire table). As a result, the absolute error in $\operatorname{Tan} x$ also increases. For example, $\operatorname{Sin} 89^\circ \approx 59;59,27,6,8$, which is rounded in Kūshyār's table to 59;59,27, and $\operatorname{Sin} 1^\circ \approx 1;2,49,43,11$, which is rounded to 1;2,50. When we calculate $\operatorname{Tan} 89^\circ$ from sine values to a total of five sexagesimal places, the result $60 \cdot 59;59,27,6,8 \,/\, 1;2,49,43,11 = 3437;23,51,56,\ldots$ equals the exact value 3437;23,51,42,44,... to the precision of seconds of Kūshyār's table. If, however, we calculate $\operatorname{Tan} 89^\circ$ from sine values to four sexagesimal places, the result $60 \cdot 59;59,27,6 \,/\, 1;2,49,43 \approx 3437;24,2$ has an error of $+10''$. Finally, if only a sine table with values to three sexagesimal places (such as Kūshyār's) is available, the result becomes $60 \cdot 59;59,27 \,/\, 1;2,50 \approx 3437;8,26$ with an error of $-926''$, which is exactly the error in Kūshyār's table. Table C shows that this is not a coincidence. Not only are the differences between Kūshyār's tangent values and an exact recomputation (in the third and seventh columns) consistently

larger than the differences from a reconstruction based on his sine table (in the fourth and eighth columns), but the differences from the reconstruction are in fact mostly zero.

Of the non-zero differences between Kūshyār's tangent and the reconstructed tangent values, three are in fact significantly larger than the errors with respect to an exact recomputation and hence need to be discussed separately. For the reconstruction of Tan 67° in Table C, I used the erroneous value 23;26,39 for Sin 23° that is found in manuscripts **FHCC$_1$**. The correct value gives Tan 67° = 60 · 55;13,49 / 23;26,38 ≈ 141;21,4, as in Kūshyār's table, so we may conclude that the corrected sine table was used for the computation of Tan 67°. This appears to be confirmed by Kūshyār's value for Tan 23°, which is given in the table as 25;28,7. This value is produced by the correct value of Sin 23°, whereas use of the incorrect value yields Tan 23° = 60 · 23;26,39 / 55;13,49 = 25;28,7,45,... ≈ 25;28,8.

Kūshyār's value for Tan 82° cannot be obtained by the division of plausible sine values to three sexagesimal places. It is accurately reproduced as the quotient of Sin 82° ≈ 59;24,58 and the value 8;21,1,50 for Sin 8° (exact: 8;21,1,23), but I do not have a plausible explanation for the use of this value. Kūshyār's value for Tan 87° is surprisingly good in comparison with the surrounding values and can only be explained by assuming that he had a more precise value for Sin 3° at his disposal than the one given in his sine table (the exact value is 3;8,24,33,59,...). Note that already the use of a sine value to four sexagesimal digits leads to the exact value 1144;51,6, which leaves the remaining error of +3″ in need of an explanation. In spite of these few anomalies, it is absolutely clear that Kūshyār's tangent table was calculated from a sine table with correct values to exactly three sexagesimal places, such as his own.

A notable error in the tangent table is the one of 10″ in the value for 37° (45;12,58 instead of the correct 45;12,48), which occurs in most of the witnesses and is confirmed by the corresponding tabular differences. Only in **L** (the table on fol. 34v) and in **B** are the tabular value and the tabular differences corrected.

The second column of tabular differences in **Y** at first sight seems nonsensical. It turns out that the minutes of these values were slid downwards by one row and further on by two rows, while the seconds were miscopied from the minutes in the first column of the cotangent table for gnomon length 7 (*sic*!) on the following page. I have not included these errors in the apparatus.

Explanatory text B1 stems in its entirety from Chapter I.3.2 of the *Jāmiʿ Zīj*, where Kūshyār also states that he tabulated the tangent only up to 45° because linear interpolation between tangent values for integer degrees larger than 45° would have yielded inaccurate results.[35] Explanatory text B2, accompanying the

[35] Bagheri, *az-Zīj al-Jāmiʿ*, p. 32 (translation) and Arabic p. 20. Kūshyār's statement that he tabulated the tangent only up to argument 45° appears in manuscripts **F** (fol. 5r), **Y** (fol. 237r),

tangent tables in **L**$_2$ and **F**, is also found in the *Dustūr al-munajjimīn*, fol. 41r, where the table is explicitly attributed to Kūshyār. Since precisely these three sources, and no others, have the table with arguments up to 45°, it may be concluded that table and explanatory text belonged together and are firmly linked to Kūshyār.

Table 11: Second tangent

All manuscripts of Book II of the *Jāmiʿ Zīj* except **C**$_2$ contain Kūshyār's tables for the second tangent (i.e., the cotangent). For the second copies of the tangent and cotangent tables in **L**, see the commentary to Table 10 above. A third copy of the table for gnomon length 7 is found among the additional tables in **L** on fol. 126v, but it has many additional scribal errors and hence has not been included in the edition. A further copy of the table for gnomon length 7 is found in the *Dustūr al-munajjimīn*, fol. 41r, without attribution to Kūshyār but appearing on the same page as a textual passage on the conversion of tangents for different gnomon lengths and the tangent table for $R = 60$, both of which are explicitly associated with Kūshyār.

This table, without attribution, is also found in the late thirteenth-century *zīj* by Ibn Maḥfūẓ al-Baghdādī.[36] The cotangent table for feet and fingers here appears in the middle of a set of much more accurate spherical-astronomical tables, of which I have shown that they were most probably extracted from Abū l-Wafāʾ's *al-Majisṭī*.[37]

The two functions displayed in this table are the shadows of gnomons of lengths 12 fingers and 7 feet, i.e., $12 \cdot \cot x$ and $7 \cdot \cot x$, where $x$ is the solar altitude. In the analysis of this table in my doctoral dissertation, I showed, among other things, that the second function was calculated by multiplying the first function by 7⁄12 and rounding the result in the standard way, rather than by an independent computation.[38] This is also the reason why the second function has 19 errors of mostly ±1′, whereas the first function has only six errors.

**L** (Chapter 11, fol. 6r), and in Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1, fol. 8r (the chapter concerned is entirely missing from **B**). Chapter I.14 in **H** (fol. 9v) does mention that interpolation for arcs larger than 45° gives inaccurate results, but does not state explicitly that the table was laid out only for arguments up to 45°.

36 Paris, BnF, arabe 2486, fol. 229r.

37 cf. the entry for Abū l-Wafāʾ's *al-Majisṭī* in the Quick reference guide for *zīj*es on pp. 523–24.

38 van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 172–73. The multiplication by 7⁄12 leaves only one error unexplained (1;6 instead of 1;7 for 81°, which is a common scribal error).

Table D: Errors in Kūshyār's cotangent tables.

| altitude | Kūshyār's cotangent for $R = 12$ | error | Kūshyār's cotangent for $R = 7$ | error |
|---|---|---|---|---|
| 1 | 687;26 | −3 | 401; 0 | −2 |
| 2 | 343;39 | +1 | 200;28 | +1 |
| 3 | 228;58 | | 133;34 | |
| 4 | 171;37 | +1 | 100; 7 | +1 |
| 5 | 137; 9 | −1 | 80; 0 | −1 |
| 6 | 114;10 | | 66;36 | |
| 7 | 97;47 | +3 | 57; 2 | +1 |
| 8 | 85;23 | | 49;48 | |
| 9 | 75;46 | | 44;12 | |
| 10 | 68; 3 | | 39;42 | |

As I have explained in more detail for the tangent table (Table 10), the computational errors in the shadow length table for gnomon length 12 mostly result from the fact that, for arguments approaching 0°, the rounding error in the value of the sine by which the corresponding cosine is divided causes an increasingly large error in the resulting cotangent value. For example, when we calculate $\mathrm{Cot}_{12}x$ from sine values to four sexagesimal places, the result $12 \cdot 59;59,27,6 / 1;2,49,43 = 687;28,48,...$ equals the exact value 687;28,46,... to the precision of minutes of Kūshyār's table. However, if we calculate $\mathrm{Cot}_{12}x$ from sine values to three sexagesimal digits, as they are given in Kūshyār's sine table, we obtain $12 \cdot 59;59,27 / 1;2,50 \approx 687;25,41,...$, which is rounded to minutes as 687;26, as found in Kūshyār's table. We may thus conclude that the table for gnomon length 12 fingers was computed from an accurate sine table with values to three sexagesimal places. Note that the error of +3′ in Kūshyār's shadow of a gnomon of 12 fingers for an altitude of 7° cannot be explained in this way. This error appears in all six copies of the table in the manuscripts of the *Jāmiʿ Zīj* as well as in the *zīj* of al-Baghdādī, and is reflected by a smaller error in the table for gnomon length 7 feet in these sources as well as in the *Dustūr al-munajjimīn*. We must conclude that this is a 'genuine' error of Kūshyār's: either a plausible scribal confusion of digits 44 and 47 or a computational error.

The explanatory texts B1, B2 and B3, found in somewhat different versions in $\mathbf{HCC_1L_2}$ and in the *Dustūr al-munajjimīn*, and hence apparently belonging to a relatively early version of the *Jāmiʿ Zīj*, give criteria for the beginning and end of prayer times expressed in shadow lengths.[39] This subject is not treated elsewhere in the *Jāmiʿ Zīj*.

39 For this topic, see King, 'A Survey of Medieval Islamic Shadow Schemes', p. 197.

## IV.4. Mean motion parameters

**Bibliography**: see the bibliography of Section IV.5.

Table 12: Preliminaries of the Mean Motions

This table is included in all the manuscripts of Book II of the *Jāmiʿ Zīj* except **$C_2$**. As has been explained in detail in Part III (p. 281), the format of this table is essentially different between manuscript **F** and the other six manuscripts that contain it. Only **F** includes the epoch positions for the Syrian and Arabic calendars found in the upper left quarter of the edition of the table (see Plate 6). The three subtables that all sources have in common are arranged differently in manuscripts **H C $C_1$ Y L B** with respect to **F** (see Plate 7).

As suggested by the headings of the subtables, the data in this table derive entirely from al-Battānī's *Ṣābiʾ Zīj*. The epoch positions for the Syrian calendar (found only in **F**) are in fact those given by al-Battānī for the year 931 Alexander in his subtable for collected years.[40] Similarly, the positions for the Hijra epoch (likewise only found in **F**) are taken from al-Battānī's mean motion tables for the Arabic calendar.[41] Note that al-Battānī gives planetary longitudes in degrees from 0 to 359 instead of in zodiacal signs plus degrees from 0 to 29, as in Kūshyār and most other Islamic *zīj*es. I have used the Escorial and Arsenal manuscripts and Nallino's edition to correct several scribal errors in the manuscripts of the *Jāmiʿ Zīj*. Al-Battānī's epoch values can be reliably checked since they are part of his mean motion tables, which have nearly constant tabular differences. In the apparatus of Kūshyār's table, I have not included the numerous scribal errors found in the Escorial and Arsenal manuscripts, most of which were corrected by Nallino.

The values for the mean motions in 20 Julian years given by Kūshyār were taken from the headings of al-Battānī's subtables for the mean motions in multiples of 20 Byzantine years.[42] Al-Battānī did not include the numbers of full rotations, which Kūshyār apparently added in the early version of his *zīj* (they do not appear in **YLB**) in order to be able to calculate the daily mean motions. Only for the lunar anomaly is there a clear difference between the two sources, from the position of the fourths onwards: $5^{iv}38^{v}55^{vi}$ in the Escorial manuscript and Nallino, as compared to $13^{iv}58^{v}15^{vi}$ in the *Jāmiʿ Zīj*. The value in the Cas-

[40] Escorial, RBMSL, árabe 908, fols 186v and 205v; edited in Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 72 and 102; Castilian translation in Paris, Bibliothèque de l'Arsenal, MS 8322, fols 50r and 64v.

[41] Escorial, RBMSL, árabe 908, fols 164v and 168r; Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 19 and 24; Paris, Bibliothèque de l'Arsenal, MS 8322, fols 36v and 38v.

[42] Escorial, RBMSL, árabe 908, fols 189r and 206r; Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 77 and 103; Paris, Bibliothèque de l'Arsenal, MS 8322, fols 50v and 65r. Note that the value for the motion of the lunar nodes in 20 years is missing from al-Battānī's zīj, both in the Arabic and in the Castilian version.

tilian translation, $53^{iv}58^{iv}55^{vi}$, suggests that this may be the result of a scribal confusion. Al-Battānī found the mean motion in lunar anomaly to be equal to Ptolemy's[43] and in fact used Ptolemy's daily mean motion for the computation of his double set of mean motion tables. Kūshyār lists this same daily value and also uses it for the computation of his own mean motion table for the Persian calendar. The value listed by Kūshyār for the lunar anomalistic motion in 20 Syrian years is an exact multiple of this daily mean motion, which leaves open the possibility that he calculated it rather than using an unreliable value from al-Battānī.

It can be verified that Kūshyār found his daily mean motions by dividing al-Battānī's motions in 20 Syrian years by 7305 days (as indicated in explanatory text D provided with Table 12 in manuscripts **FL**) and rounding them to six sexagesimal places in the standard way. The only deviation from this rule is found for the mean anomaly of Venus, for which the calculation yields $43^{v}15^{vi}$ instead of Kūshyār's $42^{v}45^{vi}$. For the lunar nodes, for which al-Battānī does not give the motion in 20 years, Kūshyār apparently made use of a different source. In al-Battānī's mean motion tables the mean motion of the lunar node is around 0;3,10,37,24°/day, while Kūshyār lists a value of 0;3,10,37,17,40,26°/day, which also underlies his table for the motion of the node. This is significantly different from the values found in the surviving *zīj*es from before Kūshyār's time, namely 0;3,10,48,22°/day in al-Khwārizmī's *Sindhind Zīj*, 0;3,10,37,35°/day in the *Mumtaḥan Zīj*, 0;3,10,37,28°/day in Ḥabash al-Ḥāsib's *Damascene Zīj*, and 0;3,10,38,41°/day in Ibn al-Aʿlam's *ʿAḍudī Zīj*.[44] I cannot currently offer any plausible explanation for Kūshyār's unattested value. Unlike the situation for Mars (cf. Section IV.8.3), we have no evidence that Kūshyār made observations of eclipses or may have adjusted some lunar parameters to improve the accuracy of the predictions of eclipses found from his *zīj*.

[43] Escorial, RBMSL, árabe 908, fol. 55r; Nallino, *al-Battānī sive Albatenii*, vol. I, p. 54 (Latin translation) and vol. III, p. 82 (Arabic).

[44] The significance of the differences between the mean motion parameters is mathematical rather than astronomical. Not considering the Indian parameter of al-Khwārizmī, the other values mentioned deviate by at most 2½ minutes in 20 solar years; this does not significantly change the lunar longitude and latitude but will be clearly noticeable in a table covering, for example, 600 years. It follows that Kūshyār could not have obtained his parameter by dividing, for example, al-Battānī's motion in 20 Byzantine or 30 Hijra years by the corresponding numbers of days. Ibn al-Aʿlam's value was obtained from his mean motion in 20 Julian years as reproduced in the *Ashrafī Zīj* (Paris, BnF, suppl. persan 1488, fol. 234r). Both Kennedy, 'The Astronomical Tables', p. 20 and Mercier, 'The Parameters', pp. 23–24 incorrectly read this motion as 26;55,59° (instead of 26;50,59°) and hence arrived at incorrect daily and annual mean motions (the correct annual motion of 19;19,45° is in fact in good agreement with Mercier's source O1). The correct value is confirmed by the recently discovered Gulpāyagānī manuscript of the *Ashrafī Zīj*, fol. 249r, as well as by comparing the value from the *Shāhī Zīj* and its differences from the values in all other *zīj*es given in the Paris manuscript on fols 233r–v.

The calculation of Kūshyār's mean positions at the Yazdigird epoch from his calculated daily mean motions and al-Battānī's positions for the beginning of the Seleucid year 931 can also be verified to be in concordance with explanatory text E, which is found in manuscripts **F** and **L**. In fact, there is a perfect agreement to the full precision of six sexagesimal fractional places for all motions if we multiply the daily mean motions by 4491 and add them to al-Battānī's positions for the beginning of the Seleucid year 931. The only deviations are the scribal errors noted in the apparatus. It can also be seen that Kūshyār used the daily mean motion for Venus that he tabulated, not the slightly different one calculated correctly from al-Battānī's mean motion in 20 Byzantine years.

The period of 4491 days between al-Battānī's epoch year 931 Alexander and the Yazdigird epoch shows that his epoch values are for a year beginning in March. Whereas al-Battānī describes the Byzantine calendar with a year beginning in September (as opposed to the more common October), his tables start the year with March in order to remove the need for adjustments in the middle of leap years. The number of days from 1 Ādhār of the year 931 Alexander to 1 Farvardīn 1 Yazdigird is in fact 4490. However, Kūshyār needed to multiply the daily mean motions by 4491 rather than by 4490 because al-Battānī's epoch positions are for noon of the day preceding 1 March 931 Alexander, i.e., 29 February. This can be seen from the instructions for the use of his table in Chapter 33 of the *Sābiʾ Zīj* in combination with the tables themselves.[45] In order to find the mean motion at noon of 1 March 931, the motion tabulated for 1 day must be added to the epoch position, i.e., the value tabulated for the Byzantine year 931 in the subtable for collected years. In al-Battānī's subtables for days the values tabulated for 1 day are in fact the mean motions in one day, which implies that the epoch of the tables must precede 1 March 931 by one day. On the other hand, Kūshyār tabulates his mean motions for current days (i.e., the value zero is displayed for argument 1 day, the motion in one day for 2 days, etc.). It follows that his epoch positions are those for noon of 1 Farvardin 1 Yazdigird rather than for the day preceding it.

In order to obtain the epoch positions included in Kūshyār's planetary tables from the highly precise Persian ones in Table 12, two further steps are necessary: first, an adjustment for the longitude difference between Raqqa and Kūshyār's meridian of 90°, and secondly an adjustment for the displacements of the planetary equations in the *Jāmiʿ Zīj*. Since Raqqa has a geographical longitude of 73;15°, the correction for the difference in longitude from Kūshyār's meridian amounts to a fraction 16;45/360 of the daily mean motion; this amount must be subtracted from the epoch positions found because Raqqa lies

[45] The instructions can be found in Escorial, RBMSL, árabe 908, fols 72r–73v; Nallino, *al-Battānī sive Albatenii*, vol. I, pp. 71–72 (Latin translation) and vol. III, pp. 106–08 (Arabic).

west of Kūshyār's meridian. The way to obtain ordinary planetary mean positions from Kūshyār's displaced mean motions is described in the section found at the end of Book II in manuscripts **FC$_1$B** (see Part III, pp. 331–34). It follows that, in order to obtain Kūshyār's displaced mean positions, 2° must be subtracted from the solar mean longitude as found from al-Battānī; 8° must be subtracted from the lunar mean longitude and 14° from the lunar mean anomaly; 14° must be subtracted from the mean longitude of Saturn and 7° added to its mean anomaly, etc. No corrections are necessary for the double elongation and the lunar nodes.

In all seven witnesses for Table 12 we find explanatory texts that appear to have been a fixed part of the manuscript tradition of the *Jāmiʿ Zīj*. In fact, the only explanatory texts that occur in a single source (namely in **F**) are A2 (al-Battānī's apogee longitudes for the Seleucid year 1191 as given in the *Sābiʾ Zīj*; these values are also given next to the table for apogee motion in **F**, see Table 14) and F2 (on the relation between solar mean longitude and solar mean anomaly, i.e., a simple extension of the text F1 found in all seven sources). The two explanatory texts that are found in only two sources (D and E, both needed for computing Kūshyār's mean motion tables from those of al-Battānī) appear in **F** and **L**, another indication that the copyist of **L** had a manuscript close to **F** available while copying the *Jāmiʿ Zīj*. Also some of the other explanatory texts are related to the determination of Kūshyār's mean motions from al-Battānī's; this suggests that they may very well have originated with Kūshyār himself.

A1, found in all seven witnesses, lists the apogee longitudes of the Sun and the five planets (where the apogee longitude of Venus is taken equal to that of the Sun) for Kūshyār's epoch, the Yazdigird era. These longitudes are in almost perfect agreement with the list in Section I.4.4 of the *Jāmiʿ Zīj*. A2 lists al-Battānī's apogee longitudes for 1 March of the Seleucid year 1191 (AD 880), which he mentions in Chapter 33 of the *Sābiʾ Zīj* for the Sun[46] and in Chapter 45 for the planets.[47] Kūshyār's values for the Yazdigird epoch in A1 differ from those he gives for al-Battānī by a constant of −3;43°. Since the Yazdigird epoch precedes 1 March 880 by 90,475 days, this difference corresponds precisely to an apogee motion of 1½° in 100 Persian years;[48] this is in perfect agreement with

[46] Escorial, RBMSL, árabe 908, fol. 73v; Nallino, *al-Battānī sive Albatenii*, vol. I, p. 72 (Latin translation) and vol. III, p. 107 (Arabic); missing from the Castilian version.

[47] Escorial, RBMSL, árabe 908, fol. 117v; Nallino, *al-Battānī sive Albatenii*, vol. I, p. 114 (Latin translation) and vol. III, p. 173 (Arabic); likewise missing from the Castilian version. The planetary apogees are also indicated in the headings of the tables for the planetary equations (Escorial, RBMSL, árabe 908, fols 208v, 211v, 214v, 217v and 220v; Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 108, 114, 120, 126 and 132); Paris, Bibliothèque de l'Arsenal, MS 8322, fols 66v, etc.

[48] Note that $1;30 \cdot 90475 / 36500 \approx 3;43,5$.

Kūshyār's table for apogee motion (Table 14), but somewhat different from the motion of 1° in 66 Byzantine years given by al-Battānī in the above-mentioned chapters of his *zīj* and underlying his table for precession.[49] Only in **F** are the same two lists of apogee longitudes, together with a third list for 331 Yazdigird (AD 962), also copied next to the table for apogee motion (see Table 14).

Explanatory text B, which states that Theon of Alexandria equated Mars's apogee longitude at the Yazdigird epoch ($4^s\,10;36°$) with the longitude of Regulus, appears in five of the seven witnesses. I have not been able to find a direct source for this statement. In the *Handy Tables* Ptolemy normed the planetary apogees with respect to the longitude of Regulus.[50] Since Ptolemy assumes planetary apogee motion to be equal to precession, the apogee longitudes will follow from the longitude of Regulus at any time by adding a constant for each planet. However, in the *Handy Tables* this constant is given as 353° (or −7°) for Mars, which contradicts Kūshyār's suggestion that the two are equal. According to Kūshyār's star table, the longitude of Regulus at the Yazdigird epoch was 135;10° (value from his table) minus 4½° (precessional motion in 300 Yazigird years), i.e., $4^s\,10;40°$, making it plausible that statement B is based on the *Jāmiʿ Zīj* itself. In comparison, the longitude of Regulus at the Yazdigird epoch is $122;30 + 180765 / 36500 \approx 4^s\,7;27°$ according to Ptolemy (using his very inaccurate rate of precession of 1°/100 Egyptian years) and $134;0 - 90475 / 24160½ \approx 4^s\,10;15°$ according to al-Battānī.

Text C, correctly stating the number of days in 20 Syrian years in sexagesimals, is needed to derive the daily mean motions from al-Battānī's precise values for the motions in 20 years. It is found in all sources except **FL**, which include the same information in text D. The two Cairo manuscripts reverse the order of the designations of the sexagesimal positions.

Text D, found only in sources **FL**, describes the calculation of daily mean motions from the motion in 20 Syrian years, as has already been explained above.

Text E, likewise found only in sources **FL**, describes the calculation of positions for the Yazdigird epoch from al-Battānī's positions for the Seleucid epoch (see above) and for the Hijra epoch. **F** gives the number of days between the Hijra epoch and 1 Farvardīn 1 Yazdigird as 3623 days, **L** as 3624. Al-Battānī uses the astronomical Hijra calendar, which has its epoch on Thursday, 15 July 622.[51] The astronomical Hijra epoch in fact precedes the Yazdigird epoch by 3624 days. However, since al-Battānī's epoch positions for the Hijra are also for

[49] This table is found in Escorial, RBMSL, árabe 908, fol. 208r; Nallino, *al-Battānī sive Albatenii*, vol. II, p. 107, and Paris, Bibliothèque de l'Arsenal, MS 8322, fol. 66r.

[50] Stahlman, *The Astronomical Tables*, p. 134.

[51] cf. Escorial, RBMSL, MS árabe 908, fols 68v–69r (text) and 157v and 158v (tables); Nallino, *al-Battānī sive Albatenii*, vol. III, p. 101 (Arabic); vol. I, p. 57 (Latin translation); vol. II, pp. 7 and 9 (tables); van Dalen, 'Dates and Eras', Table 1.

noon of the day preceding the epoch (i.e., 14 July 622), the daily mean motions need to be multiplied by 3625 and added to the positions at the Hijra epoch in order to obtain the correct mean positions for the Yazdigird epoch. We thus see that both numbers given in **F** and **L** are incorrect: 3624 is the number of days between the astronomical Hijra epoch and the Yazdigird epoch, and 3623 that between the civil Hijra epoch and the Yazdigird epoch. As already stated above, Kūshyār's epoch positions were obviously calculated from al-Battānī's values for the Byzantine epoch and not from the Hijra epoch, since the Hijra positions do not produce the same sexagesimal digits up to sixths.

Text F1, found in all six sources, presents the elementary relations between solar, lunar and planetary mean longitudes and anomalies that hold in the Ptolemaic planetary models. F2, found only in manuscript **F**, adds the relation between solar mean longitude and solar mean anomaly. The relations mentioned in F1 are needed in order to find the mean motions and positions that are not tabulated by al-Battānī from the ones given in his *zīj*.

Text G, found in somewhat different forms in **F** and in **YB**, concerns the meridian of reference of the mean motion tables in Kūshyār's *zīj* in relation to al-Battānī's.

## IV.5. Mean motion tables

**Bibliography:** van der Waerden, 'Vergleich der mittleren Bewegungen'; Pedersen, *A Survey of the Almagest*; Neugebauer, *HAMA*, Book I; Mielgo, 'A Method of Analysis'; van Dalen, 'Origin of the Mean Motion Tables'; Moesgaard, 'In Chase of an Origin'; Jones and Duke, 'Ptolemy's Planetary Mean Motions'; Bagheri, *az-Zīj al-Jāmiʿ*, Chapters I.4.1 and I.4.2, pp. 35–36 (translation), p. 44 (commentary) and Arabic pp. 22–23.

All the mean motion tables in the *Jāmiʿ Zīj* (except the table for apogee motion) have basically the same format. They consist of the following seven subtables:

1) collected years (*al-sinūn al-majmūʿa*): mean positions for the Yazdigird years 1, 21, 41, ..., 581;
2) extended years (*al-sinūn al-mabsūṭa*): mean motions in 1, 2, 3, ..., 20 Persian years of 365 days;[52]
3) 'single' years (*al-sinūn al-mufrada*): mean motions in 40, 60, 80, 100, 200, 300, ..., 500 years (in **YLB** extended to 600 years);[53]
4) months (*al-shuhūr*): mean motions for the beginnings of the months Farwardīn, Urdibihisht, Khurdādh, ..., Ābān, Ādhar, ..., Isfandārmudh (these subtables are for current months, i.e., the value for Farwardīn is always equal to zero; all manuscripts except the Cairo ones give variant values for both the old and the later version of the Persian calendar; cf. Section IV.2.1);
5) days (*al-ayyām*): mean motions for current days 1, 2, 3, ..., 30 (i.e., the value for 1 day is always equal to zero);

[52] In Latin sources these are called 'anni expansi' and are hence also translated as 'expanded years'. In many publications they are referred to as 'single years', not to be confused with *al-sinūn al-mufrada* below.

[53] According to Wehr's *Arabic-English Dictionary*, *mufrad* has the meanings 'single', 'singular', 'solitary', 'detached', 'isolated'. Nallino, *Al-Battānī sive Albatenii*, vol. II, p. 73 translates *fī al-sinīn al-rūmiyya al-mufrada* as 'in intervallis annorum Romanorum'. This type of subtable is relatively rare in Arabic and Persian handbooks and mainly appears in the *zīj*es of al-Battānī and Kūshyār and works directly influenced by them. In the *Ṣābiʾ Zīj* and the *Jāmiʿ Zīj*, as well as in the *Dustūr al-munajjimīn*, its only function would be to aid in calculating planetary positions outside the tabulated range of collected years, i.e., AD 619–1339 in al-Battānī, AD 632–1232 in Kūshyār and AD 1089–1309 in the *Dustūr*. In fact, *al-sinūn al-mufrada* are not even mentioned in Kūshyār's instructions in Section I.4.2. On the other hand, in the *Shāmil Zīj* the mean positions are given only for every year in the century from 600 to 699 Yazdigird, so the subtable for *al-sinūn al-mufrada* is essential for calculating any planetary positions before or after this period. In order to avoid confusion with the modern use of the term 'single years' for extended or expanded years (cf. the previous footnote), I will consistently render *al-sinūn al-mufrada* as "'single' years", with 'single' between quotation marks.

6) hours and their fractions (*al-sāʿāt wa-kusūruhā*): mean motions in 1, 2, 3, …, 60 hours or fractions thereof (the mean motions in minutes and seconds of time are obtained by shifting the sexagesimal point by one or two places);
7) 'between the longitudes' (*fī mā bayna al-aṭwāl*): adjustments needed to convert the mean positions obtained from the tables to different localities, for geographical longitudes 71, 72, 73, …, 100°. I will refer to these adjustments as 'longitude corrections'. All manuscripts include, in part of the mean motion tables, labels *zāʾid* 'additive' (for longitudes 71 to 89°) and *nāqiṣ* 'subtractive' (for longitudes 91 to 100°), indicating the geographical longitudes for which the corrections must be added to the mean positions obtained from Kūshyār's tables and the ones for which they must be subtracted. The corrections are always zero for the slowest mean motions, namely the mean longitudes of the lunar node, Saturn and Jupiter. However, in **F** some further subtables of this type are filled up with zeroes.

In all manuscripts, the subtable for 'single' years is placed under the extended years (note that each column in the tables has room for approximately 30 tabular values). The subtable for hours is always divided over two columns. In **FHYLB** the subtable for months is spread out vertically, with each month corresponding to two rows (three rows in **H**) in the adjacent tables for years. In this way the month names can be written diagonally in their cells and room is left to add, for the last four months, the variants for the old version of the Persian calendar in red under those for the later version (these are written in italics in the edition of the tables).[54] However, in the three Cairo manuscripts the mean motion tables were compressed onto a single page, with the longitude corrections placed in two small columns under the subtable for months and no room left for the variants in the old version of the Persian calendar.

All three Cairo manuscripts include lists of apogee motions written vertically in the margins of the tables for the mean longitudes of the Sun, Saturn, Jupiter, Mars and Mercury (for an example, see Plate 8; the apogee longitude of Venus is taken to be equal to that of the Sun). These tables, included in the general text edition of the mean motion tables as marginal note E (pp. 290–91), were copied by the main hand of each of the three manuscripts and are in full agreement with Kūshyār's apogee longitudes and motions given in Tables 12 and 14. They display the apogee longitudes for the Yazdigird years 301, 401, 501, 521, 541, …, 601, making it plausible that they date from the sixth Yazdigird century, i.e., the period from AD 1132 to 1232.

[54] In **H** the values for the old version of the Persian calendar are written in black, and the four deviating values for the later version were written *above* these in red.

Manuscript **C** shows traces of use in Yemen, in the form of longitude corrections to be applied to the mean longitudes. For the double elongation, the fastest changing mean motion, this is an additive correction (indicated as *ziyādat al-yaman*) of $0^s\,1;47,41°$, which corresponds to a longitude difference of $1;47,41 \cdot 360° / 24;22,53,23 \approx 26½°$ in the westward direction reckoned from Kūshyār's meridian, i.e., to a geographical longitude of 63½°. However, the longitude of Ṣanʿā is correctly given in Kūshyār's geographical table as 73;30° from the Fortunate Isles. It seems possible that the author of this note confused the base meridians and calculated with Yemen's longitude 63;30° from the western shore.

In the remainder of this section, I will first discuss the computation of the mean motion tables in manuscripts **YLB**, which are the most accurate ones. Then I will make an inventory of the systematic differences in certain subtables between manuscripts **FHCC$_1$C$_2$** on the one hand and manuscripts **YLB** on the other. I will take a special look at the corrections and additions that the scribe or collator of **B** made to its mean motion tables on the basis of a manuscript related to the group **FHCC$_1$C$_2$**, leading to a significant number of tabular values for which **B** does *not* follow **YL**. Using some clues that the previous topics have provided, I will then investigate the way in which the mean motion tables in the group **FHCC$_1$C$_2$** were computed. The section concludes with some comments on individual mean motion tables.

### IV.5.1. Recomputation of the tables in manuscripts YLB

In Chapter I.4.1 of the *Jāmiʿ Zīj*, Kūshyār states that he found al-Battānī's planetary tables to be the most accurate, and that he adjusted al-Battānī's mean positions for dates in the Arabic and Syrian calendars to the Persian calendar because it would make the required calculations easier.[55] In this chapter Kūshyār does not give the details of his conversion of al-Battānī's mean motion tables to his own, but the data given in Table 12 and the explanatory texts to this table found in some of the manuscripts leave no doubt about the procedure followed. As explained in the commentary to Table 12, Kūshyār took the mean motions in 20 Syrian years from the headings of al-Battānī's subtables for 'single' years and divided them by 7305 to obtain the daily mean motions to a precision of six sexagesimal fractional digits. He then multiplied these daily mean motions by 4491 and added them to al-Battānī's epoch values for the year 931 Alexander (1 March 620) in order to obtain the mean positions at the Yazdigird epoch for the meridian of Raqqa (73;15° from the Fortunate Isles), as given in Table 12. In order to convert these to his own base meridian of 90°, he multiplied the daily mean motions by $(90° - 73;15°) / 360°$ and sub-

[55] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 35–36 (translation), 44 (commentary) and Arabic pp. 22–23.

tracted the result from the epoch positions for Raqqa.[56] Finally, he subtracted or added the displacements of his planetary equations (cf. Section IV.7.1) in order to obtain the epoch values that he had to use in his mean motion tables so that the application of his displaced equations would lead to the actual true longitudes of the planets. Note that al-Battānī did not separately tabulate the double elongation and the mean anomalies of the superior planets, which Kūshyār therefore needed to calculate from the solar, lunar and planetary mean longitudes following the rules giving in explanatory note F1 to Table 12 (p. 285). The mean longitudes of the inferior planets are equal to the solar mean longitude and therefore only differ in their displacements.

The mean motion tables in manuscripts **YLB** were indeed accurately computed by this very procedure. The subtables for 'single' years, which, together with the collected years, cover the longest time span, are in full agreement with the daily mean motions listed in Table 12, with only occasional exceptions. Although in the subtables for collected years the epoch values are given to the same precision as the other values, it can be verified that more precise epoch values were used for the computation, namely the values that were calculated from al-Battānī's tables and are listed in Table 12. For each type of mean motion, Table E shows: the epoch value displayed in Kūshyār's table, the epoch value as calculated from al-Battānī's data, as well as the range of epoch values that provide the smallest possible number of errors in Kūshyār's table in combination with the daily mean motions listed in Table 12; if this number of errors is non-zero, it is indicated between parentheses. A single exclamation mark after a calculated epoch value in the third column indicates that it falls just outside the range of values in the fourth column that produce the smallest possible number of errors in Kūshyār's table. In all three cases the calculated epoch value is just one unit removed from this range and leads to only one error more than the smallest possible number of errors.[57] In only a single case, namely the lunar anomaly, indicated in the table by a double exclamation mark, does the calculated epoch value fall clearly outside the range of optimal epoch values. Here as well this causes just a single error, namely $9^s$24;57,47 for 561 Yazdigird, whereas Table 18 has $9^s$24;57; this difference cannot be explained

[56] Chapter I.24 in manuscript **L** (corresponding to Chapter I.4.3 in Bagheri, *az-Zīj al-Jāmiʿ*, pp. 36–37 (translation) and 44 (commentary) and Arabic pp. 23–24) states consistently that the mean motions found from the tables are for Raqqa, but the tabular values in **L** agree fully with **YB** and with the method of computation here outlined.

[57] In each case this is a minor rounding error: for the solar mean longitude for 81 Yazdigird, which is calculated as $2^s$5;56,29,29,47 from the epoch value and the daily mean motion found in Table 12, Table 13 gives $2^s$5;56,30 rather than the correctly rounded $2^s$5;56,29; for the mean longitude of Jupiter for 521 Yazdigird, calculated as $6^s$14;13,29,37, Table 25 gives $6^s$14;14, and for the mean anomaly of Mars for 301 Yazdigird, calculated as $9^s$19;58,29,44, Table 29a gives $9^s$19;59. For a formal definition of a minor rounding error, see the Quick reference guide on p. 530.

Table E: Recomputation/reconstruction of the epoch values in Kūshyār's mean motion tables in manuscripts **YLB**, displayed in the second column. The values in the third column were calculated from al-Battānī's data given in Table 12. The intervals in the fourth column are those for which the daily mean motions listed in, or directly derived from, Table 12 yield the smallest numbers of errors in the subtables for collected years (with the minimum number of errors given between parentheses if it is unequal to zero). ! indicates a minor deviation of the calculated value from the optimal interval, !! a larger one.

| mean motion | epoch value in table | calculated epoch value | range of optimal epoch values |
|---|---|---|---|
| solar longitude | $2^{s}$24;54,35 | $2^{s}$24;54,35,27 ! | $2^{s}$24;54,35,28 (#2) |
| lunar longitude | $11^{s}$25;33,42 | $11^{s}$25;33,41,41 | $11^{s}$25;33,41,30–42,15 (#1) |
| lunar anomaly | $9^{s}$22;28 | $9^{s}$22;27,59 !! | $9^{s}$22;27,42 |
| double elongation | $6^{s}$13;18 | $6^{s}$13;18,12 | $6^{s}$13;18,12–13 |
| lunar node | $2^{s}$ 5;42 | $2^{s}$ 5;41,49 | $2^{s}$ 5;41,48–49 |
| Saturn longitude | $7^{s}$13;42 | $7^{s}$13;41,37 | $7^{s}$13;41,37–39 |
| Saturn anomaly | $7^{s}$ 6;13 | $7^{s}$ 6;12,58 | $7^{s}$06;12,56–59 |
| Jupiter longitude | $8^{s}$15;25 | $8^{s}$15;25, 1 ! | $8^{s}$15;25, 2– 3 |
| Jupiter anomaly | $5^{s}$29;30 | $5^{s}$29;29,34 | $5^{s}$29;29,34–36 |
| Mars longitude | $8^{s}$12;34 | $8^{s}$12;33,51 | $8^{s}$12;33,51–52 |
| Mars anomaly | $4^{s}$27;21 | $4^{s}$27;20,44 ! | $4^{s}$27;20,45–46 |
| Venus longitude | $1^{s}$ 6;55 | $1^{s}$ 6;54,35 | $1^{s}$ 6;54,35 |
| Venus anomaly | $4^{s}$ 1;58 | $4^{s}$ 1;58,11 | $4^{s}$ 1;58, 9–11 |
| Mercury longitude | $1^{s}$26;55 | $1^{s}$26;54,35 | $1^{s}$26;54,35 (#1) |
| Mercury anomaly | $5^{s}$29; 3 | $5^{s}$29; 3,24 | $5^{s}$29; 3,24 |

as a minor rounding error or as a scribal error.[58] The solar mean longitude has two inevitable errors for arguments 61 and 101, and the lunar mean longitude a single inevitable one for argument 581, which could be explained as a scribal error. The long range of optimal epoch values for the lunar mean longitude is due to the fact that the motion in 20 years is very close to a whole number of seconds (namely, $2^{s}$7;42,38,0,30).

Nearly all the other subtables in all 15 mean motion tables in manuscripts **YLB** have at most occasional minor rounding errors. Only the subtables for hours in the tables for the solar and lunar mean longitudes have the type of systematic errors that is much more common in the other manuscripts, and will be discussed further in Section IV.5.4 below.

58 See the Quick reference guide on p. 530 for a formal definition of a minor rounding error and the previous footnote for several examples.

### IV.5.2. The differences between manuscripts **FHCC$_1$C$_2$** and **YLB**

As can be seen from the general variants in the apparatuses to the mean motion tables, many tables in the manuscripts **YLB** show systematic deviations from the corresponding ones in the manuscripts **FHCC$_1$C$_2$**. In the case of Mars, the tables are even so different that I had to decide to provide a separate edition of the subtables for years and months. This is discussed further in Section IV.8 together with the adjustments to the equations of Mars that we find in Kūshyār's tables. In all other mean motion tables the differences between the two groups are never larger than one minute (in the tables for the solar and lunar mean longitude: one second), but they generally have the same sign and are concentrated in particular subtables, especially often in the ones for collected years and hours. These differences obviously have no astronomical significance, since they will only influence the calculated planetary positions by amounts that are clearly smaller than the accuracy of naked-eye observations. However, since they may provide us with further information about the procedures by which Kūshyār computed and improved his tables, and thus about medieval Islamic mathematical practices in general, I will try to explain the differences here.

The most striking systematic differences in the mean motion tables (besides those for Mars) between **FHCC$_1$C$_2$** on the one hand and **YLB** on the other are found in the following subtables:[59]

- solar longitude: collected years (14 differences), hours (13 differences)
- lunar longitude: hours (6)
- lunar anomaly: collected years (14), extended years (3)
- double elongation: collected years (12), hours (26)
- lunar node: collected years (17)
- Saturn longitude: collected years (13)
- Saturn anomaly: collected years (13), extended years (5), months (3), hours (19), longitude differences (5)
- Jupiter longitude: collected years (13)
- Jupiter anomaly: collected years (18), extended years (5), hours (17)

[59] The numbers of differences are indicated between parentheses. Here also the differences in tabular values for which **B** follows **FHCC$_1$C$_2$**, often as visible corrections to the values from **YL** (cf. Section IV.5.3), are included in the count. Subtables not explicitly mentioned show at most two differences between the two groups of manuscripts. The tables of mean longitude of the interior planets Venus and Mercury do not show any differences between the two groups; this is clearly because they could be simply rounded from the more precise table of solar mean longitude (for the collected years after adjusting for the displacements of the corresponding equations).

- Venus longitude: —
- Venus anomaly: collected years (4), hours (19)
- Mercury longitude: —
- Mercury anomaly: hours (7).

We thus see that the differences occur most often in the subtables for collected years and hours, every now and then in the subtables for extended years, and only in occasional values in the remaining subtables. A closer look at the three types of subtables with the largest numbers of differences reveals the following details.

*Collected years*. In nine of the 15 mean motion tables we find significant, and nearly always systematic, differences between the subtable for collected years in manuscripts **FHCC$_1$C$_2$** and the one in manuscripts **YLB**. In five of these tables the epoch value is different, and in eight of the nine tables at least a third of the tabular values differ. Since, with the exception of Mars, the subtables for 'single' years are almost always identical in the two groups of manuscripts and, as we will see in Section IV.5.4, the epoch values of the subtables for collected years in manuscripts **FHCC$_1$C$_2$** can be chosen near the tabulated ones in such a way that the tables become error-free for the daily mean motions listed in Table 12, we conclude that the mean motion tables in **FHCC$_1$C$_2$** were also accurately computed, but for different epoch values.

*Extended years*: In the tables for the mean anomaly of Saturn and Jupiter, the subtable of extended years in **YLB** is error-free for the daily mean motions listed in Table 12. The subtables in **FHCC$_1$C$_2$** have at least two errors for any value of the daily mean motion and reach this minimum for values that cannot be derived from the parameters in Table 12 in any logical way. We may conclude that the tables in **FHCC$_1$C$_2$** were simply less accurately computed.

*Hours*: In nine of the 15 mean motion tables, the subtable for hours shows significant systematic differences between the two groups of manuscripts. This includes the two tables for Mars, whose difference in daily mean motion between the two groups of manuscripts should not affect the subtables for hours (and in fact does not influence the subtable for days and only minimally that for months). For six of these nine mean motions, the subtable for hours in **YLB** is error-free, which suggests that the less accurately computed subtables in **FHCC$_1$C$_2$** were replaced by more accurate ones in **YLB**. In the table for the solar mean longitude, only 13 of the 21 errors of $-1''$ in **FHCC$_1$C$_2$** were corrected in **YLB**; in the lunar mean longitude, one error was left uncorrected and three new ones were introduced.

## IV.5.3. The collation of **B** against a manuscript related to **FHC$C_1C_2$**

For the sake of convenience, in this section '**FHC**' refers to a group of manuscripts that includes not only **FHC$C_1C_2$** but also a set of mean motion tables, possibly found in a different manuscript of Kūshyār's *Jāmiʿ Zīj* or a preliminary version of it, which is not identical to the set of mean motion tables in **FHC$C_1C_2$** but is directly related to it. As established in the previous section, the Berlin manuscript mostly follows **Y** and **L** in their systematic deviations from the mean motion tables in manuscripts **FHC$C_1C_2$**, but also repeatedly follows the latter. A careful inspection of the colour scans made available by the Berlin Staatsbibliothek and the ISMI project (Islamic Scientific Manuscripts Initiative), as well as an inspection of the manuscript itself by Nadine Löhr, suggests that in many of these cases **B** did originally provide the same values as **YL**, but that a scribe later corrected them to values found in **FHC**. Similar occasional corrections of values in the planetary equations are particularly clear where they are carried out in a different colour from the original tabular values (usually black over red for corrections of tabular differences), but the corrections in the mean motion tables, although in the same colour, can also be recognised on the scans. Often the paper appears somewhat damaged or stained, and frequently the shape of the erased digits can still be seen under the corrected digits (several examples can be seen in Plate 9).

As explained in Section IV.5.1, the mean motion tables in **YLB** were computed in a highly accurate way on the basis of al-Battānī's parameters derived from Table 12, thus avoiding most of the large number of small computational errors (usually of only one unit) that appear in the mean motion tables in **FHC**. However, apparently the scribe or collator of **B** considered the values from a manuscript from the group **FHC** that was also available to him to be more trustworthy, and in many of the planetary tables he therefore corrected the values found in **YL** to the values from **FHC**. Since, especially in the subtables for collected years, there were too many differences to be able to correct every individual value in the table itself, the scribe wrote the minutes from **FHC** to the left of these subtables, with an indication that they were corrections from a different copy (for an example, see Plate 9). However, whenever the degrees were also different, these were corrected in the table itself, as can often be appreciated on the scans. This shows that the scribe or collator in fact intended the values from **FHC** to be used rather than the ones from **YL**. While in all cases the columns of corrected digits can be seen to be related to the mean motion tables in manuscripts **FHC$C_1C_2$**, the following overview of tables with corrections shows that only for the mean longitude of Jupiter are the corrected minutes in fact *identical* to those from **FHC$C_1C_2$**:

- Jupiter mean longitude (Table 25): the minutes found in **FHC$C_1C_2$** are written to the left of the column for collected years, headed by the letters *ṣḥ* for *ṣaḥīḥ* 'correct'. Since they are all either equal to, or one

less than, the minutes from **YL**, and since the minutes of none of the values from **YL** are equal to zero, there is no value for which also the degrees needed to be adjusted.

- Jupiter, mean anomaly (Table 26): minutes that are consistently three larger than those in **FHCC$_1$C$_2$** are written to the left of the column for collected years. Due to the 18 differences of 1′ between manuscripts **FHCC$_1$C$_2$** and **YLB**, these are sometimes 2 and sometimes 3 larger than the minutes written in the column itself. For the original value $7^s$ 11;58 for 341 Yazdigird, not only were the corrected minutes '0' written to the left of the column, the degrees were also corrected from '11' to '12'. In the subtable for extended years it can be clearly recognised that the manuscript originally had the values from **YL**, but that the minutes for 11, 12, 18 and 19 (but not for 13) years were then corrected to those from **FHC**.
- Mars mean longitude (Table 28a, see Plate 9): the minutes found in **FHC** (with two scribal errors) are written to the left of the column for collected years, headed 'copy of these minutes' (*nuskha hadhihi al-daqāʾiq*). Since, for the mean motions of Mars, **FHC** on the one hand and **YL** on the other use the same epoch value but different daily mean motions, the differences between the two groups increase by roughly three minutes in every 20 Persian years, reaching a maximum of 96 minutes for 581 Yazdigird.[60] **B** follows **YL** in the signs and minutes, but reduces all degrees by two. It can be recognised on the scans, and even more easily in the manuscript itself, that the original degrees that are also found in **YL** were later corrected to values that are two less. In the mean longitude $10^s$ 1;19 for 101 Yazdigird found in **YL**, not only were the degrees corrected from '1' to '29', but also the signs from '10' to '9'. Similarly, in the mean longitude $9^s$ 0;33 for 481 Yazdigird, not only were the degrees corrected from '0' to '28' but also the signs from '9' to '8'. On the basis of the scans it cannot be decided whether the degrees were first adjusted to those in **FHC** before they were once more corrected to values two less than those in **YL**. The minutes from **FHC** are also written next to the subtable of 'single' years, which covers a similar period as the subtable for collected years, but here only the degrees of the value for 80 Persian years were corrected from '2' to '3' (possibly by a later hand), but not those for 100 up to 500 years. No

[60] Interestingly, **L** gives the value for 581 Yazdigird found in **FHC**, which is incompatible with the rest of the subtable for collected years in this manuscript. As we have seen (cf. footnote 71 on pp. 58–59 and p. 360), this is only one of several indications that the copyist of **L** also had a manuscript close to **F** at hand.

minutes from **FHC** are indicated for the extended years, although these differ from **YL** for all 20 values except one.

- Mars mean anomaly (Table 29a): the minutes found in **FHC**, with several scribal errors including a slide of five values towards the end, are written to the left of the subtable for collected years, headed by *nuskha* ('copy' or 'manuscript'). Furthermore, similarly to the mean longitude, all degrees were visibly corrected to values that are two larger than the ones in **YL**, and in those cases where they were 28 or 29 (namely for 321 and 381 Yazdigird), also the corresponding zodiacal signs were recognisably modified. These changes are more sloppy and even more visible on the scans than those for the mean longitude of Mars and may have been made by a different hand. Also here it cannot be recognised on the scans whether the degrees were first corrected to the values from **FHC**, and only then to values two larger than the degrees in **YL**. Although no corrective minutes are written next to any of the other subtables, corrections can be clearly recognised in the minutes and the last four degrees of the subtable for 'single' years, as well as in the minutes of the last four values of the subtable for extended years. However, in these cases the corrected values are those from **YL** and not from **FHC**. For some of the corrections (esp. '22°' for 300 'single' years and '10°' for 400 'single' years) it can be seen on the scan that the original values were those from **FHC**. The same may be the case for several corrections in the subtable for months. This would indicate that **B** was collated on the basis of manuscripts from both groups right from the beginning of the copying process and that the corrections on the basis of **FHC** are not necessarily later additions.
- Venus mean longitude (Table 31): **B** writes corrective minutes next to the subtable of collected years. In this case the corrected digits are mostly 3 more, and for seven tabular values 2 more, than the minutes in the main column stemming from **YL**, which deviate only once from the values in **FHC**. For 161 Yazdigird the degrees of the original value $11^s28;58$ from **YL** were recognisibly corrected from '28' to '29', in agreement with the correction of the minutes from '58' to '1'. Note that Kūshyār's mean longitudes of Venus and the Sun, when the latter are rounded to minutes, differ by exactly 48°, i.e., the difference between the displacements of the equation of centre of Venus and the solar equation (respectively 50° and 2°).
- Venus mean anomaly (Table 32): The corrective minutes that **B** provides next to the subtable for collected years are exactly two larger than the minutes in **FHC$C_1C_2$**. The only necessary correction of the degrees in the main column can be clearly recognised for 1 Yazdigird ($4^s1;58$ corrected to $4^s2;0$, where the '1' appears to have been scratched out).

- Mercury mean longitude (Table 34): exactly the same corrective minutes as for the mean motion of Venus are indicated next to the subtable for collected years, and for the original value $0^s$ 18;58 for 161 Yazdigird (181 Yazdigird in the manuscript due to a slide of the arguments) the correction of the degrees from '18' to '19' can be recognised on the scans. Kūshyār's mean longitudes of Venus and Mercury differ by exactly 20°, namely the difference between the displacements of the equation of centre of the two planets (respectively 50° and 30°).
- Mercury mean anomaly (Table 35): in this table **B** indicates corrections that are exactly nine minutes larger than those in **FHC**. The necessary corrections in the degrees were carried out for arguments 21, 41, 301, and 321 Yazdigird, with some traces of the original numbers visible on the scans, but not for 581 Yazdigird.

All corrections of mean positions discussed above differ by a (practically) constant amount from the mean positions in **FHC$_1$C$_2$**. It is thus clear that they do not involve a change of the daily mean motion, but only a small adjustment of the epoch value. We may also note that the size of the corrections is more or less proportional to the respective daily mean motions. For example, the largest correction of +9 minutes in the mean anomaly of Mercury corresponds to the mean motion in approximately $1^h10^m$, or to a longitude difference of approximately 17° in a westward direction. In other words, it is plausible that the corrective mean motion values are intended for the meridian of Raqqa rather than for Kūshyār's meridian of 90°. This is quite easy to confirm by comparing the corrective epoch values with those derived from al-Battānī's tables in Table 12, taking into account the displacements that have been applied in Kūshyār's tables and calculating the mean anomalies of the superior planets as the difference between the solar mean longitude and the mean longitudes of these planets. In each case (except for the mean longitude of Mars, for which the longitude correction of 1 minute appears to have been ignored), we find full agreement. It thus seems clear that the corrections made by the scribe/collator of **B** are adjustments of the mean positions in **FHC$_1$C$_2$** for the meridian of Raqqa. Or, alternatively, they may stem from intermediate mean motion tables, possibly drawn up by Kūshyār himself, in which the displacements of the *Jāmiʿ Zīj* had already been applied to the epoch values, but not yet the adjustment to the meridian of 90°.[61] The fact that the corrections to the mean longitudes of Venus and Mercury are 3 minutes for 23 of the 30 values and 2 minutes for the remaining seven values is then because these tables were simply rounded

[61] Support for this possibility may be found in Chapter I.24 in manuscript **L** (cf. footnote 56), which consistently states that the mean motion tables are for the longitude of Raqqa and which hence might stem from such an intermediate version of the *Jāmiʿ Zīj*.

from a table to seconds for the solar mean longitude for Raqqa, for which the longitude correction amounts to 0;2,45°.

A clue for the above possibility is found in manuscript **B** itself. As explained in Section I.6.3 (pp. 51–52), among the additional tables in **B** we find on pp. 207–210 a set of mean motion subtables which, in combination with the remaining subtables belonging to the *Jāmiʿ Zīj* itself, produce Kūshyār's displaced mean motions for the longitude of Raqqa. This set includes the subtables for collected years and longitude corrections for the Sun, the Moon and the mean longitude of Saturn, but then switches to a different format that presents the longitude corrections for the mean anomalies of Saturn, Jupiter, Venus and Mercury and for the mean longitudes of Jupiter and Venus in parallel columns, whereas Mars is entirely omitted. The epoch values given in these tables are in full agreement with the epoch values in Table 12 with which they can be compared directly, whereas the double elongation differs by only a single minute from a value calculated from the solar and lunar mean longitudes. Interestingly, the subtables for collected years not found among these additional tables are, with the exception of the mean anomaly of Saturn, exactly those that were corrected in the mean motion tables in the main work. It thus seems possible that the scribe/collator of **B** had at hand a complete set of subtables for collected years and longitude corrections of Kūshyār's displaced mean motion tables for the Yazdigird calendar, but for the meridian of Raqqa instead of 90°. On the basis of these subtables he corrected the mean positions taken from **YL** for Jupiter, Mars, Venus and Mercury, whereas he copied the remaining subtables as additional tables after the main work. The reason for the corrections remains unclear. Since the longitude corrections for the Moon are much larger than for the five planets, it seems strange that the scribe did not correct the solar and lunar mean motion tables if his intention had been to make the whole set of tables suitable for use in Raqqa.

In the mean motion tables for Mars, presumably a later user made a second round of corrections, namely reducing all the mean longitudes in the subtable for collected years by exactly 2° and increasing all the mean anomalies by the same amount. It can be clearly seen that these corrections were copied from, or based on, a manuscript from the group **YLB**, because whenever the corrections on the basis of **FHC** and the values from **YL** differ in their degrees, the corrected degrees in **B** differ by two from those in **YL** and by three from those in **FHC**. Wherever necessary, the user also corrected the zodiacal signs in the subtable itself. I have found no explicit indications of the reason for these corrections by 2°, but will hypothesise in Section IV.8.3 that they may be linked to Kūshyār's note on his observation of the Saturn-Mars conjunction of AD 993.

Occasional corrections in **B** of values that differ from the group **FHC** can also be observed in several other tables. Some of these instances suggest that

**B** was originally copied from a manuscript close to **L**, since they correct errors only found in **L** to the values included in **FHC**.

## IV.5.4. Recomputation of the tables in manuscripts **FHCC$_1$C$_2$**

With the exception of Mars, the mean motion tables in manuscripts **FHCC$_1$C$_2$** can be seen to be based on the same values for the daily mean motion as manuscripts **YLB**. This is already clear from the fact that the subtables for 'single' years, which cover 600 Persian years, are basically identical in the two groups of sources. It can also be confirmed by applying the Least Number of Errors criterion (LNE) to the subtables for collected years, which cover a period of 580 Persian years.[62] When using the round epoch values shown in Kūshyār's tables, only the subtables for the solar and lunar mean longitudes and the mean anomaly of Venus are error-free for the daily mean motion parameters derived from al-Battānī in Table 12, whereas the lunar anomaly becomes error-free for a slightly different, unattested parameter.[63] For all other eleven mean motion tables, the minimum possible number of errors in the subtables for collected years lies between one and five, reached for daily mean motions different from the ones given in Table 12. On the other hand, for every single mean motion table it is possible to find a range of more precise epoch values for which the subtable for collected years becomes error-free for the daily mean motions derived from al-Battānī. In each case, the epoch values shown in the tables are correctly rounded from these 'optimal' epoch values. In other words, the up to 18 differences between the subtables for collected years in the two groups of manuscripts can be fully explained by the use of epoch values that are more precise than the ones shown in the tables, and are generally different between the two groups.[64]

There are several plausible ways by which Kūshyār could have obtained the epoch values given in manuscripts **FHCC$_1$C$_2$** as opposed to the ones in **YLB**, which were calculated exactly from al-Battānī's data given in Table 12 (see Sec-

[62] The Least Number of Errors criterion was introduced in van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 60–62. A related method tailored to mean motion tables was introduced in Mielgo, 'A Method of Analysis'. A full application of the LNE to a historical set of mean motion tables can be found in van Dalen, 'Origin of the Mean Motion Tables'.

[63] The subtable for collected years for the lunar mean anomaly can be reproduced exactly for the given epoch value and a daily mean motion in the range 13;3,53,56,18,5–37°, which, however, does not contain Ptolemy's daily mean motion 13;3,53,56,17,51,59° given by Kūshyār in Table 12 (for which the number of errors in the subtable is as large as 15 if the epoch value is not adjusted!).

[64] The subtables for collected years of the mean motion tables for Mars in **FHCC$_1$C$_2$** can also be recomputed without a single error, but in this case we cannot verify the correctness of the daily mean motions on the basis of the *Jāmiʿ Zīj* itself, since Table 12 lists only the daily mean motion in longitude used in **YLB**.

tion IV.5.1). In particular, he could have rounded the intermediate results of his calculation of the epoch values for the Yazdigird epoch and his meridian of 90° from those of al-Battānī. The possibilities for rounding are then in the mean positions at the Yazdigird epoch for Raqqa (given in the left bottom quarter of the edition of Table 12) and in the correction for the difference in longitude between Raqqa and 90°. Secondly, Kūshyār could have obtained his epoch values directly from the mean motion tables in al-Battānī's *Ṣābiʾ Zīj*, either from the tables for the Arabic calendar or from those for the Byzantine calendar.

In Table F the second column displays the epoch values of Kūshyār's mean motion tables as they can be calculated from those presented in Table 12 by accurately applying the longitude correction between Raqqa and Kūshyār's meridian of 90° and then incorporating the displacements of the planetary equations in the *Jāmiʿ Zīj*. As explained in Section IV.5.1, the epoch values in manuscripts **YLB** were systematically rounded from these values. The third column of Table F presents the epoch values found in manuscripts **FHCC$_1$C$_2$**, with an asterisk indicating the values that are different from **YLB**. The fourth column gives the range of epoch values for which the subtables for collected years in **FHCC$_1$C$_2$** become error-free for the daily mean motion parameters listed in Table 12 (or, in the case of Mars, for otherwise unattested daily mean motions different from those in Table 12, to be discussed further below and in Section IV.8.3). I will now investigate the method by which epoch values within the optimal ranges could have been derived.

We can note that the differences between the optimal epoch values in the fourth column and the exactly calculated ones in the second column are generally of the order of half a unit (i.e., up to approximately 30‴ for the solar and lunar mean longitudes and up to 30″ for all other tables). This makes it plausible that the differences resulted from rounding of intermediate results of the calculation of the epoch values to the precision of the tables. Table G displays the errors that occur when the quantities that appear in these calculations are rounded to the precision of the tables at different stages. The second column displays the errors caused by rounding the accurately calculated values from the second column of Table F (i.e., the rounding errors of the epoch values in **YLB**), the third column the errors caused by rounding the epoch values derived directly from al-Battānī in Table 12, and the fourth column the errors caused by rounding accurately calculated values for the longitude corrections before subtracting them from the mean positions for Raqqa.[65] Thus for

[65] Since the longitude corrections need to be subtracted from the epoch values for Raqqa, the sign of the induced error in the final result is the opposite of the sign of the rounding error in the longitude correction. For example, the exact longitude correction for the mean anomaly of Mercury is 16;45 · 3;6,24,7,44,53,13 / 360 ≈ 0;8,40°, which is rounded to 0;9. This implies a rounding error of −0;0,20°, which results in an error in the resulting epoch value of +0;0,20°.

Table F: Recomputation/reconstruction of the epoch values in Kūshyār's mean motion tables in manuscripts **FHCC$_1$C$_2$**. The second column gives the epoch values accurately calculated from those derived in Table 12, which were used in **YLB**. The third column displays the epoch values shown in **FHCC$_1$C$_2$**, with an asterisk indicating the values that differ from **YLB** and the number between parentheses giving the smallest possible number of errors in the subtables for collected years if exactly these epoch values are used for the recomputation. The fourth column shows the range of epoch values for which the daily mean motions derived in Table 12 lead to an error-free subtable for collected years, with an asterisk indicating that the range includes the rounded value given in the tables.

| mean motion | calculated epoch value | epoch value in table (**FHCC$_1$C$_2$**) | | range of optimal epoch values |
|---|---|---|---|---|
| solar longitude | $2^s$24;54,35,27 | $2^s$24;54,36 * | | $2^s$24;54,36,0–10 * |
| lunar longitude | $11^s$25;33,41,41 | $11^s$25;33,42 | | $11^s$25;33,41,30–42,15 * |
| lunar anomaly | $9^s$22;27,59 | $9^s$22;27 * | | $9^s$22;27,13–30 |
| double elongation | $6^s$13;18,12 | $6^s$13;19 * | (#4) | $6^s$13;18,34–35 |
| lunar node | $2^s$ 5;41,49 | $2^s$ 5;42 | (#4) | $2^s$ 5;42,20–22 |
| Saturn longitude | $7^s$13;41,37 | $7^s$13;42 | (#1) | $7^s$13;42, 4– 5 |
| Saturn anomaly | $7^s$ 6;12,58 | $7^s$ 6;13 | (#5) | $7^s$ 6;13,26–29 |
| Jupiter longitude | $8^s$15;25, 1 | $8^s$15;25 | (#4) | $8^s$15;24,36–37 |
| Jupiter anomaly | $5^s$29;29,34 | $5^s$29;29 * | (#1) | $5^s$29;28,57–58 |
| Mars longitude | $8^s$12;33,51 | $8^s$12;34 | (#1) | $8^s$12;33,52–55[a] |
| Mars anomaly | $4^s$27;20,44 | $4^s$27;22 * | (#3) | $4^s$27;21,38–47[b] |
| Venus longitude | $1^s$ 6;54,35 | $1^s$ 6;55 | (#3) | $1^s$ 6;54,35–37 |
| Venus anomaly | $4^s$ 1;58,11 | $4^s$ 1;58 | | $4^s$ 1;58, 0– 2 * |
| Mercury longitude | $1^s$26;54,35 | $1^s$26;55 | (#3) | $1^s$26;54,35–37 |
| Mercury anomaly | $5^s$29; 3,24 | $5^s$29; 3 | (#3) | $5^s$29; 3,18–21 |

[a] Since we do not know the exact daily motions for the Mars tables, I here give the entire range of epoch values for which the number of errors in the subtable for collected years may become zero. The corresponding intervals of daily mean motions for the four optimal epoch values to seconds are, respectively, [0;31,26,41,53,] 52,40 ±53, 50,27 ±1,58, 48,14 ±3,2 and 46,2 ±4,7, i.e., they cover the range 0;31,26,41,53,42–53.

[b] The corresponding intervals of daily mean motions for these optimal epoch values range from [0;27,41,38,] 52,56,25 ±1,56 to 53,12,8 ±27, i.e., they cover the range 0;27,41,38,52,54–53,12. It follows that the sum of the daily mean motion in longitude as found in note a and the daily mean motion in anomaly found here lies in the range 0;59,8,20,46,36–47,5, which includes Kūshyār's daily mean motion in solar longitude 0;59,8,20,46,56,14.

each type of mean motion for which the epoch positions in the two groups are equal, the entries in the third and fourth columns add up to the entry in the second column; in the other cases, marked by an asterisk in the second column, they differ by an integer number of minutes (seconds for the Sun or Moon). The fifth column, calculated as the difference of columns 4 and 2 in Table F, shows the range of errors in the epoch values for which the subtables for collected years in manuscripts **FHCC$_1$C$_2$** become error-free for the daily mean motions derived in Table 12.

Table G: Errors caused by the rounding of intermediate results of the calculation of the epoch values in manuscripts **FHCC₁C₂**. The second column shows the rounding error in the final result of an accurate calculation, the third column that in the accurate epoch value for Raqqa and the fourth column that in the longitude correction. The fifth column displays the errors contained in the epoch values used for the calculation of the subtables for collected years in **FHCC₁C₂** (i.e., those from the intervals in the fourth column of Table F) with respect to the accurately calculated ones in the second column of that table. Asterisks indicate mean motions for which the epoch values in **FHCC₁C₂** differ by one unit from those in **YLB**.

| mean motion | error in epoch value for 90° | error in epoch value for Raqqa | error in longitude correction | difference in epoch value |
|---|---|---|---|---|
| solar longitude | −0;0,0,27 * | +0;0,0,27 | +0;0,0,6 | +0;0,0,33–43 |
| lunar longitude | +0;0,0,19 | +0;0,0,16 | +0;0,0,2 | −0;0,0,11–+34 |
| lunar anomaly | +0;0, 1 * | −0;0,27 | −0;0,32 | −0;0,29–46 |
| double elongation | −0;0,12 * | −0;0,16 | +0;0, 4 | +0;0,22–23 |
| lunar node | +0;0,11 | +0;0, 3 | +0;0, 9 | +0;0,31–33 |
| Saturn longitude | +0;0,23 | +0;0,17 | +0;0, 6 | +0;0,27–28 |
| Saturn anomaly | +0;0, 2 | +0;0,23 | −0;0,20 | +0;0,28–31 |
| Jupiter longitude | −0;0, 1 | −0;0,15 | +0;0,14 | −0;0,25–24 |
| Jupiter anomaly | +0;0,26 * | −0;0, 5 | −0;0,29 | −0;0,37–36 |
| Mars longitude | +0;0, 9 | −0;0,19 | +0;0,28 | +0;0,1–4 |
| Mars anomaly | +0;0,16 | −0;0, 2 | +0;0,17 | +0;0,54–0;1,3 |
| Venus longitude | +0;0,25 | −0;0,21 | +0;0,0,6 | +0;0,0–2 |
| Venus anomaly | −0;0,11 | +0;0, 6 | −0;0,17 | −0;0,11–9 |
| Mercury longitude | +0;0,25 | −0;0,21 | +0;0,0,6 | +0;0,0–2 |
| Mercury anomaly | −0;0,24 | −0;0, 4 | −0;0,20 | −0;0,6–3 |

Under the assumption that the deviations in the epoch values in **FHCC₁C₂** from accurately calculated ones stem from intermediate rounding of the exact epoch values for Raqqa or of the longitude corrections, we expect either the errors in the third column or the errors in the fourth column of Table G to fall within the range of 'optimal errors' in the fifth column, since rounding of both intermediate results leads to epoch values that only have the precision shown in the tables, whereas rounding after the exact calculation of epoch values for Kūshyār's meridian yields the epoch values found in **YLB**.

As can be seen from Table F, for the solar mean longitude the optimal range of epoch values includes the epoch value shown in the tables in **FHCC₁C₂**. This implies that the epoch values could be obtained by rounding both the epoch position at Raqqa and the longitude correction to seconds. We thus find $2^s$26;57,21 − 0;2,45 = $2^s$26;54,36 (before the subtraction of the displacement), as in **FHCC₁C₂**, whereas in **YLB** the more accurate value from the second column of Table F was rounded to $2^s$24;54,35 (after subtraction of the displacement).

Only for the mean longitudes of Venus and Mercury are the differences between the mean positions for Kūshyār's meridian of 90° and the positions

for Raqqa provided in the additional tables and as corrections to the mean motion tables of Jupiter, Mars, Venus and Mercury in manuscript **B** (cf. Section IV.5.3) not constant. Instead we find 23 differences of 3′ and seven differences of 2′. This is in excellent agreement with the longitude correction of 0;2,45,6° for the solar mean longitude, and we can easily confirm that the subtables for Venus and Mercury in **FHCC$_1$C$_2$** were simply rounded from the more precise one for the Sun while adjusting for the different displacements.

For the lunar mean longitude the range of optimal values contains the rounded value given in the table and is furthermore so wide that it can not be decided whether any particular intermediate rounding must have taken place: in this case all roads lead to Rome. For the mean anomaly of Venus the epoch value to minutes given in Table F is contained in the range of optimal values, but in this case both rounding of the epoch value for Raqqa and rounding of the longitude correction lead to an epoch value for 90° outside of the optimal range. We must thus conclude that either both intermediate results or only the final result were rounded to minutes.

For the mean anomaly of Mercury the subtable for collected years can be obtained by subtracting an accurate longitude correction (i.e., 0;8,40 instead of 0;9) from all values of the subtable for Raqqa to which the collected years in **B** were corrected. In other words, the epoch value for Raqqa was rounded to minutes, but not the longitude correction.

All values in the subtable for collected years of the lunar mean anomaly differ by exactly 37′ from the table for Raqqa included among the additional tables in manuscript **B**. This value appears to be incorrectly rounded from 16;45 · 13;3,53,56,17,51,59 / 360 ≈ 0;36,28,23°. The subtable in **FHCC$_1$C$_2$** can be fully reconstructed by assuming that this incorrect correction was applied to accurate mean positions for Raqqa.

The values of the double elongation in manuscripts **FHCC$_1$C$_2$** differ by exactly 68 minutes from the values for Raqqa found among the additional tables in manuscript **B**, whereas the exact longitude correction is 1;8,3,54°. Both tables have values with even as well as odd numbers of minutes, making certain that the rounding to minutes took place after the doubling of the elongations.

The epoch values of the remaining mean motion tables cannot be explained in a similar way. I have made some further attempts at reproducing these, but without success. In particular, I have verified that rounding of the precise daily mean motions given in Table 12 to sexagesimal fourths (solar and lunar mean longitude) or thirds (all other mean motions) leads to errors in the epoch positions for Raqqa that are of the correct order of magnitude but in no case can be related to the errors in the epoch positions used by Kūshyār for the computation of the mean motion tables in **FHCC$_1$C$_2$**.

Another plausible method for calculating the epoch values, possibly applied by Kūshyār before he collected the very precise data in Table 12, finds them directly from al-Battānī's mean motion tables, either those for the Arabic calendar or those for the Syrian calendar. However, since al-Battānī's tables for the mean motions of the five planets all have values to minutes, the only possible cause of resulting epoch values with numbers of seconds different from zero lies in the longitude corrections. But even if these were calculated from the tabular value for a single day, the result would be so close to the exact correction that the errors listed in the fourth column of Table G remain valid. For example, for the solar mean longitude the exact correction is 0;2,45,5,48°, whereas the value 0;59,8° for one day leads to a correction of 0;2,45,4,50°.[66] All other subtables contribute at most rounding errors to the precision of the table, whose sum will always be equal to the rounding error of an exact calculation plus or minus an integer number of units. In other words, also the direct use of al-Battānī's tables cannot explain the epoch values that Kūshyār used for his mean motion tables in **FHCC$_1$C$_2$**.

## IV.5.5. Comments on individual mean motion tables

### Table 13: Solar mean longitude

As we have seen above, the epoch value in **YLB** was calculated exactly from al-Battānī's data given in Table 12, whereas for **FHCC$_1$** both the underlying epoch value for Raqqa and the longitude correction were rounded to seconds before the latter was subtracted from the former, leading to an epoch value differing by 1 second from that in **YLB**. Since both tables were computed on the basis of the precise daily mean motion from Table 12, the subtables for collected years differ between the two groups of manuscripts in fourteen values.[67]

The subtable for extended years in **FHCC$_1$** contains only a single error (3″ instead of 4″ for 13 years); this error was corrected in manuscripts **YLB**, but in **B** afterwards corrected back to the value from **FHC**. The same holds for the

[66] The longitude correction could also be calculated from the motion in a single hour from al-Battānī's *zīj* by multiplying it by 16;45 / 15 = 1;7. This would lead to larger errors than the use of the motion in a day. For example, the solar mean motion in one hour, 0;2,28°, leads to a longitude correction of 0;2,45,16. However, I have verified that also the errors in these estimates cannot explain the precise epoch positions that were used for the computation of the subtables for collected years in manuscripts **FHCC$_1$C$_2$**.

[67] The situation is somewhat confused by a number of small differences among the manuscripts **YLB**. Apparently Kūshyār did not modify the mean positions for years 61–101 when he recomputed the table of collected years; these values are not correct if the precise epoch value $2^s$ 24;54,35,27° is used, although for year 81 the error is extremely small. **B** can be seen to have corrected the original seconds of the value for 81 Yazdigird from '2' to '1'. **Y** keeps the value from **FHCC$_1$** for the year 421, which was correctly adjusted to the new epoch value in **LB**.

single error in the subtable for months (33″ instead of 34″ for Day in the late version of the Persian calendar).[68]

The 21 errors in the subtable for hours in manuscripts **FHCC$_1$** can be easily explained from the use of a rounded daily mean motion of 0;59,8°. In **YLB** only 13 of these 21 errors were corrected, leaving eight anomalous values that can only be explained from the use of the less precise daily mean motion. This characteristic is further only found in the table for the lunar mean longitude; the subtables for hours in all other mean motion tables in **YLB** are error-free with the exceptions of occasional minor rounding errors and the obvious error for 30 hours in the tables for the mean longitudes of Venus and Mercury.

**B** contains an interesting note in Persian (marginal note H, p. 292) that relates to the *Shāhī Zīj*, most likely the frequently cited but non-extant work by the thirteenth-century astronomer Ḥusām al-Dīn al-Sālār. It mentions the base longitude of the *Shāhī Zīj* as 92;30°, which is apparently for Nishapur.[69] The note then continues to calculate the time difference between Nishapur and a locality with longitude 85;37°. This longitude appears six times in Kennedy and Kennedy, *Geographical Coordinates*, of which four occurrences are for Alamut, the stronghold of the Assassins until their defeat by Mongol troops in 1256. The text appears to mix up the two localities, either misplacing the 'which is 85;37°' (as I have assumed in my translation) or mistakenly repeating 85;37° instead of 92;30°.

Another interesting marginal note in **B** (edited as note G, p. 292) likewise deals with a correction of the solar mean longitude as a result of a difference in geographical longitude. In this case 40 seconds must be added to the values for collected years in order to obtain the positions from the *Fākh⟨ir⟩ Zīj*. This was the work of Abū l-Ḥasan ʿAlī ibn Aḥmad al-Nasawī, a student of Kūshyār's, who also wrote a commentary on the *Jāmiʿ Zīj* around the year AD 1050.[70] Unfortunately, the *Fākhir Zīj* is lost, although some of its tables survive in manuscripts of other *zīj*es, for instance among the additional tables in the Leiden manuscript of the *Jāmiʿ Zīj* (see p. 47). The addition of 40 seconds mentioned in note G corresponds to a difference in geographical longitude of approximately 4° westwards, which implies that the *Fākhir Zīj* would be for a base locality with a longitude near 86°. This is used for Isfahan and Kashan,

[68] The correction of 4″ into 3″ in the subtable for extended years can quite clearly be recognised on the scans of manuscript **B**, but the correction in the subtable for months less so. See Section IV.5.3 for an extensive discussion of this type of corrections in **B**.

[69] Note that the *Nāṣirī Zīj* by Nāṣir ibn Ḥaydar ibn Muḥammad al-Shīrāzī, as contained in Rampur, Raza Library, MS 1208, gives the mean motions according to the *Malikshāhī Zīj*, but obviously referring to the *Shāhī Zīj* by Ḥusām al-Dīn al-Sālār. These mean motions are stated to be for a longitude of 92;30,45°.

[70] See the short description of the *Fākhir Zīj* on pp. 524–25.

among other sites,[71] while it is also relatively close to the longitude 85;37° mentioned in note H.

In all three Cairo manuscripts, the tables for the mean longitudes of the Sun, Saturn, Jupiter, Mars and Mercury come with a small table of apogee longitudes and motions, in each case copied sideways in the margin by the main hand (see Plate 8). These tables (edited as marginal note E in the general edition of the mean motion tables on pp. 290–91) display longitudes for the years 301, 401, 501, 521, 541, ..., 601 Yazdigird and motions in 1, 2, 3, ..., 20 years, all in full agreement with Kūshyār's Table 14.

Table 17: Lunar mean longitude

The tables for the lunar mean longitude in the two groups of manuscripts are practically identical, with the exception of the subtable for hours. This implies that the occasional minor rounding errors in the subtable for collected years in **FHCC$_1$** were not corrected in **YLB**, and that a scribal error for 581 Yazdigird was introduced in **YLB**. In the subtable for hours, four of the five errors in **FHCC$_1$** were corrected (note especially the correction of 28;0,0 to 27;59,59 for 51 hours, which makes it clear that the differences are not of a scribal nature), but two new errors were introduced. For a discussion of the subtable for the equation of time expressed in lunar mean longitude, see the commentary on Table 15b in Section IV.6.

For descriptions of, and explanations for, the errors in all other mean motion tables and the differences between the two main groups of manuscripts, see Sections IV.5.1–4. For the mean motions of Mars, see Section IV.8.3.

[71] cf. Kennedy and Kennedy, *Geographical Coordinates*, p. 640.

## IV.6. Solar and lunar equations and related tables

### Table 14: Apogee motion

The table for apogee motion, which is also used for finding the precession of the equinoxes,[72] has a setup somewhat different from the mean motion tables. Although the text in Book I makes it clear that it is intended to be used with the Yazdigird calendar, this is not obvious from the table: the type of years is not explicitly mentioned, and the months are numbered from 1 to 12 rather than indicated by their names. In source **C** the Persian month names were partially added next to the numerical arguments, apparently in a different hand; in **B** a Persian marginal note in a different hand states that the months are reckoned from Farwardīn māh. Since the difference between a Persian and a Julian year is so small, the table might just as well be used with the Syrian calendar (the difference in apogee motion would amount to 4 seconds of arc in a century; compare this with the difference of 3 minutes in a century if the table were to be used with the Arabic calendar).

As confirmed by the explanatory text in Book I,[73] the apogee motion is taken to be one full rotation in 24,000 Persian years, i.e., 1;30° in a century and 54″ per year. Hence, as indicated in explanatory note A (p. 293), which is found in **CC$_1$LB**, the motion of the apogee expressed in minutes of arc can be found from the number of Yazdigird years that have passed by subtracting a tenth of it. For example, in 350 Yazdigird years the solar and planetary apogees will have moved by (350 – 35)′, i.e., 5;15°. This rate, corresponding to a motion of 1½° in 100 years, is also confirmed by an addition to the title of the table in manuscript **C$_1$** (explanatory note B, p. 293).

As can be easily verified, the subtables for single years, tens and hundreds of years are indeed based on a motion of 54″ per year or 1;30° per century and do not contain a single error. However, the subtables for months and days were not accurately computed for this parameter. The table for months may have been calculated by dividing the annual motion of 54″ in twelve equal parts and rounding in the standard way. The subtable for days has different ranges of arguments in manuscripts **FC$_1$** (2, 4, 6, ..., 32 days) on the one hand and manuscripts **HYLB** (1, 2, 4, 6, ..., 30 days, although **H** does not give a value for 30 days) on the other (in **C** the arguments are entirely distorted and can-

72 Most Islamic astronomers followed Ptolemy by taking the motion of the solar and planetary apogees equal to precessional motion (see Mozaffari, ‘Holding or Breaking’). Kūshyār describes this in the first lines of the chapter on the fixed stars in the *Jāmiʿ Zīj* in a straightforward way as *wa-taʿdīluhā taʿdīl al-awjāt* (‘and their equation [i.e., of the fixed stars] is ⟨the same as⟩ the equation of the apogees’); see Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.8.9, pp. 107 (translation) and 113 (commentary) and Arabic p. 70.

73 Bagheri, *az-Zīj al-Jāmiʿ*, Section I.4.4 on pp. 37 (translation), 44–45 (commentary) and Arabic pp. 24–25.

not be recognized to conform to either of these ranges). Furthermore, the two Cairo manuscripts that include this table appear to have omitted the first value, effectively resulting in an upward slide of one row. It can be seen that the table is calculated for completed days rather than for current ones (like in the other mean motion tables), since in the latter case one expects consecutive series of 3 ones, 3 twos, 4 threes, and 3 fours for the even arguments (rather than 4, 3, 3, 4 for completed days), which leads to two or three errors in every possible recomputation. Under the assumption that the arguments are completed days, the table in **F** has four errors, but the table with adjusted arguments in **HYLB** and the table with slid values in **CC$_1$** only one (namely, in each case the last value). It thus seems possible that both variations were an attempt to improve on the erroneous table in **F** (or, alternatively, that **F** is an erroneous copy of a better original table that survives in **CC$_1$**). I have decided to give the range of arguments from **YLB** in the edition, so that the sequence of tabular values coincides with **F**. Of course, none of the above possible errors would have any noticeable influence on the actual calculation of a solar or planetary position.

Only manuscript **F** gives actual apogee longitudes together with the table for apogee motion, namely three sets for respectively the beginning of the reign of Yazdigird, the beginning of the Syrian year 1191 (in the text mistakenly 1091, but correctly said to be approximately equal to the beginning of the year 249 Yazdigird), and the beginning of the year 331 Yazdigird. The first set is also found, with identical values, in the Table of Preliminaries of the Mean Motions (Table 12) in each of our manuscripts. The second set is written upside down in the empty space in the middle of that table only in manuscript **F** (variants in this set of values are given in the apparatus for explanatory text A2 of Table 12 on p. 282). Both sets are explicitly attributed to al-Battānī, which has been shown to be correct in the commentary on Table 12. The third set of apogee longitudes, for 26 March 962 (i.e., relatively close to the time when Kūshyār compiled his *zīj*), is only found with the table for apogee motion in **F**. However, the same values for this year are written above the tables of mean longitude in **L** (only for the Sun) and **B** (for the five planets), apparently in different hands. Except for Jupiter, the values are larger than those for the Yazdigird epoch by 4;57°, in full agreement with Kūshyār's apogee motion of 1;30° in 100 Persian years. The value for Jupiter hence probably needs to be corrected to $5^s$ 15;42°. Note that in this case as well Kūshyār took al-Battānī's apogee longitude of Mars to have 58′ instead of the correct 18′.

## Table 15: Equation of Time

**Bibliography**: Rome, 'Le problème de l'équation du temps'; Neugebauer, 'The Astronomical Tables P. Lond. 1278'; Neugebauer, *HAMA*, vol. I, pp. 61–68; vol. II, pp. 984–86; Pedersen, *A Survey of the Almagest*, pp. 154–58; Kennedy, 'Two Medieval Approaches'; van Dalen, 'On Ptolemy's Table'; van Dalen, 'Al-Khwārizmī's Astronomical Tables'; Mercier, *Ptolemy's Handy Tables 1b*, pp. 89–119.

The equation of time (in Arabic: *taʿdīl al-ayyām wa-layālīhā*, lit. 'equation of the days and their nights') is the difference between true solar time as measured, for example, by a sundial and mean solar time as found in medieval mean motion tables and as indicated by modern clocks. The equation of time is a function of the solar position on the ecliptic and depends on the obliquity of the ecliptic, the solar eccentricity and the longitude of the solar apogee. It can be expressed in terms of the right ascension and the solar equation for any given solar longitude. As a result of the different points at which these two functions assume their maxima and minima, the equation of time has two local minima and two local maxima and a total amplitude of somewhat more than 30 minutes of time. Medieval astronomers mostly tabulated the equation of time as a function of the true solar longitude and less frequently as a function of the mean solar longitude.[74]

Kūshyār's table for the equation of time appears in two different forms in manuscripts **FY** on the one hand and manuscripts **HCC$_1$LB** on the other. In what appears to be the earliest surviving version of the *Jāmiʿ Zīj*, the Fatih manuscript, as well as in the Yeni Cami copy, the equation of time is tabulated for every six degrees together with interpolation coefficients (*ḥiṣṣat al-daraja*, lit. 'share of the degree'). The use of these coefficients is described in an explanatory text (A) only found in **F** (p. 294): 'In order to obtain the equation of time for each degree, ⟨repeatedly⟩ add the share of a degree written in the table'. In fact, the coefficients are exactly equal to one sixth of the differences between consecutive tabular values for multiples of 6 degrees. In manuscripts **HCC$_1$LB** the equation of time is tabulated for every single degree without interpolation constants. The values for non-multiples of 6 degrees were calculated by repeatedly adding the respective interpolation coefficients to the values for multiples of 6 degrees and rounding in the standard way. There is another copy of Kūshyār's table for the equation of time with values for every degree in the Escorial manuscript of the *Mumtaḥan Zīj*.[75] However, this copy contains numerous scribal errors and even a column of digits that is almost entirely displaced, so I have decided not to include it in my edition.

Kūshyār's table for the equation of time was first analysed by Kennedy with a rather unsatisfactory result, but was then fully explained by the present author.[76] Since Kūshyār's solar mean motion is displaced by 2°, also the argu-

[74] Extensive explanations of the equation of time and methods for analysing tables for this function can be found, among others, in van Dalen, 'On Ptolemy's Table' and van Dalen, 'Al-Khwārizmī's Astronomical Tables'.

[75] Escorial, RBMSL, árabe 927, fol. 61v; facsimile edition Sezgin, *The Verified Astronomical Tables*, p. 121.

[76] Kennedy, 'Two Medieval Approaches', pp. 2–4 and van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 134–141.

ment of his table for the equation of time is displaced by this amount. Taking this into account, I found the table to be based on the parameters one would expect, namely the values 23;35° for the obliquity of the ecliptic and 2;4,45$^p$ for the solar eccentricity, which were consistently used by both al-Battānī and Kūshyār, and the longitude 24° Gemini for the solar apogee, which is confirmed by additions to the title of the table in **FHCC$_1$** and which was reached in the middle of the year 365 Yazdigird (AD 996) according to Kūshyār's table for apogee motion (Table 14).[77]

## Table 15b: Equation of time expressed in lunar mean longitude

The table of lunar mean longitude in **YLB** was expanded with a subtable for the equation of time expressed in lunar mean longitude. In **Y** this subtable is headed *taʿdīl al-ayyām bi-layālīhā*, whereas **LB** have only column headers *(wasaṭ) al-shams* 'the (mean) sun' for the argument and *al-nuqṣān* 'the decrease' for the tabular values (indicating that the equation of time must be subtracted from the lunar mean longitude). The tabulated function can also be clearly recognised from the relative positions of the two local minima and maxima. With only very few exceptions the first-order differences of the subtable are multiples of 0;0,33 (in four cases they are one second less than such a multiple). This makes it highly probable that the subtable was computed by multiplying values for the equation of time expressed in minutes of time by the hourly mean lunar motion in longitude of 0;32,56,27,35,...°. In fact, a perfect agreement is obtained if we multiply the reconstructed values for the equation of time displayed in Table H by 0;32,56,28 (if the hourly motion is rounded to seconds, the recomputation produces three differences of +1″).

The instructions for the use of the tables in Book I in manuscripts **YL** do not mention the subtable for the equation of time expressed in mean lunar longitude either in the chapter on the equation of time or in the chapter on finding the true lunar position (this part of Book I is missing entirely from **B**). Another reason why this subtable may be considered extraneous is that it does not work well together with the common type of table for the equation of time found in all manuscripts of the *Jāmiʿ Zīj*: if the equation of time measured in minutes and seconds of time is first found as a a function of the solar mean longitude, the corrected time will be available when the lunar mean longitude is determined, and hence no correction of the longitude will be necessary any more. A more consistent solution would hence be to supply a similar table for

[77] Considering that the longitude is given as a whole number of degrees, it seems plausible to assume that at the time for which Kūshyār intended to calculate the table, the actual longitude of the solar apogee lie within half a degree from 2$^s$24°, which, according to Kūshyār's table for apogee motion and list of apogee positions, was the case in the period from 332 to 399 Yazdigird, i.e., AD 963–1030.

Table H: Reconstructed table for the equation of time. The small table in **YLB** for the equation of time expressed in lunar mean longitude was computed from this table by multiplying the values by the hourly lunar mean motion, 0;32,56,28°.

| mean long. | equation of time | mean long. | equation of time | mean long. | equation of time | mean long. | equation of time |
|---|---|---|---|---|---|---|---|
| 6 | 0;12 | 96 | 0;15 | 186 | 0;26 | 276 | 0;11 |
| 12 | 0;14 | 102 | 0;14 | 192 | 0;28 | 282 | 0; 9 |
| 18 | 0;16 | 108 | 0;13 | 198 | 0;30 | 288 | 0; 6 |
| 24 | 0;18 | 114 | 0;12 | 204 | 0;31 | 294 | 0; 4 |
| 30 | 0;20 | 120 | 0;12 | 210 | 0;32 | 300 | 0; 2 |
| 36 | 0;21 | 126 | 0;12 | 216 | 0;32 | 306 | 0; 1 |
| 42 | 0;22 | 132 | 0;12 | 222 | 0;32 | 312 | 0; 0 |
| 48 | 0;22 | 138 | 0;13 | 228 | 0;31 | 318 | 0; 0 |
| 54 | 0;22 | 144 | 0;14 | 234 | 0;30 | 324 | 0; 0 |
| 60 | 0;22 | 150 | 0;15 | 240 | 0;28 | 330 | 0; 1 |
| 66 | 0;22 | 156 | 0;16 | 246 | 0;26 | 336 | 0; 2 |
| 72 | 0;21 | 162 | 0;18 | 252 | 0;24 | 342 | 0; 4 |
| 78 | 0;20 | 168 | 0;20 | 258 | 0;21 | 348 | 0; 6 |
| 84 | 0;18 | 174 | 0;22 | 264 | 0;18 | 354 | 0; 8 |
| 90 | 0;16 | 180 | 0;24 | 270 | 0;15 | 360 | 0;10 |

the sun as well; such a table is found, for example, in Naṣīr al-Dīn al-Ṭūsī's *Īlkhānī Zīj* (Maragha, 1272).[78]

The reconstructed values in Table H are close, but not identical, to Kūshyār's values in Table 15. With a slide for arguments 7$^s$24°–8$^s$12°, the same values are found in an additional table on fol. 142v in **L**, together with a copy of the subtable for the equation of time expressed in mean lunar longitude itself. Unfortunately, the table in **L** does not provide further clues as to its origin. The instructions next to the table only describe how to use the interpolation coefficients for finding values of the equation of time for non-multiples of 6 degrees. The preceding pages contain tables from the *Fākhir Zīj* by Kūshyār's student al-Nasawī, and the following page the table for the equation of daylight for latitude 36° also found in **FH**, suggesting a certain nearness to the *Jāmiʿ Zīj*. However, the equation of time appears on a page together with a table for the maximum solar altitude for the latitude 29;30° of Shiraz, which cannot be assumed to be associated with Kūshyār.

The values to minutes for every six degrees of the ecliptic given in Table H do not allow for accurate estimates of the underlying parameters, but a least squares estimation does suggest that the eccentricity and apogee longitude are close to those of Kūshyār and al-Battānī,[79] while the underlying obliquity

[78] See, for example, Paris, BnF, persan 163, fols 29v and 36v and the instructions on fol. 20v. Note that al-Ṭūsī's tables for the equation of time have the true position of the sun as their argument.

[79] In fact, the 95% confidence interval for the longitude of the solar apogee includes both al-Battānī's values 82;14 and 82;17 and Kūshyār's 84;0.

appears to be Ptolemaic. Since the obliquity enters the equation of time through the right ascension, it thus seems a possibility that the small table for the equation of time expressed in lunar mean longitude comes from the same source as Kūshyār's table of the oblique ascension for latitude 35;30°, which was likewise computed for Ptolemy's value of the obliquity of the ecliptic (cf. the commentary on Table 46 in Section IV.10).

### Table 16: Solar equation

**Bibliography**: Kennedy and Muruwwa, 'Bīrūnī on the Solar Equation'; Salam and Kennedy, 'Solar and Lunar Tables'; Pedersen, *A Survey of the Almagest*, Chapter 5, pp. 122–58; Neugebauer, *HAMA*, vol. I, pp. 53–61; Kennedy, 'The Solar Equation'; Van Brummelen, *Mathematical Tables*, Chapter 8, pp. 146–54; van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 21–22, 70–76 and 137–39; van Dalen, 'A Table for the True Solar Longitude'; van Dalen, 'Al-Khwārizmī's Astronomical Tables', pp. 236–38; Mozaffari, 'Limitations of Method', parts I and II; Mozaffari, 'An Analysis of Medieval Solar Theories'; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.6, pp. 38 (translation) and 46 (commentary) and Arabic p. 26; Chapter IV.4.2, pp. 143–44 (translation) and 159–60 (commentary) and Arabic pp. 103–04.

The ninth-century astronomer Ḥabash al-Ḥāsib introduced displacements for the lunar equations in order to remove the need to decide whether the equations had to be added to, or subtracted from, the respective mean motions. Kūshyār is the first astronomer who applied such displacements to the solar equation and furthermore used it systematically for the Moon (see below) and the five planets (see Section IV.7.1). The solar equation $q$ as a function of the mean solar anomaly $a_m$ is given by

$$q(a_m) = \arctan\left(\frac{e \cdot \sin a_m}{60 + e \cdot \cos a_m}\right),$$

where 60 is the radius of the eccentric circle in the Ptolemaic solar model and $e$ the solar eccentricity. For values of $a_m$ between 0 and 180°, the equation must be subtracted from the mean solar longitude in order to obtain the true solar longitude, for values of $a_m$ between 180 and 360° it must be added. Instead of tabulating $q(a_m)$, Kūshyār provided a displaced solar equation $q'(a_m) = 2 - q(a_m)$. Since his maximum solar equation is 1;59,10°, the displaced equation thus defined is always positive and must always be added to the mean solar longitude in order to obtain the true solar longitude. Whereas Ptolemy and most early Islamic astronomers needed to tabulate the solar equation (as well as the lunar and planetary equations) only for arguments 0 to 180° and made use of the symmetry of the equations to find them for arguments between 180 and 360°, Kūshyār had to tabulate his equations for all 360 degrees of their arguments. Furthermore, in order to let the addition of the mean longitude and the displaced equation yield the actual true longitude of the Sun, he had to

adjust the tabulated mean longitude by −2°. This can be seen from the difference between the solar mean longitude at the Yazdigird epoch as found from al-Battānī's parameters in Table 12, namely $2^s$26;57,20, and the epoch value of Kūshyār's table of solar mean motion (Table 13), namely $2^s$24;54,36. This difference consists of the longitude correction between Raqqa and Kūshyār's meridian of 90° (16;45 · 0;59,8,20 / 360 ≈ 0;2,46°) and the displacement of 2°. However, since the reduction of the mean longitude by the displacement will also lead to a mean anomaly reduced by 2°, Kūshyār likewise had to shift the argument of his solar equation table by −2°, so that the original zero equations at $0^s$0° and $6^s$0° appear in the displaced table as values 2;0,0 for arguments $11^s$28° and $5^s$28°, and the original maximum value 1;59,10° for mean anomaly 92° can be recognised in the displaced table as a minimum value 0;0,50° for argument $3^s$0° and a maximum value 3;59,10° for argument $8^s$26°.

For all displaced equations, I will carry out any comparisons with other sources or analyses of the underlying parameters and methods of computation on versions of Kūshyār's equation tables that have been stripped from their displacements and shifts. In the analysis of Kūshyār's table for the equation of time in my doctoral dissertation, I made the following comments on his solar equation:[80] 'Kūshyār's solar equation table was badly computed. Like al-Battānī's solar equation table it was based on eccentricity $e = 2;4,45$ and has an irregular error pattern that may at least partially be due to interpolation. The error patterns in al-Battānī's and Kūshyār's tables are very different between arguments 90 and 135°, but almost identical outside this interval. Neither in Kūshyār's *Jāmiʿ Zīj*, nor in al-Battānī's *Ṣābiʾ Zīj* have I been able to find clues as to the possible causes of the errors.' To this the following may now be added.

In Chapter 28 of his *Ṣābiʾ Zīj*, al-Battānī describes how he derived his value 2;4,45$^p$ for the solar eccentricity, which by accurate calculation leads to his maximum equation 1;59,10°.[81] In contrast, the observations under the caliph al-Maʾmūn led to tables for the solar equation in both surviving manuscripts of the *Mumtaḥan Zīj* and in the *zīj* of Ḥabash al-Ḥāsib with a maximum value of 1;59,0°.[82] There is a small tradition of Islamic *zīj*es that adopted al-Battānī's

[80] van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 137–38.

[81] Nallino, *al-Battānī sive Albatenii*, vol. I, pp. 43–48 (Latin translation) and 212–21 (commentary); vol. III, pp. 64–73 (Arabic).

[82] For both sources, see Salam and Kennedy, 'Solar and Lunar Tables', pp. 493–95. For the *Mumtaḥan Zīj*, see also van Dalen, 'A Second Manuscript', pp. 22–23, and for Ḥabash al-Ḥāsib, Debarnot, 'The *Zīj* of Ḥabash al-Ḥāsib', pp. 41–42. For the maximum solar equation 1;59,56° associated with Yaḥyā ibn Abī Manṣūr, see Kennedy, 'The Solar Equation' and van Dalen, 'A Table for the True Solar Longitude'. Note that the solar equation table in the Berlin manuscript of Ḥabash's *zīj* with values to sexagesimal thirds and maximum 1;59,0,0 was copied in a later hand after having scratched out the original table that is also found in the Istanbul manuscript; in the *Baghdādī Zīj* this table with values to thirds is explicitly attributed to Abū l-Wafāʾ.

Table J: Comparison of al-Battānī's and Kūshyār's values of the solar equation for arguments 91–120° with their tabular differences.

| mean anomaly | al-Battānī's equation | tabular differences | Kūshyār's equation | tabular differences | Battānī–Kūshyār | exact equation | Kūshyār −exact |
|---|---|---|---|---|---|---|---|
| 91 | 1;59, 8 | 0; 0, 5 | 1;59, 8 | 0; 0, 4 | | 1;59, 8 | |
| 92 | 1;59,10 | 0; 0, 2 | 1;59,10 | 0; 0, 2 | | 1;59, 9 | +1 |
| 93 | 1;59, 8 | −0; 0, 2 | 1;59, 8 | −0; 0, 2 | | 1;59, 8 | |
| 94 | 1;59, 3 | −0; 0, 5 | 1;59, 3 | −0; 0, 5 | | 1;59, 5 | −2 |
| 95 | 1;58,57 | −0; 0, 6 | 1;58,56 | −0; 0, 7 | +1 | 1;58,59 | −3 |
| 96 | 1;58,50 | −0; 0, 7 | 1;58,48 | −0; 0, 8 | +2 | 1;58,51 | −3 |
| 97 | 1;58,40 | −0; 0,10 | 1;58,38 | −0; 0,10 | +2 | 1;58,42 | −4 |
| 98 | 1;58,28 | −0; 0,12 | 1;58,26 | −0; 0,12 | +2 | 1;58,30 | −4 |
| 99 | 1;58,13 | −0; 0,15 | 1;58,13 | −0; 0,13 | | 1;58,15 | −2 |
| 100 | 1;57,56 | −0; 0,17 | 1;57,58 | −0; 0,15 | −2 | 1;57,59 | −1 |
| 101 | 1;57,38 | −0; 0,18 | 1;57,41 | −0; 0,17 | −3 | 1;57,40 | +1 |
| 102 | 1;57,20 | −0; 0,18 | 1;57,21 | −0; 0,20 | −1 | 1;57,19 | +2 |
| 103 | 1;56,59 | −0; 0,21 | 1;56,58 | −0; 0,23 | +1 | 1;56,56 | +2 |
| 104 | 1;56,38 | −0; 0,21 | 1;56,33 | −0; 0,25 | +5 | 1;56,31 | +2 |
| 105 | 1;56, 0 | −0; 0,38 | 1;56, 5 | −0; 0,28 | −5 | 1;56, 4 | +1 |
| 106 | 1;55,28 | −0; 0,32 | 1;55,35 | −0; 0,30 | −7 | 1;55,34 | +1 |
| 107 | 1;54,58 | −0; 0,30 | 1;55, 3 | −0; 0,32 | −5 | 1;55, 3 | |
| 108 | 1;54,30 | −0; 0,28 | 1;54,29 | −0; 0,34 | +1 | 1;54,29 | |
| 109 | 1;53,54 | −0; 0,36 | 1;53,54 | −0; 0,35 | | 1;53,53 | +1 |
| 110 | 1;53,14 | −0; 0,40 | 1;53,17 | −0; 0,37 | −3 | 1;53,15 | +2 |
| 111 | 1;52,30 | −0; 0,44 | 1;52,37 | −0; 0,40 | −7 | 1;52,34 | +3 |
| 112 | 1;51,47 | −0; 0,43 | 1;51,54 | −0; 0,43 | −7 | 1;51,52 | +2 |
| 113 | 1;51, 3 | −0; 0,44 | 1;51, 8 | −0; 0,46 | −5 | 1;51, 7 | +1 |
| 114 | 1;50,20 | −0; 0,43 | 1;50,20 | −0; 0,48 | | 1;50,21 | −1 |
| 115 | 1;49,33 | −0; 0,47 | 1;49,29 | −0; 0,51 | +4 | 1;49,32 | −3 |
| 116 | 1;48,39 | −0; 0,54 | 1;48,37 | −0; 0,52 | +2 | 1;48,41 | −4 |
| 117 | 1;47,45 | −0; 0,54 | 1;47,44 | −0; 0,53 | +1 | 1;47,48 | −4 |
| 118 | 1;46,50 | −0; 0,55 | 1;46,50 | −0; 0,54 | | 1;46,53 | −3 |
| 119 | 1;45,55 | −0; 0,55 | 1;45,55 | −0; 0,55 | | 1;45,56 | −1 |
| 120 | 1;44,56 | −0; 0,59 | 1;44,58 | −0; 0,57 | −2 | 1;44,57 | +1 |

table for the solar equation, but Kūshyār appears to have started a sub-tradition in which part of the table was computed anew. In fact, Kūshyār's table is basically identical to al-Battānī's for arguments 1–90° and 150°–180° (with common errors of up to 17 seconds in the sixth sign), but differs systematically from it by up to 7 seconds in the fourth and fifth signs.

A closer inspection of al-Battānī's and Kūshyār's tables for the solar equation and their first-order differences shows a pattern of adjustments that we will also encounter in the tables of planetary latitudes and stations (see Section IV.9) and less conspicuously in some further tables. As can be seen in Table J, in between the irregularly chosen nodes for arguments 94, 99, 109, 114 and 118, for which Kūshyār copied al-Battānī's values, he appears to have smoothened the tabular values in such a way that the first-order differences became much more regular (and hence the second-order differences nearly constant over stretches of the argument). Whereas in the tables of planetary latitudes and stations the

nodes were chosen as multiples of 30°, in this table the choice of the nodes appears to have depended more on the already present pattern of first-order differences rather than on anything else: the intervals between the nodes are irregular and al-Battānī's tabular values for the nodes were usually not correct. In some cases (e.g., for arguments 95–98 and 115–117°) Kūshyār's smoothening led to somewhat larger errors in his equation values, but in most cases the adjusted values had smaller errors, and occasionally they were almost correct. Interestingly, Kūshyār did not make any modifications to the solar equation values for arguments between 150 and 180°, in the part of the table with the largest curvature, although these show a large cluster of negative errors of up to −17 minutes.[83] However, in this part of al-Battānī's table the first-order differences are entirely regular, so that Kūshyār will not have seen any reason for smoothening them. We may thus tentatively conclude that tables with regular tabular differences were a larger priority of his than accurately calculated tabular values.

The seven manuscripts of Book II of the *Jāmiʿ Zīj* that contain the solar equation table all present essentially the same tabular values. However, the layout of the table shows some variation. In **FHB** each page of the table displays the values for three signs; in **F** the three signs share a single argument column, but in **HB** each sign has its own argument column. In $\mathbf{CC_1}$ each of the three pages covered by the table contains four signs, and each sign has its own argument column. In **YL** the table covers only two pages; in **Y** the six signs on each page share one argument column, but in **L** each sign has its own argument column. Both pages in **L** repeat the heading *taʿdīl al-shams*, suggesting that the table was compressed from one spread out over four pages. Of course, the variation in the layout of the table may have been due entirely to decisions of scribes. As usual, in my edition I have reproduced the format of the table in **F**.

The tables for the equations of the Sun, the Moon and the planets in the *Jāmiʿ Zīj* include labels that indicate specific points of the apparent motion of these heavenly bodies around the Earth. These include the mean, furthest and nearest distances of the heavenly body or of its epicycle and, in the tables for the second equations of the planets, their stations and their first and last visibility (i.e., their heliacal risings and settings). In **F**, and partially in $\mathbf{CC_1B}$,

[83] It is probable that al-Battānī made use of interpolation within intervals of 6° in this part of the table: the values for 150 and 168° are correct, the value for 174° contains an error of only −1″, and in general the first-order differences between multiples of 6° of the argument vary only minimally. The large cluster of errors would then be the result of incorrect equation values for the nodes 156 and 162° (these cannot be easily explained as scribal errors). Similar patterns can be recognised in the other parts of the table that Kūshyār simply copied, although here the interpolation nodes do not appear to be separated by equal intervals.

the labels for the distances are given at the top of the columns for the signs in which they are reached, with the exact degree indicated by an *abjad* number. In **HYLB** the labels are written next to the tabular values, roughly around the place where the distances are reached. In the tables for the second equations of the planets in all sources, the labels for the planetary phases are likewise written between or next to the tabular values concerned.

For the Sun, the mean distances correspond to the positions where the solar equation reaches its maximum and minimum values (i.e., for the unshifted arguments $3^s2°$ and $8^s28°$), and the furthest and nearest distances correspond to the positions where the equation is zero (for the unshifted arguments 0° and 180°). Due to the shift of Kūshyār's solar equation, the positions of the distances indicated in the table are two degrees less, and the equation at the extreme distances is 2;0,0 rather than zero. Additionally, only in **F**, two labels *zāʾid* 'increasing' and *nāqiṣ* 'decreasing', written horizontally between the tabular values, indicate where the solar equation starts to increase after having reached its first mean distance at $3^s0°$ and where it starts to decrease after having reached its second mean distance at $8^s26°$.

Kūshyār's solar equation table was copied by al-Rīqānī in the *Zīj al-Qirānāt*, but both copies of the undisplaced solar equation table in the Cambridge manuscript of the *Mufrad Zīj* and the displaced table attributed to al-Battānī in the *Ashrafī Zīj* can be clearly recognised to have been copied from al-Battānī's *zīj* rather than from Kūshyār's.

## Table 20: Lunar Equations

**Bibliography**: Salam and Kennedy, 'Solar and Lunar Tables'; Pedersen, *A Survey of the Almagest*, Chapter 6, pp. 159–202; Neugebauer, *HAMA*, vol. I, pp. 68–98; Van Brummelen, *Mathematical Tables*, Chapter 9, pp. 155–88 (see also pp. 24–26 for a general description of Ptolemaic interpolation); Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.7 on pp. 38–39 (translation), 46 (commentary) and Arabic p. 24, and Chapter IV.4.3–5 on pp. 144–49 (translation), 160–62 (commentary) and Arabic pp. 104–09.

For the lunar model, early Islamic astronomers followed Ptolemy without any significant modifications and mostly also copied the tables for the lunar equations directly from the *Handy Tables*. Therefore their maximum equation of centre was 13;8 or 13;9°, corresponding to an eccentricity or radius of the crank circle of 10;19 units, and their maximum equation of anomaly at the apogee and perigee of the deferent was equal to respectively 5;1 and 7;40°, deriving from an epicycle radius of 5;15 units.

As we will see below, Kūshyār partially computed Ptolemy's tables for the lunar equation of centre and equation of anomaly at apogee anew. Furthermore, he followed the example of Ḥabash al-Ḥāsib in making the lunar equations of the displaced type, thus removing the need to decide whether the equa-

tions had to be added to, or subtracted from, the respective mean positions.[84] However, where Ḥabash used a displacement equal to the maximum value of the equation of centre (13;8°), Kūshyār used the next larger integer (14°). Furthermore, where for the equation of anomaly Ḥabash used a displacement of 5 degrees, namely the maximum of the equation when the epicycle centre is at the apogee of the deferent, Kūshyār used 8 degrees, which is the next higher integer of the maximum equation of anomaly for any value of the arguments, namely 7;40°. Note that, unlike the planets, no shift of the arguments of the lunar equation tables is necessary, but the tabulated lunar mean longitude had to be decreased by the displacement of the equation of anomaly (5°), and the lunar mean anomaly by the displacement of the equation of centre (14°).

As for the Sun, Kūshyār's tables for the lunar equations include labels for the mean, furthest and nearest distances, indicating where the undisplaced equations reach their extreme values and their zero points. Only **F** writes the exact degrees of these points at the top of the columns for the signs concerned (as in my edition), while the other sources write only part of the labels in the right margin of the table or between its columns. Since Kūshyār's lunar equations have no shift, the distances appear in the tables at the same places where undisplaced tables would have them. Thus the furthest distance is reached at 0° of the arguments, the nearest distance at 180°, and the mean distances at $3^s24°$ / $8^s6°$ for the first equation and at $3^s5°$ / $8^s25°$ for the second equation (corresponding to the undisplaced maxima 13;8° and 5;1° respectively).

Finally, Kūshyār implemented a method of interpolation different from Ptolemy's for calculating the lunar equation of anomaly for any position of the epicycle on the deferent. As we will see below (and more extensively for the planets in Section IV.7.2), this innovation was based on a simpler method for approximating a family of functions from one particular member. Although it was original, it was not successful enough to be called 'Kūshyārian interpolation' from now on.

As for all displaced equations, the comparisons with other sources have been carried out on, and are here formulated in terms of, versions of Kūshyār's tables that have been stripped from their displacements and shifts.

## Lunar equation of centre

Kūshyār's lunar equation of centre (the 'first equation of the Moon') as a whole differs significantly from Ptolemy's *Handy Tables* and al-Battānī's *Ṣābiʾ Zīj*. However, if we only consider multiples of 6° of the argument, Kūshyār's values agree completely with the *Handy Tables* and differ from al-Battānī only in the maximum value 13;8° for a double elongation of 114° (both the *Almagest*

[84] cf. Salam and Kennedy, 'Solar and Lunar Tables', and the more extensive discussion of Kūshyār's displaced planetary equations in Section IV.7.1.

and al-Battānī have a maximum value of 13;9°). It thus seems that Kūshyār computed new values for non-multiples of 6°. Whereas in the *Handy Tables* Ptolemy clearly used distributed linear interpolation between his values for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180° already given in the *Almagest*,[85] Kūshyār appears to have used quadratic interpolation at least when computing the tabular values for arguments between 90 and 180°. While the table is highly linear in the first quadrant and can be well explained by regular linear interpolation with some resulting values truncated rather than rounded up, the rate of change in the second quadrant is much higher and even allows us to exclude the use of several particular types of quadratic interpolation discussed in Islamic astronomical sources.[86] It turns out that, in order to compute the tabular values between two consecutive nodes at multiples of 6°, Kūshyār most probably simply used the parabola through the tabular values for these nodes and the previous one. Although slight variations of this method likewise satisfactorily explain the clustering of errors between multiples of 6° for arguments between 90 and 180°, this particular method produces the smallest number of differences from Kūshyār's table (26 out of the first 180 tabular values, as compared to 66 for linear interpolation) and the smallest standard deviation of these differences (23″ as opposed to 52″ for linear interpolation).

### Equation of anomaly at apogee

Kūshyār's 'second equation of the Moon' tabulates the lunar equation of anomaly as a function of the true anomaly for the situation that the epicycle centre is at the apogee of the deferent. The equation of anomaly is one of relatively few planetary tables in the *Ṣābiʾ Zīj* that al-Battānī computed from scratch. This can be easily recognised because his table has values to seconds that are not the result of interpolation between Ptolemy's values to minutes.[87] Kūshyār's values to minutes were not rounded from al-Battānī's values to seconds, but do

[85] cf. Section IV.7, p. 408 below; for distributed linear interpolation, see the Quick reference guide on p. 528. Van Brummelen, 'Mathematical Methods', p. 278 identified this type of interpolation in Kūshyār's tables for the planetary equation of centre and central equation of anomaly. The interpolation was in fact already carried out by Ptolemy in order to extend his tables for multiples of 3 / 6° in the *Almagest* to every degree of the argument. Van Brummelen, *Mathematical Tables*, pp. 170–73 showed that there is statistical evidence that the peculiar error pattern in the lunar equation of centre in the *Almagest*, which had previously been noticed by Toomer, is due to the use of interpolation (most probably quadratic) between values for multiples of 12°.

[86] I included the possibility of recomputing tables by means of seven different types of quadratic interpolation in my programme Table Analysis. Most of these methods are described in the work by Javad Hamadanizadeh; see especially Hamadanizadeh, 'A Survey'.

[87] van Dalen and Pedersen, 'Re-editing the Tables', pp. 421–24, provides a new apparatus for this table. Nallino gives generally correct values in his edition in *Al-Battānī sive Albatenii*, vol. II, pp. 78–83, with a commentary on pp. 223–27.

not appear to have been taken from the *Almagest* or the *Handy Tables* either. Although some clustering can be seen in the errors of Kūshyār's table, in general the differences from other historical tables and from a modern recomputation are so small and few and the tabular differences so regular that it is not possible to establish whether Kūshyār smoothened existing tables or computed the entire table anew with or without the use of interpolation. Later *zīj*es that included Kūshyār's table for the lunar equation of anomaly at apogee, although without his new type of interpolation for the equation of anomaly described below, are the *Shāmil Zīj* and the *Ashrafī Zīj*. On the other hand, the *Mufrad Zīj* and Naṣīr al-Dīn al-Ṭūsī's *Īlkhānī Zīj*, the official product of the Maragha observatory completed in 1272, incorporated al-Battānī's table.

## A new type of interpolation for the equation of anomaly

The most obvious innovation in the tables for the solar, lunar and planetary equations in the *Jāmiʿ Zīj* is the consistent use of displacements and shifts. Before Kūshyār, these had only been applied by Ḥabash al-Ḥāsib to the lunar equations, but in the centuries to follow they became the standard method for simplifying the use of the planetary tables. A less visible, but probably the most remarkable innovation in the *Jāmiʿ Zīj* is a modification of the type of interpolation that Ptolemy established in the *Almagest* for tabulating functions of two variables. Kūshyār consistently applied this new method in his tables for the lunar and planetary equations of anomaly. Glen Van Brummelen was the first to note Kūshyār's variant of Ptolemaic interpolation and to describe it in detail for the planetary tables.[88] For full details, the reader is referred to Section IV.7.2, where I explain Van Brummelen's findings on Kūshyār's method and present some additional observations. Here I will only briefly summarise the particular situation for the Moon.

Since the equation of anomaly changes more rapidly as a function of the true anomaly than of the mean centrum (i.e., the double elongation), Ptolemy computed the lunar equation of anomaly as a function of the true anomaly for the two extreme positions of the epicycle on the deferent, namely the apogee (where the epicycle distance from the Earth is 60 units) and the perigee (where the distance is 39;22 units). For epicycle positions in between these two extremes he provided, as a function of the mean centrum, an interpolation function (with values ranging from 0 to 1) that, independently from the

[88] Van Brummelen, 'Mathematical Methods'. In this commentary, I use the notation that Van Brummelen adopted from Pedersen, *A Survey of the Almagest*. As will be discussed in more detail in Section IV.7.2, Van Brummelen's reconstruction was later recognised to agree with Kūshyār's proof for the calculation of the maximum equation of anomaly and the variation in Chapter IV.4.5 of the *Jāmiʿ Zīj*; cf. Bagheri, *az-Zīj al-Jāmiʿ*, pp. 147–49 (translation), 161–62 (commentary) and Arabic pp. 107–109.

value of the true anomaly, measures the progress of the value of the equation of anomaly from the apogee of the deferent towards the larger equation at perigee. In other words, for every value of the true anomaly, the rate of change in the equation of anomaly between the two extreme positions of the epicycle is assumed to be the same. Rather than the equation of anomaly at perigee itself, Ptolemy tabulated the differences between the equations at perigee and apogee, which in Islamic astronomy became to be called the *ikhtilāf*, translated by me as 'variation' and by Bagheri as 'difference ⟨in epicyclic equation⟩'. These differences have the same general behaviour as the two equations themselves but have their maximum of 2;40° for a somewhat larger argument. The equation of anomaly can now simply be found by multiplying the interpolation function by the difference in equation and adding the result to the equation at apogee.

At first glance it may appear as if Kūshyār exchanged the roles of the strong and weak variables in the process of Ptolemaic interpolation. His table for the 'variation of the nearest distance' is a function of the double elongation and has a maximum value of 2;40° for 180° (i.e., at the quadratures of the lunar orbit), while his interpolation function, with the usual range of values from 0 to 1, is a function of the true anomaly.

Thanks to Van Brummelen's analysis of the tables for the planetary equations of anomaly, it is an easy matter to establish the functions tabulated by Kūshyār for the Moon. Analogously to the planetary tables, the variation $v$ for the Moon is defined by

$$v(2\eta) = p_{\max}(2\eta) - p_{\max}(0°),$$

where $p_{\max}$ is the maximum equation of anomaly as a function of the double elongation $2\eta$,[89] and $p_{\max}(0°)$ can be read from Kūshyār's table for the second equation as 5;1°. Kūshyār's table of the variation is in reasonable agreement with a modern recomputation according to the above formula, although there are several large clusters consisting of dozens of errors of 1 or 2 minutes, sometimes positive and sometimes negative. This points to intermediate rounding or an approximation in the process of computation, which I have not investigated further.

Also analogously to the planetary tables, Kūshyār's interpolation function for the Moon can be seen to be equal to the undisplaced equation of anomaly at the apogee of the deferent, denoted as $p_0(a_v)$, scaled down to the interval [0,1], i.e., divided by the maximum equation $p_{\max}(0°) = p_0(95°) = 5;1$. This method of recomputation gives a good agreement with Kūshyār's table with only 27 scattered deviations of ±1 minute.

[89] $p_{\max}(2\eta)$ can be calculated as $\arcsin(r/\rho)$, in which the epicycle distance $\rho$ is given by $\rho(2\eta) = \sqrt{R^2 - (e \cdot \sin 2\eta)^2} + e \cdot \cos 2\eta$. Here $e$ is the eccentricity (i.e., the radius of the crank circle) and $R = 49;41$.

In Chapter I.4.7 of the *Jāmiʿ Zīj*, Kūshyār describes how the true position of the Moon is to be found.[90] To begin with, the first lunar equation is always added to the mean anomaly in order to find the true anomaly. Then the second equation and the interpolation function are found from their respective tables with the true anomaly as the argument, and the variation as a function of the double elongation. The product of the interpolation function and the variation is added to, or subtracted from, the second equation depending on whether the true anomaly is smaller or larger than 180°. The resulting equation of anomaly is always added to the lunar mean longitude to obtain its true longitude.

If we denote the interpolation function by $f_K(a_v)$, we thus see that Kūshyār's tables for the lunar equations implement the following approximation of the lunar equation of anomaly $p$ for any value of the true anomaly $a_v$ and the double elongation $2\eta$:[91]

$$\begin{aligned} p(a_v) &\approx p_0(a_v) + f_K(a_v) \cdot v(2\eta) \\ &= p_0(a_v) + \frac{p_0(a_v)}{p_{\max}(0^\circ)} \cdot \left(p_{\max}(2\eta) - p_{\max}(0^\circ)\right) \\ &= p_0(a_v) \cdot \left\{1 + \frac{p_{\max}(2\eta) - p_{\max}(0^\circ)}{p_{\max}(0^\circ)}\right\} \\ &= p_0(a_v) \cdot \frac{p_{\max}(2\eta)}{p_{\max}(0^\circ)}. \end{aligned}$$

In other words, similarly to the five planets, Kūshyār approximates the lunar equation of anomaly by multiplying the equation of anomaly at the apogee of the deferent by the quotient of the respective maximum equations, i.e., as a homothetic function of the equation at apogee.[92] Unlike the planets, no confusion is possible concerning the argument of Kūshyār's table of the variation for the Moon, since it has no shift and it has the double elongation, i.e., twice the difference between the lunar and solar mean longitudes, as its argument. As I will discuss in more detail for the planets, Kūshyār could have more easily implemented his type of interpolation for the equation of anomaly by using only the equation of anomaly at apogee $p_0(a_v)$ and a multiplier function $g_K(2\eta) = p_{\max}(2\eta) \,/\, p_{\max}(0°)$. Then his appoximation of the equation of anomaly would have been found as $g_K(2\eta) \cdot p_0(a_v)$ for every $a_v$ and $2\eta$. However, $g_K$ would take on values between 1 and 7;40/5;1 ≈ 1;31,42, and hence would be

[90] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 38–39 (translation), 46 (commentary) and Arabic p. 24.

[91] Note that, in modern terms, the interpolation function $f_K(a_v)$ will always have the same sign as the undisplaced version of Kūshyār's function $p_0(a_v)$, so that the conditional addition or subtraction of the product does not need to be taken into account in this derivation.

[92] For homothetic functions, see the Quick reference guide on p. 529 and, for a more extensive discussion, Section IV.7.2, p. 413 below.

visibly different from the type of interpolation function that Ptolemy uses in the *Almagest*. Thus while Kūshyār introduced a significant innovation in the calculation of the equation of anomaly as such, we see that, in his implementation of this innovation, it was apparently important for him to stay within the framework of the traditional Ptolemaic tables.

As Van Brummelen already showed for the planets, Kūshyār's new method of interpolation for the equation of anomaly was not necessarily an improvement as far as accuracy is concerned. Once Kūshyār had a tabulation of the maximum equation of anomaly as a function of the double elongation, it was a simple matter for him to compute the needed tables. The use of the tables was very similar to that of Ptolemy's, except that a decision about addition or subtraction must be made at a different moment. However, the resulting approximation to the exact lunar equation of anomaly was significantly worse than Ptolemy's. In addition to near the syzygies (where the equation of anomaly is close to the base function $p_0(a_v)$) and whenever the equation of anomaly is close to zero, Kūshyār's method only provides a good approximation to the exact equation of anomaly when it is close to its maximum value (which is a natural consequence of its calculation as a homothetic function based on the ratio of the respective maxima). Kūshyār's method produces the largest deviations from the exact equation for values of the true anomaly somewhere in the middle between the zeroes and maxima, specifically near arguments 50 and 140°. These deviations reach 60″ for a mean centrum of 45°, 3′52″ for 90°, 6′29″ for 135°, and 10′59″ for a mean centrum of 180°, where the homothetic approximation is furthest removed from the reference function.[93] In comparison, the maximum error in Ptolemaic interpolation amounts to little more than 1′ for values of the mean centrum around 90°, i.e., in the middle between Ptolemy's two reference functions. Considering the accuracy of naked-eye observation (approximately 10 minutes of arc) and the general accuracy of Ptolemy's lunar model (with errors in the resulting true longitude ranging between −1 and +1 degrees), the inaccuracy of Kūshyār's method of interpolation for the lunar equation of anomaly will have had no major practical consequences, but as a theoretical innovation it cannot be considered a great success.

A peculiarity of Kūshyār's new method is also that it brings in anew the need to decide whether a component of the equation of anomaly must be added to, or subtracted from, the equation at apogee, whereas the purpose of the

[93] If Kūshyār had computed the equation of anomaly for arbitrary values of the mean centrum $2\eta$ by scaling down the equation at the perigee of the deferent by a factor $p_{\max}(2\eta)$ / 7;40 (rather than scaling up the equation at apogee by a factor $p_{\max}(2\eta)$ / 5;1), the maximum error would have been only 7′8″. However, this would have meant an even further departure from the structure of Ptolemy's tables.

introduction of displacements was exactly to avoid this.[94] This may be another reason why Kūshyār's novel tables for the interpolation of the lunar and planetary equation of anomaly were taken over by very few later authors, not even by those who incorporated other planetary tables from the *Jāmiʿ Zīj*. Whether the reason for this was that the tables were considered too untraditional, were not understood, or were seen to lead to significantly less accurate results than Ptolemaic interpolation or exact calculation of the equation of anomaly, we cannot currently decide.

[94] Note that also Ḥabash's system of displaced lunar equations involved a conditional addition or subtraction. More complicated implementations of displacements that avoided this are only found in *zīj*es from the late twelfth century onwards.

## IV.7. Planetary equations

**Bibliography**: Pedersen, *A Survey of the Almagest*, Chapters 5, 6, 9 and 10, pp. 122–202 and 261–328; Neugebauer, *HAMA*, vol. I, pp. 53–261; Van Brummelen, *Mathematical Tables*, Chapters 8, 9 and 12, pp. 146–92 and 243–313; Van Brummelen, 'Mathematical Methods'; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.8, pp. 39 (translation), 46–47 (commentary) and Arabic p. 27; Chapters IV.4.4–7, pp. 145–51 (translation), 161–63 (commentary) and Arabic pp. 105–12.

The tables of the planetary equations in the *Jāmiʿ Zīj* were studied extensively and many of its peculiar features explained by Glen van Brummelen.[95] In this commentary I will summarise Van Brummelen's results and add some additional information based on my own inspection and analysis of Kūshyār's tables.

Kūshyār's tables for the planetary equations are in principle based on Ptolemy's geocentric geometrical planetary models that dominated Islamic and western astronomy until the time of Copernicus. The planetary tables in al-Khwārizmī's *Sindhind Zīj* and several other lost *Sindhind* works of the eighth to tenth centuries were based on Indian and/or Sasanian-Iranian methods, but the earliest *zīj*es surviving in Arabic all faithfully followed Ptolemy's planetary tables as contained in the *Handy Tables*, with some adjustments of parameters. Whereas Ptolemy tabulated the equations in the *Almagest* for arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°, the *Handy Tables* presented them for every degree of the ecliptic. Thus the type of interpolation between the *Almagest* values that Van Brummelen identified in al-Battānī's and Kūshyār's tables for the equation of centre and the central equation of anomaly (since, as we will see below, Kūshyār had to compute the remaining equation tables himself), was in fact already carried out by Ptolemy.[96]

### IV.7.1. Displacements

**Bibliography**: Salam and Kennedy, 'Solar and Lunar Tables'; Saliba, 'The Double-Argument Lunar Tables'; Kennedy, 'The Astronomical Tables'; Saliba, 'Computational Techniques'; Saliba, 'The Planetary Tables'; Mercier, 'The Parameters'; Van Brumme-

[95] Van Brummelen, 'Mathematical Methods'. For readers who want to follow the numerical examples in this article, the following minor corrections may be helpful. On p. 277, the equation of centre of 14;27° should be *subtracted* from the mean anomaly in order to obtain the true anomaly rather than added to it. Furthermore, $p_K$ needs to be taken with $\hat{c}_m$ as the argument rather than with $c_m$, and its value for 132° is 5;50 instead of 5;41 (it was mistakenly read from argument 122°). In the recomputation of Kūshyār's variation in Table V on p. 275, the value for 90° has an error of –1′ rather than +1′.

[96] For the particular type of interpolation that Van Brummelen recognises in Kūshyār's planetary equations and calls 'distributed linear interpolation' (Van Brummelen, 'Mathematical Methods', p. 278), see the Quick reference guide entry on pp. 528–529.

len, 'Mathematical Methods', pp. 268–71; van Dalen, 'The *Zīj-i Nāṣirī*', pp. 840–41; Chabás and Goldstein, 'Displaced Tables in Latin'; Bagheri, *az-Zīj al-Jāmiʿ*, pp. 46–47.

As for the solar and lunar equations, the most obvious difference between Kūshyār's tables for the planetary equations and those in the *Handy Tables* and early Islamic *zīj*es such as the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr, the *Damascene* or *Arabic Zīj* of Ḥabash al-Ḥāsib and the *Ṣābiʾ Zīj* by al-Battānī is the systematic use of so-called displacements and shifts.[97] In Ptolemy's planetary model, the true longitude of a planet is found by adding or subtracting the mean longitude, the equation of centre and the equation of anomaly, where, in modern terms, the equations are positive for certain arguments and negative for others. Since the mean longitude is the sum of the longitude of the apogee and the mean centrum, we have

$$\lambda(t) = \lambda_A + c_m(t) - q(c_m) + p(a_m + q(c_m),\ c_m),$$

where $t$ denotes time, $\lambda$ the true planetary longitude, $\lambda_A$ the longitude of the apogee, $c_m$ the mean centrum, $a_m$ the mean anomaly, $q$ the equation of centre, and $p$ the equation of anomaly. Here the two equations are defined in such a way that they take on positive values for arguments between 0 and 180° and negative ones between 180 and 360°. Since Ptolemy and early Islamic astronomers tabulated the equations as positive numbers for arguments 0 to 180° (with a so-called 'double entry' for the symmetric values for arguments 360 down to 180°), the user had to decide in each particular case whether the equations had to be added or subtracted. Note that the equation of anomaly was tabulated as a function of the true anomaly, which had to be found by subtracting the tabulated value of the equation of centre from, or adding it to, the mean anomaly; as can be seen from the formula, it had to be added to the mean anomaly when it was subtracted from the mean centrum and the other way around.

By adding to the equations integer numbers of degrees that are at least as large as their respective maximum values, the equations become always additive or always subtractive, thus simplifying the procedure for finding the true position of the planet. In order to achieve this, Kūshyār replaces in the above formula $-q$ by $d_q - q$ and $p$ by $d_p + p$, where $d_q$ and $d_p$ denote the respective displacements of the two equations. To compensate for these additions, he needs to subtract $d_q + d_p$ from the mean centrum, and hence from the tabulated mean longitude, and $d_q$ from the mean anomaly. Thus he finds the true planetary longitude as

$$\lambda(t) = \lambda_A + c_m'(t) + \{d_q - q(c_m)\} + \{d_p + p(a_m' - \{d_q - q(c_m)\},\ c_m)\},$$

[97] See Van Brummelen, 'Mathematical Methods', pp. 268–71. I use the notation that Van Brummelen adopted from Pedersen, *A Survey of the Almagest*.

where the adjusted mean centrum $c_m'$ equals $c_m - d_q - d_p$, the adjusted mean anomaly $a_m'$ is $a_m + d_q$, and the quantities between bracelets are the displaced equations. However, in order to be able to use the equation tables with the adjusted mean centrum $c_m'$ rather than with the original $c_m$, Kūshyār also needs to 'shift' the values of the equation of centre backwards over $d_q + d_p$ degrees of its argument. Thus the equation of centre will be equal to its displacement $d_q$ for $c_m' = 180° - d_q - d_p$ and $c_m' = 360° - d_q - d_p$, corresponding to the zero equations of Ptolemy's original tabulation. Note that there was no need to shift his table for the equation of anomaly, since its argument, the true anomaly $a_v$, found from the adjusted mean anomaly and the displaced equation of centre as $a_v = a_m' - \{d_q - q(c_m)\}$, is equal to the true anomaly of Ptolemy's original model.

The use of displacements for the equation of centre and the equation of anomaly certainly made the calculation of planetary longitudes more straightforward. But Kūshyār was not yet entirely consistent, since the variation (*ikhtilāf*) of his equation of anomaly still needs to be added to the central equation of anomaly in some cases and subtracted from it in others. Furthermore, whenever the centrum of the planets was needed for other calculations, especially of the latitudes and stations, the actual true centrum $c$ had to be used, and hence the shifted true centrum $c'$ found from Kūshyār's tables needed to be adjusted by adding the displacement $d_p$ of the equation of anomaly. Later Islamic astronomers would make further steps in simplifying the use of their planetary tables by means of displacements, even removing the need to decide whether the variation of the equation of anomaly had to be added or subtracted.

### IV.7.2. Alternative type of Ptolemaic interpolation

**Bibliography**: Pedersen, *A Survey of the Almagest*, pp. 85–89; Pedersen, 'Logistics and the Theory', pp. 41–45; Van Brummelen, *Mathematical Tables*, pp. 24–26 and 269–71; Van Brummelen, 'Lunar and Planetary Interpolation Tables', pp. 299–301; Van Brummelen, 'Mathematical Methods', pp. 271–75; Bagheri, *az-Zīj al-Jāmiʿ*, pp. 147–49 (translation), 161–62 (commentary) and Arabic pp. 107–109.

The equation of anomaly is a function of both the centrum, i.e., the position of the epicycle centre on the deferent, and the anomaly, i.e., the position of the planet on the epicycle. Since it depends more strongly on the anomaly, Ptolemy computed the equation of anomaly $p(a_v)$ as a function of the true anomaly for three convenient positions of the epicycle centre. For the three superior planets and Venus he tabulated:

1) $p_0(a_v)$, for mean centrum $c_m = 0°$, where the distance $\rho$ of the epicycle centre from the Earth is at its maximum 60 + $e$ (here 60 is the radius of the deferent and $e$ the eccentricity);

2) $p_1(a_v)$, for the value $c_m^0$ of the mean centrum for which $\rho$ is equal to 60 (referred to as 'mean distance'; $c_m^0$ lies between 90 and 100° for the three superior planets and Venus); and

3) $p_2(a_v)$, for mean centrum $c_m = 180°$, where $\rho$ reaches its minimum value $60 - e$.

For values of the mean centrum in between these three positions, Ptolemy carries out what is now called 'Ptolemaic interpolation', in which he assumes that, for every value of $a_v$, the equation of anomaly changes as a function of the mean centrum $c_m$ with the same rate as the maximum equation of anomaly $p_{\max}(c_m)$. Thus, for a value of the mean centrum $c_m$ between 0° and $c_m^0$, he approximates the equation of anomaly by

$$p(a_v,c_m) \approx p_1(a_v) - f_P(c_m) \cdot (p_1(a_v) - p_0(a_v)),$$

where the interpolation function $f_P$ is defined by

$$f_P(c_m) = \frac{p_{\max}(c_m^0) - p_{\max}(c_m)}{p_{\max}(c_m^0) - p_{\max}(0°)},$$

and analogously when $c_m$ lies between $c_m^0$ and 180°. For the convenience of the user, in addition to $p_1(a_v)$ Ptolemy tabulates the differences $p_1(a_v) - p_0(a_v)$ and $p_2(a_v) - p_1(a_v)$, as well as the interpolation function $f_P(c_m)$. Note that the maximum equation of anomaly for a given value $c_m$ can rather easily be found as $p_{\max} = \arcsin(r / \rho)$ once the distance $\rho$ of the epicycle centre is known, but the determination of $\rho$ includes the look-up of two sine values and the calculation of two square roots:

$$\rho(c_m) = \sqrt{(\sqrt{R^2 - (e \sin c_m)^2} + e \cos c_m)^2 + (2e \sin c_m)^2}.$$

It can be verified that a sine table with values to three sexagesimal places, such as Kūshyār's, is sufficient to compute the epicycle distance with only occasional errors of one second if the arithmetic of multiplication and square root extraction is carried out with sufficient accuracy.

On the basis of Kūshyār's tables for the planetary equations and the instructions for their use in Book I of the *Jāmiʿ Zīj*, Glen Van Brummelen established that the Persian astronomer uses a different type of interpolation for finding the equation of anomaly, involving the computation of only three functions instead of four.[98] Van Brummelen found that, like Ptolemy, Kūshyār tabulates the central equation of anomaly $p_1(a_v)$ for the value of the mean centrum $c_m^0$ for which the epicycle is at its mean distance from the Earth ($\rho = 60$), but then approximates the general equation of anomaly by assuming that, for any given

[98] Van Brummelen, 'Mathematical Methods', pp. 271–75. As noted by Bagheri, *az-Zīj al-Jāmiʿ*, pp. xix and 162, when he wrote this article Van Brummelen was not aware of Kūshyār's explanation of his alternative method of Ptolemaic interpolation in Chapter IV.4.5 of the *Jāmiʿ Zīj* (see below).

value of $a_v$, it changes as a function of $c_m$ by an amount that is proportional to the change in $p_{max}(c_m)$. He implements this by tabulating in two separate tables with the mean centrum as their argument (one for $c_m < c_m^0$ headed 'of the furthest distance' and one for $c_m > c_m^0$ headed 'of the nearest distance'), the variation (*ikhtilāf*) $v$, which is defined as the difference in maximum equation of anomaly between arguments $c_m$ and $c_m^0$, i.e .,

$$v(c_m) = p_{\max}(c_m{}^0) - p_{\max}(c_m) \qquad \text{for } 0° < c_m < c_m{}^0 \quad \text{and}$$
$$v(c_m) = p_{\max}(c_m) - p_{\max}(c_m{}^0) \qquad \text{for } c_m{}^0 < c_m < 180°.$$

He supplies both variation tables with a table of the same interpolation function $f_K$ (*daqāʾiq al-nisab*, lit. 'minutes of the proportions') which, as Van Brummelen established, was computed as $f_K(a_v) = \frac{p_1(a_v)}{p_{\max}(c_m{}^0)}$, i.e., as the central equation of anomaly scaled to the interval [0,1]. Then the equation of anomaly for any pair of arguments $a_v$ and $c_m$ is approximated by

$$p(a_v, c_m) \approx p_1(a_v) \pm f_K(a_v) \cdot v(c_m),$$

the plus sign being applied when $c_m > c_m^0$ and the minus sign when $c_m < c_m^0$.

Van Brummelen's explanation of Kūshyār's implementation of the equation of anomaly makes it plausible that the Persian astronomer made a mistake in designating the argument of his tables for the variation as the 'true centrum' (*al-markaz al-muʿaddal*) and giving it the corresponding shift $d_p$. Van Brummelen shows convincingly that the variation can only have been computed on the basis of the mean centrum (and hence would have required a shift $d_q + d_p$).[99]

As Bagheri noted, Van Brummelen's explanation of Kūshyār's method of interpolation for the equation of anomaly is confirmed by Chapter IV.4.5 of the *Jāmiʿ Zīj*, which deals with the variation of the maximum equation of anomaly both for the Moon and for the planets.[100] Kūshyār first gives a proof for the calculation of the apparent 'radius of the epicycle' (*niṣf quṭr falak al-tadwīr*), i.e., the maximum equation of anomaly. He then finds the 'excess' (*faḍl*) of the maximum equation for a particular distance of the epicycle centre from the Earth over that at the largest distance and calls this 'the total variation' (*al-ikhtilāf al-kullī*).[101] The argument is formulated for the Moon, but then generalised to the five planets. Kūshyār continues by mentioning that he tabulated the (total) variation as a function of the double elongation for the Moon and of the true centrum for the planets, since it depends only on the distance

[99] Van Brummelen, 'Mathematical Methods', pp. 273–74.

[100] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 147–49 (translation), 161–62 (commentary) and Arabic pp. 107–09.

[101] Here 'total' refers to the fact that this is the maximum variation for this particular distance.

of the epicycle centre from the Earth. He then states that the interpolation function is found as the quotient of the 'partial' (*juzʾī*) and the 'total' (*kullī*) second equations, i.e., as the equation of anomaly for a given value of the true anomaly divided by the maximum equation of anomaly, both taken at the largest distance of the epicycle centre from the Earth (for the planets: at its mean distance). The product of the interpolation minutes and the (total) variation yields the 'equation' (*taʿdīl*), i.e., in this case the component of the equation of anomaly that needs to be added to, or subtracted from, the previously found second equation. Finally, Kūshyār observes the differences that he found in the calculation of the 'variation of the epicycle' of Mars between Ptolemy and his own method (for this, see further Section IV.8, pp. 427–28), but insists that his method is the correct one.

From this explanation it can already be seen that Kūshyār found the equation of anomaly for arbitrary values of the epicycle distance as multiples of the equation at the largest distance (in the case of the Moon) or at mean distance (for the planets). In fact, from the above expressions for his variation and interpolation function we find that for every value of $c_m$ the approximation is equal to

$$p_1(a_v) - p_1(a_v) \cdot \frac{p_{\max}(c_m{}^0) - p_{\max}(c_m)}{p_{\max}(c_m{}^0)} =$$
$$p_1(a_v) \cdot \left\{ 1 - \frac{p_{\max}(c_m{}^0) - p_{\max}(c_m)}{p_{\max}(c_m{}^0)} \right\} = p_1(a_v) \cdot \frac{p_{\max}(c_m)}{p_{\max}(c_m{}^0)}.$$

In other words, as for the Moon, Kūshyār basically finds the equation of anomaly for values of $c_m$ unequal to $c_m{}^0$ by scaling the central equation of anomaly $p_1(a_v)$ by the ratio of the respective maximum values. This is the type of procedure that Van Brummelen already observed in the *Almagest* for part of the lunar equation of anomaly at perigee, and that Carlos Dorce dubbed 'homothetic tables' for the six occurrences that he identified in Muḥyī l-Dīn al-Maghribī's *Tāj al-azyāj* in planetary equations with slightly modified maximum values with respect to earlier tables.[102] Van Brummelen found that for arguments from 6 to 60°, Ptolemy's lunar equation of anomaly at perigee in the *Almagest* is very well approximated by the equation at apogee multiplied by the quotient of the values of these two equations at 60°, which is close to 1.5. Even though this factor was not the quotient of the overall maximum equations, it is interesting that Kūshyār may have started his new type of Ptolemaic interpolation by applying this same principle to the very same function.

[102] Van Brummelen, *Mathematical Tables*, pp. 174–79 and Dorce, 'The *Tāj al-azyāj*', pp. 199–205. See also p. 529.

It would thus have sufficed for Kūshyār to provide tables for the central equation of anomaly and for a multiplier function $g$ defined by $g_K(c_m) = p_{\max}(c_m) / p_{\max}(c_m{}^0)$. This function would have been continuously increasing for $0° < c_m < 180°$ and reach the value 1 for $c_m = c_m{}^0$. Kūshyār undoubtedly did not implement his approximation in this way because he wanted to stay within the tradition of Ptolemy's planetary tables by using an interpolation function that takes on values exactly on the interval [0,1]. The Ptolemaic tradition may also have been the reason why he decided not to eliminate the decision whether the product of variation and interpolation coefficient needs to be subtracted from, or added to, the central equation of anomaly. He could have achieved this elimination by a somewhat less elegant but similarly straightforward solution that his approximation to the equation of anomaly also offers, namely by tabulating:

1) the approximated equation of anomaly at the furthest distance of the epicycle centre: $p_0{}'(a_v) = p_1(a_v) \cdot \frac{p_{\max}(0°)}{p_{\max}(c_m{}^0)}$;
2) an adjusted variation defined by $v'(a_v) = p_1(a_v) \cdot \frac{p_{\max}(180°) - p_{\max}(0°)}{p_{\max}(c_m{}^0)}$; and
3) an interpolation function that is proportional to $p_{\max}(c_m)$ and increases from 0 to 1 between 0 and 180°: $f'(c_m) = \frac{p_{\max}(c_m) - p_{\max}(0°)}{p_{\max}(180°) - p_{\max}(0°)}$.

The reader may verify that now, for all values of $c_m$ and $a_v$, the sum of $p_0{}'(a_v)$ and $v'(a_v) \cdot f'(c_m)$ is equal to Kūshyār's approximation $p_1(a_v) \cdot \frac{p_{\max}(c_m)}{p_{\max}(c_m{}^0)}$ to the equation of anomaly. Note that with this setup the interpolation function has the mean centrum as its argument (as in Ptolemy), rather than the true anomaly (as in Kūshyār). However, the difference is not significant, since Kūshyār could just as easily have exchanged the role of his two functions and defined the variation as

$$v''(a_v) = p_1(a_v) \cdot \frac{p_{\max}(c_m{}^0) - p_{\max}(0°)}{p_{\max}(c_m{}^0)} \qquad \text{if } 0° < c_m < c_m{}^0 \quad \text{and}$$

$$v''(a_v) = p_1(a_v) \cdot \frac{p_{\max}(180°) - p_{\max}(c_m{}^0)}{p_{\max}(c_m{}^0)} \qquad \text{if } c_m{}^0 < c_m < 180°,$$

and the interpolation function as

$$f''(c_m) = \frac{p_{\max}(c_m{}^0) - p_{\max}(c_m)}{p_{\max}(c_m{}^0) - p_{\max}(0°)} \qquad \text{for } 0° < c_m < c_m{}^0 \quad \text{and}$$

$$f''(c_m) = \frac{p_{\max}(c_m) - p_{\max}(c_m{}^0)}{p_{\max}(180°) - p_{\max}(c_m{}^0)} \qquad \text{for } c_m{}^0 < c_m < 180°$$

Table K: Ptolemy's and Kūshyār's parameters of the planetary models. Values of Ptolemy that differ from Kūshyār are underlined. Note that Kūshyār gives the double eccentricity of Mars as 12;0 in Chapter III.19 of his *zīj*, but computed his equation of centre as a homothetic table in such a way that it received a maximum of 11;30°.

| | $e$ | | $q_{\max}$ | | $r$ | $c_m{}^0$ | $p_{\max}{}^0$ |
|---|---|---|---|---|---|---|---|
| | Ptolemy | Kūshyār | Ptolemy | Kūshyār | | | |
| Saturn | 3;25 | 3;25 | $\underline{6;31}$ | 6;32 | 6;30 | 94;53 | 6;13 |
| Jupiter | 2;45 | 2;45 | 5;15 | 5;15 | 11;30 | 93;56 | 11; 3 |
| Mars | 6; 0 | [6;2,30] | 11;25 | 11;30 | 39;30 | 98;32 | 41;10 |
| Venus | $\underline{1;15}$ | 1;2,22,30 | $\underline{2;24}$ | 1;59 | 43;10 | 91;47 | 45;59 |
| Mercury | 3; 0 | 3; 0 | 3; 2 | 3; 2 | 22;30 | 67;44 | 22; 1 |

in order to obtain exactly the same result.[103] This would in fact have been more logical, since the term *ikhtilāf* ('variation') points to the differences in the equation of anomaly at different positions of the epicycle centre on the deferent. It would also have produced more accurate results for planets with large epicycle radii, since by essentially scaling down the equation of anomaly to the interval [0,1] for his interpolation coefficients, Kūshyār loses precision if the maximum equation is significantly larger than 1°. For example, for Mars the maximum rounding error of half a second in the interpolation minutes corresponds to a possible maximum error of 23½ minutes in the equation of anomaly. As a result, when approximating the equation of anomaly of Mars for mean centrum 98° by Kūshyār's method, the resulting values are sometimes larger and sometimes smaller than those of Kūshyār's central equation of anomaly, which is for a value $c_m{}^0$ that lies near 98½°. The same is the case for Kūshyār's approximation of the equation of anomaly for mean centrum 99°.

Although Ptolemy's model for Mercury is very different from that for the other planets, especially because of the planet's two perigees, in his computation of the equation of anomaly Kūshyār applies the same principles as described above. More details of these will be given below in my attempt to reconstruct the tables for the variation and the interpolation function for Mercury.

## IV.7.3. Recomputation of the tables

Kūshyār describes the Ptolemaic planetary models and gives the values of the underlying parameters in Chapters 18 and 19 of Book III of the *Jāmiʿ Zīj*. Furthermore, he explains the methods for calculating the planetary equations in Section 4 of Book IV.[104] Table K displays Kūshyār's planetary parameters.

[103] Note that the variation as such is not dependent on $c_m$, but that the value of $c_m$ with respect to $c_m{}^0$ determines which of the two forms is to be used.

[104] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 142–58 (translation), 159–66 (commentary) and Arabic pp. 101–19. The edition of Book III by Dr Hanif Ghalandari is still in progress; for Chapters III.18–19, see, for example, **F** fols 108r–111r or **L** fols 91v–92v.

Besides the eccentricity $e$ and the epicycle radius $r$, I have listed some further quantities that will be frequently referred to, namely the maximum equation of centre $q_{\max}$, the value $c_m{}^0$ of the mean centrum for which the epicycle centre is at its mean distance (60) from the Earth[105] and the maximum value $p_{\max}{}^0$ of the central equation of anomaly (i.e., when the epicycle centre is at this mean distance). Whereas Kūshyār's epicycle radii are the same as Ptolemy's throughout, I have added Ptolemy's values for the eccentricity and the maximum equation of centre, since these are very different for Venus (in both sources following the solar model) and somewhat different for Saturn and Mars.

Whereas Kūshyār had to compute the variation tables and the accompanying interpolation coefficients for his peculiar type of interpolation from scratch, his tables for the first and second equations of the five planets, i.e., the equation of centre and the central equation of anomaly, display the standard Ptolemaic functions. As I have mentioned above, Van Brummelen established that these tables, with the single exception of the equation of centre of Mars, are the same as al-Battānī's (who, however, did not use displacements). Van Brummelen also noted that the tables were generally computed by means of distributed linear interpolation within intervals of three degrees, except around the maximum equations, where linear interpolation would not have been accurate enough. Since the Mumtaḥan astronomers, Ḥabash al-Ḥasib and al-Battānī simply copied most of the planetary equations from the *Handy Tables*, we must conclude that this interpolation was already carried out by Ptolemy himself between the values from the *Almagest* (see p. 408 and footnote 96). The methods of computation of the equation tables in the *Almagest* and their dependencies on other tables were studied in detail in Van Brummelen's doctoral dissertation.[106] In the tables for the planetary equations of centre and central equation of anomaly he found clustering of errors in the tabulated values for 6, 12, 18, ..., 90, 93, 96, ..., 180°; this clustering was not the result of interpolation within the table, but its actual cause could not be determined.[107] Below I will only discuss in detail those tables for the equation of centre and the central equation of anomaly in the *Jāmiʿ Zīj* that do not ultimately stem from the *Almagest*. Furthermore, I will somewhat expand Van Brummelen's discussion of Kūshyār's variation tables and interpolation functions for the five planets.

[105] For the superior planets and Venus, these values were computed according to the formula $\tan(180° - c_m{}^0) = 120 \cdot \sqrt{1 - \frac{1}{4}e^2/3600}\Big/3e$, derived in Pedersen, *A Survey of the Almagest*, p. 293 and reproduced in Van Brummelen, *Mathematical Tables*, p. 271. For Mercury I confirmed Pedersen's estimation in *A Survey of the Almagest*, p. 323 by inspecting in the same way sexagesimal values of the epicycle distance on ever smaller intervals.

[106] In particular, for the planetary equations, see Van Brummelen, *Mathematical Tables*, Chapter 12, pp. 243–313.

[107] The errors are only incidentally as large as ±3′, but in most cases ±1′ or ±2′. Interestingly, the errors are not generally larger for equations with larger maximum values.

Kūshyār's table of the equation of centre for Saturn is basically identical to the one in Ptolemy's *Handy Tables* and in the early Islamic *Mumtaḥan Zīj* and the *zīj*es of Ḥabash al-Ḥāsib and al-Battānī, with the exception of the values for arguments 86 to 99°, which Kūshyār appears to have adjusted in order to obtain a smooth maximum of 6;32° instead of the Ptolemaic 6;31°. Kūshyār's table of the central equation of anomaly for Saturn is likewise basically identical to the one in the *Handy Tables* and the earlier Islamic *zīj*es, but in this case several values around the maximum for arguments 93 and 101–107° were adjusted by one minute, albeit without changing the maximum equation.

For Jupiter nearly all *zīj*es that I inspected have basically the same tables for the equation of centre and the central equation of anomaly that are already found in the *Handy Tables*. Kūshyār's tables have occasional differences of one minute from the *Handy Tables* and the other early Islamic witnesses. As noted by Van Brummelen, Kūshyār's table for the equation of centre is clearly different from the *Almagest*, which implies that Ptolemy computed it anew for the *Handy Tables*. In fact, the equation of centre in the *Handy Tables* is in somewhat better agreement with a modern recomputation than the one in the *Almagest*, but still shows the same kind of error clustering that we find in most equation tables.

Kūshyār's table for the equation of centre of Mars is different from that in any other *zīj* that I have inspected. Whereas most other *zīj*es again follow the *Handy Tables* with its maximum equation of 11;25°, Kūshyār's table has 11;30° as its maximum. I will investigate this table further in Section IV.8.1 in the context of Kūshyār's changes to the tables for Mars. For the central equation of anomaly of Mars, Kūshyār follows the *Handy Tables* and the early Islamic *zīj*es but occasionally deviates by a minute from most other works, including in its maximum value 41;10 (which is also found in the *Handy Tables*), for which the *Almagest* and the other early Islamic *zīj*es have 41;9.

For Venus, the early Islamic astronomers followed their improved observations of the parameters of the solar model by assigning their value for the solar eccentricity to the distance between the Earth and the centre of the deferent of Venus as well. This implied significant reductions in both the solar eccentricity (from 2;30 to 2;4,45) and the eccentricity of Venus (from 1;15 to 1;2,22½). Kūshyār's table for the equation of centre of Venus was rounded in the standard way from the solar equation. As a result, it differs significantly from the *Mumtaḥan Zīj* and the *zīj* of Ḥabash al-Ḥasib, but it also has 24 differences of ±1′ from al-Battānī's table.

In spite of some deviations, especially around 90° and from 120–123°, Kūshyār's table for the central equation of anomaly of Venus may be considered to be the same as that in the *Handy Tables* and the early Islamic *zīj*es. Finally, Kūshyār's equation of centre and equation of anomaly for Mercury show only occasional deviations from the tables in the *Handy Tables*.

As for the Sun and the Moon, the tables for the first and second equations of the five planets in the *Jāmiʿ Zīj* provide labels 'mean distance' to indicate where the equations reach their extreme values, and labels 'furthest distance' and 'nearest distance' to indicate where the equations equal zero, namely at the apogee and perigee of the deferent or epicycle respectively. In **F**, and partially in **CC$_1$B**, the exact degrees of these distances are indicated with an *ajbad* number at the top of the columns for the signs concerned; in the other sources the labels are written in between the columns of the table near the tabular values to which they refer. Since the tables for the second equation have no shift, the furthest and nearest distances occur at $0^s0°$ and $6^s0°$; in the tables for the first equation, they were shifted backwards by the sum of the displacements of the first and second equation (cf. Section IV.7.1).

The tables for the second equation of the planets have additional labels for the planetary phases, which are written horizontally just above the tabular value to which they refer (especially in **F**) or vertically next to the tabular values. In Section I.4.11 of the *Jāmiʿ Zīj*, Kūshyār discusses how to determine whether a planet is in retrograde motion and whether it is visible.[108] First an algorithm states that a planet is in retrograde motion if the daily increase in its first equation is smaller than the daily decrease in its second equation. Alternatively, one may use the tables for the first stations (Tables 38–42, combined with the planetary latitudes) to obtain the same result. Kūshyār does not provide a method for determining whether a planet is visible at a certain time, but mentions that he has indicated the approximate places of the stations, of retrograde and progressive motion, and of the first and last visibility of the planets in the tables for the second equations.

By comparing the positions of the labels in the second equation tables with the tables for the first stations, it can be verified that the pair of labels 'stationary–retrograde' roughly indicates the entire range of values of the true anomaly at which the first station may occur depending on the value of the mean centrum. Similarly, the pair of labels 'stationary–progressive' approximately indicates the possible range of values at which the second station may occur. As a result, the labels for the first station and for progression are placed symmetrically around $6^s0°$, and so are the labels for retrogradation and for the second station.[109] Kūshyār does not give any information on his method for determining the moments of the heliacal risings and settings of the planets. The positions indicated in the tables for the second equations, which are likewise symmetrical around $6^s0°$, can only be correct under certain simplifying conditions, which I have not further investigated.

[108] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 42–43 (translation) and 49 (commentary) and Arabic pp. 30–31.

[109] In the edition, I have not corrected the incorrect placement in **F** of the label for the first station of Saturn. All positions of the labels with their variants in all eight manuscripts are given in the table on p. 303.

Van Brummelen's reconstruction of Kūshyār's new type of tables for calculating the equation of anomaly, which he illustrates on the critical case of Mars, generally produces an acceptable fit with the tables in the *Jāmiʿ Zīj*. This is not surprising for the planets with small eccentricities and epicycle radii, namely Saturn and Jupiter, for which the variation has a small range of possible values, and for the interpolation function, since it is a scaled-down version of the central equation of anomaly. However, a recomputation of Kūshyār's variation for Mars, Venus and Mercury is more problematic. In principle Kūshyār could simply have derived his variation $v(c_m)$ from Ptolemy's interpolation function $f_P(c_m)$ in the *Almagest* according to

$$v(c_m) = f_P(c_m) \cdot \left(p_{\max}(c_m{}^0) - p_{\max}(0°)\right) \qquad \text{for } 0° < c_m < c_m{}^0 \quad \text{and}$$
$$v(c_m) = f_P(c_m) \cdot \left(p_{\max}(180°) - p_{\max}(c_m{}^0)\right) \qquad \text{for } c_m{}^0 < c_m < 180°.$$

To this end he would have needed to perform interpolation either between the interpolation coefficients to seconds given in the *Almagest* for mean centrum 6, 12, 18, ..., 90, 93, 96, ..., 180° or between the variation values that he could compute directly from these.[110] For Mars the result shows a reasonable agreement but certainly not good enough to conclude that this is what Kūshyār did. Alternatively we need to assume that Kūshyār calculated values for the maximum equation of anomaly for each value of $c_m$ himself (from which his variation would follow by simple subtractions). Obviously this would have been a large computational burden, which any medieval astronomer would seek to reduce by using types of interpolation and approximation at which we can still mostly only guess. We will see below that for Mars, Venus and Mercury Kūshyār's variation is indeed closer to an exact computation then to a reconstruction on the basis of the interpolation functions in the *Almagest*. However, in between stretches with correct values they each show clustering of errors, whose cause I cannot explain. Both Van Brummelen and I assume that such groups generally derive from the use of some type of interpolation schemes. However, if interpolation was in fact applied, it certainly was not on regular intervals and with a consistent algorithm.

[110] Note that Kūshyār could not have used the interpolation coefficients for every degree from the *Handy Tables* (which were adopted by early Islamic *zīj*es), since they are tabulated as a function of the true instead of the mean centrum and are given to minutes only and would therefore produce highly inaccurate values for the variation. As Van Brummelen showed in *Mathematical Tables*, pp. 300–11, the interpolation functions in the *Almagest* themselves, or the maximum equations of anomaly on which they were based, already depended on the use of linear interpolation, so that Kūshyār's reconstructed variation would have ended up being linear over rather long stretches of the argument.

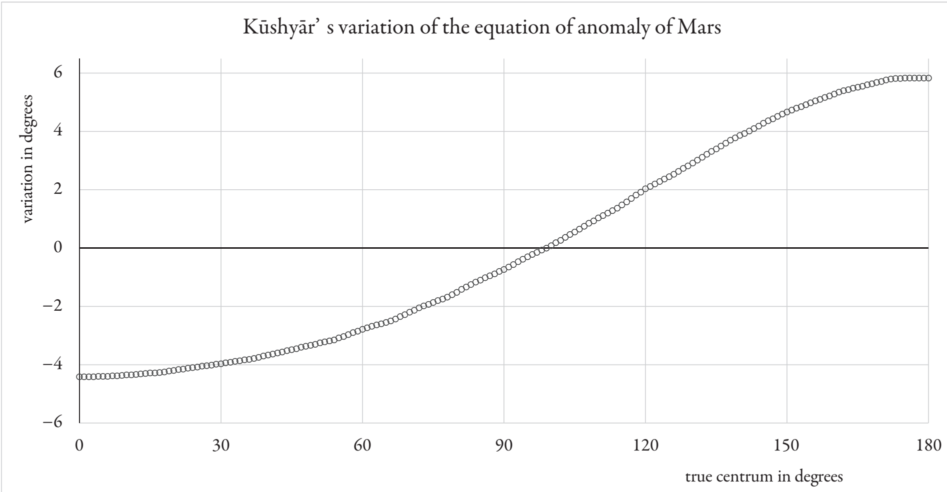


Figure 4: Kūshyār's variation of the equation of anomaly for Mars as a function of the true centrum. Van Brummelen established that the table was in fact computed as a function of the *mean* centrum.

I will now look in some more detail at the recomputation of Kūshyār's variation tables for the planets Mars, Venus and Mercury. It is reasonable to expect of a satisfactory recomputation that it produces at least the tabulated values of the variation for $c_m = 0°$ and $c_m = 180°$ and those for values of $c_m$ near $c_m^0$. The underlying parameters are the eccentricity $e$, the epicycle radius $r$, and a value $p_{\max}^0$ for the maximum central equation of anomaly; the latter depends on $e$ and $r$, but may have been rounded or chosen on other grounds (for example, to make certain tabular values zero) and thus counts as a separate parameter. Rather than to Kūshyār's shifted mean centrum and his displaced equation of centre and equation of anomaly, I will in my analysis again consistently refer to the corresponding unshifted mean motions and undisplaced equations. I will carry out my analyses on the equation and variation values corresponding to arguments 0 to 180° and only mention the symmetrical values for 180 to 360° when there is a specific reason to do so. I will speak of the variation as a single function by taking the tables for furthest and nearest distance together and removing their shift, although we will see that in some cases the two parts appear to have been computed on the basis of different parameters. Figure 4 shows Kūshyār's variation for Mars, with the values for the mean centrum between 0° and $c_m^0$ shown as negative values.

We have already seen that Kūshyār tabulated the equation of centre of Mars with a slightly increased maximum value 11;30 (instead of Ptolemy's commonly used 11;25), corresponding to a slightly increased eccentricity of 6;2,30 (rather than Ptolemy's 6;0). There is no reason to assume any adjustment of the Ptolemaic epicycle radius 39;30, which is confirmed by Kūshyār's table of the second

equation (i.e., the central equation of anomaly). When it is stripped from its displacement, the maximum tabulated value of this equation is 41;10°, which is correctly rounded from the exact value arcsin(0;39,30) ≈ 41;10,22,... For either value of the eccentricity, $c_m^0$ is slightly larger than 98½°,[111] corresponding to the shifted argument $1^s 21½°$ of the variation table. Since around these values the maximum equation of anomaly changes by 5 or 6 minutes per degree of mean centrum, one would expect the variation to be −0;3 for 98° and −0;2 or −0;3 for 99°. However, Kūshyār's table has a value 0;0 for 99° and correct tabular differences of 5 or 6 minutes beyond this. It thus seems that this part of the table for nearest distance can only be correctly recomputed if we take $p_{\max}^0$ equal to $p_{\max}(99°)$, i.e., to 41;13°. In order to reproduce Kūshyār's value $v(180°) = 5;50$, this value of $p_{\max}^0$ must be combined with $e = 6;2,30$, since $e = 6$ leads to a value 5;48. The resulting recomputation shows the usual clustering of errors; however, with 58 differences with a standard deviation of 1′13″ in the 82 tabular values between arguments 99 and 180°, it shows an improvement over Van Brummelen's recomputation for $e = 6$ and $p_{\max}^0 = 41;10°$, which yielded 72 differences with a standard deviation of 2′1″.

On the side of the furthest distance, Kūshyār's variation of Mars is in better agreement with a zero value around 98½°: for $e = 6;2,30$ and $p_{\max}^0 = 41;10$ the table shows seven differences of ±1′ for arguments 88 to 98, for $e = 6;0$ only three. Although both values for the eccentricity produce a recomputation of the variation for $c_m = 0°$ that differs by ±1′ from the tabular value 4;25, overall Van Brummelen's recomputation for $e = 6;0$ produces a somewhat better fit for this part of the table than the alternative with $e = 6;2,30$ (73 differences out of 99 values with a standard deviation of 1′39″ against 86 differences with a standard deviation of 2′22″).

For Venus, the variation at furthest distance is satisfactorily recomputed for $e = 1;2,22,30$ (half of the solar eccentricity), $r = 43;10$ and $p_{\max}^0 = 46;0$ (note that, although the table of the second equation has 45;59° as its undisplaced maximum value, an exact computation for $r = 43;10$ yields a maximum 46;0,31,...°). Like for Mars, the variation at nearest distance is not similarly well recomputed by the most plausible values of the parameters. In order to reproduce the values near the mean distance accurately, $p_{max}^0$ needs to be taken equal to 46;1°, which is acceptable as a rounded value of the exact maximum mentioned above. But for the values around perigee we need either $p_{max}^0 = 46;2°$ or an eccentricity near 1;1,30, for neither of which there is a plausible historical or mathematical justification.

[111] cf. Table K on p. 415 and footnote 105 on p. 416.

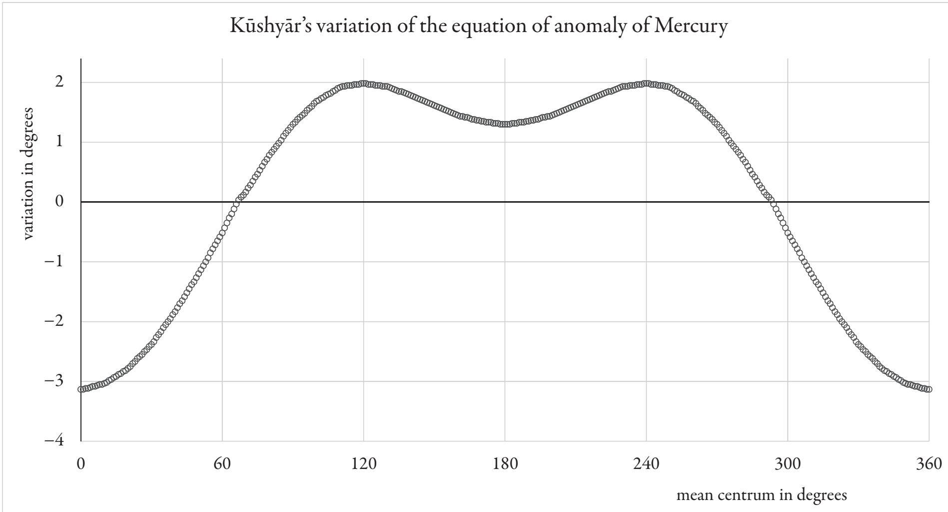


Figure 5 : Kūshyār's variation for Mercury, depicted as a function of the mean centrum with the shift removed. In Ptolemy's original type of interpolation, the typical variation of the maximum equation of anomaly with its maximum near 120° and local minimum for 180° is produced by the interpolation minutes.

Although the model for Mercury is significantly different from that of the other planets, the reconstruction proposed by Van Brummelen is also valid for this planet. Ptolemy computed the equation of anomaly at apogee ($c_m = 0°$), at mean distance ($c_m = c_m{}^0 \approx 67;44,9°$) and at one of the two perigees that his Mercury model induces ($c_m \approx 120°$). At mean distance the maximum equation of anomaly is $p_{\max}{}^0 = \arcsin(22;30/60) \approx 22;1,28$. At $c_m = 180°$ the maximum equation of anomaly has a local minimum, which is reflected in Ptolemy's interpolation function.

Following the pattern of the other planets, Kūshyār calculated the variation for Mercury as

$$p_{\max}{}^0 - p_{\max}(c_m) \qquad \text{for } 0 < c_m < c_m{}^0 \quad \text{and}$$

$$p_{\max}(c_m) - p_{\max}{}^0 \qquad \text{for } c_m{}^0 < c_m < 180°.$$

Whereas for the other four planets Kūshyār's variation increases continuously for values of $c_m$ between 0 and 180°, the variation for Mercury reaches its maximum already near 120° and then decreases to a local minimum at 180° (see Figure 5). It is reasonable to require that a satisfactory recomputation of Kūshyār's variation produces correct values for $c_m = 0°$, for values of the centrum around $c_m{}^0$ and around the perigee, as well as for $c_m{}^0 = 180°$.

A recomputation for the presumed parameter values ($e = 3$, $r = 22;30$, $p_{\max}{}^0 = 22;1$) shows systematic differences in every part of Kūshyār's variation table. These reach −9′ at the apogee, become zero at 61° (rather than at the mean distance near 67¾°), increase up to +7′ at the perigee and then slightly

decrease to 4′ at 180°. In order to reproduce the table's zero near 66½° (i.e., where it changes from furthest to nearest distance), $p_{max}{}^0$ needs to be taken equal to an unrealistic 21;57°. This also removes the errors around 90° and 180°, but leaves two very large clusters of errors in the section for nearest distance, including errors of up to +3′ near the perigee, and makes the systematic errors in the section for furthest distance even larger. Those systematic errors rather seem to point to a different value of the eccentricity. In fact, $e = 3;9$ in combination with $r = 22;30$ and $p_{max}{}^0 = 22;1$ produces a surprisingly good recomputation of Kūshyār's entire variation table with the exception of arguments 61 to 69, i.e., the area around the mean distance, which shows a group of systematic errors of up to +4′. Without these, only 50 out of 172 values have an error of ±1′. Of course, the value 3;9 for the eccentricity of Mercury is unattested, both in Kūshyār and in other Islamic sources and certainly does not underlie Kūshyār's equation of centre of Mercury. So even though the agreement of the recomputation with Kūshyār's variation is quite good, as for Ptolemy and other Islamic astronomers we are once again left with the unsatisfactory feeling not to know very well how they carried out their computations.

### IV.7.4. The sources of Kūshyār's planetary equations and their influence

When we compare Kūshyār's tables for the planetary equations with those of Ptolemy and earlier as well as later Islamic astronomers, we can recognise some clear patterns in the tradition of these tables. In general, Ptolemy's planetary equations from the *Handy Tables*, or at least his values for the eccentricity and the radius of the epicycle, continued to be used by Islamic astronomers for many centuries. The most important exception to this rule is Venus, whose eccentricity early Islamic astronomers drastically reduced by almost a fifth, along with that of the solar orbit. But the tables of the equation of centre for the other four planets and the tables for the equation of anomaly of all five planets were adopted without change in the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr, the *Arabic Zīj* or *Damascene Zīj* of Ḥabash al-Ḥāsib and the *Ṣābiʾ Zīj* of al-Battānī. Since Kūshyār relates how he adopted al-Battānī's rates of mean motion and adjusted the mean positions for use with the Persian calendar, it is most probable that the main source for his planetary equations was likewise the *Ṣābiʾ Zīj*. Although Kūshyār provided the equations with displacements and shifts (see Section IV.7.1), numerous particular error patterns found in al-Battānī's tables (and already in Ptolemy's *Handy Tables*) can also be recognised in the *Jāmiʿ Zīj*. The only adjustments that Kūshyār made to the equation of centre and the equation of anomaly were the following:

- slight adjustments around the maxima of the equation of centre and the central equation of anomaly of Saturn, which change the maximum equation of centre from 6;31 to 6;32;

- a new computation of the equation of centre of Mars for a maximum value of 10;30° instead of Ptolemy's 10;25° (this effectively changes the eccentricity from 6;0 to 6;2,30);
- occasional changes in the central equation of anomaly of Mars, including a maximum value 41;10 (as elsewhere found only in Ptolemy's *Almagest*) instead of the common 41;9; and
- substitution of the equation of centre of Venus by Kūshyār's own solar equation rounded to minutes, making this table clearly different from the tables with the same maximum value included in the *zīj*es of Yaḥyā ibn Abī Manṣūr, Ḥabash al-Ḥāsib and al-Battānī.

Of course, Kūshyār also had to compute the variation of the equation of anomaly and the corresponding interpolation functions anew for all five planets in order to implement his alternative type of Ptolemaic interpolation (cf. Section IV.7.2).

As far as the layout of the planetary equation tables is concerned, the earliest extant Islamic *zīj*es, but also the tables based on Indian and Persian methods in the *Sindhind Zīj* by al-Khwārizmī which survive in a Latin translation of an Andalusian reworking, all follow Ptolemy's *Handy Tables* in so far as they combine the five functions needed for calculating the equation of centre and the equation of anomaly in a single table with five numbered columns and a common double-entry argument (i.e., two columns for the symmetric arguments $x$ and $360 - x$ for $0 < x \leq 180°$). In this layout the argument column represents the mean centrum for the equation of centre, the true centrum for the interpolation function, and the true anomaly for the central equation of anomaly and the differences between the central equation of anomaly and the equations of anomaly at the perigee and apogee of the deferent. Since in most cases the functions were tabulated for every degree, they would typically consist of six 'blocks' of 30 degrees that covered three or six pages in the manuscripts.

Kūshyār's *Jāmiʿ Zīj* is the earliest surviving *zīj* in which this traditional Ptolemaic layout was abandoned and the four functions that Kūshyār needed for calculating the equations were tabulated in six separate tables (note that he split his variation of the equation of anomaly and the interpolation function into two separate tables, although this was not strictly necessary). Kūshyār's great contemporaries Ibn Yūnus (in Egypt) and al-Bīrūnī (in Afghanistan) maintained the original Ptolemaic layout that combined all necessary functions in a single table. But *zīj*es directly influenced by Kūshyār such as the *Zīj al-Qirānāt* by al-Rīqānī and the *Mufrad Zīj* by al-Ṭabarī followed Kūshyār's example, which then became the most common layout and was found in all major *zīj*es including the *Sanjarī Zīj* by al-Khāzinī (Marw, *c.* 1120), the *ʿAlāʾī Zīj* by al-Fahhād (Shirwan, 1176), the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsī (Maragha, 1171/2), the *Jadīd Zīj* by Ibn al-Shāṭir (Damascus, *c.* 1365) and

the *Sulṭānī Zīj* by Ulugh Beg (Samarqand, *c.* 1440). An intermediate form in which the equation of centre and the interpolation function were combined in one table and the equation of anomaly and its variation in another was first found in the anonymous *Dustūr al-munajjimīn*, and was later adopted in *zīj*es such as the one by al-Sanjufīnī, which was written in Tibet in the fourteenth century on the basis of the observational work of Iranian astronomers at the Mongol court of Khubilai Khan in China.

After Ḥabash al-Ḥāsib had introduced displaced equations for the Moon, Kūshyār was the first to apply displacements to the equations of the five planets in order to simplify the process of deciding when the equations had to be added to, or subtracted from, the mean positions. Whereas Ibn Yūnus did not yet make use of displaced equations and al-Bīrūnī did so only for the Sun and the Moon, *zīj*es directly influenced by the *Jāmiʿ Zīj* such as the *Fākhir Zīj* of Kūshyār's student al-Nasawī, the *Mufrad Zīj* of al-Ṭabarī, and the Z*īj al-Qirānāt* incorporated Kūshyār's displacements with small adjustments. Later on, displacements became common and were included in well-known *zīj*es such as those by al-Fahhād al-Shirwānī and Naṣīr al-Dīn al-Ṭūsī. Al-Fahhād also introduced different types of displaced tables involving displacements of 360° and 'mixed displacements'. The anonymous late thirteenth-century *Sulṭānī Zīj* (extant in Tehran, Majlis Library, MS 184, not to be confused with the *zīj* of Ulugh Beg) and the early fourteenth-century *Ashrafī Zīj* by Sayf-i munajjim-i Yazdī al-Kamālī, which offer the possibility of calculating planetary positions according to a dozen earlier *zīj*es as adjustments to the main tables that were in both cases adopted from the early thirteenth-century *Shāhī Zīj* by Ḥusām al-Dīn al-Sālār, also made consistent use of displaced equations. This implied that their authors had to adjust to their own displacements the planetary equations from all the *zīj*es that they included, some of which had no displacements at all and others had displacements and shifts of different amounts.

Kūshyār's alternative method of calculating the planetary equation of anomaly for any given value of the mean centrum as a homothetic function of the central equation of anomaly was apparently not considered successful by his successors, including his own student al-Nasawī, since it was not adopted even in the *zīj*es that were most directly influenced by his work. Nor was his equation of centre for Mars with a slightly increased maximum value of 11;30° used in any later *zīj*es. Kūshyār's correction to the true position of Mars was known to the twelfth-century bio-bibliographer al-Bayhaqī and was included in a slightly different form in the above-mentioned anonymous *Sulṭānī Zīj*, but does not seem to have been commonly applied.

Although the format with separate tables for all functions involved and the consistent use of displacements were the only innovative elements of Kūshyār's planetary equation tables that later became common, it can be seen that his actual tabular values for the equations (like several other tables from the

*Jāmiʿ Zīj* such as the cotangent and spherical astronomical tables) found their way into later *zīj*es. For example, the peculiar deviations around the maxima of the equation of centre and the equation of anomaly of Saturn, which were not present in Yaḥyā ibn Abī Manṣūr, Ḥabash al-Ḥāsib and al-Battānī, can be found in the mid thirteenth-century *Shāmil Zīj* (anonymous but possibly by Athīr al-Din al-Abharī) and in the official product of the Īlkhān observatory at Maragha, the *Īlkhānī Zīj* by Naṣīr al-Dīn al-Ṭūsi, finished in 1272. It can be noted that the displacements of the tables were not an essential feature in the transmission and could be removed (as in the *Shāmil Zīj*) or modified (as in the *Īlkhānī Zīj*). The same two *zīj*es also appear to have copied Kūshyār's table for the equation of centre of Venus, possibly through the intermediary of the *ʿAlāʾī Zīj* of al-Fahhād.

All in all we may conclude that, as a more accessible alternative to the contemporaneous *magnum opus*es of Abū l-Wafāʾ, Ibn Yūnus and al-Bīrūnī, Kūshyār's *Jāmiʿ Zīj* was quite influential and may be assumed to have been widely used for teaching purposes and by practising astronomers and astrologers.

## IV.8. Modifications to the tables for Mars

Due to its large eccentricity and its nearness to the Earth, which requires a very large epicycle, Mars posed a special problem for astronomers working with Ptolemy's geocentric planetary models. It is for this reason that already in early Islamic *zīj*es we find significant adjustments to the parameters and tables for Mars, more than for the other planets. For example, in both surviving thirteenth-century recensions of the *Mumtaḥan Zīj* of Yaḥyā ibn Abī Manṣūr the mean motion tables for Mars were replaced by the ones of the tenth-century Baghdad astronomer Ibn al-Aʿlam, who was famous for his observations and whose tables for Mars had the reputation to yield the best positions for this planet among all *zīj*es of the period.[112] In a joint research project, Mohammad Mozaffari and myself have started to investigate in more detail the problems in predicting the true position of Mars on the basis of Ptolemy's geocentric geometrical model, to compare the results yielded by historical tables with modern calculations, to study available historical records of observations, and to explain the rationale behind the changes that we find in the tables for Mars in Islamic *zīj*es. Here I will only describe the modifications that Kūshyār made to his tables for Mars and their effect on the positions found from the *Jāmiʿ Zīj* to the extent that I currently understand them.

Kūshyār starts Chapter I.4.1 of the *Jāmiʿ Zīj* with an extensive discussion of his scrutiny of earlier astronomical observations and planetary tables on the basis of his own observations of conjunctions and meridian altitudes.[113] He states that he found al-Battānī's *zīj* to be the most trustworthy due to its reliance on Ptolemy's observations and its nearness in time. Kūshyār then states that he set up his own mean motion tables for the meridian of 90° (instead of Raqqa) and for the Persian calendar (rather than the Arabic and Byzantine calendars that al-Battānī had used) in order to make planetary positions easier to calculate. Finally he indicates that he corrected 'the defect … in the composition and presentation of some equations' by a method he will explain in Book IV, and that the differences amount to some degrees for Mars, but are negligible for the other planets.

At the end of Chapter IV.4.5, Kūshyār states specifically that he found some divergences from Ptolemy's tables in the results of his own calculations of true positions of Mars.[114] In particular, he found the 'variation of the epicycle radius' (*ikhtilāf niṣf quṭr falak al-tadwīr*) to be smaller than that found from the *Almagest* by 1⅕ degrees at the furthest distance and by 2⅕ degrees at

112 van Dalen, 'A Second Manuscript', pp. 31–32.

113 Bagheri, *az-Zīj al-Jāmiʿ*, pp. 35–36 (translation) and Arabic pp. 22–23.

114 Bagheri, *az-Zīj al-Jāmiʿ*, pp. 148–49 (translation), 161–62 (commentary) and Arabic pp. 107–09.

the nearest distance. The discussion of Kūshyār's variant of Ptolemaic interpolation in Section IV.7.2 makes it plausible that the Persian astronomer here refers to differences that are the result of his peculiar method of calculating the equation of anomaly. The formulation 'differences in the variation of the epicycle radius' would suggest that he found differences in the maximum equation of anomaly, but this is impossible because his method of interpolation leaves the maximum equation unaltered with respect to an exact calculation. However, at the apogee of the deferent Kūshyār's method causes differences from Ptolemy of around –70′ for values of the true anomaly near 90° and even up to +135′ when the planet approaches the perigee of the epicycle. At the perigee of the deferent these numbers are respectively +122′ and –252′. It is thus possible that Kūshyār made sample calculations for different values of the true anomaly on the basis of the *Almagest* and his own tables and in this way found the differences that he mentions.

In the *Jāmiʿ Zīj* itself, Kūshyār does not give any details of observations that he made, but from a quotation attributed to him after the colophon of Book I in manuscripts **C** and $\mathbf{C}_2$ (see Plate 2b) we know that he 'observed' a Saturn-Mars conjunction in July 993.[115] Although we will see below that there are serious problems with this observational report, several types of modifications to the Ptolemaic equation tables for Mars that Kūshyār carried out in different versions of the *Jāmiʿ Zīj* do suggest that the observation in 993 was part of a larger programme. I will now describe these modifications to the tables and wherever possible try to link them to the differences between positions found from al-Battānī's tables and actual historical positions of Mars. Figure 6 displays the errors in the longitude of Mars over the course of the period AD 990–1010, produced for the parameters of the *Ṣābiʾ Zīj* by my programme Historical Horoscopes, i.e., with idealised equations of centre and anomaly. Since the equation tables in al-Battānī's *zīj* have errors of at most some minutes, the overall picture would not be significantly different if the actual tables were used. Also the effect of using Ptolemaic interpolation instead of an exact equation of anomaly is much smaller than the differences shown in the figure. Thus we see that al-Battānī's longitudes of Mars were on the average too large by roughly a degree, and that occasional spikes, always coinciding with the planet being near the perigee of the epicycle, i.e., during its retrograde motion and in opposition to the Sun, even reached differences of more than 6 degrees. Obviously this left room for improvement, be it by fitting the mean motions of Mars better with the retrograde motion of the planet or by adjusting the equations.

115 See p. 15, footnote 4 and the further discussion in Section IV.8.3.

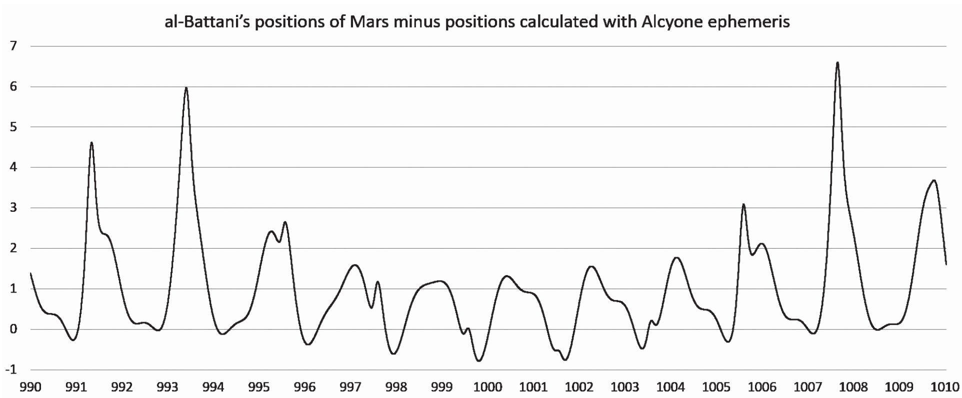


Figure 6: Errors in the longitudes of Mars in the period AD 990–1010 as produced by the tables in al-Battānī's *Ṣābiʾ Zīj*. The historical longitudes were computed by means of my programme Historical Horoscopes with idealised equations, the modern longitudes by Alcyone Ephemeris, and the graph was created in Microsoft Excel. For a similar illustration of the errors in Ptolemaic longitudes for Mars, see Voelkel and Gingerich, 'Giovanni Antonio Magini's "Keplerian" Tables', pp. 250–51.

### IV.8.1. New maximum equation of centre

As we have seen above, Kūshyār's table for the equation of centre of Mars has a maximum value of 11;30° rather than the Ptolemaic 11;25°. This difference is too small to be able to judge from an accurate table to minutes whether it was computed anew on the basis of an eccentricity of 6;2,30 or as an homothetic table based on a table for eccentricity 6;0 (the difference amounts to at most 26″ and the maximum appears for mean centrum 96° in both cases). However, Ptolemy's table for the equation of centre of Mars, found in the *Handy Tables* and also in the *Mumtaḥan Zīj* and in the *zīj*es of Ḥabash al-Ḥāsib and al-Battānī, shows a type of clustering of positive and negative errors of ±1 and ±2 minutes in the second quadrant that is very similar to the clustering in Kūshyār's table with maximum 11;30°. For the value 6;2,30 of the eccentricity that corresponds to Kūshyār's maximum equation of centre, his table has a total of 79 errors out of 180 values with a standard deviation of 45″. However, from a homothetic table obtained by multiplying al-Battānī's table by 1;0,26 (rounded from the quotient of the maximum values 11;30 and 11;25), it differs by a single minute in only 14 out of 180 values, yielding a standard deviation of only 17″. Thus it is clear that Kūshyār computed his equation of centre by rescaling Ptolemy's table, which he most probably took from al-Battānī.

The difference of 5 minutes in the maximum equation of centre leads to a change in the maximum equation of anomaly of only +1½′ when the epicycle centre is at the apogee of the deferent and somewhat less than –3′ at the perigee. However, it results in a change in the calculated true longitudes of Mars of

up to a quarter of a degree.[116] This is still a small effect in comparison with the differences of up to 6 degrees between al-Battānī's tables and the actual longitudes of Mars that occurred periodically at times when the planet was near the perigee of the epicycle. Pedersen (for Ptolemy and Saturn) and Mozaffari (for Muḥyī l-Dīn al-Maghribī and Mars) explain how a value for the eccentricity can be derived from three observations of the planet at moments when it is in opposition to the mean Sun.[117] It seems possible that Kūshyār carried out such observations, found a slightly larger maximum equation of centre than that induced by Ptolemy's tables, and accordingly modified his table for the equation of centre.

## IV.8.2. Correction of the true position of Mars

The three Cairo manuscripts as well as the Leiden and Berlin manuscripts of Kūshyār's *Jāmiʿ Zīj* include a table for the 'Correction of the true position of Mars' (*iṣlāḥ taqwīm al-mirrīkh*), here edited as Table 30b (see Plate 10 for the table in **C**). This table is mentioned by the twelfth-century bio-bibliographer al-Bayhaqī (see pp. 20–21), and a version of it is also included in the late thirteenth-century anonymous *Sulṭānī Zīj*, which like the *Ashrafī Zīj* provides tables for computing planetary positions according to a number of earlier *zīj*es.[118]

[116] The difference of at most 5 minutes in the maximum equation of centre leads to a difference of the same magnitude in the true anomaly. The highest rate of change of the equation of anomaly takes place around the perigee of the epicycle, i.e., when the true anomaly is around 180°. For Mars this maximum rate of change is 2;43° per degree of true anomaly when the epicycle centre is nearest to the Earth. However, whenever the equation of centre is close to its maximum, the epicycle centre is near its mean distance from the Earth, where the maximum daily rate of change is only 1;56° per degree of true anomaly. Hence a difference of 5 minutes in the true anomaly will lead to a difference of at most $5 \cdot 1;56°/60 \approx 10'$ in the equation of anomaly. Together with the 5 minutes contributed by the equation of centre itself, this adds up to approximately a quarter of a degree when both changes are in the same direction.

[117] Pedersen, *A Survey of the Almagest*, pp. 271–76 and Mozaffari, 'Muḥyī al-Dīn al-Maghribī's Measurements of Mars'.

[118] This work was first mentioned in Kennedy, 'A Survey of Islamic Astronomical Tables', p. 129 (no. 25). Kennedy followed the catalogue of the Majlis Library in naming the authors as Quṭb al-Dīn al-Shīrāzī and Shams al-Dīn al-Bukhārī (both late 13th/early 14th c.). As far as I know, Mohammad Mozaffari was the first to have a closer look at the *Sulṭānī Zīj* and to use it in his research; cf. especially Mozaffari, 'The Orbital Elements', p. 73, note 29 and Mozaffari, 'Muḥyī al-Dīn al-Maghribī's Measurements of Mars', pp. 218–19. Kūshyār's tables for the correction of the true position of Mars, explicitly attributed to *kiyā* ('master') *Kūshyār* and the *Jāmiʿ Zīj*, appear on fols 82v–83r. Here the correction itself and the interpolation function are tabulated separately as single functions, each on a page of their own. The interpolation coefficients agree with the *Jāmiʿ Zīj* except for a slide in the first two columns, but the correction has a shift of −53° instead of Kūshyār's −59° and a maximum value of 4;6° instead of Kūshyār's 4;12°. Both deviations are remarkable, because the *Sulṭānī Zīj* itself displaces its

Only in manuscript **C**, fol. 14r, do we find instructions for the use of this table. These appear as the third quarter of Chapter 23, which, however, is incorrectly numbered 24 because the contents of Chapter 22 (on the true position of the Moon) was repeated under the heading of Chapter 23. The passage runs as follows (with my own translation):

المرّيخ خاصّة. نأخذ بمركزه المعدّل إصلاحه وبخاصّته المعدّلة دقائق نسب الإصلاح، ونضرب بعضها في بعضٍ، وننقص من تقويمه أبدًا. وهذا الجدول موضوع بعد جداول تعاديل المرّيخ، وبه يقرب تقويمه من الصحّة.

> Mars is special. We take its correction (*iṣlāḥ*) with its true centrum and the interpolation minutes of the correction with its true anomaly. We multiply one by the other and always subtract ⟨the result⟩ from its true position. This table is drawn up after the tables of the equations of Mars, and with it its true position approaches the truth.

**L** does not include the specific instructions for Mars in Chapter I.29 (fol. 8v) although it contains the correction table. In $\mathbf{C_2}$ and **B** the chapter on the true positions of the planets (in both manuscripts also numbered 23 according to the tables of contents) is missing entirely. The chapter on true planetary positions is included as Chapter I.4.8 (following the numbering of all other manuscripts) in Bagheri's edition.[119] Bagheri did not have access to manuscript **C**, whereas Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213, which was available to him, does not contain the section on correcting the equation of Mars.

The table for the correction consists of the actual correction, divided into a part for signs 0 to 5 and a part for signs 6 to 11, and interpolation coefficients, divided into four parts for three consecutive signs each. Both the correction and the interpolation coefficients are essentially continuous functions with arguments 0 to 360°, and I will describe them thus. The correction function has the general shape of a planetary equation, i.e., it resembles a sinusoidal function, but has its extreme values somewhat displaced from the midpoints between the zero values. The zero values occur for arguments $4^s 1°$ and $10^s 1°$, which suggests that the table was shifted by −59°, just like the equation of centre. Since the argument of the table is indicated in the column headers and in the instructions to be the true centrum, and supposing that the correction

equation of centre for Mars by −59°, whereas the occurrence of a different maximum correction suggests that Kūshyār prepared more than one version of this table. The instructions for the use of the tables are given in Arabic in the headers of the tables and in Persian in the margin of the tables as well as in Chapter III.5 (fols 78v–79r), entitled 'On extracting the true position of Mars according to the *Jāmiʿ Zīj*' (*Dar bīrūn awardan taqwīm-i mirrīkh ba-ḥasb-i Zīj-i Jāmiʿ*). All three renderings state that the tables should be entered with the true centrum and the true anomaly.

[119] Bagheri, *az-Zīj al-Jāmiʿ*, p. 39 (translation) and Arabic p. 27.

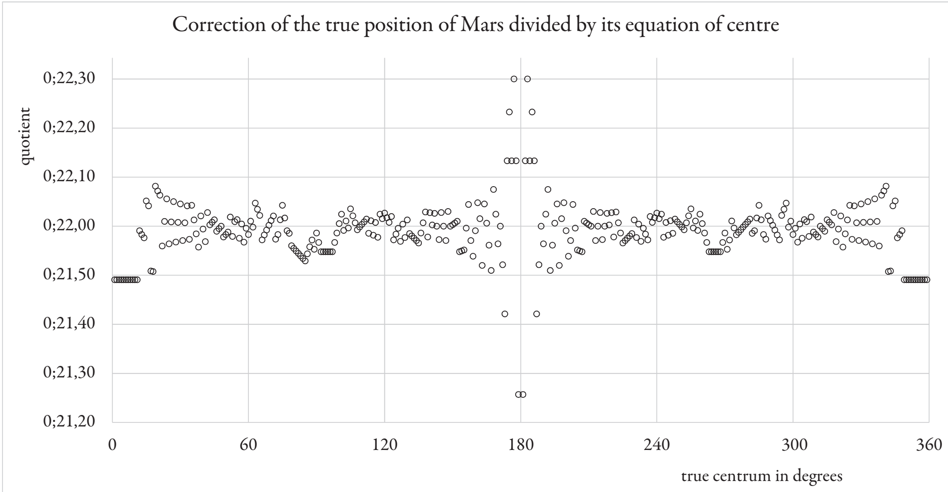


Figure 7: Kūshyār's values for the correction of the true position of Mars divided by his equation of centre. The values clearly cluster around 0;22. The outliers around argument 180° are a result of the division by numbers close to zero with a fixed number of sexagesimal fractional digits and hence a large relative error.

is symmetrical around the apogee and perigee of the deferent, we would expect the shift to be −47°. So like for the variation tables explained by Glen Van Brummelen (see Section IV.7.2) it is possible that the table was in fact computed with the mean centrum as the argument.

After having removed the shift, we find that no good agreement with the table can be reached for any parameter value if we assume that it was computed as an equation of centre or central equation of anomaly. In fact, the maximum correction of 4;12° is reached for an (unshifted) argument near 96°, although a directly computed equation of centre with this maximum value (based on an eccentricity of 2;12$^p$) would reach it near 92° and a central equation of anomaly (based on an epicycle radius of 4;24$^p$) near 94°. It thus seems plausible that the correction was calculated as a homothetic table based on an equation of centre with an eccentricity near 6$^p$ or on an equation of anomaly with an epicycle radius near 6$^p$. Here the obvious candidates are Ptolemy's or Kūshyār's own table for the equation of centre of Mars, since their underlying eccentricity 6;0 or 6;2,30 produces a maximum value near 96°. We have already seen that Kūshyār computed his own equation of centre for Mars by multiplying Ptolemy's values by 1;0,26. If we divide Kūshyār's correction of the true position of Mars by his values for the equation of centre, we observe a very clear clustering around 0;22 (see Figure 7). Indeed, if we multiply Kūshyār's equation of centre by 0;22, a very good agreement (only 27 errors of ±1′ in 180 values) is reached except between arguments 80 and 100, where 19 of the total of 27 errors appear in two negative clusters and the recomputation yields a maximum value 4;13 instead of 4;12. On the other hand, an excellent agreement

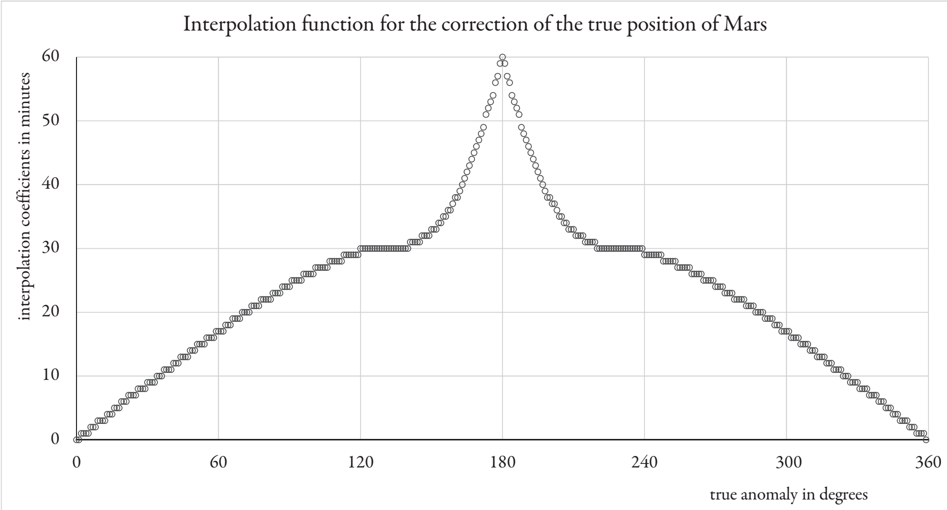


Figure 8 : The interpolation function for Kūshyār's correction of the true position of Mars. The argument of the table is 'the epicycle' (*al-tadwīr*), i.e., the true anomaly.

is reached between arguments 80 and 100 if we use the more accurate quotient 4;12/11;30 ≈ 0;21,55 of the respective maximum equations, but this leads to 90 errors of ±1 and ±2 minutes for the other arguments. We may thus conclude that Kūshyār computed his correction for the true position of Mars by multiplying his values for the equation of centre by the rounded quotient 0;22 and then either adjusting the values between arguments 80 and 100° by hand or calculating them more accurately with the multiplier 0;21,55.[120]

The interpolation function for Kūshyār's correction of the true position of Mars is split into four chunks of 90°, but, as mentioned above, the values run continuously so that it can be considered as a single function. The value zero appears at argument 12° and the maximum value 60′ at argument 192°, suggesting that the table was shifted by +12° if we may make the plausible assumption that the resulting correction is symmetric with respect to either the line connecting the epicycle centre with the Earth (i.e., around $a_v = 0° / 180°$) or the line connecting the epicycle centre with the equant point (i.e., around $a_m = 0° / 180°$). The shift of +12° would be appropriate if the argument of the table were the mean anomaly, but again the header of the argument column

[120] Theoretically it is also possible that Kūshyār calculated the correction as a homothetic table based on Ptolemy's table for the equation of centre of Mars. However, this would require a less elegant scaling factor near 0;22,8 and hence also requires a less plausible explanation of the clusters between 80 and 100°. One would expect that also the correction table with maximum 4;6° recorded in the *Sulṭānī Zīj* (see footnote 118) was computed as a homothetic table. However, I have not been able to confirm this, and even if it were, Kūshyār's equation of centre was clearly not the base table. This makes it less probable that the table stems from the Persian astronomer himself.

in the table and the instructions suggest that the interpolation function is to be taken with the true anomaly as the argument. As can be seen from Figure 8, the interpolation function also has a rather remarkable shape and obviously cannot have been computed as a single differentiable function deriving directly from a planetary model of Ptolemaic type. Most likely it was built up piecewise from more regular functions for arguments 0 to 120° on the one hand and arguments 120 to 180° on the other (with the second half of the table following from the symmetry of the function).

Taken together, Kūshyār's table for the correction of the true position of Mars and the corresponding interpolation function have the following effect on the longitude of the planet. As we have seen above, the instructions for the use of the correction table in manuscript **C** and those in the late thirteenth-century anonymous *Sulṭānī Zīj* prescribe that the correction is taken with the true centrum as the argument and the interpolation minutes with the true anomaly, and that the product of these two must always be subtracted from the uncorrected true position of Mars that was previously found. Here I will for the time being assume that the tables are to be used with the mean centrum and mean anomaly as the arguments, as the shifts suggest under the assumption that the correction has the same symmetries as the original Ptolemaic model. Since the correction itself is a multiple of the equation of centre (namely, as we have seen above, 0;22 times Ptolemy's equation of centre for eccentricity 6;0), it has the effect of *increasing* the equation of centre by a factor between 0 and 0;22 for values of the centrum between 0° and 180° (where Ptolemy's equation of centre is subtractive), and of *reducing* the equation of centre by the same factor for values of the centrum between 180° and 360° (where Ptolemy's equation is additive). The largest adjustment of the true position of Mars (always in negative direction) will therefore take place near the mean distances of the epicycle centre from the Earth, and no corrections will take place when the epicycle centre is at the apogee or perigee of the deferent.

The interpolation function assures that the magnitude of the correction is adjusted depending on the position of the planet on the epicycle. When the (unshifted) mean anomaly is zero, no correction is carried out since the interpolation coefficient is zero. When the equation of anomaly is near its maximum value, roughly for anomaly values between 120° and 140° (and symmetrically between 220° and 240°), the correction is half of its maximum, since the interpolation coefficients are equal to 30′ over a long stretch of their argument. Finally, when Mars comes close to its nearest position to the Earth, the correction spikes to its maximum value due to the spike in the interpolation function. We thus note that the total correction is zero when the epicycle centre is at the apogee or perigee of the deferent, and further when the planet is at the apogee of the epicycle. On the other hand, it reaches its maximum value when the epicycle centre is near its mean distance from the Earth and the planet is

nearest to the Earth, i.e., when the mean centrum is near 96° and the mean anomaly near 180°.

I have not yet made a detailed analysis of the effectiveness of Kūshyār's correction. A first comparison with actual longitudes around the year AD 1000 calculated by means of Alcyone Ephemeris indicated that the average error in the true longitudes of Mars becomes close to zero thanks to Kūshyār's correction, but that the spikes in the differences between the results from al-Battānī's tables and actual longitudes of Mars (cf. Figure 6) generally remain. A closer investigation will be included in the forthcoming study of tables for Mars in Islamic *zīj*es by Mohammad Mozaffari and myself.

## Change in the variation tables in the Cairo manuscripts

In the three Cairo manuscripts the tables for the variation in the equation of anomaly of Mars have been slightly adjusted (see the separate editions labelled Table 30a). Since the Leiden and Berlin manuscripts do not contain this adjustment, it is as yet unclear whether it is related to the correction of the true position of Mars. The accompanying interpolation function is the same as that in all other manuscripts, which was computed by dividing the central equation of anomaly by its maximum value (cf. Section IV.7.2). The variation tables in the Cairo manuscripts have the same maximum values as those in the other witnesses, namely 5;50° in the table for nearest distance and 4;25° in the table for furthest distance. These maximum values also occur for the same arguments as in the other manuscripts, namely at respectively 133° and 313°, around which the tables are symmetric. Thus, in this case as well, we may assume that the tables were shifted by −47°, in agreement with the fact that the true centrum is indicated to be their argument, although Van Brummelen's analysis showed that the variation tables in the other five manuscripts were computed as a function of the *mean* centrum.

The only obvious difference between the variation tables in the Cairo manuscripts and those in the five other witnesses is the location of the zero values, i.e., of the transitions from nearest to furthest distance and vice versa. In the other witnesses these were located at arguments 52° and 214° of the shifted tables, corresponding to mean centrum 99° and 261°, and hence accurately to the instances where the epicycle centre reaches its mean distance from the Earth. In the Cairo manuscripts we find the zero values at arguments 49° and 217°, corresponding to mean centrum 96° and 264° and hence to the instances where the equation of centre reaches its maximum value.

I cannot provide a plausible justification for this adjustment, since, as explained in Section IV.7.2, the original tables were set up to produce the equation of anomaly as homothetic functions of the central equation of anomaly. An obvious modification, similar to that made by Ptolemy to his planetary interpolation functions when reworking the tables from the *Almagest* into

the *Handy Tables*, would be the change of the independent variable from the mean centrum (that underlie the original tables according to Van Brummelen's analysis) to the true centrum (as indicated in the headers of the tables and in the instructions). But such a change would place the transitions from nearest to furthest distance near values 81° and 279° of the unshifted argument, and hence near arguments 34° and 232° of the shifted table, very different from what we find in the Cairo manuscripts. A closer look at the differences between the two types of variation tables also shows that the table in the Cairo manuscripts is not simply a rescaling of the table in the other witnesses, since the largest differences between the two tables do not appear at the transitions but somewhere in the middle between the maxima and the transitions.

I tried an alternative method of computation based on the observation that the transitions of the variation in the Cairo manuscripts coincide with the maximum values of the equation of centre and that the two functions have similar curvatures on certain parts of their domains:

$$v'(c_m) = -4;25 \cdot \frac{q(c_m + 96^\circ)}{q_{\max}} \qquad \text{for furthest distance, and}$$

$$v'(c_m) = -5;50 \cdot \frac{q(c_m - 84^\circ)}{q_{\max}} \qquad \text{for nearest distance}$$

(where $q$ stands for the absolute value of the equation of centre). This produces an acceptable fit if an exact equation of centre for eccentricity 6;0 is used, namely only 27 differences of ±1 minute in the 97 values for the furthest distance and 36 differences of ±1 minute in the 85 values for the nearest distance. This result cannot be significantly improved upon by starting from al-Battānī's table for the equation of centre (32/32 differences of ±1 minute) or Kūshyār's own table for the equation of centre (31/26 differences of ±1 minute). However, for comparison, if the argument of the variation tables in the other five manuscripts is rescaled linearly in order to shift the transitions between nearest and furthest distance to the arguments for which they are found in the Cairo manuscripts, the results show systematic differences reaching as much as 20 minutes from the values in the Cairo manuscripts.

We may thus tentatively conclude that the variation tables for Mars included in the Cairo manuscripts were an attempt by Kūshyār to adjust his calculation of the equation of anomaly in such a way that the constant by which the central equation of anomaly is multiplied to obtain the equation of anomaly for any given value of the mean centrum, is no longer proportional to the maximum equation of anomaly but to the equation of centre. It seems doubtful that this is a sensible strategy. It is certainly a disadvantage that this method of calculating the equation of anomaly no longer produces the central equation of anomaly when the epicycle centre is at its mean distance from the Earth but around six degrees before or after it. Since Kūshyār only carried out this mod-

ification of his variation tables for the planet Mars, and since this modification is only included in the Cairo manuscripts, it remains possible that they have some kind of relationship with his table for the correction of the true position of Mars, which needs to be investigated further.

### IV.8.3. Adjustments of the mean motions

As we have seen in Section IV.5.2, Kūshyār appears to have made numerous small adjustments to the mean motion tables during the long process of his work on the *Jāmiʿ Zīj*. The tables that I consider to be the original ones are included in manuscripts **FHCC$_1$C$_2$**, whereas the ones in **YLB** are generally more accurately computed for the parameters that Kūshyār derives from al-Battānī's *Ṣābiʾ Zīj* in Table 12. Only for Mars the differences between the two versions are so significant that the subtables for years and months had to be edited separately in Tables 28a and 29a. In particular, the underlying mean motion in longitude in **YLB** is 0;31,26,40,15,11,13 °/day, in agreement with al-Battānī's parameter listed in Table 12, whereas a value from the interval 0;31,26,41,53,42–53 °/day was used for the tables in **FHCC$_1$C$_2$**.[121] The mean anomaly of Mars conforms to the mean longitude in both versions of the tables, i.e., the mean longitude and the mean anomaly add up to Kūshyār's mean solar longitude (which is the same in both groups of manuscripts). For **FHCC$_1$C$_2$** this implies that the daily mean anomalistic motion lies in the range 0;27,41,38,53,3–14°,[122] for **YLB** that it is equal to 0;27,41,40,31,45,1°.

Since the epoch values for the Yazdigird era are the same in both groups of sources, the somewhat larger daily mean motion in manuscripts **FHCC$_1$C$_2$** accumulates to a difference in the mean longitude of Mars with respect to al-Battānī's table of exactly +1;0 degree in the year 361 Yazdigird (AD 993), the same year in which Kūshyār recorded his observation of a Saturn-Mars conjunction (on which see further below). At the moment we can only guess at the reasons why Kūshyār decided to adjust al-Battānī's daily mean motions. A comparison with modern calculations for the period AD 990–1110 indicates that this resulted in even larger errors in the longitudes of Mars than the ones produced by the *Ṣābiʾ Zīj* (cf. Figure 6 on p. 429). Therefore, reverting to al-Battānī's mean motion parameters for the version of the *Jāmiʿ Zīj* included in manuscripts **YLB** was in fact a relative improvement.

121 The daily mean motion of Mars used in manuscripts **FHCC$_1$C$_2$** is estimated in Section IV.5.4 (see the two notes to Table F on p. 385).

122 As indicated in note b to Table F on p. 385, the number of errors in the subtable for collected years in manuscripts **FHCC$_1$C$_2$** can be made zero for values of the daily mean anomalistic motion in the interval 0;27,41,38,52,54–53,12° by choosing a specific epoch value with a precision larger than shown in the table.

The difference in the parameters was noted as early as the eleventh century. Manuscript **H** reproduces on fol. 82v (see Plate 16) a note by a certain Bahrām ibn Banīmān al-munajjim, which was written in an autograph copy of the *Jāmiʿ Zīj* in the Yazdigird year 445 (1076/7). Bahrām notes that the tabulated mean motions of Mars do not agree with the daily mean motion given in Table 12 and that this leads to excessive (*fāḥish*) differences in the true position of Mars with respect to other *zīj*es. He provides a small table 'for the correction of the carelessness' (*bi-iṣlāḥ al-sahw*), which gives mean positions for collected years 381, 401, ..., 481 and mean motions for extended years 1, 10 and 20 and 'single' years 40, 60, ..., 100, 200, ..., 500. He did not simply copy these from a manuscript from the group **YLB**, since he presents the mean motions to sexagesimal thirds (as opposed to minutes in the *Jāmiʿ Zīj* itself).

As we have also seen in the commentary on the mean motion tables (Section IV.5.3), the scribe of the Berlin manuscript wrote corrected mean positions next to the subtables for collected years of the mean motion tables for the five planets (except for Saturn). These are the positions found in the manuscripts $\mathbf{FHCC_1C_2}$ (which differ from those in **YLB** by a single minute in up to half of the tabular values), but for al-Battānī's base meridian of Raqqa instead of Kūshyār's meridian of 90° (which leads to a further constant difference of up to 9 minutes for Mercury). However, in the mean motion tables for Mars there is an additional deviation from the other sources, namely a decrease of exactly 2° in all mean longitudinal positions from 1 to 581 Yazdigird and a corresponding increase of exactly 2° in all mean anomalistic positions. On the colour scans published online by the Staatsbibliothek in Berlin and the ISMI project, it can be clearly seen that all degrees of the values for collected years in both tables (as well as the zodiacal signs in the cases where these also needed to be adjusted) were corrected by erasing the original values and writing the new ones over them.

The only context in which corrections of 2° to Kūshyār's mean positions of Mars are mentioned is his note on the observation of a Saturn-Mars conjunction in AD 993, which is quoted from an autograph after the colophon of Book I in manuscripts **C** and $\mathbf{C_2}$ (see Plate 2b).[123] It states that Kūshyār would have observed such a conjunction in the evening of 6 July 993 (21 Tīr 362 Yazdigird). For noon of that day he then calculated the true position of Saturn as 0;59° Pisces and that of Mars as 1;50° Pisces, for which he decreased the mean

[123] cf. p. 15, footnote 4. The statement is quoted and translated into English in Bagheri, 'Kūshyār Ibn Labbān's Glossary', pp. 145–46 (note that Bagheri gives the *Gregorian* equivalent 11 July 993 of the Persian date in the text rather than the Julian). Bagheri, 'Mabḥath-i taqwīm', pp. 22–23 states that, according to Alcyone Ephemeris, the difference in ecliptic longitude between the two planets reached its 'relative minimum value' at the beginning of Mars's retrograde motion on 9 July 993.

longitude of Mars by 2 degrees and increased its mean anomaly by the same amount.

There are various unclarities, contradictions and possible mistakes in this statement. Most importantly, the actual Saturn-Mars conjunction of AD 993 took place only three months later on 20 October at around 1 am Universal Time. At that time the planets were in 27;9° Aquarius rather than in Pisces, and also their latitudes differed by as little as a quarter of a degree. On 6 July 993, Saturn and Mars were still separated by 5½ degrees in longitude and 4 degrees in latitude, so that it seems improbable that Kūshyār could have 'mistakenly' observed the conjunction on that day. It is true that Mars started its retrograde motion around this date, namely on 11 July (furthermore, Saturn was in retrograde motion from June to October).

Now let us have a look at what the astronomical tables available to Kūshyār would have indicated. According to al-Battānī's *Ṣābiʾ Zīj*, as calculated by my programme Historical Horoscopes on the basis of idealised equations, the Saturn-Mars conjunction would have taken place on 7 July 993 in the morning hours (local time at Kūshyār's base meridian of 90°, just hours removed from Kūshyār's purported time of observation!). At that time, again according to al-Battānī, Saturn and Mars were in 3;6° Pisces, and Mars would start its retrograde motion on 13 July.[124] It is clear that the longitudes of the two planets given by Kūshyār cannot refer to the time of the actual conjunction, since both modern and historical sources give the longitudes of Mars and Saturn in October in 26° or 27° Aquarius. It is also impossible that in less than 12 hours the distance between the two planets would reach 51 minutes: at noon of 7 July 993, just hours after the calculated conjunction, al-Battānī's *zīj* gives Mars in 3;7° and Saturn in 3;6° Pisces.

Preliminary investigations by Mohammad Mozaffari and myself have already shown that Ibn al-Aʿlam obtained a clearly better agreement with actual longitudes of Mars by reducing the mean longitude by approximately 1° in compar-

[124] Al-Battānī's longitude of Mars at this moment in fact had an error of 5½° due the planet's extreme nearness to the Earth: both its mean centrum and its mean anomaly were close to 180°. Note that the use of Ptolemaic interpolation instead of an exactly calculated equation of anomaly makes a neglegible difference of less than a minute of arc at this time. Interestingly, whereas Kūshyār's mean positions according to manuscripts **YLB** are practically the same as al-Battānī's, and his slightly larger equation of centre here makes no practical difference since the mean centrum is close to 180°, the use of Kūshyār's peculiar type of interpolation for the equation of anomaly explained in Section IV.7.2 reduces the longitude of Mars by more than 4 degrees to 328;59°, much closer than al-Battānī's result to the actual longitude 327;32°. If Kūshyār used this conjunction to verify his true positions of Mars, it may certainly have confirmed his idea that his type of interpolation provided better results. That this was only a coincidence follows from the fact that generally Kūshyār's type of interpolation produces longitudes of Mars with much larger errors than Ptolemaic interpolation or the exact equation of anomaly.

ison with the *Mumtaḥan Zīj* and al-Battānī. It thus seems plausible that the reduction by 2° hinted at in Kūshyār's note on the Saturn-Mars conjunction and carried out in the Berlin manuscript signifies an attempt to obtain a better agreement with observations. Remarkably, when we adjust the mean longitude by −2° and the mean anomaly by +2° and calculate the equation of anomaly exactly or by means of Ptolemaic interpolation, the Saturn-Mars conjunction is found to be on 20 October 993 between 6 and 7 pm (local time at Kūshyār's meridian) in 26;43° Aquarius, less than half a day and less than half an ecliptic degree removed from the time and place of the actual conjunction!

Further research is necessary to establish the role played by Kūshyār's adjustments to the mean longitude and mean anomaly of Mars and to his calculation of the equation of anomaly in his attempts to improve the true positions of Mars found from the tables in al-Battānī's *Ṣābiʾ Zīj*. At any rate it seems plausible that Kūshyār tried to adjust his tables on the basis of observations, including the observation of the Saturn-Mars conjunction in AD 993. Obviously his report of this conjunction is flawed and can only begin to be understood by assuming that the date that he gives for the conjunction is not the date of an observation but of a calculation on the basis of al-Battānī's mean motions and equations.

## IV.9. Lunar and planetary latitudes and stations

### Tables 37 and 37a: Lunar latitude

**Bibliography**: Sédillot, 'De la latitude de la lune'; Pedersen, *A Survey of the Almagest*, pp. 200–01; Neugebauer, *HAMA*, vol. I, pp. 80–84; Van Brummelen, *Mathematical Tables*, pp. 189–91; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.9, pp. 39–40 (translation), 47–48 (commentary) and Arabic pp. 27–28; Chapter IV.4.8, pp. 151–52 (translation), 163 (commentary) and Arabic pp. 112–13.

In the eight manuscripts that include the tables from Kūshyār's *Jāmiʿ Zīj* we find two different tables for the lunar latitude. Manuscripts **FHCC$_1$C$_2$** include a table with values to minutes (cf. Plate 11), and **YLB** a table with values to seconds and tabular differences. The argument of both tables is the distance of the Moon from its ascending node, called in Arabic 'share of the latitude' (*ḥiṣṣat al-arḍ*, translated by Bagheri as 'argument of the latitude'). Both tables give 5;0 (respectively 5;0,0) as the maximum latitude. Both have a quadruple entry, i.e., one and the same value is valid for four symmetrical arguments, namely $x$, $180° - x$, $180° + x$ and $360° - x$ for any $x = 1, 2, 3, \ldots, 90°$. For the first two of these arguments the latitude is northerly, and for the last two it is southerly. An exception is made by the three Cairo manuscripts, which display the latitude with a double entry for arguments 1, 2, 3, …, 180°. The tabular differences in manuscripts **YLB** correspond exactly with the latitude values to seconds.

A recomputation for a maximum lunar latitude of 5;0,0° makes it clear that Kūshyār's lunar latitude table to seconds was not computed on the basis of the exact formula $\sin\beta(\lambda_n) = \sin\beta_{\max} \cdot \sin\lambda_n$, where $\beta$ is the lunar latitude, $\beta_{\max}$ the maximum latitude and $\lambda_n$ the distance of the Moon from its ascending node, measured along the ecliptic. Kūshyār's values are systematically larger than such a recomputation, with a maximum difference of 28″ for arguments 35–39°. Since $\beta$ is relatively small, Pedersen suggested that Ptolemy may have used the approximation $\beta(\lambda_n) \approx \beta_{\max} \cdot \sin\lambda_n$.[125] This could easily be recognised from a table to seconds since it would need to have $\beta(30) = ½\beta_{\max} = 2;30,0$. However, Kūshyār's latitude value for 30° is 2;30,18, and in general his values are systematically larger than this approximation, with a maximum difference of 20″ for argument 35°.

The explanatory text in Chapter I.4.9 of the *Jāmiʿ Zīj* accounts for the unusual method of computation that was used for this table.[126] It instructs the

[125] Pedersen, *A Survey of the Almagest*, p. 201.

[126] This section is found with only small variations in **F** fols 8v–9r (Chapter I.4.9), **C** fols 14r–v (Chapter I.24), Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1, fol. 10r–v (Chapter I.24), **Y** fol. 239v (Chapter I.4.9), and **L** fols 8v–9r (Chapter I.30). The section is missing from **C$_2$**, **H** and **B**, in each of which it would have been Chapter I.24 according to

reader to calculate the lunar latitude in a way corresponding to the modern formula $\tan\beta(\lambda_n) = \tan\beta_{\max} \cdot \sin\lambda_n$ or by an equivalent second procedure. This method implies that the latitude is measured orthogonally to the lunar orbit rather than to the ecliptic, in the same way as the second declination is measured orthogonally to the ecliptic instead of to the equator. It provides a good agreement with the table, with deviations of at most ±2″. There are three large clusters of positive errors for arguments between 30 and 90° whose origin I have not investigated further.[127]

Since the differences between Kūshyār's lunar latitude table to seconds on the one hand and the exact method and Ptolemy's approximation on the other are smaller than half a minute, we need to consider the possibility that the lunar latitude table to minutes was computed by means of any of these methods. This table shows 24 differences of +1′ from an exact recomputation, partially occurring in small groups, and 16 differences of +1′ from the approximation by a sine function (that the value for 30° is exactly half the maximum lunar latitude is insignificant for a table to minutes, since the other methods produce the same value). However, since all its values can be obtained by rounding the values from the table to seconds in **YLB** in the standard way, this is very probably how the table was computed. This conclusion implies that Kūshyār already had the table to seconds at hand when he decided to include the table to minutes in an early version of his *zīj*.

Kūshyār could not have copied his lunar latitude table from the *Ṣābiʾ Zīj*, since al-Battānī includes a table to seconds based on the exact formula in his combined table of the solar and lunar equations.[128] This table generally has errors of up to ±4 seconds, but larger, apparently systematic, errors of up to +19 seconds in a faulty section for arguments 134–138. Kūshyār's lunar latitude table can also be found in three *zīj*es that frequently incorporate materials from the *Jāmiʿ Zīj*, namely the *Zīj al-Qirānāt*, the *Mufrad Zīj*, and the *Dustūr al-munajjimīn*.[129]

the respective tables of contents. Towards the end of the chapter in **F**, Kūshyār states that the method that many specialists agree upon (involving two sines instead of two tangents, i.e., the correct method) is only approximate.

[127] These are results of my work for the DFG project 'Der Dustūr al-munağğimīn als Quelle für die Geschichte der Ismaʿiliyya und ihre astronomisch-astrologischen Vorstellungen' (cf. p. xv).

[128] See van Dalen and Pedersen, 'Re-editing the Tables', pp. 424–26 for a new apparatus for this table. The edition in Nallino, *Al-Battānī sive Albatenii*, vol. II, pp. 78–83 (with commentary on p. 227) is generally faithful to the Escorial manuscript, but Nallino made several mistakes in correcting scribal errors and unsuccessfully attempted to improve the values for arguments 88–92 and 131–138 on the basis of a logarithmic recomputation by Schiaparelli.

[129] See, respectively, Paris, BnF, arabe 6913, fols 75v and 76v (two copies of the same table); Cambridge, University Library, Browne O.1, fol. 106r, and Paris, BnF, arabe 5968, fol. 98r.

## Tables 38–42: Planetary latitudes

**Bibliography**: Neugebauer, *HAMA*, vol. I, pp. 206–26 (*Almagest*) and vol. II, pp. 1006–16 (*Handy Tables*); Pedersen, *A Survey of the Almagest*, Chapter 12, pp. 355–86; Van Brummelen, *Mathematical Tables*, Chapter 14, pp. 338–72; Riddell, 'The Latitudes of Venus and Mercury'; Swerdlow, 'Ptolemy's Theories of the Latitude'; van Dalen, 'Tables of Planetary Latitude' (with a transcription of the latitude tables for Venus and Mercury from manuscript **F** on p. 322); Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.10, pp. 40–41 (translation), 47–48 (commentary) and Arabic pp. 28–29; Mozaffari, 'Planetary Latitudes'.

With the exception of the Cairo manuscripts, the surviving copies of Kūshyār's *Jāmiʿ Zīj* provide for each of the five planets a combined table for the latitudes and the first station. These tables have a double entry, and the argument is given in steps of six degrees and is expressed in signs and degrees. The tables for the superior planets have columns for the northern and southern limits, while those for the inferior planets have columns for the deviation and slant. Furthermore, the tables have a shared interpolation function for the two latitude functions and a column displaying the first station. Manuscripts **C** and $\mathbf{C_1}$ have one table for the latitudes of all five planets, which share the interpolation function, and one for their stations, but the values are the same as those in the other manuscripts.

The instructions in Chapter I.4.10 of the *Jāmiʿ Zīj* indicate that, in order to find the interpolation coefficients from the latitude tables, the 'actual corrected centrum' (*al-markaz al-muʿaddal al-ḥaqīqī*, translated by Bagheri as 'true adjusted centrum') must be used.[130] A paragraph at the end of Chapter I.4.8 explains that this quantity can be found from the true centrum obtained from the tables in the *zīj* by adding 7 degrees for Saturn, 12 degrees for Jupiter, 47 degrees for Mars, 48 degrees for Venus and 26 degrees for Mercury.[131] The same rules are indicated alongside the tables for planetary latitudes and stations in manuscripts **FHYB**. Furthermore, the argument of the interpolation function is measured from the northernmost point of the deferent of the superior planets, which is removed from their ascending node by 90°. To obtain this argument, the 'actual corrected centrum' must be decreased by the distance from the apogee of the planet to the northernmost point. This is achieved by adding 50 degrees to the actual true centrum for Saturn and subtracting

[130] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 40–41 (translation), 48–49 (commentary) and Arabic pp. 28–29.

[131] Bagheri, *az-Zīj al-Jāmiʿ*, p. 39 (translation) and Arabic p. 27. The related procedure of finding the original planetary equations from Kūshyār's displaced ones is explained extensively in the chapter appended to Book II in three of the extant manuscripts of Kūshyār's *zīj*; see the edition on pp. 331–34 and Section IV.15. For a full discussion of the displacements of Kūshyār's planetary equations, see Section IV.7.1.

20 degrees for Jupiter, whereas no adjustment is necessary for Mars, because its northernmost point coincides with its apogee. For the different model for Venus and Mercury, the argument for the interpolation function is obtained by adding 90° and 270° respectively to the actual true centrum.

In Sections IV.7.3 and IV.7.4 we have seen that Ptolemy's *Handy Tables* was the source for most of the tables of planetary equations in early Islamic *zīj*es. The planetary latitude tables in the *Handy Tables* are very different from those in the *Almagest*,[132] and so it can easily be established that for the latitudes the tables from the *Almagest* remained the standard for Islamic astronomers even up to the fifteenth century. As I have shown in an article aimed at placing the double-argument tables of planetary latitude found in some late Islamic sources in their historical context, most modifications to Ptolemy's tables in the *Almagest* up to the thirteenth century were elementary.[133] The latitude theory that has survived in the *Mumtaḥan Zīj* is Indian in type and is very different from Ptolemy's.[134] In his *Damascene* or *Arabic Zīj*, Ḥabash al-Ḥasib includes the exact tables from the *Almagest* with their peculiar range of arguments 6, 12, 18, ..., 90, 93, 96, ..., 180°.[135] Also al-Battānī adheres to the *Almagest*, as for many other aspects of his *Ṣābiʾ Zīj*, but he omits the values for odd multiples of 3° between 90 and 180° and provides only a single interpolation function for all five planets combined.[136]

132 See Swerdlow, 'Ptolemy's Theory of the Latitude'.

133 van Dalen, 'Tables of Planetary Latitude'.

134 Viladrich, 'The Planetary Latitude Tables'.

135 The tables are found in Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 178r–179v and in Berlin, SBPK, Wetzstein I 90, fols 66r–67r. Unlike the *Almagest*, Ḥabash's *zīj* does not repeat the column of interpolation coefficients for every superior planet but provides a single column to be used for each of the three. As Ibn Yūnus already noted, the instructions and therewith the examples that Ḥabash supplied were incorrect; see Debarnot, 'The *Zīj* of Ḥabash al-Ḥāsib', p. 54.

136 The tables are found in Escorial, RBMSL, árabe 908, fols 224v–225r and were edited in Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 140–41 (with commentary on pp. 247–55). As usual, Nallino corrects al-Battānī's values in order to make them correspond to the *Almagest* (cf. van Dalen and Pedersen, 'Re-editing the Tables'), but since in this case the vast majority of the differences are obvious scribal errors, this does not significantly distort the picture of the relationship between the two sources. The only differences from the Greek *Almagest* that may be systematic and intentional are the values 6;12 and 7;22 for the deviation of Venus for arguments 174 and 180° (instead of Ptolemy's 5;52 and 6;22, which were adopted by Nallino without any comment). This modification, which is confirmed by the Castilian translation of al-Battānī's *zīj* (Paris, Bibliothèque d'Arsenal, MS 8322, fol. 83r), most likely originates in the Arabic translation of the *Almagest* by Isḥāq ibn Ḥunayn that was revised by Thābit ibn Qurra. While al-Ḥajjāj's translation maintains the values from the Greek, in the Tunis and London manuscripts of Isḥāq's translation and in the Judaeo-Arabic copy in Paris we also find the values 5;12 and 6;12 for argument 174° and 7;22 for argument 180°. Interestingly, both the Judaeo-Arabic copy in Paris and the London manuscript also omit the tabular values for odd

Al-Battānī is the most likely source for Kūshyār's planetary latitude tables. Most of the tables are identical throughout in the two sources; only the final parts of the northern and southern limit of Mars and the deviation of Venus show peculiar differences that turn out to be typical of Kūshyār. In the case of the northern limit of Mars, Kūshyār adjusted the values for arguments 120–132° and 150–174°, apparently in order to smoothen the function, since he removed a number of irregularities in the second-order differences of al-Battānī's table (and hence of the table in the *Almagest*). Of the southern limit of Mars, Kūshyār adjusted the values for arguments 126–144° by negative amounts ranging from −2 to −5′ and the values for arguments 156–174° by positive amounts between +6′ and +27′, apparently for similar reasons. I have not investigated a possible association with other changes to the Mars tables that Kūshyār made, such as the small change in the maximum equation of centre. On the one hand, such changes would have influenced the entire latitude tables rather than just the values for true anomalies between 120 and 240 degrees. On the other, it is around the perigee of the epicycle (i.e., for true anomaly 180°) that the largest deviations of predicted longitudes of Mars from actual ones became apparent. But since Kūshyār left the values for the round arguments 120, 150 and 180° unaltered, I consider it more likely that his motivation lay again in an attempt to smoothen the table. Kūshyār also rounded al-Battānī's interpolation coefficients for the latitudes to minutes, but this did not have a significant bearing on the results of latitude calculations.

Finally, for the deviation of Venus, for which Kūshyār modifies the values for arguments 114 to 144° by positive amounts ranging from +1′ to +9′ and the values for 156 to 174° by negative amounts between −7′ and −20′, the motivation also appears to have been a systematic smoothening of the table, which results in practically constant second-order differences over its entire range of arguments. The plausibility of this explanation follows from the fact that in al-Battānī's *zīj* none of the other latitude tables (including the one for the deviation of Mercury, which has a similarly large value for 180°) have jumps in the second-order differences as large as the northern and southern limits of Mars and the deviation of Venus.

Kūshyār's latitude tables were less influential than certain aspects of his planetary equations. Only the *Zīj al-Qirānāt* includes the same adjustments to the northern and southern limits of Mars and the deviation of Venus that Kūshyār introduced. In the *Mufrad Zīj* the planetary latitudes are arranged in a completely different way and are closer to the *Almagest* than to the *Jāmiʿ Zīj*. The *Dustūr al-munajjimīn* copied the latitude tables for the superior planets

multiples of 3°, as do al-Battānī, Kūshyār and several later Islamic astronomers. Al-Battānī's deviating values for 174 and 180° were copied in the *Dustūr al-munajjimīn* and are also found in Ibn Isḥāq's (Tunis, fl. *c.* 1200) *Tunesian Zīj*. However, most other astronomers apparently reverted to sources that more reliably reproduced the *Almagest* values.

from al-Battānī rather than from Kūshyār and introduced the earliest known double-argument tables for the inferior planets attributed to the otherwise unknown al-Maʾmūrī. Starting with al-Khāzinī's *Sanjarī Zīj* (Marw, *c.* 1120) several new characteristics were introduced into the tables of planetary latitude, especially an explicit tabulation of the first latitude of the inferior planets, two separate tabulations of the slant of Mercury with the values already increased and decreased by a tenth as prescribed by the *Almagest*, and interpolation functions to minutes that had the planetary true centrum as their argument instead of the distance from the ascending node and hence already incorporated the longitude of the ascending node.

### Tables 38–42: Planetary stations

**Bibliography**: Stahlman, *The Astronomical Tables*, Tables 51–55, pp. 156–58 and 335–39; Pedersen, *A Survey of the Almagest*, Chapter 11, pp. 329–51; Neugebauer, *HAMA*, vol. I, pp. 190–206; Van Brummelen, *Mathematical Tables*, pp. 314–26; MacMinn, 'An Analysis of Ptolemy's Treatment'; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.4.11, pp. 42–43 (translation), 49 (commentary) and Arabic pp. 30–31; Chapter IV.4.10, pp. 157–58 (translation), 166 (commentary) and Arabic pp. 117–19.

Having developed his planetary theory, Ptolemy tackles the problem of determining the beginning and end of the retrograde motions of the five planets in Book XII of the *Almagest*. He bases his theory on the fundamental Theorem of Apollonius, which he proves and then generalises from the simple epicycle model to his own planetary model involving an eccentre as well as an epicycle. Finally, in Chapter XII.8, he tabulates the first and second station for each of the planets in a fairly simple form. For any value of the true centrum of a planet, the table gives the values of the true anomaly for which the planet is respectively in its first and second station. Since the problem is symmetrical, the tabular values for the first and second station always add up to 360°. In the *Handy Tables*, Ptolemy's approach is the same, but he tabulates the true anomalies at the time of the stations as a function of the *mean* centrum rather than the *true* centrum. While the table in the *Almagest* has values only for every six degrees of the true anomaly, those in the *Handy Tables* have values for every three degrees of the mean anomaly.

Although early Islamic astronomers adopted the tables for planetary latitudes from Ptolemy's *Almagest*, in general they resorted to the *Handy Tables* for the planetary stations (as for most of the planetary equations), possibly because they considered the tabulation as a function of the mean centrum to be a significant advantage. The only exception that I have found is the *zīj* of Ḥabash al-Ḥāsib, which reproduces the columns for the first station from the *Almagest*.[137]

[137] Debarnot, 'The *Zīj* of Ḥabash al-Ḥāsib', pp. 44–45. The table is found in Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784, fol. 123r. In the manuscript Berlin, SBPK, Wetz-

The two surviving recensions of the *Mumtaḥan Zīj* both include tables for the planetary stations that were apparently taken from the *Handy Tables*, but differ from it by ±1′ or ±2′ in as many as a third of the tabular values for each planet.[138] Al-Battānī copied the table for the stations from the *Handy Tables* faithfully in the *Ṣābiʾ Zīj*.[139]

Kūshyār roughly follows the *Handy Tables* or al-Battānī, but his tables show differences of up to 6 minutes from those sources for all planets. Here it is notable that his values for multiples of 30° are almost always identical to those in the *Handy Tables* and al-Battānī, and that in between these 'nodes' we find clusters of differences of the same sign. The table for Mars, with its relatively wide range of possible values of the true anomaly for the first station,[140] gives a good indication of what may have motivated Kūshyār to make these changes. Ptolemy's table in the *Handy Tables*, which had values for every 3 degrees of the mean centrum, was clearly calculated by means of linear interpolation in between independently calculated values for a set of nodes. The first-order differences are constant between so many of these nodes that it even seems possible that the values at the nodes were manipulated in order to produce these constant differences. The nodes cannot all be unambiguously determined, but it seems plausible that only the values for arguments 0, 12, 24, ..., 72, 90, 105, 120, 132, ..., 180° were independently calculated. Whereas the tabular differences change quite smoothly between arguments 48 and 132°, there are large jumps outside this region which may have inspired Kūshyār to smoothen the table. Since his table is equal to Ptolemy's for every multiple of 30° of the argument (except for a difference of +1′ for a true centrum of 180°), it seems plausible that Kūshyār himself interpolated between these nodes (half of which were themselves the result of Ptolemy's application of linear interpolation). This seems to be confirmed by the fact that, with some small exceptions, the second-order differences of Kūshyār's table are almost constant between multiples of 30° and show jumps at these nodes.

stein I 90, which is further removed from the original *zīj*, the columns for the first and second station from the *Almagest* have been gathered in two separate tables on fol. 61r–v.

138 See van Dalen, 'A Second Manuscript', p. 25. The tables for the first and second station appear as the sixth and seventh columns in the tables for the equations of the five planets both in the Escorial and in the Leipzig manuscript. Leipzig, Universitätsbibliothek, Vollers 821 also has a separate table for the planetary stations on fols 57v–60r, which contains basically the same tabular values.

139 Nallino, *Al-Battānī sive Albatenii*, vol. II, pp. 138–39 (with commentary on pp. 245–47). In some cases Nallino unjustifiably restored deviating values in the Escorial manuscript of al-Battānī's *zīj* to those from the *Handy Tables*, but in general his edition of this table is reliable.

140 While for the other planets the place of the stations expressed in true anomaly varies by at most little more than 3 degrees (2;43° for Saturn, 3;3° for Jupiter, 2;26° for Venus and 2;40° for Mercury), for Mars the range of possible values is 11;40°.

For the other planets the tabular differences of the tables of the stations in the *Handy Tables* are regular enough not to see any obvious reason to smoothen them, but Kūshyār nevertheless deviates as described above. Since his values for multiples of 30° are generally equal to those in the *Handy Tables* (or occasionally differ by only ±1 minute), whereas the differences in values in between these multiples are larger and appear in clusters, he is unlikely to have performed an entirely new computation of the stations based on a different algorithm from Ptolemy. Indeed, the explanatory text in Book I of the *Jāmiʿ Zīj* and the proofs in Book IV give no evidence of the use of a different method.

Kūshyār's tables for the planetary stations were copied in the *Zīj al-Qirānāt* and the *Dustūr al-munajjimīn*.[141] Both works have separate tables for the planetary stations and for the planetary latitudes. However, this does not necessarily imply that the tables were copied from a manuscript related to the Cairo group, since any scribe instructed by an astronomer could have reorganised Kūshyār's original tables into the one included in these later works. In the *Dustūr* the table is explicitly attributed to Kūshyār and is preceded by extracts from Chapter I.4.10 of the *Jāmiʿ Zīj*. The *Mufrad Zīj* reproduces the table for the planetary stations from the *Handy Tables* without Kūshyār's adjustments.

[141] See, respectively, Paris, BnF, arabe 6913, fol. 77v and Paris, BnF, arabe 5968, fol. 107v.

## IV.10. Spherical astronomy

**Bibliography**: Basic information on spherical astronomical functions tabulated in *zīj*es is included in Kennedy, 'A Survey of Islamic Astronomical Tables', pp. 140–41; Neugebauer, *HAMA*, vol. I, pp. 30–45, and van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 19–20. The right and oblique ascension tables from the *Handy Tables* are edited, transliterated and extensively commented upon in Tihon, *Les* Tables Faciles *1a* and Mercier, *Ptolemy's Handy Tables 1b*. Examples of mathematical analyses of spherical astronomical tables can be found in van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 63–70 (oblique ascension) and pp. 174–95 (Abū l-Wafāʾ's extensive set of highly accurate spherical astronomical tables copied in the *zīj* by Jamal al-Dīn Ibn Maḥfūẓ al-Baghdādī and some less accurate tables calculated by al-Baghdādī himself or taken from other sources), as well as in the forthcoming volume Husson et al., *Editing and Analysing Numerical Tables*.

Almost every *zīj* of standard type contains tables for performing calculations with arcs on the celestial sphere. These tables could be used to compute quantities such as the length of daylight, the times of rising, culmination and setting of any heavenly body, and several others used specifically for astrological purposes. In nearly every case the spherical astronomical tables in *zīj*es include at least the solar declination, the right ascension and the oblique ascension for one or more relevant latitudes. Kūshyār also provides a table for the tangent of the solar declination (which can be used to efficiently calculate the right ascension as well as the equation of daylight or its sine) and a table for the equation of daylight, the difference of the oblique and right ascensions. The main purpose of my commentary on Kūshyār's spherical astronomical tables is to establish whether he copied these from earlier sources or computed them himself, and whether his tables were adopted by later authors. Furthermore, I determine the underlying parameters of the tables and present some characteristics of the methods of computation.

For my investigation of the mathematical-astronomical tables in the anonymous *Dustūr al-munajjimīn* (an Ismāʿīlī *zīj* possibly written in Alamut, *c.* 1110) for the DFG project *Der Dustūr al-munağğimīn als Quelle für die Geschichte der Ismāʿīliyya und ihre astronomisch-astrologischen Vorstellungen*,[142] I compared the spherical astronomical tables from all extant Arabic and Persian *zīj*es up to the time of the composition of the *Dustūr* and from some later ones that include tables from early works or have an unusually large set of oblique ascension tables. Thus, I also analysed the tables from Kūshyār's *Jāmiʿ Zīj* that were copied in the *Dustūr al-munajjimīn*. Here I will use, and expand somewhat, several of the hitherto unpublished results of my investigation of the *Dustūr*. The further *zīj*es with which I have compared Kūshyār's tables are introduced in Section I.1 ('Historical background').

[142] cf. p. xv.

## Table 43: First and second declination

The first declination (i.e., the solar declination) gives the distance of a point on the ecliptic from the equator measured orthogonally to the equator, whereas the second declination gives the distance of a point on the ecliptic from the equator measured orthogonally to the ecliptic. If $\lambda$ denotes the ecliptic longitude and $\varepsilon$ the obliquity of the ecliptic, the first declination $\delta_1$ is given by $\delta_1(\lambda) = \arcsin(\sin\varepsilon \cdot \sin\lambda)$ and the second declination $\delta_2$ by $\delta_2(\lambda) = \arctan(\tan\varepsilon \cdot \sin\lambda)$. In all manuscripts of the *Jāmiʿ Zīj* except the Cairo ones, the first and second declinations are tabulated with tabular differences in four parallel columns, distributed over three pages, each covering 30° of the argument. In **$CC_1$** (**$C_2$** does not include any spherical-astronomical tables), the same values for the first and second declination and their differences are given in two separate tables that each cover a single page. The declination tables are based on al-Battānī's and Kūshyār's regular value of the obliquity of the ecliptic, namely 23;35,0°, which appears as the declination of 90° in both tables. Furthermore, for both tables the tabular differences are in full agreement with the declinations themselves.

Kūshyār's table of the first declination is clearly different from al-Battānī's table in the *Ṣābiʾ Zīj*.[143] In the new edition presented by the late Fritz S. Pedersen and the present author, in which any scribal errors that could reasonably be identified were corrected in a historically plausible way, al-Battānī's table has 53 errors with a standard deviation of nearly 3″, whereas Kūshyār's table has 47 errors that hardly ever coincide with al-Battānī's and have a standard deviation of somewhat less than 1″. Other declination tables in early Islamic *zīj*es with values to seconds and based on obliquity 23;35° also turn out to differ significantly from Kūshyār's.[144] I have made some attempts to explain the

[143] Escorial, RBMSL, árabe 908, fols 177v–178r; Paris, Bibliothèque de l'Arsenal, MS 8322, fol. 43r–v. The new edition of the table in van Dalen and Pedersen, 'Re-editing the Tables', pp. 411–13 supersedes the older one in Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 57–58, in which numerous unjustified corrections were made on the basis of a recomputation by the astronomer Schiaparelli. The author of the *Dustūr al-munajjimīn* copied al-Battānī's table with some additional errors.

[144] This holds for the following tables:

The declination table in the Leipzig recension of the *Mumtaḥan Zīj* (Leipzig, Universitätsbibliothek, Vollers 821, fols 64v–65v) appears together with a lunar latitude table with the Maʾmūnic maximum value 4;46°. The declination table in the Escorial recension (fols 53r–54r) is for obliquity 23;33,0° and was computed much more accurately.

In the Istanbul manuscript of the *zīj* of Ḥabash al-Ḥāsib (Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 99r–100r), the inaccurate declination table for obliquity 23;35° (appearing together with a lunar latitude table with maximum 4;46°, as in the Leipzig manuscript of the *Mumtaḥan Zīj*) was for the largest part computed by means of linear interpolation within intervals of 3°, leading to errors of up to 28″. In the modified Berlin version of the *zīj* (Berlin, SBPK, Wetzstein I 90, fols 82r–84v), the third column of an auxiliary table

errors in Kūshyār's table, but I have not reached any conclusive results. A plausible method of computation is to multiply Kūshyār's sine values by a value for $\sin \varepsilon$ and then find the declination by carrying out inverse linear interpolation in his sine table. This leads to a somewhat better agreement with Kūshyār's table than a modern recomputation if the value for the sine of the obliquity is obtained by means of linear interpolation in the sine table, which makes it equal to (25 · 23;26,39 + 35 · 24;24,15) / 60 = 24;0,15 instead of the exact figure of 24;0,17,49.[145]

The table of the second declination in the *Jāmiʿ Zīj* has 51 errors with a standard deviation of nearly 1¾″. The errors become more frequent and larger towards 90° and reach at most ±5″. Especially towards the end of the table they tend to cluster in groups with the same sign (sometimes positive, sometimes negative) within intervals of 10° (of the values for multiples of 10° only that for 50° has an error, namely of −1″). However, no interpolation patterns can be recognised from the first- or second-order tabular differences.

The second declination is less commonly tabulated in early *zīj*es than the first declination. No table for the second declination is included in either of the extant recensions of the *Mumtaḥan Zīj*, or in the *zīj* of al-Battānī. The Istanbul version of the *zīj* of Ḥabash al-Ḥāsib includes two tables for the second declination with values to seconds, both as part of auxiliary tables. The first of these appears in the first column of an auxiliary table on fols 147v–148v, which mentions in its title a range of spherical astronomical functions that can be calculated by means of it; this table is for obliquity 23;35,0°, but is not even accurate in the minutes.[146] The second table for the second declination in

contains a highly accurate solar declination table for obliquity 23;35°, which has only 23 errors of ±1″ and a single error of +2″.

Abū l-Wafāʾ's solar declination table as preserved in the *Baghdādī Zīj* and al-Bīrūnī's table from *al-Qānūn al-Masʿūdī* both have values to sexagesimal thirds with generally only small errors in the final position, but they are sufficiently different for us to conclude that they were independently computed. Ibn Yūnus's table in the *Ḥākimī Zīj* has values to seconds for every 10′ of the argument with only six errors for integer arguments. So none of these three tables could have been the source for Kūshyār's declination table either.

[145] Interestingly enough, this calculation involves the only value in Kūshyār's sine table that has an error in sources **FHCC$_\mathbf{1}$**. However, if the correct value 23;26,38 from manuscripts **YLB** is used, the result 24;0,14,35 would also be rounded to 24;0,15.

[146] Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 147v–148v. This auxiliary table is called *jadwal al-taqwīm li-Ḥabash al-Ḥāsib* by Abū Naṣr Ibn ʿIrāq in a letter to al-Bīrūnī extant in Patna, Khuda Bakhsh Oriental Public Library, MS 2468. The letter was edited as the fourth treatise in Abū Naṣr Ibn ʿIrāq, *Rasáil Abí Nasr* (71 pages with the defective table on pp. 2–3), and a full study appears in Irani, *The "Jadwal al-taqwīm"*. Three of the four columns in this 'table of rectification' are also included in an auxiliary table in the Berlin recension of Ḥabash's *zīj* (Berlin, SBPK, Wetzstein I 90, fols 82r–84v), but here the second declination was omitted.

Ḥabash's *zīj* (in the first column of an auxiliary table called *jadwal al-taqwīm*) is much more accurate than the first one, but is based on the obliquity value 23;33,0° also found in the *Mumtaḥan Zīj*.[147] The table for the first and second declination in the *Dustūr al-munajjimīn* indicates in its heading that it was taken from al-Battānī.[148] As mentioned above, this is true for the first declination, but the *Ṣābiʾ Zīj* does not contain a table for the second declination: rather, this table was taken from Kūshyār with only two deviations that cannot be attributed to common scribal mistakes. Ibn Yūnus does not include a table for the second declination, and al-Bīrūnī's table to sexagesimal thirds can be seen to be significantly more accurate than Kūshyār's. The *zīj* of Ibn Maḥfūẓ al-Baghdādī contains a copy of Kūshyār's second declination table as the third column in a table that also presents Abū l-Wafāʾs highly accurate solar declination to sexagesimal thirds and the lunar latitude with the Mumtaḥan maximum value 4;46,0°.[149] This copy has eleven notable differences from Kūshyār's table: the values for arguments 50 to 59° are all 4″ too large and the value for 60° is 4″ too small. Since the tabular difference for argument 50° is too large by 4″ in sources **FHCC$_1$** (with an additional scribal error in **C**), it thus appears that the values for 50–59° in the *Baghdādī Zīj* were 'reconstructed' from the tabular differences, and the correct values resumed only at argument 60°.

## Table 44: Tangent of the declination

The tangent of the solar declination was tabulated only by a minority of the authors of early Islamic *zīj*es. Its utility becomes obvious, for example, from the explanations on spherical astronomy in Abū l-Wafāʾ's *al-Majisṭī* and the *Baghdādī Zīj* and from those in Chapter 22 of al-Bīrūnī's *Exhaustive Treatise on Shadows*.[150] For example, with the same notation as above, we see that the tangent of the declination can be used to calculate the right ascension $\alpha(\lambda)$ from $\alpha(\lambda) = \arcsin(\tan\delta(\lambda)/\tan\varepsilon)$, or to calculate the equation of daylight $\Delta_\varphi$ as

147 Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 226r–227r.

148 Paris, BnF, arabe 5968, fol. 39r.

149 Paris, BnF, arabe 2486, fols 129v–130v; cf. van Dalen, *Ancient and Mediaeval Astronomical Tables*, p. 175. Abū l-Wafāʾ did include a table for the second declination in his *al-Majisṭī*. The Paris manuscript announces this table at the top of fol. 23r, but omits it like all other tables. From occasional values that Abū l-Wafāʾ gives in his examples, it can be seen that his table for the second declination also had values to sexagesimal thirds. Hence it follows that Kūshyār's table could not have been taken from *al-Majisṭī*.

150 For a general discussion of the tangent of declination and specific examples of tables, see King, *In Synchrony with the Heavens I*, pp. 145–52. Kūshyār's table is discussed on p. 148, where King already postulates that the inexact value for 90° is the result of linear interpolation in a tangent table. For Abū l-Wafāʾ, see Paris, BnF, arabe 2494, fols 23v and 30v. For the *Baghdādī Zīj*, see Paris, BnF, arabe 2486, fols 129v–130v. For al-Bīrūnī, see Kennedy, *The Exhaustive Treatise*, vol. I, pp. 161–66 (translation) and vol. II, pp. 98–115 (commentary), and Lesley, 'Bīrūnī on Rising Times'.

$\Delta_\varphi(\lambda) = \arcsin(\tan\delta(\lambda) \cdot \tan\varphi)$, where $\varphi$ is the geographical latitude. Because of this application, the tangent of the declination is referred to as 'excesses of the ascensions' (*fuḍūl al-maṭāliʿ*) besides the straightforward name 'tangent of the first declination' (*ẓill al-mayl al-awwal*).

The Leipzig version of the *Mumtaḥan Zīj* contains a quite accurate table for the tangent of the declination with values to seconds based on obliquity 23;33,0°.[151] Neither the Escorial manuscript of the *Mumtaḥan Zīj*, nor the two recensions of the *zīj* of Ḥabash al-Ḥāsib and al-Battānī's *zīj* tabulate the tangent of the declination. As usual, Abū l-Wafā's table extant in the *Baghdādī Zīj* and Ibn Yūnus's table in the *Ḥākimī Zīj* (both to sexagesimal thirds) are much more accurate than Kūshyār's and include only a single error larger than a second.[152] Al-Bīrūnī did not include a table of the tangent of the declination in his *Qānūn*. We may thus conclude that Kūshyār calculated his table for the tangent of the declination himself. The *Dustūr al-munajjimīn* includes Kūshyār's table with only occasional mistakes and a correct attribution.[153]

Already as part of my doctoral research, I noted that the use of Kūshyār's own declination values and linear interpolation in his tangent table lead to a significantly better agreement with his table for the tangent of the declination than an exact recomputation.[154] This is primarily due to the fact that the errors in his table for the solar declination are reflected in the tangent of the declination. In fact, the residuals of these two tables have a positive correlation coefficient of +0.57.

## Table 45: Right ascension

The eight manuscripts that include Book II of the *Jāmiʿ Zīj* contain two different tables for the right ascension: a less accurate one found in manuscripts **FHCC$_1$** (no spherical astronomical tables are included in **C$_2$**) and a more accurate one found in manuscripts **YLB**. For Kūshyār's obliquity value 23;35°, the less accurate table has 30 errors of $\pm 1'$ and 2 errors of $\pm 2'$ in every quadrant. As I already noted in my doctoral dissertation, this table was computed by means of linear interpolation within intervals of 6 degrees.[155] This linear interpolation is not very easy to recognise from the tabular differences, since they

[151] Leipzig, Universitätsbibliothek, Vollers 821, fol. 27r. A copy of this table is also found in Paris, BnF, arabe 2520, fol. 69v as part of a recension of the thirteenth-century Egyptian *Muṣṭalaḥ Zīj*; see King, *In Synchrony with the Heavens I*, p. 151.

[152] See, respectively, Paris, BnF, arabe 2486, fol. 235r and Leiden, Universiteitsbibliotheek, Or. 143, p. 271.

[153] Paris, BnF, arabe 5968, fol. 41v.

[154] van Dalen, *Ancient and Mediaeval Astronomical Tables*, p. 178, footnote 52. The use of Kūshyār's tables in this procedure leads to 33 differences with a standard deviation of 1″4‴, the use of tables with exact values to seconds to 67 differences with a standard deviation of 1″37‴.

[155] van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 175–76.

change very slowly. However, if I calculate the right ascension for non-multiples of 6° from those for multiples of 6° by performing ordinary linear interpolation between the latter and rounding values with 30″ downwards to the next lower number of minutes, only two deviations are left in each quadrant in comparison with the 28 errors for non-multiples of 6° with respect to a modern recomputation. It is also difficult to distinguish between the obliquity values 23;33° and 23;35°. Both values produce the same single error of ±2′ among the fourteen interpolation nodes, but the former value additionally yields five errors of ±1′ and the latter only three. I will therefore assume that the table is based on Kūshyār's obliquity value 23;35°.[156]

Kūshyār's less accurate table for the right ascension is not found in any other Islamic *zīj* that I have inspected. The Escorial and Leipzig manuscripts of the *Mumtaḥan Zīj* share a table for the normed right ascension for the Ptolemaic obliquity value 23;51, computed by means of distributed linear interpolation.[157] Furthermore, the Escorial manuscript contains a perfectly accurate right ascension table for obliquity 23;35°, whereas the Leipzig manuscript includes a table with values to seconds that were calculated for obliquity 23;35° by means of linear interpolation within intervals of 3°.[158] This same table is found as the only right ascension table in the Istanbul recension of the *zīj* of Ḥabash al-Ḥāsib.[159] The Berlin recension of Ḥabash's *zīj* also contains a right ascension table with values to seconds, but this table is for obliquity 23;33° and is attributed to Thābit ibn Qurra.[160] Furthermore, an auxiliary table in the Berlin manuscript includes a right ascension table (in its third column) and the corresponding normed right ascension (in its sixth column), which is apparently based on obliquity 23;33° (for which all values for multiples of 10° are correct) and is different from any other right ascension table discussed here.[161] Al-Battānī's *Ṣābi᾿ Zīj* tabulates the normed right ascension for obliquity 23;35° with 11 errors of ±1′ in every quadrant and is hence also clearly incompatible with

[156] For both 23;33° and 23;35° the value for the interpolation node 78° contains the only error larger than ±1′ (76;55° instead of the correct 76;57°; the same error is found in the corresponding symmetric values for 102°, 258° and 282°), so this may be considered an outlier. Note that the only other error of ±2′ in the entire first quadrant is in the value for 79°, which was calculated from the erroneous value for 78° by means of linear interpolation. A least squares estimation of the obliquity on the basis of the remaining 13 nodes yields a confidence interval ⟨23;34,20 , 23;36,44⟩ with a minimum possible standard deviation of 21½″.

[157] Escorial, RBMSL, árabe 927, fols 48v–49r; Leipzig, Universitätsbibliothek, Vollers 821, fols 103r–104r. For distributed linear interpolation, see the Quick reference on pp. 528-29.

[158] See, respectively, Escorial, RBMSL, árabe 927, fols 55r–56r and Leipzig, Universitätsbibliothek, Vollers 821, fols 25v–26r.

[159] Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 132v–133v.

[160] Berlin, SBPK, Wetzstein I 90, fols 90v–91v.

[161] Berlin, SBPK, Wetzstein I 90, fols 85r–87v.

Kūshyār's less accurate table.[162] Also in this case, the right ascension tables of Abū l-Wafāʾ and al-Bīrūnī (both to sexagesimal thirds) and of Ibn Yūnus (to seconds) are clearly more accurate than Kūshyār's table.[163] We may thus conclude that Kūshyār most likely computed his less accurate table for the right ascension himself.

For the most plausible obliquity value, 23;35°, Kūshyār's more accurate right ascension table found in manuscripts **YLB** has only eight errors of ±1′ in the first quadrant.[164] Only ten values in the entire table differ from the corresponding ones in al-Battānī's table for the normed right ascension, in most cases due to a common scribal error. We may thus assume that, after computing a less accurate table by means of linear interpolation, Kūshyār later replaced this table with a more accurate one derived from al-Battānī's table. Both al-Battānī's normed right ascension and Kūshyār's more accurate right ascension table derived from it were included in the *Dustūr al-munajjimīn* and are correctly attributed there to their respective sources.[165]

## Table 48: Equation of Daylight

**FH** contain a table for the equation of daylight for latitude 36° (which is also found among the additional tables in **L)** and **CYLB** a table for latitude 35;30°, in each case in correspondence with the latitude for which the oblique ascension is tabulated in the same manuscripts. In **F** (and hence in my edition) the equation of daylight appears next to an unfilled frame for a table for the maximum equation of daylight for latitudes 16 to 45°. This table is listed in the table of contents of Book II in **F**, and also in the table of contents in **L**, but does not appear at all in **H**. In **C** the equation of daylight is tabulated together

[162] Nallino, *Al-Battānī sive Albatenii*, vol. II, pp. 61–64 (with brief commentary on pp. 221–22).

[163] See, respectively, Paris, BnF, arabe 2486 (*Baghdādī Zīj*), fols 141v–143r; al-Bīrūnī, *al-Qānunu'l-Masʿūdī*, vol. I, pp. 379–87, and Leiden, Universiteitsbibliotheek, Or. 143, pp. 268–269. The tables for the right ascension in the *Baghdādī Zīj* and al-Bīrūnī's *al-Qānūn al-Masʿūdī* are basically identical except for a rather large number of obvious scribal errors. Both tables are in agreement with the approximately ten inexact values that Abū l-Wafāʾ quotes from his right ascension table in computational examples in *al-Majisṭi*. Therefore it is probable that for the right ascension as well the *Baghdādī Zīj* presents Abū l-Wafāʾ's table, and that al-Bīrūnī copied it from *al-Majisṭī*. We have already seen (footnote 32 on pp. 358–59) that al-Bīrūnī most probably copied Abū l-Wafāʾ's tables for the sine and the tangent. Al-Bīrūnī's table for the solar declination is certainly different from Abū l-Wafāʾ's, but his table for the second declination (for which the *Baghdādī Zīj* copies Kūshyār's table) displays the three values that are cited for this function in *al-Majisṭī*. Since two of these values have an error of nearly 60 sexagesimal thirds, it seems likely that al-Bīrūnī also copied this table from Abū l-Wafāʾ.

[164] Due to several asymmetries, the total number of errors in the table is 36. The total number of errors with regard to a recomputation for obliquity 23;33° is 108.

[165] Paris, BnF, arabe 5968, fols 43r and 43v.

with the oblique ascension. While in all other manuscripts it is tabulated for arguments from 1 to 90°, in **C** it thus shares the arguments from 1 to 360° with the oblique ascension. The additional values generally satisfy the symmetries $\Delta(180-\lambda) = \Delta(\lambda)$ and $\Delta(180+\lambda) = -\Delta(\lambda)$, where $\Delta$ is the equation of daylight, $\lambda$ is the ecliptic longitude, and the negative values are not explicitly indicated in the table.[166] Note that the oblique ascension and equation of daylight tables are missing from $\mathbf{C_1C_2}$.

Like the less accurate right ascension table that accompanies it in manuscripts **FH**, Kūshyār's table for the equation of daylight for latitude 36° can be seen to have been computed by means of linear interpolation within intervals of 6°. This can be clearly recognised from the tabular differences, especially in the part of the table between arguments 36 and 90°. In this case as well a reconstruction by means of linear interpolation is highly convincing: it reduces the number of errors in values for non-multiples of 6° from 42 to three if the same procedure is used as for the less accurate right ascension table discussed above (the remaining three deviations are due to the upward rounding, rather than truncation, of three interpolated values with 30″ between arguments 24 and 30°). Of the 15 interpolation nodes, only the one for 54° contains an error of +1′ with respect to a modern recomputation for the expected parameter values of 23;35° and 36;0°.

I have not been able to explain the method of computation of Kūshyār's table for the equation of daylight for latitude 35;30°. The value for 90° is correct for the expected obliquity value of 23;35° and the indicated latitude. However, for these parameters the table shows 74 errors in 90 values with a mean of −1′14″ and a standard deviation of 1′45″. All tabular values for arguments 8 to 63° have negative errors of up to −4′, whereas the values for arguments 84 to 89 have positive errors. No traces of linear interpolation are to be seen in the tabular differences, and no least squares estimation based on a subset of equidistant tabular values (e.g., for arguments that are multiples of 5, 6, 10 and 15°) yields a clearly smaller minimum possible standard deviation. I experimented a little with so-called homothetic tables by scaling up or down tables of similar functions by the quotient of the respective maximum values.[167] In fact, by multiplying a table for the equation of daylight for latitude 40° with values to seconds by the quotient 18;9 / 21;29,16, I obtain a much better fit

[166] For the solar declination, which satisfies the same symmetries, the direction is often indicated by the words 'northern' and 'southern'. The 'sign' of the equation of daylight would become clear from, for example, instructions to either add the equation to, or subtract it from, the right ascension in order to obtain the oblique ascension. All deviations of the equation of daylight in **C** from its expected symmetry can be explained as common scribal errors.

[167] cf. the Quick reference guide on p. 529.

with Kūshyār's table for latitude 35;30°.[168] Because of the uncertainty about the geographical latitude and the accuracy of the table that could have been used in this procedure, I have not tried to obtain an even better fit.

While most *zīj*es include tables for the oblique ascension, very few early *zīj*es include a table for the equation of daylight. The *Dustūr al-munajjimīn* includes a table different from Kūshyār's for latitude 36;21°, which is associated with Alamut but is also used as the latitude of the fourth climate.[169] The highly accurate table of the equation of daylight for Baghdad (latitude 33;25°) in the *Baghdādī Zīj* can be shown to stem from Abū l-Wafāʾ.[170] See below for a discussion of possible relationships of Kūshyār's oblique ascension tables with other sources.

### Table 46: Oblique Ascension

Kūshyār's oblique ascension table for latitude 36° included in the early versions **F** and **H** of the *Jāmiʿ Zīj* was copied in the *Baghdādī Zīj* as one of a set of oblique ascension tables for latitudes 30 to 40°.[171] I analysed this table in my doctoral dissertation as part of my investigation of all the spherical astronomical tables in the *Baghdādī Zīj*. First I extracted the tables for the right ascension and the equation of daylight from which Kūshyār computed the oblique ascension.[172] Since the extracted tables have zeroes in the position of the seconds throughout, we may conclude that both underlying tables had values to minutes only. In fact, the extracted tables are entirely identical to Kūshyār's less accurate right ascension table and his table for the equation of daylight for 36° found in **F** and **H** and among the additional tables in **L**. As explained in the commentary on those tables, both were computed by means of linear interpolation within intervals of 6°. This interpolation leads to a rather peculiar error pattern in the resulting oblique ascension, which can be seen most clearly from the differences between a reconstruction on the basis of interpolated tables and exact, unrounded recomputed values.[173] The *Baghdādī Zīj* is the only *zīj* that

[168] Note that 21;29,16° is the maximum equation of daylight for latitude 40° and obliquity 23;35°, and 18;9° the maximum of Kūshyār's table for latitude 35;30°. The resulting table has only 39 differences from Kūshyār's table with a mean of +7″ and a standard deviation of 44″.

[169] Paris, BnF, arabe 5968, fol. 161v.

[170] Paris, BnF, arabe 2486, fol. 117r. In van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 181–82, I showed that this table was calculated from the table for the sine of the equation of daylight on fol. 235v of the same manuscript by performing inverse linear interpolation in a sine table with accurate values to four sexagesimal places for every quarter degree, and that all three tables most likely stem from Abū l-Wafāʾ. See also the entry on Abū l-Wafāʾ's *al-Majisṭī* in the Quick reference guide for *zīj*es (pp. 523–24).

[171] Paris, BnF, arabe 2486, fols 155v–156r.

[172] cf. van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 67, 185 and 190–91.

[173] See van Dalen, *Ancient and Mediaeval Astronomical Tables*, p. 190 (Figure 4.4).

I have found to contain a copy of Kūshyār's oblique ascension table for 36°. In particular, the table is clearly different from the one for Raqqa included by al-Battānī in the *Ṣābiʾ Zīj* (copies of which are also found in the Escorial and Leipzig manuscripts of the *Mumtaḥan Zīj*).[174]

An analysis of the oblique ascension table for latitude 35;30° found in **C** and in the group **YLB** (see Plate 12) can proceed in a very similar way. Extraction of the underlying right ascension and equation of daylight produces two tables that have non-zero seconds in only two out of 90 entries. We may conclude that both underlying tables were originally given to minutes.

Not surprisingly, the extracted equation of daylight is basically identical to the table of this function found in the same manuscripts, which is likewise said to be for latitude 35;30°. As we have seen above, this table has a maximum value corresponding to the expected parameters, namely 23;35° for the obliquity and 35;30° for the latitude, but otherwise is rather inaccurate and was computed by a method that I do not currently understand.

Somewhat unexpectedly, the underlying right ascension table is *not* equal to the table for this function found in the same manuscripts, whose values, as we have seen, were taken from al-Battānī's table for the normed right ascension. In fact, the right ascension table underlying Kūshyār's oblique ascension for latitude 35;30° is different from any (normed) right ascension tables in early Islamic *zīj*es that I have checked. It can be seen to be a quite accurate table based on the Ptolemaic obliquity value 23;51°, but it has no relation with Ptolemy's own right ascension values (in the *Handy Tables* these are determined for every degree by means of distributed linear interpolation in between the values for every 10° from the *Almagest*), with the right ascension table in al-Khwārizmī's *Sindhind Zīj* (which has values to seconds with relatively small errors in the seconds), or with the right ascension table for obliquity 23;51° in the Escorial and Leipzig manuscripts of the *Mumtaḥan Zīj*.[175]

I am not aware of other oblique ascension tables for latitude 35;30° in early Islamic *zīj*es. The table for Rayy found in the *Zīj al-Qirānāt* and in the *Dustūr al-munajjimīn* is in both sources said to be for the latitude value 35;34,30° confirmed by al-Khujandī.[176] This Iranian astronomer, who hosted Kūshyār at the observatory in Rayy in the 990s, determined the latitude value of 35;34,38,45°

[174] See, respectively, Nallino, *Al-Battānī sive Albatenii*, vol. II, pp. 68–71 (with a brief commentary on p. 222); Escorial, RBMSL, árabe 927, fols 47r–48r, and Leipzig, Universitätsbibliothek, Vollers 821, fols 106v–180r.

[175] See, respectively, Toomer, *Ptolemy's Almagest*, pp. 74 and 100; Stahlman, *The Astronomical Tables*, pp. 206–09; Tihon, *Les* Tables Faciles *1a*, pp. 97–100, 139–45 and 200–04; Mercier, *Ptolemy's Handy Tables 1b*, pp. 10–13 and 79–89; Suter, *Die astronomischen Tafeln*, pp. 171–73, and van Dalen, 'A Second Manuscript', p. 19.

[176] See, respectively, Paris, BnF, arabe 6913, fol. 104r and Paris, BnF, arabe 5968, fol. 54r.

(together with an obliquity value of 23;32,21°) in a treatise entitled *Risāla fī l-Mayl wa-ʿarḍ al-balad* ('Treatise on the Declination and the Latitude of a City').[177] The exact parameters and the method of computation underlying the table for Rayy in the *Zīj al-Qirānāt* and the *Dustūr al-munajjimīn* are likewise obscure, but the table is clearly different from Kūshyār's table for 35;30°.

177 This treatise was edited in Cheikho, '*Risālat al-Khujandī*' (the latitude of Rayy is derived on p. 66). It was translated into German in the appendix of Schirmer, 'Studien zur Astronomie' (p. 76).

## IV.11. Parallax and eclipses

**Bibliography**: Kennedy, 'Parallax Theory'; Kennedy and Faris, 'The Solar Eclipse Technique'; Pedersen, *A Survey of the Almagest*, Chapter 7, pp. 203–35; Neugebauer, *HAMA*, vol. I, pp. 100–44; Hartner, 'Ptolemy and Ibn Yūnus'; Van Brummelen, *Mathematical Tables*, Chapters 10–11, pp. 193–242; Montelle, *Chasing Shadows*, esp. Chapter 6, pp. 285–324; Giahi Yazdi, 'Al-Khwārizmī and Annular Solar Eclipse'; Mozaffari, 'Annular Eclipses and Considerations'; Bagheri, *az-Zīj al-Jāmiʿ*, Section I.6, pp. 67–84 (translation), 85–92 (commentary) and Arabic pp. 41–58; Section IV.6, pp. 187–204 (translation), 205–08 (commentary) and Arabic pp. 128–57.

Kūshyār's *Jāmiʿ Zīj* has a relatively small set of tables to be used in eclipse calculations. Section I.6 (consisting of 20 chapters) describes these calculations in detail and refers to the tables in Book II at the appropriate places. The procedure starts by finding the daily and hourly motion of the Sun and the Moon at a time near a conjunction or opposition; these motions are tabulated in Table 49. Then the time of true syzygy is found from the true longitudes of the Sun and the Moon at noon (or another time close to the syzygy). For this purpose Kūshyār compiled an extensive table with the lunar velocity as one of its arguments, which is only included as an additional table in the version of the *Jāmiʿ Zīj* that is represented by the manuscripts **YLB**. This table is edited in Part II of this book, and is described and analysed below, as Table $49^{bis}$. I will argue that it most probably stems from Kūshyār himself and thus is the earliest known Islamic table for finding the time of true syzygies. The determination whether a lunar eclipse takes place and, if so, the calculation of its duration and magnitude, now proceed by straightforward arithmetical procedures. The diameters of the Sun, Moon and the shadow of the Earth can be found in Table 49 as a function of the solar mean anomaly and the lunar true anomaly. Furthermore, Table 52 is a small table, also found in Ptolemy and al-Battānī, whose second half can be used to convert the magnitude of a lunar eclipse expressed as a fraction of the diameter of the lunar disk to a fraction of its area.

The calculation of solar eclipses proceeds in a very similar way, except that the lunar parallax needs to be taken into account. Whereas al-Battānī included the table of solar and lunar parallax in altitude from Ptolemy's *Almagest* as well as the parallax tables for the climates from the *Handy Tables*, Kūshyār instructs his readers in Sections I.6.7–11 to find the lunar parallax in altitude by, respectively, calculating the distance of the Moon from the Earth (for which he provides Table 50), calculating the lunar altitude as explained in Section I.9, then finding the lunar parallax from a plane-trigonometrical calculation involving the lunar distance and the radius of the Earth, and finally subtracting the solar parallax (for which he gives Table 51). The first half of Table 52 can be used to convert the magnitude of a solar eclipse expressed as a fraction of the diameter of the solar disk to a fraction of its area.

## Table 49: Solar and lunar hourly motions and diameters

**Bibliography**: as-Saleh, 'Solar and Lunar Distances'; Bagheri et al., 'Kūshyār ibn Labbān Gīlānī's Treatise'; Bagheri, *az-Zīj al-Jāmiʿ*, Sections I.6.1–2, p. 67 (translation), p. 85 (commentary) and Arabic p. 41.

This table displays the hourly true solar motion (varying between 0;2,23 and 0;2,33°) and the apparent diameter of the solar disk as a function of the mean solar anomaly, as well as the hourly true lunar motion (varying between 0;30,17 and 0;36,4°), the apparent diameter of the lunar disk and the apparent diameter of the shadow of the Earth as a function of the true lunar anomaly. Chapter I.6.1 of the *Jāmiʿ Zīj* explains how the hourly true motion of the Sun or Moon may be calculated as the difference between the true longitude for the next day and that for the current day divided by 24, or as the difference between the current true longitude and the true longitude six hours earlier or later divided by 6. Chapter I.6.2 explains how to find the solar diameter by multiplying the daily solar motion by 0;33, the lunar diameter by multiplying the daily lunar motion by 0;2,26 or the hourly lunar motion by 0;58,25, and the diameter of the shadow of the Earth as 2⅗ times that of the Moon. In order to take the variation in the solar distance into account as well, Kūshyār prescribes that the diameter of the shadow should be reduced by 10 times the excess of the hourly solar motion over its minimum value, 0;2,23. For example, for a solar mean anomaly of $3^s 0°$, this correction would amount to $(0;2,28 - 0;2,23) \cdot 10 = 0;0,50°$.

Among Kūshyār's Islamic predecessors, al-Khwārizmī includes a table for the solar and lunar hourly motion and the radii of the Sun and the Moon with values for every degree, but the minimum and maximum values are clearly different and two columns for the 'first and second radius of the ascending node' make it clear that also the underlying system of computation is non-Ptolemaic.[178] In the two extant versions of his *zīj*, Ḥabash al-Ḥāsib describes methods and gives auxiliary tables for calculating the solar and lunar velocity that make use of the Ptolemaic interpolation functions for parallax from the *Almagest*.[179] Al-Battānī includes a table for the true solar and lunar motion in an hour with values to seconds of arc for every six degrees of the respective arguments.[180] This table is accompanied by a small table of positive or negative corrections to these true motions of 1, 2, …, 7 seconds of arc for respectively mean elonga-

[178] Suter, *Die astronomischen Tafeln*, Chapter 29, pp. 22 (Latin text), 78 (commentary) and 175–180 (Tables 61–66); Neugebauer, *The Astronomical Tables of al-Khwārizmī*, pp. 57 and 105–07 (English translation and commentary).

[179] as-Saleh, 'Solar and Lunar distances'.

[180] Nallino, *Al-Battānī sive Albatenii*, vol. II, p. 88 (with a brief commentary on p. 231).

tions 1, 2, ..., 7°.[181] Kūshyār appears to have copied his columns for the hourly motion of the Sun and the Moon from al-Battānī. However, in copying the column for the Sun, he omitted the value for $3^s24°$ and hence copied the values for arguments $4^s0$–$6^s0°$ one row too high (leading to a total of four deviations). Also the column for the hourly motion of the Moon is generally equal to al-Battānī's, but shows five deviations of –1 second and one of –2 seconds that cannot be explained as scribal errors.

It is obvious that the tabulated values for the solar diameter cannot all have been determined by multiplying the hourly true solar motion by 13;12, as suggested by the text of Book I, since the column for the solar motion displays only eleven different values, whereas nearly all of the 31 values for the solar diameter are different. However, whereas most of the values for the solar diameter show differences of up to ±10 seconds from the values of the hourly solar motion multiplied by 13;12, precisely the values corresponding to a change in the hourly true solar motion by a second are equal to the solar motion multiplied by 13;12 or differ from it by only a single second. With the exception of 150°, this concerns the values for arguments 0, 30, 48, 66, 78, 90, 102, 114, 126 and 138°, to which I will hereafter refer as 'nodes'. The tabular differences of the values of the solar diameter between these nodes, including 150°, are nearly constant (i.e., they differ from each other by not more than a second). It is thus very probable that Kūshyār (or his source) calculated only the solar diameters for the nodes by multiplying the hourly true solar motion by 13;12, and that he found all intermediate values by means of linear interpolation. Apparently he realised that linear interpolation would not be able to produce the curvature near the maximum of the sinusoidal function at 180° and therefore adjusted the last part of the table by making use of constant second-order differences. Possibly this adjustment also affected the value for 150°, the only node for which the solar diameter differs by more than ±1 second (namely, by –6 seconds) from the value found by multiplying the hourly solar motion by 13;12. A similar adjustment would also have been appropriate for the beginning of the table, but may have been considered less urgent because the computed value for 30° differs significantly from the value for 0°, whereas the values for 150 and 180° are identical.

Kūshyār's values of the lunar diameter were not computed from the hourly true lunar motion by multiplying the latter by 0;58,25, as suggested in Chapter I.6.2, but by multiplying it by a factor 0;58,47 (corresponding to a multiplication of the daily motion by approximately 0;2,27 instead of 0;2,26). A reconstruction with the factor 0;58,47 differs from the edited table in only two places: for $1^s18°$ the reconstruction yields 23″ as opposed to 24″ in the

[181] An explanation for these corrections is given by Schiaparelli in a commentary to Chapter 42 of the *Ṣābiʾ Zīj*; see Nallino, *al-Battānī sive Albatenii*, vol. I, p. 273.

table (which is a possible scribal error), and for $4^s\,18°$ the reconstruction has 21″ rather than the 27″ found in all manuscripts (below, 0;34,21 will be confirmed as the intended value for this argument).

Al-Khwārizmī mentions the same multipliers as Kūshyār for finding the solar diameter, but for the moon he gives 0;2,16 (error for 0;2,26 or 0;2,25?) and 0;58,10 in the text, although his table was computed using the multiplier 0;2,25.[182] Al-Battānī not only presents rules of the type given by Ḥabash for finding the diameters of Moon and shadow with the aid of an interpolation function, but alternatively suggests to find the diameters by multiplying the hourly true lunar motion respectively by (6 − ⅛) / 6 (i.e., 0;58,45, corresponding to a multiplication of the daily motion by 0;2,26,52,30) and by 2⅗.[183] The source for Kūshyār's slightly different multiplication factors for the lunar diameter and the reason why he did not follow one of his predecessors exactly must remain unknown for the time being.

Kūshyār's diameter of the shadow of the Earth is correctly reproduced by multiplying the lunar diameter by 2⅗, as explained in Book I of the *Jāmiʿ Zīj*. The only non-match is found for argument $4^s\,18°$, for which the lunar diameter is given as 0;34,27 in all manuscripts. Back-computation shows that the value 89;19 for the diameter of the shadow of the Earth for this argument was derived from a value 0;34,21; this confirms the above result of the computation of the lunar diameter from the tabulated hourly lunar motion.

Kūshyār's table for the hourly true motions and the diameters was copied in the *Dustūr al-munajjimīn*, where it is explicitly attributed to Kūshyār.[184] The table is here expanded with columns for the daily true solar and lunar motion, inserted before those for the hourly motion. However, since all values in these columns are plain 24-folds of the hourly motions, they may be a later addition. The tabular values in the five columns also found in the *Jāmiʿ Zīj* are in almost perfect agreement.

## Table 49^bis: Conjunctions and oppositions

**Bibliography**: Nallino, *al-Battānī sive Albatenii*, vol. III, pp. 138–45 (Arabic); vol. I, pp. 92–96 (Latin translation) and pp. 273–75 (commentary); Pedersen, *A Survey of the Almagest*, pp. 223–26; Neugebauer, *HAMA*, vol. I, pp. 122–24; Chabás and Goldstein, 'Computational Astronomy: Five Centuries'; Goldstein and Chabás, 'Ibn al-Ḥadib's Tables'.

182 Suter, *Die astronomischen Tafeln*, Chapters 30–30a, pp. 22–23 (Latin text) and 78–80 (commentary) and 175–180 (Tables 61–66); Neugebauer, *The Astronomical Tables of al-Khwārizmī*, pp. 58–59 and 105–107 (English translation and commentary).

183 Nallino, *Al-Battānī sive Albatenii*, vol. III, pp. 146–47 (Arabic) and vol. I, p. 97 (Latin translation).

184 Paris, BnF, arabe 5968, fol. 111v.

This table appears as an additional table in manuscripts **Y** (fols 311v–313v, see Plate 13) and **B** (pp. 211–214, 206–205 and 217; the pages in this part of the manuscript are not in the correct order). It was also copied in the *Dustūr al-munajjimīn*,[185] where it is attributed to Kūshyār, and I have briefly described and partially analysed it in my hitherto unpublished study of the tables in that work. Finally, the table was incorporated in the late thirteenth-century *zīj* of Ibn Maḥfūẓ al-Baghdādī.[186] Kūshyār's Table of Conjunction and Opposition is a double-argument table that gives, to three sexagesimal places, the 'part of the distance' (*juzʾ al-buʿd*), an auxiliary quantity to be explained below, and the 'hours of the distance' (*sāʿāt al-buʿd*), i.e., the time till the nearest true conjunction or opposition of the Sun and the Moon. The vertical argument of this table is the 'distance between the two luminaries' (*al-buʿd bayn al-nayyirayn*), i.e., the true elongation, which runs through the peculiar range 1, 2, 3, ..., 14, 15, 18, 21, ..., 60 that we have also seen in Kūshyār's sine table with values for fractions of a degree (Table 8a).[187] The horizontal argument is the 'lunar velocity' (*buht al-qamar*), which takes on the values 11;50, 12;0, 12;10, ..., 14;50 °/day.[188]

Kūshyār's Table of Conjunction and Opposition basically implements the iterative process for finding the time of true syzygy that was already described by Ptolemy in the *Almagest*. In two worked examples we will see that, thanks to improvements by Islamic astronomers, this process converges so rapidly that often only a single step is needed to obtain the longitude of the true syzygy to an accuracy of a minute of arc. The table is referred to in a paragraph added to the chapter on calculating conjunctions and oppositions in Book I of the *Jāmiʿ Zīj* in manuscripts **Y** (Chapter I.6.3, fols 244v–245r) and **L** (Chapter I.50, fols 11v–12r).[189] This passage reads as follows (note that a variation of its

185 Paris, BnF, arabe 5968, fols 112r–115r.

186 Paris, BnF, arabe 2486, fols 124v-127r.

187 In the Yeni Cami manuscript of the *Jāmiʿ Zīj* and the sole surviving Paris manuscript of the *Dustūr al-munajjimīn*, the vertical arguments are mistakenly given as 1, 2, 3, ..., 30 throughout the entire table, but the tabular values agree with those in the Berlin manuscript and the *Baghdādī Zīj*.

188 For al-Battānī's and Kūshyār's lunar parameters, which were mostly taken from Ptolemy, the true daily motion of the Moon varies between 11;37 and 15;6°. It is unclear to me why Kūshyār did not include columns for velocities 11;40 and 15;0°/day. This would also have had the advantage of fitting the table exactly on seven pages in a format with three columns per page. Al-Baghdādī left out the column for velocity 14;0, possibly to fit the table onto six pages.

189 The chapter on conjunctions and oppositions is among the chapters missing from manuscripts **H** (Chapter I.52) and **B** (Chapter I.56). The added passage that mentions the table is not included in manuscripts **F** (Chapter I.6.3 on fols 16r–17r), **C** (Chapter I.56 on fols 22v–23v), $\mathbf{C_2}$ (Chapter I.56 on fols 12v–13r) and Cairo, Dār al-kutub, *mīqāt* Muṣṭafā

second part can be found in a marginal note to the table in the *Dustūr al-munajjimīn*):

وقد وضعنا جداول[1]، كلّ جدول لبهت من بهت يوم القمر، كتبنا فيه البعد بين النيّرين وبإزائه جزء البعد وساعات البعد. فإن كان البعد بين النيّرين درجًا، كان جزء البعد درجًا ودقائق وثواني وساعات البعد ساعات[2] وما يتبعها من الكسور. فإن[3] كان البعد بين النيّرين دقائق، كان جزء البعد دقائق وما يتبعها[4] من الكسور وساعات البعد كذلك على هذا الرسم.

*Apparatus*: [1]**L** جداولا [2]**L** ساعاتا [3]**L** وإن [4]**L** بينهما.

And we compiled tables (or: columns of a table), each table (or: column) for one of the lunar velocities per day, and we wrote in it the distance between the two luminaries (i.e., the elongation) and next to it the 'part of the distance' and the 'hours of the distance'. And if the distance between the two luminaries is ⟨in⟩ degrees, the 'part of the distance' is ⟨in⟩ degrees, minutes and seconds and the 'hours of the distance' are ⟨in⟩ hours and the fractions that follow them (i.e., minutes and seconds of time). And if the distance between the two luminaries is ⟨in⟩ minutes, the 'part of the distance' is ⟨in⟩ minutes and the fractions that follow them (i.e., seconds and thirds of a degree) and the 'hours of the distance' likewise according to this pattern.

The table itself is missing from **L**, although this is the only manuscript that includes it in its table of contents of Book II, between the planetary latitudes and the lunar distance from the Earth, instead of the Hourly Solar and Lunar Motion and Diameters. In any case we may conclude that the Table of Conjunction and Opposition is another addition made by Kūshyār himself to his original version of the *Jāmiʿ Zīj*: like several other changes discussed in Section I.7, this table and the instructions for its use are found in manuscripts **YLB** but not yet in manuscripts $\mathbf{FHCC_1C_2}$.

Each column in the Table of Conjunction and Opposition displays exact multiples of the values for argument 1. These values correspond to a true elongation of 1 degree, but can also be used for 1 minute (or even 1 second) by shifting the sexagesimal point by one (or two) positions. Thus, for any given value of the elongation, the 'part of the distance' and the 'hours of the distance' can be obtained by adding together the values corresponding to the digits that make up the elongation, shifting the sexagesimal point of the values appropriately. For example, for a lunar velocity of 13°/day and a distance between the Sun and the Moon of 4;23,18°, the hours of the distance will be found as the sum of 7;56,32 (value for vertical argument 4), 0;41,41,48 (value for argument 21 with the sexagesimal point shifted one place), 0;3,58,16 (value for argument 2 with the sexagesimal point shifted one place) and 0;0,35,44,24 (value for argument 18 with the sexagesimal point shifted two places), i.e., as 8;42,48[h].

Fāḍil 213/1 (Chapter I.6.3 on fols 14v–15r). The addition was also omitted from the translation and edition in Bagheri, *az-Zīj al-Jāmiʿ*, pp. 67–68 (translation) and Arabic p. 42.

Table L: The relevant values from Kūshyār's Table of Conjunction and Opposition.

| Lunar velocity | part of the distance | hours of the distance | Lunar velocity | part of the distance | hours of the distance |
|---|---|---|---|---|---|
| 11;50 | 0;4,59 | 2;11,48 | 13;30 | 0;4,22 | 1;54,26 |
| 12; 0 | 0;4,55 | 2; 9,50 | 13;40 | 0;4,19 | 1;52,57 |
| 12;10 | 0;4,51 | 2; 7,55 | 13;50 | 0;4,16 | 1;51,30 |
| 12;20 | 0;4,47 | 2; 6, 4 | 14; 0 | 0;4,13 | 1;50, 5 |
| 12;30 | 0;4,43 | 2; 4,15 | 14;10 | 0;4,10 | 1;48,42 |
| 12;40 | 0;4,39 | 2; 2,30 | 14;20 | 0;4, 7 | 1;47,22 |
| 12;50 | 0;4,36 | 2; 0,49 | 14;30 | 0;4, 4 | 1;46, 2 |
| 13; 0 | 0;4,32 | 1;59, 8 | 14;40 | 0;4, 1 | 1;44,45 |
| 13;10 | 0;4,29 | 1;57,32 | 14;50 | 0;3,59 | 1;43,31 |
| 13;20 | 0;4,25 | 1;55,57 | | | |

In order to fully explain Kūshyār's table of conjunctions and oppositions, it thus suffices to establish the method of calculation of the values of the 'part of the distance' and the 'hours of the distance' for one degree of elongation as a function of the lunar velocity. Thus the relevant values can be extracted from the complete table as shown in Table L.

In Book I of the *Jāmiʿ Zīj* the calculation of oppositions and conjunctions is first described as follows.[190] The true longitudes of the Sun and the Moon are determined for noon of the day nearest to the conjunction or opposition (but the procedure would be exactly the same for any other time near the true syzygy, such as the mean syzygy or any intermediate step of an iterative process). For a conjunction the 'distance between the two luminaries' (*buʿd bayn al-nayyirāyn*, i.e., the elongation) is the difference between the true ecliptic longitudes of the Sun and the Moon, for an opposition it is the difference between the true solar longitude and the point opposite the true lunar longitude. The 'distance' multiplied by 5 minutes is called the 'part of the distance' (*juzʾ al-buʿd*). It is added to the distance to obtain the 'distance plus the part of the distance'. This is added to the lunar longitude if the Moon (or the point opposite it) lags behind the Sun on the ecliptic, or subtracted if the Moon precedes the Sun. The result is a first approximation of the ecliptic degree of the true conjunction or opposition. The corresponding difference in time between noon and the true conjunction or opposition (the 'hours of the distance', *sāʿāt al-buʿd*) is found by dividing the elongation by the 'lunar precedence' (*sabq al-qamar*, translated as 'lunar gain' by Bagheri, i.e., the excess of the lunar velocity over the solar velocity, referred to as 'relative lunar velocity' by Chabás and Goldstein). If, at the time thus found, the true longitudes of the Sun and the Moon are not yet sufficiently close, the process should be iterated.

190 cf. Bagheri, *az-Zīj al-Jāmiʿ*, pp. 67–68 (translation), 85–86 (commentary) and Arabic p. 42.

As Bagheri explains, this procedure is similar to that used by Ptolemy in the *Almagest*. It approximates the distance on the ecliptic covered by the Sun from the given starting point (mean syzygy in the *Almagest*, noon in the *Jāmiʿ Zīj*) up to true syzygy as 1/12 of the initial true elongation, and hence the distance to be covered by the Moon as 13/12 of the elongation. Ptolemy divides this latter distance by the lunar velocity in order to estimate the time from mean to true syzygy. However, Islamic astronomers from early on, including Kūshyār, instead divided the true elongation by the 'lunar precedence'. Al-Battānī also explains Ptolemy's method (without attributing it to him), but adds that the division by the 'lunar precedence' is more correct. Note that the 'lunar precedence' can easily be found from Kūshyār's Table 49 once the mean solar anomaly and the true lunar anomaly are known. Since this procedure not only avoids Ptolemy's rather rough approximation of the ecliptic arc covered by the Sun up to the time of true syzygy, but even takes into account the annual variation in the solar velocity, the result may be expected to be more accurate than that of Ptolemy's method.

In order to illustrate this, I have used two conjunctions of the Sun and the Moon in Kūshyār's lifetime as worked examples (see Table M). At the first conjunction, near the vernal equinox of AD 1000, the solar and lunar velocities are both near their mean value, which also implies that they change very rapidly. At the second conjunction, three months later, the lunar and solar velocities both deviate significantly from their means but in opposite direction. All true longitudes were found from my programme Historical Horoscopes by using the parameters from the *Jāmiʿ Zīj* (which, for the Sun and the Moon, are identical to those of al-Battānī). The hourly motions given in Table M were found as the difference between the true longitudes 30 minutes before and 30 minutes after the initial time (in both cases: noon). The exact time of the conjunctions according to the *Jāmiʿ Zīj* was determined by varying the time manually until the true solar and lunar longitudes were identical to a precision of seconds. The examples confirm that the method used by the early Islamic astronomers was indeed an improvement over Ptolemy's, as indicated by al-Battānī. As was to be expected, the difference becomes particularly clear when the solar velocity is not close to its mean value, and hence Ptolemy's estimate is inaccurate. Note that the problem of finding true syzygies is one of relatively few instances in which pre-modern astronomers were able to verify the accuracy of their algorithms directly on the basis of the planetary tables available to them, since the resulting solar and lunar true longitudes needed to be equal. If necessary, this could be achieved to a precision of less than a minute by carrying out another iteration.

Table M: Two worked examples for the calculation of the time and place of a true conjunction of the Sun and the Moon on the basis of Kūshyār's solar and lunar parameters. The second column displays the exact data as found from the *Jāmiʿ Zīj*, the third column the approximation used by Ptolemy, the fourth column the approximation used by early Islamic astronomers, and the fifth column the results of the method underlying Kūshyār's table (but with the lunar velocity not rounded to a multiple of 10 minutes, as in his table). Only those quantities are entered in each column that are used for the method concerned. Any intermediate results of the calculations were kept to their full precision. The exact times of the two new moons according to Alcyone Ephemeris are 1:25$^h$ on 9 March 1000 and 4:26$^h$ on 5 June 1000 at the longitude of modern-day Gorgan (54;26° East, exactly 10° east of Baghdad).

First example: solar and lunar velocity both close to their mean value

| 9 March 1000, noon at Kūshyār's meridian | exact calculation | Ptolemy | Islamic astronomers | Kūshyār's table |
|---|---|---|---|---|
| *Data for the starting point found from Kūshyār's zīj* | | | | |
| true solar longitude | 354;44,31 | | | |
| solar velocity $v_{\odot}$ (°/hr) | | | 0; 2,28 | |
| true lunar longitude | 0;11,53 | | | |
| lunar velocity $v_{☾}$ (°/hr) | | 0;33,16 | 0;33,16 | 0;33,16 |
| 'lunar precedence' $\Delta v$ (°/hr) | | | 0;30,48 | |
| *Calculated data* | | | | |
| 'distance' $\eta$ (true elongation) | −5;27,22 | −5;27,22 | −5;27,22 | −5;27,22 |
| 'part of the distance' $p_\eta$ | | −0;27,17 | | −0;24,12 |
| 'distance plus its part' $\eta+\Delta\eta$ | | −5;54,39 | | −5;51,34 |
| place of true syzygy from 'distance' plus its 'part' | | 354;17,14 | | |
| 'hours of the distance' $h_\eta$ | −10;40,51$^h$ | −10;39,39$^h$ | −10;37,44$^h$ | −10;34, 4$^h$ |
| time of the true conjunction | 1;19, 9$^h$ | 1;20,21$^h$ | 1;22,16$^h$ | 1;25,56$^h$ |
| error in the estimated time | | +0; 1,12$^h$ | +0; 3, 7$^h$ | +0; 6,47$^h$ |
| *Data for the found approximate time of the true syzygy, according to Kūshyār's zīj* | | | | |
| true solar longitude | 354;18,12 | 354;18,15 | 354;18,20 | 354;18,29 |
| true lunar longitude | 354;18,12 | 354;18,51 | 354;19,54 | 354;21,55 |
| true elongation | 0; 0, 0 | −0; 0,36 | −0; 1,34 | −0; 3,26 |

Kūshyār's Table of Conjunction and Opposition displays quantities with the same names as those used in his textual procedure, but since in the table these quantities both vary with the lunar velocity, it is clear that at least the 'part of the distance' must have been calculated in a different way. However, we may expect that it again represents the distance that the Sun moves on the ecliptic from the starting point of the procedure till the time of true syzygy, when the

Table M (*continued*)

Second example: solar and lunar velocity both close to one of their extremes

| 5 June 1000, noon at Kūshyar's meridian | exact calculation | Ptolemy | Islamic astronomers | Kūshyār's table |
|---|---|---|---|---|
| *Data for the starting point found from Kūshyār's zīj* | | | | |
| true solar longitude | 79;38,44 | | | |
| solar velocity $v_{\odot}$ (°/hr) | | | 0; 2,23 | |
| true lunar longitude | 84;13,15 | | | |
| lunar velocity $v_{☾}$ (°/hr) | | 0;36,54 | 0;36,54 | 0;36,54 |
| lunar precedence $\Delta v$ (°/hr) | | | 0;34,31 | |
| *Calculated data* | | | | |
| 'distance' $\eta$ (true elongation) | −4;34,31 | −4;34,31 | −4;34,31 | −4;34,31 |
| 'part of the distance' $p_{\eta}$ | | −0;22,53 | | −0;18,17 |
| 'distance plus its part' $\eta+\Delta\eta$ | | −4;57,24 | | −4;52,48 |
| place of true syzygy from 'distance' plus its 'part' | | 79;15,51 | | |
| 'hours of the distance' $h_{\eta}$ | −7;57,39$^{h}$ | −8; 3,34$^{h}$ | −7;57,11$^{h}$ | −7;56, 6$^{h}$ |
| time of the true conjunction | 4; 2,21$^{h}$ | 3;56,26$^{h}$ | 4; 2,49$^{h}$ | 4; 3,54$^{h}$ |
| error in the estimated time | | −0; 5,55$^{h}$ | +0; 0,28$^{h}$ | +0; 1,33$^{h}$ |
| *Data for the found approximate time of the true syzygy, according to Kūshyār's zīj* | | | | |
| true solar longitude | 79;19,47 | 79;19,33 | 79;19,48 | 79;19,50 |
| true lunar longitude | 79;19,47 | 79;16, 9 | 79;20, 4 | 79;20,44 |
| true elongation | 0; 0, 0 | +0; 3,24 | −0; 0,16 | −0; 0,54 |

Moon (or the point opposite it) overtakes the Sun. For the sake of convenience I will use the following notation:

| | |
|---|---|
| $\eta$ | 'distance', i.e., the true elongation at the starting point of the procedure |
| $v_{\odot}$ | solar velocity at the starting point |
| $v_{☾}$ | lunar velocity at the starting point |
| $\Delta v = v_{\odot} - v_{☾}$ | 'lunar precedence', i.e., the relative lunar velocity, the excess of the lunar over the solar velocity |
| $\Delta\eta$ | the ecliptic arc traversed by the Sun in the time that the Moon needs to overtake it |
| $p_{\eta}$ | 'part of the distance', estimate for $\Delta\eta$ by one of the methods discussed |
| $h_{\eta}$ | 'hours of the distance', estimate of the time from the starting point till true syzygy |

Consequently, the ecliptic arc covered by the Moon between the starting point and true syzygy (i.e., during the time $h_{\eta}$) is $\eta + \Delta\eta$, and it is estimated by $\eta + p_{\eta}$ in the methods of Ptolemy and, as we will see, Kūshyār.

If we take the solar velocity $v_{\odot}$ and the lunar velocity $v_{☾}$ to be constant on the time interval concerned, $p_{\eta}$ will in first approximation be equal to the distance travelled by the Sun in the time that the Moon needs to traverse the original elongation, i.e., to $v_{\odot} \cdot \eta / v_{☾}$. The time needed by the Moon to cover this additional stretch is $v_{\odot} \cdot (\eta / v_{☾}) / v_{☾}$, during which the Sun will pass through an additional ecliptic arc of $v_{\odot}^2 \cdot (\eta / v_{☾}) / v_{☾}$. Thus, in second approximation $p_{\eta}$ will be equal to $\eta \cdot (q + q^2)$ where $q = v_{\odot} / v_{☾}$ is the quotient of the solar and lunar velocities. Since, when expressed in degrees per day, $11.6 < v_{☾} < 15.1$ and $0.95 < v_{\odot} < 1.03$, we always have $0.062 < q < 0.089$, and therefore the second approximation will suffice to find $p_{\eta}$ to an accuracy of a minute of arc.[191]

Now, in order to calculate $p_{\eta}$ for an elongation of 1 degree as a function of the lunar velocity $v_{☾}$, Kūshyār needed to approximate the true solar velocity by a constant $c \approx 0;59,8,21$ (for example, $c = 0;59,8$ or $c = 0;59$) and compute, most likely, a function of the form $f(v_{☾}) = c/v_{☾}$. Since the second-order term $v_{\odot}^2/v_{☾}^2$ can become as large as 28 seconds and Kūshyār tabulated the 'part of the distance' to a precision of seconds, I will also consider the theoretical possibility that he used a function of the form $f(v_{☾}) = c/v_{☾} + c^2/v_{☾}^2$. The third to sixth columns of Table N show the errors in the four plausible recomputations that follow from these suppositions. The errors leave no doubt that Kūshyār tabulated the 'part of the distance' for an elongation of 1° as $p_{\eta}(v_{☾}) = 0;59/v_{☾}$. The use of a more accurate approximation to the daily solar motion, 0;59,8, produces negative errors of one second in most of the values, whereas the use of the second-order term shows differences of up to 26 seconds, in agreement with the order of magnitude of this term that I indicated above.

There are now two historically plausible methods of calculating the 'hours of the distance' $h_{\eta}$, namely by dividing the true elongation by the excess of the actual lunar velocity over the approximated solar velocity (i.e., $v_{☾} - 0;59$), as in the textual procedure explained by Kūshyār, or by dividing the sum of the elongation and the 'parts of the distance' by the lunar velocity (i.e., $(\eta + p_{\eta}) / v_{☾}$), as in the *Almagest*. Columns E and F of Table N show the errors for both possibilities. Judging from these errors, there is no doubt that Kūshyār used the second of these methods, thus in this case following Ptolemy. We can even see that he used for this his previously tabulated 'part of the distance', since a direct computation from the formula $h_{\eta} = (1 + 0;59 / v_{☾}) / v_{☾}$ leads to eight errors of ±1 second (see column G of Table N) rather than the single one shown in column F.

In the fifth column of Table M, I have added the results for the two worked examples if the method of computation underlying Kūshyār's table is used (but I have not rounded the lunar velocity to a multiple of 10 minutes per day, as in the table). The accuracy of the three methods can be compared as follows. All

191 For this argument, cf. Pedersen, *A Survey of the Almagest*, bottom of p. 224.

Table N: Recomputation of Kūshyār's Table of Conjunction and Opposition, showing the errors produced by several plausible methods of computation described in the text. For the 'part of the distance': A = one term, solar velocity approximated by 0;59,8°/day; B = one term, solar velocity approximated by 0;59°/day; C = two terms, 0;59,8; D = two terms, 0;59. For the 'hours of the distance': E = elongation divided by excess of lunar velocity over 0;59; F = elongation plus 'part of the distance' from the table divided by the lunar velocity; G = direct computation without use of the column for the 'part of the distance'.

| Lunar velocity | part of distance | rec. A | rec. B | rec. C | rec. D | hours of distance | rec. E | rec. F | rec. G |
|---|---|---|---|---|---|---|---|---|---|
| 11;50 | 0;4,59 | −1 | | −26 | −25 | 2;11,48 | −55 | | |
| 12; 0 | 0;4,55 | −1 | | −25 | −24 | 2; 9,50 | −53 | | |
| 12;10 | 0;4,51 | −1 | | −24 | −23 | 2; 7,55 | −51 | | |
| 12;20 | 0;4,47 | −1 | | −24 | −23 | 2; 6, 4 | −48 | | |
| 12;30 | 0;4,43 | −1 | | −23 | −22 | 2; 4,15 | −47 | | −1 |
| 12;40 | 0;4,39 | −1 | | −23 | −22 | 2; 2,30 | −45 | | −1 |
| 12;50 | 0;4,36 | | | −22 | −21 | 2; 0,49 | −42 | | +1 |
| 13; 0 | 0;4,32 | −1 | | −22 | −21 | 1;59, 8 | −42 | | −1 |
| 13;10 | 0;4,29 | | | −21 | −20 | 1;57,32 | −40 | | |
| 13;20 | 0;4,25 | −1 | −1 | −21 | −20 | 1;55,57 | −39 | | −1 |
| 13;30 | 0;4,22 | −1 | | −20 | −19 | 1;54,26 | −37 | | |
| 13;40 | 0;4,19 | −1 | | −19 | −19 | 1;52,57 | −35 | | |
| 13;50 | 0;4,16 | | | −19 | −18 | 1;51,30 | −34 | | |
| 14; 0 | 0;4,13 | | | −18 | −18 | 1;50, 5 | −33 | | |
| 14;10 | 0;4,10 | | | −18 | −17 | 1;48,42 | −32 | | |
| 14;20 | 0;4, 7 | −1 | | −18 | −17 | 1;47,22 | −30 | +1 | +1 |
| 14;30 | 0;4, 4 | −1 | | −17 | −17 | 1;46, 2 | −30 | | −1 |
| 14;40 | 0;4, 1 | −1 | | −17 | −17 | 1;44,45 | −29 | | −1 |
| 14;50 | 0;3,59 | | | −16 | −15 | 1;43,31 | −27 | | |

three methods have in common that they assume the solar and lunar velocity to be constant during the period between the starting point and true syzygy. For the solar velocity this is a reasonable assumption, since it changes by less than 0;0,2,8 °/day in the course of a day. The lunar velocity, however, can change by as much as 0;37 °/day in the course of a day. In order to obtain an impression of the size of the error that this may produce in the determination of the time of true syzygy, I will assume that the change in lunar velocity is linear, namely $v_{☾} = v_{☾}^0 + a \cdot t$, where $v_{☾}^0$ is the initial lunar velocity, $t$ the time reckoned in days from the starting point, and $a$ the lunar acceleration expressed in °/day per day. At the time $t$, the average lunar velocity since the starting point will have been $v_{\odot}^0 + ½\, a \cdot t$. Writing $\Delta v^0$ for the lunar precedence $v_{☾}^0 - v_{\odot}$ at the starting point, a good approximation to $h_\eta$ is given by $\frac{\eta}{\Delta v^0 + \frac{1}{2} a \cdot t}$ (avoiding a more accurate calculation with the use of integrals). The early Islamic approximation $\eta / \Delta v^0$ differs from this expression by

$$\frac{\eta}{\Delta v^0} - \frac{\eta}{\Delta v^0 + \frac{1}{2}a \cdot t} = \frac{\eta \cdot \left(\Delta v^0 + \frac{1}{2}a \cdot t\right) - \eta \cdot \Delta v^0}{\Delta v^0 \cdot \left(\Delta v^0 + \frac{1}{2}a \cdot t\right)}$$
$$= \frac{\frac{1}{2}\eta \cdot a \cdot t}{\Delta v^0 \cdot \left(\Delta v^0 + \frac{1}{2}a \cdot t\right)}$$
$$= \frac{\eta}{\Delta v^0} \cdot \frac{\frac{1}{2}a \cdot t}{\Delta v^0 + \frac{1}{2}a \cdot t}.$$

The acceleration $a$ is at most 0.615 °/day per day in absolute value, $\Delta v$ is at least 10.59 °/day, and we may assume that we apply the method only on the day of the syzygy, so that $t \leq ½$. Therefore it follows from the above that the relative error in the early Islamic approximation of the time of true syzygy is at most of the order of $\frac{1}{2} \cdot \frac{0.615\,t}{10.59 - \frac{1}{2} \cdot 0.615} \approx 0.03\,t$. Since $t$ increases roughly linearly with $\eta$, this implies that the error is quadratic in $\eta$ or $t$. As an example, if the lunar acceleration is near its maximum, the relative error in the time of true syzygy will be approximately $0.03 \cdot 9/24$ for a time nine hours before or after the syzygy, and hence the absolute error will be approximately $0.03 \cdot 9/24 \cdot 9/24 \approx 0.00422 \approx$ 0;0,15 days $\approx 6^m\,5^s$.

In the method used by the early Islamic astronomers, the variation in the lunar velocity is the only source of error if we ignore the much smaller variation in the solar velocity. In other words, in situations in which the lunar velocity is more or less constant, we expect the early Islamic method to give a very good estimation of the time to true syzygy. This is confirmed by a further series of tests that I carried out for conjunctions in the years AD 1000 and 1001 on the basis of Kūshyār's parameter values. In these tests I compared the estimates of the time of true syzygy according to all three methods discussed here at time spans of 3, 6 and 9 hours before and after each conjunction. Whenever the change in lunar velocity was close to zero, the early-Islamic estimate of the time of true syzygy was typically off by only 10 or 20 seconds even when applied to data for nine hours before or after the conjunction.

Both Ptolemy's method and the method underlying Kūshyār's table have one additional source of error. In Ptolemy's method this is the approximation of $\Delta\eta$ by $\eta/12$, which yields $\eta + \Delta\eta \approx 1.083\eta$. An approximation based on the exact mean motions of the Sun and the Moon would yield $1.080\eta$, whereas the actual value of $\eta + \Delta\eta$ lies in the range from $1.067\eta$ to $1.096\eta$ due to the variation in the solar and lunar velocities. We thus see that Ptolemy could not have gained significantly by using a different constant multiple of $\eta$ for his approximation of $\Delta\eta$, since the error caused by the variation in the lunar velocity is much larger. The resulting maximum error in the estimated time of true syzygy is $0.016\eta \,/\, \min v_{☾}$, i.e., at most $0.0014\eta$ days. For a large initial elongation of 6°, this would amount to at most 12 minutes of time. However, in situations in

which $\Delta\eta$ happens to be close to $\eta/12$ (and hence the quotient $v_{\odot}/v_{☾}$ is close to 13), Ptolemy's estimation of the time of true syzygy may be very accurate.

The method underlying Kūshyār's table only differs from Ptolemy in estimating $\Delta\eta$ by $0;59\eta/v_{☾}$ instead of by $\Delta\eta/12$. Since this takes into account the large variation of the lunar velocity, it may in general be expected to yield better estimates than Ptolemy's method. Besides the change in lunar velocity during the time between the starting point and true syzygy, the main source of error in Kūshyār's method is the omission of the second order term in the approximation of $\Delta\eta$ (see above). Considering only the first and second order terms of the approximation to $\Delta\eta$, Kūshyār's estimation of $\Delta\eta$ produces an error of $\left(\frac{v_{\odot} - 0;59}{v_{☾}} + \left(\frac{v_{\odot}}{v_{☾}}\right)^2\right) \cdot \eta$, which never exceeds $\left(\frac{\max v_{\odot} - 0;59}{\min v_{☾}} + \left(\frac{\max v_{\odot}}{\min v_{☾}}\right)^2\right) \cdot \eta \approx 0.011\eta$ days. Consequently, the maximum error in the estimated time of true syzygy will be at most $0.011\eta / \min v_{☾} \approx 0.00095\eta$ days. For an initial elongation of 6°, this amounts to little more than 8 minutes of time when the solar velocity is near its maximum and the lunar velocity near its minimum. However, due to the omission of the second-order term, Kūshyār's method will under no circumstances be able to give a result as accurate as the Islamic method.

In practice, the results of the three methods do not deviate by very large amounts. On average, the early Islamic method clearly produces the most accurate results and Ptolemy's method the least accurate ones, but in individual situations Ptolemy's and Kūshyār's methods may give equally good or even better results. Due to the rapid change of the lunar velocity, for some conjunctions (such as the one on 9 March 1000 in Table M) Ptolemy's method may provide the best estimate of the time of true syzygy for hours after the true conjunction, the early Islamic method for hours near the true conjunction, and Kūshyār's method for more than seven hours before the true conjunction. The early Islamic method only very rarely gives the worst result of the three methods, but of course also this may occur when the lunar velocity changes very rapidly and the further conditions are such that Ptolemy's and Kūshyār's methods yield near-optimal results.

As has been mentioned above, the method of finding true syzygy that divides the true elongation by the relative lunar velocity was already common in early Islamic astronomy. It is found in a section on conjunctions and oppositions contained in both extant recensions of the *Mumtaḥan Zīj* (although we cannot be entirely certain that it stems from the original work by Yaḥyā ibn Abī Manṣūr) and in the *zīj*es of Ḥabash al-Ḥāsib and al-Battānī.[192] The Latin

[192] For the *Mumtaḥan Zīj*, see Escorial, RBMSL, árabe 927, fols 81v–82v and Leipzig, Universitätsbibliothek, Vollers 821, fol. 27v. For Ḥabash al-Ḥasib, see Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2, fols 156r–v and 203v–204v, and Debarnot, 'The *Zīj* of

translation of the Andalusian reworking of al-Khwārizmī's *Sindhind Zīj* uses Ptolemy's method, but since it is for Cordoba it is probably a later addition by Maslama al-Majrīṭī.[193] Unfortunately, the sections on true syzygies from the important *zīj*es by Abū l-Wafāʾ and Ibn Yūnus are not contained in the incomplete surviving manuscripts of these works. Al-Bīrūnī extends the table for solar and lunar velocities to every degree of the respective anomalies, uses the same method as his Islamic predecessors, but does not provide a table for the calculation of true syzygies.[194]

We may thus for the time being conclude that Kūshyār's Table of Conjunction and Opposition was the earliest Islamic table for finding the time and place of true syzygies, and also the earliest double-argument table for this purpose. It was around a century earlier than the table in the *Muqtabas Zīj* by Ibn al-Kammād described by Chabás and Goldstein.[195] Ibn al-Kammād's table displays the time from mean to true syzygy (i.e., Kūshyār's 'hours of the distance', *sāʿāt al-buʿd*) and is likewise a double-argument table with the elongation (in steps of 0;30°) as its vertical argument. However, the horizontal argument is the relative velocity of the Moon with respect to the Sun expressed in degrees per hour, rather than the lunar velocity expressed in degrees per day. Thus Ibn al-Kammād's table requires a little more work by its user (calculation of the relative hourly velocity and simple linear interpolation for values of the elongation in between multiples of 0;30), but can be expected to produce somewhat more accurate results due to the incorporation of the variation of the solar velocity.[196]

Among the later eastern-Islamic *zīj*es that incorporated materials from Kūshyār's *Jāmiʿ Zīj*, the *Zīj al-Qirānāt* by al-Rīqānī includes a section that describes the early-Islamic method for finding true syzygies and gives explicit

Ḥabash al-Ḥāsib', pp. 51 and 59. For al-Battānī, see Chapter 42 in Escorial, RBMSL, árabe 908, fol. 96r–v; Nallino, *al-Battānī sive Albatenii*, vol. III, pp. 138–45 (esp. pp. 141–42, Arabic), and vol. I, pp. 92–96 (esp. p. 94, Latin translation).

193 Suter, *Die astronomischen Tafeln*, Chapter 31, pp. 23–25 (Latin text) and 81–84 (commentary); Neugebauer, *The Astronomical Tables of al-Khwārizmī*, pp. 59–63 (English translation and commentary).

194 Kennedy, 'al-Bīrūnī's Masudic Canon', p. 71 and Chapter VIII.2 in Paris, BnF, arabe 6840 (available online), fols 128r–129r or in al-Bīrūnī, *al-Qānunuʾl-Masʿūdī*, vol. II, pp. 884–90.

195 cf. Chabás and Goldstein, 'Computational Astronomy: Five Centuries', pp. 94–95; Chabás and Goldstein, 'Andalusian Astronomy: *al-Zîj al-Muqtabis*', p. 14, and, most extensively, Chabás and Goldstein, 'Ibn al-Kammād's *Muqtabis* Zij', pp. 624–25. The table is found in Madrid, Biblioteca Nacional, MS 10023, fol. 52r and in Vatican, BAV, Vat. ebr. 498, fol. 58v.

196 In the first of the two working examples in Table M above, Ibn al-Kammād's table would need to be used with a relative velocity of 0;31,0°/hr and then yields a time interval of $-10;35,11^h$, somewhat better than Kūshyār's table. In the second example no direct comparison is possible because Ibn al-Kammād only tabulates the time till true syzygy for relative velocities up to 0;33,30 ° /day.

values for the minimum and maximum solar and lunar velocities.[197] The table of contents mentions a table 'On Conjunctions and Oppositions' as Table 41, but this is unfortunately missing from the manuscript. The late eleventh-century *Mufrad Zīj* includes a table entitled Hours of Conjunctions and Oppositions (*sāʿāt al-ijtimāʿāt wa-l-istiqbālāt*).[198] This table has the same vertical arguments as Kūshyār's table, namely 1, 2, 3, …, 15, 18, 21, …, 60 degrees of the elongation (*ajzāʾ al-buʿd*), and as horizontal arguments relative lunar velocities 10;0, 11;0, …, 15;0°/day. The tabular values are given in days, hours and minutes, and the constant tabular differences are provided in order to facilitate linear interpolation between degrees of elongation. Although this table is similar to Kūshyār's in its general setup, it is clearly independent from it. As has already been mentioned above, the *Dustūr al-munajjimīn* and the *Baghdādī Zīj* include Kūshyār's table for finding true syzygies, the latter with correct instructions for its use. I have not investigated the general treatment of the problem of finding true syzygies in other Islamic *zīj*es after the time of Kūshyār.

To summarise, in a later stage of his work on the *Jāmiʿ Zīj* Kūshyār decided to include a double-argument table for calculating the place and time of true syzygy. As far as we know, this was the earliest table of this type in Islamic astronomy. The table is not yet contained in the version of the *Jāmiʿ Zīj* extant in manuscripts **FHCC$_1$C$_2$**, but is included among the additional tables in manuscripts **YB**, is inserted in the table of contents of **L**, while instructions for its use are found in Book I in manuscripts **YL**. With this table Kūshyār improved upon Ptolemy's method thanks to his more accurate estimate of the distance covered by the Sun until it was overtaken by the Moon. Like Ptolemy, he then found the time of true syzygy by adding this distance to the true elongation and dividing by the lunar velocity. Further improvements of the accuracy of this method could be achieved by also including the variation of the solar velocity, either by tabulating the time of true syzygy as a function of the relative velocity of the Moon and the Sun, as did Ibn al-Kammād, or by providing a double-argument table with the solar and lunar anomalies as independent variables, as we find in several later-medieval European tables in the Alfonsine tradition.[199]

[197] Chapter 46 in Paris, BnF, arabe 6913, fol. 29v. The list of tables in the second part of the *Zīj al-Qirānāt* is found on fol. 47v.

[198] Cambridge, University Library, Browne O.1, fol. 100v.

[199] Sets of astronomical tables in Latin show a wide variety of tables for finding the place and time of true syzygy, including double-argument ones. Besides the overview article Chabás and Goldstein, 'Computational Astronomy: Five Centuries', see for examples Chabás and Goldstein, 'Nicholaus de Heybeck'; Porres and Chabás, 'John of Murs's *Tabulae permanentes*'; Kremer, 'John of Murs, Wenzel Faber', and Chabás and Goldstein, 'The Medieval Moon in a Matrix', pp. 347–52. Mathematical analyses of the double-argument tables in the *Tabule magna* and the *Tabulae permanentes* are carried out in the articles by Matthieu Husson and

The tables in manuscript **B** and the *Dustūr al-munajjimīn* share, in the margin of the first page of the table, the explanatory text A (see p. 322). It prescribes, first, the division of the daily solar mean motion by the lunar velocity per day in order to obtain the 'increase of the part of the distance' (*tazāyud juzʾ al-buʿd*). As we have seen above, this is in fact the estimate of the 'part of the distance' for an elongation of 1° tabulated by Kūshyār (where the table uses a rounded mean motion of 0;59°/day). Next the elongation should be divided by the difference of the lunar velocity per hour and the solar mean motion in an hour in order to obtain the 'increase of the hours of the distance' (*tazāyud sāʿāt al-buʿd*). As we have seen, this is the procedure by which Kūshyār computes the 'hours of the distance'. The deviations in this text from Kūshyār's standard terminology nevertheless make it implausible that the text stems from him directly.

## Table 50: Lunar distance from the Earth

**Bibliography**: as-Saleh, 'Solar and Lunar Distances'; Pedersen, *A Survey of the Almagest*, pp. 194–95; Van Brummelen, *Mathematical Tables*, pp. 168–70; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.6.7, pp. 72–73 (translation), 87–88 (commentary) and Arabic pp. 46–47; Chapter IV.6.5, pp. 192–93 (translation), 205 (commentary) and Arabic pp. 123–24; Bagheri et al., 'Kūshyār ibn Labbān Gīlānī's Treatise'.

This table gives the distance of the Moon from the Earth for every six degrees of the true lunar anomaly and the values 0, 5, 10, …, 45° of the double elongation. Thus it is one of the earliest Islamic double-argument tables for a function related to the moon.[200] A number of treatises on the sizes and distances of the heavenly bodies have survived from the medieval Islamic period, including the one by Kūshyār that is in some manuscripts appended to Book III of the *Jāmiʿ Zīj*,[201] but these deal primarily with cosmological aspects and determine at most a maximum and minimum lunar distance from the centre or the surface of the Earth.[202] In order to attempt to understand Kūshyār's table of lunar distance, it is necessary to look in detail at Ptolemy's lunar model.

Richard L. Kremer in the forthcoming volume Husson et al., *Editing and Analysing Numerical Tables*.

200 No such tables are included in the extant *zīj*es by the earlier Islamic astronomers discussed in Section I.1 ('Historical background'). Ibn Yūnus has been associated with an extensive set of double-argument tables for the lunar equation of anomaly of the so-called *ḥabṭaq* type; see King, 'A Double-Argument Table' (pp. 130–31 of this article mention several later Islamic double-argument tables for finding the lunar position).

201 See Bagheri et al., 'Kūshyār ibn Labbān Gīlānī's Treatise'.

202 Further examples of studies of such treatises are Goldstein and Swerdlow, 'Planetary Distances and Sizes'; Langermann, 'The Book of Bodies and Distances' (based on a manuscript from the collection of Rabbi Yosef Kafaḥ (Yaḥyā al-Qāfiḥ), who also owned manuscript **H**

The lunar distance from the Earth, expressed in units for which the maximum distance of the epicycle centre from the Earth equals 60, was already needed for the calculation of the lunar equation of anomaly. As was shown by Pedersen and Van Brummelen, the distance $\rho$ of the epicycle centre from the Earth is given by

$$\rho(2\eta) = \sqrt{R^2 - (e \cdot \sin 2\eta)^2} + e \cdot \cos 2\eta,$$

where $e$ is the lunar eccentricity (i.e., the radius of the crank circle), $R$ is the radius of the lunar deferent, taken to be equal to $60 - e$, and $2\eta$ is twice the elongation of the Moon from the Sun. From this the distance $\Delta(2\eta, a_v)$ of the Moon from the Earth can be found for any value $a_v$ of the true lunar anomaly as

$$\Delta_{☾}(2\eta, a_v) = \sqrt{(\rho + r \cdot \cos a_v)^2 + (r \cdot \sin a_v)^2}.$$

The Ptolemaic radius of the lunar epicycle (5;15 units) is quite small with regard to the distance of the epicycle centre from the Earth (which varies between 39;22 at the quadratures and 60 at the syzygies). Since Kūshyār tabulates the distance of the Moon from the Earth only for values of the double elongation up to 45° (i.e., for lunar positions at most 22½° removed from the syzygies), his table involves only epicycle distances between 56;26 and 60. For these large epicycle distances, the change in the lunar distance from the Earth as a function of the lunar position on the epicycle is only minimally influenced by the double elongation, i.e., $\Delta_{☾}(2\eta, a_v)$ can be very well approximated by $\Delta_{☾}(0°, a_v) + c_{2\eta}$, where $c_{2\eta}$ is a constant that can be fixed, for example, as $c_{2\eta} = \Delta_{☾}(2\eta, 0°) - \Delta_{☾}(0°, 0°)$. By definition, this approximation produces the exact lunar distance whenever $2\eta = 0°$ (i.e., at the syzygies) or $a_v$ equals 0° or 180° (i.e., at the apogee and perigee of the epicycle). For every value of $2\eta$, the maximum error in the approximation occurs near $a_v = 96°$, i.e., when the equation of anomaly is near its maximum. Over the entire domain of the function the maximum error amounts to at most 7′ (for $2\eta$ near 180°), in the domain of Kūshyār's table the maximum error is only 52″ (for $2\eta = 45°$).

The above possibility to accurately approximate the lunar distance from the Earth was in fact used by Ḥabash al-Ḥāsib in a rule included in both extant manuscripts of his *zīj*.[203] This rule makes use of two of the three interpolation functions in the table for solar and lunar parallax in Ptolemy's *Almagest*.[204] Since

of Kūshyār's *Jāmiʿ Zīj*); Hogendijk, 'Al-Ṣaghānī's Treatise on the Distances', and Hogendijk, 'Al-Qabīṣī's Treatise on the Distances'.

[203] See as-Saleh, 'Solar and Lunar Distances', pp. 167–71.

[204] These are the seventh and ninth columns in the parallax table in Toomer, *Ptolemy's Almagest*, p. 265. See also Pedersen, *A Survey of the Almagest*, pp. 214–18 and Van Brummelen, *Mathematical Tables*, pp. 199–201 and 209–16. I avoid the use of Neugebauer's notation $c_7$ and $c_9$ (see *HAMA*, vol. I, pp. 112–15) because it is increasingly confusing in the later Islamic

these interpolation functions represent versions, scaled to the interval [0,1], of respectively the lunar distance from the Earth at the time of syzygy and the distance of the epicycle centre from the Earth, this method comes down to approximating $\Delta_{☾}(2\eta,a_v)$ by $\rho(2\eta) + \Delta_{☾}(0°,a_v) - 60$. In other words, the total lunar distance from the Earth is split up into two independently treated components, namely, the distance of the epicycle centre and the difference between 60 and the lunar distance at the time of syzygy.

Table O shows the errors in Kūshyār's table for the lunar distance from the Earth with respect to an exact computation according to Ptolemy's lunar model and his and Kūshyār's parameter values $e = 10;19$ and $r = 5;15$. Since there are systematic errors of up to −11′, it is immediately clear that Kūshyār used neither a basically exact method of computation nor the accurate approximation explained above. He describes the determination of the distance of the Moon from the Earth in Section I.6.7 of his *Jāmiʿ Zīj*, with a complete geometrical proof in Section IV.6.5.[205] As Bagheri points out in his commentary, in the calculation of the lunar distance Kūshyār unnecessarily makes use of the 'adjusted radius of the epicycle' (*niṣf quṭr falak al-tadwīr al-muʿaddal*), which he finds from his alternative tables for the lunar equation of anomaly. For any given value of the double elongation this 'adjusted radius' is the maximum equation of anomaly, i.e., the maximum apparent angle between the epicycle centre and the body of the moon as seen from the Earth, and is therefore helpful in the calculation of the equation of anomaly. However, Kūshyār continues to calculate the lunar distance as if the 'adjusted radius' were the epicycle radius itself. All in all, Kūshyār's explanations do not offer useful clues as to how he may have computed his table of the lunar distance from the Earth.

I have experimented with various other possible ways of approximating the lunar distance from the Earth or its two components. Considering that a medieval astronomer may want to avoid the cumbersome calculation of a square root, and that, for the values of the double elongation for which Kūshyār tabulated the lunar distance, the sine of the double elongation is particularly small with respect to the epicycle distance, I found that the approximation $\Delta_{☾}(0°,a_v) \approx 60 + 5;15 \cdot \cos a_v + 0;12 \cdot \sin a_v$ produces Kūshyār's lunar distance at the time of a syzygy significantly better than an exact recomputation: it removes nearly all negative errors up to −2′ for values of the true anomaly between 72 and 120° as well as the positive errors up to +3′ outside of this interval, leaving only six scattered errors of ±1′ in the 31 values. Similarly, $60 + 12;18 \cdot (\cos 2\eta - 1)$ provides a slightly better recomputation of what may be called the 'mean epicycle distance'. From this quantity, the lunar distance at the apogee of the epicycle

context, in which many of the solar, lunar and planetary tables were arranged differently from the *Almagest* and *Handy Tables*.

205 Bagheri, *az-Zīj al-Jāmiʿ*, pp. 72–73 and 192–93 (translation), 87–88 and 205 (commentary) and Arabic pp. 46–47 and 123–24.

Table O: Errors in Kūshyār's table of the lunar distance from the Earth (in minutes).

| | Double elongation | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| True anomaly | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 |
| $0^s$ 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | −1 | −2 |
| $0^s$ 6 | 1 | 0 | 1 | 1 | 0 | 0 | 2 | 2 | 1 | −1 |
| $0^s$ 12 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 | 4 | 2 |
| $0^s$ 18 | 3 | 3 | 4 | 4 | 4 | 4 | 5 | 5 | 6 | 5 |
| $0^s$ 24 | 3 | 3 | 3 | 4 | 4 | 5 | 6 | 6 | 8 | 7 |
| $1^s$ 0 | 3 | 3 | 3 | 3 | 4 | 4 | 6 | 7 | 8 | 7 |
| $1^s$ 6 | 3 | 3 | 3 | 4 | 5 | 4 | 6 | 8 | 9 | 8 |
| $1^s$ 12 | 2 | 2 | 2 | 3 | 3 | 4 | 5 | 7 | 7 | 7 |
| $1^s$ 18 | 2 | 2 | 2 | 3 | 3 | 3 | 6 | 7 | 7 | 7 |
| $1^s$ 24 | 1 | 1 | 2 | 3 | 2 | 3 | 5 | 6 | 7 | 6 |
| $2^s$ 0 | 1 | 0 | 1 | 2 | 1 | 2 | 3 | 4 | 5 | 5 |
| $2^s$ 6 | 0 | 0 | 0 | 1 | 1 | 1 | 3 | 4 | 4 | 3 |
| $2^s$ 12 | −1 | −2 | −1 | 0 | −1 | 0 | 0 | 1 | 1 | 0 |
| $2^s$ 18 | −1 | −2 | −1 | 0 | −1 | −1 | 0 | 1 | 1 | −1 |
| $2^s$ 24 | −1 | −2 | −1 | 0 | −2 | −1 | −1 | −1 | −2 | −3 |
| $3^s$ 0 | −2 | −2 | −1 | 0 | −2 | −3 | −2 | −2 | −3 | −5 |
| $3^s$ 6 | −2 | −2 | −2 | −1 | −2 | −4 | −3 | −3 | −4 | −7 |
| $3^s$ 12 | −1 | −1 | −1 | 0 | −2 | −3 | −3 | −4 | −6 | −8 |
| $3^s$ 18 | −1 | −2 | −1 | 0 | −3 | −4 | −4 | −5 | −7 | −10 |
| $3^s$ 24 | −1 | −1 | 0 | 1 | −2 | −4 | −4 | −5 | −7 | −10 |
| $4^s$ 0 | −1 | −1 | −1 | 0 | −3 | −4 | −5 | −6 | −7 | −11 |
| $4^s$ 6 | 1 | 0 | 1 | 2 | −1 | −2 | −4 | −5 | −7 | −10 |
| $4^s$ 12 | 1 | 1 | 1 | 2 | −1 | −2 | −4 | −14 | −6 | −10 |
| $4^s$ 18 | 1 | 1 | 2 | 3 | 0 | −1 | −2 | −4 | −5 | −9 |
| $4^s$ 24 | 2 | 2 | 2 | 3 | 0 | −1 | −1 | −3 | −6 | −9 |
| $5^s$ 0 | 2 | 2 | 2 | 2 | 1 | −1 | −1 | −2 | −5 | −8 |
| $5^s$ 6 | 2 | 2 | 3 | 3 | 1 | 0 | −1 | −1 | −3 | −6 |
| $5^s$ 12 | 2 | 2 | 2 | 3 | 1 | 1 | 0 | 0 | −1 | −4 |
| $5^s$ 18 | 1 | 1 | 2 | 2 | 0 | 0 | 0 | 2 | −2 | −4 |
| $5^s$ 24 | 1 | 1 | 1 | 2 | 1 | 1 | 0 | 1 | 0 | −3 |
| $6^s$ 0 | 0 | 0 | 0 | 0 | 0 | −1 | 0 | 0 | −1 | −2 |

can be found by adding 5;15 and that at the perigee by subtracting 5;15. The result has only 2 errors of ±1′ in the 10 tabular values given by Kūshyār for both extremes, thus getting rid of the errors of −2′ for $2\eta = 45°$. A linear combination of these two approximations produces an error pattern roughly symmetric around $a_v = 90°$; it leads to correct values for $a_v = 90°$ and symmetric errors of up to ±8′ for other values of $a_v$, where those for $a_v < 90°$ are positive and those for $a_v > 90°$ negative. However, it obviously cannot explain Kūshyār's clearly asymmetric error pattern.

From Table O and all other attempts described above to recompute Kūshyār's table, it is clear that Kūshyār's value 54;10 for $\Delta_{☾}(35°, 4^s 12°)$ is an outlier that should be corrected to 54;20 rather than to the value 54;17 found in manuscripts **YLB**.

## Table 51: Solar parallax

In the *Almagest* Ptolemy provided a table for parallax in the altitude circle that included one column for solar parallax besides seven for lunar parallax, all tabulated for arguments 2, 4, 6, ..., 90°. Since lunar parallax is a function of three variables (the zenith distance, the double elongation and the true lunar anomaly), Ptolemy tabulated the lunar parallax at four 'limits' and provided three interpolation functions in order to implement a two-step Ptolemaic interpolation procedure between these limits.[206] In the *Handy Tables* Ptolemy followed a very different approach and provided tables for the parallax in longitude and latitude for each of the seven climates together with a correction table.[207] Al-Battānī provided the table from the *Almagest* with the same set of arguments and the column for solar parallax as well as the set of tables for the seven climates from the *Handy Tables*.[208]

Rather than providing tables, Kūshyār gives instructions for calculating the lunar parallax in altitude in Sections I.6.7–11. After having found the lunar distance from the Earth by means of a calculation or Table 50, he first explains some auxiliary methods of spherical astronomy (Sections I.6.8–10) and then finds the lunar parallax from a plane-trigonometrical calculation involving the lunar distance and the radius of the Earth (Section I.6.11). Finally, he subtracts the solar parallax, for which he presents Table 51. Kūshyār did not simply copy

206 Pedersen, *A Survey of the Almagest*, pp. 214–18 and Van Brummelen, *Mathematical Tables*, Chapter 10, pp. 194–217, esp. pp. 199–201.

207 Stahlman, *The Astronomical Tables*, pp. 93–98 and 128–33, with Table 18 on p. 257 and Tables 28–35 on pp. 268–83. For the explanation of the tables for parallax and eclipses, Stahlman relies strongly on Rome, *Commentaires de Pappus et de Théon*, pp. xlv–lvi.

208 Nallino, *al-Battānī sive Albatenii*, vol. II, pp. 89 ('correction') and 93–101 (with commentaries on pp. 231 and 235–37). The explanatory chapters on solar and lunar parallax from the *Ṣābiʾ Zīj* (Chapters 39 and 40) are edited in vol. III, pp. 114–28 and translated into Latin in vol. I, pp. 76–85, with an extensive commentary in vol. I, pp. 251–65.

the solar parallax table from the *Almagest* or al-Battānī's *Ṣābiʾ Zīj*, but computed it anew for a slightly different parameter. Whereas the maximum parallax in Ptolemy's table is 0;2,51°, that in Kūshyār's table is exactly 0;3,0°.[209]

The solar parallax $\pi$ as a function of the geocentric zenith distance $z'$ is given by

$$\pi(z') = \arctan\left(\frac{\sin z'}{\Delta_\odot - \cos z'}\right),$$

where $\Delta_\odot$ is the distance of the Sun from the Earth expressed in Earth radii.[210] Since $\pi$ is always very small, Ptolemy replaces this by

$$\pi(z') = \arcsin\left(\frac{\sin z'}{\Delta_\odot - \cos z'}\right).$$

In the *Almagest* Ptolemy found the mean value of $\Delta_\odot$ as 1210 from his previously determined distance of the Moon, the apparent lunar diameter, and its ratio to the diameter of the shadow of the Earth. In a small section at the end of Chapter I.6.11 of the *Jāmiʿ Zīj*, Kūshyār states that the solar parallax is found similarly to his previous determination of the lunar parallax using the mean solar distance from the Earth.[211] He adds that the solar parallax is at most about 3 minutes and that it must be added to the lunar parallax for additional accuracy. Kūshyār determines minimum, mean and maximum values of the solar distance at the end of Section I.6.7.[212] Using the solar mean anomaly instead of the true anomaly, 2;1° instead of the adjusted radius of the epicycle (cf. the commentary to Table 50 above), 60 instead of the epicycle distance, and multiplying the result by 18⅘, he finds the maximum solar distance as 1255 Earth radii, the mean distance as 1208 and the minimum distance as 1161.

Kūshyār's table of solar parallax is most accurately recomputed for a solar distance near 1146½ Earth radii. Since this falls outside of the range of possible distances found by Kūshyār in Section I.6.7, it seems likely that he intended his table to have a round maximum value of 0;3,0. There are various ways in which he could have achieved this. For any values of the solar distance near 1146½, his table has a very peculiar error pattern: two groups of errors of +1′ for arguments 3–12 and 51–63 and two groups of errors of –1′ for arguments 27–36 and 69–75. These could have resulted from some type of interpola-

[209] Of course, neither table provides a realistic measure for solar parallax, which in fact is at most 9 seconds of arc and hence not perceptible in naked-eye astronomy.

[210] cf. Pedersen, *A Survey of the Almagest*, pp. 213–14, and Van Brummelen, *Mathematical Tables*, pp. 193–96.

[211] Bagheri, *az-Zīj al-Jāmiʿ*, p. 75 (translation) and Arabic p. 49.

[212] Ibid., p. 73 (translation) and Arabic p. 47.

tion, for example on intervals of 15°, but neither the tabular differences nor a reconstruction along these lines seems able to confirm this. Kūshyār could also have calculated his solar parallax from Ptolemy's table as a homothetic table (cf. p. 529) by multiplying the latter by 0;3,0 / 0;2,51 ≈ 1;3,9 and then performing interpolation between Ptolemy's values for multiples of 2° or 6° to find the missing values for odd multiples of 3°. However, no reconstruction along these lines yields a significantly better agreement with Kūshyār's table than the direct computation for a solar distance near 1146½.

### Table 52: Magnitude of eclipses

The properties of eclipses are calculated by representing the Sun and the Moon (for solar eclipses) and the Moon and the shadow of the Earth (for lunar eclipses) as circular disks passing in front of each other. The duration and magnitude can then be determined by means of simple plane geometry once the apparent lunar latitude and the diameters of the disks are known. The magnitude $d$ of a solar or lunar eclipse is most conveniently measured as the obscured fraction of the diameter of the eclipsed body at the middle of the eclipse. The magnitude is thus equal to the sum of the radii of the two disks concerned minus the distance between their centres, divided by the diameter of the obscured body. Islamic astronomers took over Ptolemy's convention to express the magnitude in 'digits' (*aṣābiʿ*, twelfths), so that a total eclipse corresponds to a magnitude of 12 digits.

However, already Ptolemy felt the need to also express the magnitude as a fraction of the area of the obscured body. For this purpose he provided a conversion table in Chapter VI.8 of the *Almagest* that gives the area digits corresponding to 1, 2, 3, ..., 12 linear digits for a solar and for a lunar eclipse. The same table is also found in the *Handy Tables*.[213] This table was copied in the *Ṣābiʾ Zīj* by al-Battānī[214] and then in the *Jāmiʿ Zīj* by Kūshyār. Kūshyār calls the linear digits 'absolute digits' (*aṣābiʿ muṭlaqa*) and the area digits 'corrected digits' (*aṣābiʿ muʿaddala*). There are very few differences between Ptolemy's table edited by Toomer,[215] the Arabic and Castilian manuscript versions and Nallino's edition of al-Battānī's table, and Kūshyār's copy. In the column for solar eclipses, Nallino reproduces Ptolemy's value for 3 digits as 1;45, although the Escorial manuscript has a somewhat distorted 1;21 or 1;51; the Arsenal manuscript and Kūshyār round the value to 1;50. Both Escorial and Arsenal have the value 3;20 for 5 digits, but Nallino and Kūshyār follow the *Alma-*

213 Stahlman, *The Astronomical Tables*, Table 19, p. 258 (with commentary on pp. 99–100).

214 Nallino, *al-Battānī sive Albatenii*, vol. II, p. 89 (with a brief commentary on p. 231). The table is found in Escorial, RBMSL, árabe 908, fol. 195r (Arabic) and Paris, Bibliothèque de l'Arsenal, MS 8322, fol. 57v (Castilian).

215 Toomer, *Ptolemy's Almagest*, p. 308.

*gest* with 3;40. In the column for lunar eclipses, al-Battānī rounded Ptolemy's value 2⅟₁₅ for 3 digits to 2;5, whereas Kūshyār rendered it as 2;0. For 7 digits the Arsenal manuscript appears to have a scribal error 6;46 for Ptolemy's 6¾, which is also found in the Escorial manuscript and in Nallino, whereas Kūshyār again avoids a number of minutes ending in 5 and rounds to 6;40.

The conversion from linear to area digits is explained in detail by Van Brummelen.[216] He found that the best agreement with the column for solar eclipses in the *Almagest* is obtained for Ptolemy's miscalculated ratio 12;20 : 12 of the lunar and solar radius. In the column for lunar eclipses, the ratio 2⅗ : 1 is used consistently. Both columns have errors up to 7 minutes, but since the values were apparently rounded to multiples of 5 or 10 minutes, this can be considered an acceptable agreement.

[216] Van Brummelen, *Mathematical Tables*, pp. 233–38.

## IV.12. Astrology

Several sections in Book I of the *Jāmiʿ Zīj* make it clear that astrological calculations were an important application for the intended users of the work. Section I.5 is entitled 'On operations related to ascendants'. After treating basic topics in spherical astronomy such as declinations and ascensions, it deals with transits, arcs of revolution and the equalisation of the houses. Section I.7 is dedicated explicitly to astrology (*aḥkām*) and explains the projection of planetary rays by several methods, prorogations (*tasyīr*s), year transfers and world years. While other *zīj*es might also include tables for the equalisation of the houses, the projection of the rays and the duration of gestation (for application of the *namūdhār* of Hermes), Kūshyār explains the procedures for these complicated calculations in words. He provides a table only for the relatively easy calculation of small and medium prorogations.

### Table 53: Prorogations *(tasyīrāt)*

**Bibliography**: Bouché-Leclercq, *L'astrologie grecque*, pp. 410–22; the *EI²* article 'Tasyīr' by O. Schirmer; Kennedy, 'Ramifications'; Yano and Viladrich, 'Tasyīr Computation'; Bagheri, *az-Zīj al-Jāmiʿ*, Chapter I.7.4, pp. 95–97 (translation), 101 (commentary) and Arabic pp. 60–63; Gansten, 'Balbillus and the Method'; Casulleras and Hogendijk, 'Progressions, Rays and Houses'.

The *tasyīr* (translated as 'prorogation', 'progression' or 'direction', from the Greek ἄφεσις, in Latin *aphesis*, *atazir* or *directio*) is an astrological concept that can be used to predict all kinds of events throughout the life span, including death (and hence the length of life). This can be applied not only to individuals, but also to dynasties, religions and even the world. It uses an arc between certain points on the celestial sphere, whose length in equatorial degrees (measured by using different types of projections onto great circles, leading to increasingly complicated mathematical methods) measures the years of life. Yano and Viladrich discussed Kūshyār's calculation of the *tasyīr* arc presented in Chapter III.21 of his *al-Madkhal fī ṣināʿat aḥkām al-nujūm*.[217] To this end they also considered the material in Chapter I.7.4 of the *Jāmiʿ Zīj* and in al-Battānī's *Ṣābiʾ Zīj*.

Kūshyār's Table 53 solves only a small part of the practical determination of a prorogation. It displays the middle prorogation (*al-tasyīr al-wusṭā*), which moves at a rate of one zodiacal sign in a solar year, and the small prorogation (*al-tasyīr al-ṣughrā*), which moves at a rate of 13 zodiacal signs in a solar year. For both types the motion is tabulated in 1, 2, 3, ..., 30 days and in the months of the Persian year, indicated as 1, 2, 3, ..., 12, 12,5. This last argument is to be understood as '12 months plus 5 days', but the 5 is missing from manuscripts

[217] Yano and Viladrich, 'Tasyīr Computation'.

**$CC_1$Y.** The tabulated functions are linear, but the tables do not have exact constant tabular differences since the underlying daily and monthly motions are more precise than shown in the table. A table with the same format is included in Kūshyār's *al-Madkhal fī ṣināʿat aḥkām al-nujūm*, which was written after his two *zīj*es.[218] The table in the *Madkhal* as it was edited by Yano differs from the table in the *Jāmiʿ Zīj* by a single minute in twelve of the total of 86 tabular values. Since most of the differences cannot be attributed to scribal errors, it seems that the tables were computed independently. Neither set is completely accurate for the parameters that follow directly from the headings of the tables.

Among their additional tables, manuscripts **Y** and **B** include an extension of the present table that gives both types of prorogations for every single day of a Persian year.[219] The month numbers in the column headers of this table are written out as 'one month' (*shahr*), 'two months' (*shahrān*), 'three months' (*thalātha ashhar*), ... 'twelve months' (*ithnā ʿashr shahran*). Only **Y** adds a narrow column for the *epagomenae* or 'stolen days' (*al-mustaraqa*), in which the digits of the tabular values are written beneath each other in order to fill up the entire height of the page. The values for 1, 2, 3, ..., 30 days in this table are in full agreement with the small table found in all the manuscripts of the *Jāmiʿ Zīj*. In the values for months, however, **B** repeatedly deviates from the small table and in a number of these deviations agrees with *al-Madkhal fī ṣināʿat aḥkām al-nujūm*.

I have considered the possibility that this more extensive table was another addition by Kūshyār himself in the version of the *Jāmiʿ Zīj* represented by manuscripts **YLB**. This appears to be confirmed by a reformulation of the sentence referring to the table in Chapter I.70 in manuscript **L**.[220] Whereas all other manuscripts only speak of tabular values 'next to the months and the days' (*bi-izāʾi l-shuhūr wa-l-ayyām*), the formulation in **L** explicitly mentions the months as the horizontal argument and the days as the vertical argument: 'We compiled for them (i.e., the *tasyīr*s) two tables ⟨and⟩ wrote in each of them the completed months horizontally and the days of the months vertically' (*wadaʿnā li-humā jadwalayn katabnā fī kull wāḥid min-humā al-shuhūr al-tāmma ʿarḍan wa-ayyām al-shuhūr ṭūlan*). Thus, **L** describes exactly the additional table found in **Y** and **B**. The same formulation (with *ṭūlan* mistakenly replaced by a repetition of *ʿarḍan*) occurs in an additional paragraph in

[218] This table was edited and transcribed in Yano, *Kūšyār Ibn Labbān's Introduction*, pp. 226/7.

[219] See Section I.6, pp. 44 and 48. The table in **Y** is found on fols 309r–310r in between the small table of prorogations and the geographical table. The table in **B** is found on pp. 63–65, to which it was moved together with the sine table for minutes of arc so that the latter would be near the other trigonometric tables but the table of prorogations would not be split.

[220] Chapter I.70 on *tasyīr*s is found on fols 17v–18v in **L**, with the mention of the horizontal and vertical arguments in fol. 17v, lines 6–5 from the bottom of the page.

Chapter I.7.4 in **F**, which is said to be based on 'another manuscript of this *zīj*'.[221] Even though any later astronomer could easily have extended Kūshyār's table of prorogations by looking at its simple mathematical structure, it thus certainly seems possible that Kūshyār added the more extensive table himself. Note that we have a very similar situation as for the Table of Conjunction and Opposition (Table 49$^{bis}$): the extended table of prorogations is found among the additional tables in manuscripts **YB** but is mentioned only in **L**.

[221] Chapter I.7.4 is found on fols 28v–30v in **F**, with the additional paragraph on fols 28v–29r. Bagheri did not translate this paragraph and considered it to be inauthentic. The chapter on *tasyīr*s, without the mention of horizontal and vertical arguments for the table, is also found in **H** fols 18v–19v (Chapter I.73), **C** fols 34v–36r (Chapter I.78), $\mathbf{C_2}$ fol. 22v (Chapter I.78, only the title, due to a missing leaf), Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 213/1, fols 21r–22r (Chapter I.7.4), **Y** fol. 252r–v (Chapter I.7.4), and **B** pp. 24–26 (Chapter I.78).

## IV.13. Geographical table

### Table 54: Longitudes and latitudes of localities

**Bibliography**: Lelewel, *Géographie du moyen âge*, Tome 1, esp. pp. 178–85; Nallino, 'Le tabelle geografiche'; von Mžik, *Das Kitāb Ṣūrat al-arḍ*; Honigmann, *Die sieben Klimata*; Haddad and Kennedy, 'Geographical Tables'; K&K = Kennedy and Kennedy, *Geographical Coordinates*; Kennedy and Kennedy, *Al-Kāshī's Geographical Table*; Regier, 'Kennedy's Geographical Tables'; King, *World-Maps* (the appendices of this work include editions of a number of Islamic geographical tables and gazetteers that are not included in K&K); Sezgin, *GAS*, vols X–XIII, for Kūshyār esp. vol. XIII, pp. 277–79; Stückelberger and Graßhoff, *Klaudios Ptolemaios. Handbuch der Geographie*; van Dalen, 'The Geographical Table'. For the table in **H**, see Plate 14.

Like many *zīj*es, Kūshyār's *Jāmiʿ Zīj* includes a table with geographical coordinates, which allows the user to calculate planetary positions and horoscopes not only for the base locality of the *zīj* but also for numerous others. Kūshyār's geographical table provides a set of 90 localities ranging from Rome in the west to modern-day Pakistan in the east, and from Ethiopia and Yemen in the south to modern-day Turkey and southern Russia in the north. However, localities in what Kūshyār describes as his homeland, namely northern Iran, are particularly well represented. Kūshyār's geographical table was first edited from the Leiden manuscript and analysed in the middle of the nineteenth century by Lelewel, who, due to an inaccurate date for the *Jāmiʿ Zīj*, placed it in a somewhat incorrect historical perspective.[222] Sezgin discussed the coordinates and originality of Kūshyār's geographical table in vol. XIII of his *Geschichte des arabischen Schrifttums*.[223]

As is common for geographical names in medieval sources (be they Arabic or Latin), it can be clearly seen from all seven manuscripts which include Kūshyār's geographical table that the scribes did not know many of the places. Diacritical dots are missing for many of the localities, and often also the shapes of the letters are 'uncertain' (i.e., they are written in forms somewhere in between the clear shapes generally used by the same scribes for unambiguously identified letters). This is especially true of lesser known localities and for names deriving from the Greek or other languages. In editing the place names

[222] Lelewel, *Géographie du moyen âge*, Tome 1, Appendix III, pp. 178–85; see also page 12 and footnote 36 in the introduction.

[223] Sezgin, *GAS*, vol. XIII, pp. 277–79, with a transcription of the table in the Fatih manuscript on pp. 278–79. Sezgin's transcription contains the following incorrect values: Mecca long. 77;20° (correct: 10′), Jerusalem lat. 32;10° (0′), Fasā long. 88;30° (minutes ill.), Shāhrazūr lat. 34;0° (37;15°), Hamadān lat. 36;0° (10′), Qum long. 85;55° (**F** omits the degrees and has 15′, while the other manuscripts have 80;15° (**L** 55′); as will be explained below, the most likely intended value is 84;15°), Rayy long. 84;40° (0′), Qūmis 88;55° (15′; 55′ is only found in **L**).

I have applied the following rules (which partially correspond to the general rules for editing the tables).

- If the shapes of the letters are correct, no variations in the diacritical dots are mentioned in the apparatus. Thus, for example, no variants will be given for Naṣībīn (نصيبين) as long as the *nūn*s at the beginning and end and the three consecutive letters ـ in the middle can be clearly read.
- If the shapes of the letters are *not* correct, the variants are quoted exactly as they appear in the sources with any diacritical dots that are provided; however, if multiple incorrect variants differ only in their diacritical dots, only the dots that appear in each of the sources concerned will be indicated. If a variant in the Judaeo-Arabic manuscript is nonsensical or incomplete, it may be given in Hebrew characters for clarity.
- The omission of diacritical dots on 80 (ف) and 100 (ق) is not mentioned in the apparatus. Mistakes of 80 for 100 and 100 for 80 are only included in the apparatus if the dots are unambiguously wrong (i.e., ف for 100 or ق for 80).
- As in the case of numbers in the edition of the tables in Part II, whenever a name or number is not entirely clear but may be read as the correct one, the source is given the benefit of the doubt and no note will be made in the apparatus. This also holds for readings that appear to have been corrected in the manuscripts.
- The addition or omission of the definite article ال is only indicated (by placing it between parentheses) in cases where at least two manuscripts deviate from at least two others.
- Since **C** also generally omits dots on *nūn* in the geographical coordinates, these omissions will not be indicated in the Apparatus.

Kūshyār's geographical table is arranged by the seven climates, and within each of the climates by increasing longitude. For ease of reference I have numbered the entries from A1 to C30, where A, B and C stand for the three columns of the table. All sources except **L** indicate the beginnings of the climates alongside the columns or (in **Y**) between the appropriate rows. In some cases the labels are misplaced by one row; this is not explicitly indicated in the apparatus (the correct positions can be easily recognized from the jumps from high to low longitudes). The scribe of **L** apparently missed out two localities in both the second and third columns and added them at the end of these columns, thus distorting the correct order of the table by increasing longitude within the seven climates. He correctly inserted two other localities which he seems to have initially skipped (A16 and C16) by adding an additional row between the ones originally copied.

As we will see below in more detail, Kūshyār's geographical table shows several peculiarities. Most strikingly, five of the seven witnesses for this table give a highly incorrect longitude for Baghdad, namely 75;0° instead of 80;0°, which is elsewhere only attributed to pseudo-Ptolemy's *Kitāb al-Malḥama* (in combination with the likewise rare latitude 34;0°) by the famous traveller and geographer Yāqūt al-Hamawī (*c.* AD 1200).[224] Since Baghdad appears in all seven manuscripts of the *Jāmiʿ Zīj* between al-Kūfa (longitude 79;30°) and Wāsiṭ (longitude 81;30°), it is probable that its longitude was in fact intended to be 80;0°, but it was corrected to this value only in **CC$_1$**. The two Cairo manuscripts also correct the erroneous longitude for Anṭākiya (B21, in all other witnesses 79;0°, correct 69;0°) and adjust its position in the table in order to roughly restore the correct order of increasing longitudes. Although six manuscripts give the longitude of Qum as 80;15° (**F** leaves the position of the degrees empty), its number of minutes and Qum's position in the table in between Hamadān (longitude 83;0°) and Iṣfahān (84;40°) make it plausible that the longitude was originally inserted as the Maʾmūnic value 84;15°. This is also in excellent agreement with the modern coordinates.

**C$_1$** writes *ṣḥ* in the margin for a total of 15 localities in the first and second columns. No actual corrections can be recognised, but apparently the scribe checked the place names and/or coordinates concerned against a second copy of the *Jāmiʿ Zīj*, probably the one that he already mentioned in a marginal note to the tangent table (see explanatory text B1 to Table 10, especially note 8 on p. 276. A user of manuscript **C** wrote the coordinates 69;30° and 35;20° for Ḥamā in the margin of the table.

The coordinates of thousands of localities contained in a variety of Islamic sources (among which numerous *zīj*es and various instruments), were published in K&K (abbreviation for Kennedy and Kennedy, *Geographical Coordinates of Localities from Islamic Sources*). This work also includes coordinates from some of the most important geographical dictionaries, such as Yāqūt's *Muʿjam al-buldān* and Abū l-Fidāʾ's *Taqwīm al-buldān*, and several otherwise unknown sources quoted in these works. Among the secondary literature, which is used in particular to locate cities that have disappeared over time and to identify the modern locations corresponding to historical cities, an important place in K&K is taken up by Le Strange, *The Lands of the Eastern Caliphate*. K&K uses a rather crude measure for investigating the dependence of sources, namely the percentage of localities in a given source whose latitude values are identical to those in another.[225] In this process longitudes were ignored because of the

[224] See Sezgin, *GAS*, vol. VII, p. 46; Wüstenfeld, *Yacut's geographisches Wörterbuch*, vol. I, p. 678; Honigmann, *Die sieben Klimata*, pp. 125–34, and K&K, pp. 55–56 and 422.

[225] K&K, p. xl.

occurrence of different meridians of reference, especially those of the Fortunate Isles and the African Shore, which differ by exactly 10°.

Already in the 1990s, I implemented the original data from K&K in a DOS computer programme KaK which allows more flexible and more extensive searches than the book. This programme now also implements a more detailed measure for interdependence of sources, which takes into account the differences due to varying base meridians (whenever these can be reliably established for a source) and the possibility of scribal mistakes in the digits of longitudes and latitudes. For example, if a given locality has longitudes 40;15° and 47;55° in two particular sources, it will contribute to the measure of dependence for these sources (either with the same amount as identical coordinates or with a certain weight depending on the statistical probability of the scribal errors concerned), since in Arabic 47 مز is a likely scribal error for 40 م and 15 يه a very common scribal error for 55 نه). The programme also shows the percentage of places in one source that are included in another. Obviously, a source of which only 50% of the localities are also found in another, cannot be fully dependent on it. Among the more than 80 sources listed in K&K there are hardly any that can be seen to be fully dependent on others, that is, in nearly all cases the best one can say is that two sources most likely depend on a common predecessor. The results of my more refined tests of dependence confirm the general conclusions in K&K. However, the programme makes it possible in particular to analyse in more detail whether multiple earlier sources were combined in a given source.

In an article in a volume on astronomical tables from various cultural areas, I edited and analysed the geographical table from the thirteenth-century *Shāmil Zīj*, possibly written by the well-known philosopher Athīr al-Dīn al-Abharī.[226] In this article I developed the concept of a 'reference table', which lists the coordinates for relevant localities as found in each of the three main traditions of Islamic geographical coordinates and several smaller ones on the basis of the complete data from K&K, supplemented with several sources not included by the Kennedys. The reference table that I compiled for the localities in the *Shāmil Zīj* made it very clear that Kūshyār's table was one of its main sources. Furthermore, it confirmed the Kennedys' judgement that Kūshyār stood in the Maʾmūnic tradition of Islamic geographical coordinates.[227] This tradition is primarily represented by the *Kitāb Ṣūrat al-arḍ* by al-Khwārizmī (*c.* AD 830, source KHU in K&K) and by two sources close to KHU, namely a geographical table in the manuscript Istanbul, Süleymaniye Kütüphanesi, Ayasofya 4830 (KHZ) which appears near material attributed to al-Khwārizmī, and the *Kitāb ʿAjāʾib al-aqālīm al-sabʿa ilā nihāyat al-ʿamāra* of Suhrāb (Sera-

[226] van Dalen, 'The Geographical Table'.
[227] K&K, p. xxiv.

pion, first half of the tenth century, SUH). Later important works in the Maʾmūnic tradition include Ibn Yūnus (*c.* AD 1000, plus several Egyptian *zīj*es based on him), Ibn al-Zayyāt and the *Toledan Tables* in the western part of the Islamic world, and the 'Shāmil group' (besides the *Shāmil Zīj* itself, al-Fārisī, Cyriacus, the *Mukhtār Zīj* and several scattered tables) in the eastern part.

The dependence of Kūshyār's geographical table on the Maʾmūnic geography can clearly be seen from the dependence measure applied in K&K and the more extensive ones implemented in my programme KaK. The combined percentages of identical digits and plausible scribal errors in Kūshyār's longitudes and latitudes with respect to al-Khwārizmī, the *Kitāb Rasm al-rubʿ al-maʿmūr* (RES), the Ayasofya manuscript, and Suhrāb are respectively 85%, 81%, 84% and 76%. Not surprisingly, Kūshyār's geographical table is also quite close to al-Battānī's table (69%), although al-Battānī is much closer to Ptolemy than Kūshyār (65% versus 43%) and clearly further removed from the Maʾmūnic geographers.

Below I will give some brief comments on all localities in Kūshyār's table and mention the most important deviations in the coordinates from other Islamic geographical tables and, whenever necessary, information on the identification of the place names. I have used the *Encyclopaedia of Islam. New Edition* throughout for information on localities, with the *Encyclopaedia Iranica* as a useful additional resource, especially for localities in Iran. Furthermore, I have made use of the Google Maps-based website http://www.geonames.org/ for checking modern geographical coordinates. From the sources listed in K&K, in addition to the Maʾmūnic sources, I will refer especially to the primary representatives of the other two main traditions of Islamic geographical coordinates, namely the mysterious but highly original *Kitāb al-Aṭwāl wa-l-ʿurūd* (ATW), which was later adopted by the astronomers of Maragha and Samarqand, and the geographical table in al-Bīrūnī's *al-Qānūn al-Masʿūḍī*, which was used by al-Khāzinī (Marw, early 12th c.) and Ibn al-Shāṭir (Damascus, mid 14th c.). The 39 entries for localities that do not appear in the *Shāmil Zīj* are marked by an asterisk. For the most part these are either legendary or historical localities for which most sources simply copied al-Maʾmūn's geography, or lesser known localities in Iran that appear in only a very small number of works. A reference table for these localities can be found as Table P on pp. 507–10. The abbreviations of the most important sources are explained in the caption to the table.[228] All longitudes below and in the reference table are given with respect to the Fortunate Isles, i.e., longitudes measured from the African shore are tacitly increased by 10°. Whenever a longitude is said to be 'accurate', this is in relation to the longitude of Baghdad. Pairs of coordinates are always given

[228] A more extensive overview of the sources included can be found in van Dalen, 'The Geographical Table'. Complete information on the sources is provided in K&K, pp. xv–xxxix.

in the form 'longitude/latitude'. For most of the coordinates quoted I follow K&K without checking the original sources. For some localities reference is made to Ptolemy's *Geography*.[229]

A1: Ḥabasha stands for Ethiopia and its people. Several of the 18 sources in K&K that include this locality refer explicitly to a kingdom and/or a city in, or the capital of, this kingdom. The Ayasofya manuscript and Suhrāb give the latitude as 19;40°, but from Ibn Yūnus onwards the commonly used value was 19;30°. Al-Khwārizmī associates the same coordinates with Jarmī, which is also frequently given a much smaller latitude. This may be a confusion with ancient Garama (modern-day Jerma) in western Libya.

A2*: (Madīna al-)Nūba (in other works also 'Dunqula') refers to Dongola in Nubia, present-day Sudan. Its ruins at what is now called Old Dongola lie at longitude 30;45° and latitude 18;13° (K&K incorrectly gives the coordinates of modern-day Dongola, which lies over 100 km further north). The coordinates in most Islamic sources, including Kūshyār, follow al-Khwārizmī and Suhrāb with only minor scribal distortions. See the *EI³* article 'Dongola' by Giovanni R. Ruffini.

A3: Nearly all sources in K&K give the latitude of Ṣanʿāʾ as 14;30°, but there appear to be two traditions for its longitude, namely the 73;30° found in Ibn Yūnus and Kūshyār (the 78;30° in al-Khwārizmī may be a scribal error for this value, while Suhrāb deviates with 75;30°) and the 77;0° given by the *Kitāb al-Aṭwāl* (source ATW) and al-Bīrūnī, both with minor further deviations.

A4: For ʿAdan (Aden) Kūshyār uses the common Maʾmūnic coordinates. ATW and al-Bīrūnī deviate clearly with a less accurate latitude of 11;0°.

A5: For ʿUmān (Oman) the *Jāmiʿ Zīj* also follows the common Maʾmūnic values. This locality is not included in ATW and al-Bīrūnī, as well as many sources that usually depend on these.

A6*: Kūshyār is the earliest source in K&K to list *Sūrī min Sarandīb*; the island Sarandīb (Sri Lanka) is already found in al-Battānī and Ibn Yūnus. As Sezgin noted, Kūshyār's coordinates for Sūrī are identical to those that the Maʾmūnic sources attach to Farsaqūrī, which may be Ptolemy's Prokouri (*Geography* VII,4,6).

A7: For Madīna (Medina) most sources, including ATW and Kūshyār, use the highly accurate Maʾmūnic coordinates. Only al-Bīrūnī increased the

[229] See the edition of the Greek with German translation in Stückelberger and Graßhoff, *Klaudios Ptolemaios. Handbuch der Geographie*.

longitude by 2°, which was a clear deterioration with respect to the longitude of Baghdad.

A8: The Maʾmūnic coordinates for Makka (Mecca) are 77;0°/21;0°. In this case Kūshyār's coordinates appear to be in the tradition of ATW, which has longitude 77;13° (from which Kūshyār's 77;10° may have been rounded). Al-Battānī uses the same non-Maʾmūnic latitude 21;40° but an erroneous longitude 71;0°.

A9*: Al-Yamāma was a region in the central part of present-day Saudi Arabia, with its centre in Ḥajr. The modern-day al-Yamāma is a town near the city of al-Kharj, around 100 km south-east of Riyādh. It is unclear whether the medieval coordinates refer to the centre of the region or a specific city, but they all go back to the Maʾmūnic sources.

A10*: Hajar is the pre-Islamic word for 'town', and as a result many localities have this name. Various sources in K&K explicitly associate it with Baḥrayn, which historically covered the eastern part of central Saudi Arabia. A comment in K&K states that its port was the modern-day al-ʿUqayr (the classical Gerra), just opposite and around 50 km south-west of present-day al-Baḥrayn, but no source for this identification is provided. Kūshyār's coordinates belong to the Maʾmūnic tradition, which includes the frequent scribal errors 25° and 44′ for the latitude.

A11*: Al-Baḥrayn used to be a region in the eastern part of modern-day central Saudi Arabia (including the island state al-Baḥrayn). Only Suhrāb explicitly mentions its capital, al-Aḥsā. Kūshyār, like most Islamic sources, uses the Maʾmūnic coordinates.

A12*: Al-Nīrūn was the ancient city Nīrūnkot in Sindh (Pakistan), which was conquered by the Muslims in the early eighth century and on whose ruins Hyderabad was built in the eighteenth century. Kūshyār's coordinates are those from al-Khwārizmī and Suhrāb, where the longitude 82;20° that appears in five of the seven manuscripts is an obvious scribal mistake for 102;20°.

A13*: Al-Manṣūra, founded early in the eighth century, was the principal city of the Sindh province under the Arabs. It was build on the site of, or is at least very close to, the ancient and present-day Brahmanabad. Kūshyār's coordinates are Maʾmūnic and clearly different from the tradition of ATW with longitude 105;0° and latitude 26;40°.

A14: The coordinates of Alexandria in the Maʾmūnic tradition were 61;20° and 31;0°. Kūshyār adopted the Ptolemaic longitude 60;30°, which he had presumably taken from al-Battānī. ATW has the peculiar latitude value 30;58°, which was also adopted by al-Battānī and al-Bīrūnī.

Kūshyār was the earliest astronomer to use 30;20°, which is unlikely to be a scribal error for any of the other common values (unless it was rounded from the scribal error 30;18 for 30;58°); this value was also used in the 'Shāmil group'.

A15*: Ramla is 15 km south-east of Tel Aviv in present-day Israel. It succeeded nearby Ludd (now Lod) as the residence of several Umayyad caliphs around AD 700. Kūshyār displays al-Khwārizmī's coordinates, which are somewhat different from the Ayasofya manuscript and Suhrāb.

A16: Kūshyār's coordinates for Jerusalem are those from al-Khwārizmī with the longitude slightly adjusted from 66;0° to 66;30° as in ATW (which, however, has 31;50° instead of 32;0° for the latitude).

A17*: Caesarea was a name given to numerous towns by the Roman emperors Augustus and Tiberius. Of these several were known to the Arabs, especially Qaysariyya in Rūm (Kayseri) and the present entry Qaysariyya in Shām (Syria). This is the classical Caesarea Maritima, now Caesarea on the Israeli coast, 40 km south of Haifa. K&K's identification with Baniyas in Syria (latitude 35;11°) is incorrect, as are the modern coordinates that K&K gives for this city (the modern coordinates of Caesarea are 34;55°/32;31°). The coordinates in Islamic sources show slight deviations around the Ptolemaic values of 67;20° and 33;20°. Kuṣhyār was the first to use a longitude of 68;30°.

A18*: Ṭabariyya is Tiberias on the Sea of Galilee (now Ṭeverya in Israel). Kūshyār uses the common latitude value 32;0° and is the only Islamic source to use the longitude 68;45° (cf. 67;45° in al-Khwārizmi and al-Bīrūnī).

A19: For Damascus Kūshyār agrees with al-Khwārizmī and the Ayasofya manuscript, whereas Suhrāb has a different latitude (32° rather than 33°).

A20: Fusṭāṭ was the earliest city founded by the Arabs in Egypt in AD 643, and now lies within the city limits of Cairo. Kūshyār's coordinates are idiosyncratic and were only taken up by the two sources that were strongly dependent on him. The longitude 73° is a mistake for 63°, and the latitude 31° is clearly worse than the common 30° (Ptolemy) or 29;55° (al-Bīrūnī and others). It seems probable that Kūshyār took these coordinates from al-Battānī.

A21: al-Kūfa (AD 638) was one of the cities founded by the Muslim conquerors in Iraq and lies just east of Najaf on the west bank of the Euphrates. Kūshyār, like most Islamic sources, uses the longitude and

latitude from al-Khwārizmī and the Ayasofya manuscript, while Suhrāb deviates from them.

A22: The longitude 75;0° for Baghdad is found in five of the seven manuscripts of the *Jāmiʿ Zīj* that contain the geographical table. A comparison with the surrounding localities shows that it is most probably a mistake for the common value 80;0°. The longitude 75;0° elsewhere only appears in the *Kitāb al-Malḥama* cited by Yāqūt, to which Sezgin assigns a historical position in between the Ptolemaic and the Maʾmūnic geographies.[230] The correct value 80;0° is given in the two Cairo manuscripts of the *Jāmiʿ Zīj*. For the latitude Kūshyār lists the value 33;0° from the Ayasofya manuscript, rather than al-Khwārizmī's 33;9° or the later standard value 33;25°, which was first found in Suhrāb and ATW and is written in the Berlin manuscript of the *Jāmiʿ Zīj* in the main hand.

A23: Wāsiṭ was a provincial capital in Iraq which is mentioned in geographical sources from the eighth to the eighteenth century. It was located on the Tigris (which, however, gradually changed its course) and is now referred to as al-Manāra after the building whose monumental entrance has survived. Thanks to nineteenth-century descriptions and archaeological excavations, its exact position was determined as 46;18°/32;11°, 70 km south-east of al-Kūt. The modern-day Wāsiṭ ('the middle') is a province of Iraq. See in particular the *EI*² article by R. Darley-Doran. Kūshyār and most other Islamic sources use the Maʾmūnic coordinates.

A24: Kūshyār gives the standard coordinates for al-Baṣra that appear in all Islamic sources with only minor deviations due to scribal errors.

A25: For al-Ahwāz, Kūshyār appears to have incorrectly copied the Maʾmūnic latitude (32;0°) as 30;0° (only corrected in manuscript **C**), which later confused the copyists of the tables that were strongly dependent on the *Jāmiʿ Zīj*.

A26*: K&K identify this locality as Sīnīz, whose ruins lie near Ḥiṣār, 20 km south of Bandar-i Daylam near the coast of the northernmost part of the Persian Gulf at a longitude of 50;16° and a latitude of 29;52°. The irregular and uncertain writing in the manuscripts suggests an alternative identification as the Iranian city Shūshtar, in Arabic Tustar, which lies at longitude 48;51° and latitude 32;3°. A possible confusion between Sīnīz and Shūshtar in Islamic geographical tables may be suggested by the fact that very few sources (namely, only the huge compilations in ATW, Ibn Yūnus and al-Bīrūnī) include both places.

[230] Sezgin, *GAS*, vol. XIII, pp. 205–09.

According to K&K, Sīnīz mostly appears with coordinates 86;45° and 30;0°, and Shūshtar or Tustar with latitude 31;30° and varying longitudes such as 86;20° and 84;30°. In the end I have decided to maintain the identification with Sīnīz, because its south-eastern position relative to al-Ahwāz is in much better agreement with the given coordinates than Shūshtar's position due north.

A27*: Jannābā is an Arabic form for Ganāfa (the modern-day Bandar-i Ganāva) on the coast of the Persian Gulf. Kūshyār's coordinates are the common Maʾmūnic ones given in most Islamic sources.

A28: Kūshyār and many other Islamic sources copied the Maʾmūnic coordinates for Shīrāz. The much more accurate latitude value 29;35° or 29;36° first appeared in al-Bīrūnī.

A29*: The coordinates given in most sources in K&K for the southern Iranian city Basā or Fasā are the Maʾmūnic ones. Kūshyār's latitude (differing by 10′ from the Maʾmūnic 33;40°) is very far removed from the actual value (only ATW and the *Aʾīn-i Akbarī* (India, late sixteenth century) give the latitude correctly as 29;0°, with a longitude one degree greater than the Maʾmūnic one).

A30*: Jūr (or Gūr) was the original name of Fīrūzābād, 80 km south of Shīrāz in the Iranian province of Fārs. On satellite images its circular form with four gates can still be easily recognized some 3 km west of the modern city with the same name. K&K mistakenly lists Jūr from Kūshyār's table and two dependent sources as Ḥur, but the identification with Fīrūzābād is highly probable since the coordinates are identical to those given for Jūr in Suhrāb (and close to those in the other Maʾmūnic sources).[231]

B1: Sābūr or Shāpūr was founded as Bishāpūr by the Sasanian king Shāpūr I in the early third century AD, and was also called Shahrastān in Islamic sources. It was under decay by the eleventh century, finally leaving a ruin *c.* 10 km north of the modern-day village Darīs. See the *EI²* article 'Shāpūr' by C. E. Bosworth. Kūshyār's geographical table is the earliest to introduce non-Maʾmūnic coordinates with a clearly better value for the latitude.

B2*: Iṣṭakhr or Stakhr was built 5 km north of Persepolis using the latter's remains after its destruction by Alexander the Great. It was the

[231] For Ḥūr, K&K gives the modern coordinates of an insignificant village in Kirmān, which Sezgin (*GAS*, vol. XIII, p. 278) then presents as an example of accurate new coordinates in Kūshyār's *zīj*, apparently without having checked the identification.

capital of the Sassanid empire for a short time in the third century AD. The city's importance decreased after the foundation of Shīrāz in AD 684, and it was finally demolished in the eleventh century. Its ruins are 11 km south of Sivand at longitude 52;55° and latitude 29;59°. For Iṣṭakhr's coordinates Kūshyār stays in the Maʾmūnic tradition, whereas ATW and al-Bīrūnī have a clearly better value for the latitude.

B3*: Sīrāf was one of the main commercial centres on the Persian Gulf in the early Islamic period. Its ruins are near the modern-day village of Ṭāhirī. Kūshyār, like most Islamic geographical tables, gives the Maʾmūnic coordinates.

B4*: Al-Sīrjān or al-Shīrajān is a former capital of Kirmān that was destroyed after a siege in the late fourteenth century. Its ruins are at the limestone hill Qalʿa-yi Sangī, 12 km south-east of modern-day Sīrjān (previously Saʿīdābād) at longitude 55;46° and latitude 29;22°. See Le Strange, *The Lands of the Eastern Caliphate*, pp. 300–01. Kūshyār's coordinates are the Maʾmūnic ones; clearly better ones are found in ATW and the sources based on it, such as al-Ṭūsī's *Īlkhānī Zīj*.

B5*: Jiruft was a city in the Iranian province of Kirmān, 75 km south-west of Bam. Its ruins are most likely near the modern-day Jiroft (previously Sabzāwārān). Kūshyār reproduces the very inaccurate Maʾmūnic coordinates, which were drastically improved upon by ATW.

B6*: Muḥammadiyya appears with the coordinates given by Kūshyār or slight scribal distortions of them in 13 sources in K&K, the earliest of which is al-Khwārizmī. Only in the *Ḥākimī Zīj* by Ibn Yūnus does a marginal note, not necessarily in the main hand, indicate that the location is identical to Rayy, which was in fact called Muḥammadiyya in early Abbasid times. Unlike what the entry in K&K suggests, this indication is not found in al-Khwārizmī, neither in the edition by von Mžik nor in the Strasbourg manuscript.[232] Because of the huge differences in longitude (15°) and latitude (4°) from Rayy, it is hardly possible that the indication in Ibn Yūnus is correct. Yāqūt lists a number of cities and townships with the name Muḥammadiyya in his *Muʿjam al-buldān*.[233] One of these, with the same cordinates as the *Jāmiʿ Zīj*, is stated to be a city in the third climate in the province of Kirmān. In fact, its coordinates are in good relative agreement with the ones for cities such as Kirmān, Jiruft and Hurmuz in the Maʾmūnic geography.

[232] von Mžik, *Das Kitāb Ṣūrat al-arḍ*, p. 23 (no. 324); Strasbourg, Bibliothèque nationale et universitaire, MS 4.247, fol. 6v (penultimate entry in first column).

[233] Wüstenfeld, *Jacut's geographisches Wörterbuch*, vol. IV, pp. 430–31.

B7: Kirmān is the name of an Iranian province and its capital, which goes back to Greek times. It already appears in Ptolemy's *Geography*, VI,8,13 as Karmana, capital of Karmania. Kūshyār, like all early Islamic authors, gives the Maʾmūnic coordinates for Kirmān.

B8: Kūshyār uses the highly inaccurate coordinates for Kābul from the Maʾmūnic tradition. Good coordinates are found in al-Bīrūnī, basically accurate ones in ATW.

B9*: ʿAmmūriyya was the Byzantine stronghold Amorion, which was captured by the Arabs in AD 838 and whose ruins are now found in Hisarköy, 13 km east of Emirdağ in the Turkish province of Afyonkarahisar. Kūshyār lists the Maʾmūnic coordinates. The modern coordinates in K&K should be corrected to 31;18°/39;1°.

B10: For the 6000-year old city Ṭarsūs, Kūshyār gives the Ptolemaic longitude and a latitude (36;15°) that may be a scribal error for either the Ptolemaic (36;50°) or the Maʾmūnic (36;55°) one.

B11*: Maṣṣīṣa is the classical city Mopsuestis and the Byzantine Mamistra, which declined to a village Misis (now renamed to Yakapınar), nearly 30 km east of Adana in Turkey. The ruins are presumably at the place now indicated as Eski (= old) Misis. Kūshyār reproduces the Maʾmūnic coordinates, which are close to Ptolemy's.

B12*: Ṭarābulus (also Aṭrābulus) is the Arabic transliteration of the Greek Tripolis. Arabic sources add *al-shām* in order to distinguish it from Ṭarābulus *al-gharb* (Tripoli in Libya). It was a harbour town in the Levant that functioned as a port for Damascus. It was destroyed after it surrendered to the Mamluks in 1289, and was rebuilt on a nearby hill. Of the classical Tripoli (now in Lebanon) only a small harbour al-Mīnā remained. Kūshyār lists the Maʾmūnic coordinates with a minor adjustment in the minutes of the longitude.

B13: For Aleppo, whose coordinate tradition in Islamic sources is rather polluted, Kūshyār appears to follow al-Battānī rather than the Maʾmūnic geography. The latitude value 35;50° may be a scribal mistake for al-Battānī's 34;50°, but it also appears in ATW.

B14: Kūshyār's coordinates for Ḥimṣ (Homs) appear to be in the Maʾmūnic tradition, with a slight adjustment of the latitude (33;40° instead of 34;0°).

B15: Not surprisingly, Kūshyār uses al-Battānī's coordinates for Raqqa, which are also found in ATW and were only minimally modified by al-Bīrūnī (latitude 36;1° instead of 36;0°).

B16: Āmid is the ancient Amida, which was already mentioned in Assyrian records of the second millennium BC. It is now Diyarbakır in Turkey, a name that likewise already appeared in some of the earliest sources in K&K. Kūshyār copies al-Battānī's coordinates, which can be clearly distinguished from the Maʾmūnic ones and are also different from ATW.

B17: For Ḥarrān, Kūshyār appears to follow the Ayasofya manuscript rather than al-Battānī's values, although the latter are much more in line with the coordinates that Kūshyār gives for Aleppo and Raqqa.

B18*: Naṣibīn is Arabic for the ancient Greek Nisibis and is now Nisaybin on the Turkish border with Syria. Kūshyār gives the Maʾmūnic coordinates; these are clearly different from the ones of al-Battānī and ATW, which are rooted in Ptolemy's *Geography*.

B19: For al-Mawṣil (Mosul) Kūshyār uses al-Battānī's coordinates with a slight adjustment of the longitude from 78;10° to 78;0°. The Maʾmūnic tradition can be clearly distinguished from both.

B20*: K&K identifies Balad with Iski Mosul, probably the ancient Mepsila mentioned by Xenophon, 40 km east-north-east of Mosul on the Tigris river. Kūshyār follows the Maʾmūnic tradition for Balad's coordinates. Other more or less clearly distinguishable pairs of coordinates are 76;40° or 77;40° with latitudes between 36;35° and 36;50°, first found in al-Battānī and ATW, and 68;25° or 68;30° with latitude 35;30° or 35;40°, first found in al-Bīrūnī.

B21: Anṭākiya is the classical Antiocheia, one of the most important Roman cities on the Mediterranean coast, founded around 300 BC. It is now Antakya in the southernmost part of Turkey. Kūshyār adopted the Ptolemaic coordinates from al-Battānī, which can be easily distinguished from the Maʾmūnic ones (71;35°/34;10°). The correct longitude 69;0° (rather than 79;0°) is only found in the two Cairo manuscripts, which also adjust the order of the entries accordingly (although not entirely correctly).

B21a (only in $\mathbf{CC_1}$)*: Takrīt was mentioned by Ptolemy as Birtha (*Geography*, V,18,9). It remained almost exclusively Christian during the first centuries of Islam and was rarely mentioned in early Islamic geographical tables. The longitude 79;40° given in the two Cairo manuscripts does not appear in any other sources, but it is close to al-Bīrūnī's 79;30°. The latitude 34;30° is very accurate and also appears in ATW.

B22*: Sāmarrāʾ (most likely based on the name of the nearby Babylonian city Sur-marrati) or Surra man raʾa ('he who sees it is delighted') lies 100 km north of Baghdad on the Tigris. It was the capital of the Abas-

sid dynasty from AD 836 to 892. The *Jāmiʿ Zīj* was the earliest source to round the Maʾmūnic longitude 79;45° to 80;0°.

B23: Shāhrazūr was a city in the district with the same name in the north-eastern part of present-day Iraq. Le Strange, *The Lands of the Eastern Caliphate*, p. 191 places its ruins at Yasīn Tepe, just north of Zarrayān, but K&K give the modern coordinates of the city Khūrmāl following later attempts at identifying the city. Most sources in K&K, including Kūshyār, appear to be using the Maʾmūnic coordinates with an increasing number of scribal mistakes, especially in the latitude.

B24: Ḥulwān (Greek Khala, Akkadian Khalmanu) was an ancient town on the Khurasan highway, whose site has been identified as the present-day village Sarpul-i Dhahāb in the western-Iranian province of Kirmanshāh. It flourished in the second half of the first millennium and was destroyed between AD 1046 and 1049 by the Seljuks and an earthquake. Kūshyār follows the Maʾmūnic tradition for the coordinates of Ḥulwān.

B25: For Nihāwand (Persian: Nahavand, in the province of Hamadan) Kūshyār uses the Maʾmūnic coordinates with an addition of 10′ to the latitude.

B26: For Hamadān Kūshyār also uses the Maʾmūnic coordinates with an addition of 10′ to the latitude.

B27: Kūshyār's pair of coordinates for Qum does not occur in any other sources. The incorrect order of Kūshyār's table suggests that the longitude may be a scribal error, for instance for the Maʾmūnic value 84;15°. The entry in two of the sources that are strongly dependent on Kūshyār (the *Mufrad Zīj* and a table in a manuscript in Gotha) likewise appear to be based on the Maʾmūnic coordinates, with a different scribal error in the longitude (83;15° instead of 84;15°). In the *Shāmil Zīj* the coordinates for Qum are similarly confused.

B28: For Iṣfaḥān, Kūshyār adopts the Maʾmūnic longitude 84;40°, but he replaces the latitude 34;30° with the significantly more accurate 32;0° (note, however, that manuscript **F** gives the latitude as 34;0°, following Suhrāb).

B29: For al-Rayy (near present-day Tehran), Kūshyār uses the Maʾmūnic coordinates with the latitude rounded from 35;45° to 36;0°.

B30: For Qazwīn basically all sources in K&K tabulate the Maʾmūnic coordinates.

C1: Daylam was the region consisting of the highlands of the Iranian province of Gīlān. Kūshyār, like most sources in K&K, tabulates the Maʾmūnic coordinates, with the latitude 38;10° rounded to 38;0°.

C2*: Dunbāwand is the earlier name for the now more common Damāwand. It is used both for the highest mountain in the Alborz range, which lies between Tehran and the Caspian Sea, and for a small town 20 km south of the mountain peak. Both variants of the name found in the manuscripts of the *Jāmiʿ Zīj* also appear in earlier Islamic sources (e.g., *dmāwnd* in the Ayasofya manuscript). Kūshyār follows the Maʾmunic coordinates, of which nearly all sources in K&K appear to have scribal variations. The modern coordinates for the town are 52;4°/35;43°.

C3*: Shālūs or Sālūs is the modern-day Iranian city Chalūs in Māzandarān on the Caspian sea. Like most other sources in K&K, Kūshyār lists the Maʾmūnic coordinates (and in particular the exact values found in the Ayasofya manuscript).

C4: Rūyān was the district comprising the Caspian coastlands in the western half of present-day Māzandarān and also the name of a town in this district. K&K identify the town with Khujūr (Kojur), apparently following the fifteenth-century commander and historian Ẓahīr al-Dīn al-Marʿashī (cf. the *EI²* article by V. Minorsky).[234] The coordinates listed by Kūshyār are the Maʾmūnic ones, which are given by nearly all sources in K&K, although in many cases with scribal errors in the degrees of both longitude and latitude.

C5*: Āmul was of pre-Islamic origin and as capital of Ṭabaristān an important city in the Islamic period. The ruins of the old town are somewhat west of the modern-day city Amul. See also the *EIr* article 'Āmol' by C. E. Bosworth, S. Blair and E. Ehlers. Kūshyār lists the Maʾmūnic coordinates. A different latitude was introduced in ATW and was combined with a slightly different longitude by al-Bīrūnī.

C6: Sāriya is the Iranian town Sārī in medieval Ṭabaristān, and was for many centuries the capital of the province of Māzandarān. Kūshyār tabulates the Maʾmūnic coordinates with the highly inaccurate latitude value 38;0°. Later sources used the much more accurate 36;15° introduced by al-Bīrūnī.

[234] There is a modern-day town Rūyān on the Caspian Sea, but this cannot be directly related to the historical town, since the latter is unambiguously situated in the mountains by the Islamic geographers (cf. Le Strange, *The Lands of the Eastern Caliphate*, p. 374).

C7*: Qūmis was a small province of Iran covering parts of the modern-day provinces Māzandarān and Simnān. Its administrative capital was Dāmghān and its other main cities Khuwār (present-day Aradūn), Simnān and Basṭām. It was of importance in pre-Islamic times and appeared in Greek sources as Κωμισηνή. Kūshyār presents the Maʾmūnic coordinates, which were included in all sources in K&K with minor scribal errors.

C8: Astarābād was a city at the south-eastern edge of the Caspian Sea, which is now called Gurgān. It should not be confused with the medieval Gurgān or Jurjān (C9), which lay 100 km to the north-east of Astarābād. Kūshyār reproduces the coordinates found in al-Khwārizmī, including the latitude 38;45° that was 2 degrees too high. More accurate coordinates were introduced by ATW and al-Bīrūnī.

C9: The capital Gurgān or Jurjān of the province with the same name (the ancient Hyrcania) lay 100 km north-east of the modern-day city Gurgān. It was destroyed during the Mongol invasions. Its remains, especially the eleventh-century tomb of Qābūs ibn Washmgīr, survive within the boundaries of the present-day town Gunbad-i Gāwūs. Kūshyār gives for Gurgān the same coordinates as ATW with the round longitude value 90;0°, which he also uses as the base meridian of his *zīj*, and the much improved latitude value 36;50°.

C10: Ṭūs was a district in present-day north-eastern Iran. Its main cities were Ṭabarān, which became to be called Ṭūs, and the nearby Nawqān, which was later absorbed by Mashhad. The city Ṭūs was destroyed in AD 1389 and never rebuilt due to the importance that Mashhad had meanwhile obtained. Kūshyār uses the coordinates found in Suhrāb and the Ayasofya manuscript (with longitude 92;0° instead of al-Khwārizmī's 92;50°).

C11: For Sarakhs in the northern part of the province of Khurāsān, Kūshyār uses the Maʾmūnic longitude 93;20°. For the latitude he may have based himself on Suhrāb, with a scribal error 36;0° for 37;0° (al-Khwārizmī has 38;0°, which is not found in any other source). ATW combined the Maʾmūnic latitude 37;0° with longitude 94;30°. A different tradition with longitude 95;0° and latitude 36;40° was started by al-Bīrūnī.

C12: The full name of the city Marw on the river Murghab was Marw-i Shāhijān. It lay somewhat north of the modern-day village Baýramaly, *c.* 10 km east of Mary in Turkmenistan (cf. the following entry). Kūshyār adopts the coordinates from Suhrāb, who again has a latitude one degree smaller than al-Khwārizmī, with a minor difference of 5′ in

the latitude. For this locality the Ayasofya manuscript starts an independent tradition with 95;27°/37;35°.

C13*: Marw al-Rūdh ('Marw on the river') lay further upstream on the river Murghab at the location of the modern-day town Bala Murghab in Afghanistan. It appears to have become ruinous in Timurid times. In this case Kūshyār follows the Ayasofya manuscript, which differs in the latitude from al-Khwārizmī (38;50° vs 37;50°) and in the longitude from Suhrāb (95;0° vs 95;40°).

C14: For Bukhāra the four Maʾmūnic sources in K&K also show some (possibly scribal) variations. Kūshyār's coordinates coincide with the ones from the Ayasofya manuscript. A much more accurate latitude value was introduced by ATW and al-Bīrūnī (with longitudes differing from each other by only 20′).

C15: Balkh was the ancient city Baktra, capital of the region Baktria, and flourished during the Middle Ages. It is now a small town in northern Afghanistan, 15 km west of the capital Mazār-i Sharīf of Balkh province. Kūshyār tabulates the Maʾmūnic coordinates with a deviation of 5′ in the longitude which is elsewhere only found in the sources strongly dependent on the *Jāmiʿ Zīj*. A clearly independent tradition was started by ATW and/or al-Bīrūnī with the coordinate pair 101;0°/36;41°.

C16: For Samarqand Kūshyār reproduces the coordinates from Suhrāb, which again include a latitude that is one degree smaller than the one in al-Khwārizmī (and hence three degrees smaller than the actual value). Much more accurate latitude values were introduced by ATW and al-Bīrūnī.

C17*: Rūmiya was the name used for Rome by most eastern Islamic geographers (western Islamic geographers used the name Rūma). Several sources in K&K add the adjective *kabīra*. Whereas both Ptolemy and the earliest Islamic sources already had a basically correct latitude for Rome, the longitude was drastically adjusted in order to conform better to the relative distance from Baghdad. Most sources, including Kūshyār, herein followed the Maʾmūnic geography.

C18: Modern-day Malatya is the capital of the province with the same name in eastern Anatolia (Turkey). Its ancient predecessor Melid (known to the Assyrians as Meliddu and to the Greeks and Romans as Melitene) lay 3 km to the north-east in the present-day village Arslantepe. The medieval Malaṭya, which was alternately in Byzantine and various other hands, lay at the location of present-day Battalgazi, 10 km north of modern-day Malatya. Like most Islamic sources, Kūshyār lists the

Maʾmūnic coordinates. ATW starts a smaller tradition with the same longitude but latitude 37;0° instead of 39;0°.

C19: Khilāṭ or Akhlāṭ is the modern-day Ahlat, a town and fortress on Lake Van. Kūshyār follows al-Khwārizmī, whereas Suhrāb has different coordinates not found in any other source. Also the values in ATW and al-Bīrūnī appear to be rooted in the Maʾmūnic geography.

C20*: Arzan was the chief city of the Roman province of Arzanene and has recently been shown to be the main candidate for the location of the classical Armenian capital Tigranocerta (see Sinclair, 'The Site of Tigranocerta I' and 'The Site of Tigranocerta II'). The extensive ruins lie on the bank of the Yanarsu river at longitude 37;58° and latitude 41;23° (see the photographs in Plontke-Lüning, 'In Search of the Late Hellenistic City', pp. 129–30). Kūshyār adopted the coordinates from al-Khwārizmi, from which Suhrāb again deviates.

C21: Bardaʿa, in Armenian Partaw, was the former capital of Caucasian Albania. It prospered in the tenth century, but declined soon thereafter. This is now the town Barda on the Terter river in Azerbaijan, which has some remains of the medieval city. See Le Strange, *The Lands of the Eastern Caliphate*, pp. 177–78 and the *EIr* article by C. E. Bosworth. Kūshyār follows Suhrāb, while the other Maʾmūnic sources have 83;0° rather than 84;0° for the longitude.

C22: The name Khwārizm was used both for the province on the lower course of the Amū-Daryā (Oxus), i.e., the classical Chorasmia or Oxeiana, and for its long-time capital Gurganj. After Mongol times it was increasingly called Khiva. Kūshyār, like most Islamic sources, lists the Maʾmūnic coordinates, but it is unclear to which point these refer. The longitude differs by 7° from the accurate longitude for Jurjāniyya/Gurganj that first appears in ATW and al-Bīrūnī (in K&K some entries for Jurjāniyya appear under Khwārizm, but should more appropriately be included under Gurganj).

C23*: The name Isfījāb (also: Isbījāb) was gradually replaced by the modern name Sayrām from the eleventh century onwards. This city is now Sayram, little more than 10 km east of Shymkent, the capital of southern Kazakhstan. Kūshyār is one of very few sources that adopt the Maʾmūnic coordinates. ATW and al-Bīrūnī introduce a clearly better latitude value but longitudes that, in their relative distance to Baghdad, err even more than the Maʾmūnic one, although in the opposite direction.

C24*: Ṭarāz is the classical Talas, a town in Central Asia on the river with the same name, known for a battle between Arabs and Chinese in AD 751. The medieval town Ṭarāz was at the same location as the modern-day city in Kazakhstan, which was first called Aulie Ata, became Dzhambul in the Soviet period, and was renamed to Taraz in 1997. In Mongol times also the name Yangī was used, thereafter the town sank into oblivion. The modern-day Talas, which is just over the border in Kirghizia, is at a different location. Kūshyār presents the Maʾmunic coordinates. ATW and al-Bīrūnī overcompensate in correcting the longitude as well as the latitude.

C25: (Al-)Qusṭanṭīniyya or Constantinople (earlier: Byzantion) was the capital of the eastern Roman empire from AD 330 onwards. It is the modern-day Istanbul (derived from the Greek εἰς τὴν πόλιν, 'to the city', since the city was often simply called ἥ πόλις). Like most Islamic sources, including ATW and al-Bīrūnī, Kūshyār used the Maʾmunic coordinates, which placed Constantinople 4° too far north and 5° too far west with respect to Baghdad. A better value for the latitude appears only in four scattered, less well-known sources.

C26*: The name Hiraqla was derived from the Greek Herakleia, used for a number of ancient Greek cities. In modern Turkish it is rendered as Ereğli. In Islamic geographical tables, Hiraqla mostly refers to Heraclea Pontica, now Karadeniz Ereğli in the Turkish province of Zonguldak on the coast of the Black Sea, just over 200 km east of Istanbul. Kūshyār uses the Maʾmunic longitude and latitude for Hiraqla with in each case a difference of 5 in the minutes. As for so many other localities, ATW and al-Bīrūnī brought the longitude with respect to Baghdad in a much better agreement with reality.

C27*: Jurzān is a name for Georgia and the Georgian people (see Minorsky, *Studies in Kaukasian History*). It occurs in al-Khwārizmī, Kūshyār and a dozen other sources that all base themselves on the Maʾmūnic geography.

C28*: The Khazar were a Turkic nomadic people in the southern Russian steps that flourished in the early Islamic period. At its zenith the Khazar Qaghanate was a vast multi-religious, multi-national state. The dozen sources in K&K that provide coordinates for 'Khazar' all follow the Maʾmūnic geography.

C29: Anqara was the ancient Greek and Roman Ankyra and is now Ankara, the capital of the Turkish Republic. Kūshyār follows the Maʾmūnic geography with its latitude of 48;0° that is 8° (*sic*!) too high. A significantly better latitude value is found in ATW and some later

sources. Note that the modern longitude given for Ankara in the *EI²* (38;55°) is one degree too small.

C30: Yājūj and Mājūj are the countries from which two mythical people hailed that are called Gog and Magog in the Bible. The Koran relates how Dhū l-Qarnayn held these people off with an iron wall. The historian al-Ṭabarī suggested that Gog and Magog lived in the most eastern part of the country of the Turks. In the work of later geographers their location shifted further to the east. Judging from K&K, the sources (all going back to the Maʾmūnic geography) distinguish clearly between Yājūj (with longitude 170;25° from the western shore of Africa and latitude 43;35° or 42;35°) and Mājūj (longitude 172;30° and latitude 63;0°, which are also consistently found for the entries 'Yājūj and Mājūj'). This means that the coordinates given by Kūshyār for Yājūj are in fact the ones normally associated with Mājūj. The geographical table in the *Mufrad Zīj*, which is strongly influenced by Kūshyār, gives the correct name *madīna Yājūj wa-Majūj* for these coordinates.

*Conclusion*. In his geographical table Kūshyār mostly follows the coordinates from the Maʾmūnic geography contained in al-Khwārizmī's *Kitāb Ṣūrat al-arḍ* (KHU), the table found together with material by al-Khwārizmi in the manuscript Istanbul, Süleymaniye Kütüphanesi, Ayasofya 4830 (KHZ), and the *Kitāb ʿAjāʾib al-aqālīm al-sabʿa* by Suhrāb (SUH). However, in quite a large number of cases Kūshyār deviates from the Maʾmūnic coordinates, often by very small amounts such as 5 or 10 minutes of arc, sometimes by amounts of around a degree. Hardly ever can a significant improvement or deterioration of the coordinates be seen. In some cases the deviations may be explained as scribal mistakes, in others as rounding to a multiple of 10 minutes. It cannot currently be decided whether Kūshyār had access to additional geographical data (e.g., based on itineraries or deriving from his contacts with al-Bīrūnī) or whether he himself made observations of latitudes at various localities in Iran that would have allowed him to make small improvements to the Maʾmūnic coordinates.

Several geographical tables in later *zīj*es were clearly influenced by Kūshyār's table. The sole surviving manuscript of the *ʿUmdat al-ḥāsib*, a draft *zīj* compiled by Muḥyī l-Dīn al-Maghribī in the early stages of the work at the Maragha observatory, contains a reliable copy of Kūshyār's geographical table (together with planetary equations based on the *Jāmiʿ Zīj*).[235] It can be seen that this copy is close to the tables in the Cairo manuscripts **C** and **$C_1$**, since it shares a number of variants with them. In particular, it includes the correct longitude 80;0 for Baghdad, the highly inaccurate latitude 28;0 for ʿAm-

[235] Cairo, Dār al-kutub, *mīqāt* Muṣṭafā Fāḍil 188, fol. 143r.

mūriyya, the incorrect longitude 81;0 for Rayy, the form Dināwand for Dunbāwand, and the incorrect longitude 78;50° for Akhlāṭ. Individual deviations in this copy of the table include: the scribal error 67;0 for the longitude but the better latitude value 31;0 for Alexandria, the Maʾmūnic longitude 64;40 with rounded latitude 30;0 for Fusṭāṭ, the longitude 63;0 (instead of 68;0) for Ankara, the place name Shushtar instead of Sīnīz, Nīsābūr instead of Sābūr, and the correct spelling of Jūr with a dot under the *jīm*.

Later geographical tables that were based strongly on Kūshyār's table can be found in the *Zīj al-Qirānāt*, the *Mufrad Zīj* and as a separate table in a manuscript kept in Gotha (source GT1 in K&K).[236] Each of these three tables contains around 100 entries, of which respectively 86, 83 and 79 were copied from Kūshyār. The arrangement of the entries in the *Zīj al-Qirānāt* and the *Mufrad Zīj* is also generally the same as that in the *Jāmiʿ Zīj*, but the localities in GT1 were ordered by increasing longitude without considering the climates. For the entries that the three works have in common with Kūshyār, respectively 86%, 95% and 87% of the digits of the coordinates agree with (or are common scribal errors of) the corresponding digits in the *Jāmiʿ Zīj*.

As already mentioned above, Kūshyār's geographical table was also the main source for the table in the *Shāmil Zīj*, which I studied in a recent article.[237] In fact, the *Shāmil Zīj* shares only 50 of its 80 localities with the *Jāmiʿ Zīj*, but the coordinates of these localities agree with Kūshyār's for an astounding 99% (as compared to an agreement of only 78% with the main source of the Maʾmūnic geography, the *Kitāb Ṣūrat al-arḍ* by al-Khwārizmī). Of course, the later copies of the geographical table in the *Shāmil Zīj* that were included as independent sources in K&K, namely the tables in the *zīj*es of the Priest Cyriacus and ʿAbd al-Qādir al-Mawṣilī and the European table which was presumably copied by Greaves at the end of an Oxford manuscript of the *zīj* of Ulugh Beg (respectively sources QIR, ABD and ULE in K&K), also have very high rates of dependence on Kūshyār, although that of ABD is reduced to 85% due to the numerous scribal errors in this copy.

Table P: Reference table for the geographical localities from the *Jāmiʿ Zīj* (excluding entries copied in the *Shāmil Zīj*)

For localities also contained in the *Shāmil Zīj*, see the reference table in van Dalen, 'The Geographical Table'. For the sake of easy comparison, the forms of the place names are the ones used in Kennedy and Kennedy, *Geographical Coordinates* (K&K) with very few exceptions indicated in van Dalen, 'The Geographical Table'. The column headed # indicates the number of entries

[236] GT1 is Gotha, Forschungsbibliothek, MS 1391, fol. 28v. It is probable that this table was copied from MS 1496, fol. 1r of the same library, which was not consulted by the Kennedys.

[237] van Dalen, 'The Geographical Table'.

from K&K that were considered for each locality (not counting modern coordinates). The next three columns are for the three main traditions of Islamic geographical coordinates:

MAM = the tradition of the caliph al-Maʾmūn (*c.* AD 830), represented by: 1) al-Khwārizmī's *Kitāb Ṣūrat al-arḍ* (abbreviated KHU); 2) a table found together with material by al-Khwārizmī in the manuscript Istanbul, Süleymaniye Kütüphanesi, Ayasofia 4830 (KHZ); the *Kitāb ʿAjāʾib al-aqālīm al-sabʿa ilā nihāyat al-ʿamāra* of Suhrāb (early 10th c., SUH), and entries associated by Abū l-Fidāʾ (1273–1331) with the *Kitāb rasm al-maʿmūr* (RES).

BIR = the geographical table in al-Bīrūnī's major astronomical work *al-Qānūn al-Masʿūdī*, from which the important tables in the *Sanjarī Zīj* of al-Khāzinī (Marw, *c.* 1120, SNJ) and the *Jadīd Zīj* of Ibn al-Shāṭir (Damascus, *c.* 1350, SHA) were derived.

ATW = the mysterious *Kitāb al-Aṭwāl wa-l-ʿurūḍ li-l-Furs*, which contains generally highly accurate coordinates different from MAM and BIR. The date of its composition is uncertain, but data taken from it are already found in the Ismāʿīlī work *Dustūr al-munajjimīn* (Alamut?, early twelfth century, DST). Most geographical tables in later Persian *zīj*es are based on ATW, especially al-Ṭūsī's *Ilkhānī Zīj* (Maragha, 1271/2, TUS), al-Kāshī's *Khāqānī Zīj* (Kashan/Shiraz, 1413/4, KAS), and Ulugh Beg's *Sulṭānī Zīj* (Samarqand, *c.* 1440, ULG). K&K abbreviates this source as ATH FID.

The last column mentions the coordinates in some smaller traditions, especially those of al-Battānī (Raqqa, *c.* AD 900, BAT) and Ibn Yūnus (Cairo, *c.* AD 1000, YUN), as well as systematic deviations from the main traditions by sources that generally follow these. A superscript + after the abbreviation of a source refers to that source plus all other sources usually depending on it; a source surrounded by angular brackets followed by a superscript + (e.g., ⟨YUN⟩$^{+}$) indicates all sources usually depending on this source *without* the source itself. Although Kūshyār's coordinates are listed in the fifth column, the abbreviation KUS is also used in the last column to clarify its relation to other sources. For abbreviations of sources not introduced here, see Kennedy and Kennedy, *Geographical Coordinates*, pp. xv–xxxix; Haddad and Kennedy, 'Geographical Tables'; van Dalen, 'The Geographical Table', and the index of works in this volume. Variants within these traditions are given between parentheses in the same column, using the abbreviations λ for longitude and ϕ for latitude. Pairs of coordinates are always given in the form longitude / latitude.

Table P: Reference table for the geographical localities from the *Jāmiʿ Zīj* (excluding entries copied in the *Shāmil Zīj*)

| place name | # | al-Maʾmūn = MAM | | al-Bīrūnī = BIR | | *Kitāb al-aṭwāl* ATW | | *Jāmiʿ Zīj* KUS | Other traditions / Remarks |
|---|---|---|---|---|---|---|---|---|---|
| Dunqula | 31 | 63;00 | 14;30 | 63;40 | 14;00 | 53;40 | 14;30 | = MAM | **SUH** 63;40/14;30. ATW: mistaken meridian? **BAT**$^{+}$ 63;0/14;15. |
| Suri / Farsaquri | 12 | 135;15 | 5;15 | - | | - | | 135;15 / 5;15 | Suri: only in ZAY, MUH, GT1 and ZAD with the coordinates of KUS. **MAM**, DIM: same coordinates for Farsaquri. |
| Yamama | 38 | 81;45 | 21;30 | = MAM | | = MAM | | = MAM | **BAT**$^{+}$ 76;0/21;30 (DIM λ 72;0). **YUN**,KUS$^{+}$=MAM. **SNJ**$^{+}$ 81;30/21;50. Several sources with λ 81;15. |
| Hajar | 35 | 83;00 | 24;55 | 83;00 | 24;15 | 83;00 | 25;15 | = BIR | **KHZ** ϕ 24;15. Not in ATW itself. **BAT**$^{+}$ 83;20/25;45. **YUN**$^{+}$=ATW. **SHA**$^{+}$ 83;5/24;55. Several further confusions of 15 and 55′. |
| Bahrayn | 25 | 84;20 | 25;45 | - | | = MAM | | = MAM | **BAT**$^{+}$,KUS$^{+}$=MAM. **YUN**$^{+}$,TAJ/MAG 85;20/25;45. |
| Nirun | 19 | 102;20 | 23;30 | 104;30 | 24;45 | 104;00 | 26;00 | = MAM | **BAT**$^{+}$,**YUN** 104;20/23;30. KUS$^{+}$=MAM. MAG,KAS$^{+}$=BIR. Not in DST,TUS$^{+}$. |
| Mansura asSind | 32 | 103;00 | 22;00 | 105;00 | 26;40 | = BIR | | = MAM | **YUN**$^{+}$,KUS$^{+}$=MAM. **SNJ**$^{+}$ 105;0/25;40. **TUS**,WAB,ULG/TMR 105;0/27;40. |
| Ramla | 42 | 65;40 | 32;40 | = MAM | | 66;50 | 32;10 | = MAM | **KHZ** 66;0/32;15, **SUH**,MAR λ 66;40. **BAT**$^{+}$ 65;50/31;35. **YUN**$^{+}$ 66;0/32;0 or 32;15. Not in SNJ$^{+}$. |
| Qaysariya Sham | 29 | 67;30 | 33;15 | 65;20 | 32;50 | 67;30 | 32;30 | 68;30 / 33;15 | **KHU** ϕ 33;55. **KHZ**,**YUN**$^{+}$,TAJ 67;45/32;45. **BAT** 67;25/33;20. Not in SNJ$^{+}$. **TUS**$^{+}$ 66;30/32;30. |
| Tiberias | 41 | 67;45 | 32;00 | = MAM | | 68;00 | 32;00 | 68;45 / 32;0 | **SUH**=ATW. **KHZ**,**YUN**$^{+}$,TAJ 69;20/32;0. Not in BAT. **DST**,**TUS**,KAS$^{+}$,MAG,MIZ/KHL 68;15/32;0 (also ϕ 32;5). |
| Siniz | 16 | 86;45 | 30;00 | 86;45 | 32;00 | 85;30 | 29;10 | = MAM | **YUN**,KUS$^{+}$=MAM. Not in DST,TUS$^{+}$. KAS 85;30/30;0, AIN 84;15/32;0. |
| Jannaba | 16 | 87;20 | 30;00 | - | | 85;45 | 28;55 | = MAM | Not in **YUN**,DST/TUS$^{+}$. KAS$^{+}$≈MAM. |
| Fasa = Basa | 17 | 88;15 | 33;40 | 88;50 | 32;20 | 89;15 | 29;00 | 88;15 / 33;30 | **YUN**=MAM. Not in SNJ$^{+}$, DST/TUS$^{+}$ (except AIN). |
| Hur = Jur / Firuzabad | 22 | 88;15 | 31;30 | - | | 87;30 | 28;10 | 88;30 / 31;30 | **KHZ**,**YUN** 89;30/31;30. **SUH**,KUS$^{+}$,GT1 88;30/31;30. Not in BAT. ATW$^{+}$ Firuzabad. |
| Istakhr | 37 | 89;30 | 32;00 | 88;40 | 30;00 | 88;30 | 30;00 | 89;0 / 32;0 | **SUH** 89;5/32;5. **KHZ**,**YUN**$^{+}$ 91;0/32;0. Not in BAT. **SNJ**$^{+}$ 85;40/32;0. |
| Siraf | 35 | 89;30 | 29;30 | = MAM | | 88;00 | 29;00 | = MAM | **BAT**,AIN=MAM. **YUN**$^{+}$ 89;0/29;30. **ATW** ϕ 26;0. |
| Sirjan | 33 | 93;00 | 32;00 | 93;00 | 32;30 | 90;20 | 29;30 | 93;0 / 32;30 | KUS,**SUH**=BIR, **KHZ**,**YUN**$^{+}$ 98;0/32;30. **BAT**$^{+}$ 93;0/34;0 (DIM ϕ 34;30). **SNJ**$^{+}$ 93;0/31;30. |
| Jiruft | 31 | 98;00 | 31;45 | 93;00 | 31;45 | 93;00 | 27;30 | = MAM | **BAT**$^{+}$=MAM (BAT/MUM ϕ 31;15). **YUN**$^{+}$ 100;0/31;45. ALA 98;0/32;45. |
| Muhammadiya | 17 | 100;00 | 31;45 | - | | - | | = MAM | **KHZ** λ 104;0. **BAT**$^{+}$=MAM. **YUN** 104;0/31;15. |
| Ammuriya | 38 | 63;00 | 38;00 | = MAM | | 64;00 | 43;00 | = MAM | **BAT**$^{+}$ 38;20/39;45 (sic!). **SNJ** 43;0/33;0 (**SHA**$^{+}$ ϕ 33;31). **YUN**$^{+}$=MAM (FAR/BNA/SAN λ 48,50/63;50 ϕ 44;14). |
| Massisa | 27 | 69;40 | 36;00 | = MAM | | 69;15 | 36;45 | = MAM | **PTO**,**BAT**$^{+}$ 68;50/36;45 (BAT λ 67;50). **YUN**$^{+}$ 69;40/36;45. |

| place name | # | al-Ma'mūn = MAM | | al-Bīrūnī = BIR | | *Kitāb al-aṭwāl* ATW | | *Jāmiʿ Zīj* KUS | Other traditions / Remarks |
|---|---|---|---|---|---|---|---|---|---|
| Tripoli Sham | 41 | 70;35 | 34;00 | 69;00 | 35;00 | 69;40 | 34;00 | 70;30 / 34;0 | **BAT**$^+$ 67;30/34;20 (**PTO** ϕ 34;30). **YUN**=MAM. TAJ/MAG 70;0/34;0. |
| Nisibin | 43 | 77;50 | 36;00 | 67;50 | 36;40 | 75;20 | 37;00 | = MAM | **SUH** 78;0/36;0. **BAT**$^+$ 75;30/37;0. KAS=BIR.<br>**YUN**$^+$ 76;50/36;20 (SHR/FAR/BNA 77;18/36;41). |
| Balad | 24 | 78;45 | 36;20 | 68;25 | 35;30 | 76;40 | 36;50 | = MAM | **BAT**$^+$ 77;40/36;35. **YUN**=MAM.<br>**SNJ**$^+$ 68;30/35;40. KAS ϕ 37;40, AIN ϕ 37;30. |
| Takrit | 19 | - | | 79;30 | 35;30 | 78;25 | 34;30 | 79;40 / 34;30 | **SUH** 82;40/35;8. KUS: only in mss **CC**$_1$. **TUS**/WAB/ULG/TMR λ 78;20. |
| Samarra | 34 | 79;45 | 34;00 | 79;45 | 34;12 | 79;00 | 34;00 | 80;0 / 34;0 | **SUH** ϕ 34;10. **BAT** 79;10/34;0. **YUN**=MAM. |
| Dunbawand | 29 | 85;30 | 36;15 | 87;30 | 36;20 | - | | = MAM | **BAT**$^+$ 135;30/36;35 (BAT ϕ 57;35, MUM λ 86;30). **SNJ**$^+$ 87;30/36;0.<br>**SUH**,**YUN**,MAG 86;30/36;15. KAS 86;0/36;15, AIN 86;20/35;55. |
| Shalus | 12 | 85;40 | 37;50 | 86;15 | 36;55 | 86;20 | 37;00 | 85;45 / 37;50 | **RES** ϕ 37;40, **SUH** λ 86;15, **KHZ**,**YUN**=KUS.<br>**BIR** also λ 86;55. Not in SNJ$^+$. Not in DST/TUS$^+$. |
| Amul | 38 | 87;20 | 37;45 | 87;10 | 36;35 | 87;20 | 36;35 | = MAM | Not in YUN. **BAT**$^+$,⟨YUN⟩$^+$=MAM.<br>MUF,ASH 86;0/36;0. ALA$^+$ 87;0/36;0. |
| Qumis | 11 | 88;15 | 36;25 | - | | - | | = MAM | **SUH** 88;40/36;0. Not in **BAT**$^+$. **YUN**$^+$ 88;15/36;24. |
| Marv Rud | 36 | 95;00 | 38;50 | 97;40 | 37;30 | 97;00 | 36;30 | = MAM | **KHU** ϕ 37;50, **SUH** λ 95,40. Not in BAT. **YUN**$^+$,ALA=MAM.<br>**SNJ**$^+$ 97;40/36;30. KAS 94;40/34;30, AIN 94;0/36;30. |
| Rome | 39 | 45;25 | 41;50 | = MAM | | - | | = MAM | PTO/BAT$^+$ 36;40/41;40. **YUN**$^+$ also λ 45;20.<br>**TUS**$^+$ 45;27/41;50 (KAS$^+$ λ 45;0). |
| Arzan | 16 | 76;00 | 39;15 | 76;00 | 34;15 | 75;00 | 38;00 | = MAM | **SUH** 77;5/39;40. **BAT**$^+$ 76;40/38;0. **YUN**=MAM. Not in **SNJ**$^+$. |
| Arzan Rum* | 16 | - | | - | | 79;00 | 41;00 | - | **RES** 76;0/39;15 (=MAM for Arzan). Not in BAT$^+$,YUN$^+$,KUS$^+$.<br>SML 76;0/39;0. Not in SNJ$^+$. **TUS**$^+$ 77;0/39;40 (KAS 79;0/41;15). |
| Isfijab | 27 | 108;10 | 39;50 | 99;20 | 43;20 | 99;50 | 43;35 | = MAM | **YUN**,ZAY 78;10 (sic!)/39;50. WAB/KAS ϕ 43;30, ULG/TMR ϕ 43;36. |
| Taraz | 36 | 110;30 | 40;25 | 99;50 | 43;35 | 99;50 | 44;25 | = MAM | **YUN**$^+$,ALA=MAM (YUN ϕ 110;40, ⟨YUN⟩$^+$ λ with severe errors).<br>**SNJ**$^+$ 99;20/43;35. KAS/TMR ϕ 44;31, ULG ϕ 44;36. |
| Heraqla | 28 | 63;25 | 46;35 | = MAM | | 67;22 | 46;30 | 63;20 / 46;30 | Not in BAT$^+$. **YUN**$^+$ 63;25/47;35. ALA=KUS.<br>Not in SNJ. Not in DST/TUS. KAS/AIN λ 67;20. |
| Jurzan | 18 | 81;00 | 44;00 | - | | - | | = MAM | **BAT**$^+$,**YUN**$^+$=MAM. Not in SNJ$^+$. Not in DST/TUS$^+$. |
| Khazar | 14 | 103;00 | 45;00 | - | | - | | 83;0 / 45;0 | Not in **BAT**. **YUN**$^+$=MAM. Not in SNJ$^+$. Not in DST/TUS$^+$. |
| Ankara | 26 | 68;00 | 48;00 | 68;00 | 48;20 | 64;00 | 41;00 | = MAM | **YUN** 48;0 or 98;0/48;0. **BIR** λ 58;0. KAS/AIN λ 64;40, AIN ϕ 41;45. |
| Majuj | 20 | 182;30 | 63;00 | - | | - | | 172;30 / 63;0 | **YUN**$^+$=MAM (for Yajūj wa-Majūj). KAS=KUS. |

* I have added this locality because it appears to have been confused with Arzan, especially in early sources.

## IV.14. Fixed stars

### Tables 55 and 55a: Star tables

**Bibliography:** Schjellerup, *Description des étoiles fixes*; Knobel, *Ulugh Beg's Catalogue of Stars*; Elwell-Sutton, *The Horoscope of Asadullāh Mirzā*; Kunitzsch, *Ibn aṣ-Ṣalāḥ*; Kunitzsch, *Der Almagest*; the *EI*[2] article 'Nuḏjūm. i. The Fixed Stars' by P. Kunitzsch; Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog.*

The extant manuscripts of Book II of the *Jāmiʿ Zīj* contain two somewhat different star tables for the beginning of the year 301 Yazdigird (2 April 932). A table with 48 stars appears in manuscripts **FHYB**, while **L** has an empty frame most likely intended for the same table. On the other hand, **CC$_1$** include a table with 60 stars of which 20 are not found in the table in the other four manuscripts (see Plate 15). The tables in **FYB** include a column with the names of the constellations. **H** has an empty column on the right side of both pages of the table, but has not written the constellation names in these columns. The two pages of the star table in **L** have a title and column headers, but no constellation and star names, coordinates and other values in the table. An empty column to the right of the column for star names could have been intended for the constellations. In any case it is probable that **L** intended to include the table also found in **FHYB**, since like these manuscripts, and unlike **CC$_1$**, it covers two pages instead of one and the column for the magnitudes appears before the one for the direction of the latitude. The 15 rows drawn on each page of **L** would provide room for the 30 localities from each column in the Cairo manuscripts, but this is also the standard format found in **Y** and **B**, in which several rows were filled with other information (in **F** the lines between the rows were clearly only drawn after the star names and coordinates were written, in such a way that they separated the constellations). Columns for constellation names are entirely missing from **CC$_1$**; in some cases these names are incorporated in the star names, which are generally shortened even further than in **FHYB**. The longitudes, latitudes, directions and magnitudes in the two tables are generally in agreement for the 40 stars that they have in common. The same holds for the temperaments or complexions (*mizāj*, pl. *mizājāt*), which are likewise found in both versions of the table.

For ease of reference I have numbered the stars on the two pages in manuscript **F** from A1 to A24 and from B1 to B24; the stars in **CC$_1$** are numbered from C1 to C60. In **B** the page break in the star table occurs after star A24 (as in **F**), while in **Y** it occurs after B4 (and in my edition of the textual elements of the table in Part III, pp. 327–28, after B2). **H** does not have any intermediate headers, has the page break only after star B6, and leaves enough room under the second part of the table for the colophon of Book II (see p. 334).

Due to space constraints, I had to split the edition of the star table in **FHYB** into two parts, namely the names of the constellations and stars on

the one hand, and the coordinates and further data on the other. To the edition of the coordinates in Part II (pp. 246–47 and 250–51), I have added the Baily numbers and modern identifications of all stars. In Table 55 as well as in Table 55a, stars that do not appear in the other table are indicated by an asterisk. I have applied the following specific editing rules for setting up the apparatuses for the two star tables. In addition to the sigla for the manuscripts of the *Jāmiʿ Zīj*, **A** denotes the *Almagest* as edited in Kunitzsch, *Claudius Ptolemäus. Der Sternkatalog* and **E** the manuscript Escorial, RBMSL, árabe 908 of al-Battānī's *Ṣābiʾ Zīj*. I have occasionally checked entries from the star catalogue in Nallino's edition of al-Battānī's *zīj*, but as for most of the tables Nallino uncritically corrected all readings of coordinates to values conforming with the *Almagest* (i.e., the exact latitudes, and longitudes that are 11;10° larger than Ptolemy's) while giving the manuscript coordinates in footnotes.[238] In general, variants in the coordinates found in sources **AE** are given in Table 55a (the table from $\mathbf{CC_1}$) only for stars that do not appear in Table 55 (the table from **FHYB**). For the longitudes the readings given in the apparatuses are the ones that Kūshyār should have had if his tables had been fully compatible with the other sources, i.e., with a difference of 12°0′ from the *Almagest* and a difference of 0°50′ from al-Battānī; the original longitudes from **AE** may be indicated between square brackets to elucidate any resulting non-trivial variants.

In the apparatuses of both Table 55 and Table 55a, I also indicate, for the stars that the two tables have in common, any deviations in the longitudes and latitudes found in the other table. However, differences in magnitudes, temperaments and in the indications of fatal and benefic stars are only given in the apparatus to Table 55. If the only difference between a magnitude in **FHYB** and in $\mathbf{CC_1}$ is the indication 'larger' (*kāf*, + in the table edition) or 'smaller' (*ṣād*, – in the edition), this is not mentioned in the apparatus unless there are other variants in the same magnitude.

In the names of constellations the Arabic words *min* and *ṣūra* are not translated in order to save space. In the direction column, N stands for North (س, شمال) and S for South (حـ, جنوب). In the column of temperaments, the names of the planets, in Arabic abbreviated with their final letter, are rendered as 'Sun', 'Moon', 'S' (Saturn), 'J' (Jupiter), 'Ma' (Mars, in the manuscripts found both as خ and as يح), 'V' (Venus), and 'Me' (Mercury). Any variants of the

[238] Al-Battānī's star tables are found on fols 226v–237r and 237v–239r of Escorial, RBMSL, árabe 908, the explanatory Chapter 51 on fols 126v–128v. In the Castilian translation in Paris, Bibliothèque de l'Arsenal, MS 8322, the star tables are included on fols 84v–91v and 92r–93r. While the first table is a catalogue of the type found in Ptolemy's *Almagest* with 533 stars, the second table gives equatorial coordinates of 75 important stars for 1 March 900. In Nallino, *al-Battānī sive Albatenii* the star tables are edited in vol. III, pp. 245–74 and pp. 275–79 and translated into Latin in vol. II, pp. 144–77 and pp. 178–86; Chapter 51 is edited in vol. III, pp. 187–90 and translated in vol. II, pp. 124–26 (with commentary on pp. 292–98).

temperaments apply to the entire entry (i.e., in many cases, to the pair of planets given in the table). The Arabic indication *qāṭiʿ* قاطع (sometimes abbreviated as ق) is rendered as '*fatal*' in the edition of the table, the red *hamza*s (looking like *dāl*s in **Y**) as '*incr*' (for 'increaser'; these do not appear in **CC$_1$**); for both concepts, see below. I have not tried to verify these indications on the basis of other sources but have accepted the ones that appear in a majority of the witnesses as the main variants.

In his major publications *Der Almagest* (1974) and *Ibn aṣ-Ṣālāh* (1975), Professor Paul Kunitzsch also discussed in some detail the two star catalogues in al-Battānī's *Ṣābiʾ Zīj* and both versions of Kūshyār's star table, whose star names and coordinates agree strongly with those of al-Battānī.[239] Kunitzsch noted that, for 13 constellations and approximately 120 stars, the names given by al-Battānī deviate in essential characteristics from the ones in the *Almagest* traditions of al-Ḥajjāj and/or Isḥāq ibn Ḥunayn. Such characteristics include the use of the word *muḍīʾ* instead of *nayyir* for 'bright' or 'a bright star',[240] or the use of the word *katif* instead of *mankib* for 'shoulder'.[241] Of the 533 entries in al-Battānī's star catalogue, 42 stars are discussed by the twelfth-century Baghdad scholar Ibn al-Ṣālāḥ in his critical assessment of the differences in coordinates, directions and magnitudes in Ptolemy's star catalogue found in the five versions of the *Almagest* that were accessible to him. For a majority of these 42 stars, al-Battānī's coordinates agree with the ones quoted by Ibn al-Ṣālāḥ from the Syriac translation of the *Almagest* and the so-called 'old Arabic' or 'old Maʾmūnic' translation against the translation by al-Ḥajjāj and the one by Isḥāq ibn Ḥunayn corrected by Thābit ibn Qurra.[242]

As Kunitzsch pointed out, the star names as well as the coordinates in both star tables of Kūshyār are generally close to al-Battānī's and frequently differ from those of al-Ḥajjāj or Isḥāq ibn Ḥunāyn. In particular, in each of the three cases in which Ibn al-Ṣālāh notices differences in the coordinates of stars included in the *Jāmiʿ Zīj*, Kūshyār joins al-Battānī in using values that differ from the surviving *Almagest* translations and are attributed by Ibn al-Ṣālāḥ to

[239] Kunitzsch, *Der Almagest*, pp. 49–51, 60–63, 79–82, 153–55, etc. for al-Battānī and pp. 57–58 for Kūshyār, and Kunitzsch, *Ibn aṣ-Ṣālāh*, Anhang II, pp. 97–108. An English summary of some of Kunitzsch's findings on al-Battānī's star catalogue may be found in Kunitzsch, 'New Light'.

[240] Kunitzsch, *Der Almagest*, p. 215.

[241] Ibid., pp. 71–73.

[242] Kunitzsch, *Ibn aṣ-Ṣālāh*, pp. 99–105. The old Arabic translation, now lost, was made by the otherwise unknown al-Ḥasan ibn Quraysh for the caliph al-Maʾmūn (r. 913–933) before the translation of al-Ḥājjāj. For traces of this translation in later works, see now also Thomann, 'The Oldest Translation'. Isḥāq's translation dates from approximately the same time when al-Battānī compiled his *zīj*.

the Syriac or the old Arabic version of the *Almagest*.[243] We may thus conclude that Kūshyār's star tables most probably also ultimately relied on the Syriac or the old Arabic translation of the *Almagest*.[244]

Because of the general dependence of the *Jāmiʿ Zīj* on the *Ṣābiʾ Zīj*, it is plausible that Kūshyār also based his star tables directly on al-Battānī's star catalogue. Al-Battānī's longitudes for the Syrian year 1191 Dhū l-qarnayn (which in his tables starts on 1 March 880) were found from the ones in Ptolemy's *Almagest* for the beginning of the reign of Antoninus (20 July 137) by adding a correction for precession of 11;10°. This corresponds to a precessional motion of 1° in 66½ Julian years, relatively close to the value of 1° in 66 Persian years found by the Mumtaḥan astronomers and to the value of 1° in 66 Julian years given by al-Battānī himself. Kūshyār's longitudes for 2 April 932 are exactly 12;0° degrees larger than Ptolemy's, and hence 0;50° larger than those of al-Battānī. The former difference corresponds to a precessional motion of 1° in approximately every 66¼ Persian or Julian years since the time of Ptolemy, and the latter to 1° in 62½ Persian or Julian years since al-Battānī's epoch. Although the latter rate is historically implausible, we cannot rule out the possibility that Kūshyār first found the precessional motion since Ptolemy as approximately 11;56°, decided to use a longitude difference of exactly 12;0° for the round epoch 301 Yazdigird, and then calculated the longitudes by adding 50′ to those of al-Battānī.

The magnitudes of the stars are in most cases indicated by an *abjad* number from 1 to 6 and generally follow the magnitudes from Ptolemy's *Almagest*. Especially in manuscripts **C**$\mathbf{C_1}$, but only once in **FYB**, the letters *kāf* and *ṣād* appended to the magnitudes are used as abbreviations for 'larger' (*akbar*) and 'smaller' (*aṣghar*) to indicate that the magnitudes of stars are somewhat larger or smaller than the numerical value assigned to them. For 'nebulous', the Arabic word *saḥābī* 'cloudy' is used in manuscripts **FHYB** and the abbreviation غ *ghayn* for *ghamāmī* 'cloudy' in **C**$\mathbf{C_1}$.

Besides the purely astronomical information (longitude, latitude, direction and magnitude), Kūshyār's star tables also provide three types of astrological information, namely the temperaments or complexions of the stars, expressed

[243] See Kunitzsch, *Ibn aṣ-Ṣalāḥ*, pp. 105–06. This concerns Kūshyār's star B12/C47 (Baily no. 670) with the correct latitude 20;20° against the Greek scribal error 23° found in al-Ḥajjāj and Isḥāq, star B13/C50 (Baily no. 734) with latitude 16;30° (in the Syriac version: 16;50°) against the correct 13;50°, and star no B23/C59 (Baily no. 969) with the longitude mistakenly in Libra instead of in Scorpius as in Isḥāq's version. Kunitzsch (ibid., pp. 106–07) lists nine more stars for which al-Battānī and Kūshyār agree on a value for longitude or latitude not found in the Greek and main Arabic traditions.

[244] Kunitzsch, *Ibn aṣ-Ṣalāḥ*, pp. 107–08, notes that the star names in the table found in the two Cairo manuscripts were partially modified to agree with the forms found in the translation by Isḥāq/Thābit.

by abbreviations of the names of the planets in the last column of the table, and the indicators *qāṭiʿ* and the letter *hamza* (in manuscript **Y**: *dāl*) after some of the star names (in **CC₁**: in the margins).[245]

The temperaments of fixed stars are qualities that are also possessed by one or two of the planets and are therefore expressed by listing the planet(s) concerned.[246] In Ptolemy's *Tetrabiblos* the temperaments are given for entire constellations or, especially within the ecliptic constellations, for small groups of stars.[247]

Fatal or malefic stars (called *qāṭiʿ* by Kūshyār, translated as 'cutter' by Yano) are stars that bring disaster when the *tasyīr* arc (prorogation, progression or direction, see Section IV.12) comes into their neighbourhood.[248] In the two star tables in the *Jāmiʿ Zīj* the fatal stars are indicated by the word *qāṭiʿ* after the name of the star, sometimes abbreviated by the letter *qāf*.

Kūshyār's table with 48 stars in the manuscripts **FYB** (but not in **H**) include another indication that is less obvious, namely the letter *hamza* ء (in **Y** looking more like a *dāl* د). Explanatory text A in manuscripts **YB** indicates that these can be used in finding ascendants of nativities, but do not add any further specification. I have not been able to find a primary source for the meaning of these abbreviations, but Mohammad Bagheri kindly drew my attention to a footnote to the section on astrological interpretation in the edition by Jalāl al-Dīn Humāʾī of the Persian translation of al-Bīrūnī's astrological manual *Kitāb al-Tafhīm li-awāʾil ṣināʿat al-tanjīm*.[249] Hūmāʾī here states that the term *zāʾid* (pl. *zawāʾid*) refers to stars that have a positive effect on life (in the sense of 'increasing' its length), and hence the opposite effect to *qāṭiʿ*. It certainly seems plausible that Kūshyār would indicate these 'luck-bringing stars' in his table besides the malefic ones. However, *hamza* is not the abbreviation one

[245] I have not compared Kūshyār' star tables with the one for the Malikshāhī epoch in the *Dustūr al-munajjimīn* (Paris, BnF, arabe 5968, fols 224r–225r), which likewise includes temperaments and indications of fatal stars.

[246] See Kunitzsch's article 'Nudjūm' in *EI²*, vol. VIII, p. 100a; Elwell-Sutton, *The Horoscope of Asadullāh Mirzā*, pp. 80–81 (note, however, that the star table with temperaments said to be from the *Malikshāhī Zīj* is in fact the one of the *Dustūr al-munajjimīn* and unlikely to be directly associated with ʿUmar al-Khayyām in spite of its Malikshāhī epoch; cf. van Dalen, 'The Malikī Calendar', pp. 118–19); Kunitzsch, 'Zum liber hermetis', esp. pp. 64–66, and Kunitzsch, 'Abū Maʿšar, Johannes Hispalensis', esp. pp. 103 and 105–06.

[247] See Robbins, *Ptolemy. Tetrabiblos*, Chapter I.9, pp. 46–59.

[248] Elwell-Sutton, *The Horoscope of Asadullāh Mirzā*, p. 86; Yano, *Kūšyār Ibn Labbān's Introduction*, pp. 17/18. For *tasyīr*, see the references given with the commentary on Table 53.

[249] Humāʾī, *Kitāb al-Tafhīm*, p. 522, footnote 5. Humāʾī here also describes a number of other concepts related to astrological interpretations of stars, e.g., *qātiʿ* and *qāsim*. Unfortunately he does not give any sources for his information. The section of the *Tafhīm* for which this information is supplied is in Wright, *The Book of Instruction*, § 522, pp. 322/23-26/27.

would expect for *zāʾid*[250] and none of the six stars that Humāʾī lists as *zawāʾid* are marked with a *hamza* in Kūshyār's star table. In his *al-Madkhal fī ṣināʿat aḥkām al-nujūm*, Kūshyār explains the role of *qawāṭiʿ*, but does not mention the term *zawāʾid*.

Chapter I.8 of Kūshyār's *al-Madkhal fī ṣināʿat aḥkām al-nujūm* contains a listing of 30 stars including, with the exception of latitudes, the same types of data as found in the *Jāmiʿ Zīj*, i.e., star names, longitudes, magnitudes, directions, temperaments, and an indication whether the stars are fatal.[251] The longitudes in this listing are given for the beginning of the year 361 Yazdigird (18 March 992, 60 years later than the epoch of the star tables in the *Jāmiʿ Zīj* and thus during Kūshyār's most likely period of activity); they differ from Ptolemy's by 13;0° and from the *Jāmiʿ Zīj* by 1;0°. All thirty stars are also found in the *Jāmiʿ Zīj* and, with the exception of the temperaments, are generally in perfect agreement with it. The entries in *al-Madkhal fī ṣināʿat aḥkām al-nujūm* that differ in some way from the *Jāmiʿ Zīj* are the following (I use the abbreviations for the temperaments introduced above and quote the data as given by Yano):

- B14/C51, right shoulder of the giant (Yano no. 6): temperament S Me (instead of S), cutter.
- B21/C56, Sirius (Yano no. 12): temperament J and a little of Ma (instead of J).
- B22/C58, Procyon (Yano no. 13): temperament Me and a little of Ma (instead of Me).
- B1/C29, Praesepe (Yano no. 16): temperament Ma Moon (instead of Ma Me).
- B2/C30, 'shoulder of the lion' (Yano no. 17): temperament S and a little of Ma (instead of S).
- B3/C31, Regulus (Yano no. 18): cutter.
- A5/C1, Arcturus (Yano no. 20): temperament Me S (instead of J Ma).
- B5/C33, Spica (Yano no. 22): temperament adds [V] 'and a little of Me'.
- B8/C36, Antares (Yano no. 23): temperament adds [Ma] 'and a little of J'.

[250] The *dāl* seen in **Y** would be a more regular abbreviation for a word ending in this letter (cf. the *dāl* used for Mercury (*ʿUṭārid*) in the column of temperaments), but the explanatory note A found in this manuscript and in **B** (see p. 325) refers explicitly to indications marked by a red *hamza*.

[251] Yano, *Kūšyār ibn Labbān's Introduction*, pp. xxiii-xxv (summarising table and commentary) and 19/20-25/26 (edition and English translation).

- B10/C38, ‘that follows the sting’ (Yano no. 24): temperaments as in **F**.
- B11/C45, ‘eye of the archer’ (Yano no. 25): temperaments S Me (instead of Sun Me).

A comparison of Kūshyār’s temperaments in the *Jāmiʿ Zīj* with those given in Ptolemy’s *Tetrabiblos* yields a very similar picture: they often agree, but also frequently show deviations.

## IV.15. Section on finding the original equations

As we have seen in Section IV.7.1, due to Kūshyār's use of displaced planetary equations the mean longitudes and mean anomalies found from his mean motion tables are not the actual mean positions, and the solar, lunar and planetary equations are not the actual equations as Ptolemy defined them. Furthermore, the true centrum of the planets, obtained by adding their equation of centre to their mean centrum, is not the actual true centrum. At the end of Section I.4.8, Kūshyār explains that due to 'the displacement of the equations' (*waḍʿ al-taʿādīl*) in his *zīj* the obtained true centrum is not the 'actual' (*ḥaqīqī*) true centrum that is needed for finding the planetary latitudes and stations from Tables 38–42.[252] To obtain the actual true centrum, the displacement of the equation of anomaly (7° for Saturn, 12° for Jupiter, etc.) must be added to the obtained true centrum. The same rule and numbers are given as an explanatory note to the tables for the latitudes and stations in all manuscripts; in the Cairo manuscripts the displacements are written in *abjad* notation at the top of the columns.

Two of the eight extant manuscripts of Book II of the *Jāmiʿ Zīj* include an additional chapter at the end of Book II (designated as *nawʿ* 56 in the table of contents of **F** and as *nawʿ* 44 in $\mathbf{C_1}$) which explains how to find the 'original equations' (*al-taʿādīl al-aṣliyya*) from Kūshyār's tables. In **B** only one page of this chapter appears after all other additional materials and is not included in the table of contents. In **L** the chapter is listed in the table of contents of Book II (see p. 263), but is not in fact included. I have edited the entire chapter in Part III (see pp. 331–34), but have translated only the paragraphs on the Sun, Moon and Saturn, since the paragraphs for the five planets are basically identical except for the numbers and some additions in $\mathbf{C_1}$ for Saturn.[253] Each paragraph gives the following information:

- The number that must be subtracted from a given actual degree of mean centrum or mean anomaly in order to obtain the argument for which the corresponding equation can be found in Kūshyār's tables, i.e., the shift. For the Sun the shift is equal to the displacement of the solar equation, for the planets the shift of the equation of centre is equal to the sum of the displacements of the equation of centre and the equation of anomaly. The lunar equations and the planetary equations of anomaly have no shift.
- The ranges of Kūshyār's arguments for which the original equations are additive and subtractive. For the original equations these ranges are

[252] Bagheri, *az-Zīj al-Jāmiʿ*, pp. 39 (translation), 46–47 (commentary) and Arabic p. 27.

[253] An edition of the text based on the same three manuscripts, together with a full translation, can be found in Kashino, *Planetary Theory*, pp. 108–09 and 89.

0–180° or 180–360°; for Kūshyār's displaced equations the ranges shift with the arguments. This means that their beginning and end become 360° minus the shift or 180° minus the shift. Note that the Ptolemaic solar equation, the lunar equation of anomaly and the planetary equations of centre are subtractive between 0 and 180°, the lunar equation of centre and the planetary equations of anomaly additive.

- How to find the actual equation from Kūshyār's displaced equations within each of the ranges specified in the previous step. Whenever the original equation is subtractive, it is found by subtracting Kūshyār's equation from the displacement. If the original equation is additive, it is found by subtracting the displacement from Kūshyār's equation. Note that the only purpose of the distinction between the ranges for subtractive and additive equations is to make certain that no larger quantity is subtracted from a smaller one, i.e., in modern terms, that the resulting equations are always positive. Using modern arithmetic and negative equations, the procedure could be described by a single operation.
- The adjustment needed for the argument of the variation of the planets. Since this argument is the true centrum,[254] the same adjustments are necessary as for the use of the tables of planetary latitudes and stations. For the variation of the Moon and the interpolation coefficients for the Moon and the planets, no adjustments need to be carried out.

Manuscript **C$_1$** makes some useful additions to the text found in **F** and (partially) in **B**. I have reproduced these in the apparatus (without standardising the numbers) and have added their translation between parentheses. They concern:

- The changes made by Kūshyār to the tabulated mean longitudes and mean anomalies (indicated immediately after the headers of the paragraphs). For the mean longitude of the Sun and the Moon these amount to a decrease by their respective displacements. For the mean longitude of the planets they amount to a decrease by the sum of the displacements of the equation of centre and the equation of anomaly. For the mean anomalies of the planets they amount to an increase by the displacement of the equation of anomaly.
- An example in the paragraph on Saturn that explains that if one subtracts Kūshyār's equation of centre 7;0 for argument $11^s 16°$ from the displacement of 7°, one obtains the equation 0;0 corresponding to an original argument 0 or 360° (*al-dawr*, 'a full circle').

[254] However, as we have seen in Section IV.7.2, Glen Van Brummelen showed that the variation was in fact tabulated as a function of the mean centrum and that its shift should have been equal to the sum of the displacements of the equation of centre and the equation of anomaly.

Table Q: Displacements and shifts of Kūshyār's equations of centre and equations of anomalies, with the ranges of his arguments for which the equations are additive (all in degrees).

| | first equation | | | second equation | | | variation |
|---|---|---|---|---|---|---|---|
| | displacement | shift | additive range | displacement | shift | additive range | shift |
| Sun | 2 | −2 | 178–358 | - | - | - | - |
| Moon | 14 | +0 | 180–360 | 8 | 0 | 180–360 | - |
| Saturn | 7 | −14 | 166–346 | 7 | 0 | 0–180 | −7 |
| Jupiter | 6 | −18 | 162–342 | 12 | 0 | 0–180 | −12 |
| Mars | 12 | −59 | 301–121 | 47 | 0 | 0–180 | −47 |
| Venus | 2 | −50 | 310–130 | 48 | 0 | 0–180 | −48 |
| Mercury | 4 | −30 | 330–150 | 26 | 0 | 0–180 | −26 |

- Another example at the end of the paragraph on Saturn indicating that the original 'variation at the furthest distance' for argument $0^s0°$ is found in Kūshyār's table for argument $11^s23°$. This comment refers to the maximum of Kūshyār's non-Ptolemaic variation function, which, for the furthest distance, is equal to the difference of the maximum central equation of anomaly and the maximum equation of anomaly at the apogee of the deferent.

A complete overview of the displacements, shifts and ranges of arguments described in this chapter can be found in Table Q. It is clear that this chapter became part of the manuscript tradition of Kūshyār's *Jāmiʿ Zīj*. However, it does not need to have been written by Kūshyār himself. Any capable astronomer with an understanding of the displacements of Kūshyār's planetary tables would have been able to write these explications.

# Quick reference guide: *zīj*es

This overview provides basic information, including shelf marks of the most important manuscripts and references to the relevant secondary literature, of the *zīj*es that are frequently referred to in my analyses of the tables in Kūshyār's *Jāmiʿ Zīj* and the comparisons with other works. They are presented here in chronological order, but may be found by their titles or authors through the indexes at the end of the book.

- The *Sindhind Zīj* by al-Khwārizmī (Baghdad, *c.* AD 825) is a mixture of materials of Indian or Persian origin and types of tables and parameters adopted from Ptolemy. This work is only extant in a twelfth-century Latin translation by Adelard of Bath of an Arabic reworking for Cordoba made by the Andalusian astronomer Maslama al-Majrīṭī around the year 1000. This Latin version was edited and translated in Suter, *Die astronomischen Tafeln* and Neugebauer, *The Astronomical Tables of al-Khwārizmī*. For an overview of all tables and their most likely origins, see van Dalen, 'Al-Khwārizmī's Astronomical Tables'. The Latin and Hebrew versions of the important commentary by Ibn al-Muthannāʾ (10th c.) were edited and translated in Millás Vendrell, *El comentario* and Goldstein, *Ibn al-Muthannâ's Commentary*. The commentary by Ibn Masrūr in Cairo, Dār al-kutub, *riyāḍa* Taymūr 99 remains unpublished.
- The *Mumtaḥan Zīj* by Yaḥyā ibn Abī Manṣūr was most probably written in collaboration with other astronomers in the service of the Abbasid caliph al-Maʾmūn (Baghdad, *c.* 830). This work is still clearly influenced by Indian and Persian astronomical theory, but is for the most part Ptolemaic. It is extant in two different recensions with later materials, which were most likely copied in the thirteenth century. Whereas the manuscript Escorial, RBMSL, árabe 927 had been known to historians of Islamic astronomy for many decades, only in 2004 was the highly disordered Leipzig, UB, Vollers 821 recognised to be another copy of the *zīj*. For tables of contents of the two manuscripts, see Vernet, 'Las «Tabulae probatae»' and van Dalen, 'A Second Manuscript'. A facsimile edition of the Escorial manuscript (with several pages from the beginning moved to the end and recto and verso exchanged) is available as Sezgin, *The Verified Astronomical Tables*.
- The Ptolemaic *zīj* of Ḥabash al-Ḥāsib (Samarra, *c.* 870) is referred to as the *Arabic Zīj* or *Damascene Zīj*. This handbook also still shows

obvious Indian and Persian influences, but in its foundations it is a Ptolemaic work. The *zīj* is extant in two manuscripts, of which Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/2 (which also includes the copy **Y** of Kūshyār's *Jāmiʿ Zīj*) is closer to the original, whereas Berlin, SBPK, Wetzstein I 90 (Ahlwardt no. 5750) was supplemented with materials from later authors. An extensive overview of both manuscripts can be found in Debarnot, 'The *Zīj* of Ḥabash al-Ḥāsib'. The introduction of the *zīj* is edited and translated into English in the bilingual article Sayılı, 'Habeş el Hâsib'in el Dimişkî', and a systematic study of Ḥabash's materials on solar and lunar distances and velocities is included in as-Saleh, 'Solar and Lunar Distances'.

- The *Ṣābiʾ Zīj* by al-Battānī (Raqqa, *c.* 900) is the earliest surviving purely Ptolemaic astronomical handbook, which was shown by Nallino and Kunitzsch to be at least partially influenced by a Syriac translation of Ptolemy's *Almagest*.[1] This *zīj* is extant in its entirety in the manuscript Escorial, RBMSL, árabe 908, as well as in the Castilian translation prepared for Alfonso X and kept in Paris, Bibliothèque de l'Arsenal, MS 8322 (available online). The text part of the *zīj* was translated into Latin by Plato of Tivoli and printed twice in the sixteenth and seventeenth centuries. The *zīj* was edited, translated and commented upon in Nallino, *al-Battānī sive Albatenii*, and the text of the Castilian version in Bossong, *Los Canones de Albateni*. The Latin translation by Plato was translated into French in Peyroux, *Albatégni. Livre sur la science des étoiles*. Improved editions and apparatuses for some of the tables are provided in van Dalen and Pedersen, 'Re-editing the Tables', while new studies on the entire tradition of al-Battānī's *zīj* are being carried out at the University of Barcelona by Emilia Calvo, Rosa Comes and Mònica Mont.
- Ibn al-Aʿlam was in the service of the Buyid emir ʿAḍud al-Dawla (r. 949–983) and was famous for the accuracy of his observations. In particular, his tables for Mars were adopted in later recensions of Yaḥyā ibn Abī Manṣūr's *Mumtaḥan Zīj* (see above) because they were reputed to produce the best positions for this planet among all *zīj*es available at the time. Unfortunately, Ibn al-Aʿlam's *Aḍudī Zīj* is lost, so our only information about it derives from tables and lists of parameters in other *zīj*es. Tihon, 'Sur l'identité' collects traces of Ibn al-Aʿlam's work in Byzantine sources, whereas Kennedy, 'The Astronomical Tables' and Mercier, 'The Parameters' establish the characteristics of his planetary mean motions and equations on the basis of information in the *Ashrafī*

[1] See Kunitzsch, *Der Almagest*, pp. 7–8.

*Zīj* (see below) and other sources. See also: footnote 44 on p. 366; van Dalen, 'The *Zīj-i Nāṣirī*', pp. 841–42, n. 7; van Dalen, 'A Second Manuscript'; Mozaffari, 'Ptolemaic Eccentricity', and Mozaffari, 'A Revision of the Star Tables'.

- Of Abū l-Wafāʾ's *al-Majisṭī* (Baghdad, *c*. 970) only part of the text is extant in the sole surviving manuscript Paris, BnF, arabe 2494. The tables are announced in the text, but are not actually provided. Schoy, 'Beiträge zur arabischen Trigonometrie', p. 393 already noted that Abū l-Wafāʾ tabulated his sine and tangent accurately for every quarter of a degree. In my doctoral dissertation (van Dalen, *Ancient and Mediaeval Astronomical Tables*, Chapter 4) I conjectured that a set of twelve highly accurate trigonometrical and spherical astronomical tables with values to sexagesimal thirds in the Paris manuscript of the *Baghdādī Zīj* (see below) derived from Abū l-Wafāʾ; only the table for the solar equation on fols 45r–47v of this manuscript is explicitly attributed to him on its first page. During a stay at the Institute for History of Science in Frankfurt am Main sponsored by the Alexander von Humboldt Stiftung (1994/95), I made a systematic analysis of the numbers and computations in the numerous worked examples for spherical astronomical problems that Abū l-Wafāʾ presents in the Paris manuscript of *al-Majisṭī*. Similar examples are also found in the *Dustūr al-munajjimīn* (*c*. 1110, see below), of which one was published in Kennedy, 'Applied Mathematics'. These examples not only confirmed my conjecture about the origin of the tables in the *Baghdādī Zīj*, but also showed that Abū l-Wafāʾ performed all his calculations entirely systematically by applying linear interpolation in his tables with values to sexagesimal thirds. Unfortunately these results remain unpublished, with the exception of some comments on the relation between the sine and tangent tables in van Dalen, 'Islamic and Chinese Astronomy', pp. 349–51. In the present book I argue that Abū l-Wafāʾ's tables for the sine, tangent and right ascension were adopted in their entirety by al-Bīrūnī (see footnote 32 on pp. 358–59 and footnote 144 on pp. 450–51). Abū l-Wafāʾs table for the oblique ascension of Isfahan (latitude 32;25°) is included in the *Dustūr al-munajjimīn*. A full study of Abū l-Wafāʾ's *zīj* is long overdue. Impressions of its trigonometric and spherical astronomical contents can be obtained from Carra de Vaux, 'L'Almageste d'Abû'lwéfa' and Moussa, 'Mathematical Methods'.
- Of Ibn Yūnus's huge *Ḥākimī Zīj* (Cairo, *c*. 1000) only slightly more than the first half is extant in the manuscripts Leiden, UB, Or. 143 and Oxford, Bodleian Library, Hunt. 331. Like his contemporaries Abū l-Wafāʾ and al-Bīrūnī, Ibn Yūnus provides a highly systematic

approach to all topics needed by a practising astronomer with detailed proofs for all necessary calculations. Ibn Yūnus computes his trigonometric and spherical astronomical tables to sexagesimal seconds or thirds and for the most important functions provides values for every 10 minutes of arc. A small but historically very relevant part of the *Ḥākimī Zīj* was edited and translated into French in Caussin de Perceval, 'Le livre de la grande table hakémite' as early as 1804. A full study of Ibn Yūnus's trigonometry and spherical astronomy, with results of recomputations of the tables, can be found in David A. King's doctoral dissertation *The Astronomical Works of Ibn Yūnus*, which unfortunately remains unpublished. Other aspects of Ibn Yūnus's work, among them especially his careful records of earlier observations and parameters and errors in tables, have been discussed by scholars such as Carl Schoy, Willy Hartner, Edward S. Kennedy and David A. King, and more recently by Mohammed Mozaffari.

- *Al-Qānūn al-Masʿūdī* (Ghazna, *c.* 1035) is the main mathematical-astronomical work by the polymath al-Bīrūnī (973–1048). Like Ptolemy's *Almagest*, it contains full treatments of all topics with results of historical observations, derivations of new parameters, instructions for calculations, and proofs. The work was edited in al-Bīrūnī, *al-Qānunu'l-Masʿūdī* and a detailed table of contents can be found in Kennedy, 'al-Bīrūnī's Masudic Canon'. The extensive set of trigonometric and spherical astronomical tables with values to sexagesimal thirds for every degree of arc include several types not found in other early *zīj*es, while the tables dependent on geographical latitude are for Ghazna with latitude 33;35°. As shown in this book, al-Bīrūnī copied several highly accurate tables from *al-Majisṭī* of Abū l-Wafāʾ.
- The *Fākhir Zīj* by Abū l-Ḥasan ʿAlī ibn Aḥmad al-Nasawī (*c.* 1050) is lost, but is known to have been based on the parameters of al-Battānī, possibly through the *Jāmiʿ Zīj*. Al-Nasawī was a student of Kūshyār, and wrote a commentary on the *Jāmiʿ Zīj* with the title *Kitāb al-Lāmiʿ fī amthilat al-Zīj al-Jāmiʿ* (*Book of Illustrative Examples of the Jāmiʿ Zīj*). Many calculations from this work were used as examples in Bagheri, *az-Zīj al-Jāmiʿ*, and part of the treatise was edited in the MSc thesis Zamani, *Wīrāyish, tarjama wa-sharḥ*. Some tables from al-Nasawī's own *zīj* are extant, for instance as additional tables in the Leiden manuscript of the *Jāmiʿ Zīj* (see p. 47). These can be seen to employ Kūshyār's type of displacements of the planetary equations, but not his alternative method of Ptolemaic interpolation.
- The *Zīj al-Qirānāt* by al-Rīqānī (possibly Rayy, *c.* 1090) is extant in the manuscript Paris, BnF, arabe 6913 and consists of two treatises of

sixty chapters each, the first giving instructions and the second the tables. The *zīj* makes use of al-Battānī's planetary parameters and his base meridian of Raqqa, but lays out the mean motion tables for the years 416, 436, 456, ... Yazdigird, corresponding to years in which Saturn-Jupiter conjunctions took place. The planetary equations involve Kūshyār's displacements (with a small adjustment for Mars), but his peculiar type of Ptolemaic interpolation is only applied to the Moon. The *Rīqānī Zīj* contains an unusually large set of oblique ascension tables for the climates as well as for specific localities. The materials on lunar crescent visibility in the *zīj* were studied in King, 'Some Early Islamic Tables', pp. 189–90 and 215–16.

- The *Mufrad Zīj* by Abū Jaʿfar Muḥammad ibn Ayyūb al-Ḥāsib al-Ṭabarī (Amul in northern Iran, *c.* 1100) is the earliest extant *zīj* written in Persian. The unique manuscript Cambridge, University Library, Browne O.1 appears to include a mixture of two overlapping sets of tables, one of which is easily recognisable by Kufic table headings, the use of the Maliki calendar and Kūshyār's displaced equations (but without his peculiar type of Ptolemaic interpolation). The *Mufrad Zīj* uses al-Battānī's mean motion parameters and has the same division into a theoretical (*ʿilmiyya*) and a practical (*ʿamaliyya*) part as the *Jāmiʿ Zīj*. A full study of the *Mufrad Zīj* remains a desideratum. Some methods and tables from the *zīj* were discussed in Kennedy and Hamadanizadeh, 'Applied Mathematics'; Hogendijk, 'Three Islamic Lunar Crescent Visibility Tables', and Saliba, 'Easter Computation'.
- The *Dustūr al-munajjimīn* (Alamut?, *c.* 1110) is an Ismāʿīlī *zīj* that includes a large historical section besides the usual contents of an astronomical handbook. It is extant in the sole surviving manuscript Paris, BnF, arabe 5968 (available online). It consists mostly of sections and tables taken from earlier works, including extant *zīj*es such as those by Abū l-Wafāʾ, Kūshyār and al-Bīrūnī but also several works that are now lost. The incomplete geographical table is the earliest witness of the highly accurate coordinates that were associated with the lost *Kitāb al-Aṭwāl wa-l-ʿurūḍ li-l-Furs* by the geographer-prince Abū l-Fidāʾ around the year 1300. The *Dustūr al-munajjimīn* was the topic of a DFG (German Research Foundation) project at Bonn University (cf. p. xv), of which the first results were published in Orthmann and Schmidl, *Science in the City of Fortune*. Recently a facsimile edition of the Paris manuscript was published by Mirās-e Maktoob in Tehran.
- None of the dozen extant manuscripts of the *Shāmil Zīj* (Iraq, *c.* 1240) mention the name of its author, but in one of the surviving draft versions of the *Kashf al-ẓunūn*, Ḥājjī Khalīfa associates the work with

the famous philosopher Athīr al-Dīn al-Abharī (d. *c.* 1264), who also wrote a *Mulakhkhaṣ Zīj* for the longitude of Damascus. The *Shāmil Zīj* stands in the tradition of the *ʿAlāʾī Zīj* by the observer al-Fahhād (Shirwan, north-western Iran, 1176) and likewise has 84° as its base meridian. van Dalen, 'The *Zīj-i Nāṣirī*' shows that several of the planetary equations in the *Shāmil Zīj* stem from Kūshyār's *Jāmiʿ Zīj*. Only incidental further tables from this popular *zīj* have been studied in the literature. van Dalen, 'The Geographical Table' collects the information that is currently known about the *Shāmil Zīj*, presents a table of contents of the work, and edits and analyses its geographical table.

- The *zīj* by Jamāl al-Dīn Abū l-Qāsim ibn Maḥfūẓ al-munajjim al-Baghdādī (Baghdad or Wāsiṭ, AD 1285, also referred to as the *Baghdādī Zīj*) is extant in the manuscript Paris, BnF, arabe 2486. Although significant parts of the *zīj* can be recognised as original contributions by al-Baghdādī, some of its contents was taken from, or based on, much earlier sources such as the *zīj* of Ḥabash al-Ḥāsib (see above). In my doctoral dissertation (van Dalen, *Ancient and Medieval Astronomical Tables*, Chapter 4) I systematically analysed all trigonometric and spherical astronomical tables in the *Baghdādī Zīj*; I found that some of these were taken from Kūshyār's *Jāmiʿ Zīj* and hypothesized that a set of twelve tables with values to sexagesimal thirds derived from Abū l-Wafāʾ's *al-Majisṭī* (cf. above). The innovative lunar tables from al-Baghdādī's *zīj* were studied in Jensen, 'The Lunar Theories'.
- The *Ashrafī Zīj* by Sayf-i munajjim-i Yazdī al-Kamālī was written around 1303 in Shiraz. It adopts the planetary mean motions and displaced equations from the well-known, but non-extant, *Shāhī Zīj* of Ḥusām al-Dīn al-Sālār, written in the early thirteenth century. However, in addition al-Kamālī presents the mean motions and the deviating equations from twelve earlier *zīj*es, which allow the reader to find planetary positions according to all these works. The *Ashrafī Zīj* thus includes a non-Ptolemaic table for the solar equation said to be from the *Mumtaḥan Zīj*, and it is the main source for the mean motion parameters of Ibn al-Aʿlam's *ʿAḍudī Zīj* (for both works, see above). The *Ashrafī Zīj* has been known for more than a century in the form of the beautiful manuscript Paris, Bibliothèque nationale de France, suppl. pers. 1488, which is now available online in colour. In recent years a second copy was identified in Qum, Gulpāyagānī Library, MS 64731.
- The *Jadīd Zīj* was written around the year 1365 by Ibn al-Shāṭir, *muwaqqit* and *muezzin* at the Umayyad Mosque in Damascus. The planetary tables in this *zīj* were based on the author's non-Ptolemaic planetary models with multiple epicycles, which he had explained in

his theoretical work *Nihāyat al-ṣūʾl fī taṣḥīḥ al-uṣūl*. These models were shown to be mathematically equivalent to Copernicus's models in Roberts, 'The Solar and Lunar Theory' and Kennedy and Roberts, 'The Planetary Theory', while Abbud, 'The Planetary Theory' investigated the computation of Ibn al-Shāṭir's tables. The *Jadīd Zīj* survives in around ten manuscripts, of which Oxford, Bodleian Library, Seld. A inf 30 is the oldest and was copied from an autograph. Numerous later *zīj*es were based on Ibn al-Shāṭir's work. The above-mentioned articles were reprinted together with several others and a short biography in Kennedy and Ghanem, *The Life & Work of Ibn al-Shāṭir*.

# Quick reference guide: technical concepts for the analysis of tables

This quick reference guide provides, in alphabetical order, definitions for the most important technical concepts that are repeatedly used in my analyses of Kūshyār's tables. For the underlying concepts and full descriptions of numerical and statistical methods for analysing astronomical tables, see Van Brummelen, *Mathematical Tables*, Chapter 3, pp. 28–45 and van Dalen, *Ancient and Mediaeval Astronomical Tables*, Chapters 1 and 2. For brief general notes on sexagesimal numbers and dates, see p. xvii.

**Clustering of errors**: a cluster is a group of errors of the same sign in consecutive tabular values with a recognisable drift.[1] These clusters are visible in a graph of the errors as humps. I have elsewhere referred to such clusters as 'interpolation groups', since in many cases we can prove, and in numerous other cases assume, that they stem from the use of **interpolation** between a set of (not necessarily equidistant) tabular values, which are referred to as **nodes**. Linear interpolation typically leads to clusters with errors of the same sign, and quadratic interpolation to error clusters with alternating signs.

**Distributed linear interpolation**: in this type of linear interpolation the tabular differences between two interpolation nodes are not distributed evenly, but the smaller differences are placed together at the beginning and the larger differences at the end of each interval, or the other way around. For example, instead of distributing a total difference of 12 units evenly as 2, 3, 2, 3, 2, it is distributed as 2, 2, 2, 3, 3 or 3, 3, 2, 2, 2, depending on the curvature of the tabulated function. An even distribution corresponds graphically to correct standard rounding of the *y*-coordinates of the points on a straight line between the tabular values for consecutive nodes, while 'distributed linear interpolation' produces a particularly smooth table if the tabular differences change at a moderate pace. Ptolemy already made use of this method to compute tables in the *Handy Tables* from tables in the *Almagest*. This process was described repeatedly by Theon of Alexandria in his commentaries on the *Handy Tables*, both for the planetary equations[2] and for other tables.[3] See Mercier, *Ptolemy's*

[1] Van Brummelen, *Mathematical Tables*, p. 33, with numerous analyses of examples of error clustering in Ptolemy's tables throughout the work.

[2] Tihon, *Le «Grand Commentaire»*, pp. 60–61, etc.

[3] cf. Van Brummelen, *Mathematical Tables*, pp. 13–14.

*Handy Tables 1b*, pp. 84–89 for a more extensive explanation of this same type of interpolation, which he calls 'stepwise interpolation'.

**Error**: the error in a tabular value is always calculated as 'tabular value minus recomputed value'. A tabular value is said to be **correct** (for a specified recomputation or reconstruction) if it has no error, i.e., it is equal to the recomputed or reconstructed value when the latter is rounded to the precision of the table.

**Homothetic tables**: this technique was first identified and named by Carlos Dorce in his analysis of the tables in the *Tāj al-azyāj*, a *zīj* written by Muḥyī l-Dīn al-Maghribī in Damascus in 1258 before he became employed at the Maragha observatory.[4] Dorce found that al-Maghribī adjusted several tables of planetary equations from earlier *zīj*es to his newly observed maximum equations by multiplying the original tables by the quotient of his new maximum value and the original one. This procedure produces equations with shifted maximum values and thus leads to a peculiar error pattern with respect to correctly computed equations in which all values before the maximum are too high and those after the maximum too low, or the other way around. In this book it is shown that Kūshyār not only computed several individual tables by means of this technique, but even used it as the basis for his alternative method for computing the planetary equation of anomaly.

**Least number of errors criterion**: this method finds the optimal parameter of a table by determining the interval of parameter values for which a table has the minimum possible number of errors. It works particularly well for accurately computed tables, such as mean motion tables. The method was introduced by the present author in his doctoral dissertation.[5] A related method for establishing mean motion parameters was presented by Honorino Mielgo.[6]

**Least squares**: this method determines the parameter(s) underlying an astronomical table by finding the values that minimise the sum of the squares of the residuals (i.e., the differences between the tabular values and an unrounded exact recomputation). There are several procedures for finding this minimum value, but for most types of astronomical tables included in *zīj*es I found that the method of Gauss-Newton works efficiently. A least squares estimation will produce a confidence interval for the estimated parameter (or a confidence area for multiple estimated parameters), in which the parameters underlying the table are expected to fall in 95% of the cases. See also: standard deviation.

4 Dorce, 'The *Tāj al-azyāj*', pp. 199–205.

5 See van Dalen, *Ancient and Mediaeval Astronomical Tables*, pp. 60–62 and the extensive examples of applications in van Dalen, 'Origin of the Mean Motion Tables'.

6 Mielgo, 'A Method of Analysis'.

**Mean error**: a measure for judging the correctness of a recomputation of a table. It is found by taking the average of all errors. A clearly positive or negative mean error (with respect to the precision of the table) is an indication that the recomputation generally produces values that are respectively too small or too large. Note that in tables with symmetries (i.e., in which a set of values follows by a simple linear relation from another set, such as $\sin(180° - x) = \sin x$, the errors will usually cancel out, making the mean error zero. Therefore, any symmetrical values should be omitted from the calculation of the mean error, or the sign of their errors adjusted.

**Rounding**: the process of reducing an exactly calculated tabular value to the number of digits of the table. The default type of rounding is standard (or modern) rounding: sexagesimal digits 30 and higher are rounded upwards and digits 29 and lower are rounded downwards. In some cases the possibility of **truncation** may be considered: that is, all digits beyond the precision of the table are simply discarded. I will speak of a **minor rounding error** if a recomputed value differs by at most $\frac{1}{120}$ of a unit from half a unit and as a result is rounded to the 'wrong side' of the half unit. In such cases the recomputed value would be correctly rounded to the tabular value if it were first accurately calculated and rounded to one additional digit and only then rounded to the precision of the table. For example, a tabular value 23;33 for a recomputed value 23;32,29,30 has a minor rounding error, since the latter is correctly rounded to 23;32, but if it were first found to seconds as 23;32,30, it would then be rounded to the precision of the table as 23;33. For practical examples, see footnote 57 on p. 374.

**Standard deviation**: a measure for judging the correctness of a recomputation of a table. It is found by calculating the square root of the sum of the squares of the differences between the tabular values and unrounded recomputed values (i.e., **residuals**) divided by the total number of tabular values (or, for some statistical purposes, one less). The errors of a correctly computed table are expected to have a standard deviation of approximately $\frac{17}{60}$ units of the table; e.g., if a table has values to minutes the standard deviation for an exact recomputation will be around 17″. This is also a criterion for judging the appropriateness of the result of a least squares estimation. If the minimum obtainable standard deviation in such an estimation is significantly larger than $\frac{17}{60}$ units, the estimation is unlikely to have produced a valid result, either because of the presence of outliers or because an incorrect algorithm was used to recompute the table.

**Tabular differences** (of first order): the differences of consecutive tabular values. Second-order tabular differences are the differences of the first-order differ-

ences. Tabular differences can be used to find irregularities (such as scribal or computational errors or nodes that were used for interpolation) in tables of regular, continuous functions. In this book it is shown that Kūshyār smoothened several tables that he took from Ptolemy or al-Battānī by adjusting the tabular values in such a way that their second-order differences became nearly constant. Note that linear interpolation in a table can be recognised from jumps in the first-order differences at interpolation nodes. Tabular differences will be called '**nearly constant**' whenever they differ by at most one unit on a given interval. For example, if four values are determined by means of linear interpolation between the nodal values 0;10 and 0;22, the resulting tabular differences 2, 3, 2, 3, 2 (expressed in minutes) are called 'nearly constant'. This implies that all linear tables (such as the subtables of mean motion tables for periods with fixed lengths) will be said to have 'nearly constant' tabular differences.

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# Glossary

The technical terms from the *Jāmiʿ Zīj* are given in transliterated form in the second column. They are sorted by their Arabic root as displayed in the first column. The abbreviation 'etc.' indicates the presence of further occurrences of the same term in similar types of tables. For the zodiacal signs and month names in the Syrian, Arabic and Persian calendars, see the general editions on pp. 265–69. The names of the planets and other terms commonly used in the tables and partially abbreviated in the edition are listed on p. 257.

| | | |
|---|---|---|
| أخذ | I *akhadha* | to take (a value from a table) — 270-74, 294-95, 331-34, 357, 431 |
| أرخ | *taʾrīkh* | era, epoch — 271-73, 280, 282, 284 |
| أرض | *arḍ* | Earth — 323 |
| أزي | *(bi-)izāʾ* | opposite, next to (said of tabular values with respect to the argument for which they are found) — 331-34, 357, 465 |
| أصل | *aṣl* pl. *uṣūl* | epoch value (lit. 'root', Latin: *radix*) — 280, 284, 286, 289;<br>the original — 331-32 |
| | *al-taʿādīl al-aṣliyya* | the original (i.e., Ptolemaic, undisplaced) equations — 260-62, 331, 518 |
| أندرجة | *Andarja* | *epagomenae* in the Persian calendar — 340 |
| أوج | *awj* | apogee — 282, 283, 285, 290, 292, 293, 294-95, 301 |
| أول | *awwal* | first sexagesimal position (i.e., unit) — 270-72, 283, 342 |
| برج | *burj* pl. *burūj* | zodiacal sign, sign, 30° — 280, 288, 294-95, 315-17, 319-20, 324<br>see also: *falak al-burūj*, *maṭāliʿ al-burūj* |
| برهن | *burhān* | proof — 334 |
| بسط | *mabsūṭ* | extended; see: *sinūn mabsūṭa* |
| بعد | *buʿd* | distance — 323;<br>elongation (distance between the Sun and the Moon) — 322<br>see also: *juzʾ al-buʿd*, *sāʿāt al-buʿd* |
| | *buʿd awsaṭ* | mean distance — 296-98, 300, 302 |
| | *buʿd aqrab* | nearest distance — 296-98, 300, 302, 304-05, 332 |
| | *buʿd abʿad* | furthest distance — 296-98, 300, 302, 304-05, 332 |
| | *buʿd bayn al-nayyirayn* | 'distance between the two luminaries', elongation — 322, 464-66 |

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|---|---|---|
| | *buʿd muḍāʿaf* | double elongation (also: *buʿd muḍaʿʿaf*) — 257, 285, 297 |
| بقي | I *baqiya* | to remain — 285, 293, 315-16, 331-32 |
| | *bāqī* | remainder (of a subtraction) — 273, 274, 285, 322, 331-34 |
| | *baqiyya* | remainder (of a table) — 287 n. 6, 308 n. 1 |
| بلد | *balad* pl. *buldān* | city, town, locality — 287 n. 5, 288, 292, 319, 324 |
| بلغ | I *balagha* | to reach, to amount to — 292, 319 |
| | *mablagh* | amount — 284 |
| بهت | *buht* | velocity — 46, 322, 465 |
| | *buht al-qamar* | lunar velocity — 322, 464, 465 |
| بوب | *bāb* pl. *abwāb* | chapter — 262-63;<br>pl. elementary quantities — 262, 276 |
| بيت | *bayt* pl. *buyūt* | (astrological) house — 46 |
| تمم | *tāmm* | complete(d) (years, months, days) — 270-72 |
| | *tamām* | complement, supplement (of an arc) — 275, 276, 277, 323 |
| | *tamām al-irtifāʿ* | complement of the altitude, zenith distance — 323 |
| ثبت | *thābita* | fixed (stars) — 325, 327-29<br>see also: *kawākib thābita* |
| | *muthbat* | what has been recorded / laid down (in a table) — 285 n. 7 |
| ثلث | *thālith* pl. *thawālith* | third (sexagesimal fractional position) — 257, 280, 294, 357;<br>third (power of 60) — 257, 270 etc., 283, 342;<br>third (stellar magnitude) — 327-29 |
| ثني | *thānī* pl. *thawānī* | second (i.e., second sexagesimal fractional position) — 257, 275-76, 280, 288, 292, 293-96, 314, 318-23, 357, 465;<br>second (power of 60) — 257, 270-72, 283, 342;<br>second (stellar magnitude) — 327-29 |
| جدول | *jadwal* pl. *jadāwil* | table — 25, 29, 46, 259 etc., 275, 284, 285 n. 7, 286, 287 n. 1 and n. 4, 292 n. 1 and n. 3, 295 n. 7, 321 n. 3, 322 n. 1, 323 n. 1, 324 n. 6, 330, 331, 334, 357, 358, 431, 451-52, 465, 485;<br>column of a table — 281, 465 |
| جزأ | *juzʾ* pl. *ajzāʾ* | 'part', (ecliptic) degree — 60, 298, 300 etc., 318 n. 8 and n. 11, 321, 357;<br>part (unit of the length of a gnomon and its shadow, used for tangents) — 257, 276, 278-79, 319 |

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|---|---|---|
| | *juzʾī* | partial — 413 |
| | *juzʾ al-buʿd* | 'part of the distance', i.e., the estimated distance till the true conjunction or opposition of the Sun and the Moon in the calculation of a syzygy — 322, 464-66, 475, 476 |
| | *ajzāʾ al-sāʿāt* | 'parts of the hours', equatorial degrees corresponding to a seasonal hour — 45 |
| | *ajzāʾ al-sawāʾ* | 'equal parts', equatorial degrees — 319 n. 3 |
| جزر | *al-jazāʾir al-khālidāt* | the Fortunate Isles (Canaries), base meridian of Kūshyār's and numerous other Islamic geographical tables — 285 n. 7, 286, 324 |
| جمع | *ijtimāʿ* | conjunction (of the Sun and the Moon) — 298, 322, 323, 475 |
| | *majmūʿ* | collected; see: *sinūn majmūʿa* |
| جنب | *janūb* | south — 314-16, 318, 325, 326, 512 |
| جوزهر | *al-jawzahar* | lunar ascending node — 257, 280, 290, 299 |
| جيب | *jayb* pl. *juyūb* | sine — 275, 276, 319, 357, 358, 359 |
| حرف | *inḥirāf* | slant (one of the three components of the latitude of the inferior planets) — 52, 316 |
| | *inḥirāf ʿan samt al-raʾs* | zenith distance — 50 |
| حرك | *ḥaraka* | motion — 280, 284, 292, 293 |
| حشو | *ḥāshiya* | marginal note, gloss — 30, 272, 283 n. 1 |
| حصص | *ḥiṣṣat al-ʿarḍ* | 'share of the latitude', latitude argument, distance between the ascending lunar node and the true position of the moon — 314, 441 |
| | *ḥiṣaṣ al-ʿarḍ* | 'shares of the latitude': in **$CC_1Y$** used for the latitude interpolation function — 316 |
| | *ḥissa daqīqa* | 'share ⟨in⟩ minute⟨s⟩', interpolation coefficient — 297 n. 10, 314 |
| | *ḥissa daraja* | 'share ⟨in⟩ degree⟨s⟩', interpolation coefficient, tabular difference (especially in **L**) — 276 n. 3, 296 n. 1, 297, 298, 300, 301, 318 n. 9 and n. 12, 360 |
| | *ḥissat al-daraja* | 'share of the degree', interpolation coefficient — 60, 294, 296 n. 1, 393 |
| حطط | *munḥaṭṭ* | 'lowered', i.e., with the sexagesimal point shifted one position to the left, divided by 60 (often needed in spherical-astronomical calculations due to the use of trigonometric functions to base 60) — 319, 322 |
| حقق | *ḥaqīqī* | actual (said of planetary positions to indicate the original, non-displaced positions) — 443, 518<br>see also: *markaz muʿaddal ḥaqīqī* |
| حكم | *aḥkām* | astrology — 484 |

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|---|---|---|
| خسف | *khusūf* | lunar eclipse — 287 n. 10, 296 n. 6, 323 |
| خصص | *khāṣṣa* | anomaly (of a planet), anomalistic position or motion — 60, 257, 280, 283 n. 1 and n. 6, 285, 295, 296, 300, 306 etc., 321, 331 n. 5 etc., 431 |
| | *khāṣṣa muʿaddala* | 'corrected anomaly', true anomaly — 321, 431 |
| خطط | *khaṭṭ al-istiwāʾ* | equator — 319, 324; see also: *maṭāliʿ al-burūj bi-khaṭṭ al-istiwāʾ* |
| خفي | I *khafiya* | to disappear, to become invisible (of a planet) — 303 |
| خلف | *ikhtilāf* | 'difference', variation (of the equation of anomaly) — 47, 298, 299, 304-05, 307 etc., 331-34, 404, 410, 412, 415 |
| | *ikhtilāf kullī* | 'total variation' — 412 |
| | *ikhtilāf al-manẓar* | parallax — 323 |
| | *ikhtilāf niṣf quṭr falak al-tadwīr* | variation of the epicycle radius — 427 |
| خمس | *khāmis* pl. *khawāmis* | fifth (sexagesimal fractional position) — 257, 280 |
| دخل | *dakhala* | to enter (a table) — 357 |
| | *madkhal* pl. *madākhil* | *nota*, weekday of a given date — 273, 274, 344 |
| درج | *daraja* pl. *daraj*, *darajāt* | degrees — 60, 257, 275, 280, 286, 288, 289, 293, 331 etc. |
| | *daraj(āt) al-sawāʾ* | 'equal degrees', ecliptical degrees (argument of right and oblique ascension tables) — 319, 320 |
| دريجان | *darījān* | (astrological concept) — 45, 47 |
| دقق | *daqīqa* pl. *daqāʾiq* | minute — 168 n. 0, 257, 275-76, 278, 280, 288, 293, 294-96, 297-99, 300-01, 304-05, 310, 314-16, 318-25, 357, 379, 465 |
| | *daqāʾiq al-ḥiṣaṣ* | 'minutes of the shares', interpolation minutes — 315, 316 see also: *ḥiṣṣa daqīqa* |
| | *daqāʾiq al-nisab* | 'minutes of the proportions', interpolation function — 299, 304-05, 307 etc., 310, 331, 412, 431 see also: *tafāḍul daqāʾiq* |
| دور | *dawr* pl. *adwār* | rotation, full circle, 360° — 56, 63, 257, 280, 284, 286, 315, 316, 332 n. 1 and n. 2, 519 |
| | *tadwīr* | epicycle; epicycle position, true anomaly, true anomalistic position — 298, 299, 301, 304, 305, 310, 321 n. 8 and n. 9, 323, 433 see also: *falak al-tadwīr* |
| ذو القرنين | *Dhū l-qarnayn* | the Two-Horned (Alexander the Great; epoch of the Byzantine or Syrian calendar, also called the Seleucid Era) — 282, 284, 293, 341 n. 2, 506 |

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|---|---|---|
| رأي | I pass. *yurā* | to be(come) visible (of a planet) — 302-03 |
| | *ruʾya* | visibility — 323 |
| | *ruʾyat al-ahilla* | visibility of the lunar crescent — 323 |
| ربع | *rābiʿ* pl. *rawābiʿ* | fourth (sexagesimal fractional position) — 257, 280; fourth (power of 60) — 257, 270 etc., 342 |
| رجع | *rājiʿ* | retrograde — 302 |
| رصد | *raṣad* pl. *arṣād* | observation — 282 n. 5, 284 |
| رفع | *irtifāʿ* | altitude — 278, 323 |
| ركز | *markaz* pl. *marākiz* | centre (of the Earth) — 263; centrum (of a planet, i.e., its angular distance from the apogee, measured in degrees) — 60, 300, 301, 306 n. 7, 307 n. 3 etc. |
| | *markaz muʿaddal* | 'corrected centrum', true centrum — 304-05, 310, 315-16, 317, 332 n. 5, 412, 431 |
| | *markaz muʿaddal ḥaqīqī* | 'actual corrected centrum', actual true centrum — 443 |
| روم | *al-rūm* | the Romans, the Byzantines, the Byzantine empire — 271 |
| | *rūmī* | Byzantine (calendar) — 266, 270 n. 2, 273 n. 3, 280, 341 n. 2 |
| زحل | *zuḥal* | Saturn |
| زهر | *al-zuhara* | Venus |
| زيج | *zīj* pl. *zījāt*, *azyāj* | *zīj*, astronomical handbook with tables — 4, 276 n. 8, 277 n. 4, 280, 282, 292, 293 |
| زيد | I *zāda* | to add, to increase — 278-79, 284, 292, 299, 300, 301, 304, 305 etc., 315-16, 317, 357 |
| | *zāʾid* | additive (indicating that an equation or other amount must be added to a given quantity) — 288, 296, 297, 332, 372, 400; increasing (in absolute value, said of a latitude) — 314 |
| | *zāʾid* pl. *zawāʾid* | benefic ('luck-bringing') star — 326, 515-16 |
| | *ziyāda* | increase, addition — 270, 289, 294, 299 n. 5, 304-05, 332 n. 1 and n. 4 etc., 373 |
| | *tazāyud* | (gradual) increase, increment — 322, 476 |
| | *tazāyud juzʾ al-buʿd* | 'increase of the part of the distance' — 476 |
| | *tazāyud sāʿāt al-buʿd* | 'increase of the hours of the distance' — 476 |
| سبع | *sābiʿ* pl. *sawābiʿ* | seventh (sexagesimal fractional position) — 281 |
| سبق | *sabq al-qamar* | lunar precedence ('lunar gain', relative lunar velocity, the excess of the lunar over the solar velocity) — 466 |
| سحب | *saḥābī* | 'cloudy', nebulous (said of a star) — 327-29, 514 |

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| سحل | *sāḥil al-baḥr* | the 'coast of the sea' (also: African shore, western shore, base meridian of the geographical coordinates of al-Maʾmūn and numerous later Islamic geographical tables) — 286 |
| سدس | *sādis* pl. *sawādis* | sixth (sexagesimal fractional position) — 257, 280 |
| سرق | *al-mustaraqa* | the 'stolen ⟨days⟩', *Andarja* (the five epagomenal days in the Persian calendar) — 272, 274, 289, 340, 485 |
| سرياني | *suryānī* | Syrian (calendar) — 266, 270, 273, 280, 283, 284, 341 n. 2 |
| سطر | *saṭr* pl. *suṭūr* | column, row — 315, 316, 317, 318 n. 4 |
| | *saṭr al-ʿadad* | argument column — 315, 316, 317 |
| | *fī mā bayn al-saṭrayn* | interpolation between consecutive tabular values |
| سقط | *saqaṭa* | to drop out — 273, 274 |
| سمت | *samt* | azimuth |
| | *samt al-raʾs* | zenith; see: *inḥirāf ʿan samt al-raʾs* |
| سنة | *sana* | year — 270, 273–74, 280, 282-84, 287, 293, 296 n. 4 and n. 9, 325 etc. |
| | *sana shamsiyya* | solar year — 324 |
| | *sinūn mabsūṭa* | extended years — 270 etc., 287, 341, 371 and n. 52 |
| | *sinūn majmūʿa* | collected years — 270 etc., 287, 292, 341, 371 |
| | *sinūn mufrada* | 'single' years (i.e., multiples of 20 or 100 years) — 287, 371 and n. 53 |
| سهم | *sahm* | versed sine (lit. 'arrow') — 275, 359; (astrological) lot — 45 |
| سوع | *sāʿa* pl. *sāʿāt* | hour — 45, 287, 288, 292, 295, 296 n. 7 etc., 321, 322, 372 |
| | *sāʿāt* | 'hours', time (of the day) — 294, 475 |
| | *sāʿāt al-buʿd* | 'hours of the distance', i.e., the estimated time till the moment of a true conjunction or opposition of the Sun and the Moon — 322, 464-66, 474, 476 |
| | *sāʿa mustawiyya* | equal hour (a twenty-fourth of a day) — 321 |
| | *sāʿa zamāniyya* | seasonal hour (a twelfth of the time between sunrise and sunset) — 321 |
| سوي | *mustawī* | straight (said of the second tangent) — 278; even, equal, regular; see: *sāʿa mustawiyya* |
| | *taswiya (al-buyūt)* | 'equalisation', calculation of the astrological houses — 46 |
| | *sawāʾ* | equality; see: *daraj al-sawāʾ*, *ajzāʾ al-sawaʾ* |
| سير | *tasyīr* pl. *tasyīrāt* | *tasyīr*, prorogation, progression (astrological concept) — 324, 484, 515 |
| | *tasyīr ṣughrā* | small prorogation — 484 |

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| | *tasyīr wusṭā* | middle prorogation — 484 |
| | *masīr* | travel, distance — 321 |
| شخص | *shakhṣ* | gnomon — 279 |
| شرق | *mashriq* | east — 303 |
| شرك | *mushārik* | partner (planet) — 36 |
| شري | *mushtarī* | Jupiter |
| شمس | *al-shams* | the Sun |
| | *shamsī* | solar — 324 |
| شمل | *shamāl* | north — 314-16, 318, 326, 512 |
| شهر | *shahr* pl. *shuhūr* | month — 49, 270 etc., 273 etc., 287-88, 293, 296 n. 4 etc., 324, 341, 371, 485 |
| صبع | *iṣbaʿ* pl. *aṣābiʿ* | finger, digit (unit for measuring the length of the shadow of a gnomon of length 12) — 257, 278; digit (for the magnitude of an eclipse) — 323, 482 |
| | *aṣābiʿ muṭlaqa* | 'absolute digits' (linear digits, for the magnitude of an eclipse measured in twelfths of the diameter of the eclipsed body) — 482 |
| | *aṣābiʿ muʿaddala* | 'corrected digits' (for the magnitude of an eclipse measured in twelfths of the area of the eclipsed body) — 323, 482 |
| صحح | *ṣaḥīḥ* pl. *ṣaḥāḥ* | whole (number) — 357 |
| صفر | *ṣifr* | zero, nothing — 298 n. 9, 332 n. 2 |
| صعد | *ṣāʿid* | ascending (changing in northern direction, said of a latitude) — 314 |
| صلح | *iṣlāḥ* | correction — 310, 431, 438 |
| | *iṣlāḥ taqwīm al-mirrīkh* | correction of the true position of Mars — 55, 310, 430 |
| صور | *ṣūra* pl. *ṣuwar* | (stellar) constellation — 325, 327-28, 330 |
| | *al-ṣuwar al-shamāliyya* | the northern constellations — 325, 327 |
| | *al-ṣuwar al-janūbiyya* | the southern constellations — 325, 326, 328 |
| | *ṣuwar al-mintaqa* | the ecliptic constellations — 325, 327 |
| صوم | *al-ṣawm al-kabīr* | the Great Lent — 347 n. 11 |
| | *ṣawm al-naṣārā* | Fast of the Christians, Lent — 274 |
| ضرب | *ḍaraba* | to multiply — 276, 277, 284, 299, 304, 305, 319, 321, 431 |
| | *ḍarb* | multiplication |
| ضعف | II *ḍaʿʿafa* | to double — 259 |
| | III *ḍāʿafa* | to double — 286 |
| | *al-muḍāʿaf* | double elongation — 297, 298, 323 see also: *buʿd muḍāʿaf* |

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| ضوأ | *muḍīʾ* | bright, bright star — 327-29, 513 |
| ضيف | IV *aḍāfa* | to add — 294, 357 |
| طلع | *ṭāliʿ* pl. *ṭawāliʿ* | ascendant — 325 |
| | *maṭaliʿ al-burūj* | ascensions (i.e., rising times) of the zodiacal signs — 319-20 |
| | *maṭāliʿ al-burūj bi-khaṭṭ al-istiwāʾ* | 'ascensions at the equator', right ascensions — 319 |
| | *maṭāliʿ li-ʿarḍ* ... | 'ascensions for latitude ...', oblique ascensions — 320 |
| | *maṭāliʿ al-sinīn* | 'ascensions of the year' — 45 |
| | see also: *fuḍūl al-maṭāliʿ* | |
| طوف | *ṭūfān* | flood (cf. Index of Subjects, Era of the Flood) — 271 |
| طول | *ṭūl* pl. *aṭwāl* | (geographical) longitude — 280, 285 n. 7, 286, 288, 292, 324; longitude (of a fixed star) — 325 see also: *wasaṭ al-ṭūl*; length (of a page) — 357; length (of a table), vertical argument — 274, 346, 485 |
| | *fī mā bayn al-aṭwāl / fī mā bayn al-ṭūlayn* | 'longitude difference', longitude correction (difference in mean position due to the difference in geographical longitude, i.e., the correction that needs to be applied to use a given mean motion table for a locality with a longitude different from the base meridian of the table) — 51, 287-88, 292 n. 4 etc., 372 |
| ظلل | *ẓill* | 'shadow' (of a gnomon), cotangent — 276–79, 319; shadow (of the Earth, in the calculation of an eclipse) — 321 |
| | *al-ẓill al-awwal* | first tangent (i.e., the modern tangent) — 276 |
| | *al-ẓill al-thānī* | second tangent, shadow (i.e., the modern cotangent) — 277, 278 |
| | *ẓill al-mayl al-awwal* | tangent of the solar declination — 319, 453 |
| ظهر | *ẓuhr* | midday prayer — 278 |
| عدد | *ʿadad* pl. *aʿdād* | number — 277, 317; argument (of a table) — 315-17, 318 see also: *saṭr al-ʿadad*; magnitude (of a star) — 46 |
| عدل | II *ʿaddala* | to equate (said of a planetary position) — 301, 307 n. 3, 308 n. 8 etc. |
| | *muʿaddal* | corrected, true (said of a planetary position and certain other quantities); see: *aṣābiʿ muʿaddala*, *khāṣṣa muʿaddala*, *markaz muʿaddal* |

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| | *taʿdīl* pl. *taʿādīl* | 'equation' — 294, 321, 323, 331-34, 391 n. 72, 413, 431, 518 |
| | *taʿdīl awwal* | first equation, equation of centre (of the moon or a planet) — 297, 300, 306 etc., 331-33 |
| | *taʿdīl thānī* | second equation, equation of anomaly (of the moon or a planet) — 298, 299, 301, 304-05, 307 etc., 331-34 |
| | *taʿdīl ajzāʾ al-sāʿāt al-zamāniyya* | 'equation of the parts of seasonal hours' (i.e., the difference between the length of a seasonal hour (a twelfth of the length of daylight) and an equal hour (i.e., a twenty-fourth of a day and night) — 321 |
| | *taʿdīl al-ayyām bi-layālīhā* | equation of time — 287 n. 8, 294-95, 296 n. 8, 393 |
| | *taʿdīl al-nahār* | equation of daylight — 319, 320-21 |
| | *taʿdīl al-sāʿāt al-mustawiyya* | 'equation of the equal hours' (i.e., the excess of the length of the day over 12 hours, expressed in equal hours) — 321 |
| | *taʿdīl al-shams* | solar equation — 295, 399 |
| عرب | *ʿarabī* | Arabic (calendar, era, years) — 271, 273, 280 |
| عرض | *ʿarḍ* | latitude (of the Moon, planet or fixed star) — 314-16, 325<br>see also: *ḥiṣṣat al-ʿarḍ*, *ḥiṣaṣ al-ʿarḍ*, *wasaṭ al-ʿarḍ*;<br>geographical latitude — 319, 320-21, 324;<br>width (of a page) — 357;<br>width (of a table), horizontal argument — 274, 346, 485 |
| عصر | *ʿaṣr* | afternoon prayer — 278-79 |
| عطارد | *ʿuṭārid* | Mercury |
| عظم | *ʿiẓam* | magnitude (of a star) — 325, 328 |
| عكس | *maʿkūs* | reversed (said of the first tangent) — 276 |
| علم | *ʿalāma* | *nota*, weekday of a given data — 344<br>see also: *madkhal* |
| | *ʿilmī* | 'on knowledge', theoretical — 10, 334, 525 |
| عمل | I *ʿamila* | to operate, to calculate, to carry out, to act (used in particular for the calculation of quantities by means of algorithms) |
| | *ʿamal* | 'operation' — 259, 357 |
| | *ʿamalī* | 'on operations', practical — 10, 334, 525 |
| عين | *bi-ʿaynihī* | 'with itself', current/incomplete (years, months, days) — 273 n. 7 and n. 24, 284, 289, 345 |
| غرب | *maghrib* | west — 286, 302-03 |

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| غمم | *ghamāmī* | 'cloudy', nebulous (said of a star) — 514 |
| فرد | *mufrad* | single; see: *sinūn mufrada* |
| فرس | *fārisī* | Persian (calendar, era, years) — 271-72, 274, 280, 284, 293 |
| فضل | *faḍl* | excess — 412 |
| | *fuḍūl al-maṭāliʿ* | 'excesses of the ascensions', tangent of declination — 453 |
| | *tafāḍul* | (tabular) differences — 60, 275, 276, 296-98, 300-01, 318, 360 |
| | *tafāḍul daqāʾiq* | (tabular) differences (in) minutes — 297-98, 300-01 |
| فلك | *falak* pl. *aflāk* | sphere, orb — 286 |
| | *falak al-awj* | 'sphere of the apogee', eccentric sphere, eccentric — 297 n. 14, 300 note c |
| | *falak al-burūj* | ecliptic — 318 n. 8 and n. 11, 319, 321 |
| | *falak al-tadwīr* | 'sphere of the epicycle', epicycle — 298 n. 6, 302 n. 2, 412, 427, 478<br>see also: *niṣf quṭr falak al-tadwīr* |
| فيد | *fāʾida* | 'useful lesson' — 29 |
| قبل | *istiqbāl* | opposition (of the Sun and the Moon) — 298, 322, 323, 475 |
| قدر | *qadr* pl. *aqdār* | magnitude (of a star) — 325, 329 |
| قدم | *qadam* pl. *aqdām* | foot (unit for measuring the length of the shadow of a gnomon of length 7) — 46, 257, 278 |
| قرب | *taqrīb* | 'approximation' (here apparently in the sense of the actual quantity being replaced by something that differs from it by a small step) — 261, 331 n. 1 |
| | *bi-l-taqrīb* | approximately — 282, 293, 322 |
| قسم | I *qasama* | to divide — 273, 274, 276-77, 284, 292, 322 |
| | *qisma* | division (Persian: *qismat*) — 292;<br>(point of) 'division', allotment (astrological concept) — 46 |
| | *qāsim* | 'divisor' (astrological concept) — 515 n. 249 |
| قطر | *quṭr* | diameter (of the solar and lunar disks and the shadow of the Earth, for the calculation of eclipses) — 321<br>see also: *niṣf quṭr falak al-tadwīr* |
| قطع | *qāṭiʿ* | 'cutter', fatal or malefic star — 69, 326, 513, 515 and n. 249 |
| قلب | *qalb al-asad* | 'Heart of the Lion', Regulus — 283, 328, 329 |
| قلم | *iqlīm* pl. *aqālīm* | climate (the seven zones of latitudes in which Ptolemy divided the inhabited world) — 324 |
| قمر | *qamar* | the Moon |

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| قوس | *qaws* | arc — 275-77, 357, 358 |
| قول | *maqāla* | treatise, 'book' — 10, 259–63, 334 |
| قوم | *taqwīm* | correction, rectification — 451-52;<br>true position (of a planet) — 25, 44, 55, 295 n. 6, 310, 430-31<br>see also: *iṣlāḥ taqwīm al-mirrīkh*;<br>calculation of the true position — 294 n. 3, 295, 317 |
| | *maqām* | (planetary) station — 315-17 |
| | *maqām awwal* | first station — 315-16 |
| | *maqām thānī* | second station — 315-16 |
| | *muqīm* | stationary — 302 |
| | *mustaqīm* | progressive — 302 |
| قيس | *miqyās* | gnomon — 276 n. 1, 278 |
| كبس | *kabīsa* | intercalation, intercalary — 270, 343 |
| كبيكج | *(yā) kabīkaj* | Oh Asiatic crowfoot! (invocation against damage by woodworm, insects etc.) — 30 |
| كتف | *katif* | shoulder (in star names) — 327-29, 513 |
| كسر | *kasr* pl. *kusūr* | part, fraction (in the case of hours: minutes, seconds, etc.) — 287-88, 296 n. 7, 357, 372, 465 |
| كسف | *kusūf* | solar eclipse, eclipse (in general) — 323 |
| | *kusufān* | solar and lunar eclipse(s) — 323 |
| كلل | *kullī* | total — 412, 413 |
| كوكب | *kawkab* pl. *kawākib* | planet, star — 25, 257, 280, 284, 285, 289, 316-17, 325, 327-29 |
| | *al-kawākib al-ʿulwiyya* | the superior planets — 285 |
| | *al-kawākib al-sufliyya* | the inferior planets |
| | *al-kawākib al-thābita* | the fixed stars — 325, 327-29 |
| كيد | *Kayd* | Kaid, the name of a comet — 25 |
| ماه | *māh* | month (Persian) — 268-69 |
| محن | *mumtaḥan* | 'verified' — 5 |
| مرخ | *mirrīkh* | Mars |
| مزج | *mizāj* pl. *mizājāt* | temperament (of a fixed star) — 325, 511 |
| مكث | *makth* | duration of the stay of the fetus in the mother's womb — 51 |
| ميل | *mayl* | declination (orthogonal distance from the equator of a point on the ecliptic) — 318, 453;<br>deviation (one of the three components of the latitude of the inferior planets) — 52, 316 |
| | *al-mayl al-awwal* | first declination, solar declination — 318-19<br>see also: *ẓill al-mayl al-awwal* |
| | *al-mayl al-thānī* | second declination — 318 |

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| نسخ | *nuskha* | manuscript, version, copy (also as siglum) — 21, 30, 168 n. 0, 171 n. 0, 277 n. 3, 332 n. 2 and n. 5, 334, 379-80 |
| نصف | *niṣf al-nahār* | noon — 278-79, 280, 284 |
| | *niṣf quṭr falak al-tadwīr* | radius of the epicycle — 412, 427, 478<br>see also: *ikhtilāf niṣf quṭr falak al-tadwīr* |
| | *niṣf quṭr falak al-tadwīr al-muʿaddal* | 'adjusted radius of the epicycle' — 478 |
| نطق | *minṭaqa* | belt, (short for: *mintaqat al-burūj*) ecliptic |
| نظر | *manẓar* | sight, appearance; see: *ikhtilāf al-manẓar* |
| نقص | *naqaṣa* | to subtract — 285, 293, 294, 295, 299, 300, 304-05, 306 n. 7 etc., 315-16, 322, 331-34, 431 |
| | *nuqṣān* | decrease, decrement, subtraction — 289, 292, 295, 304, 331 n. 2 and n. 5, 332 n. 1, 333 notes 1, 5, 9 and 15, 394 |
| | *nāqiṣ* | subtractive (indicating that an equation or other amount must be subtracted from a given quantity) — 288, 332, 372;<br>decreasing (said of an equation or other correction) — 296, 298, 400;<br>decreasing (in absolute value, said of a latitude) — 314, 400;<br>current, incomplete (years, months, days) — 270, 273, 274, 345 (cf. *bi-ʿaynihi*) |
| نكب | *mankib* | shoulder (in star names) — 327, 329, 513 |
| نهي | *intihāʾ* | 'terminal point' (astrological concept) — 35 |
| نور | *nayyir* | bright, bright star — 327, 329, 513 |
| | *al-nayyirayn* | the two luminaries, the Sun and the Moon — 294 n. 3, 295, 321, 322, 465<br>see also: *buʿd bayn al-nayyirayn* |
| نوع | *nawʿ* pl. *anwāʿ* | 'category' (used for the items in the table of contents of Book II of the *Jāmiʿ Zīj*) — 37, 259–63, 518 |
| هبط | *hābiṭ* | descending (changing in southern direction, said of a latitude) — 314 |
| هجر | *hijra* | Hijra (flight of the prophet Muḥammad from Mecca to Medina; epoch of the Arabic or Hijra lunar calendar) — 273, 284, 334 |
| هلل | *ahilla* | lunar crescent — 323<br>see also: *ruʾyat al-ahilla* |
| هيئ | *hayʾa* | cosmology, 'models', *hayʾa* — 334 |
| وجه | *jiha* pl. *jihāt* | direction (north or south, of a lunar, planetary or stellar latitude) — 46, 325 |

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| وسط | *wasaṭ* pl. *awsāṭ* | middle; mean motion, mean position — 257, 280, 285, 287, 289, 292, 294-95, 296, 299, 306 etc., 322, 331 n. 2 and n. 5, 332 n. 1, 333 notes 1, 5, 9 and 15, 394 |
| | *wasaṭ al-ʿarḍ* | ‘middle of the latitude’ (of a lunar mansion) — 46 |
| | *wasaṭ al-ṭūl* | ‘middle of the longitude’ (of a lunar mansion) — 46 |
| وضع | *waḍʿ al-taʿādīl* | displacement of the equations — 518 |
| | *mawḍiʿ* pl. *mawāḍiʿ* | position (of a planetary apogee, star or syzygy) — 282, 283, 293, 298, 325 |
| وقت | *waqt* pl. *awqāt* | (point in) time — 279 n. 2 and n. 5, 294 n. 3, 323 |
| | *mīqāt* | timekeeping — 29 |
| ولد | *mawlad* pl. *mawālīd* | birth, nativity — 325 |
| يزدجرد | *Yazdijird (ibn Shahriyār)* | Yazdigird III (last Persian king, r. 632–651; epoch of the Persian calendar) — 280, 282, 283, 284, 293, 325, 327-29 |
| يوم | *yawm* pl. *ayyām* | day, nychthemeron, 24-hour period — 270-73, 280, 282-84, 287-88, 293, 296 n. 4 etc., 306 n. 2 etc., 322, 324, 371, 465, 485<br>see also: *taʿdīl al-ayyām wa-layālīhā* |

# Indexes

References in bold are to the most relevant treatment of the topic concerned. A page number between square brackets indicates a discussion of the topic without explicit mention of the lemma.

**Subjects**

**T1**, **T2**, etc. indicate the numbers of the tables in the *Jāmiʿ Zīj* in which a function is tabulated. This index includes a small number of place names, but omits those that are only mentioned in the context of the geographical table (**T54**) or are indicated in the Index of Parameters, sections *Geographical latitudes* and *Geographical longitudes*. Similarly, the index does not include the names of stars or constellations that are only mentioned in the context of the star table (**T55**).

**Parameters**

This index does *not* include: Kūshyār's daily mean motions of the planets as listed in Table 12 on p. 116; Ptolemy's and Kūshyār's planetary eccentricities, maximum equations of centre, epicycle radii, and maximum central equations of anomaly as listed in Table K on p. 415; the shifts and displacements of the planetary equations as listed in Table Q on p. 520, and the longitudes and latitudes that are included in Kūshyār's geographical table (Table 54 on pp. 240-43) or are mentioned in the commentary on this table (pp. 487-510).

**Historical dates**

This index includes all dates associated with Kūshyār's life, the epochs of calendars and tables found in the *Jāmiʿ Zīj*, and the dates of copying and ownership of all manuscripts of the *zīj*, with the exception of the chronological notes by students in manuscript **C**. The corresponding Julian dates (Gregorian from 1583 onwards) in the column on the left are given in the form yyyy-mm-dd.

**Historical persons**

Further mentions of frequently cited authors in direct relation to one of their works (even if the title is not explicitly mentioned, as in 'the *zīj* of Ḥabash al-Ḥāsib') can be found through the Index of Works.

**Works**

Page numbers between square brackets are for references without explicit mention of the title (as in 'the *zīj* of al-Battānī'). Further references to each work may be found under the name of the author in the Index of Historical persons. Abbreviations such as KHU are the ones used in Kennedy and Kennedy, *Geographical Coordinates* and in the commentary on Kūshyār's geographical table in Section I.13; 'ff.' is used for the frequent mention of geographical sources in the description of place names and their coordinate traditions on pp. 492–506.

**Manuscripts**

Capitalised three-letter abbreviations between parentheses are those used for the geographical table in the manuscript concerned in Kennedy and Kennedy, *Geographical Coordinates*.

**Modern persons**

**Projects and institutions**

# Colour plates

Plate 1: Title page of the *Jāmiʿ Zīj* (Istanbul, Süleymaniye Kütüphanesi, Fatih 3418, fol. 1r)

a) 

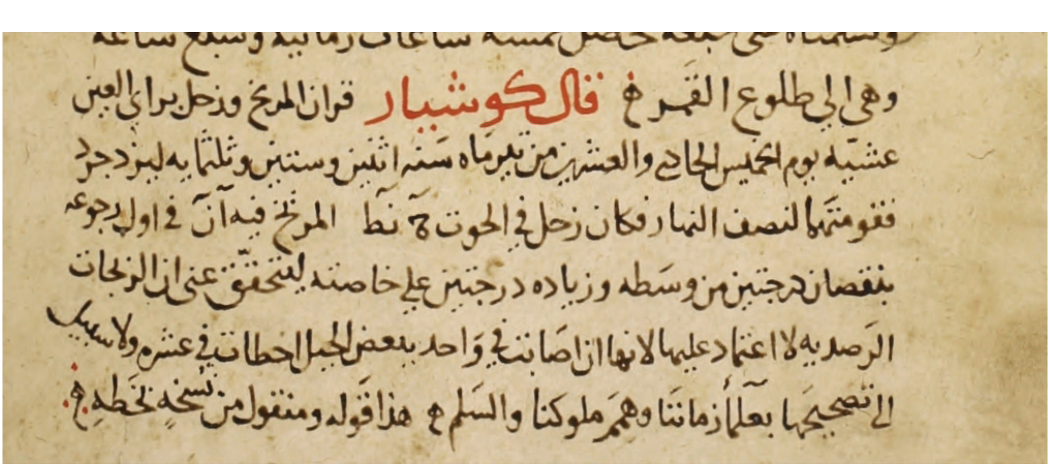
b) 

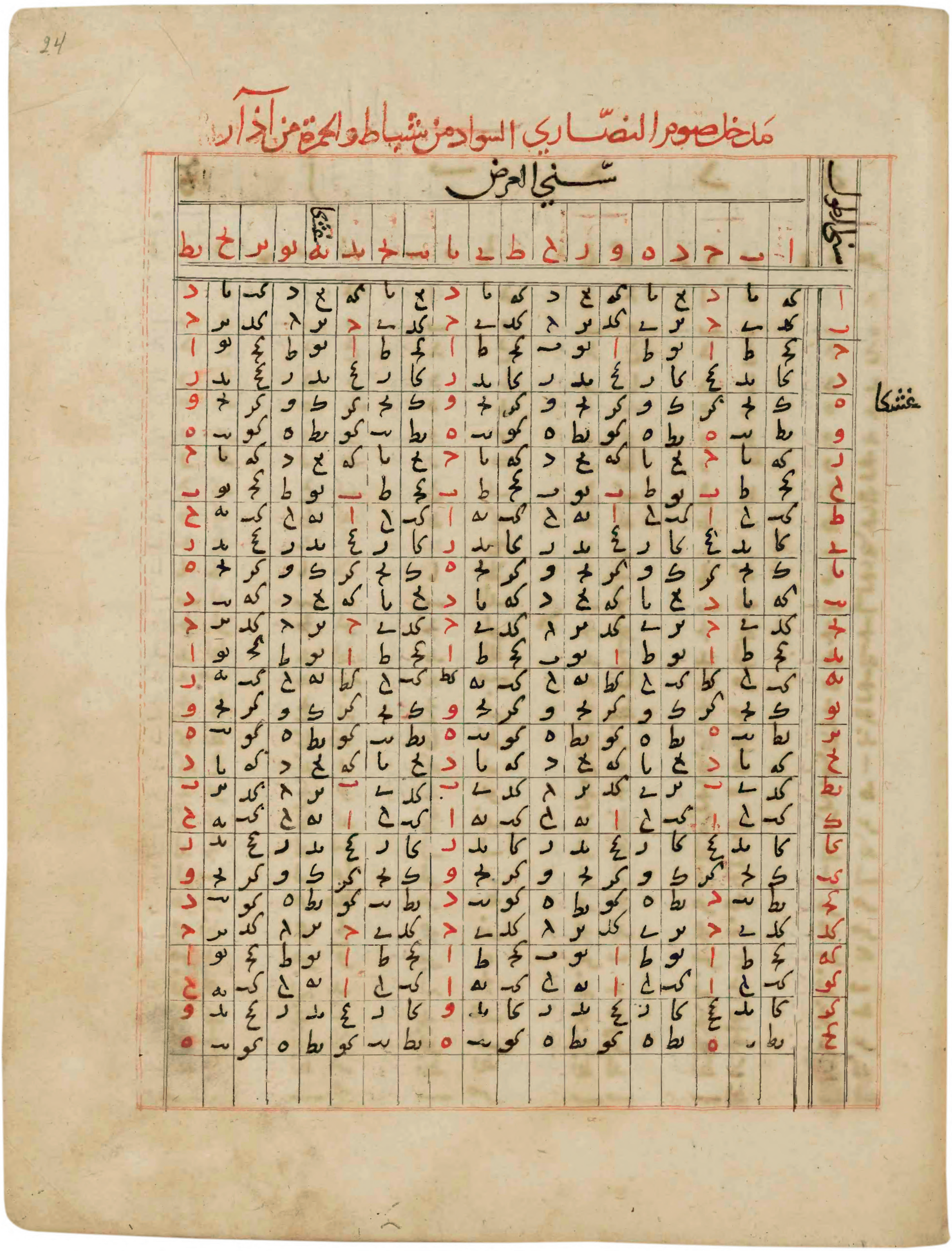
Plate 2a: Table of contents of Book II (Cairo, Dār al-kutub, *mīqāt* 691, fol. 2r)
Plate 2b: Passage on the Saturn-Mars conjunction of 993 (Cairo, Dār al-kutub, *mīqāt* 691, fol. 24v)


Plate 3: Table of the Great Lent (Leiden, Universiteitsbibliotheek, Or. 8, fol. 24r)


Plate 4: Sine table with values for minutes of arc (Leiden, Universiteitsbibliotheek, Or. 8, fol. 24v)


Plate 5: Tangent table with arguments up to 90° (Cairo, Dār al-kutub, *mīqāt* 400, fol. 47v)


Plate 6: Table of preliminaries of mean motions (Istanbul, Süleymaniye Kütüphanesi, Fatih 3418, fol. 44r)

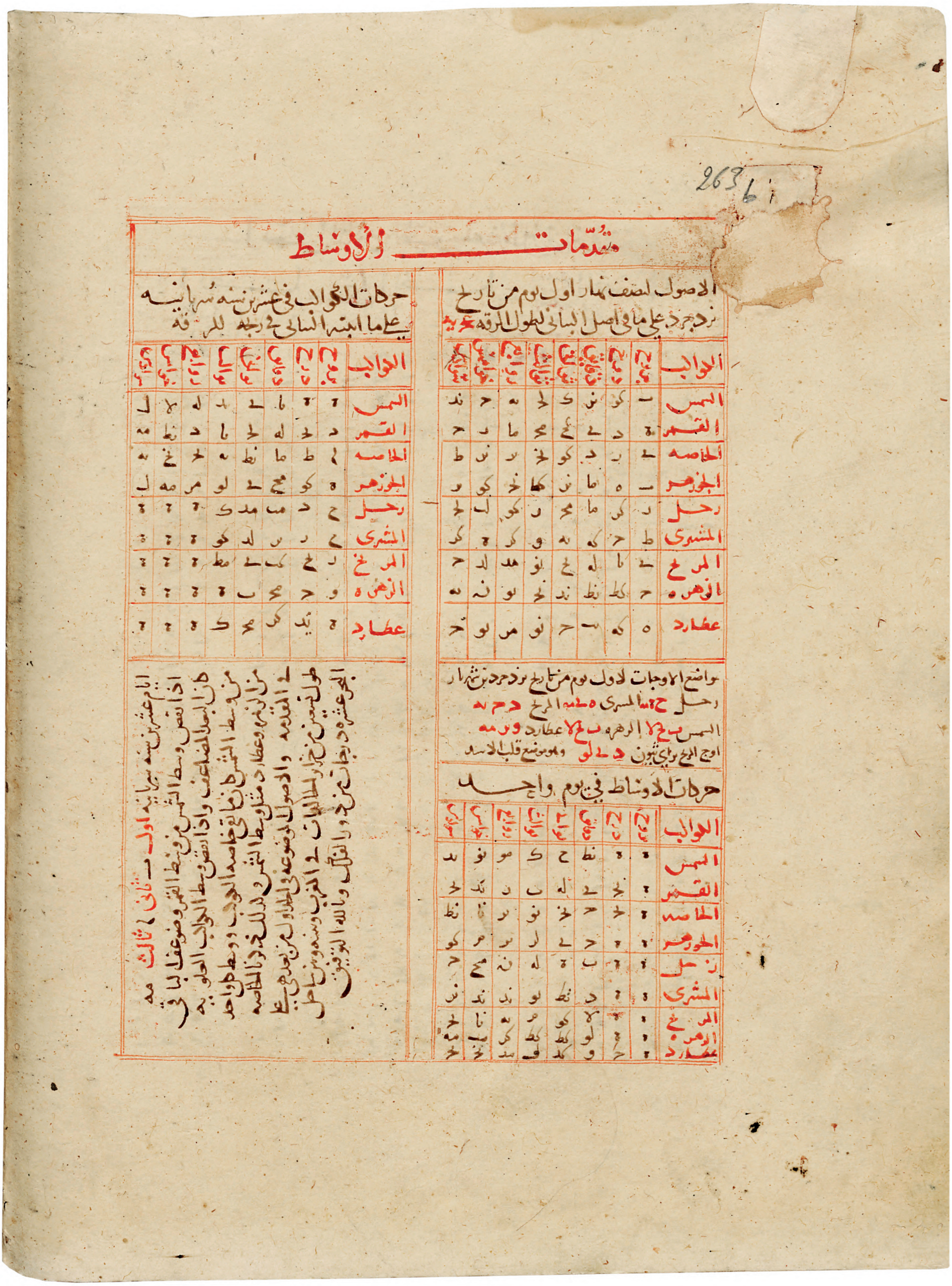



Plate 7: Table of preliminaries of mean motions (Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3, fol. 263v)




Plate 8: Solar mean motion table (Cairo, Dār al-kutub, *mīqāt* 188/2, fol. 16r)

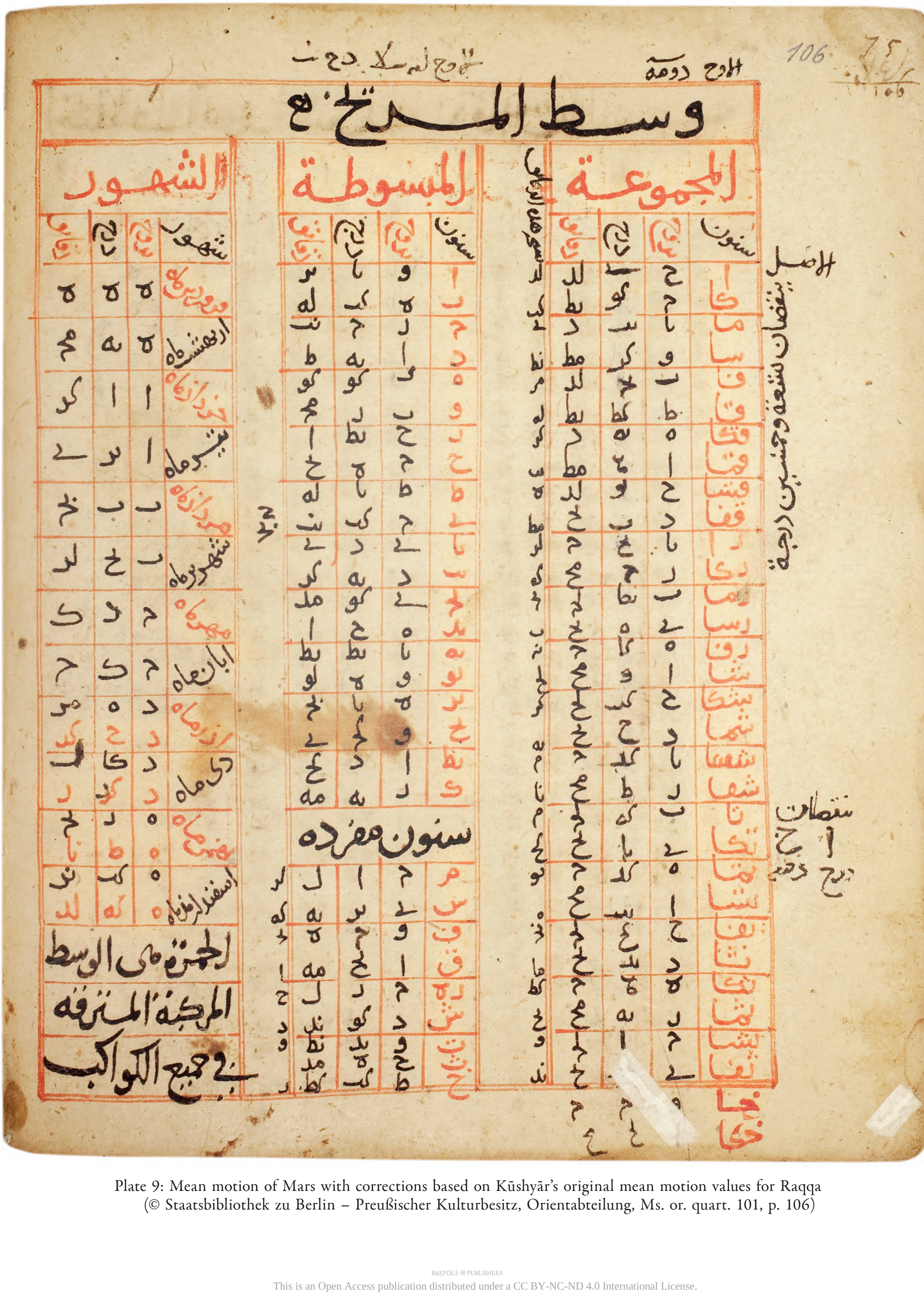



Plate 9: Mean motion of Mars with corrections based on Kūshyār's original mean motion values for Raqqa (© Staatsbibliothek zu Berlin – Preußischer Kulturbesitz, Orientabteilung, Ms. or. quart. 101, p. 106)


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Plate 10: Correction of the true position of Mars (Cairo, Dār al-kutub, *mīqāt* 400, fol. 69v)


Plate 11: Lunar latitude with library stamps (Cairo, Dār al-kutub, *mīqāt* 691, fol. 45v)

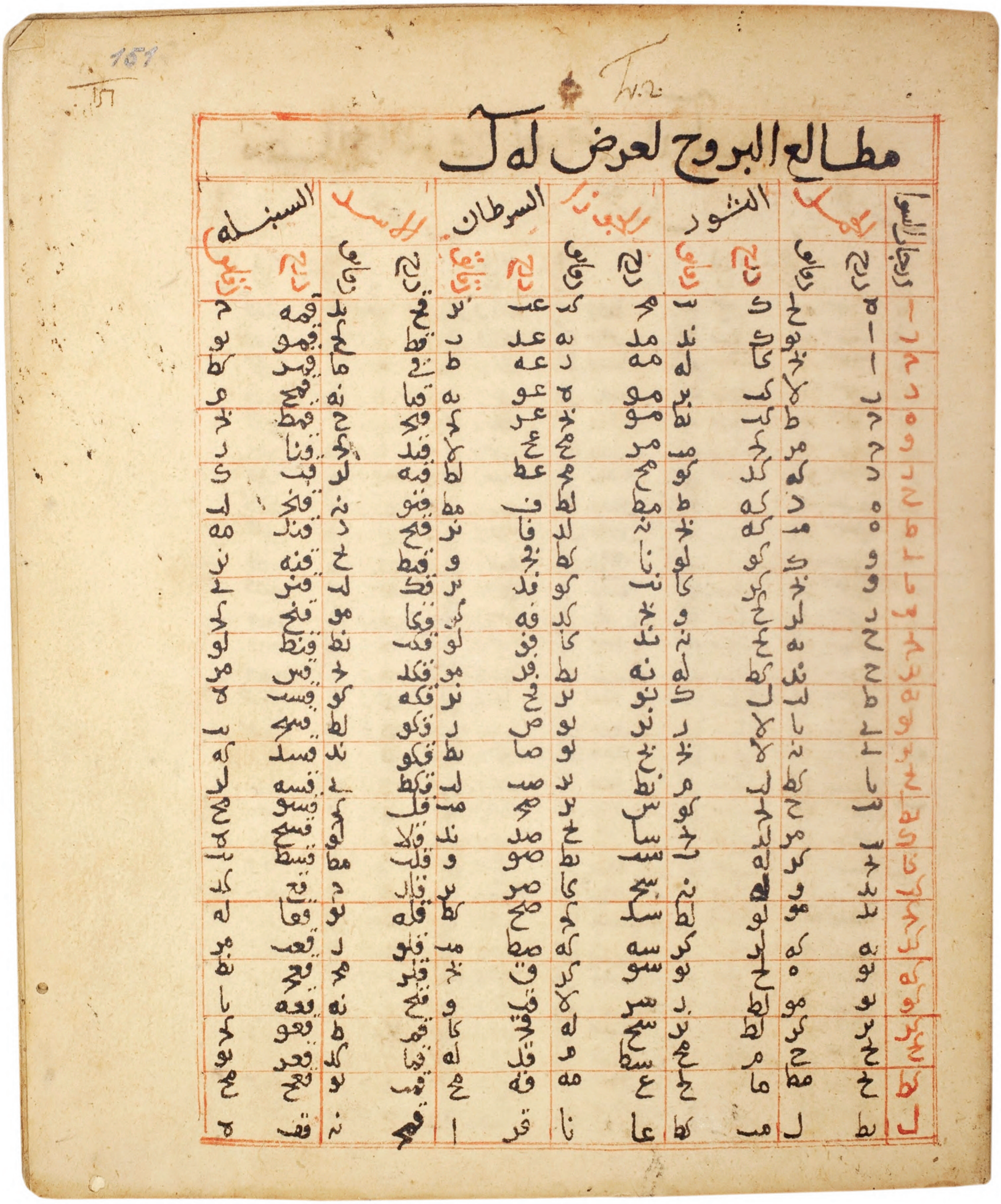


Plate 12: Oblique ascension for latitude 35;30°
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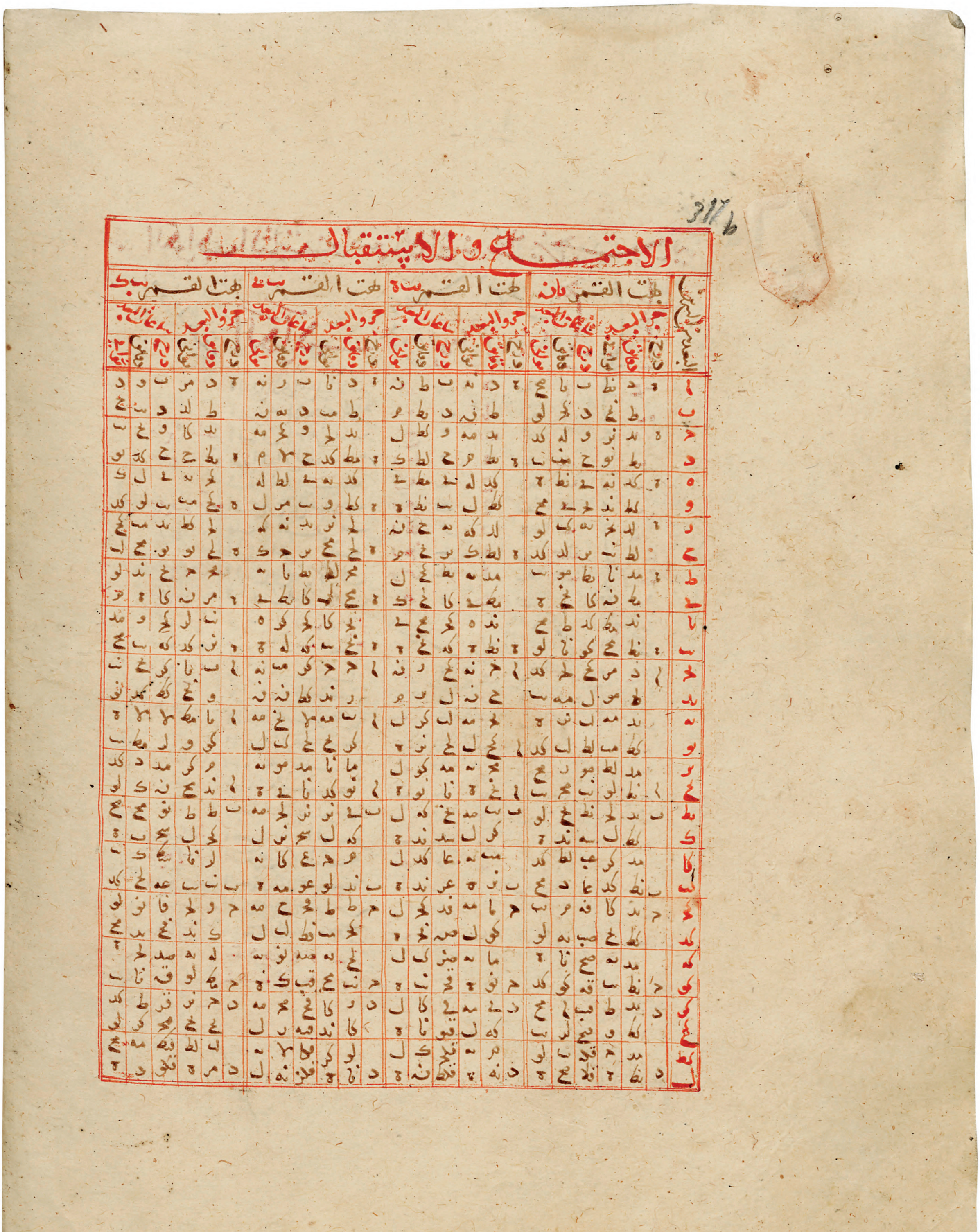



Plate 13: Table of conjunction and opposition (Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784/3, fol. 311v)

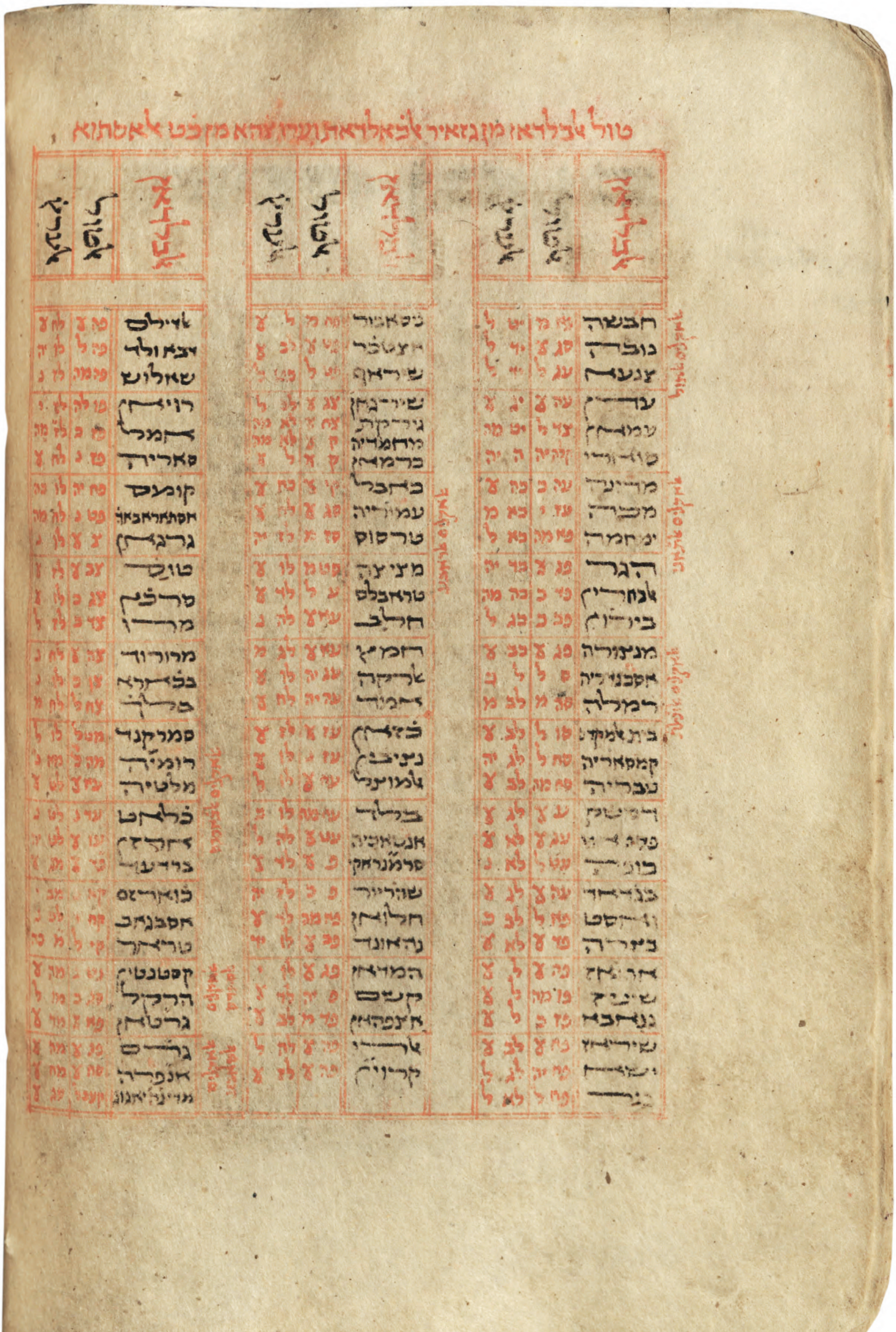
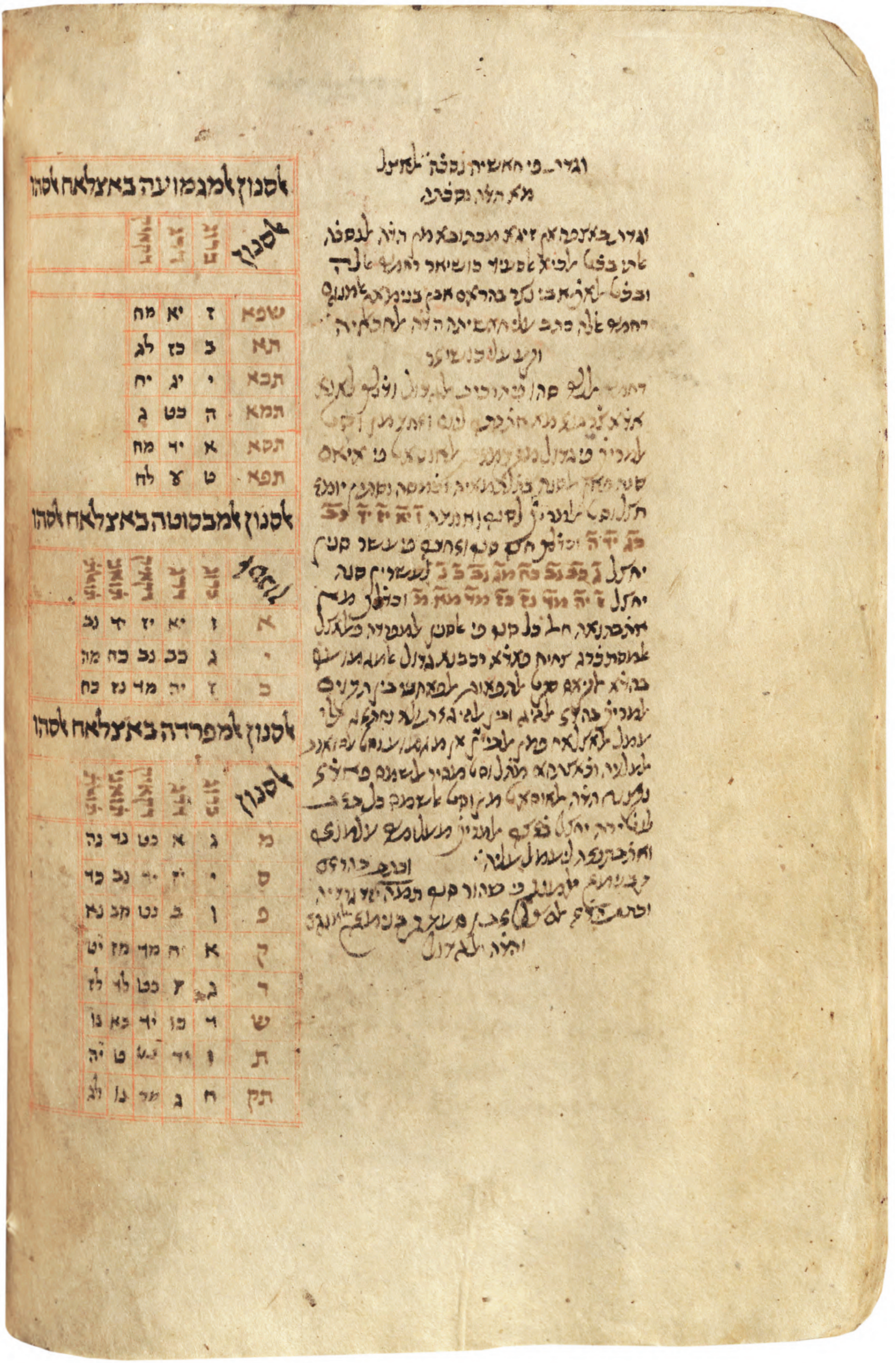




Plate 14: Geographical table (Ahuan Islamic Art, MS 40, fol. 80v).


Plate 15: Star table (Cairo, Dār al-kutub, *mīqāt* 188/2, fol. 47v)



אלסנון אלמגמועה באצלאח אלסהו

| אלסנון | ברג | גזו | דקאיק |
|---|---|---|---|
| שפא | ז | יא | מח |
| תא | ב | כז | לג |
| תכא | י | יג | יח |
| תמא | ה | כט | ג |
| תסא | א | יד | מח |
| תפא | ט | [illegible] | לח |

אלסנון אלמבסוטה באצלאח אלסהו

| אלסנון | ברג | גזו | דקאיק | תואני | תואלת |
|---|---|---|---|---|---|
| א | ז | יא | יז | יד | נב |
| י | ג | כב | נב | כח | מה |
| כ | ז | יה | מד | נז | כח |

אלסנון אלמפרדה באצלאח אלסהו

| אלסנון | ברג | גזו | דקאיק | תואני | תואלת |
|---|---|---|---|---|---|
| מ | ג | א | כט | נד | נה |
| ס | י | יז | יד | נב | כד |
| פ | ו | ב | נט | מב | נא |
| ק | א | יח | מד | מז | יט |
| ר | ג | [illegible] | כט | לד | לז |
| ש | ד | כו | יד | כא | נו |
| ת | ו | יד | [illegible] | ט | יה |
| תק | ח | ג | מד | נו | לג |

Plate 16: Eleventh-century note by Bahrām ibn Banīmān al-munajjim on the inconsistency in the mean motion tables for Mars (Ahuan Islamic Art, MS 40, fol. 82v)

The *Jāmiʿ Zīj* (*Comprehensive Zīj*) was a highly popular Arabic astronomical handbook with tables written by the Iranian astronomer Kūshyār ibn Labbān al-Jīlī around the year 1000. It belonged to an important category of works, modelled after Ptolemy's *Almagest and Handy Tables*, that allowed the practising astronomer/astrologer to carry out all necessary calculations of arcs on the celestial sphere and planetary positions, and ultimately to cast horoscopes. Around one hundred such works are extant, but only very few have been edited, translated or studied in detail.

This book contains a full treatment of Book II of Kūshyār's astronomical handbook centred around a critical edition of all the mathematical tables and their paratexts. It sets new standards for the edition of such tables by designing new types of apparatus entries for related variants in the tabular values. The introductory part describes the eight surviving manuscripts that transmit Kūshyār's tables and establishes by a detailed survey that they represent at least three different versions of the *Jāmiʿ Zīj* that in all likelihood stem from Kūshyār himself. An extensive commentary with mathematical analyses uncovers numerous new details of the methods by which the tables were computed, the astronomical parameter values on which they were based, the sources for the tables, and their influence on later *zījes*. These results show how Kūshyār, on the one hand, stayed firmly within the framework of the Ptolemaic tradition, but on the other introduced several types of innovations that later became common in Arabic and Persian astronomical handbooks.

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